EVALUATING THE EFFECTIVENESS OF EXPERIENTIAL LEARNING WITH
CONCRETE-REPRESENTATIONAL-ABSTRACT INSTRUCTIONAL
TECHNIQUE IN A COLLEGE STATISTICS AND
ALGEBRA COURSE

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EVALUATING THE EFFECTIVENESS OF EXPERIENTIAL LEARNING WITH
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ALGEBRA CLASS

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ABSTRACT

EVALUATING THE EFFECTIVENESS OF EXPERIENTIAL LEARNING WITH
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by

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A student underprepared for college-level mathematics and requiring college
algebra and statistics may need as many as three to six mathematics courses. College
algebra is an at-risk course since it has a drop, fail, and withdrawal (DFW) rate of 40-
60% (Adelman, 2004; Small, 2010). The study evaluated the effectiveness of a college
algebra class where statistics is integrated into its curriculum, denoted CA/S course. The
integration occurred through the author-developed experiential concrete-representational-
abstract (E-CRA) instructional method, a derivative of the Kolb experiential learning
model and the concrete-representational-abstract (CRA) instructional method (Kolb,
When learning new algebra and statistics topics, students brought in their own data or used real-world data, created and reflected on different representations of the data, learned the mathematics equations and functions that explain the data in the abstract stage, and then utilized mathematical modeling to further understand the data and use the model for prediction purposes. The variables, attitudes towards statistics, statistics self-efficacy, mathematics anxiety, algebra skills, statistical numeracy and reasoning skills, and persistence, were analyzed for the students in the CA/S course.

Students in CA/S course \((n=19)\) showed improved statistics self-efficacy, \((p=.002)\), and statistical numeracy and reasoning skills, \((p=.053)\), from the beginning to the end of the semester. When controlling for developmental mathematics, students in CA/S reported a stronger (i.e., practical significance \(p=.053\)) statistical self-efficacy than students in a comparable group who did not take CA/S. At the end of the study, CA/S students possessed two statistical numeracy and reasoning skills that were significantly stronger than the students in the comparable group \((p=.032, p=.006)\). No statistical significance was found for the other research variables. The research found that the CA/S curriculum utilizing the E-CRA instructional method yield positive outcomes related to the affective domain of the CA/S students. Students’ self-efficacy became significantly stronger, possibly contributing to the improvement of the students’ abilities to perform statistical numeracy and reasoning skills. Lesson plans are included as well as suggestions for implementation and recommendations for future study.
CHAPTER 1

INTRODUCTION

There is a national initiative to offer a developmental mathematics curriculum with more emphasis on statistical content, while reducing the number of developmental mathematics courses required in that curriculum (Texas Higher Education Coordinating Board [THECB], 2008). This initiative also includes a college algebra course with a statistics basis (Ganter & Barker, 2004; Gordon & Gordon, 2010). In mathematics learners who find themselves underprepared discover that statistics is ignored or omitted for the most part from developmental mathematics and college algebra syllabi (Steen, 2004). The integration of statistics into a mathematics course allows students to acquire those mathematical and statistical skills needed after their postsecondary education (Madison, 2003b). Another common problem is that many college students find that initial mathematics placement tests recommend one or more developmental mathematics courses before enrolling into their first credit-bearing mathematics course (Bailey, Jeong, & Cho, 2010). Research articles reported that college algebra has a national attrition rate of 40-60% (Adelman, 2004; Small, 2010).

Statistics, along with probability, have recently become instrumental strands of mathematics taught in secondary education mathematics along with the more common disciplines of algebra, geometry, and measurement as can be seen in the Texas College and Career Readiness Standards and Common Core Standards (Council of Chief State School Officers [CCSSO], 2009; THECB, 2008). In higher education statistics,
probability, geometry, and measurement are recommended to be included in the first 2 years of mathematics curriculum (American Mathematical Association of Two-Year Colleges [AMATYC], 1995). For numerous students not selecting a science-technology-mathematics-engineering (STEM) major, statistics has been integrated into their degree or certification plan because of the need to use statistics or quantitative reasoning in their college major (Steen, 2004). In fact, statistics can be found in the core requirements for many college majors more so than calculus, although not as a substitution. Thus, taking a statistics course, whether offered in the mathematics department or in another academic department, is an academic path followed by most postsecondary students, particularly those who are not as prepared for mathematics (Gordon & Gordon, 2010; Hassad, 2009; Onwuegbuzie & Wilson, 2003; Peck & Devore, 2008; Shaw, Jean, & Peck, 1997).

A college student is considered prepared for college mathematics upon completing the developmental mathematics course requirements that were individually prescribed to the student. In most cases this student finds that their college major requires that they take a statistics course. This same student is generally directed to take college algebra or a college-algebra equivalent as a prerequisite to their subsequent statistics course. The college-algebra equivalent courses can be business college algebra or liberal arts college algebra.

These students who became college-ready in mathematics and who now need statistics would benefit from a curriculum of mathematics that would also include another model for the college algebra course (Bressoud, 2010; Hull & Seeley, 2010; Johnson, 2004). For many students who became college-ready in mathematics, college algebra is found to be a collection of rules and procedures for manipulating symbols and numbers.
Many learners who were previously underprepared for college mathematics do not see the relevance of college algebra because they see no value or relevance from learning these mathematical rules (Johnson, 2004). Wlodkowski (1999) defines relevance as “the degree to which learners can identify their perspective and values in the course content, discussion, and methods of inquiry” (p. 112). Changing these students’ attitudes requires curricular changes that allow the use of other instructional techniques besides the traditional, lectured-based lessons still used in most college algebra classrooms. If a student perceives the relevance of college algebra, they might exhibit more positive attitudes and higher motivation (Wlodkowski, 1999; Wlodkowski, & Ginsberg, 1995). Wlodkowski (1999) wrote that as a student’s attitude improved, the student is more open to learning which can lead to higher success in achievement. Positive attitude and higher motivation result in a student being more amicable to learning mathematics and, therefore, successfully completing the college algebra course.

This chapter explores the need to integrate more statistics into the mathematics curriculum offered to students who are required to take a statistics course later in their academic career; these students who may or may not have completed a prescribed developmental mathematics program to become college-ready. The national movement towards this integration will be discussed along with why the current mathematics curriculum—which includes a college algebra course and employs lecture-based instruction—no longer meets the new demands of many postsecondary students. Thus, the justification and impetus for this study is established.

**Statement of the Problem**

Many college students find that they will be required to take several mathematics
courses when they begin their postsecondary academic career. This long chain of mathematics courses becomes tiring for many college students who often have difficulty seeing the relevance to their goals (Bressoud, 2010). This long list of mathematics courses can be intimidating to college students. Many students never complete these courses and, consequently, do not graduate from a postsecondary school (Bailey, Jeong, & Cho, 2010; Bressoud, 2010).

Many college students will be required to take statistics, which has a prerequisite of college algebra. Additionally, if a determination has been made that a student is underprepared for college-level mathematics, this student could be prescribed one to four developmental mathematics courses. Now the total number of mathematics courses has grown to a range of three to six courses for the student needing statistics for their college major. However, college algebra has a drop, fail, and withdrawal (DFW) rate of 40-60%. Consequently a student in a college algebra course has a 50% chance of repeating this course. Thus, the total number of mathematics courses needed by this student increases again.

For several decades a concern in the United States has been the growth of the postsecondary student population enrolled in developmental mathematics (Becker & Pence, 1994; Delta Project, 2008). In the past decade, national and state initiatives were established to address the need to prepare more high school graduates to be college-ready and prepared (Achieve, 2010; Mauer, 2005; THECB, 2000). These initiatives include new standards for mathematics in secondary education (CCSSO, 2009; THECB, 2008). Until the most effective initiatives and their respective standards are effectively in place throughout the United States, many high school graduates pursuing college will be placed
into developmental education courses, particularly developmental mathematics. But Cullinane and Treisman (2010) commented that “Prevention strategies alone will not solve the developmental mathematics strategies” (p. 4). There will always be students who enroll in developmental mathematics. Reasons why students will continue to require developmental mathematics are (a) college-readiness is defined by elite institutions but politics define high school graduation requirements, (b) there will always be those students who enroll in higher education after a lapse of time in their education, and (c) with the existence of open immigration in the United States, immigrants from foreign education systems will probably require some assistance from developmental education (Cullinane and Treisman, 2010).

For 2-year postsecondary institutions, the success rate of students referred to developmental mathematics and then completing gateway mathematics within 2 years of enrolling into college is 5% (Achieving the Dream, 2010). Many of these students will be required to take more than one developmental mathematics course in order to be prepared for a credit-bearing mathematics course. Several research projects found that as the number of required developmental mathematics courses increases, the probability that the student will drop out of the postsecondary school also increases (Bailey, 2009; Delta Project, 2008; Hern, 2010; Russel, 2008). This impact on attrition rates of postsecondary students occurs as a result of a multiplication principle (Hern, 2010). Hern explained the multiplication principle as the following:

Imagine you have 100 students who start the curriculum three courses below college-level, and imagine that 75% of this group passes the first course. That means 75 students are eligible for the next course in the sequence. Of course, not
all students who pass a course will enroll in the next course, so imagine that 75% of the eligible students persist to the next level. At the beginning of the second course, the pool of students has already shrunk from 100 to 56, and there are still three more semesters to complete. If your success and persistence rates stay at 75% for the rest of the sequence, only 13 of the original 100 students will pass the college-level course. (p. 3)

Still other studies found that participating in a developmental mathematics course has a positive impact on student retention, suggesting to policymakers that developmental education programs can be effective in helping to keep students enrolled in college (Johnson & Kuennen, 2004; Lesik, 2006; Waycaster, 2001).

Approximately 48% of college graduates are required to take statistical courses as part of the requirements of core courses associated with their majors (Schield, 2008). The task of preparing students for statistics while reducing the number of mathematics courses is a challenge being addressed by AMATYC and the Carnegie Foundation, as well as many other national organizations (Bressoud, 2010). Some of the national organizations concentrating on these issues are the following:

- Achieving the Dream: Community Colleges Count (ATD),
- the American Association of State Colleges and Universities (AASCU),
- the American Mathematical Association of Two-Year Colleges (AMATYC),
- Carnegie Foundation for the Advancement of Teaching, often referred to as the Carnegie Foundation,
- the Council of Chief State School Officers (CCSO),
- the College Reading & Learning Association (CLRA),
• the Mathematical Association of America (MAA),
• the Mathematicians and Education Reform (MER) Forum,
• the National Association of Developmental Education (NADE),
• Strong American Schools, and
• the Texas Higher Education Coordinating Board (THECB).

Established in 1974, AMATYC “is the only organization exclusively devoted to providing a national forum for the improvement of mathematics instruction in the first two years of college” (AMATYC, 2010, para. 1). The Carnegie Foundation is an organization charted by Congress in 1903 that is dedicated to teaching and learning. One of the Carnegie Foundation’s goals is “to double the proportion of students who, within one year of continuous community college enrollment, are mathematically prepared to succeed in further academic study and/or occupational pursuits” (Carnegie Foundation for the Advancement of Teaching, 2010, para. 8).

The AMATYC publication, Beyond Crossroads: Implementing Mathematical Standards in the First Two Years of College, comments, “The mathematics needed for successful careers and responsible citizenship continues to change” (Blair, 2006, p. 37). Society has growing demands for the use of statistical data collection and analysis in many everyday functions (Steen, 2004). Employees must possess skills with higher quantitative numeracy, quantitative reasoning, statistical numeracy, and statistical reasoning. For this reason, developmental mathematics and credit-bearing mathematics curriculum, required by non-STEM majors, needs to be expanded to include more statistics (Cullinane & Treisman, 2010; Hern, 2010).

In general, a traditional college algebra course is a part of the one-curriculum-fits-all
approach (Fischer, 2004). As stated earlier, college algebra has a high attrition rate where half of the students repeat the course. To help solve the problem, the college algebra course needs to provide relevance for those students not pursuing the calculus track, but requiring statistics. The MER Forum has objectives to reach out to more students and to expand the mathematics curriculum (Fisher, 2004). This organization also “envisages the pursuit of education reform through informed discussion of educational issues, thoughtful responses changing education conditions, and the promotion of exemplary program” (MER Forum, 2001, para. 1). As Steen (2004) has stated,

Some mathematicians and scientists assert that algebra is the gateway to higher mathematics, but this is so only because our curriculum makes it so. Much of mathematics can be learned and understood via geometry, or data, or spreadsheets, or software packages. (p. 59)

In a traditional college algebra course students are taught using the lecture modality containing an intense instruction of definitions, rules, and algorithmic procedures of formulas (Gordon, 2008; Small, 2010). MAA’s Committee on the Undergraduate Program in Mathematics (CUPM) stated that “Mathematics classes based primarily on lectures may discourage student interest and minimize opportunities for students to develop mathematical understanding” (Ganter & Barker, 2004, p. 6). Consequently, the use of hands-on activities, active learning, and representations could help students retain mathematical concepts after the completion of the course.

**Theoretical Framework**

The integration of the statistical content into the college algebra course can be taught using the concrete-representative-abstract (CRA) instructional technique and the Kolb experiential learning model (ELM). Other best practices that appeal to different
learners could be included in the lesson plans to help deliver these curricular changes needed for this course.

**CRA.** CRA is a three-stage learning process whereby students learn mathematics through physical manipulation of concrete objects, followed by learning through pictorial representations of the concrete manipulations, and ending with solving problems using abstract notation (Witzel, 2005). Figure 1 shows the three cycles of CRA. If the student is given the opportunity to develop a concrete understanding of a mathematical topic, then the student will be more successful gaining the abstract understanding.

![Concrete-Representational-Abstract (CRA) instructional technique.](image)

The effectiveness of CRA has been researched in many studies (Allsopp, 1999; Jordan, Miller, & Mercer, 1998; Paulsen & the IRIS Center; 2006; Harris, Miller, & Mercer, 1995). In regards to the National Council of Teachers of Mathematics (NCTM)
Process Standards, one researcher noted, “The CRA instructional sequence becomes a valuable intervention for students with LD (learning disabilities) to learn the NCTM process standards of problem solving, reasoning and proof, communications, connections, and representations” (Anstrom, 2006). NCTM suggests that the CRA could be beneficial to all students (Berkas & Pattison, 2007).

CRA is used primarily in elementary education. Steen, Brooks, and Lyon (2006) compared the academic achievement of a group of first grade students who used virtual manipulatives for practice in geometry instruction. These first grade students’ performance improved significantly. Another study compared the use of concrete and virtual manipulatives in third grade students studying algebraic thinking (Suh & Moyer, 2007). Both types of manipulatives were associated with higher achievement and increased flexibility in representing algebraic concepts.

In secondary education the sequential instruction of word problems from concrete, semi-concrete, and abstract is promising for algebra students who have or have no learning disabilities (Maccini, McNaughton, & Ruhl, 1999). In another CRA study, six adolescents with learning disabilities used algebra tiles to represent algebra word problems during the concrete cycle of CRA. These students were able to progress through all three cycles and successfully use the symbols of algebra (Maccini & Hughes, 2000). Another study found CRA was effective when teaching how to solve algebraic equations for sixth and seventh graders with disabilities (Witzel, Mercer, & Miller, 2003). One research study compared CRA to the representational-abstract (RA) instructional method when teaching the concept of fractions to middle school students with disabilities. Student performance improved with both the CRA and RA techniques, but
the CRA group outperformed the RA group (Butler, Miller, Crehan, Babbitt, & Pierce, 2003).

Some of the controversy associated with CRA is that secondary education teachers feel that CRA is primarily effective for elementary education (Witzel, 2003). CRA works well in small groups and for individual work, but its effectiveness in whole-class settings is unknown (Witzel, 2003). Also, the instructor cannot move to the next stage in CRA prematurely. The NCTM recommends that when using CRA teachers should make sure that students understand what has been taught at each step before moving instruction to the next stage (Berkas & Pattison, 2007).

**Kolb ELM.** According to David Kolb (1984), "*Learning is the process whereby knowledge is created through the transformation of experience* [author's italics]" (p. 38). Introduced in the 1970s by Kolb, this learning model contains four dimensions which include (a) concrete experience, (b) reflective observation, (c) abstract conceptualization, and (d) active experimentation (Kolb, 1984; Wlodkowski & Ginsberg, 1995). Figure 2 depicts the cycle associated with the Kolb ELM. In this ELM, learners observe and feel the world around them in the concrete experience dimension. The reflective observation stage is described as occurring when the learner makes sense of the concrete experience on a personal level. Abstract conceptualization allows learners to think and generate new concepts, understandings, and strategies for action. Learners test and model these new concepts and understandings in different situations in the active experimentation stage. The cycle then begins again when the learner completes the active experimentation stage which leads to the next concrete experience (Kolb, 1984). The Kolb ELM could be the learning model most applicable to learning mathematics (Knisley, 2002).
Each dimension of the Kolb ELM can represent a learning style and a preferred entry point for learning (Wlodkowsky & Gingsberg, 1995). To learn a particular topic does not necessarily imply that one has to progress through all four stages. Kinesthetic learners may want to start at the concrete stage, whereas graduate students may prefer to enter at the abstract stage. Visual and verbal learners may prefer entering in at the reflective stage. All students benefit from participating in all four stages (Harb, Durrant, & Terry, 1993; Kolb, 1984). This flexibility of multiple entry points allows the Kolb ELM to provide a theoretical configuration for selecting and organizing learning activities (Smith & Kolb, 1986).

The Kolb ELM describes students’ learning styles which are determined by two factors, (a) whether students prefer the concrete to the abstract and (b) whether students prefer active experimentation to reflective observation (Knisley, 2002; Kolb, 1984).
These learning style preferences result in a classification scheme with four learning styles. The concrete and reflective style is comprised of those learners who build on previous experience. The second learner style is concrete and active for those students who learn by trial and error. Abstract and reflective is the learning style for those learners who learn from detailed explanations. The fourth learning style abstract and active is composed of students who learn by developing individual strategies (Felder, 1993; Hartman 1995; Kolb, 1984).

The Kolb ELM centralizes the importance of experience in learning and flexibly accommodates some of the important culturally derived differences among learners (Wlodkowsky & Gingsberg, 1995). “For many so-called nontraditional students—minorities, the poor, and mature adults-experiential learning has become the method of choice for learning and personal development” (Kolb, 1984, p. 3). The Kolb ELM encourages active and passive learning. Kolb’s experiential learning model presents a way of structuring and sequencing the curriculum and indicates in particular how a lesson plan or a whole course may be taught to improve student learning (Healey & Jenkins, 2000).

The Kolb ELM has many supporters. Many researchers are discussing, utilizing, and explaining learning styles and instructional methods referencing Kolb’s ELM (Anderson & Adams, 1992; Knisley, 2002; Muro & Terry, 2007; Svinicki & Dixon, 1987; Wlodkowski, 1999; Wlodkowski & Ginsberg, 1995).

Although many researchers are support the Kolb ELM, Kolb has critics. First there is controversy over how John Dewey perceives experiential learning as compared to Kolb. Dewey wrote that observations of reality and nature were the starting point of
knowledge acquisition (Dewey, 1938). Kolb stated that experience is the starting point of knowledge acquisition (Kolb, 1984). Dewey found that observation, the non-reflective experience, is the dominant experience in learning and the reflective experience is result of contradiction (Dewey, 1938). Kolb did not address non-reflective experience or observation (Boud, Keogh, & Walker, 1985; Oxendine, Robinson, & Willson, 2004). Dewey found in ELM that in relation to reflection a number of processes can occur at once, and stages can be jumped (Dewey, 1933). Kolb described ELM with a cycle of four stages (Kolb, 1984).

Other controversies were that the claims of four different learning styles in the Kolb ELM are not accepted by some educators (Jarvis, 1995; Tennant, 1997). The four different learning styles seem to match up to the four stages of the ELM, but this matching does not necessarily validate them (Smith, 2001). One researcher disagrees with the claims that these styles can be measured reliably using currently available instruments, and that tailoring instruction to match these styles improves classroom learning performance (Marion, 2002). Other critics of the Kolb ELM argued that the concrete experience is not fully explained (Herron, 2002). Educators found that the KOLB ELM ignores observations concerning the subjective reality of the learner (Miettinen, 2000).

**Research-based best practices to be used with CRA and ELM.** The utilization of modeling, problem-based learning, using technology, implementing group work, an employing other instructional techniques are useful in a mathematics class. Johnson (2004) used mathematical modeling and statistics for non-mathematics and non-science majors in a new entry-level mathematics course. This course teaches how to analyze and
model data through the use of a graphing calculator. A drop from 45% to 25% of DFWs was reported for the new course as compared to college algebra, respectively. After teaching this course for three semesters, no students dropped out of the course as compared to an 8% drop rate in college algebra during same timeframe (Johnson, 2004). Survey results showed higher positive attitudes from students who are at-risk students. Johnson (2004) determined the skills needed by these students are the ability to read and interpret graphs, the ability to work with real world data, the ability to use mathematics to model data, and the ability to make predictions using the models.

Many research studies have reported that institutions have incorporated the successful use of technologies into mathematics classes to allow the student to do more investigations (Carter, 2004; Cohen, 1995; Forster, 2006; Mahmood, 2006; Mayes, Chase, Walker, 2008; Taylor, 2004; THECB, 2008). AMATYC, THECB, CCSSO, and NCTM are among many organizations that recommend the use of technology as an effective instructional practice (Cohen, 1995; CCSSO, 2009; NCTM, 2000; THECB, 2008).

A college algebra reform movement, investigated by one study, organized students work into small cooperative groups to discuss concepts (Jones & Balas, 2000). The goal was to allow students to construct their own understanding. The research project used real-life problems as applications in the class. An improvement in attitudes towards in mathematics was reported by the researchers (Jones & Balas, 2000). Statistics educators have found that using real-world applications and allowing students to work in groups to construct their own knowledge from their own data is a successful method in statistics courses (Moore, Peck, & Rossman, 2004). The Moore, Peck, and Rossman
(2004) study shows how the experiential aspect of ELM where students’ use their own data or real-world data that is relevant can be beneficial to students.

**Concept for the Proposed Solution**

As new topics were introduced in the college statistics and algebra course, the students in this course progressed through the four dimensions of experiential learning. The following steps describe how new mathematical concepts were introduced in the curriculum of this course:

1. Step 1. Students should obtain and bring their own data to class,
2. Step 2. Students graph, interpret, and analyze their data,
3. Step 3. Students discover which mathematical functions best describe their data, and
4. Step 4. Lastly, students model the data using mathematical models.

It should be noted that not all lesson plans started at Step 1. New statistical or mathematical topics could have been introduced starting with Step 2 or Step 3.

The fact that the students were asked to bring in their own data gives the curriculum an experiential aspect. When a student brings in their own data to class, an opportunity arises that allows students to create their own personal experiences that initiate students’ own processes of learning (Kolb, 1984). These four steps were taught in the course using the concrete-representational-abstract (CRA) instructional method. The CRA sequence of instruction provided a graduated and conceptually supported framework for students to create a meaningful connection among concrete, representational, and abstract levels of understanding (The Access Center, n. d.). The concrete stage of the CRA method maps to the first step of teaching a new mathematical
topic and the first dimension of the experiential learning model. The representational component of CRA maps to the second step of teaching a new mathematical topic and is an additive factor to the second dimension of the experiential learning model. The abstract component of CRA maps to the third step of teaching a new mathematical topic and the third dimension of the experiential learning model. When using Step 4 to teach a new mathematical topic, this step maps to the fourth dimension of the experiential learning model. For the purposes of this research project, these series of four steps will be called the experiential concrete-representational-abstract (E-CRA) instructional method used to teach the content of this course. With the combination of experiential learning and CRA, the postsecondary student will be in a position to be successful in the course. Figure 3 describes these four steps and provides examples of each step. Though research-based instructional methods have been implemented in some developmental mathematics and college algebra courses, experiential learning along with CRA is not commonly incorporated into the college algebra course.

**Proposed Solution**

The new course which included both statistics and college algebra in its curriculum was created using the E-CRA instructional model. A diagram of the instructional framework E-CRA can be found in Figure 4. Figure 4 shows how a student transitions from developmental mathematics to college algebra. The curriculum for college algebra with statistics included in the course then prepares the student for the subsequent statistics course. Appendix A contains the lesson plan template that was used for lessons associated with this course. The scope and sequence and the lesson plans for this course can be found in Appendix B and Appendix C, respectively.
**Experiential-concrete** - Students collect and bring in their own raw data defined by specific criteria to class. The data were analyzed, and in groups students compared their data with group members. This collection activity gives the students an opportunity to select data of interest to them. Also, this activity creates real-time culturally responsive opportunities for the students while doing class work.

Exponential Functions: Students use their own data that the instructor has verified is exponential, or the students could use the “history of the growth of cell phone subscribers” data provided in the lesson plan. The cell phone subscribers data is culturally relevant to students since most, if not all, of the students have a cell phone. Also, a brief history of cell phones was discussed. If students bring in their own data, they need to be ready to describe their data to their group similarly.

**Representational** - Students create multiple representations of the data, for example tables and graphs. In groups students discuss the data and look for characteristics of functions that had been previously studied in the class.

Transformations of Functions Using an Application Model: Using data previously analyzed and modeled in the class, students add or subtract values to the independent data in the table then graph the data. Students modify the dependent variable, then graph and model the data. Students discover the graph looks similar, but stretched or compressed.

**Abstract** – Instructor teaches new mathematical concept to students as explanation of new data characteristics and new graphical representations.

Logarithmic Functions: Students learn that logarithmic function is the inverse function for the exponential function. Using inverse properties, students find domain and range of logarithmic function. Students learn Properties of Logarithms.

**Mathematical Modeling** – From the mathematical topic taught in the abstract stage, students create and run mathematical models of their data and made inferences from the models.

Quadratic Equations: Students run quadratic regression on data that had been modeled in a linear regression in a previous lesson plan. Students compare results of the linear regression model to the results of the quadratic regression and make a determination of which model is stronger.

*Figure 3:* Experiential Concrete-Representational-Abstract (E-CRA) four-phase instructional technique.
The diagram in Figure 4 of the E-CRA model has a rectangle labeled “Research-based Instructional Techniques” which has arrows leading from this rectangle to each individual lesson plan. This rectangle and its arrows depict that these instructional techniques are considered and possibly used in a lesson plan. These research-based instructional techniques were discussed earlier in this chapter.

Another rectangle labeled “Standards” can be found in the diagram in Figure 4. It also has arrows leading from this rectangle to each individual lesson plan. The standards are used and/or referenced as inputs to the development of each individual lesson plan. Academic standards established by national and state organizations was another force driving the content and pedagogical strategies of each lesson plan. Standards referenced are the following:

- AMATYC’s standards found in the publication *Beyond Crossroads: Implementing Mathematical Standards in the First Two Years of College* (Blair, 2006),

- Curriculum Renewal across the First Two Years (CRAFTY) subcommittee recommendations (CRAFTY, 2007),

- Guidelines for Assessment and Instruction in Statistics Education (GAISE; Aliaga, Cuff, Garfield, Lock, Utts, & Witmer, 2010),

- Texas College and Career Readiness Standards (CCRS; THECB, 2008),

- Common Core State Standards (CCSSO, 2009), and

- National Council of Teachers of Mathematics (NCTM) standards (NCTM, 2000).
The E-CRA diagram in Figure 4 has rectangles labeled “Lesson Plan” which have a backwards arrow with the label of “Background knowledge”. This symbol represents that most, if not all, lesson plans reflect on prior content that has been discussed in the course or in prerequisite mathematics courses. In the “Lesson Plan” rectangle is another symbol labeled “Foreshadowing Content”. This symbol represents how a lesson plan hints of the upcoming content to be covered in the next lesson plan.

Each lesson plan had content objectives and other objectives, as marked by the
rectangle labeled, “Additional Lesson Plan Objectives”. Some of other objectives of the lesson plans are that (a) the students experience cultural responsiveness, (b) the students use technology, (c) students engage in real-world contextual scenarios, and (d) the students are exposed to process standards. Figure 5 shows when experiential learning and the four steps or stages are mapped to the lesson plans for this college statistics and algebra course.

With each major college algebra topic, students progressed through a cycle of learning activities. Found in Appendix C-16, the Exponential Functions Lesson Plan contains many of these instructional techniques. In Step 1, from a PBL-perspective, students were assigned to collect and bring in their own data to class that met a specific criteria. The data were analyzed, and in groups students compared their data with group members. This collection activity gave the students an opportunity to select data that were of interest to them. This part of the curriculum also created continuous, real-time culturally responsive opportunities. The raw data collection was the experiential-concrete portion of the curriculum. In the Exponential Functions Lesson Plan students were allowed to use their own data that the instructor had verified was exponential, or the students could use the “history of the growth of cell phone subscribers” data provided in the lesson plan. The cell phone subscribers data was culturally relevant to students since most, if not all, have a cell phone. Also, a brief history of cell phones was discussed. If students bring in their own data, they need to be ready to describe their data to their group similarly.
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<th>Lesson Title</th>
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<th>Reflective Observation or Representational</th>
<th>Abstract</th>
<th>Active Experimentation or Math Modeling</th>
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*Figure 5: Map of Lesson Plans to E-CRA Model.*
Step 2 involved reflecting on the raw data by using group work activities. Common questions that the students explored were “What do the distribution of the data look like?” “What are the centers?” and “What are the outliers?” As the instructor guided the students to create multiple representations of the data, such as tables and graphs, the representational side of CRA was addressed. Students graphed the data, reflected on the data, and then discussed the data to see if it looked like functions that had been previously studied in the class. In this part of the lesson the students utilized their background knowledge on linear and quadratic functions.

Next in Step 3, the students were guided to analyze the data and look for patterns. The students were asked to make a determination on whether the data collected looked or behaved like a familiar parent function that has been previously studied in class. The data may or may not look like a familiar parent function. The discussion of functions and the underlying algebraic-content was the abstract portion of the curriculum. In the Exponential Functions Lesson Plan, the students now ran linear and quadratic regressions to see which model is the better fit. The worksheets from the Exponential Functions Lesson Plan gave the students historical data that provided points on the rectangular coordinate plane that were located to the left of the y-axis. The students were asked to anticipate what the data and the graph would look like in Quadrant 3. Is the data linear? Or quadratic? Or something else? Bringing in the abstract, the lesson then introduces the topic of the exponential function and its characteristics.

The 4th step included using modeling instructional techniques when studying the functions. Students created models of their data and made inferences from the models. The students examined the impact of manipulating their data. The changes to the data
affected the spread and the centers of the data. These data changes could also produce transformations and stretches of the functions. Thus, functions were taught with the goal that students can obtain an understanding of a function by examining, analyzing, interpreting, and changing data with respect to the function. In the Exponential Functions Lesson Plan, students had not reached the 4th step for exploring exponential functions. The next lesson plan, Exponential Application Functions Lesson Plan (Appendix C-17) will cover the 4th step. At this stage, the students run mathematical models of their data and discuss the results of the exponential regression as compared to the linear and quadratic regression models.

As a result of studying these data and the underlying mathematics, students have access to more relevant mathematical and statistical content. Wlodkowski (1999) wrote that motivation and learning are by-products of relevance and positive attitudes. If the college algebra course possesses more relevance and student-centered instruction, students’ attitudes towards statistics and mathematics could possibly improve. When students have positive attitudes, then learning can occur and attrition rates can decline. The attrition rates will improve as more students complete the course without having to repeat the course. With these courses serving as the foundation of students’ mathematics education, students’ statistical literacy and reasoning skills will be improved which allows students to be more prepared for their upcoming statistics course (Wlodkowski, 1999).

**Purpose of Study**

The purpose of this study was to evaluate the effectiveness of the E-CRA instructional method, when integrating statistics into a college algebra course (a) for
those students who were required to and then subsequently completed developmental mathematics and (b) for those students who were designated college-ready without the need for developmental mathematics. Both sets of students were required to complete a college algebra course and a statistics course. The effectiveness was examined by evaluating the following changes in the affective domain as well as performance of the students in the college algebra with statistics course. In the affective domain, the variables measured were (a) the variations of attitudes towards statistics of the students, (b) the changes of statistical self-efficacy of the students, and (c) the influence on mathematics anxiety felt by the students. Changes in performance were measured by examining (a) the changes in student abilities for college algebra skills as measured by a college mathematics placement test, (b) the differences to statistical numeracy and reasoning skills of the students as measured by an instrument designed to measure these skills, and (c) the attrition rate of this college algebra course.

**Significance of the Study**

E-CRA was utilized in order to successfully integrate statistics in the college algebra curriculum. This research project contributes to existing knowledge and provides new knowledge from several aspects. There are few studies on integrating statistics into a college algebra course. Minimal research exists on the effects resulting from using the CRA instructional method for students in postsecondary mathematics. There is a void in the literature to use experiential learning with the CRA instructional method to accomplish this integration of statistics into college algebra. The study utilized at least four best practices such as modeling, PBL, group work, and technology. But, what is unique is the multiculturalism aspect of the lesson plans. The experiential learning
perspective allowed continuous, real-time cultural responsiveness that resulted from students collecting, analyzing, discussing, and modeling their own data.

**Research Questions**

The following were the research questions investigated by this study. The college algebra course with statistics is labeled CA/S.

1. Does the use of the E-CRA instructional method in a CA/S course produce changes in students’ affective domain as measured by pretest and posttest scores while controlling for whether or not the student took developmental mathematics courses?

2. Does the use of the E-CRA instructional method in a CA/S course produce different outcomes in student performance as measured by pretest and posttest scores while controlling for whether or not the student took developmental mathematics courses?

**Null Hypotheses**

The following were the null hypotheses tested by this study. For the first research question regarding the affective domain, the following three null hypotheses were analyzed.

1. The null hypothesis associated with attitudes towards statistics was the following. While controlling for whether or not developmental mathematics courses were previously taken by the students in the CA/S course, there were no statistical differences in the means for attitudes towards statistics in their field of study and the statistics course in which they are currently enrolled at the beginning of the semester as compared to the end of the semester.

2. While controlling for whether or not developmental mathematics courses were
previously taken by the students in the CA/S course, there is no statistical difference in the means for statistical self-efficacy at the beginning of the semester as compared to the end of the semester.

3. While controlling for whether or not developmental mathematics courses were previously taken by the students in the CA/S course, there is no statistical difference in the means for mathematics anxiety at the beginning of the semester as compared to the end of the semester.

For the second research question regarding student performance, the following four null hypotheses were analyzed.

1. While controlling for whether or not developmental mathematics courses were previously taken by the students in the CA/S course, there is no statistical difference of the means of college algebra skills scores at the beginning of the semester as compared to end of the semester.

2. While controlling for whether or not developmental mathematics courses were previously taken by the students in the CA/S course, there is no statistical difference of the means of the “statistical numeracy and reasoning skills” scores at the beginning of the semester as compared to end of the semester.

3. While controlling for whether or not developmental mathematics courses were previously taken by the students in the CA/S course, there is no statistical difference in the means of “statistical numeracy and reasoning skills” scores for the students in the CA/S course at the end of the semester as compared to students at the beginning of an introductory statistics course who did not take CA/S.

4. While controlling for whether or not developmental mathematics courses were
previously taken by the students in the CA/S course, there is no statistical difference of the means of the attrition rates of the CA/S course as compared to a traditional college algebra course.

**Summary**

Many postsecondary students are at-risk of not completing their postsecondary education because of their placement into developmental mathematics courses. If these students are able to successfully fulfill their developmental mathematics requirements, they usually enroll into college algebra as their first credit-bearing mathematics course. These students again find themselves in an at-risk situation because college algebra has a 40-60% DFW rate.

This chapter established the overriding need to research the use of the E-CRA instructional method to integrate statistical content into college algebra course. The study determined the impact of this curricular change to a college algebra course using the E-CRA instructional method that included research-based instructional methods such as modeling, problem-based learning, using technology, and implementing group work. The second chapter includes definitions and the background on related literature for this study. Chapter 3 describes the methods used for this research project and includes the population and sampling, research design, instrumentation, procedures, analysis plan, and limitations and assumptions. Chapter 4 contains the results of the analysis performed on the data collected from the study. Chapter 5 presents the summary of the findings, a discussion or explanation of the findings, and the recommendations based on the findings of the study.
CHAPTER 2

LITERATURE REVIEW

This section begins by elaborating on instrumental definitions associated with this research project. Next, the findings and recommendations from national and state organizations are presented. A discussion of current initiatives addressing the recommendations from these national and state organizations is provided. A literature review of the variables used to measure the research questions is provided. Interventions employed to improve college algebra are presented at the end of this section.

Students who are Underprepared for College

The definition of a postsecondary student who is considered underprepared may vary from a contextual perspective. These students are generally placed into developmental education courses at a postsecondary institution. Commenting on the definition of this specific population of college students, Arendale (2010) states,

Actually, no such thing as a ‘developmental student’ exists. Rather, it is more accurate to say that some students are not academically prepared for college-level work in one or more academic content areas (English, mathematics, or writing) or in specific skills such as reading or study strategies. (p. 2)

College academic advisors have noted that prior educational experiences such as academic failure, poor preparation, and low expectations are common factors related to this population of students who are underprepared for college (Miller & Murray, 2005).
The number of students that find themselves underprepared is growing as Tritelli (2003) commented in his editorial for the American Association of College and Universities (AAC&U); “53% of students entering our colleges and universities are academically underprepared, i.e., lacking basic skills in at least one of the three basic areas of reading, writing or mathematics” (para. 1). One problem in the definition of a college student who is underprepared is that what constitutes college-level work and placement into developmental coursework varies among postsecondary institutions (Russel, 2008). At this time, individual states as well as higher education institutions select their own placement tests, determine cutoff scores, and set policies as to whether or not developmental coursework is mandatory or voluntary. As a result Arendale (2008) writes, “Academic preparedness is not a characteristic of the student; rather, it is a condition relative to a particular academic course during the same academic term” (p. 8). Research has revealed that even the most motivated high schoolers find themselves unprepared for university-level academics (Delta Project, 2008).

These students belong to many different demographics and have various for being underprepared for college work. Students in this group may belong to previously underrepresented groups that are inclined not to be academically prepared in the past (Arendale, 2010; Cohen, 1995). These students’ families have a low-income or low social economic status (Arendale, 2010; Cohen, 1995; Delta Project, 2008; Gabriel, 2008; McCabe, 2003; Rouche & Rouche, 1999). National Center for Educational Studies (NCES) longitudinal studies show that students in the lowest socioeconomic status quintile are far more likely to enroll in remedial education than those in the highest
quintile, 63 percent compared to 25 percent respectively (Russel, 2008). These students may belong to a racial/ethnic minority (Arendale, 2010; Cohen, 1995; Delta Project, 2008; Gabriel, 2008; McCabe, 2003). These students are enrolling or re-enrolling into a postsecondary institution after an interruption in their academic studies of several years (Arendale, 2010; Cohen, 1995). These students could be first-generation students who have no family history in postsecondary education (Arendale, 2010; Cohen, 1995; Delta Project, 2008). For recent high school graduates, the level of high school coursework is a primary factor for determining if a new postsecondary student will take a developmental mathematics course(s) (Cohen, 1995; Delta Project, 2008; Gabriel, 2008; McCabe, 2003). It is not unusual to find that high school graduates who took only three years of high school mathematics are placed into developmental mathematics courses (Delta Project, 2008). These students who find themselves underprepared for college may have disabilities that require special accommodations (Cohen, 1995; McCabe, 2003). English as a second language is another factor that places students into this category of being underprepared for college (Cohen, 1995; McCabe, 2003).

**Students who are Underprepared for Mathematics**

According to American Mathematical Association of Two-Year Colleges (AMATYC), students entering the higher education environment have different backgrounds in regards to their preparation for college mathematics courses (Cohen, 1995). Though many students do graduate from high school ready to pursue calculus and higher-level mathematics courses, there have been no percentage gains in this group of students over the last couple of decades (Cohen, 1995; Lutzer, Maxwell, & Rodi, 2002; Small, 2010). More students are entering postsecondary education at a mathematics level
below calculus. The majority of the students entering below calculus are placed into college algebra or developmental mathematics (Small 2010; Stigler, Givvin, & Thompson, 2010).

For most postsecondary institutions, students who are placed into developmental mathematics courses are considered underprepared for that institution’s entry-level, college-credited mathematics course. It should be noted that in some institutions college algebra is not a credit-bearing course. In other institutions, all mathematics courses below calculus are not credit-bearing. As Arendale (2008) found, whether or not a student is college ready is conditionally determined by the postsecondary institution the students enrolls. Thus, depending on the higher education school, college algebra could be considered a developmental mathematics course.

**Developmental Mathematics**

In an article written by Vásquez-Mireles and Morales (2006), various definitions of developmental mathematics were detailed as viewed from different perspectives. Vásquez-Mireles and Morales reported developmental mathematics is defined by one or a combination of the following:

- Relationship to other mathematics courses, e.g., below college algebra;
- Topics taught, e.g., operations with whole numbers through quadratic equations; and
- Standardized exam, e.g., the Texas Higher Education Assessment (THEA). (p. 7)

As of the end of 2010, federal and state governments have no definition that is shared across all entities. State governing boards commonly list topics as a means to define
developmental mathematics. The most recognized definition used by both National Association of Developmental Education (NADE) and the College Reading & Learning Association (CRLA) is:

1: pre-collegiate mathematics courses that are designed to prepare students for the study of college-level mathematics, as defined by entrance requirements of the institution. The levels of developmental mathematics courses vary from basic arithmetic through any prerequisite course(s) for calculus. 2: instruction that may contain one or more of the following topics: arithmetic operations, math symbolism, geometry and measurement, functions, discrete math algorithms, probability and statistics, and deductive proofs. 3: specialized mathematics instruction for students who do not meet entry into a college-level mathematics course. (Arendale, 2007, p. 18)

**College Readiness - General**

Conley (2010) defines college and career readiness as “the level of preparation a student needs in order to enroll and succeed—without remediation—in a credit-bearing course at a postsecondary institution that offers a baccalaureate degree or transfer to a baccalaureate program, or in a high-quality certificate program that enables students to enter a career pathway with potential future advancement” (p. 21). Conley advocates a four-dimensional model of components that students must possess in order to be college-ready and able to successfully navigate their postsecondary experience successfully. The four-dimensions follow:

1. Key cognitive strategies are intentional behaviors students must be able to employ over time and in a variety of situations that allow content to be learned by
students by understanding, retaining, using, and applying content from a range of disciplines. These skills are meta-cognitive skills for knowing content.

