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PERIODIC SOLUTIONS FOR A DELAYED PREDATOR-PREY SYSTEM WITH DISPERSAL AND IMPULSES

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Abstract. A delayed predator-prey system with prey dispersal in n-patch environments and impulse effects is investigated. By using Gaines and Mawhin's continuation theorem of coincidence degree theory, a set of easily verifiable sufficient conditions are derived for the existence of positive periodic solutions to the system.

1. Introduction

An important and ubiquitous problem in mathematical ecology concerns the effect of environment change in the growth and diffusion of a species in a heterogenous habitat. There have been many studies in the literatures that investigate the population dynamics with diffusion process [5, 6, 10, 11].

In most of the models considered so far, it has been assumed that all biological and environmental parameters are constants in time. However, any biological or environmental parameters are naturally subject to fluctuation in time. The effects of a periodically varying environment are important for evolutionary theory as the selective forces on systems in a fluctuating environment differ from those in a stable environment. Thus, the assumptions of periodicity of the parameters are a way of incorporating the periodicity of the environment (such as seasonal effects of weather, food supplies, mating habits and so forth); on the other hand, it is generally recognized that some kinds of time delays are inevitable in population interactions. Time delay due to gestation is a common example, because generally the consumption of prey by the predator throughout its past history governs the present birth rate of the predator. Therefore, more realistic models of population interactions should take into account the seasonality of the changing environment and the effect of time delays.

There are still some other possible exterior effects under which the population densities change very rapidly. For example, impulse reduction of the population density of a given species is possible after its partial destruction by catching or by poisoning with chemical used at some transitory slots in fishing or agriculture [1, 7, 12]. Recently, many authors studies the existence of positive periodic solution in population models by using power and effective method of coincidence degree [8,

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13], In the present paper, we are concerned with the study on the combined effects of dispersion, periodicity of environment, time delays and impulses on the dynamics of predator-prey system. To do so we are devoted to the study of the following delayed periodic predator-prey system with prey dispersal in n-patch environments and impulses

$$
\begin{aligned}
\dot{x}_i(t) =& x_i(t)[r_i(t) - a_{ii}(t)x_i(t) - a_{i,n+1}(t)x_{n+1}(t)] \\
&+ \sum_{j=1, j\neq i}^{n} D_j(t)[x_j(t) - x_i(t)], \quad i = 1, \dots, n, \\
\dot{x}_{n+1}(t) =& x_{n+1}(t)\Big[- r_{n+1}(t) + \sum_{j=1}^{n} a_{n+1,j}(t)x_j(t - \sigma_j) \\
&- a_{n+1,n+1}(t)x_{n+1}(t - \sigma_{n+1})\Big], \quad t \neq \tau_k, \ k \in Z_+, \\
\Delta(x_i(\tau_k)) =& -c_{ik}x_i(\tau_k), \quad t = \tau_k, \ k \in Z_+, \quad i = 1, 2, \dots, n+1.
\end{aligned} \tag{1.1}
$$

with initial conditions

$$
x_i(s) = \phi_i(s), \quad s \in [-\sigma, 0], \quad \phi_i(0) > 0, \quad i = 1, \dots, n+1, \tag{1.2}
$$

where $x_i(t)$ $(i = 1, \dots, n)$ denote the densities of prey species in patch i, x_{n+1} denote the densities of all predator for all patches. $r_i(t)$ is the intrinsic growth rate of the prey, $a_{ii}(t)$ is the density-dependent coefficient of the prey species. $a_{i,n+1}(t)$ are the capturing rates of the predator and $a_{n+1,i}/a_{i,n+1}$ are the conversion rates of nutrients into the reproduction of predator, $r_{n+1}(t)$ are the death rates of the predator. $D_i(t)$ $(i = 1, 2, \dots, n)$ is dispersal rate of prey species, $\sigma_{n+1} \geq 0$ denotes the delay due to negative feedback of the predator species σ_i $(i = 1, 2, \dots, n)$ are the time delays due to gestation, that is, mature adult predators can only contribute to the production of predator biomass. $a_{ii}(t)$, $a_{i,n+1}(t)$, $a_{n+1,i}$ $(i, j = 1, \dots, n)$, $r_i(t)$ $(i = 1, \dots, n+1)$ and $D_i(t)$ $(i = 1, 2, \dots, n)$ are continuously positive periodic functions with period $\omega > 0$. c_{ik} are positive constants and $0 < c_{ik} < 1$, Z_+ is the set of all positive integers and there exists an integer $p > 0$ such that $c_{i(k+p)} = c_{ik}$, $\tau_{k+p} = \tau_k + \omega$.

It is well known that by the fundamental theory of functional differential equations [4], system (1.1) has a unique solution $x(t) = (x_1(t), \dots, x_{n+1}(t))$ satisfying initial conditions (1.2). It is easy to verify that solutions of (1.1) corresponding to initial conditions (1.2) are defined on $[0, +\infty)$ and remain positive for all $t \geq 0$. In this paper, the solution of system (1.1) satisfying initial conditions (1.2) is said to be positive.

We shall use the following notation: Let $J \subset R$. Denote by $PC(J, R)$ the set of function $\psi : J \to R$ which are continuous for $t \in J, t \neq \tau_k$, are continuous from the left for $t \in J$ and have discontinuities of the first kind at the points $\tau_k \in J$. Denote the Banach space of ω-periodic functions by $PC_\omega = \{\psi \in PC[0, \omega], R\} | \psi(0) = \psi(\omega)\}$ and we denote

$$
\bar{f} = \frac{1}{\omega}\int_0^\omega f(t)dt, \quad f^L = \min_{t\in[0,\omega]} f(t), \quad f^M = \max_{[0,\omega]} f(t),
$$

where $f \in PC_\omega$.

The organization of this paper is as follows. In the next section, by using Gaines and Mawhin's continuation theorem of coincidence degree theory, sufficient conditions are derived for the existence of positive periodic solutions of system (1.1) with initial conditions (1.2).

