

Detailed numerical results for paper: An approximate dynamic programming approach for the vehicle routing problem with stochastic demands

Clara Novoa ^{a,*}, Robert Storer ^b

^a *Texas State University, Ingram School of Engineering, Industrial Engineering Program, San Marcos, TX 78666, USA. E-mail address: cn17@txstate.edu*

^b *Industrial and Systems Engineering Department, Lehigh University, Bethlehem, PA 18015, USA. E-mail address: rhs2@lehigh.edu*

Following are some tables and figures with detailed results that complement the paper titled "An approximate dynamic programming approach for the vehicle routing problem with stochastic demands" submitted to the European Journal of Operations Research.

Table 1: Confidence intervals for the differences between the best rollout method found and the perfect information case for instances with 5-20 customers

	Number of customers and vehicle capacity					
	5		8		20	
	9	5	14	9	91	58
Mean	3.13%	2.24%	4.51%	3.66%	3.42%	4.86%
Std. deviation	7.12%	6.29%	4.47%	6.00%	3.67%	3.06%
95% LCI	1.54%	0.77%	3.50%	2.29%	2.61%	4.18%
95% UCI	4.71%	3.71%	5.52%	5.03%	4.24%	5.54%

*Corresponding author. Tel.: 1 512 245 4343; fax: 1 512 245 7771

Table 2: Confidence intervals for the differences between the best rollout method found and the perfect information case for instances with 30-60 customers

	Number of customers and vehicle capacity					
	30		40		60	
	137	87	183	116	274	175
Mean	3.05%	3.32%	2.79%	5.03%	2.69%	7.94%
Std. deviation	2.14%	2.18%	1.39%	3.14%	1.96%	1.94%
95% LCI	2.57%	2.83%	2.48%	4.33%	2.26%	7.47%
95% UCI	3.53%	3.80%	3.10%	5.73%	3.13%	8.42%

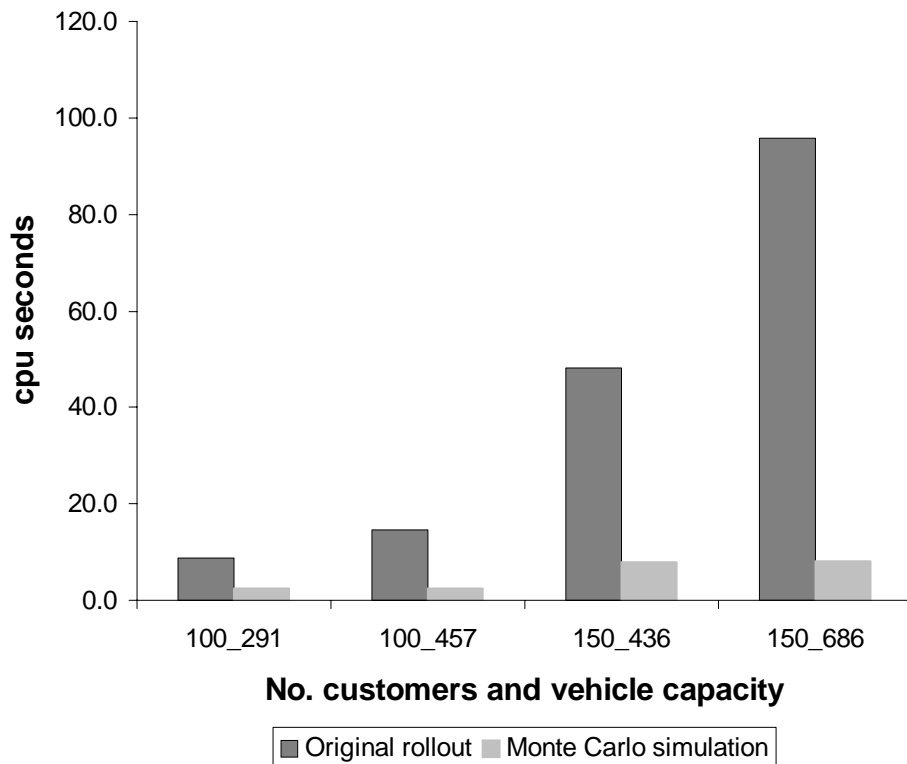


Fig. 1. Average CPU times "original rollout" vs. Monte Carlo Simulation (MCS).