2. Key content knowledge is understood and mastered by processing and applying information by means of the key cognitive strategies.

3. Academic behaviors or self-management “require greater self-awareness, self-monitoring and self-control of a series of processes and behaviors necessary for academic success” (p. 39). It should be noted that these are meta-cognitive skills that are unlike key cognitive skills because they are independent of content. Examples of academic behaviors in this dimension are reflection of a content topic or study skills.

4. Contextual skills and awareness, also referred to as college knowledge, are needed to be understood by students so that they may navigate successfully the college environment. In this dimension the student must understand the college culture. This dimension refers to students’ abilities and knowledge such as the ability to apply to college or to fill out forms for student grants or loans.

**College Readiness - Mathematics**

According to Conley (2010), students that are college-ready have content knowledge and skills “beyond the basic formulaic understanding of mathematics” (p. 37). Students need to be able to apply this knowledge, and then to solve and interpret problems.

**Recommendations of Various Mathematics Organizations**

This section will discuss what different national and state organizations are recommending for college algebra as a result of studies conducted by their respective members. The movement for more statistics in secondary education will also be
reviewed.

**Need for reform of college algebra curriculum.** The Mathematical Association of America (MAA) created a subcommittee to review the curriculum of the first two years in college. This subcommittee, called Curriculum Renewal Across the First Two Years (CRAFTY), analyzed undergraduate mathematics programs in the United States. CRAFTY conducted several forums and workshops throughout the United States to discuss mathematics curriculums from the perspectives of multiple academic disciplines in postsecondary schools.

One of CRAFTY’s sponsored forums was the Conference to Improve College Algebra held in 2002. College algebra was deemed not working by the conference attendees (Small, 2010). “Participants in the Conference rejected the traditional content – factoring linear and quadratic polynomials, radicals, absolute value, determinants, Cramer's rule, etc. – with its emphasis on algebraic manipulations as suitable for a last mathematics course” (Small, 2010, para. 3). The high attrition rates of college algebra are of great concern since success rates are generally below 50% (Cullinane & Treisman, 2010; Madison, 2003b). Even so, college algebra remains a prerequisite not only for calculus but for most general education courses (Ganter & Barker, 2004; Madison, 2003b). College algebra does prepare the student for the path to calculus. However, several studies have found that only 10% to 20% of those students enrolled in college algebra move on to enter Calculus I (Dunbar, 2006; Gordon, 2008a; Herriott & Dunbar, 2009).

The traditional college algebra course teaches solving equations and functions by using many procedural techniques and rules to manipulate variables and numbers.
Generally, contextual problems are not addressed. Most of the students in the traditional college algebra course have previously been introduced to these techniques and rules and then struggled with learning these techniques. “Repeated exercises of drill-and-skill is \textit{sic} still thought to dominate most instruction” (Goldrick-Rab, 2007). Since college algebra is a terminal course for many college degree plans, many postsecondary students will never use these skills again after completing the course.

In most college algebra classes, there is usually no real-world modeling. Regularly, examples and applications are used such as “Today Sally’s father is 3 times the age Sally was 10 years ago. Also, today Sally’s father is twice Sally’s age. What are the ages of Sally and her father?” These applications do not appeal to contemporary students. Consequently, students’ perceived relevance and value of college algebra is low. Commenting on college algebra, Gordon (2008a) noted

virtually any educated individual will need the ability to

1. examine a set of data and recognize a behavioral pattern in it,
2. assess how well a given functional model matches the data,
3. recognize the limitations (often due to uncertainty) in the model,
4. use the model to draw appropriate conclusions, and
5. answer appropriate questions about the phenomenon being studied. (p. 8)

CRAFTY has created a document called Guidelines for College Algebra which was endorsed by CUPM (CRAFTY, 2007). The new guidelines make recommendations regarding goals for the course, goals for the students, and goals for the instructors. Some of the recommendations in these guidelines are (a) use real-world data, (b) have students collect data, (c) allow students the opportunity to construct knowledge, (d) model the
data, (e) utilize small group work activities, and (f) use problem solving techniques.

Another recommendation is to refocus college algebra to meet the needs of society, work, and other disciplines (Haver, Small, Ellington, Edwards, Kays, Haddock, & Kimball, 2007).

**What students need from an introductory statistics course.** CRAFTY also held a series of workshops regarding statistics courses. Their focus was primarily on the introductory statistics course. Regarding desired outcomes from non-statistics mathematics courses that precede an introductory statistics course, the CRAFTY Curriculum Foundations Project stated that one of the highest priority needs of statistics from the mathematics department is the development of skills for problem solving and generalization. In order to accomplish this task, the following items were recommended: (a) emphasize multiple representations of mathematical objects and multiple approaches to problem solving, including graphical, numerical, analytical, and verbal; (b) be sure instruction should be learner-centered and address students’ different learning styles by employing multiple pedagogies; (c) insist that students communicate in writing and learn to read algebra for meaning; (d) use real, engaging applications through which students can learn to draw connections between the language of mathematics and the context of the application; and (e) use technology appropriately to teach students how to solve problems and explore concepts (Moore, Peck, & Rossman, 2004).

**National push for statistics education in postsecondary curriculums.**

Statistics education is becoming more and more integral in secondary and postsecondary curriculums. The Texas College and Career Readiness Standards (CCRS) and the Common Core Initiatives contain secondary education standards for mathematics.
curriculum which include an emphasis on statistics (Council of Chief State School Officers [CCSSO], 2009; Texas Higher Education Coordinating Board [THECB], 2008). It should be noted that in CCRS, statistics and probability have been uncoupled and are their own mathematics strands, along with algebra, geometry, and measurement. In secondary education, statistics has been served well by the introduction of data analysis and probability in K-12 (Madison, 2003b). In its publication *Crossroads: Standards for Introductory College Mathematics Before Calculus* (Cohen, 1995), AMATYC strongly recommends the instruction of statistics at 2-year postsecondary institutions and has included statistics, along with probability, as one of its content standards. *Crossroads’ Standard C-6 Probability and Statistics states,*

> The basic concepts of probability and descriptive and inferential statistics should be integrated throughout the introductory college mathematics curriculum at an intuitive level. Students will gather, organize, display, and summarize data. They will draw conclusions or make predictions from the data and assess the relative chances for certain events happening. Suggested topics include basic sampling techniques, tabulation techniques, creating and interpreting charts and graphs, data transformation, curve fitting, measures of center and dispersion, simulations, probability laws, and sampling distributions. (Cohen, 1995, p. 14)

Most college mathematics curriculums are linear, algebraic-based and support a calculus track. This calculus track has been proven effective for developing the knowledge and skills required for the study of mathematics and mathematical sciences. However, this algebraic-based track is not the only mathematical content needed by college students. Madison (2003b) found that for many postsecondary students, data
analysis adds valuable course content. One of CRAFTY’s recommendations is to replace traditional college algebra courses with courses that utilize statistics, problem solving, technology, and mathematical modeling (Ganter & Barker, 2004).

Mathematics courses that teach and support statistical content are needed by many of today’s college students. This requirement for statistical content is driven by the students’ college majors and future job opportunities (Arney & Small, 2004; Gordon & Gordon, 2010). Steen (2004) reported the following:

What current and prospective employees lack is not calculus or college algebra, but a plethora of more basic quantitative skills that could be taught in high school but are not. Employees need statistics and three-dimensional geometry, systems thinking and estimation skills. Even more important, they need the disposition to think through problems that blend quantitative data with verbal, visual, and mechanical information. (p. 55)

Another indication of the trend toward more statistics education is the growth of the AP-statistics examination which was first offered in 1997. In a recent report produced by the College Board (2010), the number of enrollments in AP-Statistics examinations in the years 2000-2010 more than tripled from 34,118 to 129,899. It is estimated that the total number of students who take statistics in high school is twice the number of students who actually take the AP-Statistics examination. Those high school graduates who did not take the AP-Statistic examination plus those students that did not pass this test will probably need to study statistics further when enrolled at a higher education institution.

**Why statistics in a college algebra course?** College algebra is usually the
general education course at most tertiary schools because it is a prerequisite to calculus and other general education mathematics courses (Madison, 2003b). However, other academic disciplines such as psychology, political science, and sociology need for their students to have quantitative numeracy and reasoning skills, as well as statistical numeracy and reasoning skills. In an article to be published in MAA in 2011, Gordon and Gordon commented, “college algebra is the prerequisite for introductory statistics, despite the fact that virtually none of the ideas and algebraic techniques in traditional college algebra courses are relevant to introductory statistics” (p. 10). CUPM found that “social scientists will expect students to recognize a linear pattern in a set of data, interpret the parameters of the best-fitting line, and use the equation of the line to answer questions in context” (CUPM, 2004, p. 29).

**Current initiatives addressing these issues.** In efforts to lower attrition rates of students who find themselves underprepared for mathematics or who are enrolling into their first entry-level mathematics course, several programs have been developed or are in the process of being developed. One program strategy is to reduce the number of developmental mathematics courses required by a student. This strategy utilizes a mathematics curriculum that consists of accelerated mathematics courses where statistics is integrated into the course content. Two programs that are currently implementing this curricular philosophy are Statpath and Statway. Hern (2010) wrote an article describing the accelerated developmental statistics program Statpath implemented in Los Medanos College in California. Statpath was a program offered to 29 students who (a) required a statistics course, (b) were not on a calculus track, and (c) belonged to all levels of developmental mathematics courses. In Statpath all developmental mathematics courses
were replaced with one developmental statistics course. This course was built using the backward design principle with the goal of preparing students for the introductory statistics course. This class was “… a course that had the look and feel of a course in descriptive statistics with ‘just-in-time’ development of Arithmetic or Algebra skills” (Hern, 2010, p. 9). The Statpath students successfully passed the subsequent statistics course with a percentage rate of 94% as compared to the college-wide passing rate for statistics of 61%.

Carnegie Foundation is sponsoring a collaborative effort of faculty members, researchers, community colleges, and professional groups from across the United States that is developing a statistics pathway (Statway). This group is creating an alternative path for students to follow in order to acquire mathematics and statistics knowledge and skills. Statway instruction is only 1 year long and will include developmental and credit-bearing course work (Bryk & Treisman, 2010). One of the primary advocates of this program is Uri Treisman. Treisman, the Director of the Charles C. Dana Center at the University of Texas at Austin, is a well-known mathematics educator who has been recognized multiple times in his professional career for his contributions to mathematics education (Charles A. Dana Center, 2010).

Variables

This section will review the variables that were measured as part of this research project. Besides students’ performance in this college algebra course and the attrition rate of this course, three affective domain variables were also measured. These variables were attitude towards statistics, statistics self-efficacy, and mathematics anxiety. The following paragraphs explain how students’ experiences in prior mathematics courses
affect their attitudes, anxiety, and self-efficacy as they enter a statistics course.

**Attitude towards statistics.** Olson and Zanna (1993) define attitudes towards statistics as a multidimensional concept, which is composed of affective, cognitive, and behavioral dimensions. With the increasing demand for postsecondary students to enroll in statistics courses, students’ attitudes in statistics become more critical. Attitude towards statistics is believed to impact students’ achievement, course completion, future course enrollment, and statistical thinking outside of the classroom (Gal, Ginsburg, & Schau, 1997; Rhoads & Hubele, 2000; Schau, 2003). Students’ attitudes towards statistics may affect the extent to which they will develop useful statistical thinking skills and apply what they have learned outside the classroom (Gal & Ginsburg; 1997).

Another by-product of attitudes is that students’ attitudes affect whether or not a student pursues more courses in a particular subject (Schield, 2008). In another study, attitude towards statistics was found to be related to performance on statistics examinations in a first-year statistics course (Vanhoof et al., 2006). This study also found that that this relationship between attitude towards statistics and performance on statistics examinations was content specific. Negative student attitudes towards statistics may create a major obstacle for effective learning (Araki & Schultz, 1995; Cashin & Elmore, 1997; Elmore, Lewis, & Bay, 1993; Harvey, Plake, & Wise, 1985; Schulz & Koshino, 1998; Wise, 1985).

Statistics researchers and educators need to be focused on beliefs, attitudes, and expectations students bring into the classroom or develop during their education experience (Gal & Ginsberg, 1994). Gal and Ginsberg (1994) further stated if this area is not addressed, then quantitative literacy may be impacted negatively as well.
Consequently, educators should use assessment tests to help understand students’ perspectives. Some variables found to impact students’ attitudes towards statistics and statistical performance are mathematics ability, statistics experience, and student confidence (Mills, 2004).

**Statistics self-efficacy.** Self-efficacy was defined by Albert Bandura as “the belief in one’s capabilities to organize and execute the courses of action required to manage prospective situations” (1995, p. 2). Bandura points to four factors that affect self-efficacy (Bandura, 1977; Bandura, 2006). The first factor is experience. This factor relates to mastery experience, where success raises self-efficacy and failure reduces self-efficacy. Modeling is the second factor. This factor occurs when a person compares himself or herself to an individual who is seen as a model of oneself. When the model or peer is seen succeeding, then the self-efficacy of the person increases. If the model fails, then the person becomes very unsure of himself or herself. Social persuasion is the third factor that affects self-efficacy. Social persuasions relate to encouragement or discouragement from others in a social setting. The last factor is the physiological factor. This factor addresses one’s response to his or hers physical responses to stress. These physical responses could be shaking hands, nausea, or fatigue. If one perceives these physical responses as normal and not a reflection of one’s ability, then self-efficacy remains high. If these physical responses are seen as evidence of a person’s inability to do a task, then that person’s self-efficacy is lowered (Bandura, 1977; Bandura, 2006).

Statistics self-efficacy is defined as confidence in one's abilities to solve specific tasks related to statistics (Finney & Schraw, 2003). Finney and Schraw found that statistics self-efficacy and mathematics self-efficacy had a similar relationship with
respect to learning statistics. Research studies reported that statistics self-efficacy was related to performance in statistics (Abd-El-Fattah, 2005; Blanco, 2011; Finney & Schraw, 2003; Olani, Hoestra, Harskamp, & van der Werf, 2010a; Schau, 2003; Schneider, 2011). As a student’s statistics self-efficacy increases, then the performance of the student in statistics improves. One study found that statistics self-efficacy was the most important factor that predicts a student’s performance in statistics (Abd-El-Fattah, 2005). Another study found an improvement to students’ statistics self-efficacy after providing web-based tutoring assistance (Swingler & Bishop, 2008).

Mathematics anxiety. Mathematics anxiety is a feeling of tension and anxiety that interferes with the manipulation of numbers and solving of mathematical problems in many settings (Richardson & Suinn, 1972). Mathematics anxiety affects people of all ages and can vary in severity (Ashcraft & Moore, 2009).

In several studies, anxiety and its relationship to self-efficacy have been found to impact the performance of students. Zimmerman (1995) reported that self-efficacy can heavily impact anxiety. However other studies found that anxiety can have a reconciliation quality between achievement and variables such as self-efficacy (Abd-El-Fattah, 2005). Some studies reported that statistics anxiety may result from aversive experiences with mathematics, poor mathematics self-efficacy, and poor mathematics achievement (Adams & Holcomb, 1986; Zeidner, 1991). A relationship between mathematics anxiety and statistic anxiety has been found (Benson, 1989; Maysick, 1985; Nasser, 2004; Onwuegbuzie, DaRos, & Ryan, 1997).

A causal link between statistics anxiety and student performance in a quantitative research or statistics course has also been reported (Onwuegbuzie, Leech, Murtonen, &
In fact, these researchers stated that many mixed-methods approaches to teaching statistics such as using a combination of real-world problems, cooperative learning, and effective teaching methods could reduce this anxiety.

**Interventions Applied to College Algebra**

In order to determine if this project will contribute to the existing literature, a review of the literature was conducted to determine what research has been conducted in regards to college algebra. An investigation was conducted on the EBSCO database query system to look for research similar to that which was conducted by this research project. Next, a search of dissertations and theses was analyzed for studies that resembled the intervention investigated by this research project.

**EBSCO keyword search for interventions.** Using the EBSCO database query system, a keyword search was initiated for the phrase “college algebra” for January 2006 through December 2010. The research databases included in this search were Academic Search Complete, Education Research Complete, Education Administration Abstracts, and Education Resource Information Center (ERIC). This search yielded 67 articles from these research databases.

The majority of the articles in this list of 67 articles were found to be practice-oriented or philosophical, and not researched-based. In other words, the paper did not present a research question(s), and then discuss conducting a research project to address the null hypotheses associated with the research question. Many of these practice-oriented articles published the impact to student grades or retention using basic descriptive statistics. Some of these practice-oriented articles described different techniques to teach topics such as factoring a trinomial (Donnell, 2010). Book reviews of
college algebra and general algebra textbooks were included in the list of 67 (Gregory, 2006; Miller, 2007). One article discussed administrative changes in a higher education institution to implement and embrace standards (Jacobs, Jacobs, Coe, & Carruthers, 2007). This group of articles was removed from the list because they were practice-oriented and a research study was not conducted on the intervention.

After discarding the above articles, the list of 67 became a list of 20 articles.

Seven of these articles were found to be action-research articles with no research question originally presented in the paper. Examples of these articles were studies on the implementation of online homework in college algebra (Butch & Kuo, 2010; Hagerty, Smith, & Goodwin, 2010; Mahmoud & Walsh, 2007; Thompson & McCann, 2010).

Thus, the list of 20 became a list of 13. These 13 articles were categorized by (a) whether or not they were a quantitative or qualitative research study and (b) whether or not the intervention was pedagogical or technological. Since this research project will be a mixed-methods project that is predominantly quantitative, qualitative-only articles in the list were discarded. An example of a qualitative article was a study on implementing modeling as an instructional technique in a college algebra/pre-calculus course (Gordon, 2008b). The topic of this paper was college algebra and pre-calculus reform, but the article was a qualitative study where two students were interviewed. It should be noted that these non-traditional students successfully completed the course and gained a strong conceptual understanding of the mathematical content.

In the list of quantitative articles researched, one more paper on the implementation of online homework in college algebra was found (Kodippili & Senaratne, 2008). Another technology article utilized computer-aided instruction and
online delivery as alternative delivery systems in college algebra (Wynegar & Fenster, 2009). In all these technology articles, positive attitudes were observed. Some articles saw positive gains in achievement, and a couple of research papers reported no difference in student performance.

The rest of these articles discussed different pedagogies or administrative changes in college algebra. One researcher studied the impact of supplemental instruction in college algebra (Porter, 2010). This study compared test grades and homework grades and saw overall improvement for those students who participated in the supplemental instruction program. One study used self-regulated problem solving in the classroom and reported that most of the students were memorizing methods (Cifarelli, Goodson-Espy, & Chae, 2010). Another research article investigated whether or not students learned more in an 8-week class as compared to a 16-week class (Reyes, 2010). No statistical difference was found in this article. Another article studied the impact of using guided worksheets in order to reduce scribe time and obtain more meaningful notes (Montis, 2007). The guided worksheet project did realize higher achievement scores for the students in this study.

None of the articles in the original list of 67 addressed statistical content in college algebra. Also, no articles discussed the use experiential learning or the concrete-representational-abstract (CRA) instructional methods in a college algebra class.

Other interventions for college algebra. The Journal Storage Online Research (JSTOR) database is current up to the year 2004 for the National Council of Teachers of Mathematics (NCTM)’s Journal for Research in Mathematics Education (JRME). To review years 2005-2010, six years of articles, one would have to look at microfiche
which is not a manageable task. Hence, no *JRME* articles in JSTOR were investigated.

The Dissertation and Theses database was searched for research articles regarding college algebra over the past five years. This database returned 62 articles from the keyword search. When this keyword search was further refined to “college algebra” and “modeling,” only two papers were returned from this search. Both research articles investigated integrating modeling into college algebra, but statistics was not one of the content topics. The first research paper was qualitative research in which students were interviewed. The other paper had some similar characteristics to the intervention strategy of the research project of this dissertation (Hofacker, 2006). Hofacker’s (2008) project is using modeling, discovery-based learning, and problem solving. One difference found between this research project and Hofacker’s research was that Hofacker studied the impact to the students’ understanding of exponential and logarithmic functions. Plus, Hofacker’s research question addressed the success of these students in their subsequent pre-calculus course. There was no mention of integrating statistical content into college algebra in Hofacker’s research.

Another search of the Dissertation and Theses database was conducted using the keywords, “college algebra” and “statistics.” One article was found from these search criteria. This article seemed similar to the research contained this paper. But, this dissertation was a qualitative research project in which students were interviewed (Hansen, 2010). This course was called Introduction to Mathematics and included college algebra, statistics, and computer science. There was no mention of the pedagogy and instructional techniques used in the classroom or for the lesson plans.

Thus, there were two research articles with similar research questions as this
study which was found. However, this project is different from the work performed in these two projects. The first paper examined the impact on the subsequent pre-calculus class; the second paper was a qualitative study instead of a quantitative study. Hence, this paper will be contributing to existing literature regarding integrating statistical content into college algebra using experiential learning and the CRA instructional methods.

**Operational Definitions**

Key terms for this research are listed and defined below.

- **CRA instructional approach** is a three-stage learning process in which students first learn through physical manipulation of concrete objects. Next, students follow up the concrete activity by learning through pictorial representations of the concrete manipulations. Thirdly, students complete the process by ending with solving problems using abstract notation (Witzel, 2005).

- **Culturally responsive objectives** are those that allow the instructor to transform the classroom for a diverse student body into an inclusive classroom environment. Culturally responsiveness extends beyond the common cultures of ethnicity, race, gender, and native language, and also includes religion, social class, age, and disability. These objectives can be utilized such that students can perceive themselves with multiple identities when looking at the broader society (Arendale, 2010). Thus, students can celebrate their own identity and also find identities that they have in common with other students.

- **Group work** consists of collaborative and cooperative learning. Collaborative learning is defined as “Learners are working in groups of two or more, mutually
constructing understanding, solutions, meanings, applications, or products” (Wlodkowski & Ginsberg, 1995, p. 79). Cooperative learning is an instructional technique of students working in groups, for which the success of each member is dependent of the success of all the members of the group. Pioneered by Johnson and Johnson (2009), cooperative groups consist of five basic elements. These elements are positive interdependence, individual accountability, promotive interaction, the appropriate use of social skills, and group processing.

- Mathematical modeling involves identifying and selecting relevant features of a real-world situation, representing those features symbolically, analyzing and reasoning about the model and the characteristics of the situation, and considering the accuracy and limitations of the model (NCTM, 2000).

- Problem-based learning is defined by Barrows (2002) as a student-centered approach in which students determine what they need to learn. It is up to the learners to derive the key issues of the problems they face, define their knowledge gaps, and pursue and acquire the missing knowledge.

- Process standards are defined as those skills that allow students to acquire and apply content knowledge (NCTM, 2000). These standards are problem solving, reasoning and proof, communication, connections, and representations.

- Quantitative literacy, (QL), or quantitative numeracy, is the ability to understand, do data analyses, and communicate conclusions from quantitative information (Madison, 2003a; Steen, 2004).

- Quantitative reasoning is defined “as the ability to analyze quantitative information, including the determination of which skills and procedures can be
applied to a particular problem to arrive at a solution” (Dwyer, Gallagher, Levin, & Morley, 2003, p. 12).

• Statistical literacy is the ability to understand, critically evaluate, and use statistical information and data-based arguments (Gal, 2000; Garfield & Ben-Zvi, 2005).

• Statistical reasoning is “the way people reason with statistical ideas and make sense of statistical information. This involves making interpretations based on sets of data, graphical representations, and statistical summaries” (Garfield, 2002, para. 2).

• Traditional college algebra in this paper refers to the college algebra course in which there is a strong focus on algebraic methods and techniques.
CHAPTER 3
METHODOLOGY

This chapter contains descriptions of the research design, research questions, and population and sampling of the proposed research. This chapter also describes the instruments utilized in this research. Finally, the procedures used for the collection, recording, and analysis of data are discussed. A calendar of the project is also included.

Purpose of Study

The purpose of this study is to evaluate the effectiveness of the experiential concrete-representational-abstract (E-CRA) instructional method, when integrating statistics into a college algebra course (a) for those students who were required and then subsequently completed developmental mathematics and (b) for those students who were designated college-ready without the need for developmental mathematics. Both sets of these students required college algebra and a statistics course. The effectiveness was examined by evaluating the following changes in the affective domain and performance of the students in the college algebra with statistics course. In the affective domain, the variables measured were the variations of (a) students’ attitudes towards statistics, (b) changes in students’ statistical self-efficacy, and (c) the influence of mathematics anxiety on students. Changes in performance were measured by examining, specifically, (a) the changes in college algebra skills, (b) the differences to statistical numeracy and reasoning skills of the students, and (c) the attrition rate of this college algebra course.
Research Questions

The following are the research questions investigated by this study. The college algebra course with statistics is labeled CA/S.

1. Does the use of the E-CRA instructional method in a CA/S course produce changes in students’ affective domain as measured by pretest and posttest scores while controlling for whether or not the student took developmental mathematics courses?

2. Does the use of the E-CRA instructional method in a CA/S course produce different outcomes in student performance as measured by pretest and posttest scores while controlling for whether or not the student took developmental mathematics courses?

Population and Sampling

This section will describe the population and sampling of this research project. The selection process used to place participants into the sample is described.

Population. The study was conducted at a 4-year university in central Texas during the spring 2011 semester. The university has an enrollment of over 30,000 students of which approximately 65% are white and 35% are of an ethnic minority (Hispanic, Black/non-Hispanic, Asian/Pacific Islander, American Indian/Alaskan native, other). Female students are 56% of the student population. The median age of the students at the university is 22.

This university offers two developmental mathematics courses, elementary algebra and intermediate algebra. These undergraduates could be placed into developmental mathematics as a result of several examination scores. Students could be placed into developmental mathematics courses as the result of their Scholastic
Assessment Test (SAT) quantitative examination score or their ACT mathematics score. Students can also be placed into developmental mathematics as a result of test scores from one of the following mathematics placement exams: Accuplacer, Computer Adaptive Placement Assessment and Support System (COMPASS), or Texas Higher Education Assessment (THEA). Accuplacer is provided by the College Board (2011), COMPASS is available through ACT (2011), and THEA was created and is recognized by the Texas Higher Education Coordinating Board (Pearson Education, 2011).

**Sampling.** The mathematics department received a research grant to study different curricular and instructional changes that would be beneficial to students who were placed into developmental mathematics. This research project was part of the research grant. Twenty-one postsecondary students participated in this research study. Some students had recently completed the developmental mathematics course, intermediate algebra. All students in this study were considered college-ready by virtue of their eligibility for this course. However, since college algebra historically has a 40-60% attrition rate, these students were at-risk and there was a good chance that they were still underprepared.

The participants had to fill out an application to enroll into this specific college algebra course, which was a part of the developmental mathematics grant mentioned earlier. Students in this project were all provided with textbooks required for this course, a graphing calculator, and any supplies other than pencil, paper, and notebook. The students that were accepted into this course and into this grant project met certain criteria. Some of the criteria that the students had to satisfy were the following:

1. All of the students in this study were considered college-ready since they were
enrolled in a college algebra course. Many of the students enrolled in this course had taken developmental mathematics prior to this course. Thus, several of these students were considered underprepared for college mathematics when they first began their postsecondary education.

2. College algebra is the first credit-bearing college course in mathematics required by each of these students’ respective degree plans.

3. Another course included in the degree plans of all of these students is a statistics course. This required statistics course is offered either by the mathematics department or located in another academic department, such as psychology, sociology, or criminal justice. Thus, all of the students will have to take statistics in order to successfully complete their respective degree plan.

4. Students committed to participate in certain study-skills activities and best practices. It should be noted that if a student met all of these obligations, then the student received a small stipend, which was funded by the grant. This commitment entailed complying to a strict attendance policy for this college algebra course, going to two hours of tutoring every week in which one hour of tutoring consisted of group tutoring whereby a learning community was established, completing weekly survey sheets as required by the grant, and attending monthly study-skills workshops.

**Research Design**

A quasi-experimental mixed-methods design was used to investigate the effect of the E-CRA instructional method associated with this college course. The research design is considered quasi-experimental because there was no randomization in the selection of the students in this study.
**Instrumentation.** As a part of this research study, the students took several tests at the beginning and at the end of the spring semester. The tests were administered to measure attitudes towards statistics, statistics self-efficacy, mathematics anxiety, algebra skills, and statistical numeracy and reasoning. The students’ awareness of and about learning and study strategies were also assessed as required by the grant.

**Attitudes Towards Statistics (ATS).** The instrument used in this study was the ATS created by Steven Wise. The ATS test is a paper and pencil test that was easily administered as a pretest and a posttest (Wise, 1985). This instrument is a 29-item attitudinal scale that consists of two subscales: field which is 20 items and course which is 9 items. Field subscale refers to students’ attitudes towards how statistics contributes to students’ field of study. The course subscale is used to assess students’ attitudes regarding the statistics course in which they are currently enrolled. Some of the items on the ATS tests are reverse questions. In regards to psychometric properties, the test has a high internal consistency of approximately 0.90 (Cashin & Elmore, 2005; Delmas, 2006c; Schultz & Koshino, 1998). ATS has a high correlation to the SAS test mentioned earlier (Roberts & Reese, 1987). In the ATS the 29 items are statements for which the student responds to a Likert scale of one through five for strongly disagree, disagree, neutral, agree, and strongly agree, respectively (Wise, n.d.a). It should be noted that Gal, Ginsburg, & Schau (1997) have cautioned against the indiscriminate use of paper and pencil tests that use Likert scales when studying attitudes. The scores for the field and course subscales vary from 20-100 and 9–45, respectively. For the field subscale, high scores reflect a more positive attitude from the student. Scores above 60 will have positive attitudes. The course subscale is measured where a low ATS score designates a
strong attitude towards the statistics course. For the *course subscale* a score below 24 will have leanings towards a positive attitude. Several research studies have used the ATS instrument to measure undergraduate and graduate students in introductory statistics courses (Cashin & Elmore, 2005; D’Andrea & Waters, 2002; Perepiczka, Chandler, & Becerra, 2011; Rhoads & Hubele, 2000; Schultz & Koshino, 1998).

Other instruments available to measure attitudes towards statistics are the Statistics Attitude Survey (SAS) and the Attitudes Towards Statistics (SATS). The SAS crafted by D. M. Roberts and E. W. Bildenbeck is one-dimensional with 33 items. Schau, Stevens, Dauphinee, and del Vecchio (1995) developed the SATS. The SATS instrument consists of 28 seven-point Likert-type items measuring four aspects of post-secondary students. Schau and her associates developed this survey because they believed that a good survey should exhibit seven important characteristics. A instrument measuring statistics attitudes needs to include scales that (a) measure aspects of statistics attitudes, (b) are based partly on input from introductory statistics students, (c) can be applied to most introductory statistics courses, (d) work at different times during the course, (e) are short and minimally disruptive when administered in class, (f) measure both positive and negative attitudes, and (g) the number of scales and the items that constitute them should be supported when confirmatory statistical techniques are applied to students' responses.

*Current Statistics Self-Efficacy (CSE).* The CSE instrument measures a person’s beliefs in their abilities to perform in statistics (Abd-El-Fattah, 2005; Delmas, 2006d; Finney & Shaw, 2003). The test had a Cronbach’s alpha coefficient greater than 0.90 (Delmas, 2006d). This test was administered as a pretest and posttest. It is a pencil and
paper test. The CSE contains 14 items which used a Likert scale of one through six, one meaning no confidence at all and the rating of six meaning complete confidence (Finney & Shaw, 2003). Previous research found this instrument to correlate positively to attitudes towards statistics and math anxiety. There is a negative correlation between CSE and anxiety. A score of 14 assesses a low level of statistics self-efficacy, and a score of 84 marks a high level of statistical self-efficacy (Abd-El-Fattah, 2005; Finney & Shaw, 2003). When implementing different teaching methods or implementing technology, CSE can be helpful in order to assess the effect on statistics self-efficacy (Abd-El-Fattah, 2005).

**Mathematics Anxiety Ratings Scale (MARS).** MARS was used to assess mathematical anxiety. Suinn and Winston (2003) recently authored a shortened version of the original test developed in the 1950s. Commenting on the test qualities Suinn and Winston stated, “The Cronbach alpha of .96 indicated high internal consistency, while the test-retest reliability for the MARS 30-item was .90 (p<.001). The validity data confirm that the MARS 30-item test is comparable to the original MARS 98-item scale” (p. 1). This test is a pencil and paper test and was administered as a pretest and posttest to the CA/S students. Low MARS scores reflect low mathematics anxiety, and high MARS scores relate to high mathematics anxiety.

**COMPASS.** As mentioned earlier in this chapter, the COMPASS test can be used to assess placement into college mathematics courses (ACT, 2011). This is a computer-adaptive test that students take in the testing center located at the University. For the purposes of this research study, students took the algebra level of this placement test. The algebra level is used to place students into the credit-bearing college algebra course
or into specific levels of developmental mathematics courses.

**Shortened Comprehensive Assessment of Outcomes in a First Statistics Course (CAOS).** The CAOS test is provided and maintained by the University of Minnesota with funding from the National Science Foundation. It assesses statistical reasoning after the completion of an introductory statistics course. In regards to reliability, the CAOS test has a Cronbach’s alpha coefficient of .77 (Delmas, 2006a). From the perspective of validity, there was 94% agreement among expert raters that the test measures important learning outcomes for a first statistics course” (Delmas, 2006a, para. 3).

Since the students in this research study taking this test have not yet completed an introductory statistics course, the test was shortened from 40 to 20 questions. The first 22 questions, excluding questions 7 and 9, were administered to the students. These 20 items were selected to measure the impact on statistical numerical and reasoning skills covered by the CA/S curriculum. The test was administered as a pretest and a posttest.

**Qualitative Survey.** A qualitative student survey was given to the CA/S class at the end of the semester. The survey was created as part of this research project and can be found in Appendix D. Consequently, the validity and reliability of the survey is untested. It contained questions such as “Do you like starting a new mathematics content section with a real-world application or real-world data?” or “What did you like least about the class?”

**Other Information Available.** A teaching journal was kept by the instructor of the CA/S course to record how well the lesson plans worked. Planned tasks were chronicled. The response or lack of response by the students was noted. Time management issues and any other incidentals worth noting were documented. The
teaching journal can be found in Appendix E.

The central Texas university where this research was conducted has an end-of-course survey that all instructors must give to their students. The Department of Mathematics also has its own end-of-course survey that the instructor is required to administer. These two surveys were not used in the analysis of this research.

**Analysis Plan.** The researcher used SPSS when analyzing the quantitative data collected. The databases were examined for missing data. Data were discarded for all instruments if (a) a student did not take both the pretest and the posttest or (b) a student did not finish either the pretest or the posttest. The following describes the quantitative and qualitative analysis of the data performed to address the research questions and null hypotheses. The comparable group used in several of these tests was those students entering an introductory to statistics course in the mathematics department. This group of students was coded in this document as IntroStat. For all quantitative tests performed, statistical significance was set at $p < .05$. Practical significance was determined for tests resulting in $0.05 \leq p \leq 0.10$.

**Research Question 1.** This question assessed the changes to the affective domain of the students in the CA/S course while controlling for whether or not the students took developmental mathematics courses. The three perspectives analyzed were attitudes towards statistics, statistics self-efficacy, and mathematics anxiety.

**Attitudes towards statistics.** To assess changes to the CA/S students’ attitudes towards statistics, data for this analysis came from the results of the ATS pretest and posttest. The pretest was administered to the CA/S class at the beginning of the semester, and the posttest was conducted at the end of the semester. The overall gain scores for the
field and course subscales between the pretest and the posttest for the CA/S class were analyzed by performing a paired-difference t test. This test reported any statistical significance between the pretest scores as compared to the posttest scores. An inspection of the individual ATS items, which are ordinal, was not conducted.

Afterwards an investigation was performed on whether or not the students in the CA/S course have a better attitude towards statistics than those students who do not take this course. The comparable group consisted of those students entering the IntroStat group. The ATS posttest score of the CA/S class was compared to the IntroStat group’s ATS scores at the beginning of the semester. This comparison was accomplished by utilizing a 2-sample t test on the field subscale score and the course subscale score. The standard deviations of the two samples were assumed to not be equal. No analysis of this instrument’s items was performed.

Lastly, for the CA/S posttest data and the IntroStat data, a general linear model 2x2 ANOVA was conducted, where the dependent variable was the ATS test score and the fixed categorical variables were MathCrse and DevMath. The factor, MathCrse, is referring to whether or not a student took the CA/S course or was in the IntroStat course. The DevMath factor represented whether or not a student took developmental mathematics during their collegiate career. The purpose of this test was to determine whether or not taking developmental mathematics had an impact on the performance of the students in these two courses. The qualitative data collected from the student survey was coded and analyzed to determine if there are any explanations of the changes or lack of changes in students’ attitudes towards statistics.

Statistics self-efficacy. The next aspect of the affective domain evaluated was
students’ statistical self-efficacy. To assess changes in statistical self-efficacy, data from the results of the CSE test were used. Students in the CA/S course took the CSE test at the beginning and the end of the semester. To measure any changes in students’ statistics self-efficacy due to attending the CA/S course, the overall gain score between the CSE pretest and posttest for the CA/S class was analyzed using a paired-difference t test. This test reported if any statistical significance between the pretest scores as compared to the posttest scores was found. An inspection of the individual CSE items, which are ordinal, was not conducted.

The next step for this portion of the research question was an investigation of whether or not the students in the CA/S course had a stronger statistical self-efficacy than those students who do not take this course. The CSE posttest score of the CA/S course was compared to the IntroStat group’s CSE scores at the beginning of the semester. This comparison was accomplished by utilizing a 2-sample t test. The standard deviations of the two samples were assumed to not be equal.

A general linear model using a 2x2 ANOVA was conducted to see whether or not developmental mathematics had an impact on the performance of the students in these courses. The fixed categorical variables were MathCrse and DevMath, and the dependent variable was the CSE test score. Descriptions for MathCrse and DevMath can be found earlier in this chapter. The qualitative data collected from the student survey were coded and analyzed to obtain explanations of changes or lack of changes in students’ statistical self-efficacy.

Mathematics anxiety. The third facet of the affective domain evaluated was students’ mathematics anxiety. To assess changes to mathematics anxiety, data for this
quantitative analysis came from the results of the MARS test. The pretest was given to the CA/S students at the beginning of the semester, and the posttest was administered at the end of the semester. The overall gain score between the pretest and the posttest for the whole class was analyzed. This task was performed by using a paired-difference t test. This test reported if any statistical significance between the pretest scores as compared to the posttest scores was found.

**Research Question 2.** This question assessed the changes to the performance of the students in the CA/S course while controlling for whether or not the students took developmental mathematics courses. Students’ college algebra skills, statistical numerical and reasoning skills, and persistence were analyzed.

This particular performance evaluation was conducted by comparing pretest and posttest scores from the COMPASS and CAOS tests of the CA/S student. Next, the posttest CAOS scores of the CA/S class were compared to the CAOS scores of the IntroStat group. This analysis looked for a difference in students’ preparedness for statistics from a statistical-content perspective. Finally, the attrition and DFW rates of the CA/S course were compared to other college algebra courses held during the same spring semester.

**COMPASS.** The CA/S students took the COMPASS pretest at the beginning of the semester and the posttest at the end of the semester. The overall mean gain score between the pretest and the posttest scores for the whole class was analyzed from the COMPASS test results. This evaluation was performed by using a paired-difference t test. This test reported if any statistical significance between the pretest scores as compared to the posttest scores was found.
Shortened CAOS. For an evaluation of improvement in statistical numeracy and reasoning skills, the CAOS tests given at the beginning and at the end of the semester were analyzed for the CA/S course. First, the overall gain score between the pretest and the posttest scores for the CA/S class was analyzed. This analysis was conducted by using a paired-difference t test. This test reported if any statistical significance between the pretest scores as compared to the posttest scores was found. Next, a McNemar test was run on the individual items of the instruments to compare changes from pretest to posttest responses for individual items. The McNemar test was utilized because the data are dependent, matched pairs that are binary.

Statistical numeracy and reasoning skills scores from the CAOS tests were studied to determine if the students of the CA/S course were more prepared for their following statistics course as compared to those students who did not take this course. The IntroStat classes took the CAOS test at the beginning of the semester. This group’s scores were compared to the CAOS posttest scores of the CA/S students. A 2-sample t test performed this analysis. The standard deviations of the two samples were assumed to not be equal.

A general linear model using a 2x2 ANOVA was run to see whether or not developmental mathematics had an impact on the performance of the students in these courses. The fixed categorical variables were MathCrse and DevMath, and the dependent variable was the CAOS test score. The qualitative data collected from the student survey were coded and analyzed to obtain explanations of changes or lack of changes in students’ statistical self-efficacy.

