

CONTROLLABILITY AND STABILIZATION OF A NONLINEAR HIERARCHICAL AGE-STRUCTURED COMPETING SYSTEM

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ABSTRACT. This article concerns the approximate controllability of a biological system, which is composed of two hierarchical age-structured competing species. Basing on a controllability result of linear system, we prove that the nonlinear system is approximately controllable by means of a fixed point theorem for multi-valued mappings. To fix a suitable control policy, we deal with an optimal control problem and established the existence of the unique optimal strategy. In addition, the stabilization problem of the system is also considered.

1. INTRODUCTION

We consider the dynamics of a biological system consisting of two competing age-structured populations, in which the vital rates of individuals of age a mainly depend on the size of elders. Let $p(a, t)$ be the age-specific density of a population at moment t and A the maximum age, then $\int_a^A p(r, t) dr$ represents the number of the individuals with ages larger than a . According to the balance law of continuous age-structured population system, we derive the following dynamical model for the evolution of a competing community:

$$\frac{\partial p_1}{\partial t} + \frac{\partial p_1}{\partial a} = -[\mu_1(a) + m_1(E(p_1)(a, t)) + f_1(E(p_2)(a, t))]p_1(a, t) + u_1(a, t), \quad (a, t) \in Q_1,$$

$$\frac{\partial p_2}{\partial t} + \frac{\partial p_2}{\partial a} = -[\mu_2(a) + m_2(E(p_2)(a, t)) + f_2(E(p_1)(a, t))]p_2(a, t) + u_2(a, t), \quad (a, t) \in Q_2,$$

$$p_i(0, t) = \int_0^{A_i} \beta_i(a, E(p_i)(a, t))p_i(a, t) da, \quad t \in (0, T); i = 1, 2,$$

$$p_i(a, 0) = p_i^0(a), \quad a \in [0, A_i], i = 1, 2,$$

$$E(p_i)(a, t) = \alpha \int_0^a p_i(r, t) dr + \int_a^{A_i} p_i(r, t) dr, \quad 0 \leq \alpha < 1,$$

where $Q_i = (0, A_i) \times (0, T)$, A_i is the maximum age of the individuals in population i and T the control horizon. $p_i(a, t)$ stands for the density of the population i .

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$E(p_i)(a, t)$ (with the weight α) describes the age-specific instant environment within the population i . $\mu_i(a)$ and $m_i(E(p_i)(a, t))$ denotes the natural death rate and the additional mortality caused by over-crowding, respectively. $f_i(E(p_j)(a, t))$, $j \neq i$, shows the competition effect of population j on population i , and the control variable $u_i(a, t)$ is the migration rate. $\beta_i(a, E(p_i)(a, t))$ is the fertility and $p_i^0(a)$ the initial age distribution. As one can see, in the extremal case of $\alpha = 0$, the vital rates of an individual of age a depends only on elders except its age, which reveals the internal hierarchy of ages among the individuals.

Controllability has been an interesting and challenging topic for infinite-dimensional systems. This is also the case in the study of structured population models, and a number of excellent results have been achieved by researchers, see for example [2, 3, 5, 6, 8, 12, 13, 21, 29, 30, 31] and their references. Most of the existing works in the literature focus on linear systems, which are, of course, necessary and fundamental. However, because realistic models in almost all practical situations are nonlinear, the investigation of controllability for nonlinear systems is of significance, and which is the main concern in the present article. We are also interested in stabilizing the competing populations, which makes sense in the control of pest or invading species. One can refer to [4, 18, 19, 24, 26, 28, 30] and the references cited for some related works.

During the previous four decades, the socialization in biological populations has attracted much attention of ecologists and mathematicians, see [10, 25] for ecological investigations and [1, 7, 9, 14, 15, 16, 17, 20, 22, 23, 27, 31] for analysis in mathematical modelling. To the best of our knowledge, almost all works on hierarchical population models are limited in single-species and directed to the dynamical behaviors, results in control problems on multi-species systems are quite rare. Motivated by the observation, we in this article deal with controllability and stabilization problems for a hierarchical competing model.

This paper is organized as follows: in the next section we present the normalization for the above model and assumptions posed on the parameters. The third section is devoted to the theoretic proof of the approximate controllability, which is followed by an appropriate choice in section 4. The stabilization of the zero state in a finite period is treated in section 5 by means of an optimal control problem. Some remarks are included in the final section.

2. MODEL NORMALIZATION AND ASSUMPTIONS

Denote $A = \max(A_1, A_2)$ and $Q = (0, A) \times (0, T)$. The natural zero extension to functions and parameters in the above model reaches the normal system

$$\begin{aligned} \frac{\partial p_1}{\partial t} + \frac{\partial p_1}{\partial a} &= -[\mu_1(a) + m_1(E(p_1))(a, t) + f_1(E(p_2))(a, t)]p_1(a, t) \\ &\quad + u_1(a, t), \quad (a, t) \in Q, \\ \frac{\partial p_2}{\partial t} + \frac{\partial p_2}{\partial a} &= -[\mu_2(a) + m_2(E(p_2))(a, t) + f_2(E(p_1))(a, t)]p_2(a, t) \\ &\quad + u_2(a, t), \quad (a, t) \in Q, \\ p_i(0, t) &= \int_0^A \beta_i(a, E(p_i))(a, t)p_i(a, t)da, \quad t \in (0, T); i = 1, 2, \\ p_i(a, 0) &= p_i^0(a), \quad a \in [0, A]; i = 1, 2, \\ E(p_i)(a, t) &= \alpha \int_0^a p_i(r, t)dr + \int_a^A p_i(r, t)dr, \quad 0 \leq \alpha < 1. \end{aligned} \tag{2.1}$$

To meet the need of forthcoming theoretic analysis, we propose the following assumptions for the model parameters ($i = 1, 2$), which are biologically meaningful:

- (A1) A and T are finite positive constants;
- (A2) $\mu_i(a) > 0$ for for all $a \in (0, A)$, $\mu_i \in L^1_{\text{loc}}[0, A]$ and $\int_0^A \mu_i(a)da = +\infty$;
- (A3) $0 \leq m_i(x) \leq \bar{m}_i$ for all $x \geq 0$ with constant $\bar{m}_i > 0$;
- (A4) $0 \leq f_i(x) \leq F_i$ for all $x \geq 0$ with constant $F_i > 0$;
- (A5) $u \in \mathcal{U} := \{(u_1, u_2) : |u_i(a, t)| \leq U_i, \text{ a.e. } (a, t) \in Q, i = 1, 2\}$, with constant $U_i > 0$;
- (A6) $0 \leq \beta_i(a, x) \leq B_i$ for all $(a, x) \in [0, A] \times [0, +\infty)$ with constant $B_i > 0$;
- (A7) $0 \leq p_i^0(a) \leq P_i^*$ for all $a \in [0, A]$ with constant $P_i^* > 0$.