2. Main result

In this section, by using Gaines and Mawhin's continuation theorem of coincidence degree theory, we show the existence of positive ω-periodic solutions of (1.1)-(1.2). To this end, we first introduce the following notations.

Let X, Z be real Banach spaces, let L: $\operatorname{Dom} L \subset X \to Z$ be a linear mapping, and $N : X \to Z$ be a continuous mapping. The mapping L is called a Fredholm mapping of index zero if $\dim \ker L = \operatorname{codim} \operatorname{Im} L < +\infty$ and $\operatorname{Im} L$ is closed in Z. If L is a Fredholm mapping of index zero and there exist continuous projectors $P : X \to X$, and $Q : Z \to Z$ such that $\operatorname{Im} P = \ker L$, $\ker Q = \operatorname{Im} L = \operatorname{Im}(I - Q)$, then the restriction L_P of $L \to \operatorname{Dom} L \cap \ker P : (I - P)X \to \operatorname{Im} L$ is invertible. Denote the inverse of L_P by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_P(I-Q)N : \bar{\Omega} \to X$ is compact. Since $\operatorname{Im} Q$ is isomorphic to $\ker L$, there exists isomorphism $J : \operatorname{Im} Q \to \ker L$.

For convenience, we introduce the continuation theorem of coincidence degree theory [3] and compactness criterion for set $F \subset PC_\omega$ [2] as follows.

Lemma 2.1. *Let $\Omega \subset X$ be an open bounded set. Let L be a Fredholm mapping of index zero and N be L-compact on $\bar{\Omega}$. Assume*

(a) *For each $\lambda \in (0, 1)$, $x \in \partial\Omega \cap \operatorname{Dom} L$, $Lx \neq \lambda Nx$*
(b) *For each $x \in \partial\Omega \cap \ker L$, $QNx \neq 0$*
(c) $\deg\{JQN, \Omega \cap \ker L, 0\} \neq 0$.

Then $Lx = Nx$ has at least one solution in $\bar{\Omega} \cap \operatorname{Dom} L$.

Lemma 2.2 (Compactness criterion)**.** *A set $F \subset PC_\omega$ is relative compact if and only if*

(a) *F is bounded, that is, $\|\psi\| = sup\{|\psi| : t \in J\} \leq M$ for each $x \in F$ and some $M > 0$*
(b) *F is quasiequicontinuous in J.*

We are now in a position to state our main result on the existence of a positive periodic solution to system (1.1).

Theorem 2.3. *System (1.1) with initial conditions (1.2) has at least one strictly positive ω-periodic solution provided that*

(H1)
$$\sum_{j=1}^{n} a^M_{n+1,j}(\overline{r_j} - \sum_{k=1,k\neq j}^{n} \overline{D_k} - \frac{1}{\omega}\ln\prod_{k=1}^{p}\frac{1}{1-c_{jk}})/a^M_{jj} > \overline{r_{n+1}} + \frac{1}{\omega}\ln\prod_{k=1}^{p}\frac{1}{1-c_{n+1,k}}$$

(H2) *For $i = 1, 2, \dots, n$,*
$$\begin{aligned} &\overline{r_i} - \sum_{j=1,j\neq i}^{n} \overline{D_j} - \frac{1}{\omega}\ln\prod_{k=1}^{p}\frac{1}{1-c_{ik}} \\ &> a^M_{i,n+1}\frac{A\sum_{j=1}^{n} a^M_{n+1,j} - \overline{r_{n+1}} - \frac{1}{\omega}\ln\prod_{k=1}^{p}\frac{1}{1-c_{n+1,k}}}{a^L_{n+1,n+1}}, \end{aligned}$$

where $A = \max_{1\le i\le n}\{[(r_i - \sum_{j=1,j\neq i}^n D_j)^M + \sum_{j=1,j\neq i}^n D_j^M]/a_{ii}^L\}$.

Proof. Since solutions of (1.1)-(1.2) remain positive for all $t \ge 0$, we let

$$y_i(t) = \ln[x_i(t)],\ \ i = 1,\dots,n+1. \tag{2.1}$$

Substituting (2.1) into system (1.1), we derive

$$\begin{aligned}
\dot y_i(t) =& r_i(t) - \sum_{j=1,j\neq i}^n D_j(t) - a_{ii}(t)e^{y_i(t)} - a_{i,n+1}(t)e^{y_{n+1}(t)}] \\
&+ \sum_{j=1,j\neq i}^n D_j(t)e^{y_j(t)-y_i(t)}, \quad i = 1,\dots,n, \\
\dot y_{n+1}(t) =& -r_{n+1}(t) + \sum_{j=1}^n a_{n+1,j}(t)e^{y_j(t-\sigma_j)} - a_{n+1,n+1}(t)e^{y_{n+1}(t-\sigma_{n+1})}, \\
& t \neq \tau_k,\ k \in Z_+, \\
\Delta(y_i(\tau_k)) =& \ln(1-c_{ik}), \quad t = \tau_k,\ k\in Z_+, \quad i = 1,2,\dots,n+1.
\end{aligned} \tag{2.2}$$

It is easy to see that if (2.2) has one ω-periodic solution $(y_1^*(t),\dots,y_{n+1}^*(t))^T$, then $x^*(t) = (x_1^*(t),\dots,x_{n+1}^*(t))^T = (\exp[y_1^*(t)],\dots,\exp[y_{n+1}^*(t)])^T$ is a positive ω-periodic solution of system (1.1). Therefore, to complete the proof, it suffices to show that system (2.2) has one ω-periodic solution. Let

$$X = \{x \in PC(R,R^{n+1}) : y(t+\omega) = y(t)\},$$

with the norm $\|x\| = \sup_{t\in[0,\omega]}\sum_{i=1}^{n+1}|y_i(t)|$, where $x = (y_1,y_2,\dots,y_{n+1})^T$. Let $Z = X \times R^{np}$ with the norm $\|(x,a_1,\dots,a_p)\| = (\|x\|^2 + |a_1|^2 + \dots + |a_p|^2)^{1/2}$. Then X and Z are Banach spaces. Let