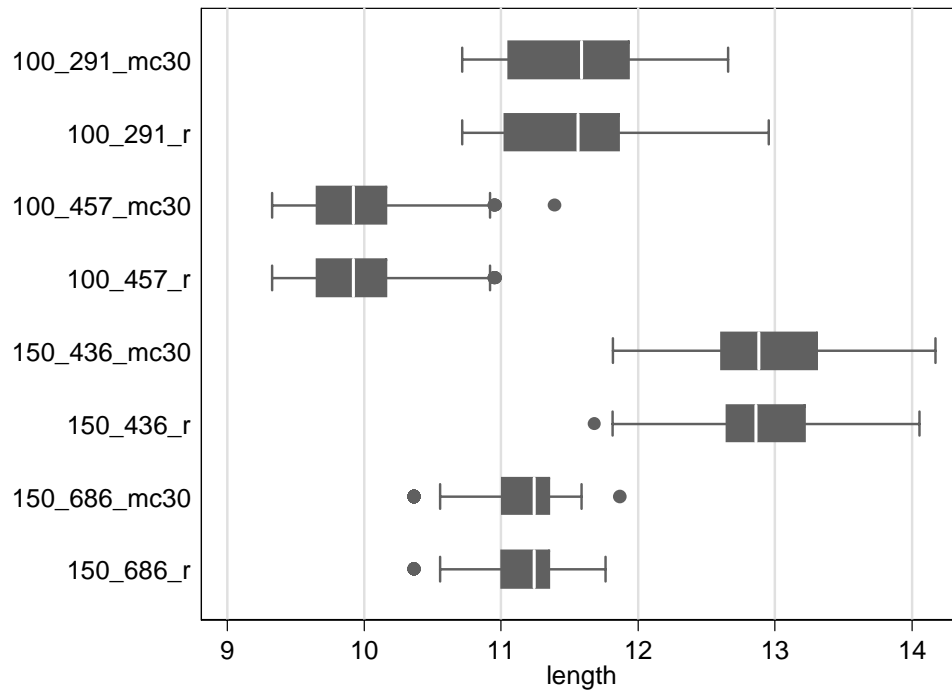


Fig. 2. Comparison of routing costs (tour lengths) "original rollout" vs. Monte Carlo Simulation (MCS)".

