

Nonexistence of solutions for quasilinear elliptic equations with p -growth in the gradient *

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Abstract

We study the nonexistence of weak solutions in $W_{loc}^{1,p}(\Omega)$ for a class of quasilinear elliptic boundary-value problems with natural growth in the gradient. Nonsolvability conditions involve general domains with possible singularities of the right-hand side. In particular, we show that if the data on the right-hand side are sufficiently large, or if inner radius of Ω is large, then there are no weak solutions.

1 Introduction

The aim of this article is to study nonsolvability in the weak sense of the quasilinear elliptic distribution equation

$$\begin{aligned} -\Delta_p u &= F(x, u, \nabla u) && \text{in } \mathcal{D}'(\Omega), \\ u &= 0 && \text{on } \partial\Omega, \end{aligned} \tag{1.1}$$

in the Sobolev space $W_0^{1,p}(\Omega)$. Note that we do not assume the solutions being essentially bounded. Here Ω is a domain in \mathbb{R}^N , $N \geq 1$, $1 < p < \infty$, $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u)$ is p -Laplacian, and $F : \Omega \times \mathbb{R} \times \mathbb{R}^N \rightarrow \mathbb{R}$ is a Carathéodory function, that is, $F(x, \eta, \xi)$ is measurable with respect to x for all (η, ξ) , and continuous with respect to (η, ξ) for a.e. $x \in \Omega$.

By $B_R(x_0)$ we denote the ball of radius R centered at x_0 . The Lebesgue measure (volume) of a subset B in \mathbb{R}^N is denoted by $|B|$, and the volume of the unit ball is denoted by C_N . The dual exponent of $p > 1$ is defined by $p' = \frac{p}{p-1}$. We define weak solutions as functions $u \in W_0^{1,p}(\Omega)$ which satisfy equation (1.1) in the weak sense:

$$\int_{\Omega} [|\nabla u|^{p-2} \nabla u \cdot \nabla \phi - F(x, u, \nabla u) \phi] dx = 0, \quad \forall \phi \in C_0^\infty(\Omega).$$

Nonsolvability for (1.1) has been studied for $F = \tilde{g}_0|x|^m + \tilde{f}_0|\nabla u|^p$, with $\Omega = B_R(0)$, $\tilde{f}_0 > 0$, $\tilde{g}_0 > 0$, and with solutions in the class of radial, decreasing

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and bounded functions, see Pašić [10] and in Korkut, Pašić, Žubrinić [7]. The aim of this article is to extend the nonsolvability results obtained in [10] and [7] for radial solutions of quasilinear elliptic problem in a ball to general domains in \mathbb{R}^N . We deal with nonexistence of unbounded solutions as well. To this end we use a combination of results obtained in [7] with the Tolksdorf comparison principle, see [12].

Existence of weak solutions for problems with strong dependence on the gradient has been studied by Rakotoson [11], Boccardo, Murat and Puel [3], Maderna, Pagani and Salsa [9], Ferone, Posteraro and Rakotoson [4], Korkut, Pašić and Žubrinić [6], [7], Tuomela [13], see also the references therein.

Our main result is stated in Theorem 2.1 below. As we have said, nonsolvability conditions involve geometry of Ω with respect to eventual singularities on the right-hand side. As an illustration, below we provide a simple consequence involving inner radius of domain Ω , that we define by

$$r(\Omega) = \sup\{r > 0 : \exists x_1 \in \Omega, B_r(x_1) \subseteq \Omega\}. \quad (1.2)$$

Here we also mention a nonsolvability result related to problem (1.1), involving inner radius of Ω , obtained in Wang and Gao [14], which complements an existence result of Hachim and Gossez [5], involving outer radius of domain. These two papers deal with quasilinear elliptic problems in which the nonlinearity on the right-hand side does not depend on the gradient. We also mention a recent paper of Bidaut-Véron and Pohožaev [2] dealing with nonexistence results for nonlinear elliptic problems with nonlinearities $\geq |x|^\sigma u^Q$, where $\sigma \in \mathbb{R}$, $Q > 0$. Here we treat nonlinearities of different type.

From Ferone, Posteraro, Rakotoson [4, Theorem 3.3] it follows, under very general conditions on $F(x, \eta, \xi)$, that if $|\Omega|$ is sufficiently small then there exists a weak solution of (1.1). We obtain a complementary result, showing that if Ω has sufficiently large inner radius, then (1.1) has no weak solutions. Equivalently, if a domain Ω is fixed, and if the data entering the right-hand side of (1.1) are sufficiently large, then (1.1) does not possess weak solutions. For the reader's convenience we state a special case of our main result formulated in Theorem 2.1.