3. APPROXIMATE CONTROLLABILITY

Definition 3.1. System (2.1) is said to be approximately controllable on $[0, T]$ if, for any given initial distribution $p^0 = (p_1^0, p_2^0) \in (L^\infty[0, A])^2$, target $\bar{p} = (\bar{p}_1, \bar{p}_2) \in (L^\infty[0, A])^2$ and ε , $0 < \varepsilon < 1$, there exists $u = (u_1, u_2) \in \mathcal{U}$, such that the solution $p^u(a, t)$ to (2.1) meets

$$\|p^u(\cdot, T) - \bar{p}\|_{\{L^2[0, A]\}^2} \leq \varepsilon.$$

Let $p^{(0,0)}$ be the solution of the system (2.1) corresponding to $u = (0, 0)$. Without loss of generality, we suppose that $\|p^{(0,0)}(\cdot, T) - \bar{p}\| > 1$. We note that if $E(p) = (E(p_1), E(p_2))$ in the functions m_i, f_i, β_i are fixed as $P = (P_1, P_2)$, then system

(2.1) reduces to

$$\begin{aligned} \frac{\partial p_1}{\partial t} + \frac{\partial p_1}{\partial a} &= -[\mu_1(a) + m_1(P_1(a, t)) + f_1(P_2(a, t))]p_1(a, t) + u_1(a, t), \\ &\quad (a, t) \in Q, \\ \frac{\partial p_2}{\partial t} + \frac{\partial p_2}{\partial a} &= -[\mu_2(a) + m_2(P_2(a, t)) + f_2(P_1(a, t))]p_2(a, t) + u_2(a, t), \\ &\quad (a, t) \in Q, \end{aligned} \quad (3.1)$$

$$\begin{aligned} p_i(0, t) &= \int_0^A \beta_i(a, P_i(a, t))p_i(a, t)da, \quad t \in (0, T); i = 1, 2, \\ p_i(a, 0) &= p_i^0(a), \quad a \in [0, A], i = 1, 2. \end{aligned}$$

It is readily seen that the system (3.1) can be divided into two independent subsystems, which are approximately controllable according to a result in the literature [8]; that is, there is u_i such that

$$\|p_i^{u_i}(\cdot, T; P) - \bar{p}_i\|_{L^2[0, A]} \leq \frac{\varepsilon}{2}, \quad i = 1, 2. \quad (3.2)$$

We hope to extend the controllability of system (3.1) to system (2.1) by means of fixed point approach. To do so, we need some analysis on the linear system (3.1). In what follows, we adopt the notation:

$$M_i(a, t; P) = \mu_i(a) + m_i(P_i(a, t)) + f_i(P_j(a, t)), \quad i, j = 1, 2, i \neq j.$$

Lemma 3.2. *For any given $P = (P_1, P_2) \in (L^\infty(Q))^2$, and all $u = (u_1, u_2) \in \mathcal{U}$, the solution $p^u(\cdot, \cdot; P) = (p_1^{u_1}(\cdot, \cdot; P), p_2^{u_2}(\cdot, \cdot; P))$ to system (3.1) satisfies*

$$\|E(p_i^{u_i}(\cdot, \cdot; P))\|_{L^\infty(Q)} + \|E(p_i^{u_i}(\cdot, \cdot; P))_t\|_{L^\infty(Q)} + \|E(p_i^{u_i}(\cdot, \cdot; P))_a\|_{L^\infty(Q)} \leq C_1,$$

where $E(p)_t$ and $E(p)_a$ stand for the partial derivatives with respect to t and a respectively, the constant C_1 is independent of P and u .

Proof. It follows from (3.1) and the method of characteristics that

$$p_i^{u_i}(a, t; P) = \begin{cases} p_i^0(a - t)\Pi_i(a, t, t; P) + \Pi_{u_i}(a, t, t; P), & a \geq t, \\ b_i^{u_i}(t - a; P)\Pi_i(a, t, a; P) + \Pi_{u_i}(a, t, a; P), & a < t, \end{cases} \quad (3.3)$$

where

$$b_i^{u_i}(t; P) = p_i^{u_i}(0, t; P), \quad (3.4)$$

$$\Pi_i(a, t, s; P) = \exp \left\{ - \int_0^s M_i(a - \tau, t - \tau; P) d\tau \right\}, \quad (3.5)$$

$$\Pi_{u_i}(a, t, s; P) = \int_0^s \exp \left\{ - \int_0^\tau M_i(a - r, t - r; P) dr \right\} u_i(a - \tau, t - \tau) d\tau. \quad (3.6)$$

By (3.3)-(3.4) and the third equation in (3.1), we claim that the population fertility $b_i^{u_i}$ solves the Volterra integral equation

$$b_i^{u_i}(t; P) = F_i(t; P) + \int_0^t K_i(t, s; P)b_i^{u_i}(t - s; P)ds, \quad t \in (0, T), \quad (3.7)$$

where

$$F_i(t; P) = \begin{cases} \int_0^{A-t} \beta_i(a+t, P_i(a+t, t)) p_i^0(a) \Pi_i(a+t, t, t; P) da \\ + \int_0^t \beta_i(a, P_i(a, t)) \Pi_{u_i}(a, t, a; P) da \\ + \int_t^A \beta_i(a, P_i(a, t)) \Pi_{u_i}(a, t, a; P) da, & 0 < t < A, \\ \int_0^A \beta_i(a, P_i(a, t)) \Pi_{u_i}(a, t, a; P) da, & A \leq t \leq T; \end{cases} \quad (3.8)$$

$$K_i(t, s; P) = \begin{cases} \beta_i(s, P_i(s, t)) \Pi_i(s, t, s; P), & s < t, \\ 0, & s \geq t. \end{cases} \quad (3.9)$$

Here we consider the case $T > A$ only. The other case has a similar process.

