EVALUATING ALGEBRA READINESS

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by

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EVALUATING ALGEBRA READINESS

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ABSTRACT

EVALUATING ALGEBRA READINESS

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Eighth grade students taking Algebra I has become an increasingly common occurrence in the United States during recent years. Still, math education in America and the placement of algebra in the curriculum differs greatly from other countries. This work examines the arguments both for and against introducing algebra to students at earlier ages and then analyzes the effectiveness of a curriculum for that purpose. The Math Explorations curriculum created by the Texas Mathworks faculty at Texas State weaves algebra throughout its curriculum. Students complete Algebra I by the end of eighth grade in a three-text series that also covers state curriculum for sixth, seventh and eighth grades. This study examines the effectiveness of the curriculum in terms of both preparing students for algebra and student learning of state-mandated standards as assessed by TAKS testing.

Introduction

Mathematics education has a history of heated debates over teaching methods and what to teach such as the "new math" movement in the 1960's and the "back to basics" movement that followed in the 1970's (L pp. 27-28.) One of the most prominent of these today is the introduction and teaching of algebra and algebra related topics. The debate centers around when it is appropriate to begin the introduction of algebra and algebraic models of thinking to students. There has been a movement towards algebra earlier, including a mandate from the California legislature that Algebra I become a mandatory class for all 8th graders (B 2008)

The historical method has been to separate elementary and middle school mathematics education from the introduction of algebra. Proponents of this model believe that algebra must be postponed until adolescence because of the abstract nature of thinking it requires (CA p. 8.) This stems from Jean Piaget's theory of cognitive development (P 1971.) Piaget theorized individuals undergo a process of cognitive development on four stages: sensorimotor, preoperational, concrete and formal operations. The majority of grade school is supposed to take place when a child is in the concrete stage. In this stage, children are not supposed to be able to make abstract observations without concrete objects. For this reason, algebra is aligned with the beginning of the formal operations stage at the start of adolescence. Although the National Math Panel recommends algebra preparation be a part of the entire K-8 curriculum leading up to Algebra I (NMAP), some believe elementary children are only able to work with specific numbers to perform arithmetic operations, but unable to extend this process to working with unknowns, a skill vital to algebraic thinking (CO 1975, K 1981 pp. 102-119, M 2001.) For example, Filloy and Rojano suggested students develop their arithmetic skills over a long period of time, making their way from specific computational examples to more general, algebraic techniques with a distinct separation of these types of skills (CA p.8.) Yet elementary children are introduced, at least somewhat, to unknowns. For example, they may solve for the unknown in the following problem, ? + 7 = 10, where the unknown is represented with a "?." However, Herscovics and Linchevski said students are able to use the counting methods they already know, or apply inverses rather than establishing and working with an unknown to solve these types of problems (CA p. 8.)

Another argument for postponing algebra until adolescence has a historical rationale. The development of algebra came chronologically after that of arithmetic, leading some to believe the mathematics curriculum ought to follow a similar pattern (CA p. 5.)

Much research has been done in recent years on just how students learn algebra and when they are capable of understanding the underlying concepts (CFL 2003, F 2003 pp. 17-23, D 2003 pp. 49-65.) The research seems to be giving momentum to a growing movement to place algebra earlier in school curriculum than previously thought appropriate.

The National Council of Teachers of Mathematics has developed standards that include algebraic ideas woven into the early mathematics curriculum. These standards

(2000) have included developing algebraic reasoning in students from early on in their schooling. (CA 7.) The NCTM proposed creating a consistent set of basic standards that would align with curriculum throughout elementary, middle and high school mathematics classes (NCTM p. 30) Among these standards are four basic algebra expectations for students at every grade level:

1) Understanding patterns, relations and functions

2) Representing and analyzing mathematical situations and structures using algebraic symbols

3) Using mathematical models to represent and understand quantitative relationships

4) Analyzing change in various contexts. (NCTM p. 92)

These new standards seek to guide mathematics curriculum away from the idea that algebra, or at least algebraic thinking, should not be imparted upon students until adolescence. The NCTM believes strongly in the importance of student exploration of mathematical concepts and processes. Their reasoning is grounded in the curiosity of young learners (NCTM p. 73.) Their vision is for a mathematics curriculum that allows students to engage in building their own conjectures and understanding of mathematics with the guidance of supportive, competent and effective educators. By encouraging curricula to include algebraic reasoning from the beginning of schooling, the NCTM aims to give early learners the opportunity to explore their intuitions about patterns while giving them an early introduction to algebra syntax and symbols. The NCTM hopes this will foster a smoother transition to more advanced algebraic learning in middle and high school, while keeping a common connection of algebraic thinking and syntax throughout the K-12 experience (NCTM p. 36.)