$$L : \operatorname{Dom} L \to Z \quad L(x)(t) = (\dot x, \Delta(y(\tau_1),\dots,\Delta(y(\tau_p))$$

where $\operatorname{Dom} L$ consist of functions $x \in X$ such that x is continuous for $t \neq \tau_k$, x is continuous from the left for $t = \tau_k$, and $\dot x(\tau_k)$ exists. Let $N : X \to Z$, be defined as

$$(Nx)(t) = (f(t,x(t)), C_1, C_2,\dots,C_p),$$

where $f(t,x)$ equals

$$\begin{pmatrix}
r_1(t) - \sum_{j=2}^n D_j(t) - a_{11}(t)e^{y_1(t)} - a_{1,n+1}(t)e^{y_{n+1}(t)}] + \sum_{j=2}^n D_j(t)e^{y_j(t)-y_1(t)} \\
\vdots \\
r_n(t) - \sum_{j=1}^{n-1} D_j(t) - a_{nn}(t)e^{y_n(t)} - a_{n,n+1}(t)e^{y_{n+1}(t)}] + \sum_{j=1}^{n-1} D_j(t)e^{y_j(t)-y_n(t)} \\
-r_{n+1}(t) + \sum_{j=1}^n a_{n+1,j}(t)e^{y_j(t-\sigma_j)} - a_{n+1,n+1}(t)e^{y_{n+1}(t-\sigma_{n+1})},
\end{pmatrix}$$

and

$$C_k = (\ln(1-c_{1k}), \ln(1-c_{2k}),\dots,\ln(1-c_{n+1,k}))^T, \quad k = 1,2,\dots,p.$$

Define two projectors P and Q as

$$P : X \to \ker L, \quad Py = \frac{1}{\omega}\int_0^\omega y\,dt,$$

$$Q : Z \to Z, \quad Q(y, C_1,\dots,Cp) = \Big(\frac{1}{\omega}\int_0^\omega y\,dt + \sum_{k=1}^p C_k, \Big\{\begin{pmatrix}0\\ \vdots \\ 0\end{pmatrix}\Big\}_{k=1}^p\Big),$$

It is clear that

$$\begin{gathered}\ker L = \{x : x \in X,\ x = h,\ h \in R^{n+1}\},\\ \operatorname{Im} L = \{z = (y, C_1, \dots, C_p) \in Z,\ \int_0^\omega y(t)dt + \sum_{k=1}^{p} C_k = 0\} \text{ is closed in } Z,\\ \dim\ker L = \operatorname{codim}\operatorname{Im} L = n+1.\end{gathered}$$

Therefore, L is a Fredholm mapping of index zero. It is easy to show that P and Q are continuous projectors such that

$$\operatorname{Im} P = \ker L, \quad \ker Q = \operatorname{Im} L = \operatorname{Im}(I - Q).$$

Furthermore, the inverse K_P of L_P exists and is given by $K_P : \operatorname{Im} L to \operatorname{Dom} L \cap \ker P$,

$$K_P(z) = \int_0^t y(s)ds - \frac{1}{\omega}\int_0^\omega \int_0^t y(s)ds - \sum_{k=1}^{p} C_k.$$

Then $QN : X \to Z$ and $K_P(I - Q)N : X \to X$ read

$$QNx = \Big(\frac{1}{\omega}\int_0^\omega f(t,x)dt + \frac{1}{\omega}\sum_{k=1}^{p} C_k, \Big\{ \begin{pmatrix}0\\ \vdots\\ 0\end{pmatrix} \Big\}_{k=1}^{p}\Big),$$

$$\begin{aligned}&K_P(I - Q)Nx\\ &= \frac{1}{\omega}\int_0^t f(t,x)dt + \frac{1}{\omega}\sum_{t>\tau_k} C_k - \Big(\frac{1}{\omega}\int_0^\omega \int_0^t f(t,x)ds + \sum_{k=1}^{p} C_k\Big)\\ &- \Big((\frac{t}{\omega} - \frac{1}{2})\frac{1}{\omega}\int_0^\omega f(t,x)dt + \frac{1}{\omega}\sum_{k=1}^{p} C_k\Big).\end{aligned}$$

Clearly, QN and $K_P(I - Q)N$ are continuous.

To apply Lemma 2.1, we need to search for an appropriate open, bounded subset Ω. Corresponding to the operator equation $Lx = \lambda Nx$, $\lambda \in (0,1)$, we obtain

$$\begin{aligned}\dot y_i(t) =& \lambda\Big[r_i(t) - \sum_{j=1, j\neq i}^{n} D_j(t) - a_{ii}(t)e^{y_i(t)} - a_{i,n+1}(t)e^{y_{n+1}(t)}\Big]\\ &+ \sum_{j=1, j\neq i}^{n} D_j(t)e^{y_j(t) - y_i(t)}], \quad i = 1, \dots, n,\\ \dot y_{n+1}(t) =& \lambda\Big[-r_{n+1}(t) + \sum_{j=1}^{n} a_{n+1,j}(t)e^{y_j(t-\sigma_j)} - a_{n+1,n+1}(t)e^{y_{n+1}(t-\sigma_{n+1})}\Big],\\ &t \neq \tau_k,\ k \in Z_+,\\ \Delta(y_i(\tau_k)) =& \lambda\ln(1 - c_{ik}), \quad t = \tau_k,\ k \in Z_+,\ i = 1, 2, \dots, n+1.\end{aligned} \tag{2.3}$$