Table 3: Estimated expected routing cost for instances in Set1

Method	Number of customers and vehicle capacity													
	(5,9)	(5,5)	(8,14)	(8,9)	(20,91)	(20,58)	(30,137)	(30,87)	(40,183)	(40,116)	(60,274)	(60,175)		
1s_combi_r	3.7463	5.4971	4.3227	5.6111	5.6803	7.6217	6.2331	7.8062	6.5814	8.1434	7.5539	9.2928		
1s_n2_mc15	3.7953	5.5759	4.3988	5.6230	5.9175	7.7166	6.5856	8.0867	7.1154	8.6554	8.2580	9.8853		
1s_n2_mc30	3.8081	5.5838	4.3961	5.5919	5.9090	7.6872	6.5653	8.1003	7.1236	8.6159	8.2372	9.9058		
1s_n2_mc30pr2	3.8100	5.5979	4.4098	5.6114	5.9231	7.8851	6.4829	8.1352	7.0584	8.6784	8.2157	9.8681		
1s_n2_mc30pr1	3.8100	5.5979	4.4098	5.6114	5.9231	7.8851	6.4829	8.1352	7.0584	8.6784	8.2157	9.8681		
1s_n2_r	3.7922	5.5902	4.3965	5.5836	5.9142	7.6626	6.5790	8.0630	7.1145	8.6298	8.2219	9.9224		
1s_n2reopt_r	3.7922	5.5902	4.3709	5.5791	6.0455	7.7453	6.7998	8.0988	7.1996	8.6378	8.2325	9.9324		
1s_stostat_r	3.6801	5.4136	4.3403	5.6002	5.7092	7.6565	6.1345	7.5759	6.6355	8.0491	7.4743	9.3184		
1s_tsp_r	3.7853	5.4440	4.3299	5.6127	5.9181	7.6478	6.4680	8.0637	6.9769	8.6719	7.9386	9.9146		
1s_ORApr_r	3.7701	5.5335	4.3270	5.6613	5.6750	7.8689	6.2530	7.9514	6.6913	8.1830	7.6413	9.5577		
1s_TRApr_r	3.7673	5.4924	4.3283	5.7325	5.6898	7.7571	6.3139	7.7941	6.5800	8.2614	7.5594	9.3643		
1s_vrp_r	3.7889	5.5335	4.3330	5.7359	5.6842	7.6733	6.2292	7.9279	6.6115	8.2416	7.5722	9.4687		
2s_combi_r	3.7875	5.7514	4.2867	5.6195	5.6474	7.5667	6.0575	7.6383	6.5162	8.0683	-	-		
2s_n2_r	3.8088	5.7440	4.4423	5.6562	5.7638	7.6881	6.3690	7.8132	6.9142	8.3951	7.9298	9.5812		
2s_stostat_r	3.7198	5.5306	4.3613	5.6445	5.6885	7.5195	6.0634	7.4718	6.5901	8.0574	7.4210	9.1819		
NN2int	4.0103	5.7736	4.7743	6.0501	6.2089	8.0664	6.8620	8.3398	7.4572	8.8008	8.8378	10.3961		
Perfmf	3.5286	5.0330	4.1364	5.3481	5.5163	7.1338	5.8610	7.2107	6.4010	7.6460	7.2201	8.2942		

Table 4: Improvement in routing costs for different methods vs. "original rollout" for instances in Set1

Method	Number of customers and vehicle capacity														
	(5,9)	(5,5)	(8,14)	(8,9)	(20,91)	(20,58)	(30,137)	(30,87)	(40,183)	(40,116)	(60,274)	(60,175)			
1s_combi_r	1.21%	1.67%	1.68%	-0.49%	3.95%	0.53%	5.26%	3.19%	7.49%	5.64%	8.13%	6.35%			
1s_n2_mc15	-0.08%	0.26%	-0.05%	-0.71%	-0.06%	-0.70%	-0.10%	-0.29%	-0.01%	-0.30%	-0.44%	0.37%			
1s_n2_mc30	-0.42%	0.11%	0.01%	-0.15%	0.09%	-0.32%	0.21%	-0.46%	-0.13%	0.16%	-0.19%	0.17%			
1s_n2_mc30pr2	-0.47%	-0.14%	-0.30%	-0.50%	-0.15%	-2.90%	1.46%	-0.90%	0.79%	-0.56%	0.08%	0.55%			
1s_n2_mc30pr1	-0.47%	-0.14%	-0.30%	-0.50%	-0.15%	-2.90%	1.46%	-0.90%	0.79%	-0.56%	0.08%	0.55%			
1s_n2_r	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%			
1s_n2reopt_r	0.00%	0.00%	0.58%	0.08%	-2.22%	-1.08%	-3.36%	-0.44%	-1.20%	-0.09%	-0.13%	-0.10%			
1s_stostat_r	2.96%	3.16%	1.28%	-0.30%	3.47%	0.08%	6.76%	6.04%	6.73%	6.73%	9.09%	6.09%			
1s_tsp_r	0.18%	2.62%	1.52%	-0.52%	-0.07%	0.19%	1.69%	-0.01%	1.93%	-0.49%	3.45%	0.08%			
1s_ORAApr_r	0.58%	1.01%	1.58%	-1.39%	4.04%	-2.69%	4.96%	1.38%	5.95%	5.18%	7.06%	3.68%			
1s_TRAApr_r	0.66%	1.75%	1.55%	-2.67%	3.79%	-1.23%	4.03%	3.33%	7.51%	4.27%	8.06%	5.62%			
1s_vrp_r	0.09%	1.01%	1.45%	-2.73%	3.89%	-0.14%	5.32%	1.68%	7.07%	4.50%	7.90%	4.57%			
2s_combi_r	0.12%	-2.88%	2.50%	-0.64%	4.51%	1.25%	7.93%	5.27%	8.41%	6.51%	-	-			
2s_n2_r	-0.44%	-2.75%	-1.04%	-1.30%	2.54%	-0.33%	3.19%	3.10%	2.81%	2.72%	3.55%	3.44%			
2s_stostat_r	1.91%	1.07%	0.80%	-1.09%	3.82%	1.87%	7.84%	7.33%	7.37%	6.63%	9.74%	7.46%			
NN2int	-5.75%	-3.28%	-8.59%	-8.35%	-4.98%	-5.27%	-4.30%	-3.43%	-4.82%	-1.98%	-7.49%	-4.77%			
Perfinf	6.95%	9.97%	5.92%	4.22%	6.73%	6.90%	10.91%	10.57%	10.03%	11.40%	12.19%	16.41%			

