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Kamenev-type oscillation criteria for forced Emden-Fowler superlinear difference equations *

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Abstract

Using Riccati transformation techniques, we establish oscillation criteria for forced second-order Emden-Fowler superlinear difference equations. Our criteria are discrete analogues of the criteria used for differential equations by Kamanev [5].

1 Introduction

Consider the forced second-order nonlinear difference equation

$$\Delta^2 x_{n-1} + q_n x_n^{\gamma} = g_n, \tag{1.1}$$

where γ is quotient of positive odd integers, n is an integer in the set $\mathbb{N} = \{1, 2 \dots\}$, $\{q_n\}_{n=1}^{\infty}$ and $\{g_n\}_{n=1}^{\infty}$ are sequences of positive real numbers, Δ denotes the forward difference operator $\Delta x_n = x_{n+1} - x_n$ and $\Delta^2 x_n = \Delta(\Delta x_n)$. In the case $\gamma > 1$, Equation (1.1) is the prototype of a wide class of nonlinear difference equations called Emden-Fowler superlinear difference equations.

In recent years there has been an increasing interest in the asymptotic behavior of second-order difference equations, see, e.g., the monographs [1, 2]. Following this trend, we study the oscillations of (1.1). It is interesting to study (1.1) because, it is the discrete version of the second order Emden-Fowler differential equation that has several physical applications [11].

We consider only nontrivial solutions of (1.1); i.e., solutions such that for every $i \in \mathbb{N}$, $\sup\{|x_n| : n \geq i\} > 0$. A solution $\{x_n\}$ of (1.1) is said to be oscillatory if for every $n_1 \geq 1$ there exists an $n \geq n_1$ such that $x_n x_{n+1} \leq 0$, otherwise it is non-oscillatory.

The oscillation of forced second order difference equations has been the subject of many publications; see for example [3, 4, 7, 8, 10, 13, 14] and references therein. In [3], the authors considered the linear forced difference equation and given some sufficient conditions for oscillation. In [8, 13], the authors considered the nonlinear forced difference equations and established some conditions for oscillation. Unfortunately, the oscillation criteria in [3, 8, 13] impose assumptions

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on the unknown solutions, which diminishes the applicability of the criteria. In [14], the authors considered the forced nonlinear delay difference equation when $\{q_n\}_{n=0}^{\infty}$ is a nonnegative sequence with a positive subsequence, and there exists a sequence $\{G_n\}_{n=0}^{\infty}$ such that $\Delta^2 G_n = g_n$ to obtain sufficient conditions for oscillations.

In the continuous case, the differential equation

$$x''(t) + q(t)f(x(t)) = 0, \quad t \geq t_0 \tag{1.2}$$

has been studied by many authors; see the survey papers [6, 12] which give over 300 references. In Kamenev [5], the average function

$$A_\lambda(t) = \frac{1}{t^\lambda} \int_{t_0}^{t} (t-s)^\lambda q(s) ds, \quad \lambda \geq 1 \tag{1.3}$$

plays a crucial role in the oscillation criteria for (1.2). Philos [9] improved Kamenev's result by proving the following result: Suppose there exist continuous functions H and h defined from $D = \{(t,s) : t \geq s \geq t_0\}$ to $\mathbb{R}$ such that:
(i) $H(t,t) = 0$, for $t \geq t_0$
(ii) $H(t,s) > 0$ for $t > s \geq t_0$, and H has a continuous and non-positive partial derivative on D with respect to the second variable and satisfies

$$-\frac{\partial H(t,s)}{\partial s} = h(t,s)\sqrt{H(t,s)} \geq 0. \tag{1.4}$$

Further, suppose that

$$\lim_{t\to\infty} \frac{1}{H(t,t_0)} \int_{t_0}^{t} [H(t,s)q(s) - \frac{1}{4}h^2(t,s)]\, ds = \infty. \tag{1.5}$$

Then every solution of (1.2) oscillates.

Using Riccati transformation techniques, we establish some new oscillation criteria, for (1.1), that are discrete analogues of (1.3) and (1.5). Our results generalized and extended the conditions (1.3) and (1.5) to the discrete case and improve the results presented in [3, 8, 13, 14].