Suppose that $(y_1(t),\dots,y_{n+1}(t))^T\in X$ is a solution of (2.3) for some $\lambda\in(0,1)$. Integrating system (2.3) over $[0,\omega]$, for $i=1,2,\dots,n$, we have

$$\begin{aligned}&\int_0^\omega a_{ii}(t)e^{y_i(t)}dt+\int_0^\omega a_{i,n+1}(t)e^{y_{n+1}(t)}dt+\ln\prod_{k=1}^p\frac{1}{1-c_{ik}}\\&=\int_0^\omega (r_i(t)-\sum_{j=1,j\neq i}^n D_j(t))dt+\sum_{j=1,j\neq i}^n\int_0^\omega D_j(t)e^{y_j(t)-y_i(t)}dt,\end{aligned}\tag{2.4}$$

and

$$\begin{aligned}&\int_0^\omega r_{n+1}(t)dt+\int_0^\omega a_{n+1,n+1}(t)e^{y_{n+1}(t-\sigma_{n+1})}dt+\ln\prod_{k=1}^p\frac{1}{1-c_{n+1,k}}\\&=\sum_{j=1}^n\int_0^\omega a_{n+1,j}(t)e^{y_j(t-\sigma_j)}dt.\end{aligned}\tag{2.5}$$

Multiplying the ith equation (2.3) by $e^{y_i(t)}$ and integrating over $[0,\omega]$ gives

$$\begin{aligned}&\int_0^\omega a_{ii}(t)e^{2y_i(t)}dt\\&\le\int_0^\omega (r_i(t)-\sum_{j=1,j\neq i}^n D_j(t))e^{y_i(t)}dt+\sum_{j=1,j\neq i}^n\int_0^\omega D_j(t)e^{y_j(t)}dt+\sum_{k=1}^p\Delta(e^{y_i(\tau_k)}),\end{aligned}$$

in which $\Delta(e^{y_i(\tau_k)})=[(1-c_{ik})^\lambda-1]e^{y_i(\tau_k)}\le 0$, then we have

$$a_{ii}^L\int_0^\omega e^{2y_i(t)}dt\le[(r_i-\sum_{j=1,j\neq i}^n D_j)^M]\int_0^\omega e^{y_i(t)}dt+\sum_{j=1,j\neq i}^n D_j^M\int_0^\omega e^{y_j(t)}dt\,.\tag{2.6}$$

Using the inequality

$$\Big(\int_0^\omega e^{y_i(t)}dt\Big)^2\le\omega\int_0^\omega e^{2y_i(t)}dt,$$

it follows from (2.6) that

$$\frac{1}{\omega}a_{ii}^L(\int_0^\omega e^{y_i(t)}dt)^2\le\big[(r_i-\sum_{j=1,j\neq i}^n D_j)^M\big]\int_0^\omega e^{y_i(t)}dt+\sum_{j=1,j\neq i}^n D_j^M\int_0^\omega e^{y_j(t)}dt.\tag{2.7}$$

Using the fact that if $i=k$,

$$\int_0^\omega e^{y_k(t)}dt\ge\max\big\{\int_0^\omega e^{y_i(t)}dt,i=1,\dots,n\big\}$$

this , together with (2.7), leads to

$$\begin{aligned}&\frac{1}{\omega}a_{kk}^L(\int_0^\omega e^{y_k(t)}dt)^2\\&\le\big[(r_k-\sum_{j=1,j\neq k}^n D_j)^M\big]\int_0^\omega e^{y_k(t)}dt+(\sum_{j=1,j\neq k}^n D_j^M)\int_0^\omega e^{y_k(t)}dt,\end{aligned}$$

which implies

$$\begin{aligned}\max\{\int_0^\omega e^{y_i(t)}dt, i=1,\dots,n\} &\le \int_0^\omega e^{y_k(t)}dt \\ &\le \frac{[(r_k-\sum_{j=1,j\neq k}^n D_j)^M + \sum_{j=1,j\neq k}^n D_j^M]\omega}{a_{kk}^L}.\end{aligned} \tag{2.8}$$

Set

$$A=\max_{1\le i\le n}\Big\{\frac{(r_i-\sum_{j=1,j\neq i}^n D_j)^M+\sum_{j=1,j\neq i}^n D_j^M}{a_{ii}^L}\Big\},$$

we have from (2.8) that

$$\int_0^\omega e^{y_i(t)}dt \le A\omega, \quad i=1,\dots,n. \tag{2.9}$$

Since $y_i\in PC_\omega$, there exists $t_i, T_i \in [0,T]\cup\{\tau_1^+,\tau_2^+,\dots,\tau_p^+\}$ such that

$$y_i(t_i)=\min_{t\in[0,\omega]} y_i(t), \quad y_i(T_i)=\max_{t\in[0,\omega]} y_i(t), \quad i=1,2,\dots,n.$$

It follows from (2.9) that

$$y(t_i)\le \ln A, \quad i=1,2,\dots,n. \tag{2.10}$$

We derive from (2.5) that

$$\begin{aligned}&\int_0^\omega a_{n+1,n+1}(t)e^{y_{n+1}(t-\sigma_{n+1})}dt \\ &\le \sum_{j=1}^n a_{n+1,j}^M\int_0^\omega e^{y_j(t-\sigma_j)}dt-\overline{r_{n+1}}\omega-\ln\prod_{k=1}^p\frac{1}{1-c_{n+1,k}} \\ &= \sum_{j=1}^n a_{n+1,j}^M\int_0^\omega e^{y_j(t)}dt-\overline{r_{n+1}}\omega-\ln\prod_{k=1}^p\frac{1}{1-c_{n+1,k}} \\ &\le (A\sum_{j=1}^n a_{n+1,j}^M-\overline{r_{n+1}})\omega-\ln\prod_{k=1}^p\frac{1}{1-c_{n+1,k}},\end{aligned} \tag{2.11}$$

which yields

$$y_{n+1}(t_{n+1})\le \ln\frac{A\sum_{j=1}^n a_{n+1,j}^M-\overline{r_{n+1}}-\frac{1}{\omega}\ln\prod_{k=1}^p\frac{1}{1-c_{n+1,k}}}{\overline{a_{n+1,n+1}}},$$

and

$$\begin{aligned}\int_0^\omega e^{y_{n+1}(t)}dt &= \int_0^\omega e^{y_{n+1}(t-\sigma_{n+1})}dt \\ &\le \frac{A\sum_{j=1}^n a_{n+1,j}^M-\overline{r_{n+1}}-\frac{1}{\omega}\ln\prod_{k=1}^p\frac{1}{1-c_{n+1,k}}}{a_{n+1,n+1}^L}\omega\,.\end{aligned} \tag{2.12}$$

