THE EFFECTS OF A MODEL DEVELOPMENTAL MATHEMATICS PROGRAM
ON ELEMENTARY AND MIDDLE SCHOOL
PRESERVICE TEACHERS

DISSERTATION

Presented to the Graduate Council of
Texas State University-San Marcos
in Partial Fulfillment
of the Requirements

for the Degree

Doctor of PHILOSOPHY

by

Lindsey N. Gerber, M.S.

San Marcos, Texas
December 2012
THE EFFECTS OF A MODEL DEVELOPMENTAL MATHEMATICS PROGRAM
ON ELEMENTARY AND MIDDLE SCHOOL
PRESERVICE TEACHERS

Committee Members Approved:

__________________________________________
Selina Vásquez Mireles, Chair

__________________________________________
Pamela Littleton

__________________________________________
M. Alejandra Sorto

__________________________________________
Alexander White

Approved:

__________________________________________
J. Michael Willoughby
Dean of the Graduate College
COPYRIGHT

by

Lindsey Gerber

2012
FAIR USE AND AUTHOR’S PERMISSION STATEMENT

Fair Use

This work is protected by the Copyright Laws of the United States (Public Law 94-553, section 107). Consistent with fair use as defined in the Copyright Laws, brief quotations from this material are allowed with proper acknowledgement. Use of this material for financial gain without the author’s express written permission is not allowed.

Duplication Permission

As the copyright holder of this work I, Lindsey N. Gerber, refuse permission to copy in excess of the “Fair Use” exemption without my written permission.
For my late father, Peter Gerber.
ACKNOWLEDGEMENTS

I would like to acknowledge my amazing family. I would not be here if it was not for my mother, Penny Gerber, and stepfather, Don Barr, for pushing me to succeed and helping whenever asked. I would also like to thank my sister, Lauren Gerber, and my brother, Andrew Gerber, for their moral support throughout the years. In addition, I would like to thank the Kennedy and Fitzsimmons families; they have been a family to me cheering me on every step of the way.

I would like to acknowledge all the people who helped make this dissertation possible. First, I would like to thank my friends, Debra Ward for dragging me to Ihop or Mochas and Javas to study and Penny Rodriquez who has become a good friend in the last year joining Debra and I on our study dates. I have great thanks to my newest friend, Michelle Elliott. I met her in the Writing Center at Texas State and I would say she is a miracle worker. If it were not for her support and her occasional yelling, I would not have completed my dissertation. She deserves more than the thanks I am writing today.

Secondly, I would like to thank all my colleagues, instructors, and faculty at Texas State. I would especially like to give thanks to Dr. Selina Mireles, who is my mentor, advisor, and friend, pushing me to my limits and never giving up on me. She has also helped me to see my potential as a mathematics educator.

This manuscript was submitted on September 12, 2012.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xiii</td>
</tr>
<tr>
<td>ACRONYMS AND ABBREVIATIONS</td>
<td>xv</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>xviii</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>Statement of the Problem</td>
<td>2</td>
</tr>
<tr>
<td>Purpose</td>
<td>6</td>
</tr>
<tr>
<td>Significance of the Study</td>
<td>8</td>
</tr>
<tr>
<td>Research Questions</td>
<td>10</td>
</tr>
<tr>
<td>II. LITERATURE REVIEW</td>
<td>13</td>
</tr>
<tr>
<td>Definitions</td>
<td>13</td>
</tr>
<tr>
<td>Preservice teacher</td>
<td>14</td>
</tr>
<tr>
<td>Quality teacher</td>
<td>14</td>
</tr>
<tr>
<td>College readiness</td>
<td>15</td>
</tr>
<tr>
<td>Developmental mathematics</td>
<td>16</td>
</tr>
<tr>
<td>Model Developmental Mathematics Program</td>
<td>16</td>
</tr>
<tr>
<td>Curriculum alignment</td>
<td>17</td>
</tr>
<tr>
<td>Process standards</td>
<td>18</td>
</tr>
<tr>
<td>Algorithmic Instruction Technique model</td>
<td>19</td>
</tr>
<tr>
<td>Concrete to Representational to Abstract model</td>
<td>21</td>
</tr>
<tr>
<td>Cooperative learning groups</td>
<td>22</td>
</tr>
<tr>
<td>Organizational frameworks</td>
<td>24</td>
</tr>
<tr>
<td>Content standards</td>
<td>25</td>
</tr>
<tr>
<td>Instructor preparation</td>
<td>25</td>
</tr>
<tr>
<td>Three-day workshop</td>
<td>26</td>
</tr>
<tr>
<td>Weekly meetings</td>
<td>27</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>--------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Faculty support</td>
<td>28</td>
</tr>
<tr>
<td>Standards</td>
<td>29</td>
</tr>
<tr>
<td>National standards</td>
<td>30</td>
</tr>
<tr>
<td>Principles and Standards for School Mathematics</td>
<td>31</td>
</tr>
<tr>
<td>Common Core State Standards</td>
<td>34</td>
</tr>
<tr>
<td>Crossroads in Mathematics</td>
<td>37</td>
</tr>
<tr>
<td>Curriculum Foundation Project</td>
<td>40</td>
</tr>
<tr>
<td>Texas standards</td>
<td>45</td>
</tr>
<tr>
<td>Texas Essential Knowledge and Skills</td>
<td>45</td>
</tr>
<tr>
<td>Texas College and Career Readiness Standards</td>
<td>46</td>
</tr>
<tr>
<td>Teacher Effectiveness</td>
<td>50</td>
</tr>
<tr>
<td>Mathematics knowledge for teaching</td>
<td>50</td>
</tr>
<tr>
<td>Content knowledge</td>
<td>51</td>
</tr>
<tr>
<td>Pedagogical knowledge</td>
<td>55</td>
</tr>
<tr>
<td>Pedagogical content knowledge</td>
<td>56</td>
</tr>
<tr>
<td>Disposition</td>
<td>60</td>
</tr>
<tr>
<td>No Child Left Behind Act</td>
<td>62</td>
</tr>
<tr>
<td>Texas certification</td>
<td>63</td>
</tr>
<tr>
<td>National certification</td>
<td>67</td>
</tr>
<tr>
<td>Professional Development and Appraisal System</td>
<td>68</td>
</tr>
<tr>
<td>Lesson plan rubrics</td>
<td>69</td>
</tr>
<tr>
<td>Learning Theories</td>
<td>71</td>
</tr>
<tr>
<td>Constructivism</td>
<td>71</td>
</tr>
<tr>
<td>Behaviorism</td>
<td>74</td>
</tr>
<tr>
<td>Cognitivism</td>
<td>78</td>
</tr>
<tr>
<td>Functions</td>
<td>79</td>
</tr>
<tr>
<td>III. METHODOLOGY</td>
<td>82</td>
</tr>
<tr>
<td>Purpose of the Study</td>
<td>82</td>
</tr>
<tr>
<td>Research Questions</td>
<td>83</td>
</tr>
<tr>
<td>Population and Sample</td>
<td>84</td>
</tr>
<tr>
<td>Demographics of the University</td>
<td>84</td>
</tr>
<tr>
<td>Population</td>
<td>85</td>
</tr>
<tr>
<td>Sample</td>
<td>87</td>
</tr>
<tr>
<td>Research Design</td>
<td>88</td>
</tr>
<tr>
<td>The appropriateness of the modified Mathematical Knowledge for</td>
<td>90</td>
</tr>
<tr>
<td>Teaching measures</td>
<td>90</td>
</tr>
<tr>
<td>The appropriateness of the lesson plan activity</td>
<td>91</td>
</tr>
<tr>
<td>How to assess the lesson plans</td>
<td>92</td>
</tr>
<tr>
<td>Model Developmental Mathematics Program surveys</td>
<td>94</td>
</tr>
<tr>
<td>Appropriateness of case study analysis</td>
<td>94</td>
</tr>
<tr>
<td>Instrumentation Design</td>
<td>95</td>
</tr>
<tr>
<td>Development of Mathematical Knowledge for Teaching assessments</td>
<td>95</td>
</tr>
<tr>
<td>Development of the modified Mathematical Knowledge for</td>
<td>95</td>
</tr>
</tbody>
</table>
Teaching assessment .................................................................................................96
Development of the lesson plan activity ............................................................96
Development of the lesson plan rubric .................................................................96
Development of the Model Developmental Mathematics Program surveys ..........100
Validity ..................................................................................................................101
Validity of the Mathematical Knowledge for Teaching test .............................101
Validity of the lesson plan rubric .........................................................................101
Analysis ..................................................................................................................102
Modified Mathematical Knowledge for Teaching scores ..................................102
Lesson plan scores ...............................................................................................105
Interviews ...............................................................................................................106
Model Developmental Mathematics Program surveys ......................................107
Case studies ..........................................................................................................107
Data Collection Procedure ....................................................................................108
Limitations ............................................................................................................110
Summary ................................................................................................................112
IV. RESULTS .........................................................................................................113

Research Questions and Hypotheses ...................................................................113
Research question 1 ...............................................................................................114
Hypothesis 1 ..........................................................................................................114
Hypothesis 2 ..........................................................................................................114
Hypothesis 3 ..........................................................................................................115
Research question 2 ...............................................................................................115
Research question 3 ...............................................................................................115
Population and Sample .........................................................................................115
Methodology Summary .........................................................................................116
Results ....................................................................................................................117
Results of the modified Mathematical Knowledge for Teaching test .................118
Participants’ content knowledge performance ....................................................118
Participants’ specialized content knowledge performance ................................120
Participants’ common content knowledge performance ...................................121
Participants’ specialized content knowledge and common content knowledge of functions ..............................................................123
Modified Mathematical Knowledge for Teaching performance on individual items .......................................................124
Results of lesson plans .........................................................................................128
Basic Mathematics course lesson plans’ performances .....................................129
Model Developmental Mathematics Program survey analysis ..........................133
Responses from the Pre-Algebra students ...........................................................133
Responses from the Basic Mathematics students ..............................................135
Participants’ lesson plan performances ...............................................................137
Interviews .................................................................................................145
Case study analyses ..................................................................................153
  Case 1: Maria .........................................................................................154
  Case 2: Carlos .........................................................................................157
  Case 3: Isabella .......................................................................................160
  Case 4: Lacy ............................................................................................163

V. DISCUSSION ..............................................................................................166
  Summary of Results ..................................................................................166
  Question Results ........................................................................................168
    Research question 1 ...............................................................................168
    Research question 2 ...............................................................................170
    Research question 3 ...............................................................................174
  Conclusions .................................................................................................176
  Limitations ..................................................................................................177
  Suggestions ..................................................................................................178
  Recommendations for Future Research ....................................................180

APPENDIX A: Sample Demographics ..............................................................182
APPENDIX B: Theoretical Framework ............................................................184
APPENDIX C: Individual Lesson Plan Rubric and Compiled Lesson Plan Rubric ...............................................................................................................................186
APPENDIX D: Item Distribution of the Modified Mathematical Knowledge for Teaching Test ..................................................................................................199
APPENDIX E: Questions Chosen for Modified Mathematical Knowledge for Teaching Test ..................................................................................................200
APPENDIX F: Interview Protocol .................................................................202
APPENDIX G: Demographic Surveys ..............................................................203
  G.1: Demographic Survey with Quiz ..........................................................204
  G.2: Demographic Survey without Quiz ......................................................207

APPENDIX H: Five-Question Survey ..............................................................210
  H.1: 1300 Five-Question Survey ..................................................................211
  H.2: 1311 Five-Question Survey ..................................................................213

APPENDIX I: Alignment Chart .........................................................................215
APPENDIX J: Alignment of Participants’ Lesson Plans to the TEKS .................225
APPENDIX K: Interviews ..................................................................................228
  K.1: Participant 21 ......................................................................................230
  K.2: Participant 22 ......................................................................................242
  K.3: Participant 23 ......................................................................................249
  K.4: Participant 24 ......................................................................................254
LIST OF TABLES

Table                                                                  Page
1. Population of Texas State Students by Class and Major .................. 86
2. Sample of Texas State Student by Class and Major .......................... 88
3. Inter-Rater Reliability Scores ........................................................... 102
4. Distribution of Preservice Teachers Sample ...................................... 104
5. Two-Way Between Groups ANOVA: CK Mean Standardized Scores .......... 120
6. Two-Way Between Groups ANOVA: SCK Mean Standardized Scores .......... 121
7. Two-Way Between Groups ANOVA: CCK Mean Standardized Scores .......... 123
8. Two-Way Between Groups ANOVA: Functions Mean Standardized Scores ... 124
9. Distribution of Correct, Incorrect, and “I’m Not Sure” Responses ....... 125
10. Scores and Totals of MDMP Function Lesson Plans .............................. 129
12. Participants’ Lesson Plan Scores and Totals by Track ......................... 138
13. MDMP Track Participants’ Learning Theory Scores .............................. 143
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Domains of Mathematical Knowledge for Teaching Model</td>
<td>52</td>
</tr>
<tr>
<td>2. Model of Effective and Successful Teachers</td>
<td>62</td>
</tr>
<tr>
<td>3. Illustration of Teacher Preparation Tracks</td>
<td>87</td>
</tr>
<tr>
<td>4. Distribution of Items by Mathematics Domains and Domains of Knowledge</td>
<td>91</td>
</tr>
<tr>
<td>5. Test Information Curve for Modified Mathematical Knowledge for Teaching Test</td>
<td>97</td>
</tr>
<tr>
<td>6. The Lesson Plan Scores for Structure and Teacher Quality were Represented as a Box Plot for MDMP Track Participants and Basic Mathematics Lessons</td>
<td>130</td>
</tr>
<tr>
<td>7. The Lesson Plan Scores for Learning Theories and Standards were Represented as a Box Plot for MDMP Track Participants and Basic Mathematics Lessons</td>
<td>132</td>
</tr>
<tr>
<td>8. College Algebra and MDMP Tracks Participants’ Performances on Lesson Plan Structure</td>
<td>139</td>
</tr>
<tr>
<td>9. College Algebra and MDMP Track Participants’ Teacher Quality Scores</td>
<td>141</td>
</tr>
<tr>
<td>10. College Algebra and MDMP Tracks Participants’ use of Learning Theory Approaches Scores</td>
<td>142</td>
</tr>
<tr>
<td>11. Number of Participants with High Scores in Each Learning Theory Approach</td>
<td>144</td>
</tr>
<tr>
<td>12. College Algebra and MDMP Track Participants’ Scores on Aligning Lesson Plans to Standards</td>
<td>145</td>
</tr>
<tr>
<td>13. Distribution of Responses to “Why do you want to become a teacher?”</td>
<td>147</td>
</tr>
<tr>
<td>14. Participants’ Various Teaching Experiences</td>
<td>148</td>
</tr>
</tbody>
</table>
15. Participants’ Inspiration for Their Lesson Plans........................................................150

16. Participants’ Self-Report of When They Learned Functions....................................151

17. Changes Participants Would Make After Lesson Plan Reflections............................152
### ACRONYMS AND ABBREVIATIONS

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIT</td>
<td>Algorithmic Instructional Technique</td>
</tr>
<tr>
<td>ALA</td>
<td>American Library Association</td>
</tr>
<tr>
<td>ALEX</td>
<td>Alabama Learning Exchange</td>
</tr>
<tr>
<td>AMATYC</td>
<td>American Mathematics Association of Two-Year Colleges</td>
</tr>
<tr>
<td>AMTE</td>
<td>Association of Mathematics Teacher Educators</td>
</tr>
<tr>
<td>ANOVA</td>
<td>Analysis of Variance</td>
</tr>
<tr>
<td>B.S.</td>
<td>Bachelor of Science</td>
</tr>
<tr>
<td>BMLP</td>
<td>Basic Mathematics Lesson Plans</td>
</tr>
<tr>
<td>CCA</td>
<td>Complete College America</td>
</tr>
<tr>
<td>CCK</td>
<td>Common Content Knowledge</td>
</tr>
<tr>
<td>CCSS</td>
<td>Common Core State Standards</td>
</tr>
<tr>
<td>CCSSO</td>
<td>Council of Chief State School Officers</td>
</tr>
<tr>
<td>CDT</td>
<td>Component Display Theory</td>
</tr>
<tr>
<td>CF</td>
<td>Curriculum Foundation</td>
</tr>
<tr>
<td>CK</td>
<td>Content Knowledge</td>
</tr>
<tr>
<td>CLDE</td>
<td>Culturally and Linguistically Diverse Exceptional Students</td>
</tr>
<tr>
<td>CRA</td>
<td>Concrete to Representational to Abstract</td>
</tr>
<tr>
<td>CRAFTY</td>
<td>Curriculum Renewal Across the First Two Years</td>
</tr>
</tbody>
</table>
EC   Early Childhood
EMS  Elementary Mathematics Specialist
ESL  English as a Second Language
GCF  Greatest Common Factor
GPA  Grade Point Average
HCK  Horizontal Content Knowledge
IEA  International Association for the Evaluation of Educational Achievement
IRB  International Review Board
KC   Knowledge of Curriculum
KCS  Knowledge of Content and Students
KCT  Knowledge of Content and Teaching
LMT  Learning Mathematics for Teaching
MAA  Mathematics Association of America
MDMP Model Developmental Mathematics Program
MKT  Mathematical Knowledge for Teaching
MT21 Mathematics Teaching in the 21st Century
NBPTS National Board for Professional Teaching Standards
NCATE National Council for Accreditation of Teacher Education
NCLB  No Child Left Behind
NCTM National Council of Teachers of Mathematics
NGA  National Governors Association
NRC  National Research Council
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PAL</td>
<td>Peer Assisted Leadership</td>
</tr>
<tr>
<td>PCK</td>
<td>Pedagogical Content Knowledge</td>
</tr>
<tr>
<td>PCK</td>
<td>Pedagogical Content Knowledge</td>
</tr>
<tr>
<td>PDAS</td>
<td>Professional Development and Appraisal System</td>
</tr>
<tr>
<td>PK</td>
<td>Pedagogical Knowledge</td>
</tr>
<tr>
<td>PPR</td>
<td>Pedagogy and Professional Responsibilities</td>
</tr>
<tr>
<td>PSSM</td>
<td>Principles and Standards for School Mathematics</td>
</tr>
<tr>
<td>SCK</td>
<td>Specialized Content Knowledge</td>
</tr>
<tr>
<td>SPSS</td>
<td>Statistical Packages for the Social Sciences</td>
</tr>
<tr>
<td>S-R</td>
<td>Stimulus and Response</td>
</tr>
<tr>
<td>TEA</td>
<td>Texas Education Agency</td>
</tr>
<tr>
<td>TEKS</td>
<td>Texas Essential Knowledge and Skills</td>
</tr>
<tr>
<td>TExES</td>
<td>Texas Examination of Education Standards</td>
</tr>
<tr>
<td>THECB</td>
<td>Texas Higher Education Coordinating Board</td>
</tr>
<tr>
<td>TIMSS</td>
<td>Trends in International Mathematics and Science Study</td>
</tr>
<tr>
<td>TX CCRS</td>
<td>Texas College and Career Readiness Standards</td>
</tr>
<tr>
<td>U.S.</td>
<td>United States</td>
</tr>
<tr>
<td>ZPD</td>
<td>Zone of Proximal Development</td>
</tr>
</tbody>
</table>
ABSTRACT

THE EFFECTS OF A MODEL DEVELOPMENTAL MATHEMATICS PROGRAM
ON ELEMENTARY AND MIDDLE SCHOOL
PRESERVICE TEACHERS

by

Lindsey N. Gerber, M.S.

Texas State University-San Marcos
December 2012

SUPERVISING PROFESSOR: DR. SELINA VÁSQUEZ MIRELES

Teacher quality is instrumental in improving student performance. Unfortunately, discrepancies between teacher preparation programs and national and state K–12 student standards have contributed to the difficult task of producing quality teachers. The contemporary mathematics education paradigm used at most colleges and universities relies on instructors transmitting mathematical content knowledge to students through didactic discourse; whereas, research suggests that preservice teachers need to be taught
using the types of instruction emphasized in grades K–12 state and national standards—
standards-based best practices. The developmental mathematics program at Texas State
University-San Marcos (Texas State) implements these standards-based best practices via
the Algorithmic Instructional Technique (AIT) and the Concrete to Representational to
Abstract (CRA) models, thereby exposing preservice teachers to pedagogy in conjunction
with mathematics content. To determine if the Model Developmental Mathematics
Program (MDMP) course(s) contribute to preservice teachers’ mathematical knowledge
for teaching and positive disposition, this study compared elementary and middle school
preservice teachers on the MDMP track to those on the College Algebra track. These
groups’ mathematical knowledge and disposition were compared based on their scores on
the Early Indicators of Effective Teachers instrument—a modified Mathematical
Knowledge for Teaching (MKT) assessment and a lesson plan writing activity. In
addition, the researcher conducted interviews with the participants to evaluate their
pedagogical, content, and pedagogical content knowledge and disposition. For further
investigation, case study analyses were performed on four selected MDMP track
participants. Overall, the results indicated that the MDMP track participants lacked
confidence, had a poor disposition towards mathematics, and had less content knowledge
than the College Algebra track participants did; interestingly though, the MDMP track
participants aligned their lesson plans, whether deliberately or instinctively, to the content
taught in the MDMP courses. This is significant because even though MDMP track
participants scored lower than the College Algebra students on the modified MKT test ($p
< .01$), due to their low confidence levels, their experiences in the MDMP course(s)
influenced their pedagogical and pedagogical content knowledge. Due to the limited
scope of this study, further research needs to be conducted to better understand the effects of the MDMP curriculum and content. Preservice teachers in developmental mathematics programs at different colleges and universities can be evaluated using the *Early Indicators of Effective Teachers* instrument and then their scores can be compared to the MDMP track participants’ scores.
CHAPTER I

INTRODUCTION

Teacher quality, measured by teachers' experience, completed preparation programs, acquired degrees, certification status, completed coursework, and pedagogical content test scores, is instrumental in improving student performance (Rice, 2003). Prior to graduating with the credentials needed to enter the teaching profession, students are classified as preservice teachers—someone “preparing to enter the teaching profession but not yet [a] classroom teacher” (Rice, 2003, p. 4). Traditional teacher preparation programs instruct preservice teachers in content knowledge, pedagogical knowledge, pedagogical content knowledge, and disposition at different stages of their education (Swar, Hart, S. Z. Smith, Smith, & Tolar, 2007). Within the first two years of education, preservice teachers at most colleges and universities learn mathematical content knowledge while learning pedagogical and pedagogical content knowledge later. In contrast, the developmental mathematics program at Texas State University-San Marcos (Texas State) exposes preservice teachers to standards-based best practices early in their education, thus teaching them content, pedagogical, and pedagogical content knowledge and positive disposition towards mathematics earlier than other teacher preparation programs (Mireles, 2010). By establishing content, pedagogical, and pedagogical content knowledge and disposition straightaway, preservice teachers develop a foundation for becoming quality teachers before enrolling in upper level courses.
The developmental mathematics program at Texas State consists of two content courses: Pre-College Algebra (Math 1300) and Basic Mathematics (Math 1311). The skills learned in Basic Mathematics build on the concepts taught in Pre-College Algebra. Enrollment in one or both of these courses is compulsory for Texas State students who do not meet the requirements to register for credit-bearing mathematics courses—including College Algebra. To help students prepare for credit-bearing mathematics courses, the developmental mathematics program provides students with mathematical content knowledge as well as additional skills needed to be successful in college (Conley, 2007).

To ensure students’ success in credit-bearing mathematics courses, the developmental mathematics curricula at Texas State adheres to state and national standards and implements standards-based best practices—the use of manipulatives, technology, cooperative learning groups, culturally relevant connections, and real-world experiences. The components of the developmental mathematics program yield a model program (Mireles, 2010); therefore, in this study, the developmental mathematics program at Texas State is referred to as the Model Developmental Mathematics Program (MDMP).

**Statement of the Problem**

College students who enroll in the Model Developmental Mathematics Program (MDMP) are weak in mathematical content knowledge, as determined by state and/or national assessments. Meaning, the MDMP students’ knowledge of mathematical content is not strong enough for them to be successful in credit-bearing mathematics courses; the MDMP’s responsibility is to prepare students to be successful in credit-bearing mathematics courses by bridging the gap between secondary and post-secondary
According to international testing agencies, the United States (U.S.) is ranked behind other developed countries in mathematics, which has become a growing concern for current education researchers (International Association for the Evaluation of Educational Achievement [IEA], 2007; “In ranking,” 2010). The IEA reported the 2007 results for the Trends in International Mathematics and Science Study (TIMSS) test—a globally distributed assessment that measures fourth and eighth grade students’ mathematical and science knowledge. The TIMSS test ranked American fourth grade students’ mathematics scores tenth behind those in Hong Kong, Singapore, Chinese Taipei, Japan, Kazakhstan, Russian Federation, England, Latvia, Netherlands, and Lithuania. In the same year, American eighth graders ranked ninth behind students in Chinese Taipei, Korea, Singapore, Hong Kong, Japan, Hungary, England, and Russian Federation (IEA, 2007). To improve the ranking of American students internationally, the U.S. government has mandated teacher education and curriculum reform (“In ranking,” 2010).