2 Main Result

Theorem 2.1 *Assume that there exists a positive sequence $\{\rho_n\}_{n=1}^{\infty}$ such that for every positive number $\lambda \geq 1$,*

$$\lim_{m\to\infty} \sup \frac{1}{m^\lambda} \sum_{n=1}^{m-1} (m-n)^\lambda \Big[\rho_n Q_n - \frac{(\rho_{n+1})^2}{4\rho_n} \Big(\frac{\Delta\rho_n}{\rho_{n+1}} - \frac{\lambda(m-n-1)^{\lambda-1}}{(m-n)^\lambda}\Big)^2\Big] = \infty \tag{2.1}$$

where

$$Q_n = \gamma\big(\frac{1}{\gamma-1}\big)^{1-\frac{1}{\gamma}} (q_n)^{\frac{1}{\gamma}} (g_n)^{1-\frac{1}{\gamma}}.$$

Then every unbounded solution of (1.1) *oscillates.*

Proof Suppose to the contrary that $\{x_n\}_{n=1}^{\infty}$ is an unbounded non-oscillatory solution of (1.1). First, we may assume that $\{x_n\}$ is a positive solution of (1.1) for $n \geq n_1 \geq 1$. Define the sequence $\{w_n\}$ by

$$w_n = \rho_n \frac{\Delta x_{n-1}}{x_n}. \tag{2.2}$$

Then in view of (1.1), we have

$$\Delta w_n = -[q_n x_n^{\gamma-1} - \frac{g_n}{x_n}] + \frac{\Delta\rho_n}{\rho_{n+1}} w_{n+1} - \frac{\rho_n}{x_n x_{n+1}}.$$

Since x_n is positive and unbounded, there exists $n_2 \geq n_1$ such that $\Delta x_n \geq 0$, for $n \geq n_2$, and $x_{n+1} \geq x_n$, so that

$$\Delta w_n \leq -[q_n x_n^{\gamma-1} - \frac{g_n}{x_n}] + \frac{\Delta\rho_n}{\rho_{n+1}} w_{n+1} - \frac{\rho_n}{(\rho_{n+1})^2} w_{n+1}^2. \tag{2.3}$$

Set

$$f(x) = q_n x^{\gamma-1} - \frac{g_n}{x}\ .$$

Using differential calculus, we see that

$$f(x) \geq \gamma\big(\frac{1}{\gamma-1}\big)^{1-\frac{1}{\gamma}} (q_n)^{\frac{1}{\gamma}} (g_n)^{1-\frac{1}{\gamma}},$$

this and (2.3) imply

$$\Delta w_n \leq -Q_n + \frac{\Delta\rho_n}{\rho_{n+1}} w_{n+1} - \frac{\rho_n}{(\rho_{n+1})^2} w_{n+1}^2. \tag{2.4}$$

Therefore,

$$\begin{aligned} &\sum_{n=n_2}^{m-1} (m-n)^\lambda \rho_n Q_n \\ &\leq -\sum_{n=n_2}^{m-1} (m-n)^\lambda \Delta w_n + \sum_{n=n_2}^{m-1} (m-n)^\lambda \frac{\Delta\rho_n}{\rho_{n+1}} w_{n+1} - \sum_{n=n_2}^{m-1} (m-n)^\lambda \frac{\rho_n}{\rho_{n+1}^2} w_{n+1}^2, \end{aligned} \tag{2.5}$$

Now, after summing by parts, we have

$$\sum_{n=n_2}^{m-1} (m-n)^\lambda \Delta w_n = -(m-n_2)^\lambda w_{n_2} - \sum_{n=n_2}^{m-1} w_{n+1} \Delta_2 (m-n)^\lambda,$$

where $\Delta_2 (m-n)^\lambda = (m-n-1)^\lambda - (m-n)^\lambda$. Then

$$\sum_{n=n_2}^{m-1} (m-n)^\lambda \Delta w_n = -(m-n_2)^\lambda w_{n_2} + \sum_{n=n_2}^{m-1} w_{n+1}((m-n)^\lambda - (m-n-1)^\lambda).$$