From (3.5)–(3.6) and (3.8)–(3.9), we have

$$\begin{aligned} \Pi_i(a, t, s; P) &\leq 1, \quad |\Pi_{u_i}(a, t, s; P)| \leq AU_i, \\ |F_i(t; P)| &\leq \max(AB_i P_i^* + ATB_i U_i, AU_i B_i), \\ 0 &\leq K_i(t, s; P) \leq B_i. \end{aligned}$$

Combining (3.7) with the above estimations, we derive that $b_i^{u_i}(t; P) \leq C_2$, with the constant C_2 independent of P, u . Therefore, the expression (3.3) and the definition of $E(p_i)$ guarantee the uniform boundedness of $E(p_i^{u_i})(a, t; P)$ and $E(p_i^{u_i})_a(a, t; P)$.

To estimate $E(p_i^{u_i})_t(a, t; P)$, we use (3.3) to obtain that, if $0 < t \leq a$ then (with the symbol P omitted)

$$\begin{aligned} E(p_i^{u_i})(a, t) &= \alpha \int_0^t [b_i^{u_i}(t-r) \Pi_i(r, t, r) + \Pi_{u_i}(r, t, r)] dr \\ &+ \alpha \int_t^a [p_i^0(r-t) \Pi_i(r, t, t) + \Pi_{u_i}(r, t, t)] dr \\ &+ \int_a^A [p_i^0(r-t) \Pi_i(r, t, t) + \Pi_{u_i}(r, t, t)] dr; \end{aligned} \quad (3.10)$$

if $0 < a < t < A$, then

$$\begin{aligned} E(p_i^{u_i})(a, t) &= \alpha \int_0^a [b_i^{u_i}(t-r) \Pi_i(r, t, r) + \Pi_{u_i}(r, t, r)] dr \\ &+ \int_a^t [b_i^{u_i}(t-r) \Pi_i(r, t, r) + \Pi_{u_i}(r, t, r)] dr \\ &+ \int_t^A [p_i^0(r-t) \Pi_i(r, t, t) + \Pi_{u_i}(r, t, t)] dr; \end{aligned} \quad (3.11)$$

and if $t \geq A$, then

$$\begin{aligned} E(p_i^{u_i})(a, t) &= \alpha \int_0^a [b_i^{u_i}(t-r) \Pi_i(r, t, r) + \Pi_{u_i}(r, t, r)] dr \\ &+ \int_a^A [b_i^{u_i}(t-r) \Pi_i(r, t, r) + \Pi_{u_i}(r, t, r)] dr. \end{aligned} \quad (3.12)$$

Making appropriate changes of integration variables, we are able to write that, if $0 < t \leq a$, then

$$\begin{aligned}
 E(p_i^{u_i})(a, t) &= \alpha \int_0^t b_i^{u_i}(\tau) \exp \left\{ - \int_\tau^t M_i(v - \tau, v) dv \right\} d\tau \\
 &\quad + \alpha \int_0^{a-t} p_i^0(v) \exp \left\{ - \int_0^t M_i(v + \tau, \tau) d\tau \right\} dv \\
 &\quad + \int_{a-t}^{A-t} p_i^0(v) \exp \left\{ - \int_0^t M_i(v + \tau, \tau) d\tau \right\} dv \\
 &\quad + \alpha \int_0^t \int_0^s \exp \left\{ - \int_v^t M_i(\theta - s, \theta) d\theta \right\} u_i(v - s, v) dv ds \\
 &\quad + \alpha \int_0^{a-t} \int_0^t \exp \left\{ - \int_v^t M_i(\theta + s, \theta) d\theta \right\} u_i(v + s, v) dv ds \\
 &\quad + \int_{a-t}^{A-t} \int_0^t \exp \left\{ - \int_v^t M_i(\theta + s, \theta) d\theta \right\} u_i(v + s, v) dv ds.
 \end{aligned} \tag{3.13}$$

Consequently, for $0 < t \leq a$, we have

$$\begin{aligned}
 \frac{\partial E(p_i^{u_i})}{\partial t} &= -\alpha \int_0^t b_i^{u_i}(\tau) M_i(t - \tau, t) \exp \left\{ - \int_\tau^t M_i(v - \tau, v) dv \right\} d\tau \\
 &\quad + \alpha b_i^{u_i}(t) - \alpha p_i^0(a - t) \exp \left\{ - \int_0^t M_i(a - t + \tau, \tau) d\tau \right\} \\
 &\quad - \alpha \int_0^{a-t} p_i^0(v) M_i(v + t, t) \exp \left\{ - \int_0^t M_i(v + \tau, \tau) d\tau \right\} dv \\
 &\quad - p_i^0(A - t) \exp \left\{ - \int_0^t M_i(A - t + \tau, \tau) d\tau \right\} \\
 &\quad + p_i^0(a - t) \exp \left\{ - \int_0^t M_i(a - t + \tau, \tau) d\tau \right\} \\
 &\quad - \int_{a-t}^{A-t} p_0(v) M_i(v + t, t) \exp \left\{ - \int_0^t M_i(v + \tau, \tau) d\tau \right\} dv \\
 &\quad + \alpha \int_0^t \exp \left\{ - \int_v^t M_i(\theta - t, \theta) d\theta \right\} u_i(v - t, v) dv \\
 &\quad - \alpha \int_0^t \int_0^s M_i(t - s, t) \exp \left\{ - \int_v^t M_i(\theta - s, \theta) d\theta \right\} u_i(v - s, v) dv ds \\
 &\quad - \alpha \int_0^t \exp \left\{ - \int_v^t M_i(\theta + a - t, \theta) d\theta \right\} u_i(v + a - t, v) dv \\
 &\quad + \alpha \int_0^{a-t} [u_i(t + s, t) \\
 &\quad - \int_0^t u_i(v + s, s) M_i(t + s, t) \exp \left\{ - \int_v^t M_i(\theta + s, \theta) d\theta \right\}] ds \\
 &\quad - \int_0^t \exp \left\{ - \int_v^t M_i(\theta + A - t, \theta) d\theta \right\} u_i(v + A - t, v) dv \\
 &\quad + \int_0^t \exp \left\{ - \int_v^t M_i(\theta + a - t, \theta) d\theta \right\} u_i(v + a - t, v) dv
 \end{aligned}$$

$$\begin{aligned}
& + \int_{a-t}^{A-t} [u_i(t+s, t) \\
& - \int_0^t u_i(v+s, v) M_i(t+s, t) \exp \left\{ - \int_v^t M_i(\theta+s, \theta) d\theta \right\} dv] ds.
\end{aligned}$$