Table 5: Estimated expected routing cost for instances in Set2

Method	Number of customers and vehicle capacity												
	(5,9)	(8,13)	(8,10)	(20,60)	(20,45)	(30,90)	(30,68)	(40,120)	(40,90)	(60,180)	(60,135)		
1s_combi_r	5.2946	6.3377	7.5841	8.0343	9.9472	8.0642	9.8995	8.6584	10.4461	9.2599	-		
1s_n2_mc15	5.4116	6.4261	7.6476	8.1216	9.6498	8.4249	9.8628	9.0373	10.7093	10.1043	11.9606		
1s_n2_mc30	5.4247	6.3369	7.5945	8.1130	9.6576	8.4107	9.8443	9.0171	10.7480	10.0631	11.8772		
1s_n2_mc30pr2	5.4707	6.3867	7.7032	8.1864	9.8556	8.4878	10.0067	9.0760	10.8649	10.0485	11.9070		
1s_n2_mc30pr1	5.5973	6.4822	7.9054	8.2124	9.9149	8.5267	10.0776	9.0953	10.9255	10.0673	11.9577		
1s_n2_r	5.4236	6.3245	7.5912	8.1122	9.6395	8.4158	9.8621	9.0342	10.7776	10.0842	11.8610		
1s_n2reopt_r	5.4236	6.3220	7.5872	8.0965	9.6386	8.4345	9.8671	9.0132	10.7828	10.0668	11.9413		
1s_stostat_r	5.2627	6.2660	7.5567	7.9183	9.5187	7.9998	9.6956	8.6120	10.1678	9.4769	11.0847		
1s_tsp_r	5.2953	6.2584	7.5373	8.0384	9.6014	8.3789	9.8288	8.9071	10.7776	9.9898	11.7739		
1s_ORApr_r	5.4362	6.4444	7.8585	7.9973	10.0312	8.1852	9.9986	8.9368	10.6072	9.4342	-		
1s_TRApr_r	5.3215	6.4086	7.6375	8.1078	10.1306	8.1298	10.2307	8.7776	10.6119	9.3554	-		
1s_vrp_r	5.4634	6.4466	7.8055	8.0309	10.1050	8.1441	9.9772	8.7526	10.5589	9.3721	-		
2s_combi_r	5.4690	6.2684	7.7008	7.8505	9.6756	7.8225	9.7030	8.5830	10.3188	-	-		
2s_n2_r	5.4889	6.3260	7.6364	8.0607	9.6818	8.1740	9.6326	8.8511	10.4821	9.7126	11.4649		
2s_stostat_r	5.2929	6.2647	7.6589	7.9019	9.4933	7.8557	9.5587	8.5212	10.1550	9.3302	10.9132		
NN2int	5.6448	6.6767	7.9095	8.3880	10.0157	8.6856	10.1153	9.3718	11.0337	10.5546	12.3009		

Table 6: Improvement in routing costs for different methods vs. "original rollout" for instances in Set2