Using the inequality, $x^\beta - y^\beta \geq \beta y^{\beta-1}(x-y)$ for all x$\geq y > 0$ and $\beta \geq 1$, we obtain

$$\sum_{n=n_2}^{m-1}(m-n)^\lambda \Delta w_n \geq -(m-n_2)^\lambda w_{n_2} + \sum_{n=n_2}^{m-1}\lambda w_{n+1}(m-n-1)^{\lambda-1}.$$

Substitute this expression in (2.5) to obtain

$$\begin{aligned}\sum_{n=n_2}^{m-1}(m-n)^\lambda \rho_n Q_n \leq &(m-n_2)^\lambda w_{n_2} - \sum_{n=n_2}^{m-1}\lambda w_{n+1}(m-n-1)^{\lambda-1}\\ &+ \sum_{n=n_2}^{m-1}(m-n)^\lambda \frac{\Delta\rho_n}{\rho_{n+1}} w_{n+1} - \sum_{n=n_2}^{m-1}(m-n)^\lambda \frac{\rho_n}{\rho_{n+1}^2} w_{n+1}^2.\end{aligned}$$

Then

$$\begin{aligned}&\frac{1}{m^\lambda}\sum_{n=n_2}^{m-1}(m-n)^\lambda \rho_n Q_n\\ &\leq (\frac{m-n_2}{m})^\lambda w_{n_2}\\ &\quad -\frac{1}{m^\lambda}\sum_{n=n_2}^{m-1}(m-n)^\lambda\Big[\frac{\rho_n}{\rho_{n+1}^2}w_{n+1}^2 - (\frac{\Delta\rho_n}{\rho_{n+1}} - \frac{\lambda(m-n-1)^{\lambda-1}}{(m-n)^\lambda})w_{n+1}\Big]\\ &= (\frac{m-n_2}{m})^\lambda w_{n_2}\\ &\quad -\frac{1}{m^\lambda}\sum_{n=n_2}^{m-1}(m-n)^\lambda\Big[\frac{\sqrt{\rho_n}}{\rho_{n+1}}w_{n+1} - \frac{\rho_{n+1}}{2\sqrt{\rho_n}}(\frac{\Delta\rho_n}{\rho_{n+1}} - \frac{\lambda(m-n-1)^{\lambda-1}}{(m-n)^\lambda})\Big]^2\\ &\quad +\frac{1}{m^\lambda}\sum_{n=n_2}^{m-1}(m-n)^\lambda\frac{(\rho_{n+1})^2}{4\rho_n}(\frac{\Delta\rho_n}{\rho_{n+1}} - \frac{\lambda(m-n-1)^{\lambda-1}}{(m-n)^\lambda})^2,\end{aligned}$$

which implies

$$\begin{aligned}&\frac{1}{m^\lambda}\sum_{n=n_2}^{m-1}\kappa(m-n)^\lambda \rho_n Q_n\\ &< (\frac{m-n_2}{m})^\lambda w_{n_2} + \frac{1}{m^\lambda}\sum_{n=n_2}^{m-1}(m-n)^\lambda\frac{(\rho_{n+1})^2}{4\rho_n}(\frac{\Delta\rho_n}{\rho_{n+1}} - \frac{\lambda(m-n-1)^{\lambda-1}}{(m-n)^\lambda})^2.\end{aligned}$$

Then

$$\begin{aligned}\frac{1}{m^\lambda}\sum_{n=n_2}^{m-1}(m-n)^\lambda\Big[\rho_n Q_n - \frac{(\rho_{n+1})^2}{4\rho_n}\left(\frac{\Delta\rho_n}{\rho_{n+1}} - \frac{\lambda(m-n-1)^{\lambda-1}}{(m-n)^\lambda}\right)^2\Big]&\\ < (\frac{m-n_2}{m})^\lambda w_{n_2},&\end{aligned}$$