Corollary 1.1 (Nonexistence) *Assume that Ω is a domain in \mathbb{R}^N and there exist positive real numbers \tilde{g}_0 and \tilde{f}_0 such that*

$$F(x, \eta, \xi) \geq \tilde{g}_0 + \tilde{f}_0 |\xi|^p \quad (1.3)$$

for a.e. $x \in \Omega$, and all $\eta \in \mathbb{R}$, $\xi \in \mathbb{R}^N$. Assume that $r(\Omega) < \infty$, where $r(\Omega)$ is inner radius of Ω , and

$$\tilde{g}_0 \cdot \tilde{f}_0^{p-1} \cdot r(\Omega)^p \geq C, \quad (1.4)$$

where C is explicit positive constant in (2.3) with $m_0 = 0$. Then (1.1) has no nonnegative weak solutions in the space $W_0^{1,p}(\Omega)$.

Remark 1. It is possible to prove another variant of nonexistence result stated in Corollary 1.1 when $r(\Omega) = \infty$. Assume that $F(x, \eta, \xi) \geq \tilde{g}_1$, where \tilde{g}_1 is a positive constant. Then it can be proved that equation (1.1) has no weak solutions in the space $W_{\text{loc}}^{1,p}(\Omega) \cap L^\infty(\Omega)$. Note that here we have a weaker assumption on $F(x, \eta, \xi)$ than in (1.3), but a smaller function space in which we claim to have nonexistence of weak solutions than in Corollary 1.1. To show this nonexistence result, assume by contradiction that there exists a solution $u \in W_{\text{loc}}^{1,p}(\Omega) \cap L^\infty(\Omega)$. It suffices to use oscillation estimate in Korkut, Pašić, Žubrinić [6, Proposition 12]:

$$\text{osc}_\Omega u \geq C \cdot r(\Omega)^{p'} \text{ess inf}_{\Omega \times (0, \infty) \times \mathbb{R}^N} F(x, \eta, \xi)^{p'-1}. \quad (1.5)$$

where C is an explicit positive constant depending only on p and N , and $\text{osc}_\Omega u = \text{ess sup}_\Omega u - \text{ess inf}_\Omega u$. Since $r(\Omega) = \infty$, we obtain that $\text{osc}_\Omega u = \infty$, which contradicts $u \in L^\infty(\Omega)$.

2 Nonexistence of weak solutions in $W_{\text{loc}}^{1,p}(\Omega)$

The main result of this paper is stated in Theorem 2.1 below. It complements the existence result stated in Ferone, Posteraro and Rakotoson [4, Theorem 3.3]. It also extends [7, Theorem 8(c)], where nonexistence result has been obtained for $\Omega = B_R(0)$, $F = \tilde{g}_0|x|^m + \tilde{f}_0|\xi|^p$, and in the class of decreasing, radial functions $u \in W_0^{1,p}(\Omega) \cap L^\infty(\Omega)$. Here we state nonexistence result for (1.1) where Ω can be arbitrary domain in \mathbb{R}^N (even unbounded), allowing more general nonlinearities than in [7], still with strong dependence in the gradient.

Theorem 2.1 (Nonexistence) *Let Ω be a domain in \mathbb{R}^N and assume that $m_0 > \max\{-p, -N\}$. Let there exist $x_0 \in \Omega$ and $R > 0$ such that $B_R(x_0) \subset \Omega$, and*

$$F(x, \eta, \xi) \geq \tilde{g}_0|x - x_0|^{m_0} + \tilde{f}_0|\xi|^p, \quad (2.1)$$

for a.e. $x \in B_R(x_0)$, and all $\eta \geq 0$, $\xi \in \mathbb{R}^N$. Assume that \tilde{g}_0, \tilde{f}_0 are positive real numbers such that

$$\tilde{g}_0 \cdot \tilde{f}_0^{p-1} \cdot R^{m_0+p} > C, \quad (2.2)$$

where

$$C = \begin{cases} [(m_0 + p)(p')^p]^{p-1}(m_0 + N) & \text{for } p > N, \\ [(m_0 + N)(p')^p]^{p-1}(m_0 + N) & \text{for } p \leq N. \end{cases} \quad (2.3)$$

Then quasilinear elliptic distribution equation $-\Delta_p u = F(x, u, \nabla u)$ has no weak solutions $u \in W_{\text{loc}}^{1,p}(\Omega)$ such that $u \geq 0$ on $\partial B_R(x_0)$.