A key to the propulsion of these ideas, in addition to the ongoing effort to create the most effective mathematics education possible, has been the economic sustainability of the United States in the evolving technological world. The NCTM acknowledges a growing need for mathematical competency in the workplace (NCTM p. 4.) Put simply, the standard of living for future generations is under severe threat due to the increase in competition of other nations in terms of creating jobs and the need for innovation from America's marketplace to keep up (AVF p. 2.) Many see a possible link between decreasing performance in math and science of U.S. students compared to their international counterparts, decreased interest in American students to pursue math and science in higher education and the increased competitiveness of the rest of the world with the United States in the marketplace (AR p. 6.) Another link is drawn between the quality of math and science teaching and the resulting interest of students. The majority of math and science teachers in fifth through eighth grade do not hold either degrees in their subject or a certification to teach it, resulting in a reluctance of teachers to effectively lead deeper exploration of concepts in the classroom and as a result turning the students away from an interest in math and science (AR p. 7.)

Research from the Tufts University Department of Education supports ideas such as algebra and pre-algebra in elementary school curriculum and changing algebra from an area of high school curriculum to something that is intertwined with other concepts throughout K-12 (CA 7.) The idea is that preventing children from exploring the concepts at a young age may hurt their mathematical development (BH 1988 pp. 20-32) and history may actually support an earlier introduction of algebra rather than suggesting otherwise, such as the placement of negative numbers in early curriculum, a topic which led to great disagreements among mathematicians less than two centuries ago.

Early mathematics curriculum has long been seen as an introduction to arithmetic, while algebra has been pushed to later in the curriculum as a generalization of the arithmetic in which the students already had a firm foundation. However, somewhat for this reason, there has been a push to intertwine the two subjects to stress to students the algebraic nature of the arithmetic and foster a deeper understanding from a younger age.

The Department of Education at Tufts University has been a leader in this movement. They view algebra as a generalized arithmetic in which the concept of functions can be emphasized from early on within the teaching of basic arithmetic operations. Their position is to make algebra a key part of early mathematics rather than postponing it until adolescence as a general explanation of arithmetic (CSB p. 3.)

The opportunity to allow students to explore algebraic generalizations within early instruction on arithmetic and build a firm understanding of mathematical algorithms and the rationale behind them is paramount to the early algebra movement. Early algebra is defined as introducing algebraic thinking and syntax in the curriculum of students before the high school age they would normally first encounter these concepts. They feel that the difficulties students experience in algebra can often relate back to the faulty logic, such as working with the associative and distributive properties or dividing terms, they established from their early mathematics experiences (CA p. 5.) This process they feel will avoid stifling the mathematical creativity and exploration of young students and offering them a narrow view of mathematical processes.

The work of Tufts is supported by numerous classroom studies (BR 2004; CSB 2000; Schliemann, Goodrow, & LaraRoth, 2001) exploring young children's abilities to perform and understand algebraic tasks. Studies of Russian children showed a higher competency with algebraic word problems upon the entrance to algebra from students who were taught algebraic representations in grades one to four versus those who had a purely arithmetic curriculum (CA p. 10.) A late 1990's study showed that Brazilian children in grades one through six were able to solve algebra problems as well as demonstrate proficiency working with linear equations (CA p. 10.) A group of fifth grade students in another study demonstrated the ability to solve linear equations for unknowns using a balance scale model with just one lesson (CA p.10.) The same study showed these students used algebra syntax rules to solve 60 percent or more of the problems on a post test. In addition, third grade students have been found to be able to work with function tables and use algebra to represent functional relations (CA p. 11.) Tufts developed a classroom study of their own in which 69 students from grades two to four in four different classrooms received at least six 90 minute lessons in algebraic concepts per semester over the span of two years. Their lessons included number lines with positive and negative numbers as well as variable representations of values. Later on in the program students explored ways to use unknown quantities to find things such as a person's height in relation to other people, or someone's current amount of money in relation to different days (CA pp.14-30.)

The study demonstrated the ability of elementary students to use unknown quantities for variable representation. The students were also able to use algebraic expressions to represent differences in values. These students worked their way from the initial belief that unknowns must have a specific value to being able to comfortably use algebraic representations and number line models to solve problems.

Tufts researches used their findings to support their claim that students can work with unknowns as early as third grade. They argue that these students were able to show these skills frequently and in a wide range of contexts rather than simply being confined to one or two examples they might have memorized within their lessons. Their studies gave evidence that students have the ability to work with general relations between sets of numbers and not just concrete and specific examples of arithmetic (CA pp. 30-34.) They cited the ability of the students to use algebraic expressions to represent functions and describe changes in quantities with algebraic expressions. They also noted an increased comfort level from students with using algebraic notation over the period of the study.