Persistence. To determine if the attrition and DFW rates of the CA/S course were
more improved than those of the college algebra courses, the attrition and DFW rates of the CA/S course were compared to the rates of all the other college algebra courses taught during the spring 2011 semester. Since these rates result from binary data giving a binomial distribution, a logistic regression was performed to obtain the analysis needed.

First, all college-credit, entry-level, mathematics courses were merged into one spreadsheet. This included (a) Math 1315, College Algebra; (b) Math 1316, A Survey of Contemporary Mathematics course for liberal arts students; and (c) Math 1319, Mathematics for Business and Economics I. The large and the small college algebra classes were assigned separate codes to allow inspection to determine whether or not their attrition and DFW rates may be significantly different from each other. The distinction used to label a college algebra class as small or large was fewer than 45 or was greater than 45, respectively. It should be noted that one college algebra class had over 350 students and another class had over 190 students.

Atypical classes were discarded from this logistical regression. This included one college algebra class that was a part of the same research grant, but conducting a different research study. Also classes that had a size of one or two students were considered special cases and were not included. Two classes of college algebra for business were discarded because all of the students were given a grade of A if the students had not dropped out of the course. All the students in these two classes received either an A or W.

**Procedures.** The study analyzed whether or not the students’ statistical numeracy and reasoning skills changed as a result of participation in the CA/S course, while controlling for whether or not the student took developmental mathematics. This
investigation evaluated possible differences in attitudes towards statistics, self-efficacy in statistics, and mathematics anxiety. Also, the attrition rate of this course was compared to that of traditional college algebra classes which were taught during the spring 2011 semester. Analyses were also performed to see if the students in the CA/S course were more prepared, mentally or performance-wise, for the introduction to statistics class than those students beginning their instruction in an introduction to statistics course the same semester the CA/S course was provided.

The calendar of events for this research project follows.

- January 18, 2011. First day of CA/S class held.
- January 21, 2011. CA/S students completed pretests required of the grant and this research project. Pretests administered for COMPASS, CAOS, CSE, ATS, and MARS.
- January 24, 2011 through May 2, 2011. CA/S follows the proposed syllabus calendar found in Appendix B.
- January 26, 2011. Students in the introduction to statistics classes took CAOS, ATS, and CSE tests as homework.
- February 15, 2011. Results of all pretests received.
- February 16, 2011 through April 30, 2011. All pretest data entered into Excel spreadsheet, ready for input into Statistical Package for the Social Sciences 17.0 for Windows (SPSS).
- May 2, 2011 through May 9, 2011. CA/S students finished all posttests
required of the grant and this research project: COMPASS, CAOS, CSE, ATS, and MARS tests.

- May 31, 2011. Results of the posttest qualitative survey reviewed and coded.
- June 20, 2011. Obtained final course grades of all students in entry-level, credit-bearing mathematics classes studied during spring 2011 semester.
  
  Data were provided by the Office of Institutional Research at the University.
- June 30, 2011. Quantitative and qualitative analyses of the all data completed.
  
  Results and conclusions of the analyses written as part of the dissertation.
- July 8, 2011. Dissertation presented to the doctoral committee.
- July 29, 2011. Defense of the research findings conducted.

**Scope and Sequence of the CA/S curriculum**

The scope and sequence for mathematical and statistical topics covered by the CA/S course can be found in Appendix B. The curriculum begins with discussion of statistical topics such as types of data and data collection, as well as distribution, measures of central tendency, and normal distribution. While these topics are being covered, students have been asked to bring to class data that they are interested in as well as graphs of these data. As students analyze data that appears to be linear, students learn to interpret representations of the graphs of linear equations, learn characteristics of linear equations, and learn to run linear regressions and use the best-fit models to predict of events. The quadratic equation and its characteristics are taught next. Quadratic data are analyzed and quadratic regressions are performed. The results of the quadratic regressions are examined. Then, the topic of functions is covered. Students are shown how common functions or family of functions help us model changes to data. So,
changes to data in the domain or range may change students’ model of their data, but the
overriding parent function—more than likely—did not change. Translations and
transformations of functions were demonstrated to create other models of students’ data.
Next, students explored operations of functions as they are connected to the real world.
Inverse functions, exponential functions, and logarithmic functions are covered in a
similar fashion. At the end of the semester, a high-level view of inferential statistics was
presented as students learned to perform statistical tests and interpret $p$ values.

The lesson plans used during this semester to deliver the CA/S curriculum with
the E-CRA instructional technique are located in Appendix C. The lesson plans
supported the use of real-world data to be analyzed by students. Students worked in
groups, so usually at least one member of the team has data that fits the pattern of the
mathematical topic to be covered. The lesson plans included using data originally
presented to the class by one of the students. In general, the students analyzed raw data
from a real-world situation. They graphed it to discover other representations of the data.
Students discussed whether or not the data had the characteristics discovered in previous
lessons or whether or not the data characterizes something else. The instructor then
presented the mathematical content that describes the characteristics of the data.
Afterwards, students created models of the data, and made interpretations and predictions
of the models created.

Even though not all college algebra content is included in the curriculum of the
CA/S course, the algebra topics that were covered needed to reflect the perspective of
college algebra curriculum. All tests given in this course contained algebraic and
statistical assessment. The instructor met with another instructor of a typical college
algebra course and discussed the algebraic assessment contained in the tests. This action was taken to ensure that similar algebraic assessments were included on the test as were found in other college algebra classes.

**Limitations and Assumptions**

This research study had several limitations. First of all, the students applied and then were selected to be in the CA/S course. There was no randomization exercised when selecting the participants in this sample. A second limitation was that the sample size of CA/S was small, where \( n=21 \). By the end of the semester, two students had withdrawn from the class, so the sample size became \( n=19 \). Third, it should be noted that some students did not complete or take all of the surveys which resulted in lower sample sizes for the statistical tests performed.

The loss of several class days limited the amount of curriculum that could be delivered. From a classroom instruction perspective, two, possibly three, 80-minute class periods were consumed to administer the CAOS, CSE, and ATS tests. An additional class day was lost because statewide, electrical-power blackouts resulting from very cold weather caused university classes to be cancelled.

Available tutoring support was limited for the CA/S students. The mathematics tutors available to the CA/S students were strong in the traditional college algebra and calculus skills and topics. Unfortunately, many of the mathematics tutors had not studied statistics or had little training in statistics. So, some tutors had limited knowledge on statistics and the use of the graphing calculator for analyzing, graphing, and modeling of statistical data. The CA/S students found this lack of knowledge frustrating. It should be noted that the CA/S students found those tutors that did know statistics and modeling to
Another limitation of this research project is that the researcher was also the 
instructor of the CA/S course. The researcher created and graded the tests, quizzes, and 
homework for the course. The grading of these items determines final course grades and 
influences whether or not students drop or withdraw from the course. To help reduce the 
effect of this limitation, another instructor of college algebra was asked to review all tests 
and the final examination for college algebra content.

It should also be noted that this semester was the first time that this course was 
taught. Consequently, an additional limitation was that as a part of the formative process, 
the proposed syllabus’s scope and sequence changed as the CA/S course was taught.

An assumption was made that the students in the CA/S and the introductory to 
statistics courses answered all questions on all pretests, posttests, and surveys honestly. 
Additionally, the differences in teaching styles and pedagogy used by instructors of the 
various college algebra classes impacts the attrition and DFW rates of the students in 
these classes. This study assumed that the differences in teaching styles and pedagogy 
used by instructors did not impact the performance of the students in these classes.

The ATS and CSE instruments are designed to be given to students after the 
completion of an introductory course in statistics at a higher education institution or after 
the completion of a Statistics-AP course in secondary education. The ATS and CSE tests 
were given to students prior to students beginning their work in this introductory statistics 
course. This research project assumed that the use of these tests at the beginning of 
introductory statistics would still provide measurements of students’ attitudes and self-
efficacy for statistics.
The CAOS test was shortened from 40 to 20 questions for two reasons. First, since the test is designed to be administered at the end of an introductory statistics course, many of the items covered topics unknown to most students at the beginning of the course. Second, the test was shortened to reduce the time required by the student to complete the test. It is assumed that the 20 items selected would measure the impact on statistical numerical and reasoning skills covered by the CA/S curriculum.

Also, the students in the introductory statistics course took the CAOS, ATS, and CSE tests home for an extra-credit quiz grade. A limited amount of basic statistical topics were covered during the first day of class in the introductory statistics course, such as types of data and their respective graphs. On the other hand, the CA/S students took the CAOS, ATS, and CSE tests in class on the first day of class. No course content was taught before the instruments were administered to these CA/S students. Extra credit was not offered to the students in the CA/S course. This research study assumed that the differences concerning how, when, and where these assessment tests were taken did not impact the results of these instruments.

Summary

This chapter contained the procedures regarding the research design of this study. The research questions were presented. A description of the study participants and other demographics were provided. Both research questions utilize several variables in order for the research questions to be adequately addressed. The methods of data collection for each variable needed for the project were documented. The statistical tests and analyses conducted on the variables were detailed.
CHAPTER 4

RESULTS

The purpose of this research is to determine if non-STEM students who were required to take statistics are more successful in a college algebra with statistics (CA/S) course that utilizes experiential concrete-representational-abstract (E-CRA) instructional method, instead of participating in the typical college algebra course offered at this university. The typical college algebra course is a part of the pre-calculus and calculus track established in the mathematics department.

First, this chapter presents the results of the curriculum and its lesson plans. Next, the chapter contains the results and findings from using the instruments and the analytical procedures described in Chapter 3. In this part of the chapter, each research question and its null hypotheses is addressed. It should be noted that the introductory to statistics course offered in the mathematics department will continue to be called IntroStat. The IntroStat students would not have taken the CA/S course.

Curriculum

A total of 20 lesson plans were taught to the CA/S class. Some lessons did not require the entire class period, and other lesson plans were long and ran over to the next class day. Two to three class days were unavailable for instruction because of the required testing for the research project and the grant associated with the research project. Another class day was lost because the university had to close as the result of power failures. As a result, the scope and sequence for the course changed as part of the
formative process associated with the course. Appendix B-1 contains the proposed scope and sequence for the CA/S course. The implemented scope and sequence for the CA/S course can be found in Appendix B-2. Appendix C contains the lesson plans associated with this course. The lessons plans include instructor’s reflections and remarks on how well each lesson plan worked.

Three lesson plans which embraced the E-CRA instructional model were the Super Bowl Linear Regression Lesson Plan, the Linear Functions Lesson Plan, and the Transformations of Functions Lesson Plan found in Appendices C-6, C-8, and C-13, respectively. These lesson plans traverse through the E-CRA model using a combination of two or more of the following: modeling, problem-based learning, using technology, and/or other instructional methods.

Instructor comments regarding classroom management and the success of the lesson plans can be found in the reflection area of the lesson plan and in the daily journal (Appendix E). A problem occurred because the students displayed limited abilities to read and interpret graphs, which, in turn, required substantial time to address. For example, the students could take concrete data and graph the data. However, if given a graph, the majority of the students could not see the concrete data in the graph. As a result, the students had difficulties finding mean and median from a graph. The students were looking at the picture, but were not reading it well enough to interpret the information that the graph was supplying. So, the lesson plans had to be adjusted to reinforce teaching the interpretation of statistical graphs, graphs of linear functions, graphs of quadratics, and graphs of exponential functions.
Another instructional area that took more time than expected to cover was using the graphing calculator to perform regressions. Additionally, the textbook was weaker in supporting the curriculum than was expected. Portions of the textbook that seem to provide supplemental material for scatter plots or modeling problems did not have enough depth to allow students to truly understand and deliberate what is happening with real-world data.

**Research Question 1**

This portion of the chapter will address the research question, “Does the use of the experiential concrete-representational-abstract (E-CRA) instructional method in a CA/S course produce changes in students’ affective domain as measured by pretest and posttest scores while controlling for whether or not students took developmental mathematics courses?” The three perspectives analyzed were attitudes towards statistics, statistical self-efficacy, and mathematics anxiety.

**Attitudes Towards Statistics (ATS).** To assess an individual’s attitudes towards statistics two areas were addressed: (a) students’ attitudes towards the field of statistics and (b) students’ attitudes towards a statistics course. The null hypothesis associated with attitudes towards statistics was the following, “While controlling for whether or not developmental mathematics courses were previously taken by the students in the CA/S course, there were no statistical differences in the means for attitudes towards the statistics field and the statistics course at the beginning of the semester as compared to the end of the semester.”

To assess changes to the CA/S students’ attitudes towards statistics after completing the CA/S course, data came from the results of the ATS pretest and posttest.
An analysis of the gain score between the pretest and the posttest for the CA/S course for the field and course subscales was conducted using a paired-difference t test. No statistical significance ($p > .05$) was found for the field and course subscales, but the students’ posttest scores showed an improved positive attitude towards statistics at the end of the semester. Table 1 provides the abbreviated results of this paired-difference t test.

To compare the CA/S students’ attitudes towards statistics to those students who did not take the CA/S course, the CA/S students’ posttest scores were analyzed against those students in the IntroStat course. A 2-sample t test of these scores was performed on the two subscales, field and course, with the assumption that the variances are not equal.

Table 1

<table>
<thead>
<tr>
<th>Subscales or significant items</th>
<th>Pretest</th>
<th>Posttest</th>
<th>$t$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field subscale</td>
<td>75.93</td>
<td>77.107</td>
<td>0.535</td>
<td>.601</td>
</tr>
<tr>
<td>Course subscale (RQs)</td>
<td>25.86</td>
<td>23.79</td>
<td>-1.288</td>
<td>.220</td>
</tr>
</tbody>
</table>

*Note. RQs denote reverse questions. $n = 14$.*

The CA/S course had a sample size of $n=14$ and the combined IntroStat courses had a combined sample size of $n=125$. No statistical significance ($p > .05$) was found for the field or course subscales. However, the CA/S students’ mean score was higher than the mean score for the students in IntroStat. Parts of the 2-sample t test are found in Table 2.

The general linear model 2x2 ANOVA was run on the CA/S posttest data as compared to the data collected from the IntroStat group. This test consisted of two categorical factors, MathCrse and DevMath. The factor MathCrse is referring to whether or not a student took the CA/S course or was in the IntroStat course. The DevMath factor
represented whether or not a student took developmental mathematics. For the two ATS subscales, field and course, no main effects or interaction of the factors were found to be statistically significant ($p > .05$) in this 2x2 ANOVA. Results of the general linear model are encompassed in Table 3.

Table 2

ATS: CA/S Posttest vs. IntroStat 2-Sample t test

<table>
<thead>
<tr>
<th>Subscales or significant items</th>
<th>CA/S posttest$^a$</th>
<th>IntroStat$^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M$</td>
<td>$SD$</td>
</tr>
<tr>
<td>Field subscale</td>
<td>77.00</td>
<td>3.430</td>
</tr>
<tr>
<td>Course subscale (RQs)</td>
<td>23.93</td>
<td>0.802</td>
</tr>
</tbody>
</table>

Note. RQs denote reverse question. $^a n = 14$. $^b n = 125$.

Table 3

ATS: CA/S course vs. IntroStat using 2x2 ANOVA with Factors MathCrse and DevMath

<table>
<thead>
<tr>
<th>Subscale</th>
<th>Took Dev Math</th>
<th>No Dev Math</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M$</td>
<td>$SD$</td>
</tr>
<tr>
<td>Field</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CA/S$^a$</td>
<td>33.79</td>
<td>4.262</td>
</tr>
<tr>
<td>IntroStat$^b$</td>
<td>33.16</td>
<td>5.179</td>
</tr>
<tr>
<td>Course (RQs)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CA/S$^a$</td>
<td>24.75</td>
<td>6.594</td>
</tr>
<tr>
<td>IntroStat$^b$</td>
<td>25.34</td>
<td>4.418</td>
</tr>
</tbody>
</table>

Note. RQs denote reverse questions. Dev Math = Developmental Mathematics. $^a n = 14$. $^b n = 125$.

**Current Statistics Self-efficacy (CSE).** The next aspect of the affective domain evaluated was students’ statistics self-efficacy. To measure changes to statistical self-efficacy, data from the results of the CSE test were used for this analysis. The null hypothesis was “While controlling for whether or not developmental mathematics
courses were previously taken by the students in the CA/S course, there is no statistical
difference in the means for statistical self-efficacy at the beginning of the semester as
compared to the end of the semester.”

The overall gain CSE score between the pretest and the posttest for the CA/S
course was analyzed by running a paired-difference t test. Table 4 contains the results of
this test. Statistical significance, \( p=.002 \), was reported for the overall gain score. So, the
students did have a higher statistics self-efficacy in the CA/S course by the end of the
semester.

Next, the CSE test results were analyzed to determine whether or not the students
in the CA/S course had a stronger statistical self-efficacy than those students in the
IntroStat course. This comparison was accomplished by using a 2-sample t test. The
standard deviations of the two samples were assumed to not be equal. A practical
significance was reported when

Table 4

\[ \text{CSE: CA/S Pretest and Posttest Paired-Difference t test} \]

<table>
<thead>
<tr>
<th>Overall and significant items</th>
<th>Pretest</th>
<th>Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( M )</td>
<td>( SD )</td>
</tr>
<tr>
<td>Overall average</td>
<td>42.00</td>
<td>19.752</td>
</tr>
</tbody>
</table>

\( \text{Note. } n = 14. \)

comparing the CSE mean scores for the two groups, \( p=.063 \). Students in the CA/S
course had higher statistics self-efficacy scores than those students in the IntroStat
course. Table 5 displays the results of the analysis of this 2-sample t test.

The general linear model 2x2 ANOVA was run on the CAS posttest data as
compared the CSE scores of the IntroStat group. This consisted of the two categorical
factors MathCrse and DevMath. These factors were described earlier in this paper.

The result of the general linear model revealed that there was an interaction

Table 5

*CASE: CA/S Posttest vs. IntroStat 2-Sample t test*

<table>
<thead>
<tr>
<th></th>
<th>CA/S posttest&lt;sup&gt;a&lt;/sup&gt;</th>
<th>IntroStat&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>Overall average</td>
<td>61.00</td>
<td>11.826</td>
</tr>
</tbody>
</table>

<sup>a</sup>n= 15. <sup>b</sup>n=132.

between the two factors (t= -1.954, p=.053). The students in the IntroStat course who took developmental mathematics had significantly lower statistics self-efficacy as compared to the students in the CA/S course who took developmental mathematics.

Table 6 contains the results of the CSE 2x2 ANOVA. Students in the IntroStat course who took developmental mathematics had a mean CSE score that was 11.19 points lower than the mean score for those students in the CA/S course.

Table 6

*CASE: CA/S Posttest vs. IntroStat using 2x2 ANOVA with Factors MathCrse and DevMath*

<table>
<thead>
<tr>
<th></th>
<th>DevMath</th>
<th>No DevMath</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>CA/S&lt;sup&gt;a&lt;/sup&gt;</td>
<td>61.00</td>
<td>12.522</td>
</tr>
<tr>
<td>IntroStat&lt;sup&gt;b&lt;/sup&gt;</td>
<td>49.81</td>
<td>16.800</td>
</tr>
</tbody>
</table>

<sup>a</sup>n= 15. <sup>b</sup>n=132.

**Mathematics Anxiety Ratings Scale (MARS).** To assess changes to mathematics anxiety to students in the CA/S course from the beginning of the semester to the end of the semester, the MARS instrument was used. The null hypothesis for this part of the research question was “While controlling for whether or not developmental mathematics courses were previously taken by the students in the CA/S course, there is
no statistical difference in the means for mathematics anxiety at the beginning of the semester as compared to the end of the semester.”

The overall gain score between the MARS pretest and the MARS posttest for the CA/S course was analyzed. This task was performed by using a paired-difference t test. No statistical significance ($p > .05$) or practical significance was found with $p = .562$. For a sample size of 16, the pretest had a mean of 111.88 with a standard deviation of 15.577 and the posttest had a mean of 115.38 with a standard deviation of 22.030.

Research Question 2

This section of the chapter will discuss the results of the study that addressed the research question, “Does the use of the E-CRA instructional method in a CA/S course produce different outcomes in student performance as measured by pretest and posttest scores while controlling for whether or not the student took developmental mathematics courses?” The purpose of this question is to assess the changes to the performance variables of the students in the CA/S course. The three perspectives were college algebra skill set, statistics numeracy and reasoning skills, and persistence.

Computer Adaptive Placement Assessment and Support System (COMPASS). For an overall analysis of improvement in college algebra content knowledge, the COMPASS test was administered to students in the CA/S course at the beginning and end of the semester. This instrument was used to address the null hypothesis, “While controlling for whether developmental mathematics courses were previously taken by the students in the CA/S course, there is no statistical difference of the means of college algebra skills scores at the beginning of the semester as compared to end of the semester.”
The overall mean for the COMPASS scores for the pretest and the posttest were analyzed for participants in the CA/S course. This evaluation was performed using a paired-difference t test. The COMPASS posttest reported that no gain in college algebra skills was measured from the beginning to the end of the semester. The COMPASS posttest also revealed that the overall mean algebra score had decreased from 30.83 to 28.11 with a sample size of \( n=18 \). Table 7 shows the information from the results of these tests. However, the median score for the course fluctuated slightly from a pretest score of \( M_{dn}=25.5 \) to a posttest score of \( M_{dn}=25 \). The paired-difference t test reported no statistical significance \( (p=.413) \) between the COMPASS pretest mean scores and the COMPASS posttest mean scores.

A correlation was run comparing the gain score, the difference between the pre-COMPASS score and the post-COMPASS score, and the final examination score. Histograms of these two data sets revealed that the COMPASS gain score was skewed left and the final examination scores had a normal distribution. These distributions can be seen in Figures 4 and 5.

Table 7

**COMPASS: CA/S Pretest and Posttest Paired-Difference t test**

<table>
<thead>
<tr>
<th></th>
<th>Pretest</th>
<th>Posttest</th>
<th>( t )</th>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebra score</td>
<td>30.83 25.5</td>
<td>28.11 25.00</td>
<td>-0.839</td>
<td>.413</td>
</tr>
</tbody>
</table>

\( n = 18 \).

Table 8 displays the results of this correlation test. Examining both Pearson correlation results and Spearman’s rho correlation results revealed that there was no significant correlation \( (p>.05) \) found between the COMPASS gain score and the final
examination grade.

Table 8

*Correlation of COMPASS Gain Score and Final Examination Score*

<table>
<thead>
<tr>
<th>Final Exam</th>
<th>COMPASS Gain Score</th>
<th>Correlation Coefficient</th>
<th>Significance (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pearson correlation</td>
<td>-.464</td>
<td>.112</td>
<td></td>
</tr>
<tr>
<td>Spearman’s rho</td>
<td>-.443</td>
<td>.129</td>
<td></td>
</tr>
</tbody>
</table>

*Figure 6.* Histogram of gain score of Post-COMPASS score and Pre-COMPASS score.
Comprehensive Assessment of Outcomes in a First Statistics course (CAOS).

For an evaluation of improvement in statistical numeracy and reasoning skills, a shortened version of the CAOS test was used. The CAOS instrument addressed the null hypothesis, “While controlling for whether developmental mathematics courses were previously taken by the students in the CA/S course, there is no statistical difference of the means of the ‘statistical numeracy and reasoning skills’ scores at the beginning of the semester as compared to end of the semester.”

For the CA/S course, the CAOS pretest and posttest scores were examined using SPSS to perform a paired-difference t test. Table 9 displays the significant results found.
in this paired-difference t test. In the classroom environment, the students in the CA/S course improved with practical significance, \( t=2.142, p=.053 \), on their statistical numerical and reasoning skills. A McNemar test was run on the individual items of the instruments. This test revealed, \( p=.016 \), that the students learned when the \( p \) value is significant while conducting a statistical test.

Table 9

*Shortened CAOS: CA/S Pretest and Posttest Paired-Difference t test*

<table>
<thead>
<tr>
<th>Overall score and item score</th>
<th>Pretest</th>
<th>Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>SD</td>
<td>M</td>
</tr>
<tr>
<td>CAOS score</td>
<td>7.00</td>
<td>9.00</td>
</tr>
</tbody>
</table>

*Note. n = 13.*

The shortened CAOS test was used to determine if the students in the CA/S course were more prepared for their subsequent statistics course as compared to IntroStat students. The comparable group’s CAOS scores were compared to the CAOS posttest scores of the CA/S students taken at the end of the semester. A 2-sample t test was administered to perform this analysis. Table 10 contains the results of this test. The outcome of the t test was that the students in IntroStat seemed to possess stronger overall statistics numerical and reasoning skills than those students in the CA/S course from a practical significance, \( t=1.923, p=.069 \), perspective.

A generalized linear model (GLM) using a binary logistic regression was performed on the individual items of the CAOS test. Table 11 contains the portions of the test outcomes from the analysis of the GLM using binary logistic regression. With statistical significance, the students in the IntroStat group scored higher in two items and responded correctly on one item with practical significance. The students in the
comparable group performed better when interpreting a histogram as indicated by Item 1, $(p < .001)$, as well as mapping an application problem’s raw data to the appropriate histogram as indicated by Item 3, $(p = .004)$. From a practical perspective as indicated by Items 14, $(p = .082)$, and 20, $(p = .069)$, the students in the comparable group were interpreting scatter plots correctly in an application problem and both selecting the histogram with the smallest standard deviation from a collection of histograms as well as understanding why they should select that histogram. There were two skills for which the students in the CA/S had a higher mean score than those students in the comparable group. The CA/S students (a) analyzed two box plots, which had the same median, and correctly determined that both box plots had the same percentage of objects above the median, $(p = .032)$; and (b) understood when a $p$ value is significant while conducting a statistical test with statistical significance as indicated by Item 19, $(p = .006)$.

The general linear model 2x2 ANOVA was run on the CA/S posttest data as compared to the same data for the IntroStat group. This consisted of the two categorical factors MathCrse and DevMath. The sample size of the CA/S students was $n = 15$; the sample size of the students in IntroStat was $n = 130$. The excerpts of the results of the 2x2 ANOVA can be found in Table 12. The DevMath factor was found to have a main effect on the CAOS scores, $(p < .001)$. When combining both CA/S and IntroStat courses, those
Table 11

**Shortened CAOS: CA/S Posttest vs. IntroStat GLM Binary Logistic Regression**

<table>
<thead>
<tr>
<th>Significant items</th>
<th>$X^2$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>01: Read and interpret a histogram&lt;sup&gt;a&lt;/sup&gt;</td>
<td>15.050</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>03: Map an application problem’s raw data to the appropriate histogram&lt;sup&gt;a&lt;/sup&gt;</td>
<td>8.516</td>
<td>.004</td>
</tr>
<tr>
<td>10: Analyze two box plots, which had the same median, and correctly determine that both box plots had the same percentage of objects above the median&lt;sup&gt;b&lt;/sup&gt;</td>
<td>4.618</td>
<td>.032</td>
</tr>
<tr>
<td>14: Select the histogram with the smallest standard deviation from a collection of histograms and understand why&lt;sup&gt;a&lt;/sup&gt;</td>
<td>3.018</td>
<td>.082</td>
</tr>
<tr>
<td>19: Understand when the $p$ value is significant when conducting a statistical test&lt;sup&gt;b&lt;/sup&gt;</td>
<td>7.516</td>
<td>.006</td>
</tr>
<tr>
<td>20: Interpret a scatter plot for an application&lt;sup&gt;a&lt;/sup&gt;</td>
<td>3.297</td>
<td>.069</td>
</tr>
</tbody>
</table>

Note.  $n = 15$ for CA/s.  $n = 133$ for IntroStat.  
<sup>a</sup>IntroStat performed better on this item.  <sup>b</sup>CA/S performed better on this item.

students who did not take developmental mathematics had a higher CAOS mean score than those students who took developmental mathematics.  Students who did not take developmental mathematics scored 11.0 out of 20 points on the test.  Whereas, students who took developmental mathematics scored 8.4 out of 20 possible points.  Also, an interaction between the factors MathCrse and DevMath was found to be statistically significant, ($p = .016$).  Students in the IntroStat course who did not take developmental mathematics scored higher than students in the CA/S course who did take developmental mathematics.
Table 12

**Shortened CAOS: General Linear Model using ANOVA 2x2 for CA/S course as Compared to IntroStat course**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Main effect model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.419</td>
<td>4.63</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>CAOS_Score = intercept + DevMath</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.542</td>
<td>2.542</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Intercept</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NoDevMath</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DevMath</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interaction model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.636</td>
<td>9.434</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>CAOS_Score = intercept + MathCrse*DevMath</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.364</td>
<td>2.448</td>
<td>.016**</td>
</tr>
<tr>
<td>Intercept</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-.293</td>
<td>-0.276</td>
<td>.783</td>
</tr>
<tr>
<td>MathCrse=IntroStat &amp; NoDevMath</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.364</td>
<td>0.769</td>
<td>.443</td>
</tr>
<tr>
<td>MathCrse=CA/S &amp; NoDevMath</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MathCrse=CA/S &amp; DevMath</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note. Dev Math = Developmental Mathematics. n = 15 for CA/S. n = 133 for IntroStat. **p < .05.

**Persistence.** To determine if the attrition rate of the CA/S course was significantly better as compared to college algebra courses, the attrition rate of the CA/S course was compared to all entry-level college algebra courses. The null hypothesis for persistence was, “While controlling for whether or not developmental mathematics courses were previously taken by the students in the CA/S course, there is no statistical difference of the means of the attrition rates of the CA/S course as compared to a traditional college algebra course.” This included the typical college algebra course, liberal arts college algebra, and business college algebra, all taught during the spring 2011 semester. A logistic regression was run on the completion rates of the CA/S against these different types of college algebra courses. No statistical significances (p > .05) were found.

Next, an analysis was performed to determine if the DFW proportion of the CA/S
was significantly improved as compared to the group of college algebra classes mentioned in the previous paragraph that were conducted during the Spring 2011 semester. A GLM using binary logistic regression was run for all the students in the entry-level college algebra courses. No statistical significance ($p > .05$) was found when comparing the DFW rates of the CA/S course as compared to the other types of college algebra courses. This GLM was then run with the fixed categorical factors of (a) in which college algebra course a student was enrolled and (b) whether or not the student took developmental mathematics. There was no interaction effect between which type of college algebra course a student was enrolled in and whether or not the student took developmental mathematics.

**Qualitative Survey**

Administered at the end of the semester, 15 CA/S students filled out the qualitative survey found in Appendix D. The first two survey questions addressed the fact that real-world data were used to teach mathematics and statistics. In the comments written for Survey Question 1, “Do you like starting a new mathematics content section with a real-world application or data?”, 13 out of 15 students reported that they liked using real-world data at the beginning of a new mathematics topic. One student wrote, “Yes, it helps you see how it will apply to your life, rather than just to get a credit.” For Survey Question 2, “Do you like how the class lessons would refer back to real-world problems that were previously used?”, all the students had positive responses. Some of the positive remarks were “Yes, it’s served as a good way to review,” “Yes, it was easier to relate,” and “Yes, easier to understand.”

The third and fourth survey questions asked the student what they liked the most
and the least about the CA/S course, respectively. The fifth question asked what changes to the class are recommended by the student. Table 13 contains the responses to these questions using the coding of curriculum, pedagogy, instructor, examinations and quizzes, homework, required work and activities associated with the grant project, or nothing. If a student wrote comments that mapped to multiple categories of the qualitative coding, then the comment would be counted toward every category referred to by the response.

What is interesting to note is that no student made the comments similar to “I don’t like mathematics” or “I am not good at math,” which are considered negative remarks. Consequently, no negative remarks were found regarding mathematics, statistics, the curriculum, or instructional methods.

Table 13

*Responses to Qualitative Survey Questions 3-5*

<table>
<thead>
<tr>
<th></th>
<th>What I liked most</th>
<th>What I liked least</th>
<th>What I would change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curriculum</td>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Pedagogy, teaching methods</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Instructor</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Exams/quizzes</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Homework</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Required grant tasks: Activities/workshops</td>
<td>0</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Tutoring</td>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Paperwork</td>
<td>0</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>“Nothing”, “N/A” was the reply</td>
<td>0</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

*Note. n = 15.*
CHAPTER 5
DISCUSSION

From an assessment of the information and research results embodied in Chapters 1 through 4, this chapter concludes this research study. Remarks on the curriculum are provided. Secondly, the conclusions to the two research questions are given. Finally, suggestions for instructing this curriculum and recommendations for future research can be found.

Curriculum

The lesson plans worked well in delivering the proposed content, but there are changes that need to be implemented to address reading and interpreting graphs. The handouts that teach how to perform regression analysis on the graphing calculators need to be rewritten. These handouts need to help students to quickly develop their modeling skills on the calculator. The lesson plans need to weave statistics into the curriculum more smoothly. More statistics topics need to be taught.

Conclusions

In this section of the chapter, conclusions derived for the research questions associated with this research project are presented. The data collected and the results of the statistical tests conducted are the premise of the conclusions made.

Research Question 1. The first research question investigated by this research project was, “Does the use of the experiential concrete-representational-abstract (E-CRA) instructional method in college algebra with statistics (CA/S) course produce changes in
students’ affective domain as measured by pretest and posttest scores while controlling for whether or not the student took developmental mathematics courses?” The three perspectives analyzed were attitudes towards statistics, statistical self-efficacy, and mathematics anxiety.

Based on the results of the Attitudes Towards Statistics (ATS) tests conducted, the CA/S students’ attitudes did not change from the beginning to the end of the semester towards the field of statistics or a statistics course. The CA/S students at the end of the semester were statistically no different than those students entering the introductory statistics (IntroStat) course who did not take the CA/S course. Whether or not students had taken developmental mathematics courses was not a contributing factor on the students’ attitudes towards statistics in either the CA/S or the IntroStat courses.

However, the Current Statistics Self-Efficacy (CSE) tests revealed that the CA/S students gained a considerable amount of self-confidence in their ability to perform statistical tasks. The improvement in the students’ self-efficacy was supported by the qualitative survey. The CA/S course consisted of a population of students from which over 80% had taken developmental mathematics. Yet, the qualitative survey analysis of all CA/S students’ responses reported an absence of negative comments regarding the students’ beliefs in their ability to perform statistics and mathematics. Negative comments were not found in the qualitative survey—comments such as “I am bad at math,” “I don’t like math,” or “I don’t like statistics” which are usually seen in a qualitative student survey for a mathematics course. Finney and Schraw (2003) reported that statistics self-efficacy was related to performance in statistics; they found that as a student’s statistics self-efficacy became stronger, the student’s ability to perform
statistical tasks also improved. Thus, one possible conclusion in this research study is that the improvement among CA/S students in performing statistical tasks relates to students’ stronger self-efficacy.

With practical significance, the CA/S students possessed stronger confidence in their abilities to perform statistical tasks as compared to the students in the IntroStat course. With statistical significance, students in the CA/S course who took developmental mathematics held stronger beliefs that they could learn and use statistics as compared to the students in the IntroStat course who took developmental mathematics courses.

One of the goals of the CA/S course was to show the relevance of mathematics and statistics to the students. The majority of the students’ responses on the qualitative survey stated that the students liked using real-world data to learn new mathematical topics. Examining concrete, real-world data; analyzing and interpreting different representations of the data; and then connecting mathematics symbols or algebraic equations to these data all give students the opportunity to comprehend and perceive more purpose in the use of statistics and mathematics. The experiential process was completed by guiding the students to model the real-world data with the algebraic equations discovered. The CA/S students made no negative comments about learning and using statistics. Students who have not taken the CA/S course do not have the same opportunity to gain knowledge of what statistics entails. Consequently, these students who have not taken CA/S have limited knowledge of whether or not they can successfully do well in statistics.

No statistical significances (p > .05) were observed from the analysis of the
Mathematics Anxiety Ratings Scale (MARS) test. The curriculum and the E-CRA instructional method, as outlined in this research study, did not alter the mathematics anxiety level of the CA/S students.

**Research Question 2.** The second research question asked, “Does the use of the E-CRA instructional method in a CA/S course produce different outcomes in student performance as measured by pretest and posttest scores while controlling for whether or not the student took developmental mathematics courses?” The college algebra skill set, statistics numeracy and reasoning skills, and persistence were the three perspectives addressed by this question.

From the context of academic performance and persistence, the mean COMPASS posttest score reported no gain in algebra skills. An examination of the COMPASS scores and final course grades revealed that students who made an A in the course had low COMPASS scores. In contrast, students making final course grades in the high 60s and low 70s had some of the highest COMPASS posttest scores in the class. Since no statistical significance \((p > .05)\) was found for a change in algebra skills, a correlation test was performed to see if there was a relationship between the final examination grade and the COMPASS gain score, in other words, the difference between COMPASS posttest and pretest. The correlation test reported no correlation with statistical significance \((p > .05)\) between the final examination grade and the COMPASS gain score. This lack of a significant correlation between these two variables implies that the COMPASS scores are not indicative of academic performance in the classroom.

The COMPASS test was administered at the University’s testing center, whereas all other instruments used in the research study were given in the classroom by the
instructor. An inspection of the qualitative survey, which was administered to the students after the students took the COMPASS posttest, disclosed no student comments regarding the COMPASS test.

This research study noted several events that could have contributed to the COMPASS mean posttest score that measured no improvement in college algebra skills. The students in the CA/S class lost a total of four class days, out of 29. Approximately three class days were consumed by administrative tests, discussions, paperwork, and tasks. Another class day was cancelled because the University was experiencing power outages as documented in Appendix E. Additionally, three test days were scheduled. These test days were recorded in the Scope and Sequence for the course located in Appendix B and the teaching journal found in Appendix E. One student did comment that she would have preferred more class tests and quizzes to reduce the amount of content covered per test. Unfortunately, losing another class day to proctor a fourth test would have put more burdens on delivering the planned curriculum.

Another explanation for the unexpected COMPASS scores was that the tutoring services available to the CA/S students were inconsistent. The tutoring aspects of the project had problems as was communicated by the many negative comments and the lack of positive comments about tutoring in the qualitative survey. Thus, tutoring support for statistics outside of the classroom was not as strong as it could have been. Additionally, the teaching journal contained comments that the students did complain in the classroom about the help they were receiving from tutors. For example, several CA/S students informed the instructor that one of the tutors informed them that their calculators were broken (they were not). The instructor had to demonstrate to the students that their
calculators were functioning, which resulted in the students’ confidence in the tutors’ abilities being compromised.

Since there was a decrease in the COMPASS scores from the beginning to the end of the semester, this outcome generated concerns about whether or not the students in the CA/S course applied themselves with sincere effort when taking the COMPASS exam. The COMPASS exam was given in the testing center for the University. All other tests associated with this study were administered by the instructor in the classroom.

With practical significance, students in the CA/S course gained stronger statistics numerical and reasoning skills after completing the course. This improvement appears to be the result of the curricular changes in the course to enhance these skill sets. Though the students in the IntroStat course exhibited higher achievement in this skill set than the CA/S students did at the end of the semester, the results were mixed when examining the individual items in the CAOS test. CA/S students demonstrated stronger knowledge on topics covered in the curriculum such as (a) reading and analyzing box plots with the same median and (b) interpreting \( p \) values. The students in the IntroStat course had more robust skills interpreting histograms and scatter plots.

Students in the CA/S course who took developmental mathematics possessed limited knowledge of these statistical numerical and reasoning skills as compared to students in the IntroStat course who did not take developmental mathematics. The curriculum and the E-CRA instructional method benefitted those students in the CA/S course who had previously taken developmental mathematics courses.

It should be noted that students who were science, technology, engineering, and mathematics (STEM) majors were enrolled in the IntroStat course. STEM majors
possess stronger mathematics skills and aptitudes and many had already taken pre-
calculus or calculus courses. As a result, STEM majors receive high scores on
mathematics skills tests. Several of the STEM degrees, such as engineering and biology,
require a statistics course or recommend statistics as an elective, which is why STEM
majors are enrolled in the IntroStat course. On the contrary, the criteria used to select
students for the CA/S course limits the number of STEM majors in order to satisfy the
purpose of the research grant. This difference in the populations of the two courses is
another explanation for why the IntroStat mean CAOS score was higher than the mean
CAOS posttest score of the CA/S course.

Another explanation for why the students in the IntroStat course scored higher on
the CAOS tests as compared to the CA/S students is that the testing conditions between
the two groups were different. The students in the IntroStat course were given the CAOS
test as a homework assignment, where the students were awarded an extra credit quiz
grade upon completing the test. Also, the IntroStat students were taught some beginning
statistics content on the first class prior to taking the CAOS test. The CA/S students took
the CAOS test during the first class day. Consequently, there was a time limit imposed
on the test. No statistics or mathematics content was taught during the first class day of
the CA/S course due to pre-testing. The assumption is that these differences in the testing
conditions had no impact to the results on this test. Further research may need to be
conducted to verify this assumption.

The attrition and DFW rates of the CA/S course were not statistically different
from other college algebra courses offered during the same semester. However, the DFW
rate of 35% recorded for the CA/S course was below the national average of 40-60% as
reported in Chapters 1 and 2.

**Overall.** In general, this research project found that the CA/S curriculum utilizing the E-CRA instructional method seemed to positively relate to the affective domain of CA/S students. Statistics self-efficacy, or students’ beliefs in their abilities to do statistical tasks, became significantly stronger. Accordingly, students’ statistics self-efficacy improved. There were no changes in attitudes towards statistics. These results are similar to other research studies where interventions were introduced into an introductory statistics class (Olani, Hoestra, Harskamp, & van der Werf, 2010a). It is possible that adjustments to the framework of the curriculum and the E-CRA instructional method may produce significant results for the perspective of students’ attitudes towards statistics.