which yields

$$\lim_{m\to\infty}\frac{1}{m^\lambda}\sum_{n=n_2}^{m-1}(m-n)^\lambda\Big[\rho_n Q_n-\frac{(\rho_{n+1})^2}{4\rho_n}\Big(\frac{\Delta\rho_n}{\rho_{n+1}}-\frac{\lambda(m-n-1)^{\lambda-1}}{(m-n)^\lambda}\Big)^2\Big]<\infty,$$

which contradicts (2.1). Next, we consider the case when $x_n<0$ for $n\geq n_1$. We use the transformation $y_n=-x_n$ is a positive solution of the equation $\Delta^2 y_{n-1}+q_n y_n^\gamma=-g_n$. Define the sequence $\{w_n\}$ by

$$w_n=\rho_n\frac{\Delta y_{n-1}}{x_n}. \tag{2.6}$$

then, $w_n>0$ and satisfies

$$\Delta w_n\leq-[q_n x_n^{\gamma-1}+\frac{g_n}{x_n}]+\frac{\Delta\rho_n}{\rho_{n+1}}w_{n+1}-\frac{\rho_n}{(\rho_{n+1})^2}w_{n+1}^2. \tag{2.7}$$

Set

$$F(x)=q_n x^{\gamma-1}+\frac{g_n}{x}.$$

Using differential calculus, we see that

$$F(x)\geq\gamma\big(\frac{1}{\gamma-1}\big)^{1-\frac{1}{\gamma}}(q_n)^{\frac{1}{\gamma}}(g_n)^{1-\frac{1}{\gamma}}.$$

and then (2.4) holds. The remainder of the proof is similar to that of the proof of the first part and hence is omitted. The proof is complete ♢

Corollary 2.2 *Assume that all assumptions in Theorem 2.1 hold, except the condition* (2.1) *which is replaced by*

$$\lim_{m\to\infty}\sup\frac{1}{m^\lambda}\sum_{n=1}^{m-1}(m-n)^\lambda\rho_n Q_n=\infty,$$

and

$$\lim_{m\to\infty}\frac{1}{m^\lambda}\sum_{n=1}^{m-1}(m-n)\frac{(\rho_{n+1})^2}{\rho_n}\big(\frac{\Delta\rho_n}{\rho_{n+1}}-\frac{\lambda(m-n-1)^{\lambda-1}}{(m-n)^\lambda}\big)^2<\infty\,.$$

Then every unbounded solution of (1.1) *oscillates.*

Theorem 2.3 *Assume that there exists a positive sequence* $\{\rho_n\}_{n=1}^\infty$*. Furthermore, we assume that there exists a double sequence* $\{H_{m,n}: m\geq n\geq 0\}$ *such that (i)* $H_{m,m}=0$ *for* $m\geq 0$ *(ii)* $H_{m,n}>0$ *for* $m>n\geq 0$*, (iii)* $\Delta_2 H_{m,n}=H_{m,n+1}-H_{m,n}\leq 0$ *for* $m\geq n\geq 0$*. If*

$$\limsup_{m\to\infty}\frac{1}{H_{m,0}}\sum_{n=1}^{m-1}\Big[H_{m,n}\rho_n Q_n-\frac{\rho_{n+1}^2}{4\rho_n}\big(h_{m,n}-\frac{\Delta\rho_n}{\rho_{n+1}}\sqrt{H_{m,n}}\big)^2\Big]=\infty, \tag{2.8}$$

where

$$h_{m,n}=-\frac{\Delta_2 H_{m,n}}{\sqrt{H_{m,n}}},\quad m>n\geq 0,$$

then every unbounded solution of (1.1) *oscillates.*

Proof We proceed as in the proof of Theorem 2.1, we may assume that (1.1) has an unbounded non-oscillatory solution $\{x_n\}_{n=1}^{\infty}$ such that $x_n > 0$ for $n \geq n_1$. Define $\{w_n\}$ by (2.2) as before, then $w_n > 0$ and satisfies (2.4) for all $n \geq n_2$. Therefore,