These research results are part of an ongoing movement to change the fundamental beliefs of math educators in the United States about students' abilities and readiness to be successful in algebra. The consensus is beginning to shift away from the notion of students not being ready for algebra until high school and a concerted effort to keep algebra separate from earlier mathematics curricula. There is evidence that students are capable of grasping algebraic thinking at earlier ages than previously assumed. The movement to make algebra a more intimate part of early mathematics curricula is part of an effort to put American students in a better position to become competent in math and go on to higher level courses to compete with graduates from other developing economies throughout the world.

This study will evaluate the effectiveness of the Texas Mathworks curriculum, Math Explorations, in preparing sixth and seventh grade students for algebra. The "algebra readiness" sought to be measured refers to the preparedness of students for the standard Algebra I course all students are required to take to start their high school math sequence. In addition, the study will examine the effect of the Math Explorations curriculum compared with the standard curriculum it replaced in teaching students the Texas state-mandated standards assessed by TAKS testing. Lastly, a comparison of Mathworks students to comparison groups will be conducted.

The Mathworks Curriculum

The Texas Mathworks faculty at Texas State University-San Marcos have developed a middle school curriculum designed to allow students to complete Algebra I by eighth grade or earlier. The curriculum is an extension of their Junior Summer Math Camp (JSMC) which began in 1996. The JSMC is a two-week camp for fourth to eighth grade students that aims to prepare students for higher level mathematics. The camp places students into levels one through five depending on their mathematical experience. The Orleans-Hanna Algebra Prognosis Test is given as a pre and post test for students in the camp. However it is not used for placement.

The JSMC curriculum was implemented in various districts across Texas with many different types of students, including groups of gifted students, groups of mixed students and groups of non-native English speaking students. According to Mathworks faculty, a concern about the effectiveness of the associated teacher training was how the teachers were using what they learned in the summer camps. Teachers had been trained in the Mathworks curriculum at the summer camps by observing the JSMC classes in the morning and attending a graduate class in the afternoon. This training was aimed to prepare teachers to integrate hands-on pre-algebra activities in their teaching. Teachers were encouraged to lead students in a thoughtful exploration of concepts. However, while some of the Math Explorations activities were being used, teachers continued to rely heavily on materials such as drill worksheets in their classrooms. As a result, the

Mathworks faculty began a curriculum project called Math Explorations that extended the Mathworks summer camps to a full year-long curriculum. Math Explorations was piloted in 2008 as one full textbook. In 2009, Mathematics Explorations was split into separate books for sixth and seventh graders and a third volume was developed for an eighth grade Algebra I course. The three volumes cover all of the topics within the Texas Essential Knowledge and Skills (TEKS) for grade levels 6-8, while taking the students from pre-algebra through the completion of Algebra I. Although Math Explorations part 1 and 2 are not designed to cover all of Algebra I, these volumes weave algebraic concepts and thinking within the students' pre-algebra curriculum, preparing all students for success in algebra by the end of eighth grade.

The Math Explorations series was piloted beginning in 2008 in McAllen ISD with all sixth and seventh grade students and in San Marcos CISD with pre-AP sixth and seventh grade classes. In the 2009-2010 school year, the curriculum pilots continued in San Marcos and McAllen and were also adopted by all pre-AP students in seventh grade in Midland ISD. In 2010-2011, San Marcos expanded its use to sixth, seventh and eighth graders in selected pre-AP classes, while Midland began sixth grade pilots for all of the students at four middle schools, along with the seventh grade pre-AP program. In addition, in 2010-2011 New Braunfels adopted the curriculum for all sixth and seventh grade classes. In 2008, the curriculum was used with 450 students and expanded to 3035 students by 2010-2011.

A review of the success of the curriculum in the 2008-2009 school year yielded intriguing results. At Fossum Middle School in McAllen, over one-third of sixth graders and two-thirds of seventh graders scored higher than eighth grade norms on the Orleans-

Hanna Algebra Prognosis test. Fossum students in sixth grade passed the TAKS at an identical rate when compared to McAllen district averages while Fossum seventh graders passed at a higher rate than the district average despite having 60 minute math classes compared to the 90 minute classes at other campuses.

Math Explorations Part 1 is designed for sixth grade classrooms and aligned with the sixth grade TEKS. Topics include: integers, multiplying and dividing, factors and multiples, fractions, decimal representations, patterns and functions, rates ratios and proportions, measurement, geometry and data analysis.

Chapter six, entitled Patterns and Functions, helps develop students' ability to see patterns within lists of numbers (which the book defines as sequences) and build upon these recognitions to produce rules for the sequence using algebraic symbols, such as "2n + 1."