It follows from (2.4), (2.5), (2.9) and (2.12) that

$$\begin{aligned}
&\int_0^\omega |\dot{y}_{n+1}(t)|dt \\
&= \int_0^\omega \lambda| - r_{n+1}(t) + \sum_{j=1}^n a_{n+1,j}(t)e^{y_j(t-\sigma_j)} - a_{n+1,n+1}(t)e^{y_{n+1}(t-\sigma_{n+1})}|dt \\
&\le \int_0^\omega [r_{n+1}(t) + \sum_{j=1}^n a_{n+1,j}(t)e^{y_{n+1}(t-\sigma_j)} + a_{n+1,n+1}(t)e^{y_{n+1}(t-\sigma_{n+1})}]dt \\
&\le 2\int_0^\omega \sum_{j=1}^n a_{n+1,j}(t)e^{y_j(t-\sigma_j)}dt - \ln\prod_{k=1}^p \frac{1}{1-c_{n+1,k}} \\
&< 2A\omega\sum_{j=1}^n a^M_{n+1,j} := d_{n+1},
\end{aligned} \tag{2.13}$$

and

$$\begin{aligned}
\int_0^\omega |\dot{y}_i(t)|dt &= \int_0^\omega \lambda|r_i(t) - \sum_{j=1,j\neq i}^n D_j(t) - a_{ii}(t)e^{y_i(t)} - a_{i,n+1}(t)e^{y_{n+1}(t)} \\
&\quad + \sum_{j=1,j\neq i}^n D_j(t)e^{y_j(t)-y_i(t)}|dt \\
&\le \int_0^\omega [r_i(t) + \sum_{j=1,j\neq i}^n D_j(t) + a_{ii}(t)e^{y_i(t)} + a_{i,n+1}(t)e^{y_{n+1}(t)} \\
&\quad + \sum_{j=1,j\neq i}^n D_j(t)e^{y_j(t)-y_i(t)}]dt \\
&\le 2\int_0^\omega [a_{ii}(t)e^{y_i(t)} + a_{i,n+1}(t)e^{y_{n+1}(t)}]dt + \ln\prod_{k=1}^p \frac{1}{1-c_{ik}} \\
&\le 2\big[a^M_{ii}\int_0^\omega e^{y_i(t)}dt + a^M_{i,n+1}\int_0^\omega e^{y_{n+1}(t)}dt\big] + \ln\prod_{k=1}^p \frac{1}{1-c_{ik}} \\
&< 2A\big[a^M_{ii} + a^M_{i,n+1}(\sum_{j=1}^n a^M_{n+1,j})/a^L_{n+1,n+1}\big]\omega + \ln\prod_{k=1}^p \frac{1}{1-c_{ik}} := d_i,
\end{aligned} \tag{2.14}$$

$i = 1, \dots, n$. From (2.10), (2.14) and (2.12), (2.13), we have

$$\begin{aligned}
&y_{n+1}(t) \\
&\le y_{n+1}(t_{n+1}) + \int_0^\omega |\dot{y}_{n+1}(t)|dt + \big|\ln\prod_{k=1}^p \frac{1}{1-c_{n+1,k}}\big| \\
&\le \ln\frac{A\sum_{j=1}^n a^M_{n+1,j} - \overline{r_{n+1}} - \frac{1}{\omega}\ln\prod_{k=1}^p \frac{1}{1-c_{n+1,k}}}{\overline{a_{n+1,n+1}}} + d_{n+1} + \big|\ln\prod_{k=1}^p \frac{1}{1-c_{n+1,k}}\big|,
\end{aligned} \tag{2.15}$$

and

$$\begin{aligned} y_i(t) \le & y_i(t_i) + \int_0^\omega |\dot{y}_i(t)|dt + \big| \ln \prod_{k=1}^p \frac{1}{1-c_{ik}} \big| \\ & \le \ln A + d_i + \big| \ln \prod_{k=1}^p \frac{1}{1-c_{ik}} \big|, \quad i = 1, 2, \dots, n. \end{aligned} \tag{2.16}$$

On the other hand, it follows from (2.4) and (2.9) that

$$\begin{aligned} a_{ii}^M e^{y_i(T_i)}\omega \ge & (\overline{r_i} - \sum_{j=1, j\ne i}^n \overline{D_j})\omega - \ln \prod_{k=1}^p \frac{1}{1-c_{ik}} - a_{i,n+1}^M \int_0^\omega e^{y_{n+1}(t)}dt \\ \ge & (\overline{r_i} - \sum_{j=1, j\ne i}^n \overline{D_j})\omega - \ln \prod_{k=1}^p \frac{1}{1-c_{ik}} \\ & - a_{i,n+1}^M \Big(A \sum_{j=1}^n a_{n+1,j}^M - \overline{r_{n+1}} - \frac{1}{\omega} \ln \prod_{k=1}^p \frac{1}{1-c_{n+1,k}} \Big) \omega / a_{n+1,n+1}^L, \end{aligned}$$

which implies

$$\begin{aligned} y_i(T_i) \ge \ln \Big[& \frac{1}{a_{ii}^M} \Big(\overline{r_i} - \sum_{j=1, j\ne i}^n \overline{D_j} - \frac{1}{\omega} \ln \prod_{k=1}^p \frac{1}{1-c_{ik}} \\ & - a_{i,n+1}^M (A \sum_{j=1}^n a_{n+1,j}^M - \overline{r_{n+1}} - \frac{1}{\omega} \ln \prod_{k=1}^p \frac{1}{1-c_{n+1,k}}) / a_{n+1,n+1}^L \Big) \Big] := \rho_i. \end{aligned} \tag{2.17}$$

From (2.13) and (2.17) it follows that for $i = 1, 2 \dots, n$,

$$y_i(t) \ge y_i(T_i) - \int_0^\omega |\dot{y}_i(t)|dt - |\ln \prod_{k=1}^p \frac{1}{1-c_{ik}}| \ge \rho_i - d_i - |\ln \prod_{k=1}^p \frac{1}{1-c_{ik}}|, \tag{2.18}$$