Method	Number of customers and vehicle capacity													
	(5,9)	(8,13)	(8,10)	(20,60)	(20,45)	(30,90)	(30,68)	(40,120)	(40,90)	(60,180)	(60,135)			
1s_combi_r	2.38%	-0.21%	0.09%	0.96%	-3.19%	4.18%	-0.38%	4.16%	3.08%	8.17%	-			
1s_n2_mc15	0.22%	-1.61%	-0.74%	-0.12%	-0.11%	-0.11%	-0.01%	-0.03%	0.63%	-0.20%	-0.84%			
1s_n2_mc30	-0.02%	-0.20%	-0.04%	-0.01%	-0.19%	0.06%	0.18%	0.19%	0.27%	0.21%	-0.14%			
1s_n2_mc30pr2	-0.87%	-0.98%	-1.48%	-0.91%	-2.24%	-0.86%	-1.47%	-0.46%	-0.81%	0.35%	-0.39%			
1s_n2_mc30pr1	-3.20%	-2.49%	-4.14%	-1.23%	-2.86%	-1.32%	-2.19%	-0.68%	-1.37%	0.17%	-0.82%			
1s_n2_r	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%			
1s_n2reopt_r	0.00%	0.04%	0.05%	0.19%	0.01%	-0.22%	-0.05%	0.23%	-0.05%	0.17%	-0.68%			
1s_stostat_r	2.97%	0.93%	0.45%	2.39%	1.25%	4.94%	1.69%	4.67%	5.66%	6.02%	6.55%			
1s_tsp_r	2.36%	1.04%	0.71%	0.91%	0.39%	0.44%	0.34%	1.41%	0.00%	0.94%	0.74%			
1s_ORApr_r	-0.23%	-1.90%	-3.52%	1.42%	-4.06%	2.74%	-1.38%	1.08%	1.58%	6.45%	-			
1s_TRApr_r	1.88%	-1.33%	-0.61%	0.05%	-5.09%	3.40%	-3.74%	2.84%	1.54%	7.23%	-			
1s_vrp_r	-0.73%	-1.93%	-2.82%	1.00%	-4.83%	3.23%	-1.17%	3.12%	2.03%	7.06%	-			
2s_combi_r	-0.84%	0.89%	-1.44%	3.23%	-0.37%	7.05%	1.61%	4.99%	4.26%	-	-			
2s_n2_r	-1.20%	-0.02%	-0.60%	0.64%	-0.44%	2.87%	2.33%	2.03%	2.74%	3.69%	3.34%			
2s_stostat_r	2.41%	0.95%	-0.89%	2.59%	1.52%	6.66%	3.08%	5.68%	5.78%	7.48%	7.99%			
NN2int	-4.08%	-5.57%	-4.19%	-3.40%	-3.90%	-3.21%	-2.57%	-3.74%	-2.38%	-4.66%	-3.71%			

Table 7: Computational times in seconds for instances in Set 1

Method	Number of Customers and Vehicle Capacity													
	(5,9)	(5,5)	(8,14)	(8,9)	(20,91)	(20,58)	(30,137)	(30,87)	(40,183)	(40,116)	(60,274)	(60,175)		
1s_combi_r	0.002	0.002	0.004	0.003	0.087	0.059	0.393	0.255	1.132	0.713	5.132	3.496		
1s_n2_mc15	0.003	0.003	0.002	0.003	0.016	0.015	0.044	0.041	0.102	0.096	0.320	0.299		
1s_n2_mc30	0.003	0.002	0.004	0.004	0.026	0.026	0.072	0.072	0.167	0.164	0.535	0.519		
1s_n2_mc30pr2	0.002	0.003	0.006	0.004	0.017	0.017	0.044	0.042	0.103	0.097	0.328	0.309		
1s_n2_mc30pr1	0.002	0.002	0.002	0.003	0.016	0.016	0.044	0.042	0.103	0.097	0.327	0.309		
1s_n2_r	0.002	0.002	0.003	0.003	0.030	0.020	0.127	0.084	0.368	0.244	1.739	1.145		
1s_n2reopt_r	0.003	0.001	0.002	0.002	0.040	0.026	0.176	0.113	0.503	0.323	2.352	1.552		
1s_stostat_r	0.006	0.006	0.007	0.007	0.042	0.034	0.141	0.101	0.375	0.255	1.686	1.120		
1s_tsp_r	0.002	0.002	0.003	0.003	0.028	0.019	0.127	0.082	0.362	0.229	1.673	1.105		
1s_ORApr_r	0.002	0.001	0.004	0.002	0.027	0.018	0.128	0.081	0.363	0.227	1.709	1.113		
1s_TRApr_r	0.002	0.001	0.002	0.002	0.029	0.019	0.127	0.083	0.366	0.229	1.698	1.135		
1s_vrp_r	0.002	0.002	0.003	0.003	0.029	0.020	0.136	0.081	0.369	0.226	1.682	1.106		
2s_combi_r	0.013	0.008	0.023	0.021	2.169	1.796	19.438	11.739	70.709	56.369	-	-		
2s_n2_r	0.003	0.002	0.006	0.005	0.652	0.477	5.345	3.906	21.840	16.620	175.311	100.676		
2s_stostat_r	0.011	0.006	0.019	0.018	0.764	0.587	5.220	3.618	21.623	15.130	150.124	81.336		
NN2int	0.007	0.007	0.006	0.007	0.007	0.006	0.009	0.009	0.013	0.013	0.031	0.030		