The chapter goes on to an introduction to equations (math sentences with an equality sign relating two expressions). In this section, students develop the skills to describe situations using unknowns and expressions to form equations. For example, students learn to translate the statement "a number is three more than 38," to the algebraic equation "N = 38 + 3." Students explore how to model these problems using a "balance scale" approach.

The last section of chapter six leads students into a basic exploration of functions. Students examine input-output tables to come up with rules for a function, make these tables given a function rule and determine inputs and outputs of functions given the other. The chapter also includes an introduction to graphing to pictorially represent functions.

This chapter is an attempt to familiarize young students with algebraic thinking, language and syntax within the pre-algebra stage to ease the transition to Algebra I and higher level math courses in general.

Math Explorations Part 2 is targeted for a seventh grade, or advanced sixth grade audience and is aligned with the TEKS for seventh grade. The book includes: integers on the number line, adding and subtracting on the number line, modeling problems algebraically, multiplication and division, patterns and functions, decimal representation and operations, number theory, adding and subtracting fractions, multiplying and dividing fractions and rates, ratios and proportions, geometry and data analysis.

Chapter three, variables and expressions, begins by defining the terms variable and expression and summarizing "the language of algebraic expressions for addition, subtraction, multiplication and division. The chapter goes on to develop a step-by-step method for students to read and interpret statements, develop equations with unknowns to model them and work to solve them using the "balance scale" approach.

Chapter four, multiplication and division, introduces students to the distributive property using both numbers and variable expressions and concludes with a section on solving equations.

Chapter five, entitled patterns and functions, extends the concepts addressed in the first volume with a much heavier emphasis on the coordinate plane and graphing functions as well as transformations. The application of linear functions is also explored in depth. For example, students evaluate a scenario where a lemonade stand is set up at a certain cost with the cost of making each cup of lemonade to sell fixed. Students are asked to fill out a function table modeling these costs, to observe patterns and then to compare the cost function with the revenue function given a fixed cost for selling each cup of lemonade.

While these and the previous examples discussed in volume one are not the only examples of algebraic syntax and language within the pre-algebra curriculum, they serve to illustrate the attempt to develop student's algebraic skills well before taking Algebra I.

Math Explorations Algebra I is the third and final volume in the Math Explorations series. This text takes students through a course in Algebra I covering all TEKS for algebra as well as addressing the TEKS for 8th grade math. The book has 12 chapters including: exploring the number line, variables expressions and equations, exploring functions, straight lines, systems of equations, linear inequalities, exponents, polynomial expressions and factoring, quadratic functions, rational expressions, radical expressions and statistics.

The JSMC, the basis for the Math Explorations series, is structured with a book for levels one through four and selected problems worked in level five. The level one book, entitled *The Mathematical Mystery Tour*, has two chapters: exploring integers on the number line and modeling problems algebraically. In chapter one, students are introduced to the concept of negative numbers using the model of a number line centered at zero. The lessons that follow help students develop the ability to model addition and subtraction problems in a number of contexts using the number line. After spending several days with the number lines, students begin to explore general rules for the addition and subtraction of integers and apply those to adding and subtracting larger numbers not as easily modeled using the number line.

The second chapter introduces students to using variables to model unknowns and forming algebraic expressions and equations to model situations where an unknown value is involved. Students go on to explore solving the equations they have created and then to an introduction to the coordinate plane. The students finish with lessons in graphing equations, applications of line equations and area and perimeter.

The level two book, Math Quest, lays out the next two chapters in the JSMC curriculum: exploring fractions and graphs and equations. Chapter three, exploring fractions, explores equivalent fractions, addition and subtraction of fractions, factorization and divisors, ratios and proportions and decimals. Chapter four, graphs and functions, extends on concepts from level one, with a more formal defining of function and exploration of function notation. The chapter also includes lessons on slopes of lines, solving equations with fractions and distance-rate-time problems including variable expressions. Below is a table outlining the parallels between JSMC and Math Explorations curriculum:

Table 1: Mathworks Curriculum Origins

JSMC Curriculum	Math Explorations Curriculum
Level one: building number lines (1.1)	(1.1, building number lines in ME 1/ME 2)
Level one: acting out addition of integers (1.3)	(2.1 addition of integers ME 2)
Level one: acting out subtraction of integers (1.4)	(2.2 subtraction of integers ME 2)
Level one: Using our rules on large numbers (1.5)	(2.3 Adding and Subtracting large numbers ME 2)