Here the condition $u \geq 0$ on $\partial B_R(x_0)$ means by definition that $u^-|_{B_R(x_0)} \in W_0^{1,p}(B_R(x_0))$, where $u^- = \max\{-u, 0\}$. The proof of Theorem 2.1 is based on iterative procedure recently introduced by Pašić in [10]. Following Korkut, Pašić and Žubrinić [7] we introduce a sequence of functions $\omega_n : (0, T] \rightarrow \mathbb{R}$,

$T = |B|$, $B = B_R(x_1)$, by $\omega_n = z_0 + z_1 + \dots + z_n$, where functions $z_k(t)$ are defined inductively by

$$z_{k+1}(t) = f_0 \int_0^t \frac{z_k(s)^\delta}{s^\varepsilon} ds, \quad z_0(t) = g_0 t^\gamma, \quad (2.4)$$

with the constants defined by

$$\gamma = 1 + \frac{m_0}{N}, \quad \delta = p', \quad \varepsilon = p'(1 - \frac{1}{N}), \quad (2.5)$$

and g_0, f_0 are positive constants:

$$g_0 = \frac{\tilde{g}_0}{C_N^{\frac{m_0+p}{N}} N^{p-1}(m_0 + N)}, \quad f_0 = \tilde{f}_0. \quad (2.6)$$

It can be shown that (see [7, Proposition 1]):

$$z_m(t) = \frac{g_0^{\delta^m} f_0^{\sum_{k=0}^{m-1} \delta^k} t^{(1-\varepsilon)\sum_{k=0}^{m-1} \delta^k + \gamma \delta^m}}{\prod_{k=1}^m [(1-\varepsilon)\sum_{j=0}^{k-1} \delta^j + \gamma \delta^k] \delta^{m-k}}. \quad (2.7)$$

It has been proved in [7, Proposition 2] that if

$$\delta > \frac{\varepsilon - 1}{\gamma} + 1, \quad \delta > 1, \quad \gamma > 0, \quad \varepsilon \in \mathbb{R}, \quad (2.8)$$

then condition

$$g_0^{\delta-1} f_0 > C_1 := \begin{cases} \frac{[\gamma(\delta-1) - \varepsilon + 1] \delta^{\delta'}}{(\delta-1) T^{\gamma(\delta-1) - \varepsilon + 1}} & \text{for } \varepsilon < 1, \\ \frac{\gamma \delta^{\delta'}}{T^{\gamma(\delta-1) - \varepsilon + 1}} & \text{for } \varepsilon \geq 1. \end{cases} \quad (2.9)$$

implies that $\omega_n(t) \rightarrow \infty$ as $n \rightarrow \infty$ for all $t \in [t^*, T]$, where

$$t^* := \begin{cases} \left(\frac{[\gamma(\delta-1) - \varepsilon + 1] \delta^{\delta'}}{(\delta-1) f_0 g_0^{\delta-1}} \right)^{1/[\gamma(\delta-1) - \varepsilon + 1]} & \text{for } \varepsilon < 1, \\ \left(\frac{\gamma \delta^{\delta'}}{f_0 g_0^{\delta-1}} \right)^{1/[\gamma(\delta-1) - \varepsilon + 1]} & \text{for } \varepsilon \geq 1. \end{cases} \quad (2.10)$$

Condition (2.2) is equivalent with (2.9), which in turn is equivalent with $t^* < T$. We obtain a more precise result in the following lemma.

Lemma 2.2 *Assume that conditions (2.8) and (2.9) are fulfilled. Then we have*

$$\omega_n(t) \geq \frac{d^{n+1} - 1}{d - 1} \cdot g_0 t^\gamma, \quad (2.11)$$

for all $t \in [t^*, T]$, where $d = \delta^{\delta'-1} > 1$.

Proof. It suffices to prove that

$$\frac{z_{n+1}(t)}{z_n(t)} \geq \delta^{\delta'-1}, \quad (2.12)$$

for all $t \in [t^*, T]$, since then

$$\omega_n(t) = z_0(t) + \dots + z_n(t) \geq g_0 t^\gamma \sum_{k=0}^n d^k = g_0 t^\gamma \frac{d^{n+1} - 1}{d - 1}, \quad (2.13)$$

Using (2.7) we obtain that

$$\frac{z_{n+1}(t)}{z_n(t)} = \frac{B(t)^{\delta^n}}{\left(\prod_{k=1}^n D_k^{\delta^{n-k}}\right)^{\delta-1} D_{n+1}}, \quad (2.14)$$

where

$$B(t) = g_0^{\delta-1} f_0 t^{\gamma(\delta-1)-\varepsilon+1}, \quad D_k = (1-\varepsilon) \sum_{j=0}^{k-1} \delta^j + \gamma \delta^k. \quad (2.15)$$