To identify teacher education and curriculum areas that need improvement, American national and state organizations and educational researchers have compared international teachers, preservice teachers, and teacher preparation programs. One such study, Mathematics Teaching in the 21st Century (MT21), compared international teacher preparation programs. The MT21 was a small-scale international comparison study that surveyed middle school/lower-secondary grades preservice teachers using three questionnaires administered during their last year of school (Schmidt et al., 2007). The American preservice teachers’ survey responses were compared to those of preservice
teachers in five other developed countries (Schmidt et al., 2007). Based on the participants’ responses, researchers determined that American middle school teacher preparation curricula are deficient in content and pedagogy, thereby producing preservice teachers who have less mathematical content and pedagogical knowledge than their international counterparts (IEA, 2007; Schmidt et al., 2007).

While researchers have evaluated middle school preservice teachers, no one has yet reported on elementary preservice teachers. On the other hand, national committees such as the Association of Mathematics Teacher Education (AMTE) have indicated that American elementary school in-service teachers do not meet the qualifications to teach mathematics. This response is due to the preparation programs established for elementary school teachers. Most elementary school teachers are generalist, meaning “they study and teach all core subjects, rarely developing in-depth knowledge and expertise with regard to teaching elementary mathematics” (AMTE, 2010, p. 1). These elementary school teachers lack content and pedagogical knowledge needed to teach mathematics as mandated by state and national standards.

One way to resolve the discrepancies between teacher preparation programs and national and state K–12 student standards is to modify the instructional methods used in content courses (Conley, Hiatt, McGaughy, Seburn, & Venezla, 2010). Instructional methods used at most colleges and universities are based on the Scholar Academic ideology—instructors transmit knowledge of mathematics to students through didactic discourse (Schiro, 2008). In contrast, research suggests that quality teachers employ a combination of instructional techniques and utilize approaches from the three main philosophical learning frameworks: constructivist, behaviorist, and cognitive learning
theories (Schiro, 2008; Swars, S. Z. Smith, Smith, & Hart, 2008). While these three learning theories have been incorporated in state and national student standards for grades K–12, most colleges and universities do not integrate all three in mathematics curricula. Incorporating the instructional techniques describe by these learning theories demonstrates to preservice teachers the type of pedagogy that should be used in their classroom.

Teacher preparation programs, according to Shulman (1986), should teach pedagogy and content as joint concepts; learning content and pedagogy separately may inhibit pedagogical content mastery. Ball, Thames, and Phelps (2008) agreed with Shulman (1986), who advised that pedagogical content knowledge is essential for bridging the gap between learning mathematics content and the practice of teaching mathematics, meaning pedagogy and content should be integrated into content courses. Whereas, government and education foundations have suggested preservice mathematics teachers complete undergraduate content courses with high grade point averages (GPAs)—measurement of students’ academic ability—prior to being taught pedagogy (Rice, 2003). Based on Schmidt et al. (2007), Shulman (1986), and Ball, Thames, and Phelps’s (2008) reports, to aid in the development of quality teachers and bridge the gap between college curricula and K–12 state and national standards, preservice teachers’ knowledge of general pedagogy and mathematics pedagogy are just as important as mathematics content knowledge.

Besides the types of instruction used at the collegiate level, the number of hours needed to complete a degree and the type of upper-level courses required by teacher preparation programs is also a concern (National Research Council [NRC], 2001).
According to Schmidt et al. (2007), American teachers are not required to take analysis courses that focus on deconstructing the basic mathematical processes taught in lower grade levels, whereas future teachers in Taiwan and Korea receive about “eighty percent or more of advanced mathematics topics typically covered in undergraduate mathematics programs” (p. 6). On the other hand, research studies report that preservice teachers who complete an increased number of advanced mathematical content courses do not necessarily improve students’ performances at the primary grade level (Ball, 1990; Ball, 1991; NRC, 2001). Nonetheless, U.S. government agencies and education foundations still advocate for an increased number of upper-level mathematics courses for grades K–6 preservice teachers (Rice, 2003; Schmidt et al., 2007).

In conclusion, teacher preparation programs that increase the number of content courses required by preservice teachers delay preservice teachers’ access to pedagogy, and programs that do not employ standards-based best teaching techniques impede the development of quality teachers (NRC, 2001). Bright (1999) insisted that K–12 preservice teachers need to be taught using the types of instruction emphasized in state and national standards; unfortunately, preservice teachers still do not experience the different standards-based best practices in college mathematics courses (Graeber, 1999; Matthews, Rech, & Grandgennett, 2010). Instead of postponing preservice teachers’ access to pedagogical knowledge, content knowledge, pedagogical content knowledge, and disposition, the Model Developmental Mathematics Program curriculum integrates content and pedagogy using standards-based best practices.

**Purpose**

The purpose of this study is to determine if the Model Developmental
Mathematics Program (MDMP) curriculum—teaching techniques and content—contributes to the development of quality elementary and middle school preservice teachers’ pedagogical knowledge, content knowledge, and pedagogical content knowledge and disposition. Preservice teachers who possess pedagogical, content, and pedagogical content knowledge and positive disposition have the foundation to become quality teachers (National Research Council, 2001; Rice, 2003; Shulman, 1986; Styliandes & Styliandes, 2009). Quality teachers gain knowledge and disposition through training and experiences with instruction that employs standards-based best practices, but conventional teacher preparation programs in the United States (U.S.) do not incorporate these types of standards-based best practices in mathematics content courses unlike the MDMP. Thus, mathematics content courses fail to support teacher preparation programs in the development of quality preservice teachers (Harris & Sass, 2007; Schmidt et al., 2007).

The MDMP instructor preparation and course curriculum and content, differ from traditional college and university developmental mathematics programs. The mathematics instructors in the MDMP are continually trained to ensure uniformity across all the developmental mathematics classrooms, and the curriculum is taught using standards-based best practices (Mireles, 2010). These attributes of the MDMP help students’ become college ready and increase preservice teachers’ access to pedagogy and pedagogical content knowledge while teaching a broad spectrum of mathematics content—probability, statistics, geometry, measurement, algebra and functions (Mireles, 2010).

The mathematical content area functions is emphasized in this study because it is
“one of the most pervasive and important topics” that is taught at all grade levels (Willoughby, 1990, p. 77). Functions are a main theme in college algebra, and the MDMP curriculum was designed to develop students’ understanding of functions. The scope and sequence of the MDMP curriculum content is numerical reasoning→ algebraic expressions→ algebraic equations→ functions. Basic functions are taught in the MDMP to foreshadow concepts taught in credit-bearing mathematics courses. Coupled with teacher preparation courses and mathematical credit-bearing courses, the MDMP curriculum increases the likelihood of preservice teachers learning content, pedagogical, and pedagogical content knowledge and developing positive disposition, thereby becoming quality teachers (Mireles, 2010; Shulman, 1986).

**Significance of the Study**

This research study is significant because it expands mathematics education scholarship by providing insight into the benefits of teaching preservice teachers mathematics using standards-based best practices in content courses. While current scholarship emphasizes the advantages of using standards-based best practices in methods courses—not content courses—the Model Developmental Mathematics Program (MDMP) highlights the use of standards-based pedagogical strategies in content courses, which may contribute to the development of high quality teachers (Mireles, 2010; Quinn, 1997; Rice, 2003). Preservice teachers enrolled in the MDMP are given the opportunity to develop a foundation for understanding K–12 mathematics content and to experience different types of instructional techniques addressed in state and national standards. If this study finds that integrating content and pedagogy in developmental mathematics courses contributes to teacher quality, then this strategy could be extended to credit-
bearing mathematics courses and other cross-disciplinary content areas including science, reading, writing, and history.

Ball, Thames, and Phelps (2008) described teachers’ pedagogical content knowledge (PCK) as the combination of pedagogical and content knowledge. For this study, participants were asked to complete a lesson plan activity that was evaluated by a lesson plan rubric to assess their PCK. To identify PCK—an indicator of quality teachers—a rubric was developed to assess preservice teachers’ knowledge and disposition as evidenced in their lesson plans. The lesson plan rubric, a viable product of this study, can be used to improve mathematics courses and interdisciplinary curriculum by guiding the development of effective lesson plans, and preservice teachers can use it as a framework for developing quality lesson plans once in their profession. Lesson plans were chosen because they are essential tools; they reflect the quality of teachers’ planning, which in turn affect the quality of their teaching (Stronge, 2007). This lesson plan rubric offers a systematic way of evaluating preservice teachers without watching them instruct, which differs from most standard lesson plan rubrics that evaluate instructors’ teaching techniques through observation.

The lesson plan rubric assessed each participant’s content, pedagogical, and pedagogical content knowledge, disposition, instructional techniques, and appropriate application of state and national standards. The rubric can be used across disciplines at a multitude of grade levels to provide a framework for developing curriculum that integrates pedagogy in content courses, and it can be used to evaluate preservice teachers’ adherence to P–16 standards, learning theories, and previous research.
Research Questions

This study examines elementary and middle school preservice teachers’ knowledge and disposition upon completion of the Model Developmental Mathematics Program (MDMP) by comparing their results on a multiple-choice content test and a written lesson plan activity. To gather additional data, each participant was interviewed and students enrolled in the MDMP during the Spring 2012 semester were surveyed about the MDMP curriculum, specific lesson plans, and activities used in the classroom. The data collected from participants was used to address three research questions.

To answer the first research question, an exam was compiled of items chosen from Mathematical Knowledge for Teaching (MKT) tests to assess participants’ subject matter knowledge. To address the second research question, a lesson plan writing activity on functions and an interview were used to assess participants’ pedagogical and pedagogical content knowledge and disposition. The lesson plan activity was scored using a rubric developed for this study. After the completion of these assessments, each participant was interviewed to discover contextual information about each participant. These research questions are:

RQ 1: Is there a significant difference in Mathematical Knowledge for Teaching scores for elementary school preservice teachers and middle school preservice teachers who completed the Model Developmental Mathematics Program at Texas State University-San Marcos compared to those who started in College Algebra—controlling for mathematics access at community college?

RQ 2: Using the lesson planrubric and interview analyses, what early indicators of effective teachers are identified in Texas State University-San Marcos?
preservice teachers who completed the Model Developmental Mathematics Program compared to those who started in college algebra?

RQ 3: How do impressions from the Model Developmental Mathematics Program affect the Model Developmental Mathematics Program track participants’ experiences, and how do these experiences inform their performances on the measures of the *Early Indicators of Effective Teachers*?

These three questions address policy makers’ concerns regarding the disparity between the “nation’s educational aspiration and student achievement” (NRC, 2001, p. 1). While the MDMP was not designed specifically for preservice teachers, the use of standards-based best practices can lessen the gap between national expectations and students’ performances. It is important to address these issues because teacher preparation programs are not consistently graduating quality teachers that can close the separation between the national expectations and students’ performances (Harris & Sass, 2007).

In summary, content courses taught with standards-based best practices support preservice teachers’ acquisition of effective tools and teaching strategies (Mathematics Association of America [MAA], 2011). It has also been found that a combination of mathematical content and pedagogical strategies in methods courses contribute to preservice teachers learning mathematical content (Quinn, 1997). Developmental mathematics courses at other college and universities are usually taught with lecture-based instruction, but the developmental mathematics courses at Texas State use alternative teaching techniques to teach the content. The MDMP teachers present preservice teachers with instruction using standards-based best practices and a wide array
of mathematical content areas. The MDMP curriculum is also rich with terminology, standards, learning theories, and content specific background that are explained in the literature review.
CHAPTER II

LITERATURE REVIEW

The purpose of this study is to determine if the Model Developmental Mathematics Program (MDMP) curriculum—teaching techniques and content—contributes to the development of elementary and middle school preservice teachers’ pedagogical, content, and pedagogical content knowledge and disposition. The MDMP program impresses upon students state and national standards, state and national policies, teacher effectiveness in the classroom, learning theories associated with classroom instruction, and the role of functions in the K–12 classrooms. These components were mapped to effective strategies that inform the development of quality teachers, as discussed in the literature.

Definitions

The vocabulary used in education has evolved over many years and varies across disciplines. For example, the term developmental education is used in contemporary literature to describe developmental education as remedial education, but prior terms used to describe developmental education include “academic preparatory programs,” “compensatory education,” and “learning assistance” (Arendale, 2005). To clarify the terminology used in this study, this section defines quality teacher, college readiness, and developmental mathematics as they are used.
**Preservice teacher.** Preservice teachers are students enrolled in a college or university teacher preparation program prior to being certified to teach in the areas of elementary, middle, and high school (Rice, 2003). Depending on their specific degree plan, Texas State University-San Marcos preservice teachers for grades K through 12 are required to take at least credit-bearing College Algebra or a quantitative mathematics course equivalent to College Algebra (Texas Higher Education Coordinating Board [THECB] & Texas Education Agency [TEA], 2009).

**Quality teacher.** Teacher quality is measured by teachers’ actions, knowledge, and creativity in the classroom; high quality teachers promote high “student achievement and success in school” (Blanton, Sindelar, and Correa, 2006, p. 115). Blanton, Sindelar, and Correa (2006) have delineated two characteristics of quality teachers: (1) they meet the criteria for state certification and they teach appropriate content using methods denoted in state and national standards, and (2) their actions positively influence student learning (Blanton et al., 2006).

Similarly, the No Child Left Behind Act (NCLB, 2008), established under the direction of President George W. Bush, described highly qualified teachers as having at least a bachelor’s degree, state teaching certification, and the ability to demonstrate competency in the subject matter they teach. The NCLB Act and Blanton, Sindelar, and Correa’s (2006) descriptions of quality teachers have similar attributes.

The developers of the NCLB Act (2008) and Blanton, Sindelar, and Correa (2006) determined that certification, the ability to teach using appropriate methods, and the capacity to competently demonstrate content knowledge are distinguishable requirements of quality teachers. Furthermore, Blanton, Sindelar, and Correa (2006) have classified
teacher effectiveness in the classroom as a defining element of quality teachers. Combined, the NCLB Act (2008) and Blanton, Sindelar, and Correa’s (2006) definitions asserted that pedagogical and pedagogical content knowledge have a positive influence on teacher effectiveness, which in turn positively affects student performance (Rice, 2003). For the purpose of this study, the definition of a quality teacher is a teacher who has mastered content, pedagogical, and pedagogical content knowledge and can effectively use these types of knowledge in the classroom.

**College readiness.** Conley (2007) claimed that college-ready students are knowledgeable in mathematical content, cognitive strategies, academic behavior, and contextual skills and awareness; these skills allow students to think and perform at a collegiate level and integrate into college life (p. 13). According to Conley (2007), students who are college-ready can succeed in credit-bearing college mathematics courses.

Many colleges and universities, including Texas State University—San Marcos (Texas State), classify students’ college-readiness as meeting the minimum requirement for enrollment in credit-bearing mathematics courses based on standardized state and national placement test benchmarks like the SAT®, ACT®, and THEA® (Atkinson & Geiser, 2009; Texas State, 2010). Students who have not attained the required minimum scores on the mathematics sections of placement exams are required to enroll in developmental mathematics, and they must successfully complete the course(s) to be college-ready. Researchers have claimed that standardized exams—SAT®, ACT®, THEA®, and other state or national exams—are the most accurate indicators of students’ mathematical proficiency and college-readiness (Atkinson & Geiser, 2009; Barlow,
On the other hand, Conley (2007) consulted students’ in-class and homework assessments, end-of-course exam scores, self-reflection surveys, and questionnaires on college system and culture to assess whether they were college-ready. These benchmarks measured students’ cognitive strategies, academic behavior, and contextual skills and awareness. However, this study uses students’ performances on formal assessments and compulsory state and national exams to determine whether they are required to enroll in developmental mathematics courses (Texas State, 2010).

Developmental mathematics. Johnson (2005) described developmental mathematics students as those who “lack adequate preparation” and need additional course(s) to develop the skills and knowledge necessary to be successful in credit-bearing mathematics courses (p. 40). At Texas State University-San Marcos, the additional courses—developmental mathematics—are those required by students, including preservice teachers, who have not achieved the required minimum scores on standardized tests to qualify for enrollment in credit-bearing mathematics (Texas State, 2010).

Model Developmental Mathematics Program

The Model Developmental Mathematics Program (MDMP) at Texas State University-San Marcos (Texas State) is comprised of two courses—Basic Mathematics (Math 1300) and Pre-College Algebra (Math 1311; Texas State, 2010). Students who do not attain minimum scores on the mathematics section(s) of college entrance exams—as defined by Texas State policy—are required to enroll in either Basic Mathematics or Pre-College Algebra based on their scores (Texas State, 2010). Pre-College Algebra is the prerequisite to Basic Mathematics; its scope consists of operations with different sets of
real numbers and introductory algebraic expressions, geometry, probability, and statistics (Mireles, 2010; Texas State, 2010). Basic Mathematics content areas include simplifying and evaluating expressions, solving equations, functions, probability, and statistics—prerequisite concepts for College Algebra (Mireles, 2010; Texas State, 2010). The two developmental mathematics courses are vertically aligned to help students transition into credit-bearing mathematics courses.

The MDMP curriculum conforms to state and national standards, incorporates various mathematical topics, and emphasizes multiple teaching models (Mireles, 2010). The program is further characterized by ongoing pedagogical training for all developmental mathematics instructors and a standardized scope and sequence of lessons across the MDMP courses (Mireles, 2010). The MDMP has standardized curriculum alignment coupled with ongoing teacher preparation that positively influences students’ success in credit-bearing mathematics (Mireles, 2010). The goal of the Texas State mathematics department is to ensure that students who complete the MDMP are equipped with the knowledge and skills necessary to master credit-bearing mathematics (Mireles, 2010).

Curriculum alignment. The Model Developmental Mathematics Program (MDMP) curriculum at Texas State University-San Marcos (Texas State) is aligned to the national standards, *Crossroads in Mathematics: Standards for Introductory College Mathematics before Calculus*, and the state standards—Texas College and Career Readiness Standards (TX CCRS; Mireles, 2010). The American Mathematics Association of Two-Year Colleges (AMATYC) developed *Crossroads in Mathematics*—the first document to detail national standards for teaching developmental mathematics.
The TX CCRS was developed to address standards and processes that lessen the gap between Texas high school and college curricula. Both sets of standards outlined the content and process standards needed by students to be successful in postsecondary mathematics. Besides listing content and pedagogy standards, *Crossroads in Mathematics* is the only document that has listed specific pedagogical standards that inform instructors of constructivist approaches that could be used in the classroom. The MDMP curriculum is aligned to the TX CCRS and *Crossroads in Mathematics*, thereby infusing standards-based best practices into the curriculum (Mireles, 2010).

**Process standards.** The American Mathematics Association of Two-Year Colleges (AMATYC), the Texas Higher Education Coordinating Board (THECB), and the Texas Education Agency (TEA) created process standards that mandate curriculum designed to promote students’ skills through the use of various instructional tools (Baroody and Coslick, 1998; Cohen, 1995). These standards included “problem solving, modeling, reasoning, connecting with other disciplines, communicating, using technology, and developing mathematical power”—students’ confidence in mathematics (Cohen, 1995, p. x). Preservice teachers can learn skills such as problem solving, reasoning, and communication in classes taught using standards-based best practices. Standards-based best practices include instructional models, teaching techniques, technology, and manipulatives (Mireles, 2010). Technology and manipulatives are essential tools for representing various mathematical operations and concepts as well as encouraging students to learn problem-solving skills. Other instructional tools and models that promote problem solving are cooperative learning groups and organizational frameworks, both of which are used in the Model Developmental Mathematics Programs.
curricula (Cohen, 1995; Mireles, 2010; THECB & TEA, 2009).

*Algorithmic Instructional Technique model.* Algorithms are step-by-step procedures used to solve mathematical problems. The Algorithmic Instructional Technique (AIT) model is a four-phase model that guides students through the processes of algorithmic writing—Modeling→Practicing→Transitioning→Independence—by progressing from a teacher-centered approach to a student-centered approach (Vásquez, 2003). At the beginning of the Pre-College Algebra and Basic Mathematics courses, the AIT model is introduced by initiating the first stage: modeling.

The four-phase model customarily begins with the instructor demonstrating mathematical examples and modeling the process of solving problems with multiple algorithms. However, it is not necessary for students to begin at the modeling stage if they are already capable of writing algorithms. Students who are required to enroll in Pre-College Algebra have algorithmic writing experience prior to enrollment in the Basic Mathematics course, meaning that they can begin the process at the practicing, transitioning, or independence stages of the AIT model (Vásquez, 2003).

In the practicing stage, students repeat the process of developing their own algorithms. The teacher assists students as they learn to write algorithms to solve mathematical problems, guiding students as they progress to the transition stage (Vásquez, 2003).

During the transition stage, students become the writers and the instructor becomes the facilitator. The students build on skills developed during the practicing stage by creating algorithms in peer groups, then presenting them to the class. Through participation in these peer led groups, students become more confident and independent
in their algorithmic writing (Vásquez, 2003).

The students become progressively autonomous through the stages of practicing, transitioning, and finally independence. Once students reach the independence stage, teachers exert minimal influence over their algorithmic writing. Upon conclusion of the MDMP, students are able to develop their own algorithms to solve complex mathematical problems (Vásquez, 2003).

In the MDMP Basic Mathematics course, the lesson plan *Linear Equations in Two Variables* introduces students to algorithmic writing. The MDMP instructors teach students the mathematical concept of determining if a point is a solution to a linear equation in two variables. First, the MDMP instructor demonstrates how to evaluate a linear equation in two variables by substituting the $x$- and $y$-coordinates for the corresponding variables in a sample problem. Next, the students work independently to evaluate a similar problem and develop their own algorithm for determining if a point is a solution of the given two-variable linear equation. The instructor guides students with the development of their algorithms. At the end of the Pre-College Algebra and the Basic Mathematics courses, the MDMP lesson plans emphasize the last two stages of the AIT model—transitioning and independence. Students’ in the independence stage develop mathematical reasoning and make connections while they develop problem-solving, modeling, and reasoning skills before transitioning to the independence stage (Mireles, 2010).

Upon conclusion of the MDMP, students are able to develop their own algorithms to solve complex mathematical problems (Vásquez, 2003). For preservice teachers using the AIT method, the experience aids in their development of pedagogical and
pedagogical content knowledge by observing peers write algorithms. Algorithmic writing displays step-by-step instructions the students used to solve their problems. This gives insight to students’ thought processes and problem-solving skills.

*Concrete to Representational to Abstract model.* The Concrete to Representational to Abstract (CRA) model is a three-stage teaching process; students learn as they progress through the stages of (1) manipulating concrete objects, (2) modeling through pictorial representation, and (3) solving problems abstractly (Witzel, 2005). Each stage of the CRA model expands on the knowledge learned in the previous stage(s) by transitioning from concrete to abstract representations of mathematical processes. For example, students use algebra tiles to learn how to count; then, they move to the representational stage of the CRA model by drawing pictures of squares to represent addition, and finally, in the abstract stage, they use symbolic representations to depict mathematical concepts (Witzel, Riccomini, & Schneider, 2008). Students who struggle with mathematics benefit from lessons that incorporate the CRA model because the beginning stage represents complex mathematical ideas with concrete tools, thereby minimizing the learning curve (Witzel et al., 2008). The CRA model is a valuable tool in the developmental mathematics program aiding struggling students with mathematical concepts (Mireles, 2010).

The first phase of the CRA model is the concrete stage—problem solving performed by means of “visual, tactile, and kinesthetic experiences” (The Access Center, 2004, p. 1-2). At this stage, students use tangible objects to understand mathematical concepts; for example, using algebra tiles to introduce multiplying polynomial expressions in the Model Developmental Mathematics Program (MDMP) lesson
Operations on Polynomial Expressions. In this lesson, students initially demonstrate how to represent three times seven—three rows of seven blocks—with unit algebra tiles. Next, students use algebra tiles to multiply linear binomials; the students are restricted to these types of polynomials because algebra tiles can only represent polynomials up to the quadratic term. Therefore, pictorial representations are used to represent polynomials with degrees larger than two.

During the representational stage—the second stage of the CRA model—students transition from concrete to pictorial representations. In the Operations on Polynomial Expressions lesson, the MDMP students utilize algebra tiles to represent multiplication of polynomials, and then the product of the polynomials is modeled with a pictorial representation known as the area model. Students can use the area model to pictorially represent polynomials with degrees higher than two; the pictorial representations instigate students’ abstract thinking by helping them multiply polynomials that cannot be represented with concrete models (The Access Center, 2004; Witzel et al., 2008).

Students who use concrete and representational methods build a foundation for understanding the abstract method, the third stage of the CRA model. Mathematics topics are abstract, so it is imperative to provide students with the skills needed to make “meaningful connections” in mathematics (Witzel et al., 2008, p. 271). An example of an abstract mathematical concept is the distributive property, which can be taught concretely using algebra tiles and representationally using the area model. The CRA model helps MDMP students advance their ability to simplify polynomials by developing their problem-solving and reasoning skills (Witzel et al., 2008).