Level one: graphing on a coordinate plane (2.3)	(graphing on the coordinate plane 5.1 ME 2)
Level one: graphing on a coordinate plane (2.3)	(1.5 graphing on the coordinate system ME 1)
Level two: adding and subtracting fractions (3.2)	(4.4 addition and subtraction of fractions ME 1)
Level two: Least common multiple and greatest common divisor (3.3)	(common multiples and the LCM 3.5 ME 1)
Level two: multiplying fractions, ratios and proportions (3.4)	(chapter 7, rates ratios and proportions ME 1)
Level two: decimal representations of fractions (3.5)	(chapter 5, decimal representations ME 1)
Level two: Inputs and Outputs (4.1)	(6.3 Functions ME 1)
Level two: slopes of lines (4.2)	(5.4graphing functions ME 2)
Level two: Inputs and Outputs (4.1)	(5.3 functions, ME 2)
Level two: adding and subtracting fractions (3.2), 3.1 equivalent fractions, 3.3 Least Common Multiple and Greatest Common Divisor	(chapter 8, adding and subtracting fractions, ME 2)
Level two: decimal representation of fractions (3.5)	(Decimal representation and operations, chapter 5 ME 2)

The basis of the Mathworks curriculum comes from the idea of teaching students to “think deeply of simple things” (Arnold Ross.) Students are encouraged to develop their own knowledge through the practice of exploring basic concepts in depth, computing examples, looking for patterns, and then understanding how and why mathematical ideas are true.

The guiding principles for Mathworks are that these practices should occur in an environment where students are not afraid of failure. This is established by encouraging students to take risks. In this model curiosity is valued and students have fun working on problems, modeled using the math camp philosophy of teaching. Students eventually become persistent in developing their own learning and feel they can build on that through hard work. All students are challenged with high expectations. Students are expected to justify their work with precise mathematical language while their teachers are encouraged to take all student ideas seriously.

The curriculum is unique in several ways:

1) The curriculum weaves in algebra throughout
2) Math is transparent and the focus is on big ideas
3) Teacher training addresses both content and other factors that impact student learning
4) The curriculum addresses Texas and national standards in a coherent way
5) There is a balance of theory and practice: students develop math fluency based on an understanding of fundamental concepts and not from being told procedures

6) Teachers develop a professional learning community to nurture and support their own growth and promote collaboration. Teachers also grow and develop as active learners.

The Math Explorations curriculum pilot project is an attempt to address the problem of how to foster teaching which leads to student success in algebra I and close the achievement gap between students of differing socioeconomic backgrounds. The curriculum does this by engaging young learners in developing algebraic ideas to build upon in future years through inquiry based learning both independently and collaboratively. There is evidence that the curriculum has begun to achieve its goals by making students more algebra ready based upon Orleans-Hanna success rates while simultaneously covering the state-mandated learning objectives as assessed by TAKS.

Study Outline and Results

We wanted to evaluate the algebra-readiness of the students who used the Math Explorations curriculum. These students included sixth and seventh grade students from McAllen ISD and seventh grade pre-AP students from Midland ISD during the 2009-2010 school year. Additionally, we evaluated algebra-readiness for sixth and seventh grade students from both New Braunfels and Midland ISD during the 2010-2011 school year. Due to the pilot status of the curriculum, difficulties existed in persuading entire schools and districts to adopt the curriculum. This led to many differing subsections of students being involved in the pilots.

The following tables show a breakdown of the pilot sites from 2008-2011:

Table 2: 2008-2009 School Year				
School District	**Schools**	**Grades**	**# of Teachers**	**# of Students**
McAllen	Fossum	6, 7	5	420
San Marcos	Miller	7	1	30
		TOTALS	**6**	**450**

Table 3: 2009-2010 School Year				
School District	**Schools**	**Grades**	**# of Teachers**	**# of Students**
Midland	Alamo, Abel, Goddard, San Jacinto	7	8	409
McAllen	Fossum, Travis	6, 7, 8	8	657
Austin	Kealing	6	3	242
San Marcos	Miller	6, 7	2	58
		TOTALS	**20**	**1,366**

Table 4: 2010-2011 School Year				
School District	**Schools**	**Grades**	**# of Teachers**	**# of Students**
Midland	8 schools	6, 7	14	1,150
New Braunfels	Oak Run, and New Braunfels	6, 7	12	1,500
Austin	Kealing	6	3	300
San Marcos	Miller	6, 7, 8	3	85
		TOTALS	**32**	**3,035**

To conduct these evaluations, we used the scores of students on the Orleans-Hanna Algebra Prognosis Test and compared them to eighth grade national averages. The Orleans-Hanna is a national test used as a metric to evaluate the algebra readiness of students and help determine student placement and potential for success in Algebra I. Factors assessed to predict potential for success in Algebra I include aptitude, achievement and affective factors. Orleans-Hanna claims reliability of their test from uniformity in test-retest score samples as well as internal consistency, or the degree to which test items statistically "hang together." The test is composed of 50 multiple choice questions divided into six sections. The first five sections present lessons followed by six assessment questions. The last section is a review of the lessons and seventh grade math topics consisting of 20 questions. Possible scores on the test items range from zero to 50, with corresponding raw scores scaled from zero to 98.