One can handle the cases $0 < a < t < A$ and $t \geq A$ similarly. The assumptions tell us that $E(p_i^{u_i}(\cdot, \cdot; P))_t$ is uniformly bounded in P and u . \square

The following three results will also be used in the proof of the controllability.

Lemma 3.3. *The set*

$$K = \left\{ (P_1, P_2) \in (L^\infty(Q))^2 : \|P_i\|_{L^\infty(Q)} + \left\| \frac{\partial P_i}{\partial t} \right\|_{L^\infty(Q)} + \left\| \frac{\partial P_i}{\partial a} \right\|_{L^\infty(Q)} \leq C_1, i = 1, 2 \right\}$$

is compact in $(C(Q))^2$, with the constant C_1 given in Lemma 3.2.

Proof. Note that $\bar{Q} = [0, A] \times [0, T]$ is compact in R^2 . The structure of K implies that every element in K must be a continuous function on \bar{Q} , and every sequence in K must be uniformly bounded and equi-continuous. The conclusion follows from the Arzela-Ascoli theorem. \square

Lemma 3.4. *The solution $p^u(\cdot, \cdot; P)$ for the linear system (3.1) depends continuously on $u \in \mathcal{U}$.*

Proof. For any given $u^r = (u_1^r, u_2^r) \in \mathcal{U}$, $r = 1, 2$, the expression (3.3) implies that, if $a \geq t$ then

$$\begin{aligned}
|p_i^{u_1^1}(a, t; P) - p_i^{u_2^2}(a, t; P)| &= |\Pi_{u_1^1}(a, t, t; P) - \Pi_{u_2^2}(a, t, t; P)| \\
&\leq \int_0^t |u_i^1(a-\tau, t-\tau) - u_i^2(a-\tau, t-\tau)| d\tau \quad (3.14) \\
&\leq T \|u^1 - u^2\|_{(L^\infty(Q))^2};
\end{aligned}$$

If $a < t$, then

$$\begin{aligned}
|p_i^{u_1^1}(a, t; P) - p_i^{u_2^2}(a, t; P)| &= |b_i^{u_1^1}(t-a; P) - b_i^{u_2^2}(t-a; P)| \\
&\quad + \int_0^a |u_i^1(a-\tau, t-\tau) - u_i^2(a-\tau, t-\tau)| d\tau. \quad (3.15)
\end{aligned}$$

On the other hand, one can see from (3.7) that

$$\begin{aligned}
& |b_i^{u_1^1}(t; P) - b_i^{u_2^2}(t; P)| \\
& \leq |F_i^{u_1^1}(t; P) - F_i^{u_2^2}(t; P)| + \int_0^t B_i |b_i^{u_1^1}(t-s; P) - b_i^{u_2^2}(t-s; P)| ds \quad (3.16) \\
& \leq AB_i \|u_i^1 - u_i^2\|_{(L^\infty(Q))^2} + B_i \int_0^t |b_i^{u_1^1}(t-s; P) - b_i^{u_2^2}(t-s; P)| ds.
\end{aligned}$$

Applying the Bellmann inequality, we derive from (3.16) that

$$|b_i^{u_1^1}(t; P) - b_i^{u_2^2}(t; P)| \leq AB_i \exp\{TB_i\} \|u^1 - u^2\|_{(L^\infty(Q))^2}. \quad (3.17)$$

Substituting (3.17) into (3.15), one gets that, if $a < t$, then

$$|p_i^{u_1^1}(a, t; P) - p_i^{u_2^2}(a, t; P)| \leq A(1 + B_i \exp\{TB_i\}) \|u^1 - u^2\|_{(L^\infty(Q))^2}. \quad (3.18)$$

Finally, combining (3.14) with (3.18), we arrive at

$$\|p^{u^1}(\cdot, \cdot; P) - p^{u^2}(\cdot, \cdot; P)\|_{(L^\infty(Q))^2} \leq C_1 \|u^1 - u^2\|_{(L^\infty(Q))^2},$$

where the constant C_1 is independent of u^1, u^2 and P . \square

For the sake of convenience, we list the following existence result [31, P. 452].

Lemma 3.5 (Ky Fan-Glicksberg). *Suppose that the following conditions are satisfied:*

- (1) *Subset K is nonempty, compact and convex in a locally convex space X ;*
- (2) *Set-valued mapping $G : K \rightarrow 2^K$ is upper-continuous;*
- (3) *$G(x)$ is nonempty, closed and convex for each $x \in K$.*

Then G has at least one fixed point.

Now we are ready to show the following result.

Theorem 3.6. *System (2.1) is approximately controllable in $(L^\infty(Q))^2$.*

Proof. Let $X = (L^\infty(Q))^2$, and K be given in Lemma 3.3. Define the set-valued mapping $G : K \rightarrow 2^X$, for $P \in K$,

$$G(P) = \{E(p^u(\cdot, \cdot; P)) : u \in \mathcal{U} \text{ such that } \|p^u(\cdot, T; P) - \bar{p}\|_{(L^\infty([0, A]))^2} \leq \varepsilon\},$$

where $p^u(a, t; P) = (p_1^{u^1}(a, t; P), p_2^{u^2}(a, t; P))$ is the solution to the linear system (3.1). The conclusion in Lemma 3.2 implies that $G(P) \in 2^K$ for each $P \in K$.