Note that

$$\int_0^\omega a_{ii}(t) e^{y_i(t)}dt + \int_0^\omega a_{i,n+1}(t) e^{y_{n+1}(t)}dt \ge (\overline{r_i} - \sum_{j=1, j\ne i}^n \overline{D_j})\omega - \ln \prod_{k=1}^p \frac{1}{1-c_{ik}},$$

which, together with (2.5), leads to

$$\begin{aligned} & a_{n+1,n+1}^M y_{n+1}(T_{n+1})\omega \\ & \ge \sum_{j=1}^n a_{n+1,j}^L \int_0^\omega e^{y_j(t)}dt - \overline{r_{n+1}}\omega \\ & \ge \sum_{j=1}^n a_{n+1,j}^L \frac{(\overline{r_j} - \sum_{k=1, k\ne j}^n \overline{D_k} - \frac{1}{\omega} \ln \prod_{k=1}^p \frac{1}{1-c_{jk}} - \overline{a_{j,n+1}} e^{y_n+1(T_{n+1})})\omega}{a_{jj}^M} \\ & \quad - \overline{r_{n+1}}\omega - \ln \prod_{k=1}^p \frac{1}{1-c_{n+1,k}} \end{aligned}$$

which yields

$$
\begin{aligned}
y_{n+1}(T_{n+1}) \geq & \frac{1}{a^M_{n+1,n+1} + (\sum_{j=1}^n a^M_{n+1,j}\overline{a_{j,n+1}})/a^M_{jj}} \\
& \times \Big(\sum_{j=1}^n a^M_{n+1,j} \big(\overline{r_j} - \sum_{k=1,k\neq j}^n \overline{D_k} - \frac{1}{\omega} \ln \prod_{k=1}^p \frac{1}{1-c_{jk}} \big)/a^M_{jj} \\
& - \overline{r_{n+1}} - \frac{1}{\omega} \ln \prod_{k=1}^p \frac{1}{1-c_{n+1,k}} \Big) := \rho_{n+1}.
\end{aligned}
\tag{2.19}
$$

From what has been discussed above, we finally derive that for $i = 1, \dots, n$,

$$
\begin{aligned}
&\max_{t\in[0,\omega]} |y_i(t)| \\
&\leq \max \big\{ |\ln A| + d_i + |\ln \prod_{k=1}^p \frac{1}{1-c_{ik}}|, |\rho_i| + d_i + |\ln \prod_{k=1}^p \frac{1}{1-c_{ik}}| \big\} := B_i,
\end{aligned}
$$

and

$$
\begin{aligned}
&\max_{t\in[0,\omega]} |y_{n+1}(t) \\
&\leq |\max\{|\ln \frac{A\sum_{j=1}^n a^M_{n+1,j} - \overline{r_{n+1}} \frac{1}{\omega} \ln \prod_{k=1}^p \frac{1}{1-c_{n+1,k}}}{\overline{a_{n+1,n+1}}}| \\
&\quad + d_{n+1} + |\ln \prod_{k=1}^p \frac{1}{1-c_{n+1,k}}|, |\rho_{n+1}| + d_{n+1} + |\ln \prod_{k=1}^p \frac{1}{1-c_{n+1,k}}| \big\} := B_{n+1}.
\end{aligned}
\tag{2.20}
$$

Clearly, B_i $(i = 1, 2, \dots, n+1)$ are independent of λ. Denote $B = \sum_{i=1}^{n+1} B_i + B_0$, here B_0 is taken sufficiently large such that each solution $(v_1^*, v_2^*, ..., v_n^*, v_{n+1}^*)^T$ of the system of algebraic equations

$$
\begin{gathered}
\overline{r_i} - \sum_{j=2}^n \overline{D_j} - \frac{1}{\omega} \ln \prod_{k=1}^p \frac{1}{1-c_{ik}} - \overline{a_{ii}} e^{v_i} - \overline{a_{i,n+1}} e^{v_{n+1}} + \sum_{j=1,j\neq i}^n \overline{D_j} e^{v_j - v_i} = 0, \\
i = 1, 2, \dots, n, \\
-\overline{r_{n+1}} - \frac{1}{\omega} \ln \prod_{k=1}^p \frac{1}{1-c_{n+1,k}} + \sum_{j=1}^n \overline{a_{n+1,j}} e^{v_j} - \overline{a_{n+1,n+1}} e^{v_{n+1}} = 0,
\end{gathered}
\tag{2.21}
$$

satisfies $\|(v_1^*, v_2^*, \dots, v_{n+1}^*)^T\| = \sum_{i=1}^{n+1} |v_i^*| < B$ (if it exists) and $\sum_{i=1}^{n+1} C_i < B$, where for $i = 1, 2, \dots, n$,

$$
\begin{aligned}
C_i = \max \big\{ |\ln A_0|, \Big| \frac{1}{\overline{a_{ii}}} \Big(& \overline{r_i} - \sum_{j=1,j\neq i}^n \overline{D_j} - \frac{1}{\omega} \ln \prod_{k=1}^p \frac{1}{1-c_{ik}} \\
& - \overline{a_{i,n+1}} (A_0 \sum_{j=1}^n \overline{a_{n+1,j}} - \overline{r_{n+1}})/\overline{a_{n+1,n+1}} \Big| \big\}
\end{aligned}
$$

and

$$
\begin{aligned}
&C_{n+1}\\
&= \max\Big\{\Big|\ln\frac{A_0\sum_{j=1}^n\overline{a_{n+1,j}}-\overline{r_{n+1}}-\frac{1}{\omega}\ln\prod_{k=1}^p\frac{1}{1-c_{n+1,k}}}{\overline{a_{n+1,n+1}}}\Big|,\\
&\Big|\frac{\sum_{i=1}^n\overline{a_{n+1,i}}(\overline{r_i}-\sum_{j=1,j\neq i}^n\overline{D_j}-\frac{1}{\omega}\ln\prod_{k=1}^p\frac{1}{1-c_{ik}})/\overline{a_{ii}}-\overline{r_{n+1}}-\frac{1}{\omega}\ln\prod_{k=1}^p\frac{1}{1-c_{n+1,k}}}{\overline{a_{n+1,n+1}}+(\sum_{j=1}^n\overline{a_{n+1,j}}\,\overline{a_{j,n+1}})/\overline{a_{jj}}}\Big|\Big\}
\end{aligned}
\tag{2.22}
$$