**Suggestions**

This section includes suggestions to instructors who plan to teach this curriculum using the E-CRA instructional method. It is strongly suggested that a college algebra textbook written with the purpose of exploring modeling be utilized for this curriculum and pedagogy. The CA/S course studied in this research project used the same college algebra textbook that was required in the regular college algebra course. Though this textbook included modeling topics and homework exercises, the students and the instructor of the CA/S course could perceive the textbook’s overall goal to prepare students for calculus.

Students should be given opportunities to bring in their own data to the classroom with guidance. Students know how data can change in real-world situations. When students make the connection of how mathematics and statistics can be used to
describe these changes, students perceive the benefits and relevance to themselves (Gordon, 2008a).

When teaching a particular topic like exponential functions, the instructor needs to provide guidance on certain characteristics that the data need to have. In other words, the instructor should tell the students to bring data that result from some kind of growth. Students should be thinking about a linear growth, though they may not be aware that their data could actually be exponential.

Many mathematics tutors do not possess statistical skills and knowledge. It is common at many postsecondary schools to hire undergraduate mathematics majors as tutors. The degree plan pursued by most of these mathematics students often does not include a statistics course. The instructor of the CA/S course should follow up and work with the mathematics tutors in advance and during the semester to ensure that the tutors can provide the services needed by the students of this curriculum.

**Recommendations**

This section of the chapter will provide recommendations for future research which should be considered when conducting similar research on the CA/S curriculum utilizing the E-CRA instructional methods. At least a class of 40 students or two classes of 20 is preferable, with the assumption that after drops and withdrawals, there will still be at least 30 students remaining till the end of the course.

In the future another aspect in the factorial design of this research study is whether or not the student is a STEM major. This categorical factor could help the researcher determine if non-STEM majors are benefitting from the CA/S with E-CRA instructional method as compared to STEM majors.
Often, students in the fall semester are more motivated to do well in their academic studies. During the spring semester many students are not as driven to perform to the best of their ability. Since this study was conducted in the spring semester at this university, a similar research project scheduled for the fall semester is recommended to scrutinize whether or not different results are found.

This course was taught under the umbrella of a grant with the objectives to improve the success of developmental mathematics students. This grant project also strives to observe and improve the success of those prior developmental students in gateway mathematics courses. To meet the requirements of the grant, classroom instruction time was used to administer instruments, ensure student time logs were filled out, and other administrative tasks. Statistical content such as confidence intervals or Type I or II errors was not taught. To regain some of this class time, the future research study is advised to assign all of the CAOS, CSE, and ATS instruments as a homework assignment on the first day of class.

The instruments used in this research study are needed in order to measure the effectiveness of this curriculum. To provide positive re-enforcement to ensure the validity of the test results, incentives need to be given so that the students will perform to the best of their ability on the COMPASS and CAOS tests. The researcher should consider including pretest and posttest COMPASS and CAOS test scores into the grading scheme for the course. Extra credit points for performance on these tests may be another avenue used to encourage students to perform to the best of their abilities.

A continuation of this research study would be to analyze the individual items in the Attitudes Towards Statistics (ATS) and the Current Statistics Self-Efficacy (CSE)
tests. There were changes at the item level that appear to be significant. An initial inspection of the pretest and posttests items showed that students are changing their responses from the pretest as to the posttest responses. To interpret this change, a generalized linear model (GLM) using ordinal regression needs to be performed. This GLM using ordinal regression of all the items, comparing pretest responses to posttest responses, could provide additional knowledge regarding CA/S students’ change of perspectives concerning attitudes and self-efficacy. The results of reviewing the individual items of the tests could provide a researcher with direct inputs to the adjustments of the curriculum and instructional methods used in the CA/S course.

This future research study should also include reviewing the 20 items selected for the shortened CAOS test. A determination needs to be made regarding which of these items should remain on this instrument for a similar study. As part of this study, the validity and reliability of the shortened CAOS test should be tested.

Subsequent research projects need to belong to an expanded longitudinal study. This longitudinal study would monitor the success of the CA/S students as they take their statistics courses. These studies should continue to use the factor of whether or not the students took developmental mathematics. The studies should measure whether or not strong statistical self-efficacy is still being reported by these students and whether or not these students’ attitudes towards statistics remain unchanged.

**Implications of this Research Study**

Offering this course could help those postsecondary students requiring both a college algebra course and a statistics course. This population of college students includes students majoring in academic disciplines such as psychology, sociology,
criminal justice, political science, and health administration. As these students complete the CA/S curriculum, they will be more prepared for their following statistics course. These students have stronger confidence in their abilities to perform tasks in their statistics course, and their performance in the statistics class may improve as the result of this stronger self-confidence in doing statistical tasks. The result of this improved performance is that a greater number of students will successfully complete the CA/S course and the following statistics course, without having to repeat either of these two courses.

Summary

The purpose of this study is to evaluate the effectiveness of the experiential concrete-representational-abstract (E-CRA) instructional method, when integrating statistics into a college algebra with statistics (CA/S) course (a) for those students who were required and then subsequently completed developmental mathematics courses and (b) for those students who were designated college-ready without the need for developmental mathematics. The students in the CA/S course were required to take a statistics course as part of their degree plan. The CA/S group did not have an academic need to pursue pre-calculus or calculus.

This research study presents the problem that in the United States, many postsecondary students find that they are required to take several mathematics courses as part of the degree or certification plan that they are pursuing. A review of the literature stated that the number of mathematics courses required of a college student is impacted by (a) whether or not a student had to take developmental mathematics, (b) the student’s success in a college algebra course, and/or (c) whether or not the student was pursuing a
statistics or calculus track as directed by their college major’s degree plan (Achieving the Dream, 2010; Bressoud, 2010; Bailey, 2009; Ganter & Barker, 2004). The literature review also reveals that college algebra curriculum does prepare postsecondary students for calculus, but only 10-20% of college algebra students go on to take calculus (Gordon, 2008a). If a college student finds that a large number of mathematics courses will be required, the number of mathematics courses is perceived as intimidating and a large hurdle to overcome (Bailey, Jeong, & Cho, 2010). These authors also commented that many postsecondary students lose interest in completing their mathematics courses when those students have a long list of mathematics courses that must be completed. They find little relevance between themselves and these mathematics courses. The combination of these two factors, intimidation and little perceived relevance, creates an environment where many students do not fulfill the mathematics requirements specified by their degree or certification plan. Consequently, these students often do not complete their collegiate goals.

This research found that the integration of statistics into the curriculum of a college algebra course while utilizing the E-CRA instructional method produced a significant, positive impact to the students’ statistical self-efficacy. With practical significance in a classroom environment, the students in the CA/S course improved their performance levels for statistical numeracy and reasoning skills. The E-CRA instructional method provided opportunities for the students to work with real-world data and discover the relevance of mathematics and statistics to real-world situations.
APPENDIX A

Daily Lesson Plan Template

File Name
- Course_Topic_Initials_YYYY_MM_DD
- Document Title

Lesson Title
- General description of the lesson

Prerequisites
- What does the student need to know prior to this lesson?

Content Objective(s)
- Breakdown of subtopics of the lesson
- What students should be able to do at the end of this lesson
- Measurable
- Observable
- How achievement of the objective will be measured
- Begins with an action word

Technology Objective(s)
- How students will use technology in the lesson

Text References
- Instructional objective should have text references, when applicable
- Include text description (XYZ textbooks, or Blitzer) and text section (in parentheses)

Culturally Relevant Objective(s)
- How is the lesson relevant to the culture of your student population?

Real-World Applications
- Anticipatory set: Real world story to engage students in the topic

Study Strategies
- Consider test preparation, test-taking, test analysis, note-taking, reading the text, etc.

Connections
• Connections to other areas of mathematics, as noted in the Texas CCRS

Process Standards
• Inclusion of Process Standards, as noted in the Texas CCRS

Lesson Agenda
• Includes outline of introduction/background, main lesson script, applications/connections, and summary/extension
• “The Lesson at a Glance”
• Maintain outline form

Materials
• List of what students need for the lesson (per student, per group…)
• List of what instructor needs for the lesson
  • Handouts
  • Technology
  • Manipulatives
  • Other creative material

Pre-Homework
• Global, mind-opening ideas to get the learner ready to receive the lesson
• What definitions/terms and other basic knowledge is needed?

Introduction/Background
• Link to previous material
• Set tone using historical or anecdotal story

Main Lesson Script
• Scripted lesson which includes:
  • Content- may be delivered through lecture, cooperative group work…
  • Terms and theorems from text in **bold** accompanied by elaboration or definition in (parenthesis)… if you use a secondary source, cite it!
  • Activities… include directions for instructor
  • Activity: Title, Italicize directions
  • HO: Worksheet Title
  • Questioning techniques: questions you may ask with anticipated responses in [brackets]
  • Examples and exercises
  • References to handouts and keys
  • Technology
  • Don’t forget to incorporate study strategies, process standards, AIT model, PBL, movement from concrete to abstract whenever possible!

Summary/Extension
• Check for Understanding
• Can use graphic organizers, questioning, and other means to help students solidify information
• Foreshadow to future lessons or courses

Possible Assessments from Today’s Lesson
• What do you think are the main ideas of the lesson?
• How would you assess these main ideas?
• Incorporate problem solving

Teacher Reflection
• Self-evaluation of the presentation of the lesson
• Class time: too much/not enough time allotted for lesson, spent too much/not enough time on a particular component of lesson, etc.
• Worked well: comment on positive aspects of lesson
• Recommendations: sequencing, flow of lesson, how would you improve the lesson itself, or how would you improve the way you taught the lesson?

Other Comments about the Lesson
• Time management
• Sequencing from previous lesson

References
• APA style (6th ed.) 3rd printing
• Cite references in text using APA style (6th ed.) 3rd printing

Attachments
• Handouts need to be attached after the lesson plan template.
• Needs to have “Adapted by…”, etc. – Cite where the material came from
<table>
<thead>
<tr>
<th>File Name</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Lesson Title</td>
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<tr>
<td>Prerequisites</td>
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<table>
<thead>
<tr>
<th>Content Objective(s)</th>
<th>E-CRA</th>
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<table>
<thead>
<tr>
<th>Technology Objective</th>
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<table>
<thead>
<tr>
<th>Culturally Relevant Objective(s)</th>
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<table>
<thead>
<tr>
<th>Real-World Applications</th>
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<table>
<thead>
<tr>
<th>Connections</th>
<th>Process Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebra</td>
<td>Communication</td>
</tr>
<tr>
<td>Geometry</td>
<td>Connections</td>
</tr>
<tr>
<td>Measurement</td>
<td>Problem-Solving</td>
</tr>
<tr>
<td>Statistics</td>
<td>Representations</td>
</tr>
<tr>
<td>Probability</td>
<td>Reasoning</td>
</tr>
<tr>
<td>Functions</td>
<td>Cross-Disciplinary</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Lesson Agenda</th>
<th>Instructor Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td>Student Materials</td>
<td></td>
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</tr>
</tbody>
</table>

Pre-Homework

Introduction/Background

Main Lesson Script

Summary/Extension

Possible Assessments from Today’s Lesson

Teacher Reflection
  - Did you use the class time wisely?
  - What worked well in the lesson?
  - What recommendations would you make to improve this lesson?

Other Comments About the Lesson

References
APPENDIX B

Scope and Sequence

B-1 Proposed Scope and Sequence
B-2 Implemented Scope and Sequence
### B-1 Proposed Scope and Sequence

<table>
<thead>
<tr>
<th>Week/Day</th>
<th>Main Topic/Textbook</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1/Day 1</td>
<td>Class Syllabus, pre-test, etc.</td>
</tr>
<tr>
<td>Jan 19, 2011</td>
<td></td>
</tr>
<tr>
<td>Week 2/Day 1</td>
<td>Statistics : Blitzer 12.1</td>
</tr>
<tr>
<td>Jan 24, 2011</td>
<td>1. Sampling, Frequency, Distribution</td>
</tr>
<tr>
<td></td>
<td>2. Describe population and sample</td>
</tr>
<tr>
<td></td>
<td>3. Select sampling technique</td>
</tr>
<tr>
<td></td>
<td>4. Organize &amp; present data</td>
</tr>
<tr>
<td>Week 2/Day 2</td>
<td>Statistics : Blitzer 12.2</td>
</tr>
<tr>
<td>Jan 26, 2011</td>
<td>1. Measures of Central Tendency</td>
</tr>
<tr>
<td></td>
<td>2. Determine mean, median, mode, and midrange</td>
</tr>
<tr>
<td>Week 3/Day 1</td>
<td>Statistics : Blitzer 12.3</td>
</tr>
<tr>
<td>Jan 31, 2011</td>
<td>1. Measures of Dispersions</td>
</tr>
<tr>
<td></td>
<td>2. Determine range of data</td>
</tr>
<tr>
<td></td>
<td>3. Find standard deviation</td>
</tr>
<tr>
<td>Week 3/Day 2</td>
<td>Statistics : Blitzer 12.4</td>
</tr>
<tr>
<td>Feb 2, 2011</td>
<td>- Normal Distribution</td>
</tr>
<tr>
<td></td>
<td>- Characteristics of normal distribution</td>
</tr>
<tr>
<td></td>
<td>- Understand Empirical Rule</td>
</tr>
<tr>
<td></td>
<td>- Use Empirical Rule</td>
</tr>
<tr>
<td></td>
<td>- Convert to z-score</td>
</tr>
<tr>
<td></td>
<td>- Understand percentiles/quartiles</td>
</tr>
<tr>
<td></td>
<td>- Use/interpret margin of error</td>
</tr>
<tr>
<td></td>
<td>- Other distributions</td>
</tr>
<tr>
<td>Week 4/Day 1</td>
<td>Statistics : Blitzer 12.5</td>
</tr>
<tr>
<td>Feb 7, 2011</td>
<td>- Problem solving with the normal distribution</td>
</tr>
<tr>
<td></td>
<td>- Solve applications problems using normal distribution</td>
</tr>
<tr>
<td>Week 4/Day 2</td>
<td>Statistics : Blitzer 12.6</td>
</tr>
<tr>
<td>Feb 9, 2011</td>
<td>- Scatter plots, correlation, and linear regression</td>
</tr>
<tr>
<td></td>
<td>- Make scatter plot from table of data</td>
</tr>
<tr>
<td></td>
<td>- Interpret information in a scatter plot</td>
</tr>
<tr>
<td></td>
<td>- Calculate correlation coefficient</td>
</tr>
<tr>
<td></td>
<td>- Use linear regression</td>
</tr>
<tr>
<td>Week 5/Day 1</td>
<td>Test 1</td>
</tr>
<tr>
<td>Feb 14, 2011</td>
<td></td>
</tr>
</tbody>
</table>
| Week 5/Day 2 | - Gather sample data that resembles quadratic patterns  
| Feb 16, 2011 | - Curve fitting |
| Week 6/Day 1 | Algebra/Stats: Dugopolski 1.5  
| Feb 21, 2011 | - Scatter Diagrams and Curve Fitting  
| | - Line of best fit  
| | - Modeling |
| Week 6/Day 2 | Algebra/Stats: Dugopolski 1.6  
| Feb 23, 2011 | 1. Quadratic Equations  
| | 2. Modeling data: quadratic regression |
| Week 7/Day 1 | Algebra/Stats: Dugopolski 2.1-2.2  
| Feb 28, 2011 | - Functions  
| | - Identify functions  
| | - Domain and range  
| | - Function notation  
| | - Square function  
| | - Square root function  
| | - Horizontal parabola  
| | - Cube and cube root  
| | - Increasing, decreasing, constant |
| Week 7/Day 2 | - Gather before and after data where an intervention caused a translation  
| Mar 2, 2011 | - Student data collected/generated that resembles linear regression before and after intervention. (Models translation)  
| | - Curve fitting |
| Week 8/Day 1 | Algebra/Stats: Dugopolski 2.3  
| Mar 7, 2011 | - Families of Functions, Transformations |
| Week 8/Day 2 | Algebra/Stats: Dugopolski 2.4  
| Mar 9, 2011 | - Operations of Functions  
| | - Modeling/applications  
| | - Composition of functions |
| Week 9/Day 1 | Algebra/Stats: Dugopolski 2.5  
| Mar 21, 2011 | - Inverse Functions |
| Week 9/Day 2 | Test 2 |
| | Mar 23, 2011 |
| Week 10/Day 1 | Algebra/Stats: Dugopolski 3.1-3.2  
| Mar 28, 2011 | - Polynomial Functions  
| | - Graphs of quadratic functions  
| | - Zeros of polynomial functions |
| Week 10/Day 2 | Algebra/Stats: Dugopolski 3.3  
| Mar 30, 2011 | - Theory of equations |
| Week 11/Day 1 | Gather data that resembles exponential and logarithmic functions  
| Apr 4, 2011 |
| Week 11/Day 2 | Algebra/Stats: Dugopolski 4.1  
| Apr 6, 2011 | Exponential Functions |
| Week 12/Day 1 | Apr 11, 2011 | Algebra/Stats: Dugopolski 4.2  
- Understand definition  
- Know Domain  
- Know graphs  
- Modeling |
<table>
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</thead>
<tbody>
<tr>
<td>Week 12/Day 2</td>
<td>Apr 13, 2011</td>
<td>Exponential and logarithmic functions (continued)</td>
</tr>
<tr>
<td>Week 13/Day 1</td>
<td>Apr 18, 2011</td>
<td>Test 3</td>
</tr>
</tbody>
</table>
| Week 13/Day 2 | Apr 20, 2011 | Algebra/Stats: Dugopolski 8.4-8.5  
- Counting, permutations, combinations |
| Week 14/Day 1 | Apr 25, 2011 | Algebra/Stats: Dugopolski 8.6  
- Probability |
| Week 14/Day 2 | Apr 27, 2011 | Z-test, t-test, confidence interval |
| Week 15/Day 1 | May 2, 2011 | Hypothesis testing |
## B-2 Implemented Scope and Sequence

<table>
<thead>
<tr>
<th>Week/Day</th>
<th>Main Topic/Textbook</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1/Day 1</td>
<td>Class Syllabus, pre-test, etc.</td>
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<tr>
<td>Jan 19, 2011</td>
<td>Statistics : Blitzer 12.1</td>
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<tr>
<td>Week 2/Day 1</td>
<td>5. Sampling, Frequency, Distribution</td>
</tr>
<tr>
<td>Jan 24, 2011</td>
<td>6. Describe population and sample</td>
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<tr>
<td></td>
<td>7. Select sampling technique</td>
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<td>Week 2/Day 2</td>
<td>Statistics : Blitzer 12.2</td>
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<tr>
<td>Jan 26, 2011</td>
<td>3. Measures of Central Tendency</td>
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<td>4. Determine mean, median, mode, and midrange</td>
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<tr>
<td>Week 3/Day 1</td>
<td>Statistics : Blitzer 12.3</td>
</tr>
<tr>
<td></td>
<td>5. Determine range of data</td>
</tr>
<tr>
<td></td>
<td>6. Find standard deviation</td>
</tr>
<tr>
<td>Week 3/Day 2</td>
<td>Class canceled because of rolling black outs.</td>
</tr>
<tr>
<td>Feb 2, 2011</td>
<td>Statistics : Blitzer 12.4</td>
</tr>
<tr>
<td>Week 4/Day 1</td>
<td>- Normal Distribution</td>
</tr>
<tr>
<td>Feb 7, 2011</td>
<td>- Characteristics of normal distribution</td>
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<tr>
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<td>- Understand Empirical Rule</td>
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<td>- Use Empirical Rule</td>
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<td></td>
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<td></td>
<td>- Use/interpret margin of error</td>
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<tr>
<td></td>
<td>- Other distributions</td>
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<tr>
<td>Week 4/Day 2</td>
<td>Statistics : Blitzer 12.6</td>
</tr>
<tr>
<td>Feb 9, 2011</td>
<td>- Scatter plots, correlation, and linear regression</td>
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<tr>
<td></td>
<td>- Make scatter plot from table of data</td>
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<tr>
<td></td>
<td>- Interpret information in a scatter plot</td>
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<tr>
<td></td>
<td>- Calculate correlation coefficient</td>
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<tr>
<td></td>
<td>- Use linear regression</td>
</tr>
<tr>
<td></td>
<td>- Linear equations</td>
</tr>
<tr>
<td>Week 5/Day 1</td>
<td>Linear equations (cont)</td>
</tr>
<tr>
<td>Feb 14, 2011</td>
<td>- Linear equations (cont)</td>
</tr>
<tr>
<td>Week 5/Day 2</td>
<td>- Linear equations (cont)</td>
</tr>
<tr>
<td>Feb 16, 2011</td>
<td>Test 1</td>
</tr>
<tr>
<td>Week 6/Day 1</td>
<td>- Gather sample data that resembles quadratic</td>
</tr>
<tr>
<td>Feb 21, 2011</td>
<td>- Gather sample data that resembles quadratic</td>
</tr>
<tr>
<td>Date</td>
<td>Topic</td>
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<td>Feb 23, 2011</td>
<td>patterns - Curve fitting</td>
</tr>
<tr>
<td>Week 7/Day 1</td>
<td>Algebra/Stats: Dugopolski 1.5/1.6 - Line of best fit - Modeling - Quadratic equations</td>
</tr>
<tr>
<td>Mar 2, 2011</td>
<td>3. Quadratic Equations (cont)</td>
</tr>
<tr>
<td>Week 7/Day 2</td>
<td>Algebra/Stats: Dugopolski 1.6 - Modeling data: quadratic regression</td>
</tr>
<tr>
<td>Mar 2, 2011</td>
<td>4. Quadratic Equations (cont)</td>
</tr>
<tr>
<td>Week 8/Day 1</td>
<td>Algebra/Stats: Dugopolski 1.6 - Modeling data: quadratic regression</td>
</tr>
<tr>
<td>Mar 7, 2011</td>
<td>4. Quadratic Equations (cont)</td>
</tr>
<tr>
<td>Week 8/Day 2</td>
<td>Algebra/Stats: Dugopolski 2.1-2.2 - Functions - Identify functions - Domain and range - Function notation</td>
</tr>
<tr>
<td>Mar 9, 2011</td>
<td>- Horizontal parabola - Cube and cube root - Increasing, decreasing, constant - Gather before and after data where an intervention caused a translation - Student data collected/generated that resembles linear regression before and after intervention. (Models translation) - Curve fitting</td>
</tr>
<tr>
<td>Week 9/Day 1</td>
<td>Algebra/Stats: Dugopolski 2.1-2.2 - Functions (continue)</td>
</tr>
<tr>
<td>Mar 21, 2011</td>
<td>- Horizontal parabola - Cube and cube root - Increasing, decreasing, constant - Gather before and after data where an intervention caused a translation - Student data collected/generated that resembles linear regression before and after intervention. (Models translation) - Curve fitting</td>
</tr>
<tr>
<td>Week 9/Day 2</td>
<td>Algebra/Stats: Dugopolski 2.3 - Families of Functions, - Transformations - modeling/applications</td>
</tr>
<tr>
<td>Mar 23, 2011</td>
<td>- Families of Functions, - Transformations - modeling/applications</td>
</tr>
<tr>
<td>Week 10/Day 1</td>
<td>Algebra/Stats: Dugopolski 2.3 - Modeling/applications of transformations of functions</td>
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<tr>
<td>Mar 28, 2011</td>
<td>- Modeling/applications of transformations of functions</td>
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<tr>
<td>Week 10/Day 2</td>
<td>Test 2</td>
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<tr>
<td>Week 11/Day 1</td>
<td>Algebra/Stats: Dugopolski 2.3-2. Transformations of functions</td>
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<tr>
<td>Apr 4, 2011</td>
<td>- Transformations of functions</td>
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<tr>
<td>Week 11/Day 2</td>
<td>Algebra/Stats: Dugopolski 2.5 - Operations of Functions</td>
</tr>
<tr>
<td>Apr 6, 2011</td>
<td>- Operations of Functions</td>
</tr>
<tr>
<td>Week 12/Day 1</td>
<td>Algebra/Stats: Dugopolski 2.5 - Composition of functions</td>
</tr>
<tr>
<td>Apr 11, 2011</td>
<td>- Composition of functions</td>
</tr>
<tr>
<td>Week 12/Day 2</td>
<td>Inverse Functions</td>
</tr>
<tr>
<td>Apr 13, 2011</td>
<td>- Inverse Functions</td>
</tr>
<tr>
<td>Week 12/Day 2</td>
<td>Review data that resembles exponential and logarithmic</td>
</tr>
<tr>
<td>Apr 13, 2011</td>
<td>- Review data that resembles exponential and logarithmic</td>
</tr>
<tr>
<td>Week</td>
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<td>Week 13/Day 1</td>
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<td>Week 13/Day 2</td>
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<td>Week 15/Day 1</td>
<td>May 2, 2011</td>
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<tr>
<td>May 9, 2-4:30pm</td>
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APPENDIX C
Lesson Plans
C-1 Syllabus; Administer Pretests
C-2 Statistics Data and Data Collection Lesson Plan
C-3 Statistical Graphs Lesson Plan
C-4 Measures of Central Tendency and Dispersion Lesson Plan
C-5 Normal Distribution Lesson Plan
C-6 Super Bowl Linear Regression Lesson Plan
C-7 Linear Equations Lesson Plan
C-8 Linear Modeling Lesson Plan
C-9 Quadratic Equations Lesson Plan
C-10 Linear and Quadratic Problems Lesson Plan
C-11 Introduction to Functions Lesson Plan
C-12 More on Functions Lesson Plan
C-13 Transformations of Functions Lesson Plan
C-14 Operations on Functions Lesson Plan
C-15 Inverse Functions Lesson Plan
C-16 Exponential Functions Lesson Plan
C-17 Exponential Application Functions Lesson Plan
C-18 Logarithmic Functions Lesson Plan
C-19 Solving Exponential and Logarithmic Functions Lesson Plan
C-20 Inferential Statistics
C-1 Course Syllabus
Syllabus for Math 1315 Section 0259: College Algebra
2:00-3:20pm Monday/Wednesday

Instructor Information
Name – Terri Westbrook
Office – 
Telephone – 
Email – 

Office Hours
Mon: 1-2 pm
Tues: 1-2 pm
Wed: 3:30-4:30 pm
Thurs: 1-2 pm
Fri: 10-11 am
Or by appointment

       These books will be provided to you, for this term only, as part of the FOCUS grant.

Calculators: A graphing calculator is required for this course. If you do not have one, one will be provided to you for the term of this course only as part of the FOCUS grant.

Description: A course covering linear and quadratic equations, inequalities, functions and their graphs, exponential and logarithmic functions, systems of equations, and applications of mathematics. Special emphasis on statistical concepts including linear and quadratic regression, distributions, confidence intervals, & hypothesis testing.

Prerequisite: Completion of 3 hours from any of the following courses: MATH 1311 with a minimum of CR, MATH 1315 or MATH 1319 with a minimum grade of D, MATH 1316 with a minimum grade of C. If a student has passed any of the following tests with the indicated score, then the prerequisites for this course will be waived: ACT math component score of 21, Math placement score of 26, TSIP Math score of 270, Recentered SAT High Math of 480. Also, approval from the FOCUS Grant Director is required.

Objectives: The objectives of this course include the following:
       o strengthen quantitative numeracy, as well as, statistical numeracy, reasoning, and thinking skills while teaching mathematical concepts.
       o stress problem solving, mathematical modeling, descriptive statistics, and applications.
       o use real-world data and applications to learn abstract mathematical concepts.
       o comprehend the connection of the mathematical concepts to the data.
       o begin modeling real-world data and begin asking inferential questions regarding the data.

Grading: Homework 15%
       Quiz Grades 15%
       Tests 1-3 (15% each) 45%
       Final exam 25% comprehensive course

Course Grades:

90-100% A
80-89% B
70-79% C
60-69% D
Below 60 F
**Dates:**
- March 14th–18th: No class, spring break
- March 24th: Last day to drop with an automatic W assigned
- April 21st: Withdrawal deadline (go to zero hours enrolled)
- May 2nd: Last day of class
- May 9th @ 2-4:30pm: Final exam

**Notes:** All aspects of this course are comprehensive in nature. The instructors reserve the right to deviate slightly from this syllabus for the purpose of better serving the needs of the students.

**Attendance Policy**

You are expected to attend class every period. Attendance will be taken.

*If you accrue more than 3 absences, you may be dropped from the grant program and loose the benefits associated with being in this program.*

If you will be absent due to a University Sponsored Function/Event or a religious holiday you must present the instructor with written notification in advance of the anticipated absence (a letter from you the student describing the circumstances and a note from an authoritative figure such as a University Staff member or the Dean of Students). When written notification is received and verified, accommodations will be made.

Note: It is the responsibility of each student to be aware of the manner in which attendance records are maintained. It is the responsibility of each student to be aware of the number of absences he/she has at any point in the semester.

**University Honor Code**

As members of a community dedicated to learning, inquiry, and creation, the students, faculty, and administration of our University live by the principles in this Honor Code. These principles require all members of this community to be conscientious, respectful, and honest.

**WE ARE CONSCIENTIOUS.** We complete our work on time and make every effort to do it right. We come to class and meetings prepared and are willing to demonstrate it. We hold ourselves to doing what is required, embrace rigor, and shun mediocrity, special requests, and excuses.

**WE ARE RESPECTFUL.** We act civilly toward one another and we cooperate with each other. We will strive to create an environment in which people respect and listen to one another, speaking when appropriate, and permitting other people to participate and express their views.

**WE ARE HONEST.** We do our own work and are honest with one another in all matters. We understand how various acts of dishonesty, like plagiarizing, falsifying data, and giving or receiving assistance to which one is not entitled, conflict as much with academic achievement as with the values of honesty and integrity.

**THE PLEDGE FOR STUDENTS**
Students at our University recognize that, to insure honest conduct, more is needed than an expectation of academic honesty, and we therefore adopt the practice of affixing the following pledge of honesty to the work we submit for evaluation:

I pledge to uphold the principles of honesty and responsibility at our University.

THE PLEDGE FOR FACULTY AND ADMINISTRATION
Faculty at our University recognize that the students have rights when accused of academic dishonesty and will inform the accused of their rights of appeal laid out in the student handbook and inform them of the process that will take place.

I recognize students’ rights and pledge to uphold the principles of honesty and responsibility at our University.

ADDRESSING ACTS OF DISHONESTY
Students accused of dishonest conduct may have their cases heard by the faculty member. The student may also appeal the faculty member’s decision to the Honor Code Council. Students and faculty will have the option of having an advocate present to insure their rights. Possible actions that may be taken range from exoneration to expulsion.

Common Student Questions:

“Can an absence be excused?” No, an absence is an absence. This means that there is no such thing as an excused absence. Absences due to athletics or other University sponsored functions and religious holidays documented by the Dean of Students are the only absences that will not be counted for the 3 absence rule. Each of these absences must be properly documented as described above in paragraph 3.

“When does attendance begin to “count”?” You are expected to follow the university schedule. You should attend each lecture regardless of which one precedes the other. Do not expect any “walks” or early releases.

“Who do I contact if I am going to miss a class day?” Your instructor.

“Can I make up an exam?” Come see your instructor if you cannot take an exam. To make up an exam, you will need an excused absence.

“Will you use an email besides my Texas State email to contact me?” Unfortunately, no. To make communication quicker and easier, I will only send emails to your University email account. I will respond to emails within 36 hours that are received between M-F.

“What if I need to reschedule my final?” Please let me know as soon as possible.

“Where can I receive tutoring?”

1. From your math instructor during her office hours.

2. Win-At-Math tutor: Times to be announced after FOCUS orientation on Jan 21, 2011.
3. The Student Learning Assistant Center (SLAC) is located on the 4th floor of the library.

4. The Math Lab is located in Derrick Hall room 233. Hours are: Mon-Thurs 8am-7pm, Fri 8am-5pm. The Math Lab can also be found at

“Where can I receive other assistance?”

- Student Support Services
- Student Affairs
- Academic Advising
- Academic Support
**C-2 Statistics Data and Data Collection Lesson Plan**

<table>
<thead>
<tr>
<th><strong>File Name</strong></th>
<th>Math 1311_Stat_Data_&amp;_Data_Collection</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Statistics: Types of Data and Data Collection</td>
</tr>
<tr>
<td><strong>Prerequisite Objective</strong></td>
<td>None</td>
</tr>
</tbody>
</table>
| **Content Objectives** | Students will be able to:  
1. differentiate between categorical versus quantitative data. | E-CRA- Experiential, concrete, representational, abstract, modeling |
| | 2. distinguish among different types of data collection. | Concrete, representational |
| | 3. distinguish between different types of bias in data collection. | Concrete, representational |

**Technology Objectives**
Students will be able to use the random number generator on a graphing calculator.

**Culturally Relevant Objectives**

Bias Examples  
Sampling Bias Activity

**Real-World Applications**

Questioning about personal experiences with data collection

**Applications/Connections**


**Process Standards**

| Communication | Connections | Problem-Solving | Representations | Reasoning | Cross-Disciplinary |
Lesson Agenda

I. Types of data
   a. Categorical Data
      i. Non-binary
      ii. Binary
   b. Quantitative Data

II. Types of data Collection
III. Definitions in Statistics
IV. Type of Bias

Instructor Materials
TI-Smartview

Student Materials
Copies of handouts
Graphing calculators

Pre-Homework

Define Statistics

Introduction/Background

We have been discussing linear equations in one-variable. Other uses for one variable data can be seen in statistics, but first we need to examine some definitions in statistics. Statistics is “a method for collecting, organizing, analyzing, and interpreting data, as well as drawing conclusion based on data.” (Blitzer, p. 680) Has anyone been in some type of data or have collected data before? Why or what was the situation? [Student responses may vary.] Do you know what type of data was collected?

Main Lesson Script

Pass out HANDOUT: Data and Data Collection and ask students to fill in the tables as you go through the definitions.

I. Types of data

In statistics (the science of reasoning from data), we refer to two types of data (number collected in a particular context): categorical (qualitative) and quantitative. (Note that the word “data” is plural.) Categorical data is data that can be put into some type of category. For example, hair color can be categorized as black, brown, blonde, etc. When there are only two categories, we call the data binary. For instance, flipping a coin yields one of two results: heads or tails. Quantitative data, on the other hand, is data that can be quantified such as height, weight, and age. Now, be careful not to think that all numbers represent quantitative data. For example, zip codes are used to classify regions, so even though the labels are numbers, it is still categorical. The type of data, categorical or quantitative, is referred to as the variable of interest. In statistics, we collect data on an item of interest and we investigate its distribution (pattern of data variability). Typically, we describe the distributions using numbers, descriptive statistics, and with
graphs. Different numerical summaries and graphical representations are used depending on whether or not the data is categorical or quantitative, which will be discussed in the next lesson.

II. Types of Data Collection

There are four main types of data collection methods: Anecdotal Data Collection, Observational Study, Survey, and Experimental. **Anecdotal data** is collected through interviews, questionnaires, or any other method where participants tell stories and researchers collect data based on what they were **told**. Can you think of a situation where you might have contributed to anecdotal data? [Student’s Response may vary.] **Observational studies** are conducted to allow researchers to collect data based on observations that they make. Do you know of any occupations where observational studies might be utilized? [Student’s Response may vary.] **Surveys** are given in a manner that the researcher asks specific questions in order to collect needed or desired data. Have you ever taken a survey at a restaurant on the service you received? [Student’s Response may vary.] Lastly, researchers use **experiments** to collect data. This is the only type of data collection technique that allows causal relationships to be drawn. What do you think it means to have a causal relationship? [Student’s Response may vary.] Usually, the researcher has some sort of **control** designated in the experiment.

III. Definitions in Statistics

Before we move forward, it is important that we understand the meanings of some terms that are commonly used in statistics. Earlier we said that, in statistics, we collect data on an item of interest. The total set of subjects of interest in our study is referred to as the **population**. Usually, it is not realistic to expect to be able to collect data on an entire population, so researchers select a **sample**, or a subset of the population, on which to collect the data. For example, if we wanted to collect data on students at Texas State University in order to determine who enjoys mathematics, would it be realistic to collect data from every single student? [Student Response: No.] What would be a sample space of Texas State University students? [Student Response: Our class would be considered a sample of Texas State students.] That is true, but it brings us to our next discussion about bias.

IV. Types of Bias

**Bias** is caused by a sampling procedure that tends to systematically over-represent or under-represent some portion of the population. Some sampling procedures that are considered to be bias are: convenience sampling, voluntary response, and non-response. **Convenience Sample** is where subjects are chosen because they are easy to reach/collect data from. So what might be the problem with sampling from this classroom to represent all
Texas State students? (Probing Question: How many students in here are math majors? How many are computer science majors?) [Student Response: Our class does not have the diversity of all the majors.] Also this is a very small sample to represent all the students at Texas State. **Voluntary Response** is where subjects volunteer to be in the sample which may lead to poor representation of some part of the intended population. Does anyone have a good example of voluntary response? [Student Response: May vary.] **Non-Response** is where some sampled subjects refuse to participate or fail to participate fully. Has anyone had those annoying telemarketers call at 8pm, asking if you wouldn’t mind answering some questions? When you start answering questions and thirty minutes later you are tired of answering questions and stop in the middle of their survey that is considered non-response. Or when you just hang up the phone, they must record that you did not answer the questions.

To investigate the Bias techniques we will perform an activity called **Sampling Rectangle – Investigating Bias.** Pass out the HANDOUT labeled: *Sampling Rectangles* and ask students to follow the directions on the handout. Some clarifications might need to be made about this handout. The first portion of the handout, the students will select five rectangles that best represent the average of all the rectangles on page of rectangles. The second portion is done using the random generator on the calculator, see handout for specific directions. Then discuss the bias that occurred due to the individual “judgment” required for part 1. (Since students’ judgments are subjective, the students are subject to their own bias.) Discuss how using random samples decreases the threat of sampling bias (due to the random, unbiased selection of the population which is, in this case, rectangles). Note that the procedure used generates a Simple Random Sample. Review the true mean of all the rectangles using the bar chart (last page) on the elmo. Make note of the graphical distribution and the fact that we used a numerical summary, mean, for this activity.

**Summary/Extension**

What types of graphs would we use to graph categorical data? Or quantitative data?

Homework assignment to be completed before next class is to complete a personal survey. We will discuss what type of data the survey questions are asking and how to graph the data.

**Possible Assessments from Today’s Lesson**

**Teacher Reflection**

1. Did you use the class time wisely? No problem.
2. What worked well in the lesson? Fine. Those students, who took developmental mathematics at this university, were very familiar with this topic. For all of the other students, this was a new topic.

3. What recommendations would you make to improve this lesson? Prior to class I had found the following websites that I might want to refer to during class:
   - Average heights of American adults: 
     http://www.cdc.gov/nchs/fastats/bodymeas.htm
   - Commute distances to work
   - Extreme commuters
   - TxState undeclared majors
     http://star.txstate.edu/content/freshmen-begin-college-undecided-majors
   - TxState Crime stats
   - TxState International student statistics
     http://www.international.txstate.edu/about/statistics.html

Was planning to bring up these websites to demonstrate data, if there was time.

Other Comments About the Lesson

References

# Data and Data Collection (KEY)

<table>
<thead>
<tr>
<th>Types of Data</th>
<th>Examples</th>
<th>Definition</th>
</tr>
</thead>
</table>
| Categorical   | Marital Status  
               | Favorite Type of music  
               | Hair Color  
               | Gender (Binary: M/F)  
               | Zip code | A variable where the measurement is on a nominal scale. (Nominal scales offer names or labels for certain characteristic.) Those with only two outcomes are called binary. |
| Quantitative  | Age  
               | Weight  
               | Height | A variable where the measurement scale has numerical values. |

<table>
<thead>
<tr>
<th>Types of Data Collection</th>
<th>Situation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anecdotal</td>
<td>Police reports from several witnesses.</td>
<td>Is collected through interviews, questionnaires or any method where participants tell stories and researchers collect data based on what they are told.</td>
</tr>
<tr>
<td>Observational Study</td>
<td>Educational studies where they are observing students in a classroom.</td>
<td>Is conducted to allow researchers to collect data based on observations that they make.</td>
</tr>
<tr>
<td>Survey</td>
<td>Card at the restaurant on service.</td>
<td>Is given in a manner that the researcher asks specific questions in order to collect needed or desired data.</td>
</tr>
<tr>
<td>Experiment</td>
<td>Drug experiments, drug and placebo.</td>
<td>Type of data collection technique that allows causal relationship to be drawn. Usually, the researcher has some sort of control designated in the experiment.</td>
</tr>
</tbody>
</table>

**Definitions:**

- **Ex.** If the students at Texas State University enjoy math?

  Population – the total set of subjects of interest in a study

  From example above, the population would be all the students of Texas State University.

  Sample – a subset of the population on which the study collects data Ex. Our classroom
Bias – Is caused by a sampling procedure that tends to systematically over-represent or under-represent some portion of the population.

- Convenience sample – where subjects are chosen b/c they are easy to reach/collect data from
- Voluntary Response – where subjects volunteer to be in the sample which may lead to poor representation of some part of the intended population
- Non-Response – where some sampled subjects refuse to participate or fail to participate fully
## Data and Data Collection

### Types of Data

<table>
<thead>
<tr>
<th>Types of Data</th>
<th>Examples</th>
<th>Definition</th>
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<tr>
<td>Categorical</td>
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<tr>
<td>Quantitative</td>
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</table>

### Types of Data Collection

<table>
<thead>
<tr>
<th>Types of Data Collection</th>
<th>Situation</th>
<th>Definition</th>
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<tbody>
<tr>
<td>Anecdotal</td>
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<td>Observational Study</td>
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<td>Survey</td>
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<tr>
<td>Experiment</td>
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</table>

### Definitions:

- **Population**: ______________________________________________________
- **Sample**: ______________________________________________________
- **Bias**: ______________________________________________________
**Sampling Rectangles**

*Investigating Bias*

Given three numbers, 12, 14, 16, how do you find the average? What if I give you a whole page of numbers? Are you going to calculate the exact average or estimate? How might you estimate?