Table 8: Computational times in seconds for instances in Set 2

Method	Number of Customers and Vehicle Capacity											
	(5,9)	(8,13)	(8,10)	(20,60)	(20,45)	(30,90)	(30,68)	(40,120)	(40,90)	(60,180)	(60,135)	
1s_combi_r	0.0017	0.0051	0.0033	0.0636	0.0585	0.2886	0.2319	0.7995	0.6476	3.8823	-	
1s_n2_mc15	0.0024	0.0043	0.0043	0.0186	0.0174	0.0419	0.0446	0.0991	0.1156	0.3104	0.2926	
1s_n2_mc30	0.0030	0.0033	0.0038	0.0256	0.0263	0.0720	0.0722	0.1651	0.1642	0.5207	0.5181	
1s_n2_mc30pr2	0.0034	0.0050	0.0056	0.0171	0.0176	0.0435	0.0435	0.1000	0.0980	0.3115	0.3070	
1s_n2_mc30pr1	0.0037	0.0052	0.0044	0.0161	0.0167	0.0428	0.0425	0.0990	0.0975	0.3103	0.3046	
1s_n2_r	0.0016	0.0035	0.0030	0.0218	0.0187	0.0960	0.0787	0.2761	0.2207	1.3188	1.0492	
1s_n2reopt_r	0.0019	0.0039	0.0038	0.0225	0.0189	0.0979	0.0792	0.2782	0.2232	1.3369	1.0426	
1s_stostat_r	0.0086	0.0077	0.0055	0.0340	0.0307	0.1137	0.0962	0.3013	0.2483	1.3599	1.0826	
1s_tsp_r	0.0017	0.0023	0.0028	0.0232	0.0185	0.0977	0.0797	0.2767	0.2224	1.3398	1.0604	
1s_ORAapr_r	0.0014	0.0025	0.0029	0.0332	0.0266	0.1347	0.1099	0.3893	0.3042	1.8880	-	
1s_TRAAapr_r	0.0021	0.0025	0.0021	0.0333	0.0284	0.1415	0.1182	0.3808	0.3065	1.8542	-	
1s_vrp_r	0.0020	0.0030	0.0028	0.0220	0.0183	0.0956	0.0759	0.2660	0.2138	1.3363	-	
2s_combi_r	0.0137	0.0382	0.0333	1.8233	1.5723	14.8878	11.2693	69.3799	57.6066	-	-	
2s_n2_r	0.0037	0.0099	0.0093	0.5874	0.5170	4.7927	3.5539	20.2258	16.9559	132.1524	107.0486	
2s_stostat_r	0.0106	0.0280	0.0263	0.7566	0.6602	4.8318	4.0517	19.4231	15.7937	111.0701	94.8676	
NN2int	0.0077	0.0076	0.0079	0.0089	0.0096	0.0154	0.0150	0.0189	0.0158	0.0369	0.0371	