Cooperative learning groups. Cooperative learning groups consist of a mixture of
students with different mathematical “performance level[s], gender[s], and ethnicit[ies]” (Moore, 2009, p.203), each accountable for his or her own education. These diverse learning groups give students the chance to communicate with peers to make and test their own unique conjectures (Cohen, 1995; THECB & TEA, 2009). An example cooperative learning group activity is the jigsaw, which is used in the Model Developmental Mathematics Program (MDMP) lesson plan *Solving Rational Equations*. The jigsaw is comprised of two stages. In the first stage, cooperative learning groups of approximately 4 to 5 students learn one mathematics concept by discussing and proving conjectures with their peers; in the second stage, each person from every group forms a new cooperative learning group with peers who were assigned a different mathematical concept or procedure (Kagan, 1994). In the second cooperative learning group, no student should have the same mathematical concept as another group member. Collaborating in cooperative learning groups promotes learning, retention, oral communication, social skills, and self-esteem (Kagan, 1994).

The *Solving Rational Equations* lesson plan requires students to divide into three cooperative learning groups where each group is assigned a method for solving: proportions, least common denominator, or least common multiple. Then, each group is given a worksheet with four rational equations that are solved using the assigned method. The students deconstruct these solved problems to conjecture about the solving process and to develop an algorithm for solving rational equations. After this process is completed, each group splits into new groups such that everyone in the new groups has different methods for solving rational equations. Each person in the group presents their method of solving to their peers. Cooperative learning groups, like the jigsaw, provide
students with an environment that fosters problem solving, reasoning, and communication, all essential skills that “promote student learning and academic achievement” (Kagan, 1994, Why use cooperative learning? section).

Organizational frameworks. Flow charts, tables, and other pictorial representations are tools used for arranging written and oral mathematics information; these organizational frameworks are easy-to-read displays of complex information that help students understand mathematics concepts and problem solving processes (Moore, 2009). The Model Developmental Mathematics Program (MDMP) lessons emphasize the use of flow charts to help organize problem-solving processes for multi-step problems such as factoring four-term polynomials (Tarquin & Walker, 1997). Tables can be used to organize coordinate pairs for graphing or for analyzing problem-solving processes between mathematical concepts such as permutations and combinations (Tarquin & Walker, 1997). Pictorial representations are also considered organizational frameworks because students can use illustrations to clarify the various components of application problems, for example, identifying the variables for mixture or distance problems (Witzel et al., 2008). Organizational frameworks aid students in arranging complex mathematical information (Tarquin & Walker, 1997).

Introduction to Factoring and ac-Method are two MDMP lesson plans that engender students’ organization of the factoring process using a flowchart. The structure of the flowchart assists students with the order in which to factor two, three, and four-term polynomials. Introduction to Factoring presents the first two factoring processes—greatest common factor (GCF) and factor by grouping. The ac-Method lesson addresses the remaining factoring processes taught in the MDMP program—difference of squares,
sum and difference of cubes, and the ac-method. Organizational frameworks, such as the flowchart, are incorporated in the MDMP curriculum to help students make concrete connections, develop reasoning, and organize ideas for solving and simplifying complex problems (Mireles, 2010; Tarquin & Walker, 1997).

**Content standards.** The mathematical content in the Model Developmental Mathematics Program (MDMP)—algebra, numerical reasoning, geometry, measurement, probability, statistics, and functions—conforms to state and national content standards (Mireles, 2010). The content strands addressed in the Texas College and Career Readiness Standards (TX CCRS) are not taught discretely in the MDMP, but instead the strands are integrated to connect algebraic reasoning to numerical, geometric, measurement, functional, probabilistic, and statistical reasoning (Mireles, 2010).

The MDMP lesson *Parallel Lines* focuses on solving algebraic equations to determine the measurement of angles by exemplifying connecting algebraic reasoning with geometry. Students use the properties of parallel lines cut by a transversal to set up and solve algebraic equations to determine the degree of the missing angles. First, students develop an algebraic equation based on geometric properties, then they solve for the variable algebraically, and finally they substitute the missing value to determine the degree of the angle. Combining multiple mathematical content areas, such as algebra and geometry, allows students to make connections and develop algebraic proficiency (THECB & TEA, 2009).

**Instructor preparation.** The developmental mathematics courses in the Model Developmental Mathematics Program (MDMP) are taught by adjunct faculty and graduate and doctoral teaching assistants—some having little to no teaching experience
(Mireles, 2010). Preparing the MDMP instructors to teach effectively is important in order to ensure students’ success and positive mathematics disposition (Mireles, 2010). The MDMP necessitates ongoing teacher preparation for developmental mathematics instructors; they are required to continue training throughout each semester to prepare them to teach using non-traditional methods and to maintain program continuity (Mireles, 2010). Training consists of a three-day workshop, weekly meetings throughout the semester, and faculty support; this training provides MDMP instructors with the mathematical content, the process standards, and the disposition needed to teach the MDMP lesson effectively.

**Three-day workshop.** All adjunct faculty and graduate and doctoral teaching assistants instructing at least one developmental mathematics course are required to attend a three-day workshop before the Spring and Fall semesters. During the workshop, the Model Developmental Mathematics Program (MDMP) director discusses program objectives, instructional materials used for teaching, and program protocols and documents used throughout the semester. The workshop attendees also demonstrate lesson plan activities by presenting an updated MDMP lesson plan to their peers (Mireles, 2010). The lesson objectives include integrating probability, statistics, measurement, geometry, and numerical reasoning into the algebraic framework of the curriculum. The program objectives include the infusion of cultural relevance, technology, manipulatives, and real-world applications throughout the curriculum to help the MDMP students’ develop a positive disposition towards mathematics (Mireles, 2010). The instructional materials introduced to the teachers in the three-day workshop consist of lesson plans, textbooks, and accoutrements needed for the activities—manipulatives and technology.
Besides discussing the curricula, there is discourse about the MDMP objectives and instructional methods; also veteran staff, faculty, and teaching assistants demonstrate how lessons should be taught, while new instructors practice teaching the lesson plans. The three-day workshop provides instructors with the scope and sequence of the MDMP curriculum and prepares them to use a myriad of techniques and instructional models in their classrooms (Mireles, 2010).

**Weekly meetings.** Weekly meetings are mandatory for all instructors who teach at least one developmental mathematics course in the Model Developmental Mathematics Program (MDMP) to assure effectiveness and continuity in the classes (Mireles, 2010). These meetings last approximately one hour, which is divided into two sections: class and administration. The new graduate and doctoral teaching assistants are required to attend the full hour—class and administration—for three consecutive Fall/Spring semesters. The class segment is dedicated to learning the theories and logistics of the non-traditional teaching methods used in the MDMP. In the class segment, the graduate and doctoral teaching assistants are required to research instructional models, standards-based teaching techniques, and current research in mathematics education to gain a deeper understanding of mathematics education and to develop pedagogical and pedagogical content knowledge (Mireles, 2010). Adjunct faculty and graduate and doctoral teaching assistants who have attended the full hour session for three consecutive semesters are only required to attend the administrative portion of the meetings. This segment is dedicated to discussing upcoming assessments, potential issues with students, and improvements and/or modifications to the MDMP curriculum (Mireles, 2010). Continuing administrative discourse ensures uniformity among all the MDMP classes and
helps inform the MDMP director of classroom complications. The input from all instructors teaching the MDMP content is instrumental to the curricular revision process.

**Faculty support.** Faculty members, veteran teaching assistants, and Model Developmental Mathematics Program (MDMP) staff guide graduate and doctoral teaching assistants throughout the semester to help build their pedagogical and pedagogical content knowledge and maintain continuity of the lessons (Mireles, 2010). The graduate and doctoral teaching assistants’ pedagogical training prior to the MDMP is limited, so each of them is paired with a mentor—a veteran teacher (Mireles, 2010). The mentor visits the graduate or doctoral teaching assistant’s classroom at least two times during the semester to evaluate the instructor’s methodology and to give guidance on how to improve their pedagogy (Mireles, 2010). Teaching assistants can also approach their mentor to receive support and assistance with classroom management or any issues that may arise.

The MDMP is a departmental program, so consistency and reliability in the classrooms are essential. To guarantee that instructors are teaching the lessons appropriately and on schedule, the faculty mentors, the lead developmental mathematics instructors, and the MDMP staff observe the teaching assistants and give them feedback based on their performances (Mireles, 2010). Faculty mentors and colleagues who observe instructors provide constructive criticism on their teaching, and they monitor the content and types of instruction used in the classrooms to ensure that the lesson plans are being taught correctly. The chosen senior doctoral students or lead developmental mathematics instructors observe graduate teaching assistants to make sure the lessons are taught accurately and consistently, but also to evaluate the effectiveness of the lessons in
the classroom environment (Mireles, 2010). The instructor and her/his assessor meet shortly after each observation to discuss the effectiveness of the observed lesson and for the assessor to give constructive feedback to the instructor on their implementation of the lesson. Moreover, the MDMP staff video records the instructor teaching and a digital copy is given to the instructor for self-observation and reflection, and a copy is retained by the director of the MDMP to assist in future training (Mireles, 2010). Feedback from observations and support through mentoring help the MDMP teaching assistants improve their pedagogical and pedagogical content knowledge to become quality teachers (Mireles, 2010).

The MDMP at Texas State has evolved over the last decade in response to feedback from students and to maintain alignment to changing state and national standards, which has shown to increase students’ content knowledge based on pre/post-test scores (Mireles, 2010). The MDMP team’s goals include making students college-ready, aiding students with mathematics anxiety, providing university instructors and graduate students training on standards-based best practices—technological, hands-on, and real-world activities—and helping other colleges and universities expand their developmental mathematics programs (Mireles, 2010).

Standards

State and national standards mandate content and process standards for grades prekindergarten through twelve and the first two years of college. These standards outline the types of instructional techniques and the content needed to “enable students to know and do” mathematics (National Council of Teachers of Mathematics [NCTM], 2000, p. 29). These standards are necessary to ensure that students learn the same subject
matter at the same grade level and that it is appropriate for that grade level. The standards are also a reference or resource for teachers to help guide effective instruction in the classroom (NCTM, 2000).

The *Principles and Standards for School Mathematics* (PSSM) and the *Common Core State Standards* (CCSS) defined the national standards for grades P–12, whereas the *Crossroads in Mathematics for Introductory College Mathematics before Calculus* (*Crossroads in Mathematics*) outlined the national standard for postsecondary education. In addition, the Mathematics Association of America (MAA) developed a 10-page report on their findings from a Curriculum Foundation Project (CF Project) workshop that lists essential recommendations for developing effective post-secondary curriculum (MAA, 2012). The national standards and the MAA report outlined the required content and process standards used to develop state standards (NCTM, 2000). The Texas state standards, the Texas Essential Knowledge and Skills for grades P–12 and the Texas College and Career Readiness Standards for postsecondary education are the basis for the Model Developmental Mathematics Program (MDMP) curriculum (Mireles, 2010). The proceeding section describes the standards consulted in the development of the MDMP curriculum.

**National standards.** National standards aid educators and policymakers in establishing state requirements for mandatory courses, including mathematics. The *Principles and Standards for School Mathematics* (PSSM) and the *Common Core State Standards* (CCSS) are the national mathematics standards for the primary and secondary grade bands. The PSSM vertically aligns mathematics curriculum for grades K–12 using five process standards and five mathematical content objectives (NCTM, 2000). The
CCSS established “what students should understand and be able to do in their study of mathematics” to ensure student readiness before enrolling in college credit-bearing mathematics courses (NGA Center & CCSSO, 2010, p. 4).

The American Mathematics Association of Two-Year Colleges (AMATYC), developers of Crossroads in Mathematics, and Curriculum Renewal across the First Two-Years (CRAFTY), established national collegiate standards and recommendations for mathematics curriculum development. The introductory mathematics courses discussed in these documents were “college algebra, trigonometry, introductory statistics, finite mathematics, and precalculus as well as all courses presently characterized as developmental mathematics” (Cohen, 1995, p. ix). CRAFTY, a subcommittee of the Mathematics Association of America (MAA), developed workshops under the Curriculum Foundation (CF) Project to aid colleges and universities in creating curriculum for mathematics courses for non-mathematics majors (MAA, 2011). The eleven professional workshops gathered feedback from mathematicians and specialists in disciplines under consideration about mathematics curriculum for non-mathematics degree-seeking students (MAA, 2011). The recommendations complied after the workshops addressed suggestions for developing a credit-bearing mathematics courses for preservice teachers. The Model Developmental Mathematics Program (MDMP) implemented curriculum that connected mathematics to students’ interests and instructional practices to motivate students to learn (MAA, 2011).

Principles and Standards for School Mathematics. In 1989, the National Council of Teachers of Mathematics (NCTM) outlined ten standards and six principles “to ensure quality, to indicate goals, and to promote change” in grade bands P–2, 3–5, 6–
The ten objective standards for each grade band were divided into mathematical content areas (data analysis and probability, numbers and operations, algebra, geometry, and measurement) and process standards (problem solving techniques, reasoning and proofs, connections, communication, and representations; NCTM, 2000). Both sets of objectives were instrumental in the development of the Model Developmental Mathematics Program (MDMP) curriculum (Mireles, 2010).

The framework of the MDMP curriculum progresses from numerical concepts to algebraic functions, which are prominent topics addressed in the *Principles and Standards for School Mathematics* (PSSM) for grades P–12 (NCTM, 2000). While functions as a discrete mathematical topic are not taught in lower grade levels, the basic concepts of functions are touched upon at all grade levels. For example, in the P–2 grade band, students learn functions informally when they are taught how to identify and analyze mathematical patterns (NCTM, 2000). Students are formally introduced to functions in the 3–5 grade band via tables, words, and graphs; they expand on their knowledge of pattern recognition to develop a basic understanding of functions (NCTM, 2000). Students’ knowledge of functions is further expounded upon in grades 6–8 when they learn to identify the differences between linear and non-linear functions by means of tables, graphs, and equations (NCTM, 2000). By the 9–12 grade band, students should have the ability to identify a function, use and transition from one representation to another, identify parts of a function, and understand different function classes (NCTM, 2000). While the NCTM standards emphasize the significance of teaching functions in grades K–12, this importance also extends to credit-bearing collegiate courses (NCTM, 2000; THECB, 2012). According to the *Lower-Division Academic Course Guide*
functions are a prominent subject taught in entry-level mathematics courses at the collegiate level (THECB, 2012). As suggested by the PSSM, the MDMP curriculum facilitates learning functions through patterning and the use of multiple representations (Mireles, 2010; NCTM, 2000).

The six NCTM-endorsed principles that provide the base structure for the MDMP mathematics courses are equity, curriculum, teaching, learning, assessment, and technology. Equity refers to “high expectations and strong support for all students” (NCTM, 2000, p. 11). The MDMP instructors teach lesson plans that blend multiple strands of mathematics including algebra, geometry, and measurement (Mireles, 2010). These lesson plans are designed to make clear and concise connections between mathematical content areas as well as between courses—vertical alignment—through the use of instructional techniques addressed in state and national standards.

The instructional techniques that are infused in the MDMP curriculum include standards-based teaching methods that are essential for preservice teachers’ understanding of mathematical concepts: instructional teaching models, hands-on activities, technology, and cooperative learning groups (Mireles, 2010). According to the PSSM, these standards-based best practices are essential instructional tools for grades K–12, but the lack of using them at the collegiate level delays preservice teachers’ ability to gain knowledge and access to these types of standards-based best practices (NCTM, 2000). To help preservice teachers gain knowledge of discovery-based learning, cooperative learning groups, and hands-on activities, the MDMP incorporates standards-based best practices in the classroom through instructors that have been trained in how to use them. In addition to classroom instruction, assessments are important tools in
teaching mathematics; teachers must assess students’ mathematical knowledge to
determine what students understand as well as their misconceptions (Ball, Hill, & Bass,
2005; NCTM, 2000). Assessing students’ knowledge benefits both the teacher and the
students; it informs instructors of mathematical topics the students may be struggling
with (Ball, Hill, & Bass, 2005). Once the misunderstanding or misconception has been
identified, the teacher can incorporate the appropriate resources into their lessons, such as
technology, to help students succeed (NCTM, 2000).

Technology provides instructors with alternative means for displaying models,
tables, and pictorial representations of mathematical data, functions, and figures. The
PSSM policy-makers recognized the importance of using technology to teach students
real-world mathematics applications (NCTM, 2000). The MDMP capitalizes on
technology—graphing calculators, Geometer’s Sketchpad®, Microsoft Excel®, and
interactive educational websites—to promote mathematical connections. For example,
demonstrating various representations of a function with a graphing calculator—
algebraic, graphical, and tabular forms—helps students identify that the solution to a
quadratic equation is also the intersection of the graph and the x-axis. Technology also
helps make solving complex problems accessible to students with difficulties
understanding mathematics (Mireles, 2010).

**Common Core State Standards.** In June 2010, the National Governors
Association (NGA) Center and the Council of Chief State School Officers (CCSSO) met
with teachers, administrators, and experts from 48 states—including Texas and Alaska—to
develop the Common Core State Standards (CCSS; Burke, 2010; Confrey & Krupa,
2010; NGA Center & CCSSO, 2010). The CCSS were cultivated to help states advance
their P–12 and collegiate developmental in order to prepare students who will succeed in credit-bearing college courses (Confrey & Krupa, 2010; NGA Center & CCSSO, 2010).

The mathematical standards described in the CCSS are a combination of the Principles and Standards for School Mathematics (PSSM) process standards and the mathematical proficiencies described in the National Research Council’s (NRC) report Adding It Up (NGA Center & CCSSO, 2010; NRC, 2001); the MDMP curriculum has incorporated all of these standards. The process standards in the CCSS are the same as the National Council of Teachers of Mathematics (NCTM) process standards: “problem solving, reasoning and proof, communication, representation, and connections” (NGA Center & CCSSO, 2010, p. 6). The mathematical proficiencies include “adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition” (NGA Center & CCSSO, 2010, p. 6).

In the MDMP, preservice teachers obtain the first mathematical proficiency—adaptive reasoning—by reflecting, explaining, and justifying newly developed and/or modified algorithms (NCR, 2001, p. 116). Students who generate mathematical algorithms achieve this proficiency level by developing strategies for solving complex problems, which results in the power to “formulate, represent, and solve mathematical problems,” the second mathematical proficiency, strategic competence (NCR, 2001, p. 116).

The third mathematical proficiency—conceptual understanding—is the “comprehension of mathematical concepts, operations and relations” (NGA Center & CCSSO, 2010, p. 6). Students, specifically preservice teachers, gain conceptual understanding of the mathematical content taught in the MDMP through the Algorithmic
Instructional Technique (AIT) and Concrete to Representational to Abstract (CRA) models; the application of these models in the classroom positively influences preservice teachers’ fourth mathematical proficiency, procedural fluency—skills to carry out procedures seamlessly with minimal mistakes (NGA Center & CCSSO, 2010). Another tool to help build preservice teachers’ procedural fluency skills are organizational frameworks such as flow charts and diagrams (Witzel et al., 2008). These instructional models and organizational methods reduce MDMP students’ mathematics anxiety and contribute to their mathematics aptitude and the last mathematical proficiency, productive disposition—a “habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (NGA Center & CCSSO, 2010, p. 6). Students’ positive attitudes and beliefs towards learning mathematics are contingent upon teachers’ positive disposition, so it is important for preservice teachers to construct their own positive mathematics disposition while they are students themselves (Quinn, 1997; Schussler, 2006).

The instructional models used in the MDMP combine multiple mathematical proficiencies that help preservice teachers conceptualize how students understand mathematics. For example, the AIT model helps preservice teachers build two mathematical proficiencies—adaptive reasoning and procedural fluency. Preservice teachers gain adaptive reasoning skills when they revise algorithms as the problems become more complex, and they develop procedural fluency during the process of creating an algorithm so they can carry out tasks with minimal errors (Mireles, 2010). Therefore, the MDMP preservice teachers have an opportunity to be introduced to mathematical proficiencies such algorithmic modeling, real-world applications, and
hands-on activities (NGA Center & CCSSO, 2010).

Real-world mathematics application problems help preservice teachers gain strategic competence through formulating, algebraically representing, and problem solving (Mireles, 2010). Conceptual understanding is achieved when students, specifically preservice teachers, make connections between various forms of representations. According to the CCSS, these mathematical proficiencies and standards are tools associated with quality teachers (NGA Center & CCSSO, 2010).

**Crossroads in Mathematics.** The American Mathematics Association of Two-Year Colleges (AMATYC) developed content, process, and pedagogical standards called the *Crossroads in Mathematics* to improve mathematics curriculum for the first two years of college and to encourage more students to pursue mathematics (Cohen, 1995). The *Crossroads in Mathematics* content strands focus on “college algebra, trigonometry, introductory statistics, finite mathematics,” precalculus, and developmental mathematics curriculum; the standards focus on curriculum for students who are not pursuing careers requiring upper level mathematics (Cohen, 1995, p. ix). AMATYC’s suggested content, process, and pedagogical standards were designed to expand students’ intellectual development and motivation in learning mathematics, coincidentally also providing preservice teachers a foundation for developing a positive disposition towards mathematics (Cohen, 1995). The process, content, and pedagogical standards in *Crossroads in Mathematics* are integrated into the Model Developmental Mathematics Program (MDMP) curriculum and instruction (Mireles, 2010).

*Crossroads in Mathematics* process standards outline curricula that would enable students to develop skills such as “problem solving,” “modeling,” “reasoning,”
“connecting with other disciplines,” “communicating,” “using technology,” and “developing mathematical power” (Cohen, 1995, p. x). The cognitive processes are similar to the skills addressed in the Principles and Standards for School Mathematics (PSSM) and the Common Core State Standards (CCSS).

The *Crossroads in Mathematics* described entry-level mathematics course content as “number sense,” “symbolism and algebra,” “geometry,” “function[s],” “discrete mathematics,” “probability and statistics,” and “deductive proof[s]” (Cohen, 1995, p. x). According to Cohen (1995), these content areas support students’ success in credit-bearing mathematics. The MDMP curriculum emphasizes all the topics needed to build a foundation for learning mathematics, such as functions, whereas traditional topics taught in developmental mathematics are generally restricted to number sense, symbolism and algebra, and discrete mathematics (Cohen, 1995).

Mathematical functions are a key topic taught in credit-bearing mathematics courses, especially College Algebra. Developmental mathematics programs ought to help students build a solid foundation for learning functions so that the students can be successful in credit-bearing mathematics courses (Mireles, 2010). As described in the Pedagogical Standards chapter in *Crossroads in Mathematics*, different types of instructional techniques are needed to teach mathematical content such as functions (Cohen, 1995). The MDMP curriculum builds a foundation for learning functions with various instructional techniques such as multiple representations and discovery-based learning.

The pedagogical standards in *Crossroads in Mathematics* focus on creating student-constructed knowledge through peer interactions and engagement in classroom
activities (Ball, 1988; Cohen, 1995). These standards illustrate teaching techniques that use technology and various approaches to teaching mathematical content, describe collaboration between peers and instructor, and emphasize teaching by connecting mathematical content to concrete experiences to provide students with an environment that fosters their conceptual understanding of mathematics (Cohen, 1995).

Technology, a tool emphasized in *Crossroads in Mathematics*, is used in the MDMP courses to help students make connections between different forms of representations. The MDMP instructors use technology such as Geometer’s Sketch Pad®, graphing calculators, and Microsoft Excel® to teach mathematical concepts through visualizations, multiple representations, and patterning. They also teach mathematical topics using varied approaches; for example, instructors represent the addition of polynomials using algebra tiles, pictures, and symbolic notation.

The MDMP lesson plans also support student-constructed knowledge through the use of interactive and cooperative learning groups, such as the jigsaw method, so that students can communicate, conjecture, and work together to solve mathematical problems. When students work in collaboration with their peers, they develop mathematical connections by discussing their own ideas. Preservice teachers can use this opportunity to gain pedagogical and pedagogical content knowledge by understanding which content areas are difficult for students and how different students interpret or misinterpret different mathematical processes.

Another way students can construct knowledge is by manipulating concrete objects (Witzel, 2005; Witzel, Riccomini, & Schneider, 2008). Connecting mathematical content to tangible concepts allows students to manifest their own problem solving skills
The MDMP curriculum uses concrete manipulatives such as dice, cards, algebra tiles, and colored chips. Manipulatives add an element to traditional collegiate level course instruction.

The instructional views described in the *Crossroads of Mathematics* pedagogical standards were “based on the premise that knowledge cannot be ‘given’ to students,” but must be constructed by students; student-constructed knowledge is gained through student-centered learning (Cohen, 1995, p. 15). The lesson plans used in the MDMP support student-centered learning with hands-on activities, cooperative learning groups, and collaborative learning groups to afford students the chance to construct their own knowledge (Mireles, 2010).

**Curriculum Foundation Project.** In 2004, the Curriculum Foundation Project (CF Project) established workshops to gather feedback from mathematics professionals on college-level curricular content suggestions for non-mathematics majors (Mathematics Association of America [MAA], 2011). The CF Project’s results were synthesized by Curriculum Renewal Across the First Two Years (CRAFTY), a subcommittee of MAA, and compiled into recommendations for curriculum for college mathematics courses for non-mathematics majors (MAA, 2011).