It is clear that K is convex. Lemma 3.3 shows that the condition (1) in Lemma 3.5 is satisfied. According to the main result in [8], $G(P) \neq \emptyset$. Since the control variable $u = (u_1, u_2)$ serves as the nonhomogeneous term, $G(P)$ is convex. Moreover, Lemma 3.4 assures the closedness of $G(P)$. Hence condition (3) in Lemma 3.5 is true.

It remains to prove that G is upper-continuous. Let $P^n = (P_1^n, P_2^n) \rightarrow P = (P_1, P_2)$ in $(L^\infty(Q))^2$, and $E(p^{u^n}) = (E(p_1^{u_1^n}), E(p_2^{u_2^n})) \rightarrow h = (h_1, h_2)$ in $(L^\infty(Q))^2$ for some sequence $\{u^n = (u_1^n, u_2^n)\}$ such that $E(p^{u^n}) \in G(P^n)$. We need to show $h \in G(P)$. By (3.3), we derive that

$$p_i^{u_i^n}(a, t; P^n) = \begin{cases} p_i^0(a-t)\Pi_i(a, t, t; P^n) + \Pi_{u_i^n}(a, t, t; P^n), & a \geq t, \\ b_i^{u_i^n}(t-a; P^n)\Pi_i(a, t, a; P^n) + \Pi_{u_i^n}(a, t, a; P^n), & a < t, \end{cases} \quad (3.19)$$

Note that $u_i^n \in L^\infty(Q) \subset L^2(Q)$ and $|u_i^n(a, t)| \leq U_i, i = 1, 2$. There exists a subsequence (denoted still by $\{u_i^n\}$) such that $u_i^n \rightarrow u_i$ weakly. On the other hand, it follows from (3.17) that $\{b_i^{u_i^n}\}$ is bounded uniformly. One can extract a subsequence (denoted still by $\{b_i^{u_i^n}\}$) such that $\{b_i^{u_i^n}\}$ converges to some b_i in the weak-star sense. Passing into the limit in the right side of (3.19), we define the function

$$p_i(a, t) := \begin{cases} p_i^0(a-t)\Pi_i(a, t, t; P) + \Pi_{u_i}(a, t, t; P), & a \geq t, \\ b_i(t-a)\Pi_i(a, t, a; P) + \Pi_{u_i}(a, t, a; P), & a < t. \end{cases} \quad (3.20)$$

Consequently, $p_i(0, t) = b_i(t) = \int_0^A \beta_i(a, P(a, t))p_i(a, t)da$. We are sure that the function given by (3.20) is the solution for the system (3.1).

Furthermore, $E(p^{u^n}) \in G(P^n)$ implies that the control functions u_i^n must meet

$$\|p^{u^n}(\cdot, T; P^n) - \bar{p}\|_{(L^\infty([0, A]))^2} \leq \varepsilon.$$

Passing to the limit in the above inequality, we have

$$\|p(\cdot, T; P) - \bar{p}\|_{(L^\infty([0, A]))^2} \leq \varepsilon.$$

In a word, $h = E(p(\cdot, \cdot; P))$ and $h \in G(P)$. Therefore, the mapping given above satisfies all the conditions in Lemma 3.5 and has at least one fixed point, which establishes the approximate controllability of system (2.1). \square

4. CHOICE OF CONTROLS

Theorem 3.6 implies that there is at least one control $u = (u_1, u_2) \in \mathcal{U}$, such that the corresponding state of the system (2.1) at the moment T approaches a prescribed target \bar{p} arbitrarily. Generally speaking, there are many (even infinite) controls of such type. Which control should we choose to adjust the population states?

To seek an appropriate control, we consider the following optimal control problem:

(P1) Find $u^* = (u_1^*, u_2^*) \in \mathcal{U}$, such that $J(u^*) \leq J(u)$ for all $u \in \mathcal{U}$, with

$$\mathcal{U} = \{(u_1, u_2) \in (L^2(Q))^2 : |u_i(a, t)| \leq U_i, \text{ a.e. } (a, t) \in Q; i = 1, 2\},$$

and

$$J(u) = \frac{1}{2} \sum_{j=1}^2 \left\{ \int_Q u_j^2(a, t) da dt + k \int_0^A [p_j(a, T) - \bar{p}_j(a)]^2 da \right\}, \quad (4.1)$$

where k is the penalty parameter, (u, p) is subject to the system (2.1), and \bar{p} is the target.

As in the proof of Lemma 3.4, we can show the following result.

Lemma 4.1. *The solutions p^u to the system (2.1) are continuous in u .*

Without loss of generality, we in what follows assume that $m_1(x) = m_2(x) \equiv 0$.

Theorem 4.2. *Any optimal pair (u^*, p^*) of problem (P1) must be in the form $u_j^* = \mathcal{F}_j(k|q_j|)$, $j = 1, 2$, where the function \mathcal{F}_j is given by*

$$\mathcal{F}_j(x) = \begin{cases} 0, & x < 0, \\ U_j, & x > U_j, \\ x, & 0 \leq x \leq U_j, \end{cases}$$

and the adjoint variables q_j solve the system

$$\begin{aligned}
 \frac{\partial q_1}{\partial t} + \frac{\partial q_1}{\partial a} &= [\mu_1 + f_1(E(p_2^*))]q_1 - q_1(0, t)[\beta_1(a, E(p_1^*)) + \tilde{E}_1] + \bar{E}_2, \\
 \frac{\partial q_2}{\partial t} + \frac{\partial q_2}{\partial a} &= [\mu_2 + f_2(E(p_1^*))]q_2 - q_2(0, t)[\beta_2(a, E(p_2^*)) + \tilde{E}_2] + \bar{E}_1, \\
 q_j(a, T) &= \bar{p}_j(a) - p_j^*(a, T), \quad a \in (0, A), \quad j = 1, 2, \\
 q_j(A, t) &= 0, \quad t \in (0, T), \quad j = 1, 2, \\
 \tilde{E}_j(a, t) &= \int_0^a p_j^*(r, t) \frac{\partial \beta_j}{\partial x}(r, E(p_j^*)(r, t)) dr \\
 &\quad + \alpha \int_a^A p_j^*(r, t) \frac{\partial \beta_j}{\partial x}(r, E(p_j^*)(r, t)) dr, \\
 \bar{E}_i(a, t) &= \int_0^a [f'_i(E(p_j^*)) p_i^* q_i](r, t) dr + \alpha \int_a^A [f'_i(E(p_j^*)) p_i^* q_i](r, t) dr, \\
 &\quad i \neq j, \quad i, j = 1, 2,
 \end{aligned} \tag{4.2}$$

in which the variables (a, t) are omitted in the main equations.