in which

$$A_0=\max_{1\le i\le n}\{\overline{r_i}/\overline{a_{ii}}\}.$$

Now, we take $\Omega=\{y\in X:\|y\|<B\}$. Thus, the condition (a) of Lemma 2.1 is satisfied. When $y\in\partial\Omega\cap\ker L=\partial\Omega\cap R^{n+1}$, $y=(y_1,y_2,...,y_{n+1})^T$ is a constant vector in R^{n+1} with $\|y\|=B$. If system (2.21) has solutions, then

$$
\begin{aligned}
&QN(y_1,\dots,y_n,y_{n+1})^T\\
&=\Big(\begin{pmatrix}\overline{r_1}-\sum_{j=2}^n\overline{D_j}-\frac{1}{\omega}\ln\prod_{k=1}^p\frac{1}{1-c_{1k}}-\overline{a_{11}}e^{y_1}-\overline{a_{1,n+1}}e^{y_{n+1}}+\sum_{j=2}^n\overline{D_j}e^{y_j-y_1}\\ \vdots\\ \overline{r_n}-\sum_{j=1}^{n-1}\overline{D_j}-\frac{1}{\omega}\ln\prod_{k=1}^p\frac{1}{1-c_{nk}}-\overline{a_{nn}}e^{y_n}-\overline{a_{n,n+1}}e^{y_{n+1}}+\sum_{j=1}^{n-1}\overline{D_j}e^{y_j-y_n}\\ -\overline{r_{n+1}}-\frac{1}{\omega}\ln\prod_{k=1}^p\frac{1}{1-c_{n+1,k}}+\sum_{j=1}^n\overline{a_{n+1,j}}e^{y_j}-\overline{a_{n+1,n+1}}e^{y_{n+1}}\end{pmatrix},\\
&\quad\Big\{\begin{pmatrix}0\\ \vdots\\ 0\end{pmatrix}\Big\}_{k=1}^p\Big)\neq(0,\dots,0,0)^T.
\end{aligned}
$$

If system (2.21) does not have a solution, then we can directly derive

$$QN\begin{pmatrix}y_1\\ \vdots\\ y_{n+1}\end{pmatrix}\neq\Big(\Big\{\begin{pmatrix}0\\ \vdots\\ 0\end{pmatrix}\Big\}_{i=1}^{n+1},\Big\{\begin{pmatrix}0\\ \vdots\\ 0\end{pmatrix}\Big\}_{k=1}^{p}\Big).$$

Thus, the condition (b) in Lemma 2.1 is satisfied.

Finally, we will prove that the condition (c) in Lemma 2.1 is satisfied. To this end, we define $\phi: DomL\times[0,1]\to X$ by

$$
\begin{aligned}
&\phi(y_1,\dots,y_{n+1},\mu)\\
&=\begin{pmatrix}\overline{r_1}-\sum_{j=2}^n\overline{D_j}-\frac{1}{\omega}\ln\prod_{k=1}^p\frac{1}{1-c_{1k}}-\overline{a_{11}}e^{y_1}\\ \vdots\\ \overline{r_n}-\sum_{j=1}^{n-1}\overline{D_j}-\frac{1}{\omega}\ln\prod_{k=1}^p\frac{1}{1-c_{nk}}-\overline{a_{nn}}e^{y_n}\\ -\overline{r_{n+1}}-\frac{1}{\omega}\ln\prod_{k=1}^p\frac{1}{1-c_{n+1,k}}+\sum_{j=1}^n\overline{a_{n+1,j}}e^{y_j}-\overline{a_{n+1,n+1}}e^{y_{n+1}}\end{pmatrix}\\
&\quad+\mu\begin{pmatrix}-\overline{a_{1,n+1}}e^{y_{n+1}}+\sum_{j=2}^n\overline{D_j}e^{y_j-y_1}\\ \vdots\\ -\overline{a_{nn}}e^{y_n}-\overline{a_{n,n+1}}e^{y_{n+1}}+\sum_{j=1}^{n-1}\overline{D_j}e^{y_j-y_n}\\ 0\end{pmatrix},
\end{aligned}
$$

where μ is a parameter. When $(y_1,y_2,\dots,y_{n+1})^T\in\partial\Omega\cap R^{n+1}$, $(y_1,y_2,\dots,y_{n+1})^T$ is a constant vector in R^{n+1} with $\|y\|=B$.

We will show that when $(y_1, y_2, \dots, y_{n+1})^T \in \partial\Omega \cap \ker L$, $\phi(y_1, y_2, \dots, y_{n+1}, \mu) \neq 0$. Otherwise, there is a constant vector $(y_1, \dots, y_{n+1})^T \in R^{n+1}$ with $\|y\| = B$ satisfying $\phi(y_1, y_2, \dots, y_{n+1}, \mu) = 0$, that is

$$\overline{r_i} - \sum_{j=1, j\neq i}^{n} \overline{D_j} - \frac{1}{\omega} \ln \prod_{k=1}^{p} \frac{1}{1-c_{ik}} - \overline{a_{ii}} e^{y_i} - \mu \overline{a_{i,n+1}} e^{y_{n+1}} + \mu \sum_{j=1, j\neq i}^{n} \overline{D_j} e^{y_j - y_i} = 0,$$
$$i = 1, 2, \dots, n,$$
$$-\overline{r_{n+1}} - \frac{1}{\omega} \ln \prod_{k=1}^{p} \frac{1}{1-c_{n+1,k}} + \sum_{j=1}^{n} \overline{a_{n+1,j}} e^{y_j} - \overline{a_{n+1,n+1}} e^{y_{n+1}} = 0.$$