In a page that will be handed out in a few minutes, there are 100 rectangles of different sizes. Here’s an example of a rectangle which has an area of 12;

```
[Rectangle representation]
```

Suppose we wanted to take a sample of five from the page to estimate the average area of all 100 rectangles. You will perform this task in two different ways.

1. **Judgment Sample:** Select five rectangles that, in your judgment, are representative of the rectangles on the page. Write the 2-digit names of the five representative rectangles, and then write the area of the rectangle. Remember: the GOAL is to find five rectangles so that their average is close to the true average of all 100 rectangles.

<table>
<thead>
<tr>
<th>2-digit name of rectangle</th>
<th>Area of rectangle</th>
</tr>
</thead>
<tbody>
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2. **Simple random sample:** Use your calculator to generate 5 random 2-digit numbers. (See the directions on the back of this handout.) Write your five 2-digit numbers in the table below and then determine the area of this rectangle by referring back to the rectangle sheet.

<table>
<thead>
<tr>
<th>2-digit name of rectangle</th>
<th>Area of rectangle</th>
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Random Number Generator

The TI-83 Plus and TI-84 Plus graphics calculators have a random number generator, which can be used in place of a random number table. Press [MATH]. Use the arrow keys to highlight PRB. Notice that 1:rand is selected.

When you press [ENTER], rand appears on the screen. Press [ENTER] again and a random number between 0 and 1 appears.

Use the first two digits of the decimal to be your random “2-digit number.”
Distribution of rectangles (population)

<table>
<thead>
<tr>
<th>area</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>12</th>
<th>15</th>
<th>16</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>freq</td>
<td>16</td>
<td>2</td>
<td>6</td>
<td>16</td>
<td>8</td>
<td>6</td>
<td>8</td>
<td>5</td>
<td>7</td>
<td>10</td>
<td>1</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

N = 100 rectangles

mean area $\mu = 7.42$

standard deviation $\sigma = 5.228$
Read Sections 12.1, 12.2, 12.3
Do problems Sect 12.1 # 1, 2, 19, 20, 21-29, 33-37
Sect 12.2 # 49-51 (don’t do mode), 58-62

Fill survey form.

1. Your height in inches: ____________________
2. Your ethnicity: __________________________
3. Your classification: _____________________
4. The college your major belongs to:
   - College of Applied Arts
   - College of Business Administration
   - College of Education
   - College of Fine Arts & Communication
   - College of Health Professionals
   - College of Liberal Arts
   - College of Science
   - University College

5. Housing type: (out-of-town, live on San Marcos and not on-campus, live on-campus) ________________________
## C-3 Statistical Graphs Lesson Plan

<table>
<thead>
<tr>
<th>Lesson Title</th>
<th>Statistics: Data Graphs</th>
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<tbody>
<tr>
<td><strong>Prerequisite:</strong></td>
<td>Statistical Data and Data Collection</td>
</tr>
<tr>
<td><strong>Content Objectives</strong></td>
<td><strong>E-CRA</strong>- Experiential, concrete, representational, abstract, modeling</td>
</tr>
<tr>
<td>Students will be able</td>
<td>1. Graph categorical data into bar graphs</td>
</tr>
<tr>
<td></td>
<td>2. Graph quantitative data into histograms</td>
</tr>
<tr>
<td><strong>Technology Objectives</strong></td>
<td>Students will be able to</td>
</tr>
<tr>
<td></td>
<td>1. Use Excel to store data and create graphs</td>
</tr>
<tr>
<td></td>
<td>2. Use graphing calculators to create statistical graphs</td>
</tr>
<tr>
<td><strong>Culturally-Relevant Objectives:</strong></td>
<td>Students examined their own data (a sample) and compare to the overall population.</td>
</tr>
<tr>
<td><strong>Real-World Applications:</strong></td>
<td>Students used real world data and examined how to create visual representations of the data. Students were also shown how visual representations of the exact same data can appear to communicate different information.</td>
</tr>
<tr>
<td><strong>Applications/Connections</strong></td>
<td><strong>Process Standards</strong></td>
</tr>
<tr>
<td>Algebra</td>
<td>Communication</td>
</tr>
<tr>
<td>Geometry</td>
<td>Connections</td>
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<tr>
<td>Measurement</td>
<td>Problem-Solving</td>
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<tr>
<td>Statistics</td>
<td>Representations</td>
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<tr>
<td>Probability</td>
<td>Reasoning</td>
</tr>
<tr>
<td><strong>Lesson Agenda</strong></td>
<td><strong>Instructor Materials</strong></td>
</tr>
<tr>
<td>I. Graphs of categorical data</td>
<td>Excel files #1 and #2.</td>
</tr>
<tr>
<td>II. Graphs of quantitative data</td>
<td></td>
</tr>
</tbody>
</table>
Introduction/Background

- Recall that last class we discussed the two types of data: categorical and quantitative.
- Does anyone remember what categorical data are?
  [data that can be put in some type of category]
- Can anyone recall what quantitative data are?
  [data that can be quantified such as height, weight, and age]
- Now, remember that not all numbers represent quantitative data. For example, last class we looked at zip codes. Zip codes are used to classify regions. So even though the labels are numbers, it is still categorical.
- Different numerical summaries and graphical representations are used depending of whether or not the data are categorical or quantitative.
- When describing and summarizing data we often look at numerical summaries and/or graphical summaries.

Main Lesson Script

I. Graphs of categorical data

- For categorical data, statisticians use bar graph and pie charts. Pie charts we will discuss in later in the course.

So, let’s say we have some survey data on people’s favorite colors. Is “favorite color” categorical or quantitative data? [categorical] We sum up the number of people who liked each individual color. A table of data would look roughly like the following table. Pull up Excel file, this example is on rows 1-21 in the spreadsheet.

<table>
<thead>
<tr>
<th>Color</th>
<th># people who picked this color</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>625</td>
</tr>
<tr>
<td>Green</td>
<td>800</td>
</tr>
<tr>
<td>Red</td>
<td>700</td>
</tr>
<tr>
<td>Yellow</td>
<td>300</td>
</tr>
<tr>
<td>Total</td>
<td>2,425</td>
</tr>
</tbody>
</table>

- A bar graph can be either horizontal or vertical. Draw either a horizontal or vertical bar graph from the above data above. Either the x-axis or the y-axis will have the numeric information with the descriptive information on the other axis. Notice that the bars do not touch. This is not quantitative data. This data is not continuous like the data we will see in a histogram (which we haven’t discussed yet, but will talk about it later in this lesson). The bars can be re-arranged and the graph will still make sense.
II. Graphs of quantitative data

- Open the Excel spreadsheet for this lesson plan.
- There are several different graphical summaries that we can construct for quantitative data.
- First, we will look at its histogram. Remember that the data on the histogram is continuous data so the bars touch. In other words, you cannot scramble the order of these bars and the graph still has the same meaning, which is the case for bar charts.
- Show students the first worksheet which contains data about Boys' Ages of Maximum Yearly Growth (a sample problem from the Blitzer textbook). Work together as a class to construct the histogram by hand. This data’s graph resembles a bell, or bell curve. This is called a normal distribution which we will discuss more later in this course. The data has a mean of 14.
- Now show the students the data and graphs from worksheet 2, Boys Max Growth2. The graph has two towers, instead of one. We call this bi-modal. Can you guess why? This data does not have a normal distribution. The mean is still 14 but at the spread of the data (Foreshadow dispersion and spread).
- Now show the students the data and graphs from worksheet 3, Boys Max Growth3. The graph has uniform towers, where we have multiple bars with the same height. We call this uniform data. Can you guess why? This data does not have a normal distribution. The mean is still 14 but look at the spread of the data (Foreshadow dispersion and spread).
- Show students how to use Excel to create graphs of this data.
• Show students how to graph data using the graphing calculator.
• We could also look at a Box and Whisker plot, Dot plot, or Time plot (depending on what type of data we have).

III. Group Activity:

Have students pull out their personal survey data that was assigned as homework.
Distribute the handout: Class Data. This handout will be used to merge all class data.
Break up the class into groups. Have the groups discuss the personal information that they wrote down for homework. The groups need to:
1. Review each data item collected and determine if the item is categorical or quantitative.
2. Create/draw graphs of the data

IV. Whole class discussion.

Have each group send a member to the document camera and show the graphs that they created and discuss how they constructed them. After students showed their graphs, remind students that their data is a sample.
What is the population that your sample belongs to? [the University]. Now worksheets, Ethnicity, Student Classifications, College Majors, and Housing-Type. Discuss how their samples are similar or not similar to the graphs of the populations. To look at the population for height go to the website, http://www.analytictech.com/mb313/sd.htm

Summary/Extension

Foreshadow distributions, dispersion, and spread.

Possible Test Questions from Today’s Lesson

Teacher Reflection

1. Did you use the class time wisely? Started losing some time to discuss reading and interpreting graphs.

2. What worked well in the lesson? I noticed that the students were weak with creating, reading, and interpreting graphs. This really came out during the student presentations of their group work. Some groups were simply creating graphs, but not building it from the perspective of “Can someone else read it and understand what you are trying to communicate?”

3. What recommendations would you make to improve this lesson? Need to begin this lesson with teaching students to build graphs with the perspective of “What do I want my graph to communicate?” Start asking the students the question, “What are your data communicating to you?”
<table>
<thead>
<tr>
<th>Other Comments About the Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>References</td>
</tr>
</tbody>
</table>
Hair Color Activity
Categorical Data

Create a data table that summarizes the number of people in the class with the following hair color:

<table>
<thead>
<tr>
<th>Hair Color</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brown</td>
<td></td>
</tr>
<tr>
<td>Black</td>
<td></td>
</tr>
<tr>
<td>Blonde</td>
<td></td>
</tr>
<tr>
<td>Red</td>
<td></td>
</tr>
</tbody>
</table>

Construct a bar graph to describe the data you collected in the table above.
Screenshot of Excel worksheet, Boys Max Growth

Example of entering data, Column A – Ages, Column B - # boys of that age

Graph of data. Appears to look like a normal distribution (which we have not discussed yet, but will foreshadow the topic)
Screenshot of Excel worksheet, Boys Max Growth2

Example of entering data, Column A – Ages, Column B - # boys of that age

Graph of data. Appears to look like a bi-modal (which we have not discussed yet, but will foreshadow the topic)
Screenshot of Excel worksheet, Boys Max Growth3

Example of entering data, Column A – Ages, Column B - # boys of that age

Graph of data. Appears to look like a uniform (which we have not discussed yet, but will foreshadow the topic)
Class Data

<table>
<thead>
<tr>
<th>Height: Men</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Height: Women</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ethnicity:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>African American</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hispanic</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Caucasian</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Other</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Classification:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Freshman</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sophomore</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Junior</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Senior</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>College your major belongs to:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>College of Applied Arts</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>College of Business Administration</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>College of Education</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>College of Fine Arts &amp; Communication</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>College of Health Professionals</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>College of Liberal Arts</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>College of Science</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>University College</td>
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</tr>
<tr>
<td>Housing type:</td>
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<tr>
<td>live on-campus</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>live in San Marcos and not on-campus</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>out-of-town</td>
<td></td>
<td></td>
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</tbody>
</table>
# Class Data

**Height: Men**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>66</td>
<td>64</td>
<td>72</td>
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</tbody>
</table>

**Height: Women**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
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<td>68</td>
<td>62</td>
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<td>68</td>
<td>65</td>
<td>64</td>
<td>63</td>
<td>65</td>
<td></td>
</tr>
</tbody>
</table>

**Ethnicity:**

- African-American: 2
- Hispanic: 8
- Caucasian: 1
- Other: 1

**Classification:**

- Freshman: 5
- Sophomore: 6
- Junior: 4
- Senior: 5

**College your major belongs to:**

- College of Applied Arts: 74
- College of Business Administration: 0
- College of Education: 3
- College of Fine Arts & Communication: 6
- College of Health Professionals: 1
- College of Liberal Arts: 1
- College of Science: 1
- University College: 1

**Housing type:**

- live on-campus: 4
- live in San Marcos and not on-campus: 10
- out-of-town: 6
Graphs of the Population

<table>
<thead>
<tr>
<th>Ethnicity</th>
<th>Count</th>
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</thead>
<tbody>
<tr>
<td>African American</td>
<td>2053</td>
</tr>
<tr>
<td>Hispanic</td>
<td>7908</td>
</tr>
<tr>
<td>White</td>
<td>20735</td>
</tr>
<tr>
<td>Other</td>
<td>1896</td>
</tr>
</tbody>
</table>

University 2010 Enrollment by Ethnicity

- African American: 2053
- Hispanic: 7908
- White: 20735
- Other: 1896
<table>
<thead>
<tr>
<th>Class</th>
<th>Number</th>
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</thead>
<tbody>
<tr>
<td>Freshman</td>
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<td>Sophomore</td>
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<td>Junior</td>
<td>6812</td>
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<td>Senior</td>
<td>9092</td>
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</table>

![Pie chart showing class distribution at University 2010](chart.png)
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<thead>
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<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
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<th>I</th>
<th>J</th>
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<th>M</th>
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</tbody>
</table>

**AppArts**

**Biology**

**Chemistry**

**Economics**

**FA&Comm**

**Health**

**LA**

**Science**

**UC**

**University 2010**

- UC
- Science
- LA
- Health
- FA&Comm
- Ed
- BA
- AppArts

![Bar Chart](chart_image.png)
### C-4 Measures of Central Tendency and Dispersion Lesson Plan

<table>
<thead>
<tr>
<th>File Name</th>
<th>Math 1315_Measures_of_Central_Tendency_and_Measures_of_Dispersion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson Title</td>
<td>Measures of Dispersions</td>
</tr>
<tr>
<td>Prerequisite Objective</td>
<td>Statistical Graphs</td>
</tr>
<tr>
<td>Content Objectives</td>
<td>Students will be able to:</td>
</tr>
<tr>
<td></td>
<td>1. understand differences between mean and median</td>
</tr>
<tr>
<td></td>
<td>2. understand when to use mean vs median</td>
</tr>
<tr>
<td></td>
<td>3. calculate the standard deviation.</td>
</tr>
<tr>
<td>Technology Objectives</td>
<td>Students will be able to use a graphing calculator to find mean and standard deviation.</td>
</tr>
<tr>
<td>Applications/Connections</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Algebra</td>
</tr>
<tr>
<td></td>
<td>Geometry</td>
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<td>Measurement</td>
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<td>Statistics</td>
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<td>Probability</td>
</tr>
<tr>
<td>Process Standards</td>
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<td>Communication</td>
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<td>Connections</td>
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<td>Problem-Solving</td>
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<td>Representations</td>
</tr>
<tr>
<td></td>
<td>Reasoning</td>
</tr>
<tr>
<td>Lesson Agenda</td>
<td>I. Mean, Median, Midrange, and Five Number Summary</td>
</tr>
<tr>
<td></td>
<td>II. Define standard deviation</td>
</tr>
<tr>
<td></td>
<td>a. Calculation S.D. by hand</td>
</tr>
<tr>
<td></td>
<td>b. Calculating large list with calculator</td>
</tr>
<tr>
<td></td>
<td>c.</td>
</tr>
<tr>
<td>Instructor Materials</td>
<td></td>
</tr>
<tr>
<td>Student Materials</td>
<td></td>
</tr>
</tbody>
</table>

### Introduction/Background

We have been reviewing statistical data, deciding if the data is categorical or quantitative, and graphing it. Now we need to look closer at the measures of central tendency, the spread and range of the quantitative data.
Main Lesson Script

I. Measures of Central Tendency
Pass out the handout “Monthly Rainfall in Austin, Texas”. Can anyone tell me the center or the middle of the data on this handout? [answers will vary]. How would you describe the rainfall in Austin? [answers will vary]

Does anyone recall what mean is? [average or arithmetic average.] How do we calculate the mean? [sum up all the numbers in a data set and divide the sum by the number of elements in the data set.] Does anyone recall what median is? [is the middle number in the data set, or the average of the two middle numbers] How do we calculate the median? [Sort the data from smallest to largest. Then find the middle number if data set size is an odd number; average the two middle numbers if the data set size is an even number.] Does anyone know what the midrange is? [Find the smallest and largest numbers in the dataset. The midrange is the average of the minimum and maximum numbers.]

Pair and share activity. Go ahead and complete this handout and share/compare your answers with one of your classmates.

II. Measure of Dispersion: Standard Deviation

In probability and statistics, standard deviation (S.D.) is a measure of the variability or dispersion of a population, a data set, or a probability distribution, symbolized as $\sigma$. A low standard deviation indicates that the data points tend to be very close to the same value (the mean), while high standard deviation indicates that the data are “spread out” over a large range of values. Remember the graphs of Maximum Growth Years for Boys from the last lesson. (If needed go back to Excel file from last lesson, and demonstrate how mean can be the same, but the distribution of the data is very different.)

Pass out the HANDOUT: Standard Deviation and develop the following algorithm, how to calculate the standard deviation, for the class.
1) Calculate the mean of the data set
2) Find all the squares of all the deviations from the mean
3) Add all the squares of the deviations from the mean
4) Divide this sum by the total number of data value
5) The standard deviation is the square root of this quotient. It is denoted by the symbol, $\sigma$, for the population. The standard deviation for a sample is denoted by the symbol, $s$. The formulas for $\sigma$ and $s$ are the following:

$$\sigma = \sqrt{\frac{\text{sum of (deviations from the mean)}^2}{\text{total number of data values}}}$$
NOTE: If you are analyzing a sample of a population, then we are finding a sample standard deviation then the denominator needs to be reduced by 1, in our case dividing by a 7 instead of an 8. Reducing the denominator by 1 is an action resulting from determining the degrees of freedom of the data. Degrees of freedom will not be covered in this class.

Example: Consider a population consisting of the following values

2, 4, 4, 4, 5, 5, 7, 9.

1) There are eight data points in total, with a mean (or average) value of 5:

\[
\frac{2 + 4 + 4 + 4 + 5 + 5 + 7 + 9}{8} = 5.
\]

2) Compute the difference of each data point from the mean, and square the result:

\[
(2 - 5)^2 = (-3)^2 = 9 \quad (5 - 5)^2 = 0^2 = 0 \\
(4 - 5)^2 = (-1)^2 = 1 \quad (5 - 5)^2 = 0^2 = 0 \\
(4 - 5)^2 = (-1)^2 = 1 \quad (7 - 5)^2 = 2^2 = 4 \\
(4 - 5)^2 = (-1)^2 = 1 \quad (9 - 5)^2 = 4^2 = 16
\]

3-5) Next we average these values and take the square root, which gives the standard deviation:

\[
\sqrt{\frac{9 + 1 + 1 + 1 + 0 + 0 + 4 + 16}{8}} = \sqrt{4} = 2.
\]

Therefore, the population above has a standard deviation of 2, \( \sigma = 2 \). We are considering that the population is complete.

Allow your students to find the standard deviation for the following population:

10, 12, 12, 14, 15, 20, 22.
Summary/Extension

Graphing calculators are important tools that can be used when determining standard deviation. We have previously discussed how to place the data in our calculator so at this time proceed with entering in the data set from our first example. {2, 4, 4, 4, 5, 5, 7, 9} We will use 1-VAR STATS in the CALC section of the STAT feature on the calculator.

1-VAR STATS program in the CALC section will calculate the 5-number summary, mean, standard deviation for a quantitative data set. When the read out appears on the home screen you will notice “σx=” this is the standard deviation for the population, if this was not all the data and only a sample of the population then you would use Sx = which represents the sample standard deviation.

Possible Assessments from Today’s Lesson

Teacher Reflection

1. Did you use the class time wisely? No problems.

2. What worked well in the lesson? Lesson plan worked fine in general. Standard deviation is confusing for them at first. I had students only do a few calculations by hand. Then do all other calculations using the STAT feature on the calculator. I made sure to emphasize the difference between mean and median.

3. What recommendations would you make to improve this lesson? Need to change the lesson plan in the future to calculate the standard deviation for a sample, as well as for a population. Next time I teach this class, I may have them bring their own quantitative data. Then go through measures of central tendency topic. Then ask the students to get in groups and decide which is better mean or median for the data that they brought to class.
Other Comments About the Lesson
  Measures of central tendency was easy for students.

References
Monthly Rainfall in Austin, Texas

Since the beginning of 2009 through August of 2010, the following is the monthly rainfall (in inches) for Austin, Texas. What is the center of this data? Or the middle of this data? How would you describe the rainfall in Austin?


<table>
<thead>
<tr>
<th>JAN</th>
<th>FEB</th>
<th>MAR</th>
<th>APR</th>
<th>MAY</th>
<th>JUN</th>
<th>JUL</th>
<th>AUG</th>
<th>SEP</th>
<th>OCT</th>
<th>NOV</th>
<th>DEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0.9</td>
<td>3.2</td>
<td>3.7</td>
<td>1.7</td>
<td>1.0</td>
<td>1.3</td>
<td>2.6</td>
<td>7.0</td>
<td>6.9</td>
<td>2.8</td>
<td>2.5</td>
</tr>
<tr>
<td>3.3</td>
<td>2.8</td>
<td>2.8</td>
<td>1.4</td>
<td>1.0</td>
<td>4.2</td>
<td>5.4</td>
<td>0.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. What is the median of this data? 1. _______

2. What is the mean of this data? 2. _______

3. What is the midrange of this data? 3. _______

The 5-number summary consists of the minimum, maximum, median, and first quartile (the median of the lower half of the data) and third quartile (the median of the upper half of the data) of a quantitative data set.

4. Find the 5-number summary of this data set.
Standard Deviation

Algorithm to calculate the standard deviation:

1) 
2) 
3) 
4) 
5) 

Example 1: Consider a population consisting of the following values 2, 4, 4, 4, 5, 5, 7, 9.

Example 2: Find the standard deviation of the population: 10, 12, 12, 14, 15, 20, 22.
# C-5 Normal Distribution Lesson Plan

<table>
<thead>
<tr>
<th>File Name</th>
<th>Math 1315_Normal Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson Title</td>
<td>Normal Distribution</td>
</tr>
<tr>
<td>Prerequisite Objective</td>
<td>Measures of Central Tendencies and Dispersion</td>
</tr>
<tr>
<td><strong>Instructional Objectives</strong></td>
<td><strong>E-CRA</strong> - Experiential, concrete, representational, abstract, modeling</td>
</tr>
<tr>
<td>Students will be able to:</td>
<td></td>
</tr>
<tr>
<td>1. Define normal distribution.</td>
<td>concrete, representational</td>
</tr>
<tr>
<td>2. Apply the bell curve to real world applications.</td>
<td>modeling</td>
</tr>
<tr>
<td>3. Use the 68-95-99.7% Rule</td>
<td>abstract, modeling</td>
</tr>
<tr>
<td>4. Calculate z-score</td>
<td>abstract</td>
</tr>
<tr>
<td>5. Skewed distributions impact to mean and median</td>
<td>concrete, representational</td>
</tr>
<tr>
<td>6. Margin of error</td>
<td>concrete, abstract</td>
</tr>
</tbody>
</table>

**Technology Objectives**
Students will be able to use Excel to demonstrate the impact of skewed distribution to mean and median.

**Culturally-relevant lesson:**
Students saw how the normal distribution can be used to analysis problems and questions.

**Real-world applications:**
Students saw how the normal distribution can be used to analysis problems and questions.

<table>
<thead>
<tr>
<th>Applications/Connections</th>
<th>Process Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebra</td>
<td>Communication</td>
</tr>
<tr>
<td>Geometry</td>
<td>Connections</td>
</tr>
<tr>
<td>Measurement</td>
<td>Problem-Solving</td>
</tr>
<tr>
<td>Statistics</td>
<td>Representations</td>
</tr>
<tr>
<td>Probability</td>
<td>Reasoning</td>
</tr>
</tbody>
</table>

**Lesson Agenda**

I. Define normal distribution
II. Applying S.D. to the normal curve

**Instructor Materials**
III. The 68%, 95%, and 99.7% Rule

IV. Using the curve in real life applications

V. Z-score

VI. Skewed Distributions impact to mean and median

VII. Margin of error

Introduction/Background

A couple of weeks ago we were looking at graphs of data and found that some look like they have the shape of a bell. Today we are going to discuss this shape further.

Main Lesson Script

I. Define Normal distribution

The normal distribution is a continuous, probability distribution that describes data that clusters around a mean or average. The graph is bell-shaped, with a peak at the mean, and is known as the bell curve. In a perfect, symmetrical normal distribution, what is the relation between mean and median? [They are equal.]

Note: that another measure of central tendency, mode, is also equal to mean and median in a normal distribution.

III. Applying Standard Deviation to Normal Curve

Can the class state some conditions for a curve to be normal:
1) Most values are near the mean
2) Data values are spread evenly around the mean
3) Larger deviations from the mean become more rare
4) Individual data values result from a combination of different factors

IV. The 68, 95, 99.7 Rule

Since we know that standard deviation (S.D.) gives a measure of the spread of the data. It is also used to give us a description of a normal distribution. 68% of the data points fall within 1 S.D. of the mean. 95% falls within 2 S.D. and 99.7% falls within 3 S.D.
Normal distribution with a mean of 50 and standard deviation of 10. 68% of the area is within one standard deviation of the mean.

Example:

Male adult heights in North America are approximately normally distributed with a mean of 70 inches and a standard deviation of 4 inches.

1) Find the percentage of men in North America with heights
   a. Between 66 inches and 74 inches
   b. Between 70 inches and 74 inches
   c. Above 78 inches
   d. Less than 66 inches

The mean of a new car is $17,000 and the S.D. is $500. Find the percentage of buyers who payed:

1) Between $16,500 and $17,500
2) Between $16,000 and $18,000
3) Between $17,000 and $17,500
4) Between $15,00 and $17,500
5) More than $17,500
6) More than $18,000
7) Less than $16,000

V. Z-score.

When you want to know how far number is from the mean, we are referring to this number’s z-score. The z-score is the number of standard deviations the number is from the mean. In the last problem, $16,500, would have a standard deviation of what? But, what about the number 17,200? How many standard deviations is this number from the mean, 17,000? To find this we calculate the z-score.

\[ z = \frac{data - mean}{Std \ dev} \]

So what is the number of standard deviations or z-score for 17,200 is from the mean 17,000?
\[ z = \frac{17,200 - 17,000}{500} = 0.4 \] What is the z-score for 19,300? [z=4.6]

When is this useful? Let’s see. There are two different Intelligent Quotient tests, Stanford-Binet and Wechler, that are use in our society. Both of these tests have a mean of 100, but the standard deviation for the Stanford-Binet test is \( \sigma_{SB} = 16 \) and the standard deviation for the Wechler test is \( \sigma_W = 15 \). If one person receives an IQ score of 128 from the Stanford-Binet test and another person receives an IQ score of 127, who is smarter (from an IQ perspective)? Let’s calculate the z-score and find out which individual is more standard deviations above the mean.

\[ z_{SB} = \frac{128 - 100}{16} = 1.75 \]
\[ z_W = \frac{127 - 100}{15} = 1.80 \]

The person who made 127 on the Wechler test is the smartest.

VI. Skewed Distributions and the impact to mean and median.

We discussed earlier that in a normal distribution, the mean and median are equal. But, what happens if the data is no longer normally distributed. Let’s go back to our Excel file with the data with the Boys Max Growth years.

Show worksheet with normal distribution and values of mean and median.
Look below for sample of worksheet from Excel worksheet.

So, what happens to our distribution if I increase some of the Now skew the data to left by increasing larger numbers.

VII. Margin of error.

When one has a collection data, one is concerned with the sampling error that occurs with the data is collected. This sampling error is called margin of error.

Let’s say that you want to know what Texans believe is bad about being a child. You conduct a survey of 100 people who live in Texas. 17% of the respondents selected “Getting bossed around”. Since you did not polled everyone in Texas, we know there is some amount of error in the data. To find out what that error is, we will calculate the margin of error.

\[ me = \pm \frac{1}{\sqrt{n}} \times 100\% , \text{ where } n \text{ is the sample size} \]

Summary/Extension

Suppose 2000 people bought a new car this week. How many paid more than $17,500? How many paid less than $16,000?

Possible Assessments from Today’s Lesson

Teacher Reflection

1. Did you use the class time wisely? Loss time discussing extracting information from histograms.

2. What worked well in the lesson? Normal distribution discussion went fine.

3. What recommendations would you make to improve this lesson? Review of homework problems/questions, showed that the class had a hard time looking at a histogram, where the x-axis has integer tick marks, and calculating mean, median, Q1, and Q3. The class had no problem calculating the information from a list of numbers (concrete data). Many students had a lot of difficulty extracting the data from a histogram and calculating this information.
Later, found that many students had problems entering \( \frac{1}{\sqrt{n}} \) into the calculator. Need to make sure to spend more time on this topic next time this lesson is taught.

Other Comments About the Lesson
Merged Normal distribution and z-score, normal distribution, and margin of error problems into one lesson. Lost a class day because of power failures. Went into a stronger lecture mode than had previously planned, in order to make up for lost time.

References
Normal distribution (KEY)

The normal distribution is a continuous probability distribution that describes data that clusters around a mean or average. The graph is bell-shaped, with a peak at the mean, and is known as the bell curve.

Some conditions for a curve to be normal:

1) Most values are near the mean
2) Data values are spread evenly around the mean
3) Larger deviations from the mean become more rare
4) Individual data values result from a combination of different factors

The 68, 95, 99.7 Rule

Answer the following questions and label the normal distribution curve to the right of each problem to help.

Example 1:

Male adult heights in North America are approximately normally distributed with a mean of 70 inches and a standard deviation of 4 inches.

Find the percentage of men in North America with heights

1) Between 66 inches and 74 inches = 68%
2) Between 70 inches and 74 inches = 34%
3) Above 78 inches = 2.5%
4) Less than 66 inches = 16%
Example 2:

The mean of a new car is $17,000 and the S.D. is $500. Find the percentage of buyers who payed:

1) Between $16,500 and $17,500 = 68%
2) Between $16,000 and $18,000 = 95%
3) Between $17,000 and $17,500 = 34%
4) Between $15,500 and $17,500 = 83.85%
5) More than $17,500 = 16%
6) More than $18,000 = 2.5%
7) Less than $16,000 = 2.5%
Some conditions for a curve to be normal:

1)  
2)  
3)  
4)  

The 68, 95, 99.7 Rule

Answer the following questions and label the normal distribution curve to the right of each problem to help.
Example 1:

Male adult heights in North America are approximately normally distributed with a mean of 70 inches and a standard deviation of 4 inches.

Find the percentage of men in North America with heights

5) Between 66 inches and 74 inches
6) Between 70 inches and 74 inches
7) Above 78 inches
8) Less than 66 inches

Example 2:

The mean of a new car is $17,000 and the S.D. is $500. Find the percentage of buyers who payed:

9) Between $16,500 and $17,500
10) Between $16,000 and $18,000
11) Between $17,000 and $17,500
12) Between $15,500 and $17,500
13) More than $17,500
14) More than $18,000
15) Less than $16,000
Screenshots of Excel Worksheet showing Boys Max Growth years with normal distribution.

Mean = Median = 14
Screenshots of Excel Worksheet showing Boys Max Growth years with normal distribution.

Mean = 14.32, Median = 14, Mean ≠ Median
<table>
<thead>
<tr>
<th>File Name</th>
<th>Math 1315_Linear Regression_Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson Title</td>
<td>Linear Regression and Correlation</td>
</tr>
<tr>
<td>Prerequisite:</td>
<td>Normal Distribution</td>
</tr>
<tr>
<td>Content Objectives</td>
<td>Students will be able to:</td>
</tr>
<tr>
<td></td>
<td>E-CRA - Experiential, concrete,</td>
</tr>
<tr>
<td></td>
<td>representational, abstract, modeling</td>
</tr>
<tr>
<td>1.</td>
<td>determine whether or not data is</td>
</tr>
<tr>
<td></td>
<td>positively or negatively correlated.</td>
</tr>
<tr>
<td>2.</td>
<td>distinguish between strong versus weak</td>
</tr>
<tr>
<td></td>
<td>correlation of the data.</td>
</tr>
<tr>
<td>3.</td>
<td>find the line of best fit of a given</td>
</tr>
<tr>
<td></td>
<td>set of data points.</td>
</tr>
<tr>
<td>4.</td>
<td>predict new values based on the line of</td>
</tr>
<tr>
<td></td>
<td>best fit.</td>
</tr>
<tr>
<td>Technology Objectives</td>
<td>Students will be able to use the</td>
</tr>
<tr>
<td></td>
<td>graphing calculator to perform a linear</td>
</tr>
<tr>
<td>Cultural-Rellevant Lesson:</td>
<td>Super Bowl is this week.</td>
</tr>
<tr>
<td></td>
<td>Super Bowl advertising dollars and # TV</td>
</tr>
<tr>
<td>Real-World Application:</td>
<td>viewers were analyzed.</td>
</tr>
<tr>
<td></td>
<td>Students were shown how linear</td>
</tr>
<tr>
<td></td>
<td>regression to be used to analyzed real</td>
</tr>
<tr>
<td>Applications/Connections</td>
<td>Algebra</td>
</tr>
<tr>
<td></td>
<td>Geometry</td>
</tr>
<tr>
<td></td>
<td>Measurement</td>
</tr>
<tr>
<td></td>
<td>Statistics</td>
</tr>
<tr>
<td></td>
<td>Probability</td>
</tr>
<tr>
<td>Process Standards</td>
<td>Communication</td>
</tr>
<tr>
<td></td>
<td>Connections</td>
</tr>
<tr>
<td></td>
<td>Problem-Solving</td>
</tr>
<tr>
<td></td>
<td>Representations</td>
</tr>
<tr>
<td></td>
<td>Reasoning</td>
</tr>
</tbody>
</table>
### Lesson Agenda

<table>
<thead>
<tr>
<th>I. Correlation and Regression Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. A Few Definitions</td>
</tr>
<tr>
<td>b. Correlation and Regression Analysis handout</td>
</tr>
<tr>
<td>II. Regression Analysis on the Calculator</td>
</tr>
<tr>
<td>a. Activity – Line of Best Fit: Calculator Activity</td>
</tr>
<tr>
<td>III. Super Bowl TV Viewers</td>
</tr>
</tbody>
</table>

### Introduction/Background

**Background**

We have been discussing equations of lines and these types of graphs can be seen in statistics. Before now, we have discussed data with one variable, but today we will be expanding our data to contain two variables which is known as **bivariate data**, meaning two-variables. One way that we will use to represent quantitative bivariate data is graphically using a scatter plot. Since bivariate data involves two variables we represent the variable as a point \((x, y)\), where \(x\) represents one variable and \(y\) represents the other. When we place points on a coordinate plane it is called a **scatter plot**.

**Introduction**

Scatter plots can be very useful in determining whether or not there is a relationship among the data. When relationships are present, we can construct predictive equations, often called **lines of best fit**, which can help us “predict” what is likely to happen in the future. Today we will be exploring what statisticians call **regression analysis**, the technique of modeling and analyzing data.

### Main Lesson Script

**Group Activity: Super Bowl Statistics.**

Well Super Bowl time is almost here. Let’s look at some interesting data about Super Bowl games. Pass out the **HANDOUT: Correlation and Regression Analysis**. Break up the class into groups of two or three students. Have the students work through the handout. Walk around to answer questions as they arrive.

When the groups finished the handout, showing the class SPSS graphs of the data.

**Regression Analysis on the Calculator**

Pass out the **HANDOUT: Line of Best Fit (Calculator Activity)**. The handout is very detailed so follow along with the class using the overhead TI-Viewscreenshot or the ELMO to help navigate the students. Allow the students to work through the practice problems in pairs or groups then go over as a class.

<table>
<thead>
<tr>
<th>Instructor Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>TI Smartview</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Student Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>TI calculator</td>
</tr>
</tbody>
</table>
TO ENSURE THAT STUDENTS WILL GET R AND R² WHEN USING THE LINREQ PROGRAM ON THEIR CALCULATORS THE FOLLOWING MUST BE PERFORMED ON THEIR TI CALCULATORS.

1. Hit 2nd 0 (to get access to CATALOG)
2. The buttons on the calculator are now in alpha-numeric mode. Push X⁻¹ button. (to enter the letter “D”)
3. The viewing screen will display CATALOG with programs starting with the letter D. Scroll down till you find DiagnosticOn. Select DiagnosticOn.
4. DiagnosticOn will now show on your viewing screen. Hit Enter to run this program.
5. We are now finished. R and R² will now appear when the student runs LinReg.

Summary/Extension
Summary

Possible Assessments from Today’s Lesson

Teacher Reflection

1. Did you use the class time wisely? Ok. Have to watch out for homework questions taking up too much time.

2. What worked well in the lesson? Students like talking about the Super Bowl data.

3. What recommendations would you make to improve this lesson?

Other Comments About the Lesson

I used Super Bowl statistics that a student had previously turned from lesson three homework assignment.

I showed other student-produced graphs generated from lesson three homework assignment to the class.

References
Correlation and Regression Analysis

Example 1:
The table below shows the cost of a 30-second TV advertisement during the Super Bowl from 1967-2010. Note that years contained in the table are the number of years since 1967. Create a scatter plot then use the scatter plot to answer the questions.

<table>
<thead>
<tr>
<th>Years Since 1967</th>
<th>AdCostPer30Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>$2,800,000</td>
</tr>
<tr>
<td>40</td>
<td>$2,385,365</td>
</tr>
<tr>
<td>36</td>
<td>$2,200,000</td>
</tr>
<tr>
<td>32</td>
<td>$1,600,000</td>
</tr>
<tr>
<td>28</td>
<td>$1,150,000</td>
</tr>
<tr>
<td>24</td>
<td>$800,000</td>
</tr>
<tr>
<td>20</td>
<td>$600,000</td>
</tr>
<tr>
<td>16</td>
<td>$400,000</td>
</tr>
<tr>
<td>12</td>
<td>$185,000</td>
</tr>
<tr>
<td>8</td>
<td>$107,000</td>
</tr>
<tr>
<td>4</td>
<td>$72,500</td>
</tr>
<tr>
<td>0</td>
<td>$40,000</td>
</tr>
</tbody>
</table>

1. Describe the correlation among the data. (positive, negative, or neither)

2. Draw a line of fit for the data. (Approximate!)
Line of Best Fit
Calculator Activity

Now, the line of fit we found in the last example was an approximation based on our estimation. The calculator has the ability to use a method called least squares to estimate the line of best fit, meaning the line that best approximates the behavior of the data.

In the following activity, we will be using the calculator to find the line of best fit.

**Step 1: Make a scatter plot**
- First, we need to enter in the data. Press \( \text{STAT} \) to enter the statistics menu. You will see the screen shown below.

![Calc TESTS Menu](image)

- We want to edit the data so press \( \text{ENTER} \).

<table>
<thead>
<tr>
<th>L1</th>
<th>L2</th>
<th>L3</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \text{L1(1) =} \)

- If there are any numbers in L1 or L2 use the up arrow to highlight L1 or L2, then press \( \text{CLEAR} \), then \( \text{ENTER} \). This will clear out any previous data.
- Now, L1 will be the independent variable, so we will put the years since 1967 in this list. In L2 will be the dependent variable, so we will put the ad cost in this list.

<table>
<thead>
<tr>
<th>L1</th>
<th>L2</th>
<th>L3</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>2.3E6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>2.3E6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>1.6E6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>1.1E6</td>
<td>800000</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>800000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \text{L2(1) =} 2800000 \)
• In order to view the scatter plot, we must turn the plot on using the following key strokes: 2nd Y= ENTER ENTER.
• Then press ZOOM 9. The scatter plot should look like this:

```
Step 2: Find the equation of the line of best fit
• To turn on correlation coefficient: Enter 2nd 0 ENTER ENTER.
• We can determine the equation of the line of best fit using the following key strokes: STAT CALC, 4 ENTER.
```

<table>
<thead>
<tr>
<th>EDIT</th>
<th>CALC</th>
<th>TESTS</th>
<th>LinReg</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:1-Var Stats</td>
<td>2:2-Var Stats</td>
<td>3:Med-Med</td>
<td>y=ax+b</td>
</tr>
<tr>
<td>4:LinReg(a+b)</td>
<td>5:QuadReg</td>
<td>6:CubicReg</td>
<td>a=66341.83544</td>
</tr>
<tr>
<td>7:QuartReg</td>
<td></td>
<td></td>
<td>b=-425669.8101</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>r²=.9169213514</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>r=.9575601033</td>
</tr>
</tbody>
</table>

• The resulting screen will give you the linear regression, or the equation of the line of best fit.
• From the LinReg screen we see that the equation of the line of best fit is \( y = 66,341.8x - 425,669.8 \)
• The number \( r \) is called the linear correlation coefficient. The closer the value of \( r \) is to 1 or -1, the closer the data points are to the line. In this case, since \( r \) is 0.957, this data is very close to being a line!
• The data has a strong positive correlation.
**Step 3: Graph the regression equation**

- In order to graph the regression equation, we need to copy it into the \( Y = \) screen. Then, when we hit the \( \text{GRAPH} \) key, we will see the following scatter plot with the regression line:

![](image)

**Step 4: Predict using the regression equation**

- We could use the regression equation to predict the Super Bowl Ad Cost for the year 2013 by using the following key strokes: (2013 is 46 years since 1967)
  
  \[
  \text{2nd, TRACE, 1} \quad \text{(type in 46), \ ENTER}
  \]

  - This shows us that the predicted Ad Cost will be $2,626,054.6 per 30 seconds.