The following table shows summary data for national seventh and eighth graders taking Orleans-Hanna:

Table 5: Orleans-Hanna Summary Data

Grade	Mean	N
7	29.1	9165
8	31.3	6743

We compared our students to eighth grade averages because students traditionally advance to Algebra I as ninth grade students the following year, making them "algebra ready" by default. "Algebra ready" is something that has no clear-cut definition, due to the various factors influencing student success in the course. The method of examining eighth grade students is not an exact indicator of algebra readiness, but it gives a reasonable starting point for the discussion. In evaluating eighth grade students, we identified algebra ready students as those who scored highest on the Orleans-Hanna post test, as these students would be expected to be the ones who will be successful in Algebra I as ninth graders. By comparing our students to eighth graders nationally, we were able to describe in a tangible way whether we would expect these students to be algebra ready.

Our evaluation began by converting our students' OH post test scores into a Z-score (standardized score indicating how many standard deviations an observation or datum is above or below the mean) for an eighth grade student. This Z-score was then converted into a percentile rank against 8th grade national performance on OH by taking the area underneath the normal curve to the left of the corresponding Z-score. From there, we determined a student to be "ready for Algebra I" if they scored in the top 58% versus 8th graders.

We developed the 58% benchmark from the 8th grade distribution because 8th grade students are considered "algebra ready" due to the fact that most of them will be enrolled in that course as 9th graders. We used 58% because, according to the Texas Education Agency, approximately 58% of students statewide pass the Algebra I End of Course Exam, identifying these students to us as having been "algebra ready."

The results of this evaluation using pre and post OH test data for the 2009-2010 sites are shown in the following table as percentages of students deemed "algebra ready" before and after using the math explorations curriculum during the school year using our criteria:

Table 6: Percent of Students Algebra Ready 2009-2010

		Pre		Post	
District	Grade	Percent	N	Percent	N
McAllen	6	4.67	321	44.37	311
	7	46.03	239	79.64	221
	Total	22.32	560	59.02	532
Midland	7	43.90	426	91.64	347
Total	6	4.67	322	44.37	311
	7	44.66	665	86.97	568
	Total	31.71	987	71.90	879

Table 7: Percent of Students Algebra Ready 2010-2011

		Pre		Post	
District	grade	Percent	N	Percent	N
Midland	6	15.63	256	58.59	256
	7	51.59	252	91.27	252
	Total	33.46	508	74.80	508
NB	6	6.22	434	68.89	434
	7	38.60	544	73.90	544
	Total	24.23	978	71.68	978
Total	6	9.71	690	65.07	690
	7	42.71	796	79.40	796
	Total	27.39	1486	72.75	1486

From the tables we see a more than 40 percent increase in algebra readiness between pre-and post-test scores from our students overall in 2009-2010 and a 45 percent increase 2010-2011. More specifically, there were significant increases at each grade level in each district for both years.

The results showed over 70 percent of students were deemed “algebra ready” during both 2009-2010 and 2010-2011. In 2009-2010, approximately 59 percent of students in McAllen ISD were algebra ready, or 44 and 80 percent in sixth and seventh grades respectively, along with approximately 92 percent pre-AP of the Midland ISD seventh grade students. No sixth grade students used the curriculum in Midland during 2009-2010. Additionally, 75 percent of the Midland students who used the Math

Explorations curriculum were algebra ready in 2010-2011, or 59 and 91 percent in sixth and seventh grades respectively, while approximately 72 percent of New Braunfels ISD students using the curriculum were algebra ready, or 69 and 74 percent in sixth and seventh grades respectively. (NOTE: This data included all 6th and 7th graders from New Braunfels, while from Midland it included 6th graders from 4 out of 23 elementary schools, and all pre-AP 7th graders)

Another item we assessed was TAKS testing scores. The TAKS test is a statewide test in Texas designed to measure student learning on designated standards outlined by the TEKS (Texas Essential Knowledge and Skills) which students are expected to be taught and learn throughout the school year. Yearly TAKS tests are administered for math and language arts during grades three through eleven. The eleventh grade tests are exit-level tests students are required to pass to graduate high school. Students receive a score ranking them as not meeting standard, meeting standard or commended. The TAKS test will be phased out and replaced by the STAAR test, which also aims to assess student learning of TEKS standards.