Many college students lose motivation in mathematics courses due to a lack of interest because they cannot make connections between the mathematical content and their majors (MAA, 2011). As a result, CRAFTY suggested developing credit-bearing mathematics courses for specific career choices (Senk, Keller, and Ferrini-Mundy, 2000): for example, credit-bearing mathematics courses specifically designated for elementary and middle school preservice teachers (MAA, 2011). These mathematics courses would
provide preservice teachers an opportunity not only to gain content knowledge but also pedagogical and pedagogical content knowledge. Traditional developmental mathematics programs, unlike the Model Developmental Mathematics Program (MDMP), do not provide standards-based best practices for preservice teachers to acquire pedagogical and pedagogical content knowledge.

According to CRAFTY, the recommended credit-bearing mathematics course(s) for preservice teachers should include opportunities to (1) develop a deeper understanding of the mathematical content that they will teach, (2) learn different instructional techniques with the use of manipulatives, (3) understand the process of learning mathematical content, and (4) communicate with school districts, colleagues, and administrators (Senk et al., 2000). The MDMP implements CRAFTY’s four recommendations by emphasizing grades P–12 mathematics content and by teaching students using various standards-based instructional techniques.

CRAFTY’s first recommendation is that preservice teachers should have a conceptual understanding of the mathematical content that they will eventually teach (Senk et al., 2000); the MDMP content and pedagogy is similar to the content and pedagogy used in grades P–12. A majority of the MDMP content aligns to the ninth grade concepts, but some of the more complex concepts are taught with mathematical examples that reflect elementary and middle school content standards. For example, the MDMP instructors teach the lesson *Operations on Rational Expressions* by adding and subtracting rational expressions using similar steps to adding and subtracting rational numbers. Grades K–12 pedagogical standards are also reflected in the MDMP curricula; the mathematical content in the MDMP courses are taught with strategies used in the
lower grades, such as representing problems concretely or numerically instead of algebraically. Teaching grades P–12 mathematical concepts to learn collegiate level mathematical content provides preservice teachers in the MDMP the opportunity to assimilate new knowledge with prior knowledge, thus solidifying their understanding of the content they will one day teach.

The supposition that teachers should learn diverse instructional techniques with the use of manipulatives is CRAFTY’s second recommendation (Senk et al., 2000). The MDMP uses manipulatives and technology via the Concrete-Representational-Abstract (CRA) model, Algorithmic Instruction Technique (AIT) model, and discovery-based learning (Mireles, 2010; Witzel, 2005; Witzel, Riccomini, & Schneider, 2008). According to Bright (1999), using these tools in content courses provides preservice teachers with an advantage. She stated that teachers needed to experience learning with manipulatives before using them to teach in their classrooms. The use of manipulatives and technology in the classroom aids in not only the development of preservice teachers’ content knowledge, but also their pedagogical and pedagogical content knowledge.

The CRA model incorporates the use of manipulatives in the early stages of the learning process. The other stages of the CRA model build on concrete concepts to aid students in developing a solid foundation for abstract mathematical concepts. Witzel, Mercer, and Miller (2003) found that algebra students who used the CRA model outperformed those who did not (The Access Center, 2004). The MDMP utilizes the CRA model to help prepare developmental mathematics students for credit-bearing mathematics courses.

Another model that can be used to teach diverse groups of students is the AIT
model. The AIT model is used to help students develop systematic procedures for mathematical operations. By the time students advance to the independence stage of the model, they are capable of developing their own algorithms, which may also help preservice teachers develop an understanding of students’ thought processes while they share their algorithms with their peers.

CRAFTY’s third recommendation is that preservice teachers need to understand the process of learning so they are able to explain mathematical content and identify students’ misconceptions (Senk et al., 2000). The AIT model requires students to write step-by-step instructions for performing a mathematical task. These algorithms provide preservice teachers and mathematics instructors with a source for understanding the learning process of others—how they learn mathematics. Preservice teachers can compare peers’ algorithms to learn how to identify strengths and weaknesses in the students’ learning processes and misconceptions of the students. The MDMP has exercises for students to identify misconceptions and explain why they are incorrect. In the MDMP lesson, *Operations on Rational Expressions*, the MDMP students are given solved problems, some with common errors, and they are asked to identify the mistakes and why they were incorrect: for example, “can \((x + 2)/x\) be simplified to 2, why or why not?” Recognizing misconceptions is the first step in helping students thrive in the classroom because misunderstandings can be identified and alternative explanations can be given to the students (Rittle-Johnson & Star, 2007). By comparing different methods of solving problems and identifying and correcting common errors, preservice teachers become familiarized with different ways to solve problems (Rittle-Johnson & Star, 2007).

Effective communication skills, CRAFTY’s fourth recommendation, are
necessary for teachers to effectively interact with students, parents, colleagues, and administrators to ensure student achievement (Senk et al., 2000). To foster students’ communication skills, the MDMP curriculum requires them to converse in collaborative and cooperative peer-learning groups to discuss mathematical problems (Mireles, 2010). This classroom discourse not only promotes communication between peers, but it also encourages communication between the students and their instructor; similar communication strategies will be needed by the preservice teachers once they join the workforce—discourse between co-workers and workers and their supervisor(s).

CRAFTY recommended that a mathematics content course focused on developing preservice teachers’ content, pedagogical, and pedagogical content knowledge should contain five elements: (1) preservice teachers “should develop ‘deep understanding’ of mathematics,” (2) the course should include technology and manipulatives for preservice teachers to use when problem solving, (3) preservice teachers should learn using various instructional methods, (4) preservice teachers should understand “how people learn mathematics,” and (5) preservice teachers should be able to communicate among colleagues, students, administration, and parents (Ganter & Barker, 2004, p. 145). Even though the MDMP was not developed based on the recommendations from CRAFTY, the MDMP curriculum contains aspects of the recommendations, which makes the MDMP an ideal course for preservice teachers. The MDMP may give preservice teachers an advantage over those who go directly into credit-bearing mathematics courses because they are exposed to curricular activities and concepts not taught in traditional classrooms (Mireles, 2010).
**Texas standards.** *Texas Essential Knowledge and Skills* (TEKS) and the Texas College and Career Readiness Standards (TX CCRS) are the Texas education standards for grades P–12 and postsecondary schools. Principles and Standards for School Mathematics (PSSM) and *Crossroads in Mathematics* contributed to the design of both the TEKS and the TX CCRS, the basis of the curriculum for prekindergarten through the first two years of college in Texas (TEA, 2011; THECB & TEA, 2009). These standards outline the content and pedagogy that preservice teachers must use to be effective in the classroom (TEA, 2011; THECB & TEA, 2009).

**Texas Essential Knowledge and Skills.** The *Texas Essential Knowledge and Skills* (TEKS), developed by the Texas Education Agency (TEA), “provide leadership, guidance and resources to help schools meet the educational needs of all students” (TEA, 2010, Mission, para. 1). The content and process standards in the TEKS influence Texas’s textbook content, curriculum, and teacher preparation for grades P–12. The TEKS for grades P–12 incorporated the Principles and Standards for School Mathematics (PSSM) content standards for numbers, operations, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurements; and probability and statistics (NCTM, 2000; TEA, 2011).

The TEKS mandate that grades K–12 curricula be designed to make connections between mathematics and real-world problems and use problem-solving models, technology, and manipulatives to explain content (TEA, 2011). The TEKS also describe instructional methods and tools that can be used in the classroom to encourage students to develop a deeper understanding of the mathematical content by comparing, justifying, and explaining the reasons for performing the specific mathematical operations (TEA,
The tools recommended by the TEKS—technology and manipulatives—can be used by students to develop problem solving skills, make connections between various forms of representations, and understand mathematics through visual representations (NCTM, 2000).

The majority of the MDMP students’ knowledge and skills in mathematics are underdeveloped, meaning they were not prepared for college in their secondary education courses. To maintain scaffolding between secondary and postsecondary education, the MDMP curriculum addresses content and process standards from the TEKS (Mireles, 2010). The alignment of the secondary and post-secondary curricula helps preservice teachers make connections between the content areas. In short, Texas preservice teachers need to be exposed to the concepts in the TEKS because they will ultimately be teaching students content outlined in the TEKS (TEA, 2011).

**Texas College and Career Readiness Standards.** The Texas Higher Education Coordinating Board (THECB) and the Texas Education Agency (TEA) established vertical teams consisting of collegiate and secondary educators to formulate the Texas College and Career Readiness Standards (TX CCRS). The collaboration between secondary and post-secondary educators guided the vertical alignment of content and process standards in the TX CCRS. The content, process, and cross-disciplinary standards of the TX CCRS describe the knowledge and skills to be taught in primary and secondary grades to enhance the students’ probability of success in college (THECB & TEA, 2009). Since the TX CCRS outlines the requirements needed to be successful in credit-bearing courses, these standards are essential for preparing developmental mathematics students for credit-bearing mathematics courses. The Model Developmental
Mathematics Program (MDMP) lessons are aligned to the TX CCRS content standards for numeric reasoning, algebraic reasoning, geometric reasoning, measurement reasoning, probabilistic reasoning, statistical reasoning, and functions, and the process standards—problem solving and reasoning, communication and representation, and connections (Mireles, 2010; THECB & TEA, 2009). With the exception of functions, the content and the process standards in the MDMP mirror the national standards described in the Principles and Standards for School Mathematics (PSSM), the Common Core State Standards (CCSS), and the Crossroads in Mathematics (Cohen, 1995; Mireles, 2010; NCTM, 2000; NGA Center & CCSSO, 2010).

The TX CCRS emphasis on functions is distinct from other national mathematical content standards (THECB & TEA, 2009). While the PSSM, the CCSS, and the Crossroads in Mathematics include functions in the Algebraic section of the content standards, functions are a separate category in the TX CCRS (THECB & TEA, 2009). In the TX CCRS, the function strand proposes that college-ready students should be able to distinguish between functions, non-functions, and different types of functions (THECB & TEA, 2009). College students should also be able to determine different characteristics of functions—minimum and maximum points; increasing, decreasing, and constant slopes; x- and y-intercepts; and transformations of parent functions (THECB & TEA, 2009). Besides the content itself, it is also important to make connections between content areas and within the content area, so students can build conceptual understanding of mathematical concepts and how those concepts relate to the real world. These connections between mathematical content and additional domains are referred to as cross-disciplinary standards (THECB & TEA, 2009).
Mathematical connections to historical, scientific, and culturally relevant topics are emphasized in the cross-disciplinary section of the TX CCRS to help students’ understanding of the importance of mathematics (THECB & TEA, 2009). These cross-disciplinary standards are organized into two categories: key cognitive skills and foundational skills (THECB & TEA, 2009). Key cognitive skills are reflected in students’ intellectual curiosity about mathematics, mathematical concept reasoning, problem-solving abilities, and responsibility for academic behaviors—such as self-monitoring to achieve optimal study habits and reach goals, working independently or with others, and having academic integrity (THECB & TEA, 2009). Students attain foundational skills through interdisciplinary reading, writing, and researching, which includes collecting data and interpreting results (THECB & TEA, 2009).

Students develop key cognitive skills in classrooms that utilize standards-based best practices such as discovery-based learning, scaffolding, and deconstruction and reconstruction of knowledge (THECB & TEA, 2009). The MDMP lessons facilitate discovery-based learning through patterning and the use of manipulatives that expand intellectual curiosity about mathematics (Mireles, 2010). When the MDMP students build on their prior knowledge through scaffolding in conjunction with deconstruction and reconstruction of mathematical knowledge, they are conceptually learning mathematics (Mireles, 2010). For example, the MDMP lessons on simplifying polynomials deconstruct then reconstruct the use of the acronyms PEMDAS and FOIL. Students’ problems with operations on polynomials using PEMDAS is evident in their solutions; they often multiply before divide, or add before subtract, regardless of the actual order of operations necessary to simplify the problem correctly. Likewise,
students who remember FOIL do not know how to transfer their knowledge of multiplying binomials to multiplying polynomials with more than two terms. In these situations, the mathematical concepts of order of operations and the distributive property need to be deconstructed and reconstructed. Deconstruction and reconstruction is used in the MDMP to inform students of the correct methods for simplifying and solving problems otherwise misconstrued due to erroneous prior knowledge (Ball, 1988; Mireles, 2010).

The last key cognitive skill is responsibility for academic behaviors. Successful students study and monitor their own progress, which enables them to be organized and responsible for their academic success. To help guide students in developing positive academic behaviors, the MDMP students are required to maintain organized notebooks containing all completed class documents, graded homework assignments, graded quizzes, and collected notes (Mireles, 2010). The students that maintain a notebook receive quiz grades. The notebooks are tools for helping students monitor their progress in the class; they can calculate their grade because they should have all their graded assignments. They are accountable for their own academic success throughout college (Mireles, 2010).

The next cross-disciplinary cognitive skills category—students’ foundational skills—includes reading, writing, researching, and analyzing data across the curriculum (THECB & TEA, 2009). The MDMP students practice these skills by reading application problems and writing algorithms and surveys. The students must also be able to perform statistical analysis on collected data; they must have the ability to develop surveys, collect data, and perform statistical analysis correctly. The MDMP emphasizes
the use of statistics and data collection. MDMP students have the ability to develop both key cognitive and foundational skills needed by students to successfully complete entry-level credit-bearing mathematics courses (THECB & TEA, 2009).

The MDMP curriculum is aligned to the process, content, and cross-disciplinary standards described in the TX CCRS (Mireles, 2010). These standards outline curriculum development for grades P–16 in Texas, consequently easing students’ transition from high school to postsecondary mathematics courses and enabling preservice teachers to gain the skills necessary to become quality teachers (TEA, 2011; THECB & TEA, 2009).

**Teacher Effectiveness**

Teacher effectiveness is contingent upon teachers’ quality and qualifications in conjunction with students’ progress (Rice, 2003). Teachers’ “experience, preparation programs [attended], degrees, type of certification, coursework taken in preparation for the profession, and teachers’ own test scores” all contribute to teacher quality, a component of teacher effectiveness (Rice, 2003, p. 9). Preservice teachers cannot be evaluated based on their students’ progress because they are not yet teachers. Other qualifications that are required are preservice teachers’ knowledge and disposition. Therefore, teacher effectiveness is based on preservice teachers’ well-developed knowledge of pedagogy, content, and pedagogical content and positive disposition towards teaching.

**Mathematical knowledge for teaching.** While in-service teachers’ effectiveness is assessed by students’ progress, preservice teachers’ effectiveness is measured by their content, pedagogical, and pedagogical content knowledge and disposition as evidenced in
their coursework and test scores. Preservice teachers gain knowledge and disposition through class experiences and by teaching others mathematical content (Ball, 1988). According to research models developed by Cavallo (2010) and Hill, Ball, and Schilling (2007), high quality teachers have content, pedagogical, and pedagogical content knowledge and positive disposition. In the Effective and Successful Teachers model, Cavallo (2010) described high quality teachers as having a combination of content knowledge, teaching pedagogy, and personal disposition; whereas Hill, Ball, and Schilling’s (2007) model, Mathematical Knowledge for Teaching (MKT), termed quality teachers as having subject matter and pedagogical content knowledge. This section presents in-depth descriptions of content, pedagogical, and pedagogical content knowledge and disposition as illustrated in the MKT model (Hill, Ball, & Schilling, 2008) and Effective and Successful Teachers model (Cavallo, 2010).

**Content knowledge.** Content knowledge (CK) is the “understanding of the key facts, concepts, principles, and explanatory frameworks” of mathematics (Lee, 2000, p. 24). Ball, Thames, and Phelps (2008) recognized three areas of content knowledge collectively referred to as subject matter knowledge: common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (HCK). These three components of subject matter knowledge, essential for developing high quality teachers, all inform the Model Developmental Mathematics Program (MDMP) curriculum (see Figure 1).

Common content knowledge (CCK) is mathematical knowledge shared by people who know and use mathematics in their occupations (Ball et al., 2008). For example, mathematicians, engineers, accountants, and construction workers all use applied
mathematics to solve problems in their workplaces. Preservice teachers will someday be teaching students who will use mathematics in their respective professions, so they need the ability to contextualize mathematics by adding real-world problems, understand mathematics in historical contexts, and defend mathematical reasoning processes (Ball, Hill, & Bass, 2005). Preservice teachers with these abilities have the capacity to teach mathematical content thoroughly. The MDMP provides preservice teachers opportunities to learn and teach mathematical concepts through historical relevance, formula derivation, and real-world applications (Mireles, 2010).

![Figure 1](image)

*Figure 1: Domains of Mathematical Knowledge for Teaching (MKT) Model (Hill et al., 2008, p. 377). This model was developed from the Learning Mathematics for Teaching (LMT) project at University of Michigan and permission was sought out and granted for use in this dissertation. This model describes the components of subject matter knowledge and pedagogical content knowledge needed by quality teachers.*

Preservice teachers need to have CCK to be able to explain mathematics to P–12 students and the students need to be proficient in P–12 CCK to ensure success in college credit-bearing mathematics courses (Ball et al., 2005). To aid P–12 students’
development of CCK, the MDMP instructors (re)build P–12 mathematical content knowledge and skills. The MDMP is comprised of two lower proficiency mathematics courses—Pre-College Algebra and Basic Mathematics. The MDMP courses give preservice teachers who took at least one MDMP course an advantage over preservice teachers who have not taken the course, because the MDMP provides preservice teachers with opportunities to build a deeper understanding of mathematical content through relearning, reconstructing, and refreshing P–12 mathematical content with standards-based best practices (Mireles, 2010). Current literature discusses the ineffectiveness of upper level mathematics courses on developing primary grade quality teachers, but there is no literature discussing the benefits of reconstructing lower level mathematics courses to benefit preservice teachers, such as the MDMP courses (Ball, 1990; Ball, 1991; NRC, 2001). The preservice teachers’ exposure to standard-based best practices in the MDMP may aid in the development of mathematical knowledge for teaching (MKT) before they enroll in pedagogical content courses (Texas State, 2010).

Teachers who possess SCK are proficient in the mathematical content they teach, know the appropriate methods for teaching a specific mathematical topic, and can decipher students’ methods for working problems (Hill et al., 2008; Swars, Hart, Smith, S., Smith, M., & Tolar, 2007). To build students’ SCK, the MDMP curriculum incorporates various instructional models, which includes cooperative learning groups, using multiple representations to solve one problem, and provides students with the opportunity to discover mathematical errors and correct them. Students participating in cooperative learning groups are required to work with peers teaching each other mathematical concepts; the students must work together as a team to learn because they
are responsible for each other’s learning. Students learn how to use appropriate methods for solving because they are taught how to perform mathematical tasks multiple ways, such as solving systems of equations using graphing, substitution, and addition methods, one method maybe more effective than the other methods.

Horizon content knowledge (HCK) is “an awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (Ball et al., 2008, p. 403). Vertical alignment, an example of HCK, is the “alignment of standards, tests, and even curriculum from one grade to the next” (Wise & Alt, 2005, p. 61). Scaffolding, a form of vertical alignment, is a process of learning new knowledge from existing knowledge by building on mathematics concepts learned in prior grades (NCTM, 2000). Vertical alignment allows preservice teachers and students to visualize the overall sequence of the curriculum and understand connections between prior knowledge and new topics; this curriculum alignment to current and prior courses is necessary for effective teaching and learning (Ball et al., 2008). The MDMP participants who have enrolled in both developmental mathematics courses and the College Algebra course may notice the alignment of the curriculum. The MDMP curricula was developed to align not only to state standards, but also to subsequent credit-bearing mathematics courses taught at Texas State University-San Marcos (Texas State).

Preservice teachers’ knowledge of the mathematical content is an important component for developing MKT. The content knowledge needed by elementary and middle school preservice teachers are in the areas of number and operations, geometry, early algebra thinking, and data analysis (National Council of Teachers of Mathematics [NCTM], 2000; NGA Center & CCSSO, 2010; THECB & TEA, 2009). The content
taught in the MDMP includes algebra, geometry, measurement, statistics, probability, and functions, which are similar content areas needed by preservice teachers. Teachers must have CK in multiple grade levels to understand their students’ prior knowledge and the knowledge their students will need to be successful in proceeding course(s).

**Pedagogical knowledge.** Pedagogical knowledge (PK), understanding the process of learning to teach, is an important characteristic of quality teachers. According to Graham and Fennell (2001), preservice teachers begin learning PK in “the methods course or courses that are the singular responsibility of mathematics educators” (p. 322). These courses offer standards-based mathematics instruction to support understanding of “how students learn, what interests them mathematically, and curricular issues important at various grade and instructional levels” (Graham & Fennell, 2001, p. 322).

Instructional level and type of instruction is dependent on the students’ knowledge of mathematics, because primary, secondary, and post-secondary grade students’ learning is contingent upon age, maturity, intellectual ability, and learning preference. Understanding how students learn can be achieved through participating in cooperative learning groups or group work. Cooperative learning groups, used in the Model Developmental Mathematics Program (MDMP) classes, permit preservice teachers to observe and make conjectures about how students learn (Ball et al., 2008). In developing student’s mathematical comprehension, teachers must relate mathematics to students’ interests through personal and real-world experiences. Instructors teach the MDMP using various pedagogical techniques that correspond with students’ interests, such as drawing, trivia, sports, and gaming. These types of activities promote discussion that instructors need to anticipate.
Teachers are required to anticipate and correct curricular issues at varying instructional levels. Some issues include deconstructing and reconstructing acronyms and mnemonics that were incorrectly taught or retained by students. These shortcuts are often misunderstood, so preservice teachers need to have the ability to understand the needs of their students to teach the mathematical content appropriately and to be able to correct students’ misconceptions.

**Pedagogical content knowledge.** Grossman defined preservice teachers' pedagogical content knowledge (PCK) as their comprehension of (1) the purpose of teaching mathematics, (2) knowledge of students, (3) knowledge of curriculum, and (4) knowledge of teaching strategies for particular mathematical topics (as cited in Sowder, 2007, p. 164). Similarly, Ball and Bass (2000) defined PCK as a combination of content knowledge (CK) and pedagogical knowledge (PK)—knowledge of mathematics intertwined with the “knowledge of [the] learner, learning, and pedagogy” (p. 88). Prior to learning PCK, preservice teachers need to make connections between the content and the learner; these connections can be developed by the preservice teachers in the MDMP through the utilization of manipulatives to develop mathematical understanding and examples that aid with understanding mathematical misconceptions, connection of mathematics to other disciplines and real-world experiences, and students’ learning processes through algorithmic writing.

Preservice teachers who understand the purpose of teaching mathematics, Grossman’s first PCK component, can convey the importance of mathematics to students by connecting mathematical content to real-world applications. For instance, imaginary numbers are introduced in the MDMP through discussing the importance of mathematics
in the fields of electrical engineering and signal processing. These include cell phones applications, radars, and other telecommunication devices. Instructors linking mathematical applications to students’ interests provide a way for students to make meaningful connections, but the instructor must know their students’ interests to make these essential correlations.

Instructors’ knowledge of their students, the second component of PCK as defined by Grossman, helps in determining students’ knowledge, ideas, and possible misconceptions (Ball et al., 2008; Sowder, 2007). Effective teachers make clear mathematical connections in lessons because they have “knowledge of students’ understandings, conceptions, and potential misunderstandings” of mathematical content (Sowder, 2007, p. 164). To develop students’ mathematical proficiency content is deconstructed and reconstructed, a form of instruction used in the MDMP. Preservice teachers who can deconstruct and reconstruct misinterpreted mathematical concepts have a deep understanding of mathematical content and pedagogy.

Quality teachers develop and teach effective lessons with an understanding of the curricular materials needed for teaching a diverse population of learners, effective instructional techniques, and grade appropriate standards, Grossman’s third component of PCK (Sowder, 2007). Teachers need to be able to use varied instructional techniques and materials when explaining mathematical content to students with diverse learning styles—kinesthetic learners versus auditory learners. For example, graphing calculators, computer programs, and manipulatives are helpful teaching tools for kinesthetic learners, whereas explaining concepts verbally helps auditory learners (Snyder, 2000). While learners’ differing cognitive abilities require various teaching tools and methods, content
must be considered when deciding what tools are needed to teach the content appropriately.

Preservice teachers need to know which instructional methods are appropriate to use when teaching a particular mathematical topic, Grossman’s fourth component (Sowder, 2007). For example, graphing calculators should be used at the high school level when discussing functions, but not at the third grade level when teaching times tables. Materials and instructional techniques should be used appropriately to engage and instruct mathematics students (Mireles, 2010). Using different instructional methods and the use of various manipulatives by the teachers helps support conveying the importance of mathematics to students, the understanding students’ knowledge, ideas, and possible misconceptions, and their understanding of the mathematics curriculum. These four components of Grossman’s definition of PCK differ slightly from Hill, Ball, and Bass’s (2007) definition.

Hill, Ball, and Bass’s (2007) description of PCK is separated into three elements: knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of curriculum (KC). KCS is “content knowledge intertwined with knowledge of how students think about, know, or learn” mathematics, which includes understanding students’ “mistakes and misconceptions,” prior and current knowledge, and schema for learning mathematics (Hill et al., 2008, p. 375). An example MDMP lesson plan that can build KCS is *Simplifying Rational Expressions*. The lesson contains various examples of common mathematical mistakes, and students are responsible for identifying common errors and explaining the correct procedures for simplifying.