By similar argument in (2.10), (2.12), (2.17) and (2.19), we have

$$|y_i| \le C_i, i = 1, \dots, n+1,$$

where C_i is defined by (2.22). Thus, we have $\sum_{i=1}^{n+1} |y_i| \le \sum_{i=1}^{n+1} C_i < B$, which is leads to a contradiction. Using the property of topological degree and taking

$$J : \operatorname{Im} Q \to X, \quad \Big(\begin{pmatrix} y_1 \\ \vdots \\ y_{n+1} \end{pmatrix}, \Big\{ \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} \Big\}_{k=1}^{p} \Big) \to \begin{pmatrix} y_1 \\ \vdots \\ y_{n+1} \end{pmatrix},$$

we have

$$\begin{aligned}
&\deg(JQN(y_1, \dots, y_{n+1})^T, \Omega \cap \ker L, (0, \dots, 0)^T) \\
&= \deg(\phi(y_1, \dots, y_{n+1}, 1), \Omega \cap \ker L, (0, \dots, 0)^T) \\
&= \deg(\phi(y_1, \dots, y_{n+1}, 0), \Omega \cap \ker L, (0, \dots, 0)^T) \\
&= \deg\Big(\Big(\overline{r_1} - \sum_{j=2}^{n} \overline{D_j} - \frac{1}{\omega} \ln \prod_{k=1}^{p} \frac{1}{1-c_{1k}} - \overline{a_{11}} e^{y_1}, \dots, \overline{r_n} - \sum_{j=2}^{n} \overline{D_j} \\
&\quad - \frac{1}{\omega} \ln \prod_{k=1}^{p} \frac{1}{1-c_{nk}} - \overline{a_{nn}} e^{y_n}, -\overline{r_{n+1}} - \frac{1}{\omega} \ln \prod_{k=1}^{p} \frac{1}{1-c_{n+1,k}} \\
&\quad + \sum_{j=1}^{n-1} \overline{a_{n+1,j}} e^{y_j} - \overline{a_{n+1,n+1}} e^{y_{n+1}}\Big)^T, \Omega \cap \ker L, (0, \dots, 0)^T\Big).
\end{aligned}$$

Under assumption (H1), one can easily show that the system of algebraic equations

$$\begin{gathered}
\overline{r_i} - \sum_{j=1, j\neq i}^{n} \overline{D_j} - \frac{1}{\omega} \ln \prod_{k=1}^{p} \frac{1}{1-c_{ik}} - \overline{a_{ii}} u_i = 0, \quad i = 1, 2, \dots, n, \\
-\overline{r_{n+1}} - \frac{1}{\omega} \ln \prod_{k=1}^{p} \frac{1}{1-c_{n+1,k}} + \sum_{j=1}^{n} \overline{a_{n+1,j}} u_j - \overline{a_{n+1,n+1}} u_{n+1} = 0,
\end{gathered} \tag{2.23}$$

has a unique solution $(u_1^*, \dots, u_{n+1}^*)^T$ which satisfies

$$u_i^* = \frac{\overline{r_i} - \sum_{j=1, j\neq i}^{n} \overline{D_j} - \frac{1}{\omega} \ln \prod_{k=1}^{p} \frac{1}{1-c_{ik}}}{\overline{a_{ii}}} > 0$$

for $i = 1, \dots, n$, and

$$
\begin{aligned}
u^*_{n+1} &= \frac{1}{\overline{a_{n+1,n+1}}}\Big(\sum_{j=1}^{n}\overline{a_{n+1,j}}\big(\overline{r_j} - \sum_{j=1,j\neq i}^{n}\overline{D_j} - \frac{1}{\omega}\ln\prod_{k=1}^{p}\frac{1}{1-c_{ik}}\big)/(\overline{a_{jj}}) \\
&\quad - \overline{r_{n+1}} - \frac{1}{\omega}\ln\prod_{k=1}^{p}\frac{1}{1-c_{n+1,k}}\Big) \\
&\geq \frac{1}{\overline{a_{n+1,n+1}}}\Big(\sum_{j=1}^{n}a^M_{n+1,j}\big(\overline{r_j} - \sum_{j=1,j\neq i}^{n}\overline{D_j} - \frac{1}{\omega}\ln\prod_{k=1}^{p}\frac{1}{1-c_{ik}}\big)/(\overline{a_{jj}}) \\
&\quad - \overline{r_{n+1}} - \frac{1}{\omega}\ln\prod_{k=1}^{p}\frac{1}{1-c_{n+1,k}}\Big) > 0.
\end{aligned}
$$

A direct calculation shows that

$$
\begin{aligned}
&\deg(JQN(y_1, y_2, \dots, y_{n+1})^T, \Omega \cap KerL, (0, 0, \dots, 0)^T) \\
&= \operatorname{sgn}\big(\prod_{i=1}^{n+1}(-\overline{a_{ii}})\big) = (-1)^{n+1} \neq 0.
\end{aligned}
$$

Finally, easily we show that the set $\{K_P(I-Q)Nu | u \in \bar{\Omega}\}$ is equicontinuous and uniformly bounded. Using Lemma 2.2 and the Arzela-Ascoli Theorem, we see that $K_P(I-Q)N : \overline{\Omega} \to X$ is compact. Moreover, $QN(\bar{\Omega})$ is bounded. Consequently, N is $L-$compact.

We have proved that Ω satisfies all the requirements in Lemma 2.1. Hence, system (2.2) has at least one ω-periodic solution. Accordingly, system (1.1) has at least one positive ω-periodic solution. This completes the proof. □

Remark. In this paper, by borrowing Gaines and Mawhin's continuation theorem of coincidence degree theory, we have established sufficient conditions for the existence of positive periodic solutions to system (1.1) with initial conditions (1.2). We would like to mention here that it is interesting but challenging to discuss the existence of positive periodic solutions of (1.1) when we incorporate time delays to the self-regulated terms of the prey in n-patch environments. We leave this for our future work.

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