The following table shows summary data for sixth and seventh grade TAKS mathematics testing in 2010-2011:

Table 8: TAKS Summary Data

Grade	Percent Commended	Percent Passing	N
6	31	83	339483
7	23	81	334319

While we were unable to obtain individual scores for students in McAllen and New Braunfels, we compared scores for students in two segments of Midland. We split the Midland schools into a group including the schools which used Math Explorations curriculum and those which did not. We looked at the scores of these sets of schools from fifth to sixth grade during the 2009-2010 school year when sixth graders did not use the curriculum and the 2010-2011 results when the curriculum was implemented. The results are shown in the following charts:

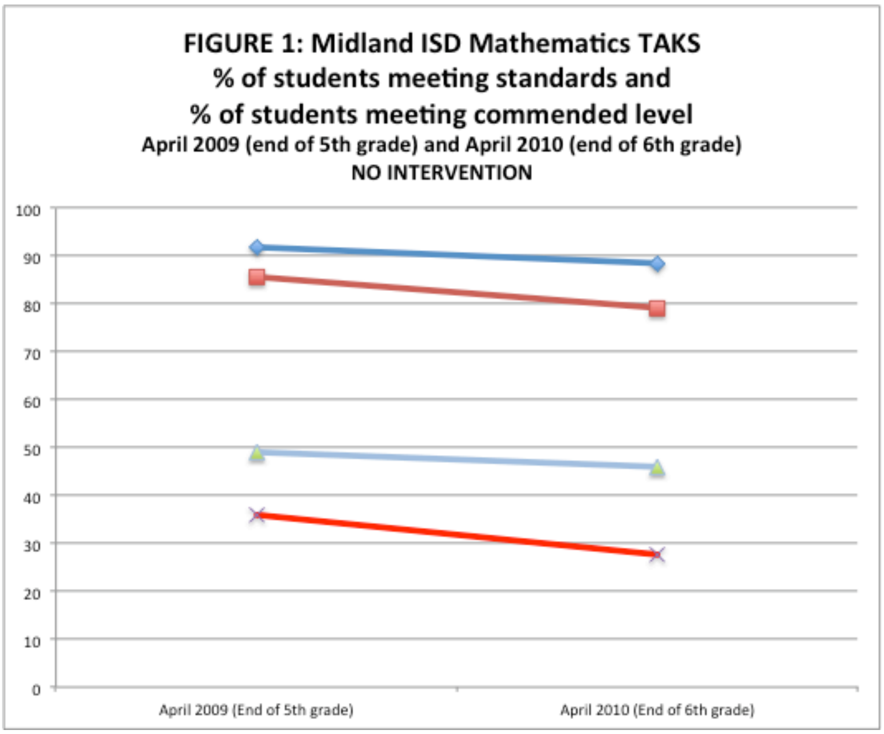

Figure 1: Midland TAKS 2009-2010

(Note: Mathworks schools denoted in blue and other schools in Midland ISD are represented in red. The top of the charts show percentages of students passing and the bottom represents percentages of students earning commended status)