Another component of PCK is KCT—a combination of “knowing about teaching
and knowing about mathematics” (Hill, Ball, and Bass, 2008, p. 401). Cooperative learning groups encourage students in the MDMP courses to teach and learn mathematical content while interacting with peers, which is a beneficial exercise for preservice teachers to help them learn about mathematics and teaching it to others (Mireles, 2010). For example, the MDMP lesson Solving Rational Equations incorporates cooperative learning, specifically the jigsaw activity. Students learn a specific mathematics concept with their peers then regroup and teach each other their assigned concept; these types of activities promote knowledge of both mathematics and teaching (Kagan, 1994).

Lastly, KC encompasses the lateral and vertical scope and sequence of the curriculum (Ball et al., 2008). Lateral curriculum knowledge is the comprehension of curriculum in subjects other than mathematics at the same grade level and how they are related to mathematics; for example, how mixing chemicals in science are related to mixture problems in mathematics (Ball et al., 2008). Whereas, vertical curriculum knowledge is teachers’ familiarity of curriculum in the same subject matter but for consecutive grade levels. An example of vertical curriculum knowledge is the MDMP instructor’s knowledge of their curriculum and the curriculum of the College Algebra course, since the MDMP courses are prerequisite for students who are considered not college ready (Ball et al., 2008). Prerequisite courses are designed to prepare students for the higher-level course(s) they need for their majors; the MDMP course Pre-College Algebra (Math 1311) is the prerequisite to credit-bearing mathematics courses including College Algebra. To prepare students for College Algebra at Texas State, the MDMP curriculum is aligned to College Algebra content and state and national standards.
The three components of pedagogical content knowledge—KCS, KCT, and KC—were not deliberately incorporated in the MDMP lesson plans, but the MDMP encompassed both pedagogy and content—unlike upper level mathematics courses that only focus on content (Mireles, 2010). Educators and researchers have argued that teachers only need to know the content they are planning to teach (Ball et al., 2005; Kajander, 2010); however, many other educators and theorists have ascertained it is not sufficient to know subject matter knowledge they will teach (Ball & Bass, 2000; Graham & Fennell, 2001). Teachers must be educated in the students’ prior knowledge, and in the knowledge the students will learn in upper level courses (Ball et al., 2008; Kajander, 2010).

Ball and Bass (2000) asserted that learning PCK is dependent on comprehending KCS, KCT, and KC. Preservice teachers can learn KCS, KCT, and KC in content courses by observing their instructors teach and by instructing students in peer groups (Ball et al., 2005; Kajander, 2010). Preservice teachers who are able to merge knowledge of mathematical content and pedagogy into PCK show early signs of effective teaching (Ball and Bass, 2000).

**Disposition.** Teachers’ disposition, beliefs and attitudes, affect “student learning, motivation, and development as well as the educator’s own professional growth” (National Council for Accreditation of Teacher Education [NCATE], 2002, p. 53). The National Council for Accreditation of Teacher Education (NCATE) defined disposition as “the values, commitments, and professional ethics that influence behaviors toward students, families, colleagues, and communities” (NCATE, 2002, p. 53). When teachers relinquish the opportunity to “consider where [their] decisions originate, how they think
through [their] decisions, or the underlying assumptions that affect [their] decisions” (Schussler, 2006, p. 252), they lack disposition. These teachers are unable to reflect on their own thought processes and choices concerning curriculum, understanding of student knowledge, and lesson planning, which affects students’ disposition and learning (Swarz et al., 2007; Quinn, 1997). Preservice teachers in the Model Developmental Mathematics Program (MDMP) are afforded a supportive environment in which to develop and maintain a positive disposition about mathematics and teaching mathematics that will one day be transferred to their students.

Vacc and Bright (1999) discovered that preservice teachers changed their beliefs in response to the instructional techniques of their instructors and professors, which in turn affects their future students’ learning and disposition. The preservice teachers constructive disposition is developed from their classroom observations while they are students (Ball, 1988; Hill et al., 2005). The MDMP supports preservice teachers development of positive disposition by providing them with an encouraging environment so they can develop their own knowledge, teach peers, and engage in exploratory activities in mathematics (Mireles, 2010).

In the Effective and Successful Teachers model, Cavallo (n.d.) emphasized that teachers need “professional personal disposition” along with “sound content knowledge” and a “high level of teaching skill” to be highly qualified (slide 23; see Figure 2). Teachers lacking one or more of these attributes can negatively influence their students’ performances. Quinn (1997) observed that primary grade teachers have “less favorable attitudes toward mathematics,” and their attitudes contributed to their students’ poor disposition towards mathematics (p. 108). The lack of positive disposition teachers have
towards mathematics negatively affects students’ positive disposition, so earlier development of positive disposition may contribute to the preservice teachers’ positive disposition in the classroom ultimately encouraging students to have promising disposition (Quinn, 1997).

According to the Mathematical Knowledge for Teaching (MKT) and the Effective and Successful Teachers models, pedagogical, content, and pedagogical content knowledge and disposition are attributed to high quality teachers (Ball, Hill, & Bass, 2005; Cavallo, 2010). In addition, the No Child Left Behind Act (NCLB, 2008) described highly qualified teachers as someone who has teacher certification, a degree, and can demonstrate competency in the content area they will be teaching.

Figure 2: Model of Effective and Successful Teachers (Cavallo, n.d.)

**No Child Left Behind Act.** The No Child Left Behind Act (NCLB, 2008) defined a highly qualified teacher as one who (1) has a state certification or has passed the state teacher licensing examination and holds a license to teach in the state, (2) has at least a bachelor’s degree, and (3) can demonstrate competency in the subject areas they
will teach (U.S. Department of Education, 2004). Texas teachers meet the NCLB Act requirements by mandating all Texas teachers must pass two certification exams and have a bachelor’s degree to be defined as highly qualified; however, these requirements do not necessarily reflect overall teacher quality (Ball et al., 2005).

**Texas certification.** In Texas, the two exams preservice teachers must pass to become certified are the Texas Examination of Education Standards (TExES)—content and pedagogical content knowledge assessment—and the Pedagogy and Professional Responsibilities (PPR) exam, pedagogical and classroom ethics assessment. These two exams are aligned to K – 16 state and national standards. The PPR is a general pedagogy test that all Texas preservice teachers must pass, whereas, the TExES focuses on mathematical domains and mathematical pedagogy domains to some degree. This section discusses the intensity at which mathematics is assessed by TExES exams.

The TExES focuses on preservice teachers’ competency for specific content areas for the grade bands Early Childhood through six (EC – 6), four through eight (4 – 8), and eight through twelve (8 – 12). The percentage of mathematical content and pedagogical content covered on the TExES exams are dependent on the type of certification the preservice teachers are seeking. The mathematics portion of the TExES exam for EC – 6 Generalists focuses on four mathematical content domains and five pedagogical content domains. The four content domains are number concepts, patterns and algebra (including functions), geometry and measurement, and probability and statistics; the pedagogical content domains are mathematical processes, mathematical perspectives, mathematical learning and instruction, and mathematical assessment, and professional development (TEA, 2012). Similarly, the Generalist for grades 4 – 8 exam focuses on the same
content and pedagogical content domains as the EC – 6 Generalist, except professional development. The 4 – 8 mathematics and mathematics/science composite certification tests focus on more mathematical content and pedagogical content items than compared to the Generalists’ exams. The test given to preservice teachers seeking certification in grades 4 – 8 mathematics concentrates solely on mathematics; the content domains are the same as the Generalists’ content domains, but the two pedagogical content domains emphasized on the 4 – 8 mathematics test is mathematical processes and perspectives and mathematical learning, instruction, and assessments. The 4 – 8 mathematics/science composite test has the same domains as the 4 – 8 mathematics test but with fewer mathematics questions. The low percentage of mathematical content and pedagogical content knowledge assessed on some of these exams, for example EC – 6 Generalist, can result in passing a preservice teacher who has not mastered the mathematical content and pedagogy.

For grade band EC–6, elementary school preservice teachers on a traditional certification track at Texas State, as opposed to an alternative or emergency certification route, must receive a Bachelor of Science (B.S.) in Interdisciplinary Studies (EC–6; Texas State, 2010). At Texas State, there are two B.S. degrees in Interdisciplinary Studies with an emphasis in early childhood to grade six: English as a Second Language (ESL) Generalist and Bilingual Generalist (Texas State, 2010). These degrees do not necessitate upper level mathematics courses unless the preservice teacher is specializing in mathematics, but both degrees require at least College Algebra or Mathematics for Business and Economics (Texas State, 2010). Upon graduation, EC–6 preservice teachers must pass the mathematics content and pedagogical content sections of the
TExES test. However, there is little emphasis on mathematics on this exam; meaning, a teacher is not required to master mathematics for certification.

The content areas assessed by the TExES for ESL Generalist EC–6 certification are language concepts and language acquisition, ESL instruction and assessment, foundations of ESL education, cultural awareness and family and community involvement, English, language arts and reading, mathematics, social studies, science, and fine arts, health, and physical education (TEA, 2011b, p. 10). The mathematics section of the TExES exam comprises only 13% of the total test questions. The content areas assessed by the TExES for Bilingual Generalist EC–6 certification are “bilingual education, English, language arts and reading, mathematics, social studies, science, and fine arts, health and physical education” (TEA, 2011c, p. 12). For this certification, the mathematics section of the TExES exam is also only 13% of the total test questions. For both EC–6 certification exams, the percentage of mathematics questions are less than 20%; therefore, preservice teachers failing the mathematics domains can still become certified to teach as long as they receive 240 points out of 300 points on the overall exam (TEA, 2012). Preservice teachers receiving certification who have not passed the mathematics domains are not necessarily qualified to teach K–6 (Ball, Hill, & Bass, 2005; Cavallo, 2010).

In Texas, middle school preservice teachers on the traditional certification track must complete a B.S. in Interdisciplinary Studies with an emphasis in grades 4–8 that requires taking College Algebra prior to Precalculus (Texas State, 2010). At Texas State, there are three certifications for B.S. in Interdisciplinary Studies with emphasis in grades 4–8: Generalist, Mathematics, and Science and Mathematics Composite (Texas State,
2010). The content areas assessed by the TExES for 4–8 Generalist certification are “English, language arts and reading, mathematics, social studies, and science” (TEA, 2011d, p. 12). The mathematics section is 23% of the total test questions, meaning 4–8 Generalist can pass the overall exam even if they fail the mathematics portion. On the other hand, the TExES rigorously assesses grades 4–8 mathematics and mathematics/science composite preservice teachers in mathematics; the mathematics preservice teachers are assessed exclusively in mathematics, and the 4–8 mathematics/science composite TExES test is 51% mathematics. If students do not pass the mathematics section, they fail the test.

To aid middle school mathematics and mathematics/science preservice teachers in the development of mathematical competency, the MDMP courses provide an environment for preservice teachers to learn content knowledge assessed on the mathematics portion of the certification exams. Past research has shown that students who complete the MDMP have a higher rate of passing credit-bearing mathematics courses and the Texas Higher Education Assessment (THEA), meaning they are college ready (Mireles, 2010).

The Texas state certification assessments, the TExES and the PPR, do not prioritize mathematical knowledge for preservice teachers seeking certification in EC–6 ESL, EC–6 Bilingual, and 4–8 Generalist. The lack of mathematical knowledge assessment is detrimental when trying to classify the overall quality of a teacher, or preservice teacher, based on certification alone. While preservice teachers who do not pass all content areas on the TEKS (TEA, 2011) exam are considered qualified, they do not meet the standards for quality teachers—possessing content and pedagogical content
knowledge for mathematics (Ball, Hill, & Bass, 2005).

**National certification.** National certification does not replace state certification, but instead complements it. State standards apply to all teachers, whereas national certification applies to teachers with at least three years of experience (NBPTS, 2002). The National Board for Professional Teaching Standards (NBPTS, 2002) accepts applications from teachers who meet the minimum teaching requirements and “effectively enhance students learning and demonstrate the high level of knowledge, skills, abilities, and commitments reflected in the five core propositions” for national certification (NBPTS, 2010, p. 3). The five national certification standards stipulate that teachers must (1) be “committed to students and their learning,” (2) have content knowledge and pedagogical content knowledge, (3) be “responsible for managing and monitoring students learning,” (4) be reflective, and (5) be a “member of learning communities” (NBPTS, 2010, pp. 3-4). The MDMP guides preservice teachers on the path to meeting NBPTS requirements by teaching them with a multitude of instructional teaching techniques, using activities that involve communication between peers, and practicing deconstruction and reconstruction of mathematical concepts.

These requirements are similar to the recommendations given by the Association of Mathematics Teacher Educators (AMTE). The AMTE (2010) advocates for elementary teachers who specialize in mathematics instead of generalists since most generalists are weak in mathematical content. Elementary mathematics specialists (EMSs) are required to complete extra course work in elementary and middle school mathematics, pedagogy, and administration. According to the MET report, mathematics preservice teachers need “courses that develop a good understanding of the mathematics
they will teach” (CBMS, 2012, p. 7). The MDMP supports the understanding of grades K – 12 mathematical content by making connections of the content to algebraic reasoning, such as dividing polynomial expressions using the long division algorithm. The elementary preservice teachers are required to complete at least 12 hours on “fundamental ideas of elementary mathematics, their early childhood precursors, and middle school successors,” while the middle school preservice teachers are required to take at least 24 hours of mathematics (p. 7). The MDMP is another mathematics course that can provide preservice teachers with experience in becoming “mathematical thinker[s]” (p. 8). The MDMP students have opportunities to collaborate with peers to model and explain mathematical concepts and develop algorithms to understand structure and processes. The types of instruction, activities, and opportunities to collaborate with peers provide MDMP preservice teachers extra time to develop mathematical knowledge recommended by the CBMS (2012). Along with the requirements from the CBMS (2012), teachers who are recent graduates do not yet have all the qualifications required by the NBPTS or AMTE, but their experiences in the MDMP and teacher preparation programs help develop their effective teaching skills and knowledge of content and pedagogy (AMTE, 2010; NBPTS, 2010).

**Professional Development and Appraisal System.** Texas school districts have the option to adopt the Professional Development and Appraisal System (PDAS), developed by the TEA, to aid administrators and principals in the evaluation of in-service teachers. The eight domains PDAS appraise are (1) “active, successful student participation in the learning process,” (2) learner-centered curriculum, (3) “evaluation and feedback on student progress,” (4) “management of student discipline, instructional
strategies, time and materials,” (5) collaboration between colleagues and teachers and administrators, (6) teacher growth and teacher improvement through years of educating, (7) curriculum aligned to state standards, and (8) high standard of proficiency for all students on campus (TEA, 2005, p. 20-21). The eight domains are evaluated with separate criteria as indicated by the PDAS; the composite evaluation is used to assess the teacher’s overall performance in the classroom.

The purpose of evaluating teachers’ efficacy is to identify how “to improve student performance through the professional development of teachers” (TEA, 2005, p. 6). Preservice teachers can learn the qualities specified in the eight domains before they graduate and pursue careers in teaching. The MDMP exposes preservice teachers to standards-based lessons that incorporate learner-centered activities that may aid in the development of preservice teachers’ content and pedagogical knowledge. The skills gained by preservice teachers in the MDMP can be identified and evaluated in their lesson plans using the lesson plan rubric designed for this study.

**Lesson plan rubrics.** Rubrics are commonly used by teachers to appraise students’ coursework and assessments (Moskal, 2000). Specifically designed lesson plan rubrics can furthermore be used to evaluate teachers’ content, pedagogical, and pedagogical content knowledge (Stronge, 2007). Milkova (2011) described lesson plans as including an introduction, a body, and a conclusion; whereas Stronge (2007) expanded on Milkova’s description to incorporate standards, objectives, and assessments to define an effectual lesson plan.

First, in lesson plan introductions, anticipatory sets should present mathematical topics or real-world problems to initiate classroom discussion prior to teaching the lesson
topic(s) (Milkova, 2011). Second, the body of the lesson plan ought to be scripted in enough detail so that another teacher can recreate the lesson. Then, the conclusion needs to revisit the material covered in the lesson (Stronge, 2007). If teachers present an unanswered question at the beginning of class, then students should be able to determine the solution by the end of class (Milkova, 2011). Finally, lesson plans must clearly state objectives—what the students should know at the completion of the lesson—and incorporate standards, objectives, and assessments that align to state and national standards (Milkova, 2011).

Lesson plan rubrics developed by Alabama Learning Exchange (ALEX, 2012), Culturally and Linguistically Diverse Exceptional Students (CLDE, 2012), and American Library Association (ALA, 2012) have provided insightful criteria for evaluating lesson plans. These lesson plan rubrics are not published, but their corresponding organizations use them to qualify lesson plans on a scoring scale.

While these lesson plan rubrics are distinct from one another, they all concede there are four basic characteristics of quality lesson plans. All of these rubrics ascribe higher value to lesson plans that are (1) clear, concise, and have standards-based objectives that explain what students are required to know, understand, or be capable of achieving; (2) age and ability appropriate; (3) scripted to detail the types of instruction, examples, and scaffolding that should be included in the instruction; and (4) appropriately designed to assess students’ content knowledge (ALEX, 2012; AASL, 2012; CLDE, 2012; Milkova, 2011; Stronge, 2007). These rubrics also established rudimentary foundations for evaluating lesson plans; however, they failed to formally address teachers’ content, pedagogical, and pedagogical content knowledge and
disposition, as well as informing the various learning philosophies used in the lesson plans.

Learning Theories

The learning theory component of the theoretical framework (see Appendix B) design for this study is comprised of three philosophical learning theories—constructivism, behaviorism, and cognitivism—all of which are observed in the Model Developmental Mathematics Program curriculum. These are key constructs when determining the type(s) of instruction to employ in the classroom and can be detected in lesson plans by the rubric designed for this study. This section discusses the philosophers who contributed to the development of these learning theories and clarifies how these theories can be used in the classroom by presenting examples of MDMP lesson plans incorporating constructivism, behaviorism, and cognitivism.

Constructivism. Constructivists generally believe “individuals construct their own knowledge or mental versions of the world” (Harlow, Cummings, & Aberasturi, 2006, p. 41). Jean Piaget, a constructivist theorist, believed children constructed knowledge based on their experiences in the environment in which they interacted (Harlow et al., 2006). According to Piaget, individuals can construct knowledge through either physical experiences by moving objects, or mental experiences by building on preexisting knowledge (Harlow et al., 2006, p. 44); meaning, individuals learn while interacting with objects or having ideas that she or he explores, developing an understanding by making connections. Similarly, constructivist theorist John Dewey referred to constructivism as the understanding of the “fundamental concepts and methods of the respective disciplines in accessible, engaging, and powerful ways,” also
called psychologizing (as cited in Smith & Girod, 2003, p. 295). On the other hand, Lev Vygotsky and Jerome Bruner defined constructivism as an interactive environment where learning is a product of discussion (Kearsley, 2011). Vygotsky claimed that students should interact with society first, then reflect on their experiences to develop knowledge and understanding; whereas Bruner thought learning was an active process by which students learn based on their current and past knowledge (Kearsley, 2011).

Unlike Vygotsky and Bruner, Piaget was “interested in why children fail[ed] to learn,” not how they learned (Pass, 2003, p. 9). Piaget believed construction of new knowledge could only be completed when new ideas “could not be assimilated into prior knowledge” (Harlow et al., 2006, p. 45). During the process of learning, an individual first experiences disequilibrium (Harlow et al., 2006). Disequilibrium occurs when information does not match current schema; the person will search for meaning “through the process of accommodation, a new schema is constructed into which the information can be assimilated and equilibrium can be temporarily reestablished” (Harlow et al., 2006, p. 45). The Model Developmental Mathematics Program (MDMP) students deconstruct prior knowledge and reconstruct new connections by assimilating new knowledge into a new cognitive schema to develop an understanding of the content.

In addition, Bruner concluded that learners rely on existing cognitive structures, like schemas or mental models, to explore their experiences to derive in-depth meaning (Kearsley, 2011). Bruner also claimed that intelligence starts at infancy and develops not only in response to personal thoughts, but due to the manner in which people share understanding—known as intersubjectivity (Levorato, 2008). Students sharing their thoughts with others promote “various modes of meaning-making and communicating”
that are needed by students to learn (Takaya, 2008, p. 2). Communication leads to creating a “community in which multiple ways of learning take place as opposed to the largely cultureless mode of learning which dominates schools” (Takaya, 2008, p. 2). The MDMP does not incorporate a cultureless mode of learning; instead, the MDMP curriculum integrates communication through group work, cooperative learning groups, and classroom discussions. These activities allow preservice teachers to communicate with their classmates and interpret their peers’ ideas, which may help preservice teachers gain a perspective of pedagogy and the mathematical content.

Similarly to Bruner, Vygotsky concentrated on educational psychology and focused “on the social rather than individual aspects of pedagogy” (as cited in Pass, 2003, p. 11). Vygotsky’s social development theory claimed “that social interaction plays a fundamental role in the development of cognition” (as cited in Kearsley, 2011; Social Development section, para. 1). He developed the zone of proximal development (ZPD), “a level of [cognitive] development attained when . . . [people] engage in social behavior” (as cited in Kearsley, 2011; Social Development section, para. 2). The MDMP incorporates ZPD through cooperative learning groups. Discussion between peers helps lower performing students because they hear explanations from someone at the same grade level and with similar classroom experiences. The higher performing students also benefit from discussion groups because they become the teachers when they explain the concepts to their peers.

According to constructivist theorists Dewey, Piaget, Bruner, and Vygotsky, discovery-based instruction, including communication among peers in the classroom or in other social environments, promotes learning (Kearsley, 2011; Levorato, 2008; Pass,
The elements of constructivist theory, incorporating hands-on activities and communication between students, are infused in the MDMP curriculum through interactive classroom activities and group discussions.

**Behaviorism.** Unlike constructivists, behaviorists believe changes in human and animal behaviors are indicators of the learning process (Ertmer & Newby, 1993).

According to behaviorists, learning is accomplished when an appropriate response is achieved after an environmental stimulus has occurred (Ertmer & Newby, 1993). The noticeable changes that occur in the students’ behavior is known as learning.

Behaviorists describe students “as being reactive to conditions in the environment as opposed to taking an active role in discovering the environment” (Ertmer & Newby, 1993, p. 55).

The different types of stimulus-response (S-R) theories that affect students’ learning are classical conditioning, operant conditioning, and connectionism. Watson, the first known behaviorist, derived his understanding of behaviorism in learning processes from Ivan Pavlov’s research (Kearsley, 2011). Pavlov first observed classical conditioning during his research on dogs’ response to stimuli prior to feeding time. He noticed that when the dogs ate, they produced excessive amounts of saliva compared to their state before eating. He then exposed the dogs to a conditioned stimulus: a bell ringing to indicate feeding time; after exposing them to the combined stimuli for several weeks, the dogs produced excess saliva when he rang the bell (Schacter, Gilbert, & Wegner, 2011, pg. 265). Pavlov determined, through his research, there were four steps of classical conditioning: (1) demonstration of the neutral stimulus, (2) demonstration of the unconditional stimulus, (3) demonstration of the neutral stimulus and unconditional
stimulus together, and (4) demonstration of the conditioned stimulus (as cited in Schacter et al., 2011, pg. 266). An example of classical conditioning in education is the process by which students’ performance is changed through rewards or other types of positive reinforcements—stimuli that encourage them to increase the frequency of wanted behavior. For example, teachers can give students rewards for completing tasks successfully; through consistency of combined stimuli, students will then perform the tasks successfully without rewards from their teachers.

Another S-R response was B. F. Skinner’s operant conditioning theory, which suggested that adjustments in behavior occur as a result of reinforcement, “anything that strengthens the desired response” (Kearsley, 2011, Operant Conditioning section, para. 2). In other words, positive reinforcements promote acceptable behaviors, whereas undesired behaviors are discouraged through negative reinforcements or punishments. Negative reinforcement occurs when the number of desired responses increases as the stimulus is being removed (Kearsley, 2011). When reinforcements are used consistently and positive responses occur, then without the stimulus the expected responses still occur (Kearsley, 2011).

Another interpretation of cognitive theory, Thorndike’s connectionism theory, is divided into three laws: (1) the law of effect, (2) the law of readiness, and (3) the law of exercise (as cited in Kearsley, 2011). The law of effect assumes a response will follow a certain situation—S-R theory. The law of readiness states that only when a specific sequence of events have occurred is the learner ready to reach their goal (Kearsley, 2011). If the learner is not ready, they are “blocked” from learning the content. The law of exercise is characterized by the strengthening of connections resulting from practice or
the loss of cognitive networks due to lack of practice (Kearsley, 2011). Thorndike’s three laws are represented by vertical alignment of curriculum, scaffolding of simple to complex concepts, and continuous practice with online and paper homework in the MDMP.