$$\sum_{n=n_2}^{m-1} H_{m,n}\rho_n Q_n \leq -\sum_{n=n_2}^{m-1} H_{m,n}\Delta w_n + \sum_{n=n_2}^{m-1} H_{m,n}\frac{\Delta\rho_n}{\rho_{n+1}}w_{n+1} - \sum_{n=n_2}^{m-1} H_{m,n}\frac{\rho_n}{\rho_{n+1}^2}w_{n+1}^2,$$

which yields, after summing by parts,

$$\begin{aligned}
\sum_{n=n_2}^{m-1} H_{m,n}\rho_n Q_n
&\leq H_{m,n_2}w_{n_2} + \sum_{n=n_2}^{m-1} w_{n+1}\Delta_2 H_{m,n} + \sum_{n=n_2}^{m-1} H_{m,n}\frac{\Delta\rho_n}{\rho_{n+1}}w_{n+1} \\
&\quad - \sum_{n=n_2}^{m-1} H_{m,n}\frac{\rho_n}{\rho_{n+1}^2}w_{n+1}^2 \\
&= H_{m,n_2}w_{n_2} - \sum_{n=n_2}^{m-1} h_{m,n}\sqrt{H_{m,n}}w_{n+1} + \sum_{n=n_2}^{m-1} H_{m,n}\frac{\Delta\rho_n}{\rho_{n+1}}w_{n+1} \\
&\quad - \sum_{n=n_2}^{m-1} H_{m,n}\frac{\rho_n}{\rho_{n+1}^2}w_{n+1}^2 \\
&= H_{m,n_2}w_{n_2} \\
&\quad - \sum_{n=n_2}^{m-1}\Big[\frac{\sqrt{H_{m,n}\rho_n}}{\rho_{n+1}}w_{n+1} + \frac{\rho_{n+1}}{2\sqrt{H_{m,n}\rho_n}}\Big(h_{m,n}\sqrt{H_{m,n}} - \frac{\Delta\rho_n}{\rho_{n+1}}H_{m,n}\Big)\Big]^2 \\
&\quad + \frac{1}{4}\sum_{n=n_2}^{m-1}\frac{(\rho_{n+1})^2}{\bar{\rho}_n}\big(h_{m,n} - \frac{\Delta\rho_n}{\rho_{n+1}}\sqrt{H_{m,n}}\big)^2.
\end{aligned}$$

Then

$$\sum_{n=n_2}^{m-1}\Big[H_{m,n}\rho_n Q_n - \frac{\rho_{n+1}^2}{4\rho_n}\big(h_{m,n} - \frac{\Delta\rho_n}{\rho_{n+1}}\sqrt{H_{m,n}}\big)^2\Big] < H_{m,n_2}w_{n_2} \leq H_{m,0}w_{n_2},$$

which implies

$$\sum_{n=1}^{m-1}\Big[H_{m,n}\rho_n Q_n - \frac{\rho_{n+1}^2}{4\rho_n}\big(h_{m,n} - \frac{\Delta\rho_n}{\rho_{n+1}}\sqrt{H_{m,n}}\big)^2\Big] < H_{m,0}\sum_{n=1}^{n_2-1}\rho_n Q_n + H_{m,0}w_{n_2}.$$

Hence

$$\limsup_{m\to\infty}\frac{1}{H_{m,0}}\sum_{n=1}^{m-1}\Big[H_{m,n}\rho_n Q_n - \frac{\rho_{n+1}^2}{4\rho_n}\big(h_{m,n} - \frac{\Delta\rho_n}{\rho_{n+1}}\sqrt{H_{m,n}}\big)^2\Big] < \infty,$$

which contradicts (2.8). Next, we consider the case when $x_n < 0$ for $n \geq n_1$. We use the transformation $y_n = -x_n$ is a positive solution of the equation $\Delta^2 y_{n-1} + q_n y_n^\gamma = -g_n$. Define the sequence $\{w_n\}$ by (2.6), then (2.7) holds. The remainder of the proof is similar to that of the proof of the first case and hence is omitted. The proof is complete. ◇