Proof. Let (u^*, p^*) be an optimal pair for (P1). Then for any given $v \in \mathcal{T}_{\mathcal{U}}(u^*)$ (the tangent cone to \mathcal{U} at u^*), one infers that $u^* + \varepsilon v \in \mathcal{U}$ for ε small enough. Consequently $J(u^*) \leq J(u^* + \varepsilon v)$; that is,

$$\begin{aligned}
 &\sum_{j=1}^2 \int_Q u_j^{*2}(a, t) da dt + k \sum_{j=1}^2 \int_0^A [p_j^*(a, T) - \bar{p}_j(a)]^2 da \\
 &\leq \sum_{j=1}^2 \int_Q [u_j^*(a, t) + \varepsilon v_j(a, t)]^2 da dt + k \sum_{j=1}^2 \int_0^A [p_j^\varepsilon(a, T) - \bar{p}_j(a)]^2 da,
 \end{aligned} \tag{4.3}$$

where $p^\varepsilon = (p_1^\varepsilon, p_2^\varepsilon)$ denotes the solution of (2.1) corresponding to $u = u^* + \varepsilon v$. By Lemma 4.1, we derive that

$$\sum_{j=1}^2 \left\{ \int_Q (u_j^* v_j)(a, t) da dt + k \int_0^A z_j(a, T) [p_j^*(a, T) - \bar{p}_j(a)] da \right\} \geq 0, \tag{4.4}$$

where $z(a, t) = (z_1(a, t), z_2(a, t)) := \lim_{\varepsilon \rightarrow 0^+} \varepsilon^{-1} [p^\varepsilon(a, t) - p^*(a, t)]$ solves the following system (with (a, t) in the main equations omitted)

$$\begin{aligned}
 \frac{\partial z_1}{\partial t} + \frac{\partial z_1}{\partial a} &= -[\mu_1(a) + f_1(E(p_2^*))]z_1 - f'_1(E(p_2^*)) p_1^* E(z_2) + v_1, \\
 \frac{\partial z_2}{\partial t} + \frac{\partial z_2}{\partial a} &= -[\mu_2(a) + f_2(E(p_1^*))]z_2 - f'_2(E(p_1^*)) p_2^* E(z_1) + v_2, \\
 z_i(0, t) &= \int_0^A [p_i^* E(z_i) \frac{\partial \beta_i}{\partial x}(a, E(p_i^*)) + \beta_i(a, E(p_i^*)) z_i](a, t) da, \quad i = 1, 2, \\
 z_i(a, 0) &= 0, \quad a \in [0, A], \quad i = 1, 2.
 \end{aligned} \tag{4.5}$$

We note that, for given p^* , the existence of the limit $\lim_{\varepsilon \rightarrow 0^+} \varepsilon^{-1} [p^\varepsilon(a, t) - p^*(a, t)]$ and the well-posedness of linear system (4.5) can be treated by a standard way [3].

Multiplying the i -th equation in the system (4.5) with $q_i(a, t)$ and integrating over Q , we obtain that (with a use of (4.2))

$$\sum_{j=1}^2 \int_0^A z_j(a, T) [\bar{p}_j(a) - p_j^*(a, T)] da = \sum_{j=1}^2 \int_Q q_j(a, t) v_j(a, t) da dt. \quad (4.6)$$

Combining (4.6) with (4.4), we see that

$$\sum_{j=1}^2 \int_Q [(kq_j - u_j^*) v_j](a, t) da dt \leq 0 \quad (4.7)$$

holds for every $v \in \mathcal{T}_{\mathcal{U}}(u^*)$. Therefore, $(kq_1 - u_1^*, kq_2 - u_2^*) \in \mathcal{N}_{\mathcal{U}}(u^*)$, the normal cone to \mathcal{U} at u^* . A use of the characteristics of normal vectors derives the conclusion. \square

Theorem 4.3. *The optimal control problem (P1) has one and only one solution.*

Proof. Let $u^i = (u_1^i, u_2^i) \in \mathcal{U}$, $i = 1, 2$ be fixed and arbitrary, with $u^1 \neq u^2$. For $\varepsilon \in (0, 1)$, define the real-valued function

$$H(\varepsilon) := J(\varepsilon u^1 + (1 - \varepsilon) u^2).$$

Next we show that $H'(\varepsilon)$ is strictly monotone increasing.

Denote by p^ε and $p^{\varepsilon+\delta}$ (with $\delta > 0$ small enough) the solutions of (2.1) corresponding to $\varepsilon u^1 + (1 - \varepsilon) u^2$ and $(\varepsilon + \delta) u^1 + [1 - (\varepsilon + \delta)] u^2$, respectively. Then

$$\begin{aligned} H'(\varepsilon) &= \lim_{\delta \rightarrow 0} \frac{1}{\delta} \{ J[(\varepsilon + \delta) u^1 + (1 - (\varepsilon + \delta)) u^2] - J(\varepsilon u^1 + (1 - \varepsilon) u^2) \} \\ &= \lim_{\delta \rightarrow 0} \frac{1}{\delta} \left\{ \frac{1}{2} \sum_{j=1}^2 \int_Q [(\varepsilon + \delta) u_j^1 + (1 - (\varepsilon + \delta)) u_j^2]^2(a, t) da dt \right. \\ &\quad + \frac{k}{2} \sum_{j=1}^2 \int_0^A [p_j^{\varepsilon+\delta}(a, T) - \bar{p}_j(a)]^2 da \\ &\quad - \frac{1}{2} \sum_{j=1}^2 \int_Q [\varepsilon u_j^1 + (1 - \varepsilon) u_j^2]^2(a, t) da dt \\ &\quad \left. - \frac{k}{2} \sum_{j=1}^2 \int_0^A [p_j^\varepsilon(a, T) - \bar{p}_j(a)]^2 da \right\} \\ &= \sum_{j=1}^2 \int_Q [(\varepsilon u_j^1 + (1 - \varepsilon) u_j^2)(u_j^1 - u_j^2)](a, t) da dt \\ &\quad + k \sum_{j=1}^2 \int_0^A z_j^\varepsilon(a, T) [p_j^\varepsilon(a, T) - \bar{p}_j(a)] da, \end{aligned} \quad (4.8)$$