- We could have also calculated this value by substituting 46 in for \( x \) in the equation,
  
  \[
  y = 66,341.8x - 425,669.8 \\
  y = 66,341.8(46) - 425,669.8 \\
  y = 3,051,722.8 - 425,669.8 \\
  y = 2,262,053
  \]
**Practice**
For the two data sets that follow, use you calculator to answer these questions:

1. Find a scatter plot of the data.
2. Find a regression equation for the data.
3. Use the equation given by your calculator to predict other values.

**Data Set 1**
The following table shows the Average number of attendance for TV viewers for the Super Bowl from 1967.

Note: Years contained in the table are since 1967
Use the data to predict the number of TV Viewers in 2013.

<table>
<thead>
<tr>
<th>Years Since 1967</th>
<th>TV Viewers in Thousands</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>106,500</td>
</tr>
<tr>
<td>40</td>
<td>93,184</td>
</tr>
<tr>
<td>36</td>
<td>88,637</td>
</tr>
<tr>
<td>32</td>
<td>83,720</td>
</tr>
<tr>
<td>28</td>
<td>83,420</td>
</tr>
<tr>
<td>24</td>
<td>79,510</td>
</tr>
<tr>
<td>20</td>
<td>87,190</td>
</tr>
<tr>
<td>16</td>
<td>81,770</td>
</tr>
<tr>
<td>12</td>
<td>74,740</td>
</tr>
<tr>
<td>8</td>
<td>56,050</td>
</tr>
<tr>
<td>4</td>
<td>46,040</td>
</tr>
<tr>
<td>0</td>
<td>51,180</td>
</tr>
</tbody>
</table>

**Data Set 2**
The table below shows the total league baseball games in recent years the data to predict attendance in the year 2015.

<table>
<thead>
<tr>
<th>Year</th>
<th>Attendance (mill)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>18.4</td>
</tr>
<tr>
<td>1990</td>
<td>25.2</td>
</tr>
<tr>
<td>1995</td>
<td>33.1</td>
</tr>
<tr>
<td>2000</td>
<td>37.6</td>
</tr>
</tbody>
</table>
Regression Homework

For exercises 1-5, use Table 1 that shows the percent of music Sales made in record stores in the United States for the period 1995-2004.

1. Make a scatter plot of the data. Is the correlation of the data positive or negative? Explain.

2. Find a regression equation for the data.

3. According to the regression equation, what was the average rate of change of record store sales during this period of time?

4. Use the regression equation to predict the percent of sales that will be in record stores in the year 2015.

5. How accurate do you think your prediction is? Explain.

Table 1

<table>
<thead>
<tr>
<th>Year</th>
<th>Sales (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>52</td>
</tr>
<tr>
<td>1996</td>
<td>49.9</td>
</tr>
<tr>
<td>1997</td>
<td>51.8</td>
</tr>
<tr>
<td>1998</td>
<td>50.8</td>
</tr>
<tr>
<td>1999</td>
<td>44.5</td>
</tr>
<tr>
<td>2000</td>
<td>42.4</td>
</tr>
<tr>
<td>2001</td>
<td>42.5</td>
</tr>
<tr>
<td>2002</td>
<td>36.8</td>
</tr>
<tr>
<td>2003</td>
<td>33.2</td>
</tr>
<tr>
<td>2004</td>
<td>32.5</td>
</tr>
</tbody>
</table>

For exercises 6-10, use Table 2 that shows the amount of money spent on sporting footwear in some recent years.

6. Find a regression equation for the data.

7. Use the regression equation to predict the sales in the year 2010.

8. Delete the outlier (1999, 12546) from the data set and find a new regression equation for the data.

9. Use the new regression equation to predict the sales in the year 2010.

Table 2

<table>
<thead>
<tr>
<th>Year</th>
<th>Sales (mill)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>13,068</td>
</tr>
<tr>
<td>1999</td>
<td>12,546</td>
</tr>
<tr>
<td>2000</td>
<td>13,026</td>
</tr>
<tr>
<td>2001</td>
<td>13,814</td>
</tr>
<tr>
<td>2002</td>
<td>14,144</td>
</tr>
<tr>
<td>2003</td>
<td>14,446</td>
</tr>
<tr>
<td>2004</td>
<td>14,752</td>
</tr>
</tbody>
</table>
SPSS graph of Super Bowl advertisement cost per 30 second commercial over the past 45 years.
SPSS graph of # Super Bowl TV viewers over the past 45 years

Scatterplot

Dependent Variable: TV Viewers

Regression Standardized Predicted Value

TV Viewers
### C-7 Linear Equations Lesson Plan

<table>
<thead>
<tr>
<th>File Name</th>
<th>Math_1315_Linear_Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson Title</td>
<td>Linear Equations</td>
</tr>
<tr>
<td>Prerequisites</td>
<td>Linear Regression</td>
</tr>
</tbody>
</table>

#### Content Objective(s)

<table>
<thead>
<tr>
<th>Students will be able to</th>
<th>E-CRA -  Experiential, concrete, representational, abstract, modeling</th>
</tr>
</thead>
<tbody>
<tr>
<td>- derive, calculate, and find the slope of a line</td>
<td>concrete, representational, abstract</td>
</tr>
<tr>
<td>- calculate and find the slope between two points</td>
<td>concrete, representational, abstract</td>
</tr>
<tr>
<td>- Understand what slope means in an equation of a line and how it applies to an application problem</td>
<td>concrete, representational, abstract, modeling</td>
</tr>
</tbody>
</table>

#### Technology Objective(s)

None.

#### Culturally Relevant Objective(s)

None

#### Real-World Applications

Sales at record store were plotted and discussed what the data is communicating to us.

#### Connections

- Algebra
- Geometry
- Measurement
- Statistics
- Probability
- Functions

#### Process Standards

- Communication
- Connections
- Problem-Solving
- Representations
- Reasoning
- Cross-Disciplinary
<table>
<thead>
<tr>
<th>Lesson Agenda</th>
<th>Instructor Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Derive the formula for the slope of a line</td>
<td></td>
</tr>
<tr>
<td>II. Interpret slope of a line</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pre-Homework</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Introduction/Background</th>
</tr>
</thead>
<tbody>
<tr>
<td>Last class we performed a linear regression on a set of statistical data. When reviewing the results of the linear regression and the line of best fit provided by a linear regression, we discussed the fact that the when the data on the x-axis is increasing at the same time data on the y-axis was increasing, then we have a positive correlation. When the data on the x-axis is decreasing at the same time data on the y-axis was increasing, then we have a negative correlation. Referring to past math knowledge, many of you said the slope is positive or negative depending on the correlation. Let’s investigate the slope further.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Main Lesson Script</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Derive the slope formula.</td>
</tr>
<tr>
<td>Group Activity: Break up the class into groups of two. Pass out the handout: Reflecting on the Sales at the Record Store. Students will derive the slope formula from the homework problem’s data and results.</td>
</tr>
<tr>
<td>II. Equation of a line: y-intercept.</td>
</tr>
<tr>
<td>Group Activity: Have the students stay with their partner. Have them now work on the handout: Lines.</td>
</tr>
<tr>
<td>* Students should notice that the line is moving up and down depending if a linear equation has a constant term in the equation.</td>
</tr>
<tr>
<td>* Students should also take note that all of these lines are parallel. Make sure students derive this from the various equations.</td>
</tr>
<tr>
<td>* Have the students discuss the property of perpendicular lines that the product of their slope is equal to -1.</td>
</tr>
<tr>
<td>III. Equations of a line.</td>
</tr>
<tr>
<td>Have the students now refer to the Handout: Equations of a Line. Discuss the</td>
</tr>
</tbody>
</table>
differences between Stand Form, Slope-Intercept form, and Point-Slope form.

**Summary/Extension**

**Possible Assessments from Today’s Lesson**

**Teacher Reflection**

1. Did you use the class time wisely? No. *Had many questions on use of graphing calculator to perform linear regression from previous lesson.*

2. What worked well in the lesson? *Lesson went fine. The instruction of the parts of the line and the various equations of the line went well. This is familiar mathematical topics for many of the students.*

3. What recommendations would you make to improve this lesson? *Make sure to spend enough time discussing what slope means in application problems. Require the students to write out what the slope of applications was communicating to us. Instead of saying “... the line is going down”, I wanted them to say that “% sales was decreasing over the years 1995-2000.” This was difficult for the students at first.*

**Other Comments About the Lesson**

**References**
Reflecting on the Sales at the Record Store

Using the information from the regression homework problem, from problem #2 we obtained the linear regression, \( y = 4823.05 - 2.39x \).

a) Using the model for the year 1995, we calculate % of sales at a record store to be:

b) Using the model for the year 1996, we calculate % of sales at a record store to be:

c) How much the % sales change from 1995 to 1996?

d) Using the model for the year 2000, we calculate % of sales at a record store to be:

e) How much the % sales change from 1995 to 2000?

f) Compare the answers received in questions c and e. What did you notice?

g) Can we construct a formula for these changes?

h) What do we call this formula?
Lines

Exercise 1.
Using your graphing calculators,
Graph the equation, \( y = \frac{1}{2}x + 5 \), then
graph the equation, \( y = -2 + \frac{1}{2}x \).

What do you notice from reviewing your viewscreen?

Now, graph the equation, \( y = 0.5x \).

What do you notice? Can we make a generalize statement?

If you enter the following equations, what do you notice?
\[
\begin{align*}
y &= 0.5x \\
y &= \frac{1}{2}x + 0
\end{align*}
\]

Can we make a generalize statement?

Exercise 2.

Lines are perpendicular if the product of their slopes is equal to \(-1\).
What does this mean?

Give an example.
Equations of a Line

Standard Form  \[ Ax + By = C \]  
A and B cannot both be zero.  
Describes all lines.

Slope-intercept Form  \[ y = mx + b \]  
\( m \) = slope of line;  
\( b \) = y-intercept

Point-slope Form  \[ y - y_1 = m(x - x_1) \]  
(\( x_1, y_1 \)) is a point on the line;  
\( m \) = slope of line

Using information from above, what are the coordinates represented by (a) and (b). Can you find an equation of a line from this information in slope-intercept form?

Can you change this equation so that it is in Standard Form?

Using information from (a) and (d), can you find an equation of a line in standard form?

Can you change this equation so that it is in slope-intercept form?
C-8 Linear Mathematical Models Lesson Plan

<table>
<thead>
<tr>
<th>File Name</th>
<th>Modeling_with_linear_regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson Title</td>
<td>Testing and modeling linear regression</td>
</tr>
<tr>
<td>Prerequisites</td>
<td>Correlation</td>
</tr>
<tr>
<td>Instructional Objectives</td>
<td>E-CRA - Experiential, concrete, representational, abstract, modeling</td>
</tr>
<tr>
<td>Students will be able to</td>
<td>experiential, concrete, representational</td>
</tr>
<tr>
<td>- Read data from editorial graphs</td>
<td>abstract, modeling</td>
</tr>
<tr>
<td>- Compare results of linear regressions</td>
<td>abstract, modeling</td>
</tr>
<tr>
<td>- Predict with regression model</td>
<td>modeling</td>
</tr>
<tr>
<td>- Interpret if results from regression model makes sense</td>
<td>modeling</td>
</tr>
</tbody>
</table>

| Technology Objectives | Student will be able to use graphing calculator to analyze bivariate data from using regression models |

| Real World Applications | Used data on the number work hours and leisure hours during the week as reported by a national newspaper. |

| Culturally-Relevant Curriculum and/or Pedagogy | Work in groups. Use of technology. Active learning. |

<table>
<thead>
<tr>
<th>Connections</th>
<th>Process Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebra</td>
<td>Communication</td>
</tr>
<tr>
<td>Geometry</td>
<td>Connections</td>
</tr>
<tr>
<td>Measurement</td>
<td>Problem-Solving</td>
</tr>
<tr>
<td>Statistics</td>
<td>Representations</td>
</tr>
<tr>
<td>Probability</td>
<td>Reasoning</td>
</tr>
<tr>
<td>Functions</td>
<td>Cross-Disciplinary</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lesson Agenda</th>
<th>Instructor Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Activity:</td>
<td>TI-84 Calculator</td>
</tr>
<tr>
<td>a. Reading editorial graph to extract data.</td>
<td></td>
</tr>
<tr>
<td>b. Build linear models.</td>
<td></td>
</tr>
<tr>
<td>c. Perform predictions from model.</td>
<td></td>
</tr>
<tr>
<td>II. Whole group discussion on results of the activity.</td>
<td></td>
</tr>
<tr>
<td>Pre-Homework</td>
<td>Student Materials</td>
</tr>
<tr>
<td></td>
<td>TI-84 calculator</td>
</tr>
<tr>
<td></td>
<td>Student handouts</td>
</tr>
</tbody>
</table>
**Introduction/Background**

Previously we have looked at some raw data that we had collected and created scatter plots. We then explored the use of algebra to create equations of a line from the data. Then we discussed and used a graphing calculator to create line regression models. Today we are going to look at a graph printed in USA Today of the median work hours and leisure hours in a week for most Americans. We are going analyze the data, create a model of the data, then perform some predictions with the model.

---

**Main Lesson Script**

**Activity:**

[Pass out the Hand out: Is Leisure Time Really Shrinking?]

Do you recall the difference from median and mean? The data in this activity are median work hours and median leisure hours. I need for you to break up into groups of two. This activity where we will be looking a Working as partners, I want you to complete this activity.

[This activity takes up most the class time. Make sure to allow time to conduct a whole class discussion where groups can share their results with the class.]

**Whole-class discussion:**

What do you think about the activity? Is this a strong linear model? [Student response: When we saw the actual data versus what we predicted for years 2004, 2007, & 2008? Did it bother you that the predictions were not exactly what the “real data” was? What about the year 2030? 2050? Keep this handout, because we will be coming back to these results later in the course.

---

**Summary/Extension**

Foreshadow quadratic equations: Do you think the data looks like something else you have seen in a prior math class? [Student response: looks like a parabola]

---

**Possible Assessments from Today’s Lesson**

Can you this regression model for other years?

Why or why not is the model a strong linear regression?

Is there a point when the model will or will not be reliable?

---

**Teacher Reflection**

1. Did you use the class time wisely?  
   *Yes. Time management was not a problem.*

2. What worked well in the lesson?  
   *Group work went well. Students Learned much from the modeling exercise. Students still are unaware (Some students are suspicious because of my foreshadowing comments.) that quadratic regression will produce stronger model. But, some students are okay with the predictions when the model begins to deteriorate in its predictions in future years.*

3. What recommendations would you make to improve this lesson?  
   *Need to be prepared to direct students on how to determine the number of decimal places to use for “slope” and y-intercept. Had to go group to group to discuss this topic. Need to discuss ahead of time before group activity begins; or incorporate this topic into the handout.*
Two Latino students asked me for the definition of Leisure. So, I need to make sure to define, leisure, the next time I teach this lesson.

Other Comments About the Lesson
Some students had problems reading this editorial graph with two sets of bivariate data. They could not see two sets of coordinate pairs. So, this lesson plan strengthened their graph reading/interpreting skills.

References

Is Leisure Time Really Shrinking?

The below chart was provided by USA Today on data collected in a series of surveys conducted by Harris Poll. Describe the trend on the median number of hours a week that Americans work (including commuting, housework and school).

Is this data categorical or quantitative? Is it univariate or bivariate?

What are the points (coordinates) in the graph for work hours?

Enter the data into your calculator and create a scatter plot of the work hours.

Do the data look linear?

Run a linear regression and record the results below.

Record: regression equation: \[ y = 0.329821x - 607 \]

\[ r = 0.91 \quad r^2 = 0.82 \]

Graph the linear regression. Describe the regression line.
Predict the number of work hours for the years 2004, 2007, and 2008 using the linear regression program on your calculator.

2004 **53.96**  
2007 **54.95**  
2008 **55.28**

A review of later versions of the Harris Poll we find the median average of work hours in a week for the years 2004, 2007, and 2008.

2004 **50**  
2007 **45**  
2008 **46**

Calculate the percent error in the predictions.

Percent error = \( \frac{(\text{predicted value}) - (\text{real value})}{\text{real value}} \times 100\% \)

2004 percent error = \( \frac{53.96 - 50}{50} \times 100\% = 7.92\% \)

2007 percent error = \( \frac{54.95 - 45}{45} \times 100\% = 22.11\% \)

2008 percent error = \( \frac{55.28 - 46}{46} \times 100\% = 20.17\% \)

How well do you think the model did for the predictions of these three years?

What does the model predict for the year 2015? 2030?

For 2015, 57.59 hours, For 2030, 62.54 years

Does this seem reasonable?

Now repeat these steps for the average number of leisure hours.

Describe the trend on the median number of hours a week that Americans spend on leisure.

Is this data categorical or quantitative? Is it univariate or bivariate?

What are the points (coordinates) in the graph for leisure hours?

Enter the data into your calculator and create a scatter plot of the leisure hours by doing the following steps.

1. Enter leisure hour into List 3 or L3. Remember Years are already in L1.
2. Turn on Stat Plot2 for a scatter plot.
3. Make sure that for Plot2 that Xlist is L1 and Ylist is L3.
4. Hit **GRAPH** button. You will see both sets of data.

Do the leisure hours data look linear?

Run a linear regression and record the results below. When doing the linear regression do the following **LinReg(ax+b) L1,L3,Y2**

Record: regression equation: \( y = -163382x + 345 \)

\( r = -0.56 \)  \( r^2 = 0.31 \)

Graph the linear regression. Describe the regression line.
Predict the number of leisure hours for the years 2004, 2007, and 2008.

2004  17.69  2007  17.20  2008  17.04

A review of later versions of the Harris Poll we find the median average of leisure hours in a week for the years 2004, 2007, and 2008.

2004: 19 hours  2007: 20 hours  2008: 16 hours

Calculate the percent error in the predictions.
2004 percent error = \((17.69 - 19) / 19 \times 100\% = -6.89\%\)
2007 percent error = \((17.20 - 20) / 20 \times 100\% = -14.00\%\)
2008 percent error = \((17.04 - 16) / 16 \times 100\% = +6.50\% (note this is a positive answer, not negative!\)

How well do you think the model did for the predictions of these three years?

What will happen in the year 2015? Does this seem reasonable?
For 2015, 15.90 leisure hours
What about the year 2030?
For 2030, 13.45 leisure hours
Is Leisure Time Really Shrinking?

This chart was provided by *USA Today* on data collected in a series of surveys conducted by Harris Poll.
Describe the trend on the number of hours a week that Americans work (including commuting, housework and school).

Is this data categorical or quantitative? Is it univariate or bivariate?

What are the points (coordinates) in the graph for work hours?

Enter the data into your calculator and create a scatter plot of the work hours.

Do the data look linear?

Run a linear regression and record the results below.
Record: regression equation: _____________________________
\[ r = \ldots \quad r^2 = \ldots \]

Graph the linear regression. Describe the regression line.
Predict the number of work hours for the years 2004, 2007, and 2008 using the linear regression program on your calculator.

2004 __________ 2007 __________ 2008 __________

A review of later versions of the Harris Poll we find the median average of work hours in a week for the years 2004, 2007, and 2008. Is the following:
2004: 50 hours  
2007: 45 hours  
2008: 46 hours

Calculate the percent error in the predictions.
Percent error = \( \frac{(predicted\ value)-(real\ value)}{real\ value} \times 100\% \)

2004 percent error = _____________________________
2007 percent error = _____________________________
2008 percent error = _____________________________

How well do you think the model did for the predictions of these three years?

Now repeat these steps for the median number of leisure hours.
Describe the trend on the number of hours a week that Americans spend on leisure.

Is this data categorical or quantitative? Is it univariate or bivariate?
What are the points (coordinates) in the graph for leisure hours?

Enter the data into your calculator and create a scatter plot of the leisure hours by doing the following steps.
1. Enter leisure hour into List 3 or L3. Remember Years are already in L1.
2. Turn on Stat Plot2 for a scatter plot.
3. Make sure that for Plot2 that Xlist is L1 and Ylist is L3.
4. Hit GRAPH button. You will see both sets of data.
Do the leisure hours data look linear?

Run a linear regression and record the results below. (When doing the linear regression do the following LinReg(ax+b) L1,L3,Y2
Record: regression equation: ________________________________
        r = _____       r^2 = _____

Graph the linear regression. Describe the regression line.

Predict the number of leisure hours for the years 2004, 2007, and 2008.
2004 _______ 2007 _______ 2008 _______
A review of later versions of the Harris Poll we find the median average of leisure hours in a week for the years 2004, 2007, and 2008.
2004: 19 hours 2007: 20 hours 2008: 16 hours

Calculate the percent error in the predictions.
2004 percent error = ______________________
2007 percent error = ______________________
2008 percent error = ______________________

How well do you think the model did for the predictions of these three years?

What will happen in the year 2015? Does this seem reasonable?

What about the year 2030? Does this answer seem reasonable?
C-9 Quadratic Equations Lesson Plan

<table>
<thead>
<tr>
<th>File Name</th>
<th>Math_1315_Quadratic Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson Title</td>
<td>Characteristics of Quadratic Equations</td>
</tr>
<tr>
<td>Prerequisites</td>
<td>Linear Modeling</td>
</tr>
<tr>
<td>Content Objective(s)</td>
<td>Students will be able to understand characteristics of quadratic equations</td>
</tr>
<tr>
<td>Technology Objective(s)</td>
<td>Use graphing calculator to graph quadratic equations, and then find vertex and zeroes.</td>
</tr>
<tr>
<td>Culturally Relevant Objective(s)</td>
<td></td>
</tr>
<tr>
<td>Real-World Applications</td>
<td></td>
</tr>
<tr>
<td>Connections</td>
<td>Process Standards</td>
</tr>
<tr>
<td>Algebra</td>
<td>Communication</td>
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<td>Connections</td>
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<td>Problem-Solving</td>
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<tr>
<td>Statistics</td>
<td>Representations</td>
</tr>
<tr>
<td>Probability</td>
<td>Reasoning</td>
</tr>
<tr>
<td>Functions</td>
<td>Cross-Disciplinary</td>
</tr>
<tr>
<td>Lesson Agenda</td>
<td>Instructor Materials</td>
</tr>
<tr>
<td>I. Quality of linear model</td>
<td>TI-Smartview</td>
</tr>
</tbody>
</table>
II. Quadratic Equations

Student Materials

graphing calculator

Pre-Homework

None

Introduction/Background

In the previous class, we were graphing and analyzing the median of average # work hours and median of average # of leisure hours in a week. We performed a linear regression on real data. We received a correlation coefficient of r=0.91 for the linear model. But, for what period of time is this going to work? Will the prediction still seem reasonable?

Main Lesson Script

I. Linear Model

From this linear model, we found out the following:

<table>
<thead>
<tr>
<th>YEAR</th>
<th>PREDICTION</th>
<th>ACTUAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>52.96 hours</td>
<td>50 hrs</td>
</tr>
<tr>
<td>2007</td>
<td>53.95 hours</td>
<td>45 hrs</td>
</tr>
<tr>
<td>2008</td>
<td>54.28 hours</td>
<td>46 hrs</td>
</tr>
</tbody>
</table>

Maybe losing some of the integrity of the model. Can something else model this data with more integrity?

If we examine the graph of the real data, what do we intuitively believe is happening? [the data is not continuing to increase; it is starting to come down] Why? [answers may vary, economic recession, depression] Let’s talk more about it. Can we possible use something else to model the data?

II. Quadratic Equations

Quadratic equations have a graph that is parabolic in shape. Algebraically, it has a standard form of

\[ ax^2 + bx + c = 0, \text{ where } a \neq 0. \]

Can anyone give me some examples?

Look at the greatest exponent for the variables. What is it? [2]

- Discuss the geometric characteristics such as
  - parabolic shape,
  - its vertex,
  - line of symmetry,
absolute minimum or maximum,
# y-intercepts (1),
# x-intercepts, (0, 1, 2)

Graph \(2x^2 + 3x-4 = 0\). Discuss its characteristics.

The vertex occurs when \(x = \frac{-b}{2a}\). The y coordinate of the vertex is equal to the quadratic equation evaluated with this value for \(x\).

Let’s find the vertex for the \(2x^2 + 3x-4 = 0\). This is the same coordinate as we found earlier for the vertex.

Show students how to graph quadratics using their calculators.

- Solving quadratic equations.

When solving quadratic equations, we need for the quadratic equation to be in the standard form. How can we solve it? [factoring, square root property, completing the square, or the quadratic formula]

Factoring: Use the following examples
\[(x+3)(x-4) = 8\]
\[x^2 - x = 12\]

Square Root Property:
\[x^2 = k\] or \[x^2 - k = 0\]
\[(x - \sqrt{k})(x + \sqrt{k}) = 0\]
difference of squares
\[x = \pm\sqrt{k}\]
If \(k>0\), then \(x^2 = k\) has two real solutions.
If \(k<0\), then \(x^2 = k\) has two imaginary solutions.

Ask the students to work the following examples:
\[x^2 - 16 = 0\]
\[2x^2 - 1 = 0\]
\[(3x-4)^2 = 0\]

Quadratic formula. Does anyone remember what the quadratic formula is?

For \(ax^2 + bx + c = 0\), where \(a\neq0\), the quadratic formula is
\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

Can someone come to the board and show us how to find the solutions to
\[x^2 + 8x + 6 = 0\]?

Now I want everyone to solve the following problem:
\[4x^2 + 9 = 12x\]
• Discriminants. The discriminant is the expression $b^2 - 4ac$ where $a,b,c$ are the coefficients and constant term of the quadratic equation in standard form.
  
  If $b^2 - 4ac > 0$, then the quadratic equations has 2 real solutions.
  If $b^2 - 4ac = 0$, then the quadratic equations has 1 real solution.
  If $b^2 - 4ac < 0$, then the quadratic equations has 2 imaginary solutions, or 0 real solutions.

Let’s find the number of real solutions for the equation, $2x^2 + 3x - 4 = 0$.

$a=2, b=3, c=-4$

$\text{discriminant is } (3)^2 - 4(a)(-4) = 41$

$41 >0$, so this equations has 2 real solutions.

Let’s go back and try this on some of the problems that we have previously worked.

III. Quadratic Regression

Please get back into your groups. Now take your data from the group activity we did earlier and run a quadratic regression on your calculator. What does the graph look like? Did you get an $r$? a $r^2$? Which model is stronger the linear or the quadratic? Or does it depend on which values of $x$ you are interested in?

Summary/Extension

Possible Assessments from Today’s Lesson

Teacher Reflection

1. Did you use the class time wisely? No problems.

2. What worked well in the lesson? Discussion of characteristics of quadratic equations.

3. What recommendations would you make to improve this lesson? None.

Other Comments About the Lesson

References
### C-10 Linear and Quadratic Problems Lesson Plan

<table>
<thead>
<tr>
<th>File Name</th>
<th>Math_1315_Linear_vs_Quad_Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson Title</td>
<td>Linear and Quadratic Problems</td>
</tr>
<tr>
<td>Prerequisites</td>
<td>Quadratic Equations</td>
</tr>
<tr>
<td>Content Objective(s)</td>
<td>Students will be able to</td>
</tr>
<tr>
<td></td>
<td>E-CRA - Experiential, concrete, representational, abstract, modeling</td>
</tr>
<tr>
<td></td>
<td>Solve linear and quadratic application problems</td>
</tr>
<tr>
<td></td>
<td>concrete, representational, abstract, modeling</td>
</tr>
<tr>
<td></td>
<td>Identify if a scatter plot is linear or quadratic</td>
</tr>
<tr>
<td></td>
<td>representational, abstract, modeling</td>
</tr>
<tr>
<td></td>
<td>Perform a quadratic regression</td>
</tr>
<tr>
<td></td>
<td>concrete, representational, abstract, modeling</td>
</tr>
<tr>
<td>Technology Objective(s)</td>
<td>Use graphing calculator graph data and perform quadratic regression</td>
</tr>
<tr>
<td>Culturally Relevant Objective(s)</td>
<td>None</td>
</tr>
<tr>
<td>Real-World Applications</td>
<td>Students will see use of quadratic equations to solve application problems.</td>
</tr>
<tr>
<td>Connections</td>
<td>Process Standards</td>
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<tr>
<td>Algebra</td>
<td>Communication</td>
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<td>Geometry</td>
<td>Connections</td>
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<tr>
<td>Measurement</td>
<td>Problem-Solving</td>
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<tr>
<td>Statistics</td>
<td>Representations</td>
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<tr>
<td>Probability</td>
<td>Reasoning</td>
</tr>
<tr>
<td>Functions</td>
<td>Cross-Disciplinary</td>
</tr>
</tbody>
</table>
Lesson Agenda

I. Differences between linear and quadratic equations
II. Formulas and solving problems
III. Quadratic Problems
IV. Linear application

Instructor Materials
TI-Smartview

Student Materials
graphing calculator

Pre-Homework
None

Introduction/Background

Last class we discussed the characteristics of the quadratic equation. Today we are going to use the quadratic equations to solve application problems.

Main Lesson Script

I. Differences between linear and quadratic equations.
   Activity: I want you to find a partner and list the differences between linear and quadratic equations.

   When groups are complete, ask students to present their findings.

II. Formulas and solving problems.
   Note: For linear equations, \( y = mx + b \), \( y \) is a linear function of \( x \). This will be discussed in more detail later.

   Linear Example: page 124.
   From ABC Wireless the monthly cost for a cell phone with 100 minutes per month is $35, or 200 minutes per month for $45. The cost in dollars is a linear function of the time in minutes.

   Find the formula for the cost, \( C \).
   Find slope for the points (100,35) and (200,45).
   \[
   m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{200 - 100}{35 - 25}
   \]
   \[
   m = 0.10
   \]
   The slope is $0.10 per minute.

   Using point-slope formula, we find that the formula is \( C = 0.10x + 25 \).

   b) What is the cost for 400 minutes?
199

\[ C = 0.10(400) + 25 = 65 \text{ or } $65 \]

II. Scatter diagrams.

Look at the scatter plots on page 131. In Example 1, what kind of equations do you believe would best describe these scatter plots? \(\text{[linear]}\)

Now look at the two scatter plots on the bottom of the page. Which type of equations would best describe these scatter plots? \(\text{[linear for the first scatter plot; quadratic for the second plot]}\)

III. Quadratic Problem

Now let’s go to page 146 and look at the example 8 which uses the Pythagorean Theorem. I want you to work with a partner and do discuss this problem.

IV. Linear application

Now I want you and your partner to work problem #105 on page 149.

When groups finish, have a student come to the board and show how to work this problem.

Summary/Extension

Possible Assessments from Today’s Lesson

Teacher Reflection

1. Did you use the class time wisely? \(\text{yes}\)

2. What worked well in the lesson? \(\text{By utilizing mathematical modeling, the demonstrations of the differences between mathematical equations with real world data.}\)

3. What recommendations would you make to improve this lesson? \(\text{I am having problems finding good homework assignments associated with modeling. I need to look at other available textbooks.}\)
<table>
<thead>
<tr>
<th>Other Comments About the Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students are still struggling with using regression analysis on calculators.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>References</th>
</tr>
</thead>
</table>


## C-11 Introduction to Functions Lesson Plan

<table>
<thead>
<tr>
<th><strong>File Name</strong></th>
<th>Math1315_IntroToFunctions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Introduction to Functions</td>
</tr>
<tr>
<td><strong>Prerequisite Objective</strong></td>
<td>Solving Linear and Quadratic Equations</td>
</tr>
<tr>
<td><strong>Instructional Objectives</strong></td>
<td><strong>E-CRA</strong> - Experiential, concrete, representational, abstract, modeling</td>
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<tr>
<td>Student’s will be able to:</td>
<td>Concrete, representational</td>
</tr>
<tr>
<td>- build a general definition of function through various representations; the graphical and tabular</td>
<td></td>
</tr>
<tr>
<td>identify when linear equations are functions</td>
<td>Representational, abstract</td>
</tr>
<tr>
<td>identify the identity and constant functions</td>
<td>Representational, abstract</td>
</tr>
<tr>
<td><strong>Technology Objectives</strong></td>
<td>None</td>
</tr>
<tr>
<td><strong>Connections</strong></td>
<td>Algebra, Geometry, Measurement, Statistics, Probability, Functions</td>
</tr>
<tr>
<td><strong>Process Standards</strong></td>
<td>Communication, Connections, Problem-Solving, Representations, Reasoning, Cross-Disciplinary</td>
</tr>
<tr>
<td><strong>Lesson Agenda</strong></td>
<td>Instructor Materials, Handouts</td>
</tr>
<tr>
<td>I. Introduction to Relations and Functions</td>
<td></td>
</tr>
<tr>
<td>II. Introduction to Linear Functions</td>
<td></td>
</tr>
<tr>
<td>III. Common Parent Functions</td>
<td></td>
</tr>
<tr>
<td>a. Identity</td>
<td></td>
</tr>
<tr>
<td>b. Constant</td>
<td></td>
</tr>
<tr>
<td><strong>Introduction/Background</strong></td>
<td>Student Materials, None</td>
</tr>
</tbody>
</table>
We have been discussing linear equations in two variables. Common forms that we found were:

- \( Ax + By = C \) which we denote as **Standard Form**, and
- \( y = mx + b \) which we denote as **Slope Intercept Form**.

Can anyone think of any others? [Student’s Response: The point-slope formula.]

Today we want to discuss another aspect of linear equations. They are also called linear functions. You may have heard the term **linear function**. We are going to learn about what it means to be a function in general and the ways to represent functions.

**Main Lesson Script**

**I. Introduction to Functions**

Have students refer to the HANDOUT: Functions. When we talk about functions, there are some new definitions that we need to review. A set of ordered pairs is called a **relation**. Can anyone give me an example of a relation? [Student’s answers will vary.] The set of all x-coordinates is called the **domain** of the relation, and the set of all y-coordinates is called the **range** of a relation.

So for the following relation, what is the domain and range?

\[ \{(0,1),(2,3),(4,5),(8,9)\} \]

Answer: [Domain is the set \{0,2,4,8\} and the range is the set \{1,3,5,9\}.]

Can you find the domain and range for this relation?

\[ \{(-3,5),(-3,1),(4,6),(10,0)\} \]

Answer: [Domain is the set \{-3,4,10\} and the range is the set \{0,1,5,6\}.]

Some special relations are, also, called functions. A **function** is a set of ordered pairs in which each x-coordinate has exactly one y-coordinate. In other words, each and every x-coordinate of a relation can only have one y-coordinate paired with it. Observing the relations above, are these relations functions? [Student’s Response: The first relation is a function and the second relation is not a function.]

For the following relations, which ones are functions?

- \( \{(0,1),(2,3),(4,5),(8,9)\} \) [A function, why?]
- \( \{(-3,5),(-3,1),(4,6),(10,0)\} \) [Not a function, only a relation, why?]
- \( \{(-2,1),(2,0),(3,0),(4,1)\} \) [A function, why?]
- \( \{(0,5),(2,1),(3,4),(5,2)\} \) [A function, why?]

Note: that the domain has other names, such as input to the function or the independent variables; the range has other names, such as the output of the function or the dependent variables. These are function terms that will be expanded on in later mathematics courses.

Besides set notation, a function can be represented numerically, graphically,
II. Linear Functions
Have students work in small groups on the HANDOUT: More on Functions. Let them discuss and find that there different representations of relations, such as tables and graphs, that students can use to determine if a relation is a function.

When students have completed the handout, note the following.

- The function notation is $f(x)$.
- It is important to note that all the lines we have talked about are functions with the exception of the vertical line. Based on your previous knowledge, can anyone make a conjecture as to why vertical lines are not linear functions? [Student’s Response: wait for responses – guide discussion towards “each x is mapped to an infinite number of y’s.”]
- Discuss the use of the vertical line test on graphs to determine if a graph is a function or not.

III. Common Parent Functions

Have students refer to HANDOUT: Common Functions.

A. Identity Function
Complete the box related to $f(x) = x$ also known as the identity function. Any idea why this function is, also, known as the identity function? [Student’s Response: y has the same value as x.] It turns out that functions have identities just like 0 is the additive identity and 1 is the multiplicative identity. Another reason is that the output is the same as the input – again, function terms that we will expand on later. For now, use your calculator and what you know about creating tables and graphs to complete this part of the handout regarding what is the domain and range of this function?

B. Constant Function
Also complete the top right box, $f(x) = k$, to show the horizontal line is a linear function with slope equal to zero. Let’s use the example of $f(x) = 2$. What is the domain? [{x: x ∈ R } ] The range? [ y=2 , so range is {2}] Ask other questions such as what is the domain and range for $f(x) = -1$? What can we say about the domain and range of constant functions? [ Student’s Response: Domain is always all real numbers and range is equal to the constant.]
Inform the students to keep this page handy because we will be discussing some of these other functions later.
Summary/Extension
Can you think of an example of another type of function? [Student’s Response: quadratic, power functions (from examples of polynomials), answers will vary.] What are other characteristics of functions? Hint: increasing, decreasing, and constant.

Possible Test Questions from Today’s Lesson

Teacher Reflection
1. Did you use the class time wisely? Yes.

2. What worked well in the lesson? Yes.

3. What recommendations would you make to improve this lesson? None.

Other Comments About the Lesson

References
FUNCTIONS

Relation - A set of ordered pairs

Examples:

Domain - The set of all x-coordinates

Range - The set of all coordinates

Find the domain and range of the relation \{(0,1),(2,3),(4,5),(8,9)\}.

Domain: \{0,2,4,8\}  
Range: \{1,3,5,9\}

Can you find the domain and range for this relation? \{(-3,5),(-3,1),(4,6),(10,0)\}

Domain: \{-3,4,10\}  
Range: \{0,1,5,6\}

Function - a set of ordered pairs in which each x-coordinate has exactly one y-coordinate

Examples

For the following relations, which ones are functions? Explain your answers.

a) \{(0,1),(2,3),(4,5),(8,9)\} answer: "a function"

b) \{(-3,5),(-3,1),(4,6),(10,0)\} answer: not a function

c) \{(-2,1),(2,0),(3,0),(4,1)\} answer: a function

d) \{(0.5,2),(1.3),(1.5,4),(5.2)\} answer: a function, (the same ordered pair was listed twice)

Can you think of an example of a relation that is a function?
Relations and functions can be represented in many ways. Examine the following relation.
Numerical/Ordered pairs: \{(2,5), (1,4),(4,2)\}  Is this a function? \textit{Answer: yes}

\textbf{Tabular:} Fill in the table accordingly for the ordered pairs. Does this relation meet the definition of a function? \textit{Answer: yes}

\begin{tabular}{|c|c|}
\hline
x & y \\
\hline
2 & 5 \\
1 & 4 \\
4 & 2 \\
\hline
\end{tabular}

\textbf{Graphical:} graph the above ordered pairs on the rectangular coordinate plane

\textit{Graphically, does the relation still meet the definition of a function?} \textit{Answer: yes}

How about the following relation?
Numerical/Ordered pairs: \{(2,5), (-2,1), (2,0),(6,-3),(2,-5)\}  Is this a function? \textit{Answer: no}

\begin{tabular}{|c|c|}
\hline
x & y \\
\hline
2 & 5 \\
-2 & 1 \\
2 & 0 \\
6 & -3 \\
\hline
\end{tabular}
**Tabular:** Fill in the table accordingly for the ordered pairs. Does this relation meet the definition of a function?

*Answer: no*

**Graphical:** Graph the above ordered pairs on the rectangular coordinate plane.

Graphically, does the relation still meet the definition of a function?

*Answer: no*
Linear equations and Linear Functions

Examine the linear equation in two variables, \( y = 2x + 5 \)
List some ordered pairs for \( y = 2x + 5 \). _Answers will vary_

Create a table of these ordered pairs. Graph these ordered pairs or the line on the coordinate plane.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Is \( y = 2x + 5 \) a function? In particular, is it a linear function? _yes_

Can all linear equations be linear functions? _Let the students think about this and look at next exercise._

How about \( x = 5 \)?
List some ordered pairs for \( x = 5 \). _Answers will vary_

Create a table of these ordered pairs. Graph these ordered pairs or the line on the coordinate plane.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Is $x = 5$ a function? In particular, is it a linear function?

___no________

Is there anything graphically that we could do to determine if a graph represents a function or not? *Vertical line test*
FUNCTIONS

Relation - ____________________________________________________________

Examples - __________________________________________________________

Domain - ____________________________________________________________

Range - ______________________________________________________________

Find the domain and range of the relation \{(0,1),(2,3),(4,5),(8,9)\}.

Domain: _______________ Range: _______________

Can you find the domain and range for this relation? \{(-3,5),(-3,1),(4,6),(10,0)\}

Domain: _______________ Range: _______________

Function - ____________________________________________________________

_____________________________________________________________

Examples - __________________________________________________________

For the following relations, which ones are functions? Explain your answers.

e) \{(0,1),(2,3),(4,5),(8,9)\}

f) \{(-3,5),(-3,1),(4,6),(10,0)\}

g) \{(-2,1),(2,0),(3,0),(4,1)\}

h) \{(0.5,2),(1,3),(1.5,4),(5,2)\}

Can you think of an example of a relation that is a function?
More on Functions

Relations and functions can be represented in many ways. Examine the following relation.
Numerical/Ordered pairs: \{ (2,5), (1,4),(4,2) \}  Is this a function?