**Cognitivism.** Cognitive theorists have claimed that “learning is equated with discrete changes between states of knowledge rather than with changes in the probability of response” (Ertmer & Newby, 1993, p. 55). They concentrate on “what [students] know and how they acquire[d] it” (Ertmer & Newby, 1993, p. 55). Robert Gagne, a cognitive theorist, listed five major types of learning: “verbal information, intellectual skills, cognitive strategies, motor skills, and attitudes” (as cited in Kearsley, 2011, Conditions of Learning section, para. 1). Each type of learning requires different types of external and internal influences. Gagne concentrated on intellectual skills, because he thought intellectual skills was the basis for learning all subject areas through the development processes or frameworks (Kearsley, 2011). According to Gagne, learning tasks that range from simple to complex tasks develop intellectual skills. An example of tasks ranging from simple to complex are “stimulus recognition, response generation, procedure following, use of terminology, discriminations, concept formation, rule application, and problem solving” (as cited in Kearsley, 2001, Conditions of Learning section, para. 2). Drawing from cognitive theory, the Model Developmental Mathematics Program (MDMP) curriculum progresses from simplistic to complex tasks; for example, lessons that incorporate the Algorithmic Instructional Technique (AIT) method shifts from teacher-centered instruction to student-centered instruction, thereby making students fully responsible for the algorithmic models they develop. David Merrill and
Charlie Reigeluth established two cognitive theories: the component display theory (CDT) and the elaboration theory (Kearsley, 2011).

The CDT classified two types of learning: content and performance. Content learning relies on “facts, concepts, procedures, and principles,” while performance learning consists of “remembering, using, and generalities” (as cited in Kearsley, 2011, Component Display Theory section, para. 1). These two learning processes are needed by students to be successful in credit-bearing mathematics courses. The CDT is used in the MDMP lessons by teaching MDMP students how to use mathematical facts and concepts to develop procedures for solving algebraic equations and simplifying algebraic expressions. A tool utilized by the MDMP is the AIT method—procedures are developed to complete mathematics tasks. The performance-learning component is taught to the MDMP students though formula memorization, such as the quadratic formula, and how to use them correctly through deconstruction and reconstruction.

The CDT describes primary and secondary presentations as tools for presenting mathematical content. Primary presentations include rules, examples, recall, and practice; the secondary presentations are comprised of “prerequisites, objectives, helps, mnemonics, and feedback” (Kearsley, 2011, Component Display Theory section, para. 1). Based on the CDT, a “complete lesson would consist of objective followed by some combination of rules, examples, recall, practice, feedback, helps, and mnemonics appropriate to the subject matter and learning task” (Component Display Theory section, para. 2). However, the most effective combination of presentations are dependent upon on the content and the learners’ knowledge. In the MDMP, both types of presentations are used to teach and reconstruct mathematical content, such as the deconstruction and
reconstruction of misused mnemonics (Mireles, 2010).

The CDT acknowledges different types of memory such as algorithmic memory. Students construct algorithmic memory by remembering and using schema or by finding new schema to retain the process of completing a task (Kearsley, 2011, Component Display Theory section, para. 3). An important “aspect of the CDT framework is learner control,” meaning the students choose the presentation and content components that coincide with their preferred method of learning (Kearsley, 2011, Component Display Theory section, para. 4). The CDT manifests in the MDMP curriculum when students are required to write algorithms; deconstruct and reconstruct mnemonics; and explain mathematical definitions, examples, and rules.

Reigeluth developed the elaboration theory which is composed of seven major strategies: “(1) an elaborative sequence, (2) learning prerequisite sequences, (3) summary, (4) synthesis, (5) analogies, (6) cognitive strategies, and (7) learner control” (as cited in Kearsley, 2011, Elaboration Theory section, para. 2). According to Kearsley (2011), the elaborative sequence is the most important since it represents the progression of complexity within a lesson. The MDMP curricula examples for simplifying expressions and solving equations evolve from simple one to two step examples to complex six to seven step examples. The concepts presented to the MDMP students also advances in complexity; for instance, concrete concepts are introduced before abstract concepts. An MDMP example of progressing from concrete to abstract is the use of algebra tiles in the operations on polynomials lesson plan.

Sequential ordering of content within lessons is an additional key construct for developing effective lessons. Organized lessons summarize and review concepts before
proceeding to new content. Reviewing previously learned mathematical concepts helps stimulate learners’ memories so they are able to establish “meaningful context into which subsequent ideas and skills can be assimilated” (Kearsley, 2011, Elaboration Theory section, para. 1). The MDMP lesson plans are vertically aligned so each new lesson elaborates or introduces new content by expounding on the previous lesson.

These three learning theories—constructivism, behaviorism, and cognitivism—were intended to provide educators with a framework to help interpret learning processes and develop strategies to teach (Kearsley, 2011). Accordingly, the lesson plan rubric used in this study was designed to detect elements of these theories in preservice teachers’ lesson plans (see Appendix C). Preservice teachers who effectively use a variety of theories in their lesson plans embody the attributes of a quality teacher.

Functions

According to Willoughby (1990), functions are “one of the most pervasive and important topics” in mathematics and should be taught at all grade levels, K–12, to some degree (p. 77). Willoughby (1990) discussed that students can develop an understanding of functions in earlier grades by using concrete demonstrations, and then extending the knowledge to abstract representations in upper level grades. Furthermore, he described ways to implement functions at each grade level from K through 12.

Elementary preservice teachers must have a solid foundation in the knowledge of functions because they will need to teach the topic in grades K–6. According to Willoughby (1990), grades K–6 should be taught functions using concrete and pictorial representations and activities, such as a function machine as a concrete model in grades K–2 and as a pictorial model in grades 3–6 (Willoughby, 1990).
When teaching functions, teachers should devise and use creative and effective lesson plans that correspond to the abilities and knowledge of their students. An example lesson, recommended by Willoughby (1990), includes a function machine activity for kindergarteners. The tangible function machine is a large box with two openings: one at the top for the input and one at the bottom for the output. The student inside the box is given a function rule; she/he is responsible for taking the items the other students deposit into the input slot and returning the correct number of items through the output opening. For example, based on the rule of plus two—a function rule—the student inside the box would return three pencils if another student put one pencil into the function machine (Willoughby, 1990, p. 78). Similarly, first graders would use a function machine, but instead of using objects for inputs and outputs, they would use numbers on slips of paper; the concept of functions gradually becomes more abstract.

In grade 2, students should use function rules that include multiplication and division. Second grade teachers do not have to use the function machine box, but they should use pictorial representations of the function machine to teach the same concept. At the third grade level, the function box is omitted and another pictorial representation is introduced—the arrow diagram—along with variables. Arrow diagrams illustrate two disjoint sets and arrows describe the relationship between elements in the independent and dependent sets. Next, grades 4–6 students evaluate functions and compute inverse functions by doing and undoing given operations.

In the earlier grades, K–6, the concept of functions is gradually becoming more abstract. Grades K–3 use concrete and pictorial representations to teach functions; whereas, the latter elementary grades, 4–6, use symbolic notation to represent functions.
Inverse functions are also introduced in grades 4–6 by doing and undoing operations.

The mathematical content and instruction used to teach middle school and high school students are abstract compared to the lessons described for the elementary school students (TEA, 2011; Willoughby, 1990). At the middle school grade levels, 6–8, function notation is introduced and functions are described using graphical, tabular, and pictorial representations (Willoughby, 1990). The MDMP uses similar methods to teach functions—graphical representations that help students draw connections between different types of functions and their characteristics.

The MDMP curriculum consists of five function topics—linear, quadratic, polynomial, rational, and radical functions. The MDMP students create different modes of representation, identify function characteristics, and connect functions to other content areas. The MDMP supports the content knowledge and pedagogical content knowledge of functions by preservice teachers learning functions before enrolling into college algebra.
CHAPTER III

METHODOLOGY

This chapter provides complete descriptions of the questions addressed in this study, the details of the population and sample, research design, and instrumentation used to collect and evaluate the participants’ results on the measures of the Early Indicators of Effective Teachers. This mixed-methods research study evaluated the potential differences between Model Developmental Mathematics Program (MDMP) track and College Algebra track elementary and middle school preservice teachers’ knowledge and disposition; this chapter concludes with the scope and limitations of this study.

Purpose of the Study

The purpose of this study is to determine if the Model Developmental Mathematics Program (MDMP) curriculum—teaching techniques and content—contribute to the development of quality elementary and middle school teachers through various data collecting methods—multiple-choice test, lesson plan writing activity, surveys, and interviews—to evaluate preservice teachers’ pedagogical, content, and pedagogical content knowledge and disposition upon completion of this Texas State program. While standards-based best practices have been consistently associated with the instructional methods used by quality teachers, preservice teachers often do not experience different standards-based best practices in college mathematics courses (Graeber, 1999; Matthews, Rech, & Grandgennett, 2010). Whereas if the lesson plans of
the MDMP are implemented as intended, employs a range of standards-based best practices, while most college curriculum continues to rely primarily on lecture-based instruction (Mireles, 2010; Schiro, 2008). Opponents of Scholar Academic practices, such as didactic discourse, have insisted preservice teachers need to be taught using the types of instruction emphasized in K–12 state and national standards, so the preservice teachers are familiar with the tools and instruction that will be expected in the classroom (Bright, 1999). This study assessed preservice teachers’ knowledge and disposition based on participants’ results on the Early Indicators of Effective Teachers instrument to investigate whether the standards-based best practices used in the MDMP contribute to an increased number of observable characteristics typically attributed to quality teachers.

**Research Questions**

This study asked the following questions to determine if the standards-based best practices incorporated into the Model Developmental Mathematics Program (MDMP) curriculum affect preservice teachers’ content, pedagogical, and pedagogical content knowledge and disposition. The first question addressed the participants’ scores on the modified Mathematical Knowledge for Teaching (MKT) assessment. To understand the differences between the groups’ content knowledge, the scores of the participants who completed the MDMP course(s) were compared to those who entered directly into College Algebra. The second question asked to identify the attributes of quality teachers observed in the participants’ lesson plans and interviews compared to the College Algebra track participants; the last question examined and compared four MDMP participants having varying levels of MKT.

**RQ 1:** Is there a significant difference in Mathematical Knowledge for Teaching
scores for elementary school preservice teachers and middle school preservice teachers who completed the Model Developmental Mathematics Program at Texas State compared to those who started in College Algebra—controlling for mathematics access at community college?

RQ 2: What early indicators of effective teachers were identified in preservice teachers at Texas State who completed the Model Developmental Mathematics Program compared to those who started in college algebra using the lesson plan rubric and interview analyses?

RQ 3: How do impressions from the Model Developmental Mathematics Program affect the Model Developmental Mathematics Program track participants’ experiences, and how do these experiences inform their performances on the measures of the Early Indicators of Effective Teachers?

Population and Sample

Demographics of the University. The enrollment size of Texas State University-San Marcos (Texas State) for the Summer 2011 semester was 12,697 students; for the Fall 2011 semester was 32,572 students; and for the Spring 2012 semester was 32,001 students (Texas State, 2012). Demographics for these semesters were collected. Approximately 57% of the school’s population was female and 43% male. The approximate age distribution of Texas State students was 70% 25 years old and younger, 22% between the ages of 25 and 34, 5% between the ages of 35 and 44, and 3% over the age of 45. Approximately 63% of the students were white, 24% were Hispanic, 7% were black, and 6% had other ethnic backgrounds.

The Texas State demographics reflected a diverse student population;
approximately “32 percent of student body is ethnic minority” (The Hispanic Outlook: In Higher Education Magazine [The Hispanic Outlook], 2012). According to The Hispanic Outlook, Texas State is “ranked 16th in the United States (U.S.) for the number of bachelor’s degrees awarded to Hispanic students” (The Hispanic Outlook, 2012).

**Population.** The population of the study consisted of all the College Algebra students during the Summer 2011, Fall 2011, and Spring 2012 semesters. This population was comprised of students who took the Model Developmental Mathematics Program (MDMP) and those who started directly into College Algebra. Nearly 12.5% of Texas State undergraduate students were enrolled in the Model Developmental Mathematics Program (MDMP); during the Spring 2012 and Summer 2011 semesters, an average of 5.4% of students were enrolled in the MDMP. Specifically, in the Spring 2011 semester, there were 1899 total students enrolled in the MDMP, 492 in the Summer 2011 semesters, 3441 in Fall 2011, and 1596 in Spring 2012. Table 1 provides the breakdown of the numbers of students in each developmental mathematics course, College Algebra course, and those majoring as either preservice teachers for grades Early Childhood (EC)–6 or preservice teachers for grades 4–8 for each semester.
Table 1

*Population of Texas State Students by Class and Major*

<table>
<thead>
<tr>
<th>Math Course</th>
<th>Spring 2011</th>
<th>Summer 2011</th>
<th>Fall 2011</th>
<th>Spring 2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>1300 (Pre-Algebra)</td>
<td>366</td>
<td>57</td>
<td>471</td>
<td>234</td>
</tr>
<tr>
<td>1311 (Basic Math)</td>
<td>1533</td>
<td>435</td>
<td>2970</td>
<td>1362</td>
</tr>
<tr>
<td>1315 (College Algebra)</td>
<td>3666</td>
<td>978</td>
<td>6702</td>
<td>4188</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Major</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Preservice EC–6</td>
<td>1494</td>
<td>638</td>
<td>1549</td>
<td>–</td>
</tr>
<tr>
<td>Preservice 4–8</td>
<td>194</td>
<td>76</td>
<td>207</td>
<td>–</td>
</tr>
</tbody>
</table>

*Note:* The Spring 2012 data for majors will not be available until after August of 2012. This date is after the publication of this document.

The elementary and middle school preservice teachers enrolled in College Algebra took either one developmental mathematics course, two developmental mathematics courses, or enrolled directly into College Algebra at Texas State University-San Marcos (Texas State; see Figure 3). All Texas State elementary school preservice teachers are required to take at least one credit-bearing mathematics course—College Algebra or Mathematics for Business and Economics. Middle school preservice teachers must take College Algebra or be exempted as a prerequisite to the upper-level mathematics courses required by their respective certifications (Texas State, 2010). The Elementary Generalist and Middle School Generalist, Mathematics, and Mathematics/Science composite preservice teachers in College Algebra were chosen because (1) of the short duration of time between completion of the developmental mathematics course(s) and enrollment in College Algebra, (2) each degree plan requires College Algebra or an equivalent course, and (3) College Algebra is a prerequisite for their pedagogical content courses—method courses that focus on mathematical pedagogy.
(see Figure 3).

### Figure 3: Illustration of Teacher Preparation Tracks.

The diagram represents two timelines each representing one mathematical track.

### Sample.
The sample size for this study was 22 elementary and middle school preservice teachers. The participants included 11 elementary preservice teachers, EC–6, 10 middle school preservice teachers, 4–8, and one all grade levels preservice teacher. Table 2 lists the sample by class—MDMP track or College Algebra track—and major.

The elementary and middle school preservice teachers enrolled in College Algebra at Texas State during the Summer 2011, Fall 2011, and Spring 2012 semesters were asked to participate in this study. A minimum of 30 participants was sought in order to decrease the potential statistical margin of error. All the participants who indicated they were seeking an elementary or middle school certification were contacted, and the respondents were selected based on convenience. The 22 elementary and middle school preservice teachers volunteered to complete the Early Indicators of Effective Teachers instrument—an instrument consisting of two assessments that measured preservice teachers’ content, pedagogical, and pedagogical content knowledge and disposition.
### Table 2

**Sample of Texas State Students by Class and Major**

<table>
<thead>
<tr>
<th>Major</th>
<th>MDMP</th>
<th>College Algebra</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>EC–6 ESL Generalist</td>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>EC–6 Bilingual Generalist</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>4–8 Generalist</td>
<td>6</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>4–8 Mathematics</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4–8 Mathematics-Science Composite</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>All Grade Levels</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>14</td>
<td>8</td>
<td>22</td>
</tr>
</tbody>
</table>

### Research Design

This study is a mixed-methods approach to compare the pedagogical, content, and pedagogical content knowledge and disposition of elementary and middle school preservice teachers who took the Model Development Mathematical Program (MDMP) to those who entered directly into College Algebra. A mixed-methods research design was used in this study because there was a lack of developed instruments to test the appropriate variables needed to evaluate preservice teachers’ knowledge and disposition.

Hill, Schilling, and Ball (2004) developed the Mathematical Knowledge for Teaching (MKT) assessments, a product of the Learning Mathematics for Teaching (LMT) project at the University of Michigan. The test items developed were based on the researchers’ experiences in the classroom and teachers’ responses to MKT surveys. The surveys were composed of various types of questions about subject matter knowledge. Hill, Schilling, and Ball (2004) utilized factor analysis and scaling
techniques on the survey questions to determine unobserved variables—types of knowledge. The K–6 number and operation test, the first MKT assessment developed, was comprised of common content knowledge (CCK) and subject content knowledge (SCK) items (Hill, Schilling, & Ball, 2004). These items were placed into three comparable tests—forms A, B, and C. These tests were piloted then statistical analyses were performed on in-service teachers’ results. There were three types of analysis performed on the three MKT tests: (1) “exploratory factor analyses of the three forms,” (2) factor analysis with removed items, and (3) “bi-factor analyses, to further assess the issue of multidimensionality and to resolve questions regarding knowledge of students and content items” (Hill et al., 2004, p. 19).

Quantitative analysis was used to determine preservice teachers’ common content knowledge (CCK) and specialized content knowledge (SCK) through comparison of participants’ modified Mathematical Knowledge for Teaching (MKT) test scores (see Figure 1). The modified MKT test scores were compared to determine any significant difference in the content knowledge score means. According to researchers, pedagogical and pedagogical content knowledge, important traits of quality teachers, are not assessed by the MKT test. In addition, each participant was asked to write a lesson plan on functions. Functions were selected because they are a prominent topic at all grade levels, emphasized in the state and national standards, and are foreshadowed in the MDMP curriculum. The lesson plan activity was scored using criteria indicated in the developed rubric (see Appendix C). The scores given to each lesson plan were ordinal in nature; further interviews and surveys were used to explain the various results of the lesson plans. Following completion of the MKT test and the lesson plan activity, participants
were interviewed. The participants were asked questions to better understand their disposition towards learning and teaching mathematics, and clarify any uncertainties regarding their lesson plans. These instruments, the modified MKT and the lesson plan activity, are collectively referred to as the *Early Indicators of Effective Teachers* instruments.

**The appropriateness of the modified Mathematical Knowledge for Teaching measures.** The COMPASS®, THEA®, PRAXIS®, and MKT tests were instruments considered for this study’s quantitative assessment to measure participants’ mathematical content knowledge. While the COMPASS® and THEA® are convenient assessments to administer, they cost monies; furthermore, they only concentrate on testing common content knowledge, not specialized content knowledge. The PRAXIS® measures preservice teachers’ content knowledge and teaching skills for a variety of certifications. The assessment considered was the Early Childhood: Content Knowledge exam; however, this exam would have been inconvenient to administer because there are no testing sites at Texas State. Moreover, the results would not have been available for approximately 4 weeks after being administered. The most appropriate assessment was the MKT test. The MKT assessments measure common content knowledge (CCK) and specialized content knowledge (SCK) for elementary and middle school geometry, probability, statistics, proportional reasoning, rational numbers, place value, functions, and algebraic concepts. The MKT test is free to administer and grade as long as the researcher has completed the LMT training program. The researcher of this study completed the Learning Mathematics for Teaching (LMT) workshop at the University of Michigan on Monday, May 23, 2011. The MKT test could also be adapted to answer the
questions posed in this research study since each reported item had a difficulty and a
discriminant value for developing new assessments. For this study, items were chosen to
correlate to the mathematical topics numerical expressions, algebraic expressions,
equations, and functions (see Appendices D and E) because these items reflect the
mathematical domain of the MDMP curriculum. The concepts taught in the MDMP
courses are the necessary topics needed by developmental mathematics students to
become college-ready versus College Algebra students who are already deemed college-
ready, so the items chosen for the MKT test aligned to the MDMP curriculum (see
Appendix I). In effect, the modified MKT assessment quantitative diagnostic tool met
this study’s requirements—it was accessible and provided the information needed to
answer the research questions posed.

<table>
<thead>
<tr>
<th>Mathematical Domains</th>
<th>Common Content Knowledge (CCK)</th>
<th>Specialized Content Knowledge (SCK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Expressions</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Equations</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Functions</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

*Figure 4: Distribution of Items by Mathematical Domains and Domains of Knowledge. The values in the table represent the number of items in each subcategory.*

**The appropriateness of the lesson plan activity.** The MKT test assessed only
preservice teachers’ subject matter knowledge (SMK) because there were a limited
number of MKT items that assessed preservice teachers’ pedagogical content knowledge.
Due to the lack of questions, alternative assessments were researched. The Pedagogy and Professional Responsibilities (PPR) test, developed by Texas Education Agency (TEA), is an assessment for evaluating preservice teachers’ pedagogical knowledge, not pedagogical content knowledge. Another unfavorable characteristic of the PPR is the cost to administer the assessment. The PPR was not used because of the restriction of the type of knowledge it evaluated and the cost of the assessment; instead, a lesson plan activity was used to capture some aspects of preservice teachers’ knowledge and disposition.

**How to assess the lesson plans.** To analyze participants’ lesson plans uniformly, a rubric focused on assessing lesson plans and not participants’ teaching performance of the lesson plans was developed. The participants’ lesson plans were evaluated on the presence of key components of the lesson plan structure, the knowledge and disposition of the participants, the use of different learning theory approaches, and the alignment of the lesson plan to state and national students’ content and process standards. The participants’ scored lesson plans were used to compare the MDMP participants’ and College Algebra participants’ scores that reflected their pedagogical and pedagogical content knowledge and disposition. The participants’ pedagogical knowledge—understanding the process of learning to teach—was portrayed in the lesson plan’s subcategories: pedagogical knowledge, the type of instruction used based on the learning theory approaches, and lesson plan structure. The pedagogical content knowledge—knowledge of mathematics intertwined with the “knowledge of [the] learner, learning, and pedagogy” (Ball & Bass, 2000, p. 88)—was represented in the subcategories’ pedagogical content knowledge and the type of instruction used based on the different
learning theories. Disposition—beliefs “that influence behaviors toward students, families, colleagues, and communities” (NCATE, 2002, p. 53)—was captured by the disposition subscale of teacher quality.

In addition to participants’ lesson plans, designated MDMP lesson plans were also evaluated using the same lesson plan rubric. Since the participants were developing lesson plans on functions, the five function lesson plans from the highest-level developmental mathematics course, Basic Mathematics (BM), was assessed with the lesson plan rubric. The chosen MDMP lesson plans include *Introduction to Linear Functions, Graphing Quadratic Functions, Graphing Polynomial Functions, Rational Functions,* and *Radical Functions.* The BM function lesson plans were developed and have evolved within the past 10 years, whereas the participants were undergraduate students who only had approximately 30 minutes to develop their function lesson plan (Mireles, 2010). Because the BM lesson plans were developed by professional educators, evolved to accommodate the MDMP students, and aligned to the subcategories evaluated by the lesson plan rubric, the function lesson plans were considered the ideal lesson plans. The BM lesson plans results were compared to the MDMP participants’ lesson plan results to see how they fared against the model lesson plans.

The *Early Indicators of Effective Teachers* instrument—the modified MKT test and lesson plan activity—were used to evaluate preservice teachers’ content, pedagogical, and pedagogical content knowledge and disposition. Individual interviews were also utilized to help support the findings and expand on the information not captured by the modified MKT test and lesson plan activity. The *Early Indicators of Effective Teachers* instrument and the interviews were evaluated with case study analysis
to identify specific details about MKT of four MDMP participants with varying levels of MKT.

**Model Developmental Mathematics Program survey.** In addition to the *Early Indicators of Effective Teachers* instrument, a five-question multiple-choice survey was administered to the Basic Mathematics classes and the Pre-College Algebra classes in the Spring 2012 semester. Since the content taught in both courses was different, two surveys were developed to reflect the different instruction and content in each class (see Appendix H). The purpose of these surveys was to determine the validity of the program to ensure all Model Developmental Mathematics Program (MDMP) instructors were teaching using the MDMP lesson plans developed for their course(s) and confirm that the students realize the various teaching methods used in the classroom. The results of these surveys reflected the students’ interpretations of the type of instruction administered in the MDMP classes, their disposition towards the MDMP instruction, and their understanding of the structure of the MDMP curriculum.

**Appropriateness of case study analyses.** To further understand the results of the *Early Indicators of Effective Teachers* instrument and interviews, four Model Developmental Mathematics Program (MDMP) track participants were selected for case study analyses. The participants selected for the case study analyses had varying levels of Mathematical Knowledge for Teaching (MKT), meaning they scored high or low on the modified MKT measures and lesson plan activity compared to other MDMP participants. The different levels of MKT chosen—high or low on both measures or high on one and low on the other measure—were compared to determine similarities and differences amongst the MDMP participants.
Instrumentation Design

Development of Mathematical Knowledge for Teaching assessments. The Learning Mathematics for Teaching (LMT) project developed the Mathematical Knowledge for Teaching (MKT) assessments to evaluate elementary and middle school in-service teachers’ subject-matter knowledge—common-content knowledge (CCK) and specialized-content knowledge (SCK; Hill, Schilling, & Ball, 2004). These MKT items “investigate the mathematical knowledge needed for teaching, and how such knowledge … [is] develop[ed] as a result of experience and professional learning” (Hill & Ball, 2007, What is LMT?, para. 1). The subcategories CCK and SCK represent two of the three subject matter knowledge domains described in the Mathematical Knowledge for Teaching (MKT) model; the third subcategory—knowledge at the mathematical horizon—was not assessed because no items are yet developed.