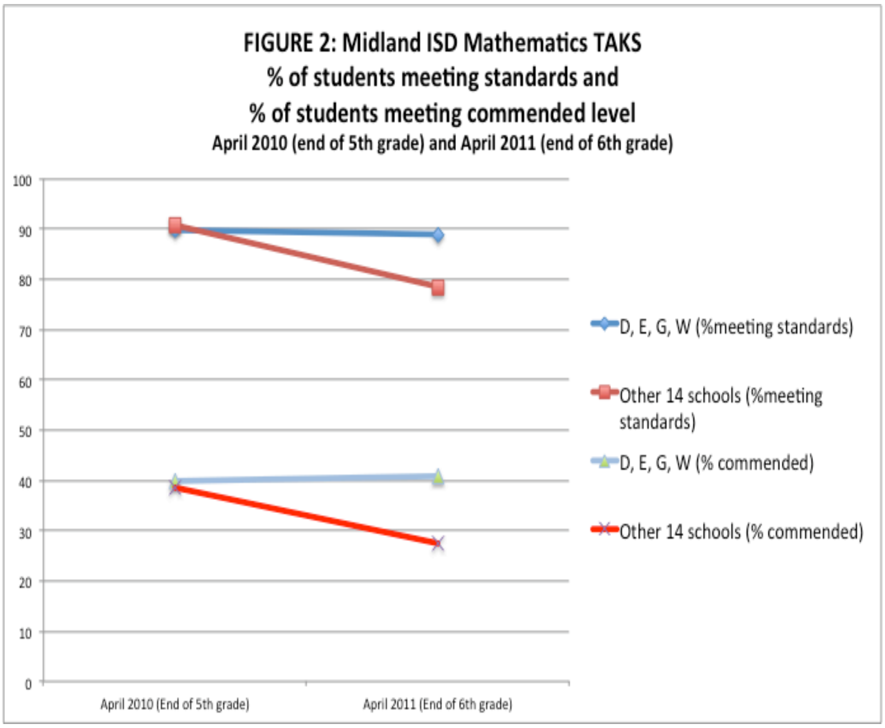

Figure 2: Midland TAKS 2010-2011

(Note: Mathworks schools denoted in blue and other schools in Midland ISD are represented in red. The top of the charts show percentages of students passing and the bottom represents percentages of students earning commended status)

In addition, we evaluated performance on Orleans-Hanna for the 2010-2011 New Braunfels students against that of New Braunfels students from 2008-2009 who did not use the Math Explorations curriculum. We found both sixth and seventh grade students using the curriculum in 2010-2011 outperformed their 2008-2009 counterparts on post test scores by average scores of 31.92 to 28.75 and 34.56 to 31.39, respectively.

Also, we compared Orleans Hanna scores of students using the Math Explorations curriculum in McAllen in 2008-2009 with students in New Braunfels the same year who did not receive the curriculum. While sixth graders in McAllen scored lower on both the pre-and post-test, the gap between these students and the ones in New Braunfels shrunk from about six points to about two points. Furthermore, seventh graders in McAllen went from a disadvantage on pre-test scores compared to their New Braunfels counterparts to outperforming these students on the post-test by an average score of 34.16 to 31.39.

Conclusions

The data lead to two significant conclusions: students who used the Math Explorations curriculum ended up ready for algebra in large proportion with a significantly higher percentage of them meeting this criterion after using the curriculum during the school year. Also, these students were able to maintain reasonable levels of performance on state-mandated TAKS testing standards.

Our conclusion regarding algebra readiness stems from student performance on the Orleans-Hanna test. We feel this is an appropriate tool to measure algebra readiness because it is designed and used nationally to do just that. This test helps schools, teachers and administrators across the country to place students in the correct math class as well as predict the potential for success in Algebra I.

Using Orleans-Hanna as our metric, it appears the Math Explorations curriculum is effective in preparing students for Algebra I. This is shown in the significant increase in Orleans-Hanna success after implementation of the curriculum as well as the favorable performance of these students when compared to eighth grade students nationally. In addition, our comparisons of students using the Math Explorations curriculum to students who did not shows evidence that the curriculum may be more effective than what may already be in place in preparing students for Algebra I.

Also, while the evidence is still preliminary and more data will need to be assessed to make a stronger claim, there is evidence that students who use this curriculum are not impeded in learning state mandated topics as assessed by TAKS based upon our results in Midland. The debate will continue as to what is the most effective way to get students prepared for and have them be successful in taking algebra courses. There are no doubt many crucial concepts that are currently taught as part of middle school or pre-algebra curriculum that must not be overlooked. However, the success of American students in the areas of mathematics as well as science is something that has been called into serious question over recent years. We believe a key to boosting the success of students in mathematics is to provide a strong foundation in earlier courses that can be built upon and allow students to go on and take more advanced mathematics courses.

Placing algebra in a more prominent role earlier in the lives of students allows students to become more familiar with the thinking and syntax associated with it and ease their transition into algebra and later advanced mathematics courses. If students are able to take and master algebra in earlier grades, they will have the opportunity to go on and explore and be more successful in advanced math courses by the end of high school.

Not only is it important to give students the tools and opportunity to reach high levels of mathematics, but it is also important to inspire them to want to do so. The Math Explorations curriculum seeks to combat this issue by providing an instructional style that allows students to explore concepts for themselves and take ownership of their own learning. Also, the activities are meant to be fun and interactive so the students will hopefully be stimulated and intrigued by the concepts and develop a desire to want to learn more.

More research must be done to measure at what ages it is appropriate to introduce algebraic thinking and syntax. Definitive answers to questions of students' cognitive abilities at various stages will need to be addressed. The ability of students to process basic arithmetic skills and perhaps simultaneously learn a more generalized version of these concepts in the form of algebraic thinking must be more thoroughly examined. Comparisons of traditional curricula and algebra preparation strategies with alternative strategies in this country as well as others will continue to shed light on the most effective practices. Lastly, future studies must be conducted about student results in Algebra I after going through various different programs to evaluate the effectiveness of these programs.

Early results of the Math Explorations curriculum pilots, along with the early algebra work of Tufts researchers provides evidence of potential success in accelerating the timeline to convey algebraic thinking to our students.

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