Let us consider the case $\varepsilon \geq 1$ (it is equivalent with $p \leq N$). In this case we have $D_k \leq \gamma \delta^k$, which enables us to estimate the denominator on the right-hand side of (2.14):

$$\begin{aligned} \left(\prod_{k=1}^n D_k^{\delta^{n-k}}\right)^{\delta-1} D_{n+1} &\leq \left(\prod_{k=1}^n (\gamma \delta^k)^{\delta^{n-k}}\right)^{\delta-1} \gamma \delta^{n+1} \\ &= \gamma^{\delta^n} \delta^{\frac{(2\delta-1)(\delta^n-1)}{\delta-1} - \delta^n + 2}. \end{aligned}$$

Here we have used the identity $\sum_{k=1}^n k \cdot \delta^{n-k} = \frac{(2\delta-1)(\delta^n-1)}{(\delta-1)^2} - \frac{\delta^n+n-1}{\delta-1}$. Therefore

$$\frac{z_{n+1}(t)}{z_n(t)} \geq \delta^{\delta'-1} \left(\frac{B(t)}{\gamma \cdot \delta^{\delta'}}\right)^{\delta^n} \geq \delta^{\delta'-1},$$

since $B(t) \geq \gamma \cdot \delta^{\delta'}$ is equivalent with $t \geq t^*$.

It is easy to see that (2.14) holds also with modified $B(t)$ and D_k :

$$B(t) = g_0^{\delta-1} f_0 t^{\gamma(\delta-1)-\varepsilon+1} (\delta-1), \quad D_k = (\gamma(\delta-1) - \varepsilon + 1) \delta^k + \varepsilon - 1. \quad (2.16)$$

Therefore if we assume that $\varepsilon < 1$ (that is, $p > N$) we obtain $D_k \leq (\gamma(\delta-1) - \varepsilon + 1) \delta^k$, and we can proceed in the same way as above, by noting that $B(t) \geq (\gamma(\delta-1) - \varepsilon + 1) \delta^{\delta'}$ is equivalent with $t \geq t^*$ also in this case. \square

Lemma 2.3 *We have*

$$\frac{d\omega_n}{dt} \leq g_0 \gamma t^{\gamma-1} + f_0 \frac{\omega_n(t)^\delta}{t^\varepsilon} \quad (2.17)$$

for all n and for all $t \in (0, T)$.

Proof. Using (2.4) we obtain

$$\begin{aligned} \frac{d\omega_n}{dt} &= g_0\gamma t^{\gamma-1} + f_0 \frac{z_0(t)^\delta}{t^\varepsilon} + \dots + f_0 \frac{z_{n-1}(t)^\delta}{t^\varepsilon} \\ &\leq g_0\gamma t^{\gamma-1} + f_0 \frac{z_0(t)^\delta + \dots + z_{n-1}(t)^\delta + z_n(t)^\delta}{t^\varepsilon} \\ &\leq g_0\gamma t^{\gamma-1} + f_0 \frac{(z_0(t) + \dots + z_{n-1}(t) + z_n(t))^\delta}{t^\varepsilon}, \end{aligned}$$

where in the last inequality we have used $\delta > 1$. \square

Lemma 2.4 Let $m_0 > \max\{-p, -N\}$ and let us define, $B = B_R(x_0)$,

$$u_n(x) = \int_{C_N|x-x_0|^N}^{|B|} \frac{\omega_n(s)^{p'-1}}{s^{p'(1-\frac{1}{N})}} ds. \quad (2.18)$$

(a) We have $u_n \in W_0^{1,p}(B) \cap C^2(B \setminus \{x_0\}) \cap C(\bar{B})$, and

$$\begin{aligned} -\Delta_p u_n &\leq \tilde{g}_1 |x - x_0|^{m_0} + \tilde{f}_0 |\nabla u_n|^p \quad \text{in } B \setminus \{x_0\}, \\ u_n &= 0 \quad \text{on } \partial B, \end{aligned} \quad (2.19)$$

(b) Furthermore, if condition (2.2) is satisfied, then

$$u_n(x) \rightarrow \infty \quad \text{as } n \rightarrow \infty, \quad \text{for all } x \in B_R(x_0). \quad (2.20)$$

Proof. (a) Inequality in (2.19) is equivalent with (2.17), see [7, Lemma 1]. Regularity of u_n follows from $0 \leq \omega_n(t) \leq Mt^\gamma$ for $t \in [0, T]$, in the same way as it was deduced in the proof of [7, Proposition 11].