The LMT (Hill & Ball, 2007) project researchers equated multiple assessments and performed pre-test and post-test evaluations on in-service teachers in the areas of elementary (EC–6) and middle school (4–8) place value; geometry; probability, data, and statistics; proportional reasoning; rational numbers; number concepts and operations; patterns, functions, and algebra to determine validity of each item and to obtain difficulty and discriminant values. The items from these different content area tests assessed in-service teachers’ knowledge of solving mathematical problems, using appropriate definitions, identifying different solution techniques, and justifying different mathematical explanations. Items aligned to the MDMP curriculum were selected and compiled to create the modified MKT assessment.
Development of the modified Mathematical Knowledge for Teaching assessment. This research study used the Mathematical Knowledge for Teaching (MKT) test differently than the developers intended; instead of using a pre-test and post-test, a modified MKT test was developed and given to two comparison groups. The modified MKT test assessed multiple areas of the K–8 subject matter knowledge needed by preservice teachers who planned to teach in that grade band. The questions for the adapted MKT test were selected from the Learning Mathematics for Teaching (LMT) project’s query of MKT items because they corresponded to the MDMP curricular framework: Numerical→Expressions→Equations→Functions. The test items selected included 10 numerical examples, 16 on algebraic expressions, 16 on algebraic equations, and 14 on functions. To organize the chosen items into mathematical domains and teaching tasks—common content knowledge (CCK) and specialized content knowledge (SCK)—a matrix was developed (see Figure 4). The items were distributed evenly in each teaching task category to have an equal number of SCK and CCK items for each mathematical domain; for example, five of the numerical examples were CCK, and the other five were SCK. The distribution of modified MKT test items are displayed in Figure 4 and a detailed item distribution list is in Appendix E. These display the different items for both subscales, CCK and SCK, and for each mathematical domain—numerical expressions, algebraic expressions, algebraic equations, and functions.

In addition to verifying the distribution of SCK and CCK items for each mathematical topic, individual item difficulty was also considered when selecting the items for the compiled assessment to ensure instrument reliability. For a non-equated assessment, like the modified MKT test, the difficulty scale should range from −1 to 1
and the slope should be larger than .5 (Hill & Ball, 2007). The range of difficulty was selected around the mean 0, denoting that “the average teacher in the study [with a score of zero] had a 50-50 chance of getting an item of average difficulty correct” on the test (Garet et al., 2010, p. 14). The test was designed for in-service teachers but the participants for the study were preservice teachers, so easier test items were selected ranging from –2 to 0. The developers of the MKT assessments also suggested choosing items with slopes or discriminate values above 0.5. The test items with higher discriminate values can distinguish between participants’ knowledge (see Appendix E).

A developed MKT test must also satisfy the minimum number of items required by the developers, 15, for the scores to be considered reliable which was fulfilled by the 56-item modified MKT test (Hill & Ball, 2007). Hill and Ball (2007) recommended an information curve, which represents the information explained by the test item(s), with a value above 2.5 to ensure reliability of the participants’ modified MKT test scores; the modified MKT test had an information curve above 8.5 (see Figure 5).

![Figure 5: Test Information Curve for Modified Mathematical Knowledge for Teaching Test.](image)
Development of the lesson plan activity. The modified Mathematical Knowledge for Teaching (MKT) test was used to determine participants’ content knowledge, whereas the lesson plan writing sample was used to evaluate their pedagogical and pedagogical content knowledge and disposition. The lesson plan activity completed by each participant was evaluated using a lesson plan rubric.

The participants were given 1.5 hours to complete the Early Indicators of Effective Teachers instrument—the modified MKT test and lesson plan activity. The lesson plan activity was initiated with the instruction statement “In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.” The space provided for participants to write the lesson plan was blank and extra sheets of paper were provided if needed. During a pilot study completed before this study, it was discovered that lines restricted participants’ responses—the format limited the space available for participants to draw images, therefore, the format was changed for this study.

Development of the lesson plan rubric. Prior to the development of the lesson plan rubric, a pilot study consisting of three case studies was used to determine what types of knowledge can be reflected in preservice teachers’ lesson plans. One participant involved in the pilot study was enrolled in College Algebra and the other two participants were enrolled in the pedagogical content course Principles of Mathematics I. All three participants completed the Model Developmental Mathematics Program (MDMP) at Texas State University-San Marcos (Texas State).

During the pilot study, three participants developed plans on functions and were interviewed to understand how their experiences in the MDMP influenced their
Mathematical Knowledge for Teaching (MKT). Common themes—such as types of instruction and teaching experiences—between the lesson plans and interviews were identified and coded. These themes along with research on MKT were building blocks for developing the theoretical framework (see Appendix B) that was used to cultivate the lesson plan rubric. The research topics investigated were learning theories, state and national standards and policies, and characteristics of teachers’ knowledge and disposition. Lesson plan structuring was also performed to aid with the development of the theoretical framework and lesson plan rubric.

Other than the lesson plan activity, each participant in the pilot study completed an interview. The interviews were used to validate the interpretation of the preservice teachers’ lesson plans. Upon completion of the pilot study, an interview protocol was designed (see Appendix F) and a lesson plan rubric was developed (see Appendix C).

The lesson plan rubric was divided into four sections: structure of lesson plan, four characteristics of teacher quality, learning theories, and standards/policies. Each element of the lesson plan rubric was scaled using ordinal values ranging from 0 to 2. A value of 0 was assigned to items that did not appear in the lesson plan, 1 was ascribed to items that appeared in the lesson plan but were underdeveloped, and 2 was assigned to items that met the criteria specified in the lesson plan rubric. The maximum score possible for each segment was contingent on the number of items within that section. On the first section, the maximum score was 24; 22 on the second section; 24 on the third section; and 12 on the fourth section.

In this study, each participant was interviewed after they completed the Early Indicators of Effective Teachers instrument; all participants were asked the same nine
questions (see Appendix F). There were three lesson plan specific questions; participants’ responses to these questions were deconstructed on an individual basis for case study analysis.

**Development of the Model Developmental Mathematics Program surveys.**

Each Model Developmental Mathematics Program (MDMP) course—Basic Mathematics and Pre-College Algebra—was surveyed with five multiple-choice questions. Since the lessons are different for both courses, there were two surveys with similar questions (see Appendix H). The surveys were developed to verify whether the lessons were being taught correctly, to investigate the MDMP students’ disposition towards the lesson, and to distinguish if the MDMP students realized they were being taught with various standards-based best practices.

The surveys were developed and administered in one semester. The questions asked on both surveys were directed toward specific lesson plan curriculum, content, and instructional methods. The questions asked about certain scenarios, definitions, and feelings towards activities used in the classroom.

One use of this survey was to ensure validity of the program; in other words, all MDMP instructors were teaching the correct content using appropriate standards-based best practices. The MDMP courses are components of a departmental mathematics program, meaning all the students are taught with the same content and pedagogy and must complete the same exams, quizzes, and assignments. The MDMP lessons include standards-based best practices: using manipulatives, technology, and various learning models, such as the Algorithmic Instructional Technique (AIT) model and the Concrete to Representational to Abstract (CRA) model. These approaches are different from
traditional instructional practices making the MDMP unique.

Validity

This study was developed to compare the mathematical knowledge for teaching (MKT) of the elementary and middle school preservice teachers on the Model Developmental Mathematics Program (MDMP) track and the College Algebra track using the participants’ results on the *Early Indicators of Effective Teachers* measures. The *Early Indicators of Effective Teachers* was a two-part assessment—the modified MKT multiple-choice test and a lesson plan writing activity.

**Validity of the Mathematical Knowledge for Teaching test.** The first component completed by the participants was the modified Mathematical Knowledge for Teaching (MKT) test. The modified MKT test was developed based on previously substantiated requisites from the Learning Mathematics for Teaching (LMT) Project, so there was no pilot test conducted (Hill, Schilling, & Ball, 2004). To ensure content validity an alignment chart corresponding the test items and the MDMP curriculum was developed. The alignment chart was used (see Appendix I) to verify that the questions chosen aligned to the upper-level MDMP course—Basic Mathematics—and College Algebra. Also to validate that the lesson plans were taught in the MDMP classes, a five-question survey was administered to all the MDMP students in the Spring 2012 semester to discern their perspectives on the type of instruction and content in the program.

**Validity of the lesson plan rubric.** The second component of the *Early Indicators of Effective Teachers* instrument, the lesson plan activity, required preservice teachers to develop a lesson on functions. To measure the performance of the participants on the lesson plans a lesson plan rubric was developed. A focus group of
preservice teachers was assembled to develop a lesson plan on functions. Each lesson plan was reviewed for underlying themes, then the participants were interviewed to verify face validity of the lesson plan rubric and the interview protocol. The interviews were also used to validate the interpretations of the participants’ lesson plans as well as to identify strengths and weaknesses of the rubric. The final lesson plan rubric was developed to facilitate unbiased scoring of the participants’ lessons.

Table 3

*Inter-Rater Reliability Scores*

<table>
<thead>
<tr>
<th>Lesson Plan Rubric Section</th>
<th>Inter-rater Reliability Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part 1: Lesson Plan Structure</td>
<td>0.78</td>
</tr>
<tr>
<td>Part 2: Four Characteristics of Teacher Quality</td>
<td>0.82</td>
</tr>
<tr>
<td>Part 3: Learning Theories</td>
<td>0.64</td>
</tr>
<tr>
<td>Part 4: Standards/Policies</td>
<td>0.54</td>
</tr>
</tbody>
</table>

The lesson plans were also scored by two instructors at Texas State University-San Marcos (Texas State) to determine inter-rater reliability (see Table 3). The reliability scores were calculated for each of the four sections of the lesson plan rubric. The scores on the first two sections of the rubric were reliable, 0.78 and 0.82. On the other hand, the two latter categories had average reliability, 0.64 and 0.54. Since the scores varied, both instructors discussed the results of each item for each participant and a new score was agreed upon. Also, the lesson plan rubric was updated to clarify any misconceptions perceived by the lesson plan evaluators.

**Analysis**

**Modified Mathematical Knowledge for Teaching scores.** The modified Mathematical Knowledge for Teaching (MKT) test was a 56-item multiple-choice test that addressed two types of knowledge: common content knowledge (CCK) subscale and
specialized content knowledge (SCK) subscale. There were four different mathematical domains represented in the modified MKT test: numerical, expressions, equations, and functions. Each item had a difficulty and discriminate score, which was used to determine the probability that a participant would answer the item correctly. These values were used to determine the standardized scores for each participant since the raw scores or percentages are not a linear measure and participants could not be compared in this manner. For example, “the difference between 10% and 20% is a more substantial difference in true ability than [the] difference between 50% and 60%” (Hill & Ball, 2004, p. 339).

This study focused on comparing the group means of two main groups, major and mathematics track, and four interaction groups—elementary MDMP, middle school MDMP, elementary College Algebra, and middle school College Algebra track participants. The mean standardized-scores for the overall content knowledge (CK) and subscales specialized-content knowledge (SCK), common-content knowledge (CCK), and content knowledge of functions were compared in four different two-way between groups analysis of variance (ANOVA; see Table 4). This analysis was used to determine if the average standardized scores for each group were significantly different.

Each participant’s individual item responses on the modified MKT test were entered into a Microsoft® Excel spreadsheet. The items on the MKT test had different difficulty levels, so the raw scores were converted into standardized scores. This was performed by importing the scores into Wolfram Mathematica® 8. There were four types of standardized scores reported: overall CK and subscales CCK, SCK, and functions. The subscale functions was chosen for further investigation because the
participants’ developed lesson plans on functions. The standardized scores were entered into the Microsoft® Excel spreadsheet with the raw data and the file was imported into Statistical Packages for the Social Sciences (SPSS). A two-way between groups ANOVA was performed on the recorded data.

Table 4

*Distribution of Preservice Teachers Sample*

<table>
<thead>
<tr>
<th></th>
<th>Elementary School Preservice</th>
<th>Middle School Preservice</th>
</tr>
</thead>
<tbody>
<tr>
<td>MDMP Track</td>
<td>n = 7</td>
<td>n = 7</td>
</tr>
<tr>
<td>College Algebra Track</td>
<td>n = 4</td>
<td>n = 4</td>
</tr>
</tbody>
</table>

The two-way between groups ANOVA compared the dependent variable—the MKT standardized scores—to the two independent variables—course track and major. The main effects and interaction between the factors were investigated to determine any statistically significant differences in mean scores between the groups (see Table 4). Factors with no significance, meaning their p-value was larger than .05, were eliminated one at a time starting with the largest p-value to see if there was any variation in the results. This process was repeated for the dependent variables SCK, CCK, and functions standardized scores. The participants’ results on the subscales—SCK and CCK items—were analyzed separately to determine if there were any significant differences with different types of MKT. The knowledge of functions subscale was also examined independently due to the participants’ topic for the lesson plan activity, functions.

When scoring the modified LMT assessments, a logit metric was used to determine the standardized scores for the participants to make the scores comparable. The preservice teachers’ scores “represent the log of the odds of correctly answering test item of average difficulty” (Garet et al., 2010, p. 14). For example, a score of zero means
“the average teacher in the study had a 50-50 chance of getting an item of average
difficulty correct” on the test (Garet et al., 2010, p. 14). The scores in this study ranged
from –4 to 1, meaning a participant with a score of 1, a high score, is more
knowledgeable than a participant scoring a –2 (Hill & Ball, 2004). To calculate the
scores, the participants’ responses to each test item were saved in a Microsoft® Excel—
correct responses were indicated with a 1 and incorrect with a 0. The difficulty and
discriminant of each item was used to develop the logistic formula:

\[ p = \frac{e^{a(z-b)}}{1+e^{a(z-b)}} \]  

(1)

for \( p \) is the probability correct, \( a \) is the discriminant or slope, \( b \) indicates the difficulty
level, and \( z \) is the maximum likelihood estimator. For example, the difficulty for item 2
is –0.377 and the discriminant is 0.660, so the difficulty and discriminate scores were
used to develop the logistic function (See Equation 1) for a standardized score of 1:

\[ p = \frac{e^{0.660(1-(-0.377))}}{1+e^{0.660(1-(-0.377))}} = 0.713 \]  

(2)

meaning in this study a pre-service teacher with a standardized score of 1 has a 71.3%
chance of answering item 2 correctly. Wolfram Mathematica® 8 was used to estimate
the knowledge level of each participant, their standardized score, based on the given
responses to each item. This process was repeated for the three different subscales to
determine the participants SCK, CCK, and functions standardized scores. The
participants’ scores ranged from 4 standard deviations below mean 0 to 4 standard
deviations above mean 0.

**Lesson plan scores.** Each participant’s lesson plan was evaluated with the lesson
plan rubric to assess their pedagogical and pedagogical content knowledge (see Appendix
C). The lesson plans were coded and scored based on observed criteria addressed in the
lesson plan rubric. Two reviewers scored the lesson plans using the same lesson plan rubric. Then the reviewers compared scores and came to an agreement of the participants’ finalized score results. After all the lesson plan scores were confirmed, the results were transferred to a condensed lesson plan template to aid in data comparison (see Appendix C). The lesson plan scores were entered into a Microsoft® Excel spreadsheet developed by Vertex42® to make box plots of the results. Box plots were used to observe the distribution of responses and aid in comparing between the participants on each mathematics track. This process was also completed for the five Basic Mathematics course lesson plans, and they were compared against the MDMP participants’ results.

**Interviews.** After the completion of the *Early Indicators of Effective Teachers* instrument, individual interviews were conducted to justify the interpretation of the lesson plans that were evaluated by the rubric. Based on the interviews from the pilot study and the theoretical framework (see Appendix B), an interview protocol was developed (see Appendix F). In the interviews, the participants were asked to explain their reasons for becoming teachers, their experiences in the education field, and the sources for their understanding of functions. The explanation for becoming a teacher helped identify the participants’ disposition towards teaching. The participants’ experiences in the classroom are influential to their understanding of the mathematical content and the pedagogy needed to teach mathematics. The participants could have gained Mathematical Knowledge for Teaching (MKT) from various resources, including the MDMP (Model Developmental Mathematics Program) courses, so the responses to this question allowed the researcher to identify which experiences might have been more
instrumental in developing MKT than other experiences and determine commonalities between participants’ responses.

After the interviews were completed, they were transcribed and analyzed. First, the individual interview transcriptions were compiled into a single document; then, the transcribed responses to the six general questions in the interview protocol were arranged to align each of the participants’ responses with the questions. After all the responses were processed, common themes were identified and coded accordingly to identify correlations between the participants’ responses and their lesson plans.

**Model Developmental Mathematics Program surveys.** In the last week of the Spring 2012 semester, all developmental mathematics instructors were given surveys to administer in their classrooms. These five-question surveys were given to the Basic Mathematics and Pre-College Algebra students currently enrolled in the courses to examine the Model Developmental Mathematics Program (MDMP) students’ interpretations of the type of instruction administered in the MDMP classes, their disposition towards the MDMP instruction, and their understanding of the MDMP curriculum. The results of the survey should reflect the Mathematical Knowledge for Teaching (MKT) and disposition of the MDMP participants. The surveys were given back to the researcher, the results were inserted into a spreadsheet, and bar graphs were developed to illustrate the various responses. These responses were linked to the types of instruction used by the MDMP participants in their lesson plan activity.

**Case studies.** The modified Mathematical Knowledge for Teaching (MKT) test scores and the lesson plan activities were vital pieces in deciding which participants should be selected for the case study analysis. There were four participants that were
chosen; one participant made high scores on both assessments, one participant made low scores on both assessments, and the other two either did well on the modified MKT or the lesson plan activity but not both. Each of the participants’ background information and experiences with formal and informal training in mathematics were described. The results from the interview questions were communicated in detail to help the researcher identify what factors influenced the participants’ lesson plans, modified MKT results, and disposition towards mathematics to understand the participants’ MKT and disposition. The instruments also helped detect what, if any, impressions from the Model Developmental Mathematics Program (MDMP) were imposed on the participants, meaning the MDMP course influenced the development of the participants’ MKT and disposition.

**Data Collection Procedure**

The proposal for this dissertation was accepted on May 27, 2011. An Institutional Review Board (IRB) application was submitted on June 1, 2011; the author was granted an IRB exemption from the Office of Research Compliance at Texas State on June 7, 2011: exemption approval number EXP2011M9626. Federal regulations have described six research categories that qualify for exempt status. This study was categorized as research conducted in established or commonly accepted educational settings, involving normal educational practices such as (ii) research on effectiveness of or the comparison among instructional techniques curricula, or classroom management methods (Public Welfare, 2011, p. 130).

Participants were contacted via email during the Summer and Fall semesters of 2011, but few students replied; there were only three respondents. The process of
collecting participants for this study was extended into spring of 2012 when 19 additional preservice teachers volunteered to participate in the study.

In the first 2011 Summer semester session, preservice teachers in College Algebra whose registered major was Interdisciplinary Studies in EC–6 ESL, EC–6 Bilingual, 4–8 Generalist, 4–8 Mathematics, or 4–8 Mathematics/Science Composite were contacted by letter and email. Nine potential participants were contacted with a letter describing this research study and the incentives for participating; two days later, a follow up email was sent to preservice teachers enrolled in College Algebra. Two students responded, but neither one had time to take the *Early Indicators of Effective Teachers* instrument.

During the second Summer semester session in 2011, five preservice teachers were enrolled in College Algebra and were contacted by email based on their registered degree. There were no respondents due to time constraints and no formal introduction from the researcher. The following semester an alternative approach was taken to attract more participants.

In Fall 2011 and Spring 2012 semesters, the researcher contacted all of the College Algebra instructors at Texas State and asked them to administer a 10-minute demographic survey to their students to determine which College Algebra students met the sample criteria. Some instructors gave an extra quiz grade as an incentive to complete the in-class survey. The surveys collected data on the demographics of the students—age, gender, ethnicity, classification, students’ past mathematics courses, major, and students’ contact information (see Appendix G). Those students majoring in Interdisciplinary Studies for EC–6 Generalist or 4–8 Generalist, Mathematics, or Mathematics/Science Composite who took either developmental mathematics or College
Algebra as their first mathematics course at Texas State were contacted via email or phone.

For the Fall 2011 semester, three different rooms were reserved on three different days for the participants convenience. They were allotted 1.5 hours to complete the *Early Indicators of Effective Teachers* instrument. Only one participant showed up for each session, totaling 3 participants from the Fall 2011 semester. When they completed the assessment, each participant scheduled an interview time that fit both the participant and the interviewer’s schedules. Since there was a lack of participants at each session in the fall, the following semester, Spring 2012, no group sessions were established. Each participant chose a time that was convenient for them and after the completion of their assessment an interview time was set up for only eight participants. As the Spring 2012 semester was ending, participants were interviewed directly after the administration of the *Early Indicators of Effective Teachers* instrument.

The interviews were either conducted in the researcher’s office or a closed room in the mathematics building. At the end of each interview, the participants were given a $20.00 gift card of their choice for contributing their time to the project.

**Limitations**

The limitations of this study include the small sample size, lack of pre-tests and post-tests, the duration of administering the *Early Indicators of Effective Teachers* instrument, and the reliability of the lesson plan rubric. The first limitation, sample size should be approximately 30 participants for a quantitative statistical analysis to have minimal error. A low number of participants could result in a Type II Error—failure to reject the null hypothesis—because the sample may not represent the population only
students with similar Mathematical Knowledge for Teaching (MKT). Since the sample size was small not all of the five-certification tracks were represented, so the participants’ results were condensed into two groups—elementary education and middle school education—which limited the detection of potentially significant variables in the individual certification areas.

The second limitation, the MKT tests were initially designed to be pre-test and post-test assessments, but they were not used in this manner. The participants’ pretest and post-test scores could have been compared, instead of comparing two groups with different experiences. The results from this study may not be representative of the MKT the participants gained in the MDMP because of time, current teacher pedagogy, and content of the course. The MDMP participants were enrolled in College Algebra at the time of the study and it had been at least a semester since they were enrolled in the MDMP course(s), because of the time lapse the MKT gained in the class could have been forgotten. In addition, the current teacher’s instructional methods would influence the participants’ MKT and disposition since it was used most recently in the College Algebra course.

The third limitation was the length of time required to take the *Early Indicators of Effective Teachers* measurements, which ultimately affected the number of participants involved in this study. Many respondents were contacted via email or phone. Those who were unable to participate indicated it was due to the time commitment to the project. The MKT assessments are not released, so converting to an online assessment was inappropriate. So there was no alternative administration for the *Early Indicators of Effective Teachers* measurements.
The fourth limitation was the reliability of the lesson plan rubric. The lesson plan rubric was developed specifically for this project so it has not been widely tested. The inter-rater reliability was low for two categories of the rubric. Changes to the rubric have been made to satisfy miscommunications between lesson plan evaluators, but more research needs to be completed on the lesson plan rubric.

Summary

In conclusion, a mixed-methods research design was used to describe the approaches employed to evaluate the types of knowledge and disposition acquired by elementary and middle school preservice teachers on the Model Developmental Mathematics Program (MDMP) track. The quantitative component of the design was used to answer RQ 1 by assessing the preservice teachers’ content knowledge and comparing elementary and middle school preservice teachers scores and the MDMP track participants and College Algebra track participants’ scores. The qualitative section of the research design was used to answer RQ 2 by evaluating the preservice teachers’ pedagogical and pedagogical content knowledge and disposition, as evidenced in the participants’ lesson plans and interviews. Based on the mixed-methods results, four MDMP track participants with varying levels of MKT and disposition were selected and compared to determine what experiences and impressions informed their performances on the Early Indicators of Effective Teachers instrument. The results from this mixed-methods design, discussed in the next chapter, were analyzed using a two-way between groups analysis of variance (ANOVA) with Statistical Package for the Social Sciences (SPSS) and exploring connections between the data collected in the interviews, lesson plans, and MDMP surveys.
CHAPTER IV

RESULTS

This mixed-methods study was designed to determine if the standards-based best practices used in the Model Developmental Mathematics Program (MDMP) at Texas State University-San Marcos (Texas State) contributed to preservice teachers’ pedagogical, content, and pedagogical content knowledge and disposition. The *Early Indicators of Effective Teachers* instrument, which consisted of two assessments, a modified Mathematical Knowledge for Teaching (MKT) test and a lesson plan activity, were used to measure the preservice teachers’ knowledge and disposition. The research questions and null hypotheses, the population and the sample of the participants at Texas State, the methods used to analyze the collected data, and the *Early Indicators of Effective Teachers* instrument, interviews, and survey results are presented in this chapter.