where $(z_1^\varepsilon, z_2^\varepsilon)$ is the solution of (4.5) corresponding to $u^* = \varepsilon u^1 + (1 - \varepsilon) u^2$. Treating in a similar manner as in the proof of (4.6), we have

$$\sum_{j=1}^2 \int_0^A z_j^\varepsilon(a, T) [p_j^\varepsilon(a, T) - \bar{p}_j(a)] da = \sum_{j=1}^2 \int_Q [(u_j^1 - u_j^2) p_j^\varepsilon q_j^\varepsilon](a, t) da dt, \quad (4.9)$$

where q^ε is solution of the system (4.2) corresponding to $u^* = \varepsilon u^1 + (1 - \varepsilon)u^2$ and $p^* = p^\varepsilon$. Combining (4.8) and (4.9) yields

$$H'(\varepsilon) = \sum_{j=1}^2 \int_Q \{(u_j^1 - u_j^2)[kp_j^\varepsilon q_j^\varepsilon + (\varepsilon u_j^1 + (1 - \varepsilon)u_j^2)]\}(a, t) da dt. \quad (4.10)$$

For any $\varepsilon_i \in (0, 1)$, $i = 1, 2$, with $\varepsilon_1 \neq \varepsilon_2$, we have

$$\begin{aligned} & (\varepsilon_1 - \varepsilon_2)[H'(\varepsilon_1) - H'(\varepsilon_2)] \\ &= (\varepsilon_1 - \varepsilon_2) \sum_{j=1}^2 \int_Q \{(u_j^1 - u_j^2)[kp_j^{\varepsilon_1} q_j^{\varepsilon_1} - kp_j^{\varepsilon_2} q_j^{\varepsilon_2} \\ &\quad + (\varepsilon_1 - \varepsilon_2)(u_j^1 - u_j^2)]\}(a, t) da dt \\ &= k(\varepsilon_1 - \varepsilon_2) \sum_{j=1}^2 \int_Q [(u_j^1 - u_j^2)(p_j^{\varepsilon_1} q_j^{\varepsilon_1} - p_j^{\varepsilon_2} q_j^{\varepsilon_2})](a, t) da dt \quad (4.11) \\ &\quad + (\varepsilon_1 - \varepsilon_2)^2 \sum_{j=1}^2 \int_Q (u_j^1 - u_j^2)^2(a, t) da dt \\ &=: \Delta + (\varepsilon_1 - \varepsilon_2)^2 \sum_{j=1}^2 \int_Q (u_j^1 - u_j^2)^2(a, t) da dt. \end{aligned}$$

From the continuity and bounded-ness of p and q (let the upper bounds be C_3 and C_4), it follows that

$$\begin{aligned} \Delta &= k(\varepsilon_1 - \varepsilon_2) \sum_{j=1}^2 \int_Q [(u_j^1 - u_j^2)(p_j^{\varepsilon_1} q_j^{\varepsilon_1} - p_j^{\varepsilon_2} q_j^{\varepsilon_2})](a, t) da dt \\ &\geq -k|\varepsilon_1 - \varepsilon_2| \sum_{j=1}^2 \int_Q [|u_j^1 - u_j^2| |p_j^{\varepsilon_1} q_j^{\varepsilon_1} - p_j^{\varepsilon_2} q_j^{\varepsilon_2}|](a, t) da dt \\ &= -k|\varepsilon_1 - \varepsilon_2| \sum_{j=1}^2 \int_Q [|u_j^1 - u_j^2| |p_j^{\varepsilon_1} (q_j^{\varepsilon_1} - q_j^{\varepsilon_2}) + q_j^{\varepsilon_2} (p_j^{\varepsilon_1} - p_j^{\varepsilon_2})|](a, t) da dt \\ &\geq -k|\varepsilon_1 - \varepsilon_2| \left\{ \sum_{j=1}^2 \int_Q [|u_j^1 - u_j^2| p_j^{\varepsilon_1} |q_j^{\varepsilon_1} - q_j^{\varepsilon_2}|](a, t) da dt \right. \\ &\quad \left. + \sum_{j=1}^2 \int_Q [|u_j^1 - u_j^2| q_j^{\varepsilon_2} |p_j^{\varepsilon_1} - p_j^{\varepsilon_2}|](a, t) da dt \right\} \\ &\geq -k|\varepsilon_1 - \varepsilon_2| \left\{ C_3 \sum_{j=1}^2 \left[\int_Q (u_j^1 - u_j^2)^2(a, t) da dt \int_Q (q_j^{\varepsilon_1} - q_j^{\varepsilon_2})^2(a, t) da dt \right]^{1/2} \right. \\ &\quad \left. + C_4 \sum_{j=1}^2 \left[\int_Q (u_j^1 - u_j^2)^2(a, t) da dt \int_Q (p_j^{\varepsilon_1} - p_j^{\varepsilon_2})^2(a, t) da dt \right]^{1/2} \right\} \\ &\geq -k(\varepsilon_1 - \varepsilon_2)^2 (C_3 \sqrt{C_2 T} + C_4 \sqrt{C_1 T}) \sum_{j=1}^2 \int_Q (u_j^1 - u_j^2)^2(a, t) da dt. \end{aligned}$$

Inserting the above expression into (4.11), we see that

$$\begin{aligned}
 &(\varepsilon_1 - \varepsilon_2)[H'(\varepsilon_1) - H'(\varepsilon_2)] \\
 &\geq (\varepsilon_1 - \varepsilon_2)^2[1 - k\sqrt{T}(C_3\sqrt{C_2} + C_4\sqrt{C_1})] \sum_{j=1}^2 \int_Q (u_j^1 - u_j^2)^2(a, t) da dt.
 \end{aligned}$$