(b) Condition (2.2) is equivalent with $g_0^{\delta-1} f_0 > C_1$, (C_1 is defined in (2.9)), which in turn is equivalent with $t^* < T$ (see (2.10)), that is, $r^* < R$, where $r^* = (t^*/C_N)^{1/N}$. Assume that $x \in B_R(x_0)$ is such that $r^* \leq |x - x_0| < R$. Using (2.11) and (2.18) we have

$$u_n(x) \geq \left(g_0 \frac{d^{n+1} - 1}{d - 1} \right)^{p'-1} \int_{C_N|x-x_0|^N}^{|B|} s^{\gamma(p'-1) - p'(1-\frac{1}{N})} ds \rightarrow \infty \quad (2.21)$$

as $n \rightarrow \infty$, since $d = \delta^{\delta-1} > 1$ and the integral is > 0 . If $x \in B_{r^*}(x_1)$ then by (2.18) obviously $u_n(x) \geq u_n(r^*) \rightarrow \infty$ as $n \rightarrow \infty$, where we identify radial function $u_n(x)$ with $u_n(|x|)$. \square

Proof of Theorem 2.1. Assume by contradiction that there exists a distribution solution $u \in W_{\text{loc}}^{1,p}(\Omega)$ of $-\Delta_p u = F(x, u, \nabla u)$, such that $u \geq 0$ on $\partial B_R(x_0)$. Then due to condition (2.1) the function $\bar{u} := u|_B$ is a supersolution of problem

$$\begin{aligned} -\Delta_p v &= \tilde{g}_0 |x - x_0|^{m_0} + \tilde{f}_0 |\nabla v|^p \quad \text{in } B, \\ v &= 0 \quad \text{on } \partial B, \end{aligned} \quad (2.22)$$

where $B = B_R(x_0)$, and \bar{u} is essentially bounded on B .

On the other hand, u_n defined by (2.18) is a subsolution of (2.22), and $u_n \in W_0^{1,p}(B) \cap L^\infty(B)$ by Lemma 2.4. Since we have $-\Delta_p u_n \leq -\Delta_p \bar{u}$ in B and $u_n = 0 \leq \bar{u}$ on ∂B , then by the Tolksdorf comparison principle, see [12], we get $u_n \leq \bar{u}$ a.e. in B . Letting $n \rightarrow \infty$ and using (2.20) we obtain that $\bar{u} \equiv \infty$, which is impossible. \square

Proof of Corollary 1.1. The claim follows from Theorem 2.1 with $m_0 = 0$, and taking a ball $B_R(x_0)$ in Ω such that $R = r(\Omega)$. \square

Open problems.

It has been proved in Tuomela [13] (using a suitable reduction from [7]) that for $F(x, u, \nabla u) = \tilde{g}_0 |x|^{m_0} + \tilde{f}_0 |\nabla u|^p$, $\Omega = B_R(0)$, $m_0 > \max\{-N, -p\}$, there exists a critical value $C_0 > 0$ such that if $\tilde{g}_0 \tilde{f}_0^{p-1} < C_0$, then equation (1.1) possesses a radial, decreasing and bounded solution, while for $\tilde{g}_0 \tilde{f}_0^{p-1} \geq C_0$ there are no radial, decreasing and bounded solutions. It would be interesting to know if analogous result holds for general bounded domains Ω .

We note by the way that in the radial case the above mentioned solution corresponding to case $\tilde{g}_0 \tilde{f}_0^{p-1} < C_0$ is unique in the class of radial, decreasing functions in $W_0^{1,p}(B_R(0)) \cap L^\infty(B_R(0))$, provided $m_0 > \max\{-N, -p\}$, see [7]. We do not know anything about uniqueness of solutions of equation (1.1) with p -growth in the gradient, in the case of general bounded domains Ω . For $p = 2$ this question was treated in [1].

It has been shown in Ferone, Posteraro, Rakotoson [4] that if $|\Omega|$ is sufficiently small, then a class of quasilinear elliptic problems with p -growth in the gradient is solvable. We do not know if for domains Ω having sufficiently large Lebesgue measure we have nonexistence result in general. In this paper we have shown that this is so only for the class of domains with sufficiently large inner radius. One can imagine a domain Ω with large measure, but with very small inner radius. The question of existence or nonexistence of solutions in this case is an open problem.

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