Corollary 2.4 *Assume that all the assumptions of Theorem 2.3 hold, except the condition* (2.7) *which is replaced by*

$$\lim_{m\to\infty} \sup \frac{1}{H_{m,0}} \sum_{n=0}^{m-1} H_{m,n}\rho_n Q_n = \infty,$$

and

$$\lim_{m\to\infty} \sup \frac{1}{H_{m,0}} \sum_{n=1}^{m-1} \frac{\rho_{n+1}^2}{4\rho_n}\big(h_{m,n} - \frac{\Delta\rho_n}{\rho_{n+1}}\sqrt{H_{m,n}}\big)^2 < \infty\,.$$

Then every unbounded solution of (1.1) *oscillates.*

By choosing the sequence $\{H_{m,n}\}$ appropriately, we can derive several oscillation criteria for (1.1). For instance, consider the double sequence

$$H_{m,n} = \Big(\ln\big(\frac{m+1}{n+1}\big)\Big)^\lambda, \quad \lambda \geq 1,\ m \geq n \geq 0. \tag{2.9}$$

Then $H_{m,m} = 0$ for $m \geq 0$ and $H_{m,n} > 0$ and $\Delta_2 H_{m,n} \leq 0$ for $m > n \geq 0$. Hence we have the following result.

Corollary 2.5 *Assume that the assumptions in Theorem 2.3 hold, except the condition* (2.7) *which is replaced by*

$$\limsup_{m\to\infty} \frac{1}{\ln^\lambda(m+1)} \sum_{n=0}^{m-1} \Big[\Big(\ln\frac{m+1}{n+1}\Big)^\lambda \rho_n Q_n - B_{m,n}\Big] = \infty \tag{2.10}$$

where

$$B_{m,n} = \frac{\rho_{n+1}^2}{4\rho_n}\Big(\frac{\lambda}{n+1}\big(\ln\frac{m+1}{n+1}\big)^{\frac{\lambda-2}{2}} - \frac{\Delta\rho_n}{\rho_{n+1}}\sqrt{\big(\ln\frac{m+1}{n+1}\big)^\lambda}\Big)^2$$

for every positive number $\lambda \geq 1$. Then every unbounded solution of (1.1) *oscillates.*

Another choice for a sequence is

$$H_{m,n} = \phi(m-n), \quad m \geq n \geq 0,$$

where $\phi : [0,\infty) \to [0,\infty)$ is a continuously differentiable function which satisfies $\phi(0) = 0$, $\phi(u) > 0$, and $\phi'(u) \geq 0$ for $u > 0$.

Yet another choice for a sequence is

$$H_{m,n} = (m-n)^{(\lambda)} \quad \lambda > 2,\ m \geq n \geq 0,$$

where $(m-n)^{(\lambda)} = (m-n)(m-n+1)\dots(m-n+\lambda-1)$ and

$$\Delta_2(m-n)^{(\lambda)} = (m-n-1)^{(\lambda)} - (m-n)^{(\lambda)} = -\lambda(m-n)^{(\lambda-1)}.$$

For these two sequences we can state corollaries similar to the one above. Note that our results can be extended to the equation

$$\Delta(a_n \Delta x_n) + q_n x_n^{\gamma} = g_n$$

where $\{a_n\}_{n=1}^{\infty}$ is a sequence of positive real numbers. However, our results can not be applied in the case when $\gamma = 1$ and also it remains an open problem to give sufficient conditions for the oscillation of all bounded solutions in this case.

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Addendum posted by a managing editor on June 13, 2012.

A reader informed us of two inaccuracies in this article: In the proof of Theorem 2.1, the statement

> Since x_n is positive and unbounded, there exists $n_2 \geq n_1$ such that $\Delta x_n \geq 0$ for $n \geq n_2$

is incorrect. The sequence $x_n = n + (-1)n + 1$ provides a counterexample. Also in the same proof, the statement

$$f(x) \geq \gamma\Big(\frac{1}{\gamma - 1}\Big)^{1-\frac{1}{\gamma}} (q_n)^{\frac{1}{\gamma}} (g_n)^{1-\frac{1}{\gamma}}$$

is incorrect.

Regarding these inaccuracies, Prof. Saker informed us that the results in this paper have been corrected and improved in later publications, by the same author.