**Tabular:** Fill in the table accordingly for the ordered pairs. Does this relation meet the definition of a function?

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Graphical:** graph the above ordered pairs on the rectangular coordinate plane

Graphically, does the relation still meet the definition of a function?

How about the following relation?
Numerical/Ordered pairs: \{ (2,5), (-2,5), (2,0),(6,-3),(2,-5) \}  Is this a function?

**Tabular:** Fill in the table accordingly for the ordered pairs. Does this relation meet the definition of a function?

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Graphical:** graph the above ordered pairs on the rectangular coordinate plane

Graphically, does the relation still meet the definition of a function?
Linear equations and Linear Functions

Examine the linear equation in two variables, \( y = 2x + 5 \)
List some ordered pairs for \( y = 2x + 5 \). ________________________________

Create a table of these ordered pairs. 
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
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</tbody>
</table>

Graph these ordered pairs or the line on the coordinate plane.

Is \( y = 2x + 5 \) a function? In particular, is it a linear function? ________________

Note: Functions use the notation, \( f(x) \). So the above linear equation is the linear function, \( f(x) = 2x + 5 \).

Can all linear equations be linear functions?

How about \( x = 5 \)?
List some ordered pairs for \( x = 5 \). ________________________________

Create a table of these ordered pairs. 
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

Graph these ordered pairs or the line on the coordinate plane.

Is \( x = 5 \) a function? In particular, is it a linear function? ________________

Is there anything graphically that we could do to determine if a graph represents a function or not?
## Common Functions

<table>
<thead>
<tr>
<th>Function</th>
<th>Domain (D)</th>
<th>Range (R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = x$</td>
<td>$x \in \mathbb{R}$</td>
<td>$y \in \mathbb{R}$</td>
</tr>
<tr>
<td>$f(x) = x^2$</td>
<td>$x \in \mathbb{R}$</td>
<td>$y \geq 0$</td>
</tr>
<tr>
<td>$f(x) = x^3$</td>
<td>$x \in \mathbb{R}$</td>
<td>$y \in \mathbb{R}$</td>
</tr>
<tr>
<td>$f(x) = \sqrt{x}$</td>
<td>$x \geq 0$</td>
<td>$y \geq 0$</td>
</tr>
<tr>
<td>$f(x) = k$</td>
<td>$x \in \mathbb{R}$</td>
<td>$y = k$</td>
</tr>
<tr>
<td>$f(x) =</td>
<td>x</td>
<td>$</td>
</tr>
<tr>
<td>$f(x) = \frac{1}{x}$</td>
<td>$x \neq 0$</td>
<td>$y \neq 0$</td>
</tr>
<tr>
<td>$f(x) = \frac{1}{x^2}$</td>
<td>$x \neq 0$</td>
<td>$y \geq 0$</td>
</tr>
</tbody>
</table>
\[ f(x) = \sqrt[3]{x} \]

D: \[ \mathbb{R} \]

R: \[ \mathbb{R} \]

\begin{tabular}{|c|c|}
\hline
x & y \\
\hline
\end{tabular}

\[ f(x) = [x] \]

D: \[ \mathbb{R} \]

R: \[ \mathbb{Z} \]

\begin{tabular}{|c|c|}
\hline
x & y \\
\hline
\end{tabular}
# C-12 More on Functions Lesson Plan

<table>
<thead>
<tr>
<th>File Name</th>
<th>Math_1315_More on Functions</th>
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<tbody>
<tr>
<td>Lesson Title</td>
<td>More on Functions</td>
</tr>
<tr>
<td>Prerequisites</td>
<td>Introduction on Functions</td>
</tr>
<tr>
<td>Content Objective(s)</td>
<td>E-CRA - Experiential, concrete, representational, abstract</td>
</tr>
<tr>
<td>Students will be able to describe when a function is increasing and decreasing</td>
<td>concrete, representational</td>
</tr>
<tr>
<td>Construct Functions from Formulas</td>
<td>abstract</td>
</tr>
<tr>
<td>Calculate rate of change</td>
<td>concrete, representational, abstract</td>
</tr>
</tbody>
</table>

## Technology Objective(s)

## Culturally Relevant Objective(s)
None

## Real-World Applications

- Change in population in Texas

## Connections
- Algebra
- Geometry
- Measurement
- Statistics
- Probability
- Functions

## Process Standards
- Communication
- Connections
- Problem-Solving
- Representations
- Reasoning
- Cross-Disciplinary

## Lesson Agenda

I. When is a function Increasing/decreasing
II. Constructing functions from formulas

## Instructor Materials

## Student Materials

## Pre-Homework
Now that we have functions from an introductory perspective, let’s discuss why and how functions are important to us.

Main Lesson Script

I. When a function is increasing or decreasing
Now let’s go back look at the linear model we built for the sales at a record store. I am referring to the first problem in the homework set assigned on February 9. We ran a linear regression and found that the line of best fit or the linear model was \( y = 4823.05 - 2.39x \), where \( y \) is percent of music sales made in record stores in the United States.

From reviewing our notes on this problem, was the correlation positive or negative? [negative] Was the slope positive or negative? [negative from \(-2.39x\)]

Using the model from the year 2000 to 2005 to 2010 to 2015, are the % of music sales at the record increasing or decreasing? [If we substitute 2000, 2005, and 2010, and 2015 into the model, we will progressively get smaller amounts for \( y \).]

From the years 2000 to 2005, we will find that slope is \(-2.39\). The records sales for the year 2000, sales \( = y = 4823.05 - 2.39(2000) = 43.05 \) or 43.05%
for the year 2005, sales \( = y = 4823.05 - 2.39(2005) = 31.10 \) or 31.10%
for the year 2010, sales \( = y = 4823.05 - 2.39(2010) = 19.15 \) or 19.15%
for the year 2015, sales \( = y = 4823.05 - 2.39(2015) = 7.20 \) or 7.20%
Answer: the % sales is decreasing as time goes by. In fact, what two years we look at, the model will show that the % sales is decreasing as you move into the future.

This linear function \( P(x) = 4823.05 - 2.39x \) is decreasing for any value of \( x \). Thus, the function is decreasing from \((-\infty, \infty)\). In fact all linear functions will be either increasing or decreasing over the entire domain of the functions.

What is an example of an linear function that is increasing? [answers will vary]
Example: \( c(m) = 0.10m + 25 \)

What about a quadratic equation that has the following graph? Is this function
increasing? decreasing?

This quadratic function is doing both in two different intervals.
This function is increasing from \((2, \infty)\) and it is decreasing from \((-\infty, 2)\).

How about the following function? When is it increasing and decreasing?

This function is doing both in two different intervals.
This function is increasing from \((-\infty, -5) \cup (0,4)\); and it is decreasing from (-
II. Constructing Functions from Formulas

Remember that functions have a domain and a range. Or an input and output. When thinking about input and output, we can construct and reconstruct formulas to solve problems and described everyday processes. Let’s take the following example:

The area of a square has the formula \( A = s^2 \). The area is calculated by squaring the lengths of the sides of the square. In this formula, \( A \) is the output and \( s \) is the input. So, we can define the process of calculating the area as \( A(s) = s^2 \). \( A(s) \) represent the output and \( s \) is the input; \( A(s) \) is dependent on the independent variable \( s \); \( A(s) \) is the range and \( s \) is the domain of the function \( A \).

Now let’s construct some other functions from formulas.

Example: Find the area of a square if the length of the diagonal, \( d \), is known. Construct an area function where area is a function of \( d \).

\[
A(d) = \text{We know from PythagoreanTheorem that } d^2 = s^2 + s^2 = 2s^2.
\]
\[
d = \sqrt{2}s
\]
\[
\text{then } \frac{d\sqrt{2}}{2} = s
\]

If \( A(s) = s^2 \), then \( A(d) = \left(\frac{d\sqrt{2}}{2}\right)^2 \) by substitution
\[
A(d) = \frac{d^2}{2} \text{ by simplifying}
\]

Example: Find the \( C = .1m + 25 \) is the cost of your cell phone bill is calculated, where \( m \) is the number of minutes you use on your cell phone. Create the cost function where cost is a function of minutes.

Answer: \( C(m) = .1m + 25 \)

Create a function for calculating the number of cell phone minutes from a cell phone bill.

\[
C = .1m + 25
\]
\[
C-25 = .1m
\]
\[
\frac{c-25}{10} = m
\]

Answer: \( M(c) = \frac{c-25}{10} \)

II. Average Rate of Change

Slope does not necessarily always apply to a line. You can also calculate the slope
between points in a function.
For example, if you purchased a Honda Element in 2005 for $12,800 and replaced
with a new Honda Element in 2010 for $15,750. How much did the price of the
Honda Element increase during this time frame?
    The change in the price over these five years is simply the slope between
these two points.
\[
\frac{15750-12800}{2005-2000} = 590 \quad \text{or} \quad $590/\text{year}
\]

Example: For the quadratic function, \( f(x) = x^2 \), what is the average rate of change
of the function over the interval, \([1,3]\)?
\[
\frac{f(3) - f(1)}{3 - 1} = \frac{9 - 1}{2} = \frac{8}{2} = 4
\]

Example: How much did the population of Texas change from 20.9 million in year
2000 to 24.8 million in 2009? What is the average rate of change in the population
of Texas?

Average rate of change = \( \frac{\Delta p}{\Delta t} = \frac{\text{change in population}}{\text{change in time}} = \frac{24.8-20.9}{2009-2000} \)

= 0.43 million people per year


Note: All of these examples are averages!

Summary/Extension

Possible Assessments from Today’s Lesson

Teacher Reflection

1. Did you use the class time wisely? yes

2. What worked well in the lesson? Increasing and decreasing intervals of functions

3. What recommendations would you make to improve this lesson? Need to spend more
time with constructing function from formulas. Students did struggle with this.

Other Comments About the Lesson
| References |
## Transformations of Functions Lesson Plan

### File Name
Math_1315_Transformations_of_Functions

### Lesson Title
Transformations of Functions

### Prerequisites
Introduction to Functions

### Content Objective(s)

<table>
<thead>
<tr>
<th>Objective</th>
<th>E-CRA - Experiential, concrete, representational, abstract, modeling</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Understand a horizontal translation of a function</td>
<td>Experiential, concrete, representational, abstract, modeling</td>
</tr>
<tr>
<td>- Understand a vertical translation of a function</td>
<td>Experiential, concrete, representational, abstract, modeling</td>
</tr>
<tr>
<td>- Shrink/stretch of a function</td>
<td>Experiential, concrete, representational, abstract, modeling</td>
</tr>
</tbody>
</table>

### Technology Objective(s)
Use a graphing calculator to perform translations of functions

### Culturally Relevant Objective(s)
None

### Real-World Applications
Referring to our previous linear and quadratic regression, we discussed the impact of real-life circumstances to a model. These circumstances caused transformations of the model.

### Connections

- Algebra
- Geometry
- Measurement
- Statistics
- Probability
- Functions

### Process Standards

- Communication
- Connections
- Problem-Solving
- Representations
- Reasoning
- Cross-Disciplinary
Lesson Agenda

I. Transformations of Functions Using an Application Model
II. Transformations of functions from a algebraic/symbolic perspective

Instructor Materials

TI-Smartview

Student Materials

calculators

Pre-Homework

None

Introduction/Background

So, we have learned that the linear and quadratic equations and models, that we create and use, are also functions. What happens to the functions if the data changes? Or the incorrect years were used when generating model where years is the independent variable? What do you think will happen?

Main Lesson Script

I. Transformations of Functions Using an Application Model

Group Activity:
We are going to go back and look at the data for the “Is Leisure Time Shrinking?” exercise that we did earlier this semester. Pass out the handout: “Another Look at Work Hours vs Leisure Hours”. I want you to work on exercises # 1-5.

I want you to break up into groups and do this activity.

[Walk around and guide the groups into discovering how the parent function of this model transforms with the changes in the data, but is still the same function. Students should see how functions can have horizontal translation, and stretch/shrink of a function.]

Exercises 2 and 3 are examples of horizontal stretches. Exercises 4 and 5 are vertical shrink and stretches.

II. Transformations of functions from a algebraic/symbolic perspective

Now discuss with the class horizontal and vertical translations of functions. Next, discuss vertical shrink and stretch of functions. Last, discuss the reflection of functions.
Make sure to do linear and quadratic functions.
**Summary/Extension**

What do you think happens if we add the work hours and leisure hours? What would happen to these two functions? We will discuss this in the next class.

**Possible Assessments from Today’s Lesson**

**Teacher Reflection**

1. Did you use the class time wisely? Yes.

2. What worked well in the lesson? The handouts include a story where previous model the students have worked on. The story caused the data to be changed. Students discover that the change did not cause the function to go away, but transform and/or translate. The students make the connection regarding the idea of parent functions. The students realize that the characteristics of the data did not change from the transformations.

3. What recommendations would you make to improve this lesson? I had some errors on my handouts. The errors have been corrected and the corrected version is in this lesson plan.

**Other Comments About the Lesson**

**References**
Another Look at Work Hours vs Leisure Hours

1. Enter in Work time vs Leisure time data. (First spreadsheet table). Run Quadratic regression on work hours. Store quadratic regression model in Y1.

2. Consider what would happen if the years provided to you were in error and they should have been five years later? What would happen to the regression graphs for work and leisure? Will the shape of the graph change? Will the graph move?
   - Fill in Table B for this problem.
   - Run quadratic regression and store the regression model in Y2.
   - Look at the graphs. How are they different?
   - Now enter in Y3=Y1-5

3. Consider what would happen if the years provided to you were in error and they should have been ten years earlier? What would happen to the regression graphs for work and leisure? Will the shape of the graph change? Will the graph move?
   - Fill in Table B for this problem.
   - Run quadratic regression and store the regression model in Y2.
   - Look at the graphs. How are they different?
   - Now enter in Y3=Y1-5

4. Consider what would happen if you were asked to create a model where people are expected to work 10% more hours in a week. How would the model be affected? Would the shape change? Would the graph move?
   - Fill in Table C for this problem.
   - Run quadratic regression and store the regression model in Y3.
   - Look at the graphs. How are they different?
   - Now enter in Y4=Y1+10

5. Consider what would happen if you were asked to create a model where people are expected to work 50% fewer hours in a week. How would the model be affected? Would the shape change? Would the graph move?
   - Fill in Table D for this problem.
   - Run quadratic regression and store the regression model in Y4.
   - Look at the graphs. How are they different?
   - Now enter in Y3=.5Y1
## Work vs Leisure Time Exercise

<table>
<thead>
<tr>
<th>Table A</th>
<th>Years</th>
<th>Work Hrs</th>
<th>Leisure Hrs</th>
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<tbody>
<tr>
<td>1973</td>
<td>40.6</td>
<td>26.2</td>
<td></td>
</tr>
<tr>
<td>1980</td>
<td>46.9</td>
<td>19.2</td>
<td></td>
</tr>
<tr>
<td>1987</td>
<td>48.8</td>
<td>16.6</td>
<td></td>
</tr>
<tr>
<td>1994</td>
<td>50.7</td>
<td>19.5</td>
<td></td>
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<tr>
<td>1997</td>
<td>50.8</td>
<td>19.5</td>
<td></td>
</tr>
<tr>
<td>1998</td>
<td>49.9</td>
<td>19.4</td>
<td></td>
</tr>
<tr>
<td>1999</td>
<td>50.2</td>
<td>19.8</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table B</th>
<th>Years + 5</th>
<th>Work Hrs</th>
<th>Leisure Hrs</th>
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<tbody>
<tr>
<td>40.6</td>
<td>26.2</td>
<td></td>
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<tr>
<td>46.9</td>
<td>19.2</td>
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<td>16.6</td>
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<tr>
<td>50.7</td>
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<td></td>
</tr>
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<td>19.4</td>
<td></td>
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<td></td>
<td></td>
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</table>

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<thead>
<tr>
<th>Table C</th>
<th>Years - 10</th>
<th>Work Hrs</th>
<th>Leisure Hrs</th>
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<td>50.8</td>
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<tr>
<td>49.9</td>
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<table>
<thead>
<tr>
<th>Table D</th>
<th>Years</th>
<th>Work Hrs</th>
<th>Leisure Hrs</th>
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<td>26.2</td>
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<td>46.9</td>
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<table>
<thead>
<tr>
<th>Table E</th>
<th>Years</th>
<th>Work Hrs</th>
<th>Leisure Hrs</th>
<th>10% more Work Hrs</th>
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<td></td>
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<tr>
<td>1980</td>
<td>46.9</td>
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# C-14 Operations on Functions Lesson Plan

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<tbody>
<tr>
<td>Lesson Title</td>
<td>Operations on Functions</td>
</tr>
<tr>
<td>Prerequisites</td>
<td>Transformations of Functions</td>
</tr>
<tr>
<td>Content Objective(s)</td>
<td>Students will be able to</td>
</tr>
<tr>
<td>- perform operations (add, subtract, multiply, and division) of functions</td>
<td>E-CRA - Experiential, concrete, representational, abstract, modeling</td>
</tr>
<tr>
<td>- perform composition of functions</td>
<td>abstract</td>
</tr>
<tr>
<td>Technology Objective(s)</td>
<td>Use calculator to learn about operations on functions</td>
</tr>
<tr>
<td>Culturally Relevant Objective(s)</td>
<td>Students worked with real-world data, instead of raw numbers. Data was median leisure time.</td>
</tr>
<tr>
<td>Real-World Applications</td>
<td>Students worked on the Leisure Time data where work hours and leisure hours were added; hours could, also, be converted to minutes.</td>
</tr>
<tr>
<td>Connections</td>
<td>Process Standards</td>
</tr>
<tr>
<td>Algebra</td>
<td>Communication</td>
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<td>Geometry</td>
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<td>Reasoning</td>
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<td>Functions</td>
<td>Cross-Disciplinary</td>
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</tbody>
</table>
Lesson Agenda

I. Application that utilizes operations on functions
II. Operations on Functions
III. Composition of Functions

Instructor Materials

TI-Smartview

Student Materials

Calculator

Pre-Homework

None

Introduction/Background

Last time we took did a group activity that demonstrated how functions can be transformed. Today we will discuss another way that functions can be utilized.

Main Lesson Script

I. Application that utilizes operations on functions.

Group Activity: I want you to go back into the groups you were in last time. I want you do one more problem that is somewhat to similar to what we did last time. [Pass out the Handout: Summing up Hours]. Let students see and discuss what happens if you sum two functions.

II. Operations on Functions

With functions we can perform addition, subtraction, multiplication, and division. Let’s explore this further.

Addition Example: Let $f(x) = x^2 + 5$ and $g(x) = x + 3$.

$$(f+g)(x) = f(x) + g(x) = (x^2 + 5) + (x + 3) = x^2 + x + 8$$

What is the domain of $f(x)$? [all real numbers] What is the domain of $g(x)$? [all real numbers]

For $(f+g)(x) = f(x) + g(x)$ what do you think the domain is? [all real numbers]. In fact it is the intersection of the two domains.

What do you think $(f+g)(2)$ is? $[2^2+2 + 8 = 14]$

Subtraction example (for same $f(x)$ and $g(x)$)

$$(f-g)(x) = f(x) - g(x) = (x^2 + 5) - (x + 3) = x^2 - x + 2$$

What do you think the domain is for $(f-g)(x)$? [still all real numbers]
\[(f-g)(5) = f(5) - g(5) = 5^2 - 5 + 2 = 22\]

**Multiplication example:**
\[
(f \cdot g)(x) = f(x) \cdot g(x) = (x^2 + 5)(x + 3) = x^3 + 3x^2 + 5x + 15
\]

What do you think the domain is for \((f \cdot g)(x)\)? [still all real numbers]

\[(f \cdot g)(4) = 4^3 + 3(4)^2 + 5(4) + 15 = 147\]

**Division example:**
\[
(f/g)(x) = \frac{f(x)}{g(x)} = \frac{(x^2 + 5)}{(x + 3)}
\]

What do you think the domain is for \((f/g)(x)\)? Is it still all real numbers?

[No, we need to be concerned about the denominator being equal to zero.]

So \(g(x) \neq 0\). Thus, \(x \neq -3\). The domain for \(g(x)\) is all real numbers, except \(x \neq -3\).

The intersection of these two domains is all real numbers except \(-3\)
or

\[(-\infty, -3) \cup (-3, \infty)\]

\[
(f/g)(3) = \frac{f(3)}{g(3)} = \frac{(3^2 + 5)}{(3 + 3)} = \frac{14}{6} = \frac{7}{3}
\]

Another set of examples: Let \(f(x) = \{(1,3), (2,8), (3,6), (5,9)\}\)
\(g(x) = \{(1,6), (2,11), (3,0), (4,1)\}\)

What is the domain of \((f+g)(x)\)? [1,2,3, the intersection of f’s and g’s domain]

What is the range of \((f+g)(x)\)? [6, 9, 19]

What is \((f+g)(x)\)? [\{1,9), (2,19), (3, 6)\}]

What is \((f+g)(5)\)? [no solution, not in domain]

Now do the following problems:

Find \((g-f)(2)\).

Find \((f \cdot g)(3)\).

What is the domain of \((f/g)(x)\)? \(g(x) \neq 0\). So, domain is \{1,2\}. Can’t use \(g(3)\).

So, what is \((f/g)(1)\)? \(\frac{3}{6} = \frac{1}{2}\)

### III. Composition of Functions
Let \(f(x) = x^2 + 5\) \(g(x) = x + 3\)
(f ∘ g)(x) = f(g(x))

**Domain**

\[
x \Rightarrow \begin{align*}
g(x) & : x + 3 \\
f(x) & : x^2 + 5, \text{ so } (g(x))^2 + 5
\end{align*}
\]

**Range**

What is \(f(g(1))\)?
What is \(f(g(7))\)?

**Summary/Extension**

**Possible Assessments from Today’s Lesson**

**Teacher Reflection**

1. Did you use the class time wisely? yes

2. What worked well in the lesson? *By the previous lesson where data was being transformed, the students now performed arithmetic operations on the data (which could occur in the real world).*

3. What recommendations would you make to improve this lesson? *Ask the students to take one of their own collections of data and determine what arithmetic operations could be performed on the data collected.*

**Other Comments About the Lesson**

**References**
Summing up Hours

What if we want to a model that predicts the total hours for work and leisure? What could we do?

- Fill in Table F for this problem.

- Run quadratic regressions so that the Work hours regression model is in Y1 and Leisure hours model is in Y2.

- Add up the work hours and leisure hours and put the sum into the last column of Table F.

- Run quadratic regression for Work+Leisure hours and store regression model in Y3.

- Look at the graphs. How are they different?

- Now enter in Y4=Y1+Y2

<table>
<thead>
<tr>
<th>Years</th>
<th>Work Hrs</th>
<th>Leisure Hrs</th>
<th>Work + Leisure Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>1973</td>
<td>40.6</td>
<td>26.2</td>
<td></td>
</tr>
<tr>
<td>1980</td>
<td>46.9</td>
<td>19.2</td>
<td></td>
</tr>
<tr>
<td>1987</td>
<td>48.8</td>
<td>16.6</td>
<td></td>
</tr>
<tr>
<td>1994</td>
<td>50.7</td>
<td>19.5</td>
<td></td>
</tr>
<tr>
<td>1997</td>
<td>50.8</td>
<td>19.5</td>
<td></td>
</tr>
<tr>
<td>1998</td>
<td>49.9</td>
<td>19.4</td>
<td></td>
</tr>
<tr>
<td>1999</td>
<td>50.2</td>
<td>19.8</td>
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C-15 Inverse Functions Lesson Plan

<table>
<thead>
<tr>
<th>File Name</th>
<th>Math_1315_Inverse_Functions</th>
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<tbody>
<tr>
<td>Lesson Title</td>
<td>Inverse Functions</td>
</tr>
<tr>
<td>Prerequisites:</td>
<td>Operations on Functions</td>
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<table>
<thead>
<tr>
<th>Content Objective(s)</th>
<th>E-CRA - Experiential, concrete, representational, abstract, modeling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students will be able to</td>
<td>abstract</td>
</tr>
<tr>
<td>• Understand definition of one-to-one</td>
<td>abstract</td>
</tr>
<tr>
<td>• how to use horizontal in tests</td>
<td>abstract</td>
</tr>
<tr>
<td>• find inverse of a function</td>
<td>abstract</td>
</tr>
<tr>
<td>• know what methods to use to prove that a function is the inverse of another function</td>
<td>abstract</td>
</tr>
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<table>
<thead>
<tr>
<th>Technology Objective(s)</th>
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</thead>
<tbody>
<tr>
<td>Culturally Relevant Objective(s)</td>
<td>None</td>
</tr>
<tr>
<td>Real-World Applications</td>
<td>Charging the purchase of items onto a credit card account, then backing out the transaction.</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Connections</th>
<th>Process Standards</th>
</tr>
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<td>Algebra</td>
<td>Communication</td>
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<td>Geometry</td>
<td>Connections</td>
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<tr>
<td>Measurement</td>
<td>Problem-Solving</td>
</tr>
<tr>
<td>Statistics</td>
<td>Representations</td>
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<tr>
<td>Probability</td>
<td>Reasoning</td>
</tr>
<tr>
<td>Functions</td>
<td>Cross-Disciplinary</td>
</tr>
<tr>
<td>Lesson Agenda</td>
<td>Instructor Materials</td>
</tr>
<tr>
<td>--------------------------------------------------</td>
<td>----------------------</td>
</tr>
<tr>
<td>I. One-to-one correspondence of a function</td>
<td></td>
</tr>
<tr>
<td>II. Inverse function</td>
<td></td>
</tr>
<tr>
<td>III. Finding the inverse function</td>
<td></td>
</tr>
<tr>
<td>IV. Proving a function is an inverse of another function.</td>
<td></td>
</tr>
<tr>
<td>Pre-Homework</td>
<td>Student Materials</td>
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<tr>
<td>None</td>
<td></td>
</tr>
</tbody>
</table>

**Introduction/Background**

Sometimes a function can undo what another function performs. We have functions like this in everyday life. You can drive your car from your home to the post office. Then, drive your car back home. You bought a shirt at a department store with your credit card. The purchase cost is placed onto your account as a credit. If you decide that you do not want the shirt and return it, the store backs out or reverses the charge placed onto your account. Mathematical functions can sometime, not always, have inverse functions or functions that undo the rule applied by the function.
Main Lesson Script

I. One-to-one correspondence of a function

Does anyone recall what the definition of a function? [A function is a rule that assigns each element in one set, the domain, to a unique element in a second set, the range. Also, a function is a set of ordered pairs in which no two ordered pairs have the same first coordinate mapped to different second coordinates.]

An one-to-one function is a function where “no two ordered pairs with different first coordinates and the same second coordinate”.

Now, let’s discuss some an example of a one-to-one functions.

Example 1: If you pick up a table, carry 5 feet to the left and 4 feet forward. Can you put the table back where it was originally? [Yes. You can walk backwards for 4 feet, and to your right 5 feet.]

Which of the following sets of ordered pairs are one-to-function?

(a) \( f(x) = \{(1,3), (2,4), (7,10), (11,13)\} \)
(b) \( g(x) = \{(1,3), (3,1), (4,2), (5,3)\} \)
(c) \( h(x) = \{(2,5), (5,2), (4,1), (6,3), (8,9)\} \)

What about graphs of functions? Can we tell when a function is a one-to-one function by looking at its graphs? Let’s look at some examples.

\( f(x) = \{(1,-4), (3,-2), (-5,0), (6,4), (-3,6), (-9,8), (4,2)\} \) Each point passes the vertical and horizontal line test.

\( g(x) = 2x + 3. \) Each point passes the vertical and horizontal line test.
\[ h(x) = x^2 + 6x + 8. \] Each point passes the vertical and horizontal line test.

\[ m(x) = x^3 + 4. \] Each point passes the vertical and horizontal line test.

\[ c(x) = \pm \sqrt{4 - x^2} \] which is the equivalent to \( x^2 + y^2 = 4 \) (a circle of radius 2
fails the horizontal line test. So does not have an inverse.
So, we can use the horizontal line test to determine if a function is, also, an one-to-one function.

II. Inverse function
If a function is one-to-one, then it is invertible. This means that the function has an inverse function.
The notation for the inverse function of the function, $f(x)$, is $f^{-1}(x)$.

The definition of an inverse function: The inverse of an one-to-one function, $f(x)$, if $f^{-1}(x)$, where the domain of $f$ is the range of $f^{-1}(x)$; the range of $f$ is the domain of $f^{-1}$.

So, for the example (a) that we used previously, what is the $f^{-1}(x)$? $\{ (3,1), (4,2), (10,7), (13,11) \}$

What about for example (c) that we examined earlier, what is $h^{-1}(x)$? $\{ (5,2), (2,5), (1,4), (3,6), (9,8) \}$

III. Finding the inverse function
For a function that is invertible, we can find its inverse function.

Let $f(x) = 2x + 3$. $f(x)$ is invertible. We know it is a 1-1 function and its graph passes the horizontal line test. Its rule is simply: take a variable, multiply it by two, then add 3.
How can we back out of this rule? [We need to subtract 3 and then divide by 2.]

What about the function $g(x) = 7x - 4$. What is the rule? [multiply variable by 7, then subtract 4]. How can we back out this rule? [add 4, then divide by 7]

Symbolically or algebraically this is called the Switch and Solve Method. Let’s
find \( f^{-1}(x) \) algebraically, for \( f(x) = 2x + 3 \)

Step 1. \( y = 2x + 3 \) Replace \( f(x) \) with \( y \)

Step 2. \( x = 2y + 3 \) Interchange \( x \) and \( y \)

Step 3. \( \frac{x-3}{2} = y \) Solve for \( y \)

Step 4. \( \frac{x-3}{2} = f^{-1}(x) \) Replace \( y \) with \( f^{-1}(x) \)

Note: check if domain of \( f \) is range of \( f^{-1}(x) \); and range of \( f(x) \) is domain of \( f^{-1}(x) \).

Now try an example on your own. \( f(x) = \frac{2x+1}{x-3} \). Check your answer with one of your neighbors.

\[ \text{Answer: } f^{-1}(x) = \frac{3x+1}{x-2} \]

A visual view of a function and its inverse: discuss with the class the visual picture below of the mapping of a function, \( f \), and its inverse, \( f^{-1}(x) \).

IV. Proving a function is an inverse of another function.

To show or prove that a function is the inverse of another function the following compositions must be true:

\[ (f \circ g)(x) = x \] and \( (g \circ f)(x) = x \)

Walk through the example of \( f(x) = 2x + 3 \) and \( f^{-1}(x) = \)

Summary/Extension

Possible Assessments from Today’s Lesson

Teacher Reflection

1. Did you use the class time wisely? No time management problems.

2. What worked well in the lesson? Lesson was taught in the more typical ways of college algebra.

3. What recommendations would you make to improve this lesson? I would like to add an application problem to this lesson. Additionally, it would be nice to make a homework assignment where the students write up a real-world problem which
can be represented by a function and its inverse function.

<table>
<thead>
<tr>
<th>Other Comments About the Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students were struggling with using composition functions and the concept of composition functions.</td>
</tr>
</tbody>
</table>

| References |
Visual Picture of an Invertible Function and its Inverse Function

\[ f(x) = 2x + 3 \quad f^{-1}(x) = \frac{x - 3}{2} \]
## C-16 Exponential Functions Lesson Plan

<table>
<thead>
<tr>
<th>File Name</th>
<th>Math_1315_Exponential_Functions</th>
</tr>
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<tbody>
<tr>
<td>Lesson Title</td>
<td>Exponential Functions</td>
</tr>
<tr>
<td>Prerequisites</td>
<td>Inverse Functions</td>
</tr>
<tr>
<td>Content Objective(s)</td>
<td>Student will be able to</td>
</tr>
<tr>
<td></td>
<td>- visualize differences between exponential function and linear and quadratic functions.</td>
</tr>
<tr>
<td></td>
<td>- know domain and range of exponential function</td>
</tr>
<tr>
<td></td>
<td>- understand difference between growth and decay exponential functions</td>
</tr>
<tr>
<td></td>
<td>E-CRA - Experiential, concrete, representational, abstract, modeling</td>
</tr>
<tr>
<td>Technology Objective(s)</td>
<td>Students will be able to use graphing calculator to produce graphs of exponential functions.</td>
</tr>
<tr>
<td>Culturally Relevant Objective(s)</td>
<td>Students will study cell phone growth. Plus, students will discuss the history of cell phones. This example was provided to the class by one of the students.</td>
</tr>
<tr>
<td>Real-World Applications</td>
<td>Students will study cell phone growth.</td>
</tr>
<tr>
<td>Connections</td>
<td></td>
</tr>
<tr>
<td>Algebra</td>
<td>Communication</td>
</tr>
<tr>
<td>Geometry</td>
<td>Connections</td>
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<td>Measurement</td>
<td>Problem-Solving</td>
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<td>Representations</td>
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<td>Probability</td>
<td>Reasoning</td>
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<td>Functions</td>
<td>Cross-Disciplinary</td>
</tr>
<tr>
<td>Lesson Agenda</td>
<td>Instructor Materials</td>
</tr>
<tr>
<td>------------------------------------------------------------------------------</td>
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</tr>
<tr>
<td>I. Analysis of data pertaining to the number of cell phone subscribers in US</td>
<td>TI-Smartview</td>
</tr>
<tr>
<td>II. Exponential Functions characteristics</td>
<td>Handouts</td>
</tr>
<tr>
<td>III. Exponential Regression</td>
<td>Student Materials</td>
</tr>
<tr>
<td></td>
<td>calculator</td>
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</table>

<table>
<thead>
<tr>
<th>Pre-Homework</th>
</tr>
</thead>
<tbody>
<tr>
<td>Previously students were assigned to bring data and graphs of something that</td>
</tr>
<tr>
<td>they are interested in, where the data is growing. Instructor then needs to</td>
</tr>
<tr>
<td>select student data that can be used for demonstrating exponential data.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Introduction/Background</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Instructor selected the following data and had the students analyze it.]</td>
</tr>
</tbody>
</table>

Handheld cell phone

On April 3, 1973, Martin Cooper, a Motorola researcher and executive, made the first analogue mobile phone call using a heavy prototype model. He called Dr. Joel S. Engel of Bell Labs. There was a long race between Motorola and Bell Labs to produce the first portable mobile phone. Cooper is the first inventor named on "Radio telephone system" filed on October 17, 1973 with the US Patent Office. John F. Mitchell, (Cooper's boss) was also named on the patent. He successfully pushed Motorola to develop wireless communication products that would be small enough to use anywhere and participated in the design of the cellular phone. Source: http://en.wikipedia.org/wiki/History_of_mobile_phones

<table>
<thead>
<tr>
<th>Main Lesson Script</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Analysis of data pertaining to the number of cell phone subscribers in US.</td>
</tr>
</tbody>
</table>

    Pass out the Handout: United States Cell Phone Subscribers. I want you to break up into groups and work on this handout. [After students complete this handout, discuss why the data will not be linear (data points will never go into Quadrant 3 or go with negative y values) or quadratic (data points will be parabolic to the left of x=1973).]

II. Exponential Functions characteristics.

    Pass out the Handout: Exponential Functions. Discuss the characteristics of an exponential function. Definition, domain, range, growth vs decay.
### III. Exponential Regression

Ask the students to go back into their groups and try an exponential regression on the cell phone data or their own data (that is exponential).

### Summary/Extension

### Possible Assessments from Today’s Lesson

### Teacher Reflection
1. Did you use the class time wisely? *No time management problems.*

2. What worked well in the lesson? *The activity where the students were allowed to further explore the cell phone data or their own exponential data.*

3. What recommendations would you make to improve this lesson? *I feel that if I have given more guidance to the students during the previous lesson, then more students would have brought exponential data. Many of the data sample that were brought to class appeared to be linear.*

### Other Comments About the Lesson

For students who brought their own data, student were asked to
- Describe their data
- Create tables of their data.
- Graph their data. Does your data look linear or quadratic? Why or why not?
- Run linear regressions. Find the line of best of fit. Examine the r and r².
- Run quadratic regressions. Find the regression model. Examine r².
- Which model is stronger? Why? How can you tell?
- What characteristics do the graphs of your regression model display for your x-values (of your data) as x approaches smaller values?
- Do these graphs make sense for your data? Why or why not?

Examples of data brought in by students were number of members of Facebook and number of itunes downloads.

### References
United States Cell Phone Subscribers (page 1)

Data:

<table>
<thead>
<tr>
<th>Year</th>
<th>Cell phone Subscribers (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>5.3</td>
</tr>
<tr>
<td>1995</td>
<td>33.8</td>
</tr>
<tr>
<td>2000</td>
<td>109.4</td>
</tr>
<tr>
<td>2005</td>
<td>207.9</td>
</tr>
<tr>
<td>2006</td>
<td>233</td>
</tr>
<tr>
<td>2007</td>
<td>255.4</td>
</tr>
<tr>
<td>2008</td>
<td>271.6</td>
</tr>
</tbody>
</table>

**Graphical:** graph the above ordered pairs on the rectangular coordinate plane. Let x=0 represent 1990, x=1 represent 1991, etc.
1. Does this graph look linear? ______________ Run a linear regression. What is r? What is $r^2$?

2. Does this graph look quadratic? ______________ Run a quadratic regression. What is $r^2$?

3. Which is the stronger model?

4. What if we enter in the data for the years 1985, 1986, and 1988? In 1985, US had 340,000 (0.34 million) cell phone subscribers, in 1986, US had 680,000 (0.68 million) cell phone subscribers, and in 1988, US had 2,100,000 (2.10 million) cell phone subscribers.

Graph this data into the graph. Be careful with the units of measure.

5. Does this graph look linear now? _____________ Run a linear regression. What is r? What is $r^2$?

6. Does this graph look quadratic now? ______________ Run a quadratic regression. What is $r^2$?

7. Which of the two new models is the stronger model? __________

Is the new or the previous linear model stronger? __________

Is the new or the previous quadratic model stronger? __________

8. Where do you think the point for 1973 would be on the graph? For 1975? For 1980?

9. When graphing data after 1973, will the graph have points in the third quadrant? Which model shows the data going into the third quadrant? Does this make sense for your data?
United States Cell Phone Subscribers (page 3)

10. Will the data after 1973 have $y$-values that will increase as $x$-value decrease? Which model shows the data after 1973 having $y$-values that will increase as $x$-value decrease? Does this make sense for your data?

Let’s see if there is another mathematical explanation for this data.
Examining Student Data (page 1)

1. What kind of data did you bring to class?

2. Describe the characteristics of the raw data.

3. Create a table of your data.

4. Graph your data. Does your data look linear or quadratic? Why or why not?
Examining Student Data (page 2)

5. Run a linear regressions. Write the line of best of fit. Interpret the slope of the model. Examine the r and r^2.

6. Run a quadratic regressions. Write the regression model. Examine r^2.

7. Which model is stronger? Why? How can you tell?

8. What characteristics do the graphs of your regression model display for your x-values (of your data) as x approaches smaller values?

9. Do these graphs make sense for your data? Why or why not?
Exponential Functions

Exponential Function with base $a$: _________________________________

_______________________________________________________________

Domain of exponential function: _________________________________

Range of exponential function: _________________________________

When $a>1$, _________________________________

Example: $2^x$, find some points, and graph them

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>
Exponential Functions (page 2)

When $0 < a < 1$, _____________________________________________________________

Example: $2^x$, find some points, and graph them

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
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# C-17 Exponential Application Functions Lesson Plan

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<th>Math_1315_Exponential_Applications_and_e</th>
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<tbody>
<tr>
<td>Lesson Title</td>
<td>Exponential Applications and e</td>
</tr>
<tr>
<td>Prerequisites</td>
<td>Exponential Functions</td>
</tr>
<tr>
<td>Content Objective(s)</td>
<td>Students will be able to</td>
</tr>
<tr>
<td>Use exponential functions to solve application problems</td>
<td>concrete, representational, abstract, modeling</td>
</tr>
<tr>
<td>Use exponential functions for financial applications</td>
<td>abstract, modeling</td>
</tr>
<tr>
<td>Use e in exponential functions</td>
<td>abstract, modeling</td>
</tr>
<tr>
<td>Technology Objective(s)</td>
<td>Students will be able to use e on their calculator.</td>
</tr>
<tr>
<td>Culturally Relevant Objective(s)</td>
<td>None.</td>
</tr>
<tr>
<td>Real-World Applications</td>
<td>Use exponential functions to determine balance of savings account.</td>
</tr>
<tr>
<td>Connections</td>
<td>Algebra, Geometry, Measurement, Statistics, Probability, Functions</td>
</tr>
<tr>
<td>Process Standards</td>
<td>Communication, Connections, Problem-Solving, Representations, Reasoning, Cross-Disciplinary</td>
</tr>
<tr>
<td>Lesson Agenda</td>
<td>I. Group activity: Run exponential regression on the data used in</td>
</tr>
<tr>
<td>Instructor Materials</td>
<td>TI-Smartview</td>
</tr>
</tbody>
</table>
the previous class.

| II. Review characteristics of exponential functions | Student Materials |
| III. Applications of the exponential function | Calculator |
| IV. The number, $e$ |

Pre-Homework
None

Introduction/Background
We are going to go back to the data we were analyzing in our previous class. What function do you now believe best describes the characteristics of the data? [exponential]

Main Lesson Script

I. Group Activity.
- Have the students to back into the same groups that they were in the previous.
- Ask them to run an exponential regression.
- Discuss the results of the linear, quadratic, and exponential regressions.
- Determine which function best describes the characteristics of the cell phone subscribers data.