If T is small enough, then $(\varepsilon_1 - \varepsilon_2)[H'(\varepsilon_1) - H'(\varepsilon_2)] > 0$. Therefore $H(\varepsilon)$ is strictly convex down, and there exists only one $\varepsilon^* \in (0, 1)$, which minimizes $H(\varepsilon)$. Consequently, the functional $J(u)$ will be minimized at $\varepsilon^*u^1 + (1 - \varepsilon^*)u^2$. The proof is complete. \square

5. STABILIZATION OF THE ZERO STATE

Firstly, we consider the stability of the zero solution to the system

$$\begin{aligned}
 \frac{\partial p_1}{\partial t} + \frac{\partial p_1}{\partial a} &= -[\mu_1(a) + f_1(E(p_2)(a, t))]p_1(a, t), \quad (a, t) \in Q, \\
 \frac{\partial p_2}{\partial t} + \frac{\partial p_2}{\partial a} &= -[\mu_2(a) + f_2(E(p_1)(a, t))]p_2(a, t), \quad (a, t) \in Q, \\
 p_i(0, t) &= \int_0^A \beta_i(a, E(p_i)(a, t))p_i(a, t)da, \quad t \in (0, T); i = 1, 2, \\
 p_i(a, 0) &= p_i^0(a), \quad a \in [0, A]; i = 1, 2, \\
 E(p_i)(a, t) &= \alpha \int_0^a p_i(r, t)dr + \int_a^A p_i(r, t)dr, \quad (a, t) \in Q.
 \end{aligned} \tag{5.1}$$

The linearization of this system about $(0, 0)$ is

$$\begin{aligned}
 \frac{\partial p_1}{\partial t} + \frac{\partial p_1}{\partial a} &= -[\mu_1(a) + f_1(0)]p_1(a, t), \quad (a, t) \in Q, \\
 \frac{\partial p_2}{\partial t} + \frac{\partial p_2}{\partial a} &= -[\mu_2(a) + f_2(0)]p_2(a, t), \quad (a, t) \in Q, \\
 p_i(0, t) &= \int_0^A \beta_i(a, 0)p_i(a, t)da, \quad t \in (0, T); i = 1, 2, \\
 p_i(a, 0) &= p_i^0(a), \quad a \in [0, A], i = 1, 2.
 \end{aligned} \tag{5.2}$$

Considering the solutions to (5.2) in the form $p_i(a, t) = w_i(a) \exp\{\lambda t\}$, $i = 1, 2$, we have

$$\lambda w_i(a) + w_i'(a) = -[\mu_i(a) + f_i(0)]w_i(a), \quad a \in (0, A), i = 1, 2. \tag{5.3}$$

Consequently,

$$w_i(a) = w_i(0) \exp \left\{ - \int_0^a [\lambda + \mu_i(r) + f_i(0)]dr \right\}. \tag{5.4}$$

Substituting this expression into the third equation of (5.2), we derive the characteristic equation

$$\int_0^A \beta_i(a, 0) \exp \left\{ - \int_0^a [\lambda + \mu_i(r) + f_i(0)]dr \right\} da = 1. \tag{5.5}$$

It can be readily seen that if there exists an $i \in \{1, 2\}$, such that

$$\int_0^A \beta_i(a, 0) \exp \left\{ - \int_0^a [\mu_i(r) + f_i(0)]dr \right\} da > 1, \tag{5.6}$$

then equation (5.5) has at least one positive real solution, which means the zero solution to (5.1) is unstable.

Next, we investigate the problem of how to keep the zero state stable by means of appropriate migration strategies. In a finite period $[0, T]$, its specific form is

(P2) Find $u^* \in \mathcal{U}$, such that $J(u^*) \leq J(u)$ for every $u \in \mathcal{U}$, with

$$J(u) = \frac{1}{2} \sum_{j=1}^2 \int_Q \{u_j^2(a, t) + kp_j^2(a, t)\} da dt,$$

and (u, p) is subject to (5.1), and k is the penalty parameter.

As in Theorem 4.2, we are able to describe the optimal policies as follows.

Theorem 5.1. *Problem (P2) has a unique solution. If (u^*, p^*) is an optimal pair to the problem, then there are variables (q_1, q_2) solving the adjoint system (with arguments (a, t) omitted in the main equations)*

$$\begin{aligned} \frac{\partial q_1}{\partial t} + \frac{\partial q_1}{\partial a} &= [\mu_1 + f_1(E(p_1^*))]q_1 - q_1(0, t)[\beta_1(a, E(p_1^*)) + \tilde{E}_1] + \bar{E}_2 + kp_1^*, \\ \frac{\partial q_2}{\partial t} + \frac{\partial q_2}{\partial a} &= [\mu_2 + f_2(E(p_2^*))]q_2 - q_2(0, t)[\beta_2(a, E(p_2^*)) + \tilde{E}_2] + \bar{E}_1 + kp_2^*, \\ q_j(a, T) &= q_j(A, t) = 0, \quad a \in (0, A), t \in (0, T), j = 1, 2, \end{aligned} \quad (5.7)$$

such that $u_i^* = \mathcal{F}(q_i), i = 1, 2$. Here, the functions $\mathcal{F}, \tilde{E}_i, \bar{E}_i$ are given by (4.2).

Concluding remarks. We have established the approximate controllability for the hierarchical competitive system (2.1) in Theorem 3.6, which means that there exist migrations (u_1, u_2) such that the populations distribution $(p_1(a, T), p_2(a, T))$ approaches arbitrarily to the prescribed target. With a proper selection of k , Theorems 4.2 and 4.3 demonstrate that the migrations can be uniquely determined by equations (2.1), (4.2) and the feedback control law in Theorem 4.2. Furthermore, the control policy may be approximated by means of the conjugate gradient algorithms [31, P. 29]. Therefore, we have sufficient theoretic and computational preparations for the population adjustment. A similar understanding applies to the stabilization of the trivial state.

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