II. Review characteristics of exponential functions.

In the previous class, we discussed exponential functions and their characteristics. Do you recall any of these characteristics? [Answer: the variable is in the exponent. If base, $b$, is greater than 1, then we have a growth function. If $0<b<1$, then we have a decay function. Domain is all real numbers. Range is $f(x) > 0$.] So, what kind of data would be best modeled by exponential functions? [Answer: non-linear growth where the function is always increasing or decreasing.] What does the graph of an exponential function look like?

![Graph of an exponential function]

$y = 0$
$y = 0$
III. Growth applications of the exponential function.

Today we are going to discuss another use of exponential functions. Exponential functions are used heavily in financial applications.

If we want to invest money into a saving account, certificates of deposits (CDs) or Individual retirement arrangement (IRA) account, can we determine the new balance of this account after a few years?

Let’s say you inherited $12,000 and you want to put this money away till you buy a home in three years. Your local credit union has 3-year CDs available with 2.08% annual interest rate compounded monthly. How much money will you have at the end of the 3-year period?

To find the new balance of a savings account, the following formula is used by all financial institutions

\[ A = P \left( 1 + \frac{r}{n} \right)^{nt} \],

where

- \( A \) is the new balance of your saving account
- \( P \) is the principal or original amount invested
- \( r \) is the percentage rate of the account (in decimal form, for example 4.15% use 0.0415)
- \( n \) is the number of times the account is compounded annually
- \( t \) is the number of years the principal will be invested

Note: To determine how many times the account is compounded annually, do the following:

- If compounded daily, \( n = 365 \).
- If compounded monthly, \( n = 12 \).
- If compounded quarterly, \( n = 4 \).
- If compounded semi-annually, \( n=6 \).
- If compounded annually, \( n=1 \).

For our savings account problem let’s see what our new balance will be. Set \( r = 0.0208 \). What will \( n \) be? \([Answer: n=12]\)
\[ A = 12000 \left(1 + \frac{.0208}{12} \right)^{12 \times 3} = $12,771.97. \]

Be very careful entering this formula into your calculator. You should enter the following into your calculator.

\[ 12000 \left(1 + \frac{.0208}{12} \right)^{(12 \times 3)} \]

Notice that the exponent \(12 \times 3\) is in parenthesis, this because if you do not use parenthesis then the exponent will be 3 and then the total amount would subsequently by multiplied by 3 (per the order of operations rules).

What if our account is compounding daily or quarterly? Would either option generate more savings? Let’s see.

For daily compounding,

\[ A = 12000 \left(1 + \frac{.0208}{365} \right)^{365 \times 3} = $12,772.63. \]

For quarterly compounding,

\[ A = 12000 \left(1 + \frac{.0208}{4} \right)^{4 \times 3} = $12,770.59. \]

So, the daily compounding gave us more money than quarterly. Can we conclude anything about this result? [Answer: The greater the number of compounding in a year, the more money is generated by the savings account.]

IV. The number, \(e\)

The number, \(e\), is an irrational number similar to \(\pi\). It is naturally occurs in many areas of science and nature. \(e \approx 2.71828\)

You can find \(e\) on your calculator. Enter in \(e\) (on calculator hit 2\(^{nd}\) key, \(\text{then} \div \text{key}\)) and you will obtain this value.

Mathematicians get the value of \(e\) by using calculus. But, we are not going to discuss. We are going to trust that the mathematicians know this \(e\) quite well, which they do.

\(e\) is used when there is continuous compounding occurring on an amount.

So, for our savings account example we get what?

\[ A = 12000 \times e^{(.0208 \times 3)} = $12,772.66. \text{ About 65 cents more than compounding daily!} \]

V. Continuous growth and decay problems

If we go back to our savings account example, what if our savings account
was compounding and infinite number of times per year? What would the new balance be? We can’t divide and multiply by $\infty$. Why? [Answer: We don’t know what $\infty$ is. It is not a definitive number. So, how can we perform arithmetic operations using it.]

We use the number, $e$, instead.

The formula for compounding continuously is

$$A = Pe^{rt} \quad \text{where}$$

$A$ is the new balance of your saving account
$P$ is the principal or original amount invested
$r$ is the percentage rate of the account (in decimal form, for example 4.15% use 0.0415)
$t$ is the number of years the principal will be invested

Summary/Extension

Possible Assessments from Today’s Lesson

Teacher Reflection

1. Did you use the class time wisely? *I felt that I needed a little more time to go over more application problems.*

2. What worked well in the lesson? *The financial problems worked well.*

3. What recommendations would you make to improve this lesson? *Practice or guided worksheets would have been a positive impact to the lesson. Their use provides more instructional time because I feel that I can go faster.*

Other Comments About the Lesson

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C-18 Logarithmic Functions Lesson Plan

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<td>• Understand that logarithmic functions are inverse functions of exponential functions</td>
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<tr>
<td>• Understand characteristics of logarithmic functions</td>
<td>abstract</td>
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<tr>
<td>• Draw graph of logarithmic function</td>
<td>representational</td>
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<tr>
<td>• Use properties of logarithms</td>
<td>abstract</td>
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<tr>
<td>• Use change of base formula</td>
<td>Concrete, abstract</td>
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| Technology Objective(s) | Students will be able to use the calculator when working with logarithmic problems. |

| Culturally Relevant Objective(s) | None |

| Real-World Applications | None |

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II. Logarithmic Functions
III. Characteristics of logarithmic functions
IV. Graphs of logarithmic functions
V. Properties of Logarithms
VI. Change of base formula

Student Materials

Pre-Homework
None.

Introduction/Background

We have been solving application problems using exponential functions. Also, we discussed the fact that sometimes functions have an inverse. This is true for exponential functions. Their inverse function is the logarithmic function.

Main Lesson Script

I. Exponential functions are one-to-one.

Does anyone remember what a one-to-one function is? [Answer: An one-to-one function is a function where “no two ordered pairs with different first coordinates and the same second coordinate.”]

If a function is one-to-one this means that what also exist? [Answer: The inverse of the function.] For the function, \( f(x) \) that is an one-to-one function, has an inverse function, denoted \( f^{-1}(x) \).

Do exponential functions have the one-to-one characteristic? How can we find out? [Answer: Yes, it is one-to-one. Its graphs pass the horizontal line test. No two x-values have the same y-value.]

So, does the exponential function have an inverse function? [Answer: yes]
II. Logarithmic Functions

The inverse of the exponential function is called the logarithmic function. The definition of a logarithmic function is “For \( a > 0 \) and \( a \neq 1 \), the logarithmic function with base \( a \) is denoted \( f(x) = \log_a(x) \), where 
\[
y = \log_a(x) \iff a^y = x.
\]

Let’s look at some examples.

Example 1. \( 3^2 = 9 \), so \( \log_3(9) = y \). What is \( y \) equal to? \([Answer \ y=2.]\) Why? \([Answer: 3^y = 9, \ or \ 3^2 = 9.]\)

Example 2. \( \log_2(1/4) = y \).
\[
y = -2 \quad \text{What is } y \text{ equal to? [Answer: We know } 2^y = 1/4, \text{ so } y = -2.\]

Example 3. \( \log_{\frac{1}{2}}(8) = y \).
\[
y = -3 \quad \text{What is } y \text{ equal to? [Answer: We know } \left(\frac{1}{2}\right)^y = 8, \text{ so } y = -3.\]

Example 4. \( e^2 \approx 7.389 \) so \( \log_e(7.389) \approx y \), \( y \approx 2 \)

Note: For the number, \( e \), we use the natural log as its inverse function.

For \( \log_e \), the notation \( \ln \) is used. You will find \( \ln \) on your calculator on the far left, third button from the bottom.

Example 5. \( \ln(1) = y \). What is \( y \)? What power of \( e \) gives you an answer of \( 1 \)? \([Answer: Zero.]\)

Example 6: \( \ln(-6) = y \) What is \( y \)? What power of \( e \) gives you an answer of \( -6 \)?: \([Answer: Nothing.]\)

Why? \([Answer: There is no power of } e \text{ that gives you an answer of } -6. \ -6 \text{ is not in the domain of } e^x.\]

III. Characteristics of logarithmic functions.

What do you believe is the domain of a log function? Why?

*Domain: \( x > 0 \) because range of the exponential function is \( f(x) > 0 \) and exponential and log functions are inverse functions.*

What do you anticipate is the range of a log function? Why?

*Range: \( f(x) \in \mathbb{R} \) or the range is all real numbers. This is because domain of the exponential function is \( x \in \mathbb{R} \); and exponential and log functions are inverse functions.*
IV. Graphs of logarithmic functions.

Now let’s look some graphs of log functions. To graph \( y = \log_2(x) \), let’s first build a table of the inverse function for \( f(x) = 2^x \) or \( y = 2^x \).

I want you to do the following.
1. Build a table for \( y = 2^x \) then graph \( y = 2^x \).
2. Build a table for \( y = \log_2(x) \).
3. Graph \( y = \log_2(x) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 2^x )</th>
<th>( x )</th>
<th>( y = \log_2(x) )</th>
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\[ \text{Note: To graph } y = \log_2(x) \text{ on TI-SmartView or the graphing calculators use change of base, } y = \frac{\log(x)}{\log(2)} \]

V. Properties of Logarithms

We have exponent rules. Such as \( x^a \cdot x^b = x^{a+b} \). There are rules for logarithms, as well. What do you think happens when you have \( \log_a(b) + \log_a(c) \)?

**Product Rule:**
For \( m > 0 \) and \( n > 0 \), \( \log_a m + \log_a n = \log_a (mn) \)

*Example:* \( \log_3(x) + \log_3(6) = \log_3(6x) \)

*Example:* \( \ln(3) + \ln(x^2) + \ln(y) = \ln(3x^2y) \)

**Quotient Rule:**
For $m > 0$ and $n > 0$, $\log_a m - \log_a n = \log_a (m/n)$

Example: $\log_7(24) - \log_7(4) = \log_7(24/4) = \log_7(6)$

Example: $\ln(x^6) - \ln(x^2) = \ln \left( \frac{x^6}{x^2} \right) = \ln(x^4)$

Example: $\log_9(x) + \log_9(y) - \log_9(z) = \log_9 \left( \frac{xy}{z} \right)$

Power Rule:
For $m > 0$, $\log_a m^n = n \log_a (m)$

Example: $\log(3^8) = 8 \log(3)$

Example: $\log \left( \sqrt[3]{3} \right) = \log \left( 3^{\frac{1}{3}} \right) = \frac{1}{2} \log(3)$

Example: $\log \left( \frac{1}{3} \right) = \log \left( 3^{-1} \right) = -1 \log(3)$

Inverse Rules:
1. If $a > 0$ and $a \neq -1$, $\log_a (a^x) = x$ for any real number $x$
2. $a^{\log_a x} = x$ for $x > 0$

Practice problems:

a) $\ln \left( \frac{3x^2}{yz} \right) =$

b) $\log_3 \left( \frac{(x-1)^3}{z^2} \right) =$

VI. Change of Base:

$\log_a (x) = \frac{\log_b (x)}{\log_b (a)}$ where $a > 0, a \neq 1, b > 0, b \neq 1, x > 0$

Example: $\log_9(53) = \frac{\log(53)}{\log(9)} \approx 1.807$ or $\log_9(53) = \frac{\ln(53)}{\ln(9)} \approx 1.807$
We have exponent rules. Such as $x^a \cdot x^b = x^{a+b}$. There are rules for logarithms, as well. What do you think happens when you have $\log_a(b) + \log_a(c)$? We will discuss in our next lesson.

<table>
<thead>
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<th>Possible Assessments from Today’s Lesson</th>
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**Teacher Reflection**

1. Did you use the class time wisely? *This lesson is very full. So, the pedagogy was predominantly lecture, so that I could cover all the topics that were planned.*

2. What worked well in the lesson? *The sections of the lesson plan that worked well were (1) characteristics of logarithms and (2) the fact that they are inverses of exponential functions.*

3. What recommendations would you make to improve this lesson? *The lesson needs to spread over more than one lesson. Because I have already lost 2-3 instructional days because of power failures, grant testing, and test for the research project, I decided to compress the topics in this lesson. Worksheets for this lesson would help this lesson move faster.*

**Other Comments About the Lesson**

**References**
### C-19 Solving Exponential and Logarithmic Functions Lesson Plan

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</table>
III. Solve application problems using exponential and log functions

Student Materials

- Calculator

Pre-Homework

None

Introduction/Background

Today, we are going to see how exponential and logarithmic functions can be used. First, let’s finish discussing how the two functions can be used to solve each other’s equations.

Main Lesson Script

I. Find inverse functions.

If we have the following exponential function, what is its inverse function or its logarithmic function? How could we find it?

\[ f(x) = \frac{1}{2}(6)^{x-3} \]

I want you to get with a partner and use the knowledge you have gained regarding exponential and logarithmic functions and find the inverse of this function.

Let student work in pairs and find the inverse function.

**Answer:**

\[ f(x) = \frac{1}{2}(6)^{x-3} \]

\[ y = \frac{1}{2}(6)^{x-3} \]

\[ x = \frac{1}{2}(6)^{y-3} \]

\[ 2x = (6)^{y-3} \]

\[ \log(2x) = \log(6)^{y-3} \]

\[ \log(2x) = (y-3)\log(6) \quad \text{using Logarithmic Power Rule} \]

\[ \frac{\log(2x)}{\log(6)} = y - 3 \quad \text{Note: } \frac{\log(2x)}{\log(6)} \neq \log \left( \frac{2x}{6} \right), \text{ why?} \]

\[ \frac{\log(2x)}{\log(6)} + 3 = y = f^{-1}(x) \]

Note: \( \frac{\log(2x)}{\log(6)} \neq \log \left( \frac{2x}{6} \right), \text{ why? } [\text{Answer: This not how the Logarithmic Quotient Property is defined.}] \]

Now, I want you and your partner to find the inverse of the following logarithmic function.

**Example:** \( f(x) = \log_2(x-1) + 4 \)

**Answer:**

\[ f(x) = \log_2(x-1) + 4 \]

\[ y = \log_2(x-1) + 4 \]

\[ x = \log_2(y-1) + 4 \]
\[ x - 4 = \log_2(y - 1) \]
\[ 2^{x-4} = 2^{\log_2(y-1)} \quad \text{exponential both sides of equation with a base of 2} \]
\[ 2^{x-4} = y - 1 \quad \text{using Inverse Logarithmic Property} \]
\[ 2^{x-4} + 1 = y = f^{-1}(x) \]

II. Solve exponential and logarithmic equations.
Using logarithmic properties I want you and your partner to solve the following logarithmic and exponential equations.

Example: \[ \log_3(x) = -2 \]
\[ 3^{\log_3(x)} = 3^{-2} \quad \text{exponential both sides of equation with a base of 3} \]
\[ x = 3^{-2} \quad \text{using Inverse Logarithmic Property} \]
\[ x = \frac{1}{9} \]

Example: \[ \log_5(5^x) = 2 \]
\[ x^{\log_5(5)} = x^2 \quad \text{exponential both sides of equation with a base of 3} \]
\[ 5 = x^2 \quad \text{using Inverse Logarithmic Property} \]
\[ \pm \sqrt{5} = x \]
But, \(-\sqrt{5}\) is not in the domain for logarithmic functions. Domain of logarithmic functions have a domain of \(x > 0\). So, answer is \(x = \sqrt{5}\).

Example: \[ 5^x = 9 \]
\[ \log_5(5^x) = \log_5 9 \quad \text{take the log of both sides of the equation} \]
\[ x = \log_5(9) \quad \text{using Inverse Logarithmic Property} \]

Example: \[ \ln(x^2) = \ln(3x) \]
\[ e^{\ln(x^2)} = e^{\ln(3x)} \quad \text{exponential both sides of equation with a base of e} \]
\[ x^2 = 3x \quad \text{using Inverse Logarithmic Property} \]
\[ x^2 - 3x = 0 \]
\[ x(x-3) = 0 \]
\[ x = 0, 3 \]
But, 0 is not in the domain for logarithmic functions. Domain of logarithmic functions have a domain of \(x > 0\). So, answer is \(x = 3\).

Example: \[ 2e^{0.5x} = 6 \]
\[ e^{0.5x} = 3 \]
\[ \ln(e^{0.5x}) = \ln(3) \quad \text{take the natural log of both sides of the equation} \]
\[ 0.5x = \ln(3) \quad \text{using Inverse Logarithmic} \]
\[ \begin{align*}
\text{Property} \\
x &= 2 \ln(3) \\
x &= \ln(3^2) \\
x &= \ln(9)
\end{align*} \]

III. Solve application problems using exponential and log functions.

Pass out the Handouts: Population Growth Models and Logarithmic Application Problem.
Have the students work in small groups to solve the problems on this handout.

After students have completed the two handouts, have students present their work on the board or using the document camera.

Summary/Extension

Possible Assessments from Today’s Lesson

Teacher Reflection
1. Did you use the class time wisely? Lesson plan went well. It would have been beneficial to have time to do more application problems.

2. What worked well in the lesson? The application problems worked the best for the students once they started to understand how to solve exponential and logarithmic equations.

3. What recommendations would you make to improve this lesson? Textbook does not have enough problems that are for modeling and predicting. So I need to look at other available textbooks.

Other Comments About the Lesson

References
Population Growth Problems

The following are some population growth models, that use the continuous growth model, $A = Pe^{rt}$.

**Note:** that $A$ and $P$ are in millions; $t$ is years after 2006.

India: $A = 1095.4e^{0.014t}$
Iraq: $A = 26.8e^{0.027t}$
Japan: $A = 127.5e^{0.001t}$
Russia: $A = 142.9e^{-0.004t}$

1. What was the population of Japan in 2006?

2. Which country has a decreasing population?

3. When will India’s population be 1238 million?

4. What was the population of Iraq in 2006?

5. Which country has the greatest growth rate? By what percentage is that country increasing each year?

6. When will India’s population be 1416 million?
Logarithmic Application Problem

Let the function $f(x) = 8 + 38\ln(x)$, where $f(x)$ is the percentage of new cellphones with cameras $x$ years after 2002. Use the function to determine the percentage of new cellphones with cameras in 2008.

When will the percentage of cellphones with cameras reach 87%.
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Lesson Agenda

I. Review of mean and standard deviation
II. Activity. Review of student data
III. Statistical Testing
IV. Activity. Z test of student data

Instructor Materials

Handouts
TI-Smartview

Student Materials

Calculator

Pre-Homework

Bring in quantitative data that you are interested in and are curious about whether or not the latest data reflects some kind of significant change. You need to bring at least ten data items to reflect what you believe is status quo or the norm for the topic you are investigating. Plus, at least one or more data items that you believe represent the change.

Introduction/Background

I asked all of you to bring to class data that you wanted to use to determine if something that you are interested in has changed. This is data that you think will tell you whether “this something you are curious about” is still the same, maybe going through a cycle.

So, today we are going to discuss performing statistical tests. Statistics tests are used to answer these very questions. Such as, “Is it really hotter than normal?”, “Is the rainfall amount really low?”, “Are people wearing larger shoe sizes?”, “Are professional football players taller than they used to be?”

Today we will discuss a simple statistical test, called the t test. There are many more statistics, but you will more about these tests in your upcoming statistics class.

Main Lesson Script

Review of mean and standard deviation. Does anyone recall what is the definition of mean in statistics? [It is the arithmetic average of a data set of numbers.] Does anyone recall what is the definition of standard deviation? [Standard deviation describes the spread or dispersion of the distribution of a sample or a population.]

Activity. Review of data.

I want you to get into groups of two or three. Discuss the data that you brought to class. Decide on whether the data brought by each member of the group seems to indicate sometime atypical may be occurring for this topic.

Statistical Testing.
Pass out the Handout: Statistical Testing.
Go through hand out explaining hypotheses testing.
Walk through the example of average temperatures in Austin.

Activity.  Z test of student data.

I want you to get to your groups and now perform the z test on your data.
Determine if you get statistical significance with your data.  How did your hypotheses go?

Summary/Extension

The z test is the first and simplest of many statistical tests available to us. There are many more tests available to use depending on your data (whether its data are categorical or quantitative), the distribution of the data, whether the data’s standard deviation is known, etc. These topics are covered in your upcoming statistics class.

Possible Assessments from Today’s Lesson

If you get a $p$ value of 0.501, do you have statistical significance?

Teacher Reflection

1. Did you use the class time wisely? *There were no time management problems.*

2. What worked well in the lesson? *Students were interested in how hypotheses testing is conducted.*

3. What recommendations would you make to improve this lesson? *It would be preferable to move this content across more than one lesson. It seems that this statistical content could be better understood if students were given more time to absorb the material.*

Other Comments About the Lesson

References
Statistical Inferences

- Usually we are not interested in describing the performance of the subjects in our various different groups.

- One group is usually considered the control group. The other groups are treatment groups or groups under study.

- The goal is make statistical inferences about the behavior of the population or those subjects not included in the experiment.

- We can conduct an analysis which can be quantitative, qualitative, or both.

- For today’s class, we are going to do a z test.

Hypothesis Testing

- Need to formulate two statistical hypotheses that are mutually exclusive of each other.

- Null hypothesis, denoted $H_0$, takes the position that no treatment effects or changes were present in the population.

- Alternative hypothesis, denoted $H_a$, is the statistical hypothesis that represents the position that the population was significantly affected by the treatment or change.

- Generally the parameter measured is the dependent variable. Usually the dependent variable is the mean, denoted $\mu_1$, $\mu_2$, etc.
• If $\mu_1$ is the mean of the control group and $\mu_2$ is the mean of the treatment group, then

<table>
<thead>
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<th>Null Hypothesis</th>
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<tbody>
<tr>
<td>$H_0$: $\mu_1 \leq \mu_2$</td>
<td>$H_a$: $\mu_1 &gt; \mu_2$</td>
<td>Right–tail test</td>
</tr>
<tr>
<td>$H_0$: $\mu_1 \geq \mu_2$</td>
<td>$H_a$: $\mu_1 &lt; \mu_2$</td>
<td>Left–tail test</td>
</tr>
<tr>
<td>$H_0$: $\mu_1 = \mu_2$</td>
<td>$H_a$: $\mu_1 \neq \mu_2$</td>
<td>2-tail test</td>
</tr>
</tbody>
</table>
Normal Distribution

Today we are going to assume our data has a normal distribution. So what do we mean by right-tail, left-tail, or 2-tailed tests?

If $H_0: \mu_1 = \mu_2$ and $H_a: \mu_1 \neq \mu_2$, then the statistical test is performed to determine whether the treatment group or new numbers or either less than or greater than the mean or the value claimed for the control group. Figure 1 below depicts this situation. If the treatment group is significantly different, the treatment mean would be found in either of the two shaded areas in the tails of the normal distribution.

![Two tails](image1)

If $H_0: \mu_1 = \mu_2$ and ($H_a: \mu_1 < \mu_2$ or and $H_a: \mu_1 > \mu_2$), then the statistical test is performed to determine whether the treatment group or new numbers or either less than or greater than the mean or the value claimed for the control group as specified by the alternate hypothesis. Figure 2 below depicts this situation. If the treatment group is significantly different, then the treatment mean would be found in one of the two shaded areas in the tails of the normal distribution that is described by the alternate hypothesis.

![Two tails](image2)

It should be noted that:

We can’t prove a null hypothesis is true.
What we try to do is:
  - Reject the null hypothesis; or
  - Fail to reject the null hypothesis
Now the next question is where do the tails begin on the normal distribution? We use two key variables 
- *P* value
- *α*

**P value**
- Conditional probability
- The probability of the observed statistic given that the null hypothesis is true.
- Not the probability that the null hypothesis is true
- Not the conditional probability that null hypothesis is true given the data

If *p* value is big,
- does not prove null hypothesis is true,
- But definitely does indicate that it is not true
- Can only conclude that we “fail to reject the null hypothesis”

If *p* value is small,
- Need to decide if something rare occurred
- Is it rare enough to reject the null hypothesis?

But, what makes the determination of big or small?

**Significance Level, α**
- *α* is used to make the determination if the *p* value is small enough.
- Generally, when *α* ≤ 0.05, statistical significance has been found.
- *α* ≤ 0.10, statistically interesting or practical significance

**Hypothesis Decision Time**

Compare the *p* value to *α*
- If the *p* value is greater than *α*, we say that there is insufficient evidence to reject *H₀*. You would fail to reject *H₀*.
- If the *p* value is less than *α*, we say there is sufficient evidence to reject *H₀* in favor of *H₀*(alternate), you would reject *H₀*. 
Example of a t test

Examples of performing a t test. A t test is performed if the standard deviation is not known for the population that you are examining.

Example: The average temperature in Austin, Texas for all of 2010 was 69°. Is the temperature getting hotter than normal in Austin, Texas? Let’s see. Let’s compare the average temperature for the decades of 1850s through 1990s to the temperatures for the 2000s.

<table>
<thead>
<tr>
<th>Avg March temperature for:</th>
<th>1850s: 66.3</th>
<th>2000: 70.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1860s: 67.1</td>
<td>2001: 68.7</td>
<td></td>
</tr>
<tr>
<td>1870s: 67.2</td>
<td>2002: 68.8</td>
<td></td>
</tr>
<tr>
<td>1880s: 68.0</td>
<td>2003: 69.5</td>
<td></td>
</tr>
<tr>
<td>1890s: 67.9</td>
<td>2004: 69.3</td>
<td></td>
</tr>
<tr>
<td>1900s: 67.8</td>
<td>2005: 69.9</td>
<td></td>
</tr>
<tr>
<td>1910s: 67.9</td>
<td>2006: 71.6</td>
<td></td>
</tr>
<tr>
<td>1920s: 68.2</td>
<td>2007: 69.0</td>
<td></td>
</tr>
<tr>
<td>1930s: 68.8</td>
<td>2008: 70.9</td>
<td></td>
</tr>
<tr>
<td>1940s: 67.6</td>
<td>2009: 70.5</td>
<td></td>
</tr>
<tr>
<td>1950s missing data.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1960s: 67.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1970s: 67.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980s: 68.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1990s: 69.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: http://www.srh.noaa.gov/ewx/?n=ausclidata.htm

We need to enter the data first and get the mean and standard deviation of the population.

1) Enter the above list of temperatures for 1850s through 1990s into the list, L1, in your calculator.
2) Now run the 1-variable stats program.
   a. Hit “STAT” button.
   b. Select “CALC”.
   c. Select option 1, 1-Var Stats
3) Find the mean, denoted \( \bar{x} \). For this example, the mean is 68.01.
4) Find the standard deviation, \( \sigma \), of this population data. \( \sigma = 0.86 \).
5) Enter the above list of temperatures for 2000-2009 into the list, L2, in your calculator.
Now we are ready to run the z test.

1) Hit “STAT” button.
2) Select “TEST”.
3) Select option 1, Z-test.
4) For Inpt: Select “Data”
   \( \mu_0 = 68.01 \)
   \( \sigma = 0.86 \)
   List = L_2
   Freq: 1
   \( >\mu_0 \)
   Calculate

The answers you receive are
Z-test
\( \mu > 68.01 \)
\( z = 6.76 \quad (\text{which is } 6.76 \text{ standard deviations above the mean}) \)
\( p = 6.666588E-12 \quad (\text{which is the same as } 6.66 \times 10^{-12}) \)
\( \bar{x} = 69.85 \)
\( S_x = 0.964 \)

Since \( p < .05 \), we have statistical significance. In the past decade Austin had hotter than normal temperatures.

Now, examine your data and see what you find.
APPENDIX D
Qualitative Survey

Name: ______________________

- Do you like starting a new mathematics content section with a real-world application or data?

- Do you like how the class lessons would refer back to real-world problems that were previously used?

- What did you like most about the class?

- What did you like the least about the class?

- What would you change in the course?
January 19, 2011
18 students in attendance.
The class time was completely allocated to administrative tasks. I called and reviewed the first day class roster. I went over the class syllabus. Since this class is a part of a THECB grant, I discussed the grant program and requirements from the students. Discussed tutoring requirements and what tutoring services are available. Students were told that this is a special college algebra class where statistics curriculum will integrated into the instruction. Students were told that the proposed calendar of topics and homework is subject to change, since this is the first time this course has been taught. Students were reminded of the Attendance Policy and were told to sign the Attendance Sign-In Sheet every day. The students then took the following tests:

- MARS
- CSE
- CAOS

Once they completed these test they were allowed to leave. No homework assigned.

January 24, 2011
18 students in attendance. JB started today. RG moved to another college algebra class. I called out the class roster because I want to learn the names to the faces. Lost a couple of students, but gained a couple of new students through the Adds and Drops process.

I taught the lesson plan for statistics data and data collection. The lesson went quite. Those students who took developmental mathematics at this university were very familiar with this topic. For all of the other students, this was a new topic. I assigned homework from the textbook, but also asked the students to fill out a survey form, where the data in the survey would be used in the next class.

Note to self:
1. Prior to class I had found the following websites that I might want to use later:
   - Commute distances to work

- Extreme commuters

- TxState undeclared majors
  http://star.txstate.edu/content/freshmen-begin-college-undecided-majors

- TxState Crime stats

- TxState International student statistics
  http://www.international.txstate.edu/about/statistics.html

2. Emailed CAOS, CSE and ATS test to JB. Asked him to take these tests and bring them to the next class.

**January 26, 2011**

20 students in attendance. Two new students.

Students mentioned that the calculators supplied to them through the grant project had dead batteries. Made a note to myself to notify the Math Grant Office.

We went over the homework questions. I taught the lesson plan regarding “Graphing Statistical Data”. I noticed that the students were weak with creating, reading, and interpreting graphs. This really came out during the student presentations of their group work. Some groups were simply creating graphs, but not building it from the perspective of “Can someone else read it and understand what you are trying to communicate?”

Assigned students to find some statistical data, both categorical and quantitative, that they are interested in. Then the students were asked to create graphs of this data.

Received an email from AL saying she has been very ill the past few days. She missed yesterday’s class (Jan 24). Sent yesterday’s hw assignments to her.

After class, sent handouts and syllabus to new students.

**January 31, 2011**

Prior to class, I had stopped by the Math Grant Office and told them that my several of my students had dead batteries in their calculators. I was given all the batteries that they had in stock and was told to tell the students that they would order more.
14 students in attendance (Weather is very cold and because hilly region where we are located, students were concern by travelling to and from the university with rain and ice in the weather forecast.). One more new student in the class today. Now have a total of 21 students. I distributed the batteries that were given to me.

We went over the homework questions. I taught the lesson plan regarding Measures of Central Tendency and Measures of Dispersions”. Measures of central tendency was easy for them. Standard deviation is confusing for them at first. I am having them only do a few calculations by hand. Then do all other calculations using the calculator. I made sure to emphasize the difference between mean and median. Need to change the lesson plan in the future to calculate the standard deviation for a sample, as well as for a population.

Next time I teach this class, I may have them bring their own quantitative data. Then go through measures of central tendency topic. Then ask the students to get in groups and decide which is better mean or median for the data that they brought to class.

February 2, 2011
No class today. The university had canceled all classes this afternoon and evening because of rolling power blackouts resulting from cold winter weather.

February 7, 2011
21 in attendance. Am 1x(class period) behind because of no class on Feb 2. Merged Normal distribution and z-score, normal distribution problems into one lesson. Margin of error and linear correlation will be the next lesson. Postponed a quiz till next class. Went into a stronger lecture mode than had previously planned, in order to make up for lost time.

First I went over homework problems. Discovered that the class had a hard time looking at a histogram, where the x-axis has integer tick marks, and calculating mean, median, Q1, and Q3. The class had no problem calculating the information from a list of numbers (concrete data). But, many students had a lot of difficulty extracting the data from a histogram and calculating this information. I planned to quickly go through the homework. Unfortunately, I spent more time on hw because of this problem.

I taught Normal distribution lesson plan. Also, covered margin of error.

February 9, 2011
21 in attendance. Gave students a few minutes to fill out mandatory timesheets for the grant project. Collected timesheets from the students.
Several homework questions. A couple of questions on margin of error. Students had problems entering $\frac{1}{\sqrt{n}}$ into the calculator. Gave quiz today. Students had problems reading graphs of histograms again. ARTIST test graphs were different than the textbook histograms and some students could not figure out this difference. Again I am seeing student difficulty with reading statistical graphs.

Taught the linear regression and line of best fit lesson plan. I used Super Bowl statistics that a student had previously turned from Jan 26 hw assignment. Showed how to use the graphing calculator to perform a linear regression. Students seems to enjoy working with Super Bowl statistics.

Showed other student-produced graphs generated from the Jan 26 homework assignment to the class.

**February 14, 2011**

19 in attendance.

I had a lesson all planned for teaching the topic, “Equation of a line”. Had only 10 to cover this topic. I had several unexpected tasks,

- Grant Project administrator came in to discuss the problems with setting a mandatory tutoring time that all students can attend and meet with the tutor assigned to the class. (15 minutes)
- Questions about the time monitors (5 minutes)
- Reviewed Quiz 1 (5-8 minutes)
- Hw Questions.
- Retaught stddev for population vs sample. Saw that students were missing this in the hw. That’s why in journal entry, Jan 31, I reminded myself to change lesson plan to more thoroughly cover this topic. (5 minutes)
- Many comments about not being able to do the linear regression on their calculators outside of class. Since this is taught in developmental mathematics at this university, I was surprised by these comments. Apparently, many of the math tutors do not know how to do a linear regression. In particular, they were unable to correct problems that can occur while using the graphing calculator to do a regression analysis. One tutor told several students that their calculator was broken! Needless to say, I had to re-teach how to use the graphing calculator for linear regressions. Students were frustrated, but I was able to build their confidence back up. (20 minutes)

Spent about 10 minutes on equations of a line. Discussed slope and what it means in particular to application problems. Required the students to write out what the slope of applications was communicating to us. Instead of saying “… the line is going down”, I wanted them to say that “% sales was decreasing over the years 1995-2000.” This was difficult for the students at first. (Note: Many students had problems with this all of the semester. Students are still having problems interpreting graphs and what is being communicated by the graphs.)
After the class, contacted tutor assigned to my class. I found out that he has never taken statistics. I made a request for a tutor that know statistics and know how to use graphing calculators for statistics.

**February 16, 2011**
20 students in attendance. Since I covered very little new ground in the previous class, I planned to cover both lesson plans on linear equations.

Still some hw questions on how to use the calculator for linear regressions. Students were not adjusting their Viewscreen or a couple of students deleted their lists from their calculators. For the latter problem, I had to reset RAM to correct the problems. Told students that they need to know how to use their calculators for Test 1.

Taught the lesson plan on Linear Equations. The instruction of the parts of the line and the various equations of the line went well. This is familiar mathematical topics for many of the students.

Note: Test 1 is scheduled for the next class. All students were to be on-campus for a Grant Workshop. I offered to stay after the workshop to answer any questions. Only one student, JT remained after the workshop for some one-on-one tutoring.

**February 21, 2011**
Administered Test 1. First student to complete test, completed the test in 45 minutes. Some students did use all of the class time to complete the test. One student commented that she was surprised that the test was not all multiple-choice. She had a hard time taking the test. She is used to taking the answers and picking the best choice. No comments/problems with the test questions themselves. All 21 students took Test 1.

Before class, I received a call from MD, the Grant Project coordinator. My student, TO, was in her office asking for tutoring assistance for the test. She told the coordinator that she is unable to do the homework. She stated she is too busy at home to dedicate the time needed to do the homework. MD asked me if I could tutor TO. I said I am leaving in 10 minutes to administer the exam. I, also, mentioned that TO never came by my office for tutoring even though we had previously set up appointments for this topic.

**February 23, 2011**
19 students in attendance. TO did not come to class today; she never did return to class and eventually withdrew from the University. She made the lowest grade, 44, on Test 1.
Moved on to the lesson plan. Still had more comments regarding students not knowing how to perform a linear regression on the calculator. The weak tutoring support is hurting morale on this topic.

I taught a lesson plan that was modeling supposedly linear data. The data is actually quadratic, but the students do not know this. I used a lesson plan that I found from a Texas Instruments site that was intended to be used for quadratic regressions. Lesson plan is called “Is Leisure Time Really Shrinking?” I modified it to meet the needs of the curriculum for this course. I modified the lesson plan to first be an exercise for a linear regression. I wanted the students to see if the data looked linear or not. I thought the lesson plan would take 30-40 minutes, but the time was closer to 50 minutes. One problem was that the linear regression produced many decimal places. Students did not know how many to use. This particular problem you needed to use 5 decimal places. Students were used to rounding off at 2 decimal places. I had two Latina students ask me for the definition of “leisure”. I was not expecting that.

Note to self: Define “Leisure” when teaching this lesson next time.

AL informed me that she was in a car accident a week ago. She believes that she is still having problems from a concussion she received from the accident. She said she will be tested soon for side effects sustained from the concussion. She remembers very little of newly taught content material after leaving a class. She told me she had difficulty concentrating while taking Test 1.

Note to self: Rewrite the portion of the lesson plan that addresses how to do a Linear Regression on the calculator.

February 28, 2011

19 students in attendance.

Discussed error analysis from previous lesson and the fact that linear regression is strong and positive ($r = .91$), but after year, 2007, model’s integrity is questionable. In year 2050, 61 hours is average # of weekly work hours. Does this seem reasonable?

Began lesson on quadratic equations and their characteristics. Would have like to utilized an activity, but am concerned that I am behind. Covered everything, except quadratic regression. Few questions on quadratic equations. Told students to come see me or go to Math Lab or see a tutor if they had questions.

Returned Test 1 to students. Grades range from 44-97, median was 77 and median was 75.

March 2, 2011
18 students in attendance. As part of the grant project, an individual from the Counseling Center gave a 15 minute presentation on the services that they provide.

Went over hw questions and problems. Taught the lesson on difference between Linear and Quadratic Equations.

March 7, 2011
12 students in attendance. This is the Monday before Spring Break. Many students have exams this week in their other classes.

Went over hw questions and problems. Found that several problems in the textbook were too messy for what I wanted the students to do. Also, there are few modeling and regression problems.

Gave a quiz today. Covered quadratic regressions. Went back to the “Is Leisure Time is Shrinking?” problem and asked students to do a quadratic regression. Found we get $R^2 = .98$ for quadratic regression, where $R^2 = .83$ for linear regression. Discussed what $R^2$ meant.

Note to self: Look at other college algebra textbooks that may have more math/statistics modeling friendly exercises.

March 9, 2011
15 students in attendance. Wednesday before Spring Break (Many students have exams this week in their other classes; several students sent emails stating that they are sick; other students are taking off early.) MD came to the beginning of class and distributed Livescribes to those students who wanted to use the Smartpens.

Taught Intro to Function lesson plan, “concrete to representation” portion. Gave another quiz.

March 21, 2011
19 students. Monday after Spring Break.

Went over hw questions and problems.

Taught the remainder of the Intro to Function lesson plan. Discussed the representational to abstract perspective of the lesson plan. Then covered the parent functions or common functions. Next, will discussed when functions are increasing and decreasing.
**March 23, 2011**
15 students in attendance. Reminded students that Test 2 is scheduled for next week, March 30.

Went over hw questions and problems. Discussed constructing functions from formulas.

**March 28, 2011**
17 students in attendance. 19 students now enrolled in the class. AL officially dropped from class because of medical reasons (The head injury in car accident gave her a concussion. After running tests on her, doctors believe she is suffering from swelling of the brain that is affecting her ability to retain new knowledge and concentrate. She needs to take some time off and let her brain heal.)

Went over hw problems and then taught lesson on transformations of functions. Some discussion on what will be on Test 2.

**March 30, 2011**
A couple of days before this class, I met one of the lecturers who have taught college algebra many times over ten years. I wanted to ensure that the college algebra topics were covered within reason on the Test 2 I had created. We decided that since I only used multiple-choice questions on a few questions, that I may have too many open-response questions.

I administered Test 2 today. All 19 students took the test.

**April 4, 2011**
15 students in attendance.

Went over hw problems and questions. I then continued the lesson on transformations of functions. Covered the symbolic perspective of transformations. Also, covered reflection of functions.

**April 6, 2011**
14 students in attendance.

Returned Test 2. Went over hw problems and questions. Taught lesson on Composition of Functions, one-to-one correspondence, and inverse functions.

**April 11, 2011**
14 students in attendance.
Went over hw problems and questions. Taught lesson on Operations on Functions and Inverse Functions.

**April 13, 2011**
15 students in attendance.

Went over hw problems and questions. Taught lesson Exponential Functions. Students modeled cell phone growth by reviewing look at the data through the lens of a linear regression model, quadratic regression model, and then explored the Exponential Regression model.

**April 18, 2011**
16 students in attendance.

Went over hw problems and questions. Taught lesson on Log properties.

**April 20, 2011**
16 students in attendance.

Went over hw problems and questions. Taught lesson on Log functions, solving exponential and log equations. Worked and solved application problems using exponential and log functions.

**April 25, 2011**
14 students in attendance.
Went over hw problems and questions. Taught lesson an Introductory lesson on statistical tests, *p* values, hypothesis testing.

**April 27, 2011**
Similar to what I had done for Test 2, I again met with the same lecturer and discussed the Test 3 that I had created.

I administered Test 3 today.

**May 2, 2011**
16 in attendance. Returned Test 3. Discussed the upcoming final exam. Continued lesson plan on statistical hypothesis testing.

Administered the CAOS posttest.

**May 9, 2011**
Administered the final exam today. Also, administered the MARS posttest, CSE posttest, and ATS posttest.
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VITA

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This dissertation was typed by Thersa R. Westbrook.