Research Questions and Hypotheses

The modified Mathematical Knowledge for Teaching (MKT) test was compiled from the common content knowledge (CCK) and specialized content knowledge (SCK) items, subscales of subject matter knowledge, as described by Hill and Ball (2007). The participants’ results on the tests were used to compare the average score of each group—MDMP track versus College Algebra track and elementary versus middle school preservice teachers. The participants’ raw scores were converted to standardized scores
because the items were not weighted equally with respect to their difficulty levels. The overall content knowledge (CK) and subscales CCK, SCK, and the mathematical topic functions standardized scores were dependent on the participants’ majors and mathematics tracks (independent variables or fixed factors) and assessed by the Statistical Package for the Social Sciences (SPSS). Note there are three sets of hypotheses for a two-way analysis of variance (ANOVA): (a) hypothesis 1 when the population means for one factor (major) are equal, (b) hypothesis 2 when the population means for the second factor (mathematics track) are equal, and (c) there is no interaction between the two factors (major*mathematics track).

**Research question 1.** Is there a significant difference in MKT test scores for elementary school preservice teachers and middle school preservice teachers who completed the Model Developmental Mathematics Program (MDMP) at Texas State compared to those who started in College Algebra (CA) controlling for mathematics access at community college?

**Hypothesis 1.** \( H_0: \) There is no difference between middle school preservice teachers’ mean CK/CCK/SCK/functions standardized score and elementary school preservice teachers’ mean CK/CCK/SCK/functions standardized score on the modified Mathematical Knowledge for Teaching test \( (H_0: \mu_{\text{middle}} = \mu_{\text{elementary}}) \).

\( H_A: \) There is a significant difference between elementary and middle school preservice teachers’ mean CK/CCK/SCK/functions standardized scores on the modified Mathematical Knowledge for Teaching test \( (H_A: \mu_{\text{middle}} \neq \mu_{\text{elementary}}) \).

**Hypothesis 2.** \( H_0: \) There is no difference between Model Developmental Mathematics Program track preservice teachers’ mean CK/CCK/SCK/functions
standardized scores and College Algebra preservice teachers’ mean CK/CCK/SCK/functions standardized scores on the modified Mathematical Knowledge for Teaching test (H0: \( \mu_{CA} = \mu_{MDMP} \)).

\( H_A: \) There is a significant difference between Model Developmental Mathematics Program track and College Algebra track preservice teachers’ mean CK/CCK/SCK/functions standardized scores on the modified Mathematical Knowledge for Teaching test (H0: \( \mu_{CA} = \mu_{MDMP} \)).

**Hypothesis 3.** H0: Based on CK/CCK/SCK/functions standardized scores on the modified MKT test, there is no interaction between the two factors major and mathematics track.

\( H_A: \) There is interaction between the two factors major and mathematics.

**Research question 2.** Using the lesson plan rubric and interview analyses, what early indicators of effective teachers were identified in preservice teachers at Texas State who completed the MDMP compared to those who started in college algebra?

**Research question 3.** How do impressions from the Model Developmental Mathematics Program affect the Model Developmental Mathematics Program track participants’ experiences, and how do these experiences inform their performances on the measures of the *Early Indicators of Effective Teachers*?

**Population and Sample**

The data for this study was collected in the Summer 2011, Fall 2011, and Spring 2012 semesters. The total sample size was 22. The sample consisted of 11 elementary school preservice teachers, 10 middle school preservice teachers, and one all level preservice teacher whose data was combined with the middle school preservice teachers’
data.  From the sample of elementary school preservice teachers, seven were on the Model Developmental Mathematics Program (MDMP) track and four went directly into College Algebra; there were seven middle school preservice teachers who took the MDMP course(s) and three that went directly into College Algebra and one all-level certification College Algebra track participant (see Table 4).

Four participants were male, 18 were female.  Most of the participants were under the age of 20.  There were sixteen participants 18 or younger, five participants between 21 and 30 years of age, and one participant over the age of 30.  The participants’ classification ranged from freshman to senior; there were 7 freshman, 12 sophomores, 2 juniors, and 1 senior.  As for ethnic background, half of the participants were Hispanic, 11, followed by 10 participants who categorized themselves as White, non-Hispanic, and 1 African-American participant.  The demographics of each individual in the sample is displayed in Appendix A.

Methodology Summary

The College Algebra students who participated in this study completed the Early Indicators of Effective Teachers instrument—modified Mathematical Knowledge for Teaching (MKT) test and a writing sample consisting of a lesson plan activity on functions.  The participants were also required to complete a short interview about their disposition towards mathematics and teaching and answer questions regarding their lesson plan.  The interviews were transcribed and a coding protocol was implemented to determine participants’ level of pedagogical and pedagogical content knowledge.  The statistical package, SPSS, was used to analyze the participants’ MKT test standardized scores with a two-way between groups analysis of variance (ANOVA).  The writing
samples were scored using a lesson plan rubric developed in this study (see Appendix C).

A survey was also given to the Spring 2012 semester Model Developmental Mathematics Program (MDMP) students. This five-question survey was used to verify the standards-based best practices were being implemented in the MDMP and how the MDMP students responded to these different instructional practices. The survey supported the validation of the MDMP participants receiving standards-based best practices as well as their disposition of the instructional methods used in the MDMP.

From the collected data—modified MKT tests, lesson plan activities, interviews, and MDMP surveys—four MDMP track participants were chosen for case study analysis. These participants were selected based on their performance on the Early Indicators of Effective Teachers instrument.

Results

This section describes the results of the modified Mathematical Knowledge for Teaching (MKT) assessment, the lesson plan activity, the interviews, and the case study analyses on 4 Model Developmental Mathematics Program (MDMP) participants’ results. There were three different types of results reported for the modified MKT test: 4 two-way between groups analysis of variance (ANOVA) as well as descriptive outcomes on selected items, such as items with the answer choice “I’m not sure” and problems where the majority of the participants selected the same wrong answer choice. Another assessment, the lesson plan activity, was scored with a rubric developed for this study. The scores were reported and the MDMP participants’ scores were compared to the Basic Mathematics course function lesson plans’ scores and the College Algebra participants’ scores. Furthermore, one-on-one interviews were conducted. The interviews were
transcribed and common themes were identified and coded. The identified themes for six of the nine interview questions were reported and connections were identified. In addition, four MDMP track participants were selected based on their *Early Indicators of Effective Teachers* scores for case study analyses, which was reported in the following section.

**Results of the modified Mathematical Knowledge for Teaching test.** The results presented in this section, captured using the modified Mathematical Knowledge for Teaching (MKT) test, answered RQ 1: “Is there a significant difference in MKT test scores for elementary school preservice teachers and middle school preservice teachers who completed the Model Developmental Mathematics Program (MDMP) at Texas State compared to those who started in College Algebra (CA) controlling for mathematics access at community college?” To evaluate the difference in the mean standardized scores for overall content knowledge (CK) and the three subscales specialized content knowledge (SCK), common content knowledge (CCK), and content knowledge of functions, a two-way between groups analysis of variance (ANOVA) was performed. The participants’ MKT scores were compared between MDMP track participants and College Algebra participants as well as between the different majors—elementary and middle school.

**Participants’ content knowledge performance.** The 56-item modified Mathematical Knowledge for Teaching (MKT) test assessed the participants’ overall subject matter knowledge, or content knowledge (CK). A two-way between groups analysis of variance (ANOVA) was used to determine if there is a significant difference between the different majors—elementary and middle school preservice teachers—and if
there is a significant difference between the two track groups—Model Developmental Mathematics Program (MDMP) and College Algebra. The main effects—major and mathematics track—and the interaction variable based on overall content knowledge are displayed in Table 5. Based on the results, there is a significant difference (p < .05) between the mean scores of the participants on the MDMP track and the College Algebra track. The average overall score for the MDMP track participants is \(-1.4226\), which is \(1.4226\) standard deviations below the average mean 0, and the mean score for the College Algebra track students is \(-0.74624\), meaning they were only \(0.74624\) standard deviations below the mean. The significant difference between the MDMP and College Algebra students was equivalent to \(0.67636\) \((1.4226 – 0.74624)\). Meanwhile, there is no significant difference between the elementary preservice teachers and the middle school preservice teachers’ CK group means; the elementary school preservice teachers scored an average of \(1.1666\) standard deviations below the mean zero, whereas the middle school preservice teachers scored an average of \(1.18668\) standard deviations below the mean. The results indicate that there is no significant difference between the elementary and middle school preservice teachers’ average scores and similarly there is no significant difference between the interaction variables. Therefore, the MDMP track participants performed statistically lower than the College Algebra track participants on the modified MKT test for overall CK, but there was no significant difference between the elementary and middle school preservice teachers.
Table 5

Two-way Between Groups ANOVA: CK Mean Standardized Scores

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected Model</td>
<td>2.468</td>
<td>3</td>
<td>0.823</td>
<td>3.151</td>
<td>0.05</td>
</tr>
<tr>
<td>Intercept</td>
<td>23.947</td>
<td>1</td>
<td>23.947</td>
<td>91.713</td>
<td>0.000</td>
</tr>
<tr>
<td>MDMP</td>
<td>2.329</td>
<td>1</td>
<td>2.329</td>
<td>8.919</td>
<td>0.008**</td>
</tr>
<tr>
<td>MSMajor</td>
<td>0.021</td>
<td>1</td>
<td>0.021</td>
<td>0.082</td>
<td>0.778</td>
</tr>
<tr>
<td>MDMP*MSMajor</td>
<td>0.137</td>
<td>1</td>
<td>0.137</td>
<td>0.525</td>
<td>0.478</td>
</tr>
<tr>
<td>Error</td>
<td>4.700</td>
<td>18</td>
<td>0.261</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>37.627</td>
<td>22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>7.168</td>
<td>21</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

R Squared = 0.344 (Adjusted R Squared = 0.235)

Note: * p < 0.05, ** p < 0.01, and *** p < 0.001

Participants’ specialized content knowledge performance. The participants’ specialized content knowledge (SCK)—knowledge of the mathematical content they will teach, knowing the appropriate teaching methods for a specific mathematical topic, and ability to decipher students’ methods for working a problem—was assessed using a 28-item subscale of the modified Mathematical Knowledge for Teaching (MKT) test (Hill et al., 2008; Swars, Hart, Smith, S., Smith, M., & Tolar, 2007). The participants’ scores were compared using a two-way between groups analysis of variance (ANOVA) to determine the significance between the mean scores of the two groups—major and mathematics track. The results of the specialized content knowledge comparison of main effects and the interaction variable are displayed in Table 6. Based on the results, there is no significant difference (p < 0.05) between the SCK mean scores of the participants on the Model Developmental Mathematics Program (MDMP) track and the College Algebra track. The average SCK score for the MDMP participants is 1.52541 standard deviations below mean zero, and the mean score for the College Algebra track participants is
1.09391 standard deviations below the mean. Even though the difference is not significant, the College Algebra participants performed about half a standard deviation better than the MDMP participants on the SCK items. There is also no significant difference between the elementary preservice teachers and the middle school preservice teachers’ SCK group means; the elementary school preservice teachers scored an average of 1.46148 standard deviations below mean zero, whereas the middle school preservice teachers scored an average of 1.27552 standard deviations below mean. The results indicated that there were no significant differences among any of the different groups’ SCK mean scores. This signifies that the groups appear to be homogenous when applying mathematical content they teach, knowing appropriate methods for teaching specific mathematical topics, and deciphering students’ methods for working problems (Hill et al., 2008; Swars, Hart, Smith, S., Smith, M., & Tolar, 2007).

Table 6

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected Model</td>
<td>1.661</td>
<td>3</td>
<td>0.554</td>
<td>0.665</td>
<td>0.584</td>
</tr>
<tr>
<td>Intercept</td>
<td>34.928</td>
<td>1</td>
<td>34.928</td>
<td>41.954</td>
<td>0.000</td>
</tr>
<tr>
<td>MDMP</td>
<td>0.948</td>
<td>1</td>
<td>0.984</td>
<td>1.139</td>
<td>0.300</td>
</tr>
<tr>
<td>MSMajor</td>
<td>0.049</td>
<td>1</td>
<td>0.049</td>
<td>0.059</td>
<td>0.810</td>
</tr>
<tr>
<td>MDMP*MSMajor</td>
<td>0.523</td>
<td>1</td>
<td>0.523</td>
<td>0.629</td>
<td>0.438</td>
</tr>
<tr>
<td>Error</td>
<td>14.986</td>
<td>18</td>
<td>0.833</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>57.849</td>
<td>22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>16.647</td>
<td>21</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

R Squared = 0.100 (Adjusted R Squared = 0.050)

Participants’ common content knowledge performance. The participants’ common content knowledge (CCK) was evaluated by analyzing their results on the 28-item CCK subscale of the modified Mathematical Knowledge for Teaching (MKT) test.
Based on a two-way between groups analysis of variance (ANOVA), there is a significant
difference ($p < 0.05$) between the mean CCK scores of the participants on the two
mathematics tracks (see Table 7). The average CCK score for the Model Developmental
Mathematics Program (MDMP) participants is 1.39237 standard deviations below mean
zero, and the mean score for the College Algebra students is 0.46804 standard deviations
below the mean. The College Algebra participants scored approximately one standards
deviation above the MDMP track participants on the common content knowledge section
of the MKT test. There is no significant difference between the elementary preservice
teachers and the middle school preservice teachers’ CCK group means; the elementary
school preservice teachers scored an average of 0.98229 standard deviations below mean
zero, whereas the middle school preservice teachers scored an average of 1.13021
standard deviations below mean. The results indicate that there is no significant
difference between the elementary and middle school preservice teachers’ average scores
and similarly there is no significant difference between the interaction variables.
Therefore, the MDMP track participants performed statistically lower than the College
Algebra track participants on the modified MKT test subscale CCK, whereas there was
no significant differences between the elementary and middle school preservice teachers.
Table 7

Two-way Between Groups ANOVA: CCK Mean Standardized Scores

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected Model</td>
<td>4.499</td>
<td>3</td>
<td>1.500</td>
<td>7.146</td>
<td>0.001</td>
</tr>
<tr>
<td>Intercept</td>
<td>17.620</td>
<td>1</td>
<td>17.620</td>
<td>83.972</td>
<td>0.000</td>
</tr>
<tr>
<td>MDMP</td>
<td>4.350</td>
<td>1</td>
<td>4.350</td>
<td>20.728</td>
<td>0.000***</td>
</tr>
<tr>
<td>MSMajor</td>
<td>0.144</td>
<td>1</td>
<td>0.144</td>
<td>0.688</td>
<td>0.418</td>
</tr>
<tr>
<td>MDMP*MSMajor</td>
<td>0.029</td>
<td>1</td>
<td>0.029</td>
<td>0.132</td>
<td>0.716</td>
</tr>
<tr>
<td>Error</td>
<td>3.777</td>
<td>18</td>
<td>0.210</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>32.820</td>
<td>22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>8.276</td>
<td>21</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

R Squared = 0.544 (Adjusted R Squared = 0.468)

Note: * p < 0.05, ** p < 0.01, and *** p < 0.001

Participants’ specialized content knowledge and common content knowledge of functions. The lesson plan activity topic chosen was functions due to its prominent role in grades P–12 and post-secondary education. The modified Mathematical Knowledge for Teaching (MKT) test also assessed participants’ common content knowledge (CCK) and specialized content knowledge (SCK) on functions with 14 test items. Table 8 displays the two-way between groups analysis of variance (ANOVA) results for the dependent variable functions and independent variables major and mathematics track. Based on the reported significance values in Table 8, there is no statistical significance (p > 0.05) between the main effects—major and mathematics track—and the interaction variables group means on the function items. The average function score for the Model Developmental Mathematics Program (MDMP) participants is 1.07015 standard deviations below mean zero, and the mean score for the College Algebra students is 0.71252 standard deviations below the mean. The elementary school preservice teachers scored an average of 0.93047 standard deviations below mean zero, whereas the middle
school preservice teachers scored an average of 0.94973 standard deviations below mean. The results indicated that there were no significant differences among any of the different groups’ mean scores on the mathematical content area functions. This implies the groups seem to be homogenous when using common and specialized content knowledge for functions.

Table 8

Two-way Between Groups ANOVA: Functions Mean Standardized Scores

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected Model</td>
<td>0.766</td>
<td>3</td>
<td>0.256</td>
<td>0.677</td>
<td>0.578</td>
</tr>
<tr>
<td>Intercept</td>
<td>16.178</td>
<td>1</td>
<td>16.178</td>
<td>42,755</td>
<td>0.000</td>
</tr>
<tr>
<td>MDMP</td>
<td>0.651</td>
<td>1</td>
<td>0.651</td>
<td>1.721</td>
<td>0.206</td>
</tr>
<tr>
<td>MSMajor</td>
<td>0.002</td>
<td>1</td>
<td>0.002</td>
<td>0.006</td>
<td>0.937</td>
</tr>
<tr>
<td>MDMP*MSMajor</td>
<td>0.115</td>
<td>1</td>
<td>0.115</td>
<td>0.303</td>
<td>0.589</td>
</tr>
<tr>
<td>Error</td>
<td>6.811</td>
<td>18</td>
<td>0.378</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>27.022</td>
<td>22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>7.579</td>
<td>21</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

R Squared = 0.101 (Adjusted R Squared = –0.048)

Modified Mathematical Knowledge for Teaching performance on individual items. Since the results from the overall content knowledge and common content knowledge (CCK) scores indicated the College Algebra track participants were more successful, further investigations occurred to observe specific patterns between the groups. The modified Mathematical Knowledge for Teaching (MKT) test consisted of 56 multiple-choice items. Thirty-seven of the test items had an “I’m not sure” option for students who did not feel confident enough to answer. Table 9 displays the percentages of correct, incorrect, and “I’m not sure” responses for each participant for the 37-items. The percentage of times “I’m not sure” was selected by the participants ranged from 2.7% to 62.16%, meaning participants that answered, “I’m not sure” on 2.7% of the test
items had more confidence in answering the questions correctly than the other participants. Ten of the participants selected “I’m not sure” on more than 18% of the 37-items; nine of those ten participants were Model Developmental Mathematics Program (MDMP) track participants.

Table 9

Distribution of Correct, Incorrect, and “I’m not sure” Responses

<table>
<thead>
<tr>
<th>Participant No.</th>
<th>% Correct</th>
<th>% Incorrect</th>
<th>% “I’m not sure”</th>
</tr>
</thead>
<tbody>
<tr>
<td>44</td>
<td>24.32</td>
<td>13.51</td>
<td>62.16*</td>
</tr>
<tr>
<td>33</td>
<td>37.84</td>
<td>29.73</td>
<td>32.43*</td>
</tr>
<tr>
<td>30</td>
<td>35.14</td>
<td>35.14</td>
<td>29.73*</td>
</tr>
<tr>
<td>34</td>
<td>35.14</td>
<td>35.14</td>
<td>29.73*</td>
</tr>
<tr>
<td>23</td>
<td>45.95</td>
<td>27.03</td>
<td>27.03</td>
</tr>
<tr>
<td>42</td>
<td>37.84</td>
<td>35.14</td>
<td>27.03*</td>
</tr>
<tr>
<td>28</td>
<td>51.35</td>
<td>27.03</td>
<td>21.62*</td>
</tr>
<tr>
<td>37</td>
<td>45.95</td>
<td>32.43</td>
<td>21.62*</td>
</tr>
<tr>
<td>45</td>
<td>43.24</td>
<td>35.14</td>
<td>21.62*</td>
</tr>
<tr>
<td>40</td>
<td>56.76</td>
<td>24.32</td>
<td>18.92*</td>
</tr>
<tr>
<td>25</td>
<td>54.05</td>
<td>32.43</td>
<td>13.51</td>
</tr>
<tr>
<td>26</td>
<td>45.95</td>
<td>40.54</td>
<td>13.51</td>
</tr>
<tr>
<td>29</td>
<td>43.24</td>
<td>43.24</td>
<td>13.51</td>
</tr>
<tr>
<td>36</td>
<td>29.73</td>
<td>56.76</td>
<td>13.51*</td>
</tr>
<tr>
<td>35</td>
<td>64.86</td>
<td>24.32</td>
<td>10.81</td>
</tr>
<tr>
<td>22</td>
<td>56.76</td>
<td>35.14</td>
<td>8.11</td>
</tr>
<tr>
<td>24</td>
<td>59.46</td>
<td>32.43</td>
<td>8.11</td>
</tr>
<tr>
<td>31</td>
<td>51.35</td>
<td>43.24</td>
<td>5.41*</td>
</tr>
<tr>
<td>32</td>
<td>43.24</td>
<td>51.35</td>
<td>5.41*</td>
</tr>
<tr>
<td>39</td>
<td>45.95</td>
<td>48.65</td>
<td>5.41*</td>
</tr>
<tr>
<td>43</td>
<td>54.05</td>
<td>40.54</td>
<td>5.41*</td>
</tr>
<tr>
<td>21</td>
<td>72.97</td>
<td>24.32</td>
<td>2.70</td>
</tr>
</tbody>
</table>

Note: * MDMP participants who marked “I’m not sure” on the 37 modified MKT items

The participants’ results on the modified MKT test for items 2, 7, 11, 31, 32, 33, 37, 38, 42, 53, 54, and 56 required further investigation because a majority of the
participants responded to these questions with the same incorrect answer. As mandated by MKT developers, the MKT items are not allowed to be released; hence, the investigated questions are described rather than stated.

Item 2, a SCK item, asked participants to determine a fraction equivalent to 0/0. Fifteen of the twenty-two participants responded that 0/0 = 0, whereas three stated it was equivalent to 1. Only 14% of the participants answered this question correctly.

Test item 7, a CCK item, required participants to identify if a word problem was translated correctly into a mathematical sentence. The correct response was “no,” because the word problem was to be solved using multiplication not subtraction, which was given as the answer choice; nineteen of the twenty-two students marked “yes.”

Item 11, a CCK item, asked if the given example was a mathematical justification for the distributive property. Seven participants responded correctly, while fourteen participants thought substituting values for the variables was enough mathematical justification to determine that two expressions were equivalent.

Item 31, a SCK question, required participants to solve an equation in the fewest number of steps. Fifteen students chose a mathematically correct method, but it was not an efficient method for solving. The participants who selected the wrong option (68%) knew how to solve an equation one way, but they did not understand all the mathematical field properties.

Questions 32 and 33 were CCK items that presented inequalities for participants to determine the number of solutions in the solution set. Twelve participants chose the incorrect answer for item 32, more than one solution, even though the solution set was empty. On item 33, a compound inequality written similar to \( 7 < x < 9 \) was given to the
participants and eleven participants replied that the solution set was exactly one value.

Test item 37 was also a CCK item where the participants had to identify the number of solutions in the solution set. This item differed from items 32 and 33, because these questions asked for the number of solutions for a quadratic equation with the general form $x^2 = a$ where $a$ is a perfect square. Out of the 22 participants, 13 participants selected responded that it only had one solution.

Item 38 was a SCK question that asked participants how they would explain to students why it was correct to divide an equation by a number, for example dividing $2x = 6$ by 2, but not a variable, dividing $x^2 = 6x$ by $x$. Thirteen of the participants chose the answer that explained using the square root principle incorrectly. The square root principle could not have been performed on a mathematics sentence similar to $x^2 = 6x$ to receive the correct solution. Only five participants answered this item correctly.

Question 42, an SCK item, focused on the use of appropriate examples to teach proportions in the classroom. The question asked which example, if any, is the most challenging proportion to use when first teaching proportions. Two of the answer choices were easier than the third because the solutions were integers. The fourth option was “all problems provide the same level of difficulty,” which was also incorrect because one option had a more complex solution. Half of the participants answered incorrectly; they chose “all problems provide the same level of difficulty.”

Test items 53 through 56 were testlets of one problem. Each item was a graphical representation of a mathematical sequence. The participants were supposed to identify the type of growth function—linear, quadratic, or exponential—represented by the diagram. Items 53, 54, and 56 had more incorrect responses than correct responses for
one answer choice. The sequence for item 53 represented a quadratic function, but half of the participants marked linear. The sequences for items 54 and 56 represented linear and quadratic functions, but a majority of the participants chose “I’m not sure,” which may be due to the participants’ lack of confidence or insufficient knowledge of sequences and functions.

There were four SCK items and eight CCK items that were further investigated. A similar characteristic between these items was that eight items had the option of “I’m not sure,” but only two of those eight items had a high percentage of participants marking “I’m not sure.” These two items were also function items. The CCK items had the most struggle and they are also the subscale that resulted in a significant difference between MDMP track and College Algebra track participants’ mean scores.

**Results of lesson plans.** The qualitative portion of the mixed-methods approach used in this study—the lesson plan activity on functions—was analyzed using a lesson plan rubric and an interview. The lesson plan rubric was divided into four sections: structure of lesson plan, four characteristics of teacher quality, learning theories, and standards/policies (see Appendix C). The rubric assigned ordinal values ranging from 0 to 2 for each comparison item. The value of 0 was assigned to items that were not represented in the lesson plan, 1 was assigned to items that somewhat met the requirement, and 2 was assigned to items that met the criteria listed in the lesson plan rubric. A total ordinal score of 82 could be achieved on the lesson plan rubric. The score of 82 would mean the developed lesson plan would exhibit all the components listed on the lesson plan rubric. The maximum score earned on the lesson plans may be difficult to achieve because of the preservice teachers’ instructional preference. Preservice teachers
could possibly earn a maximum score of 24 on component three (learning theories), but preservice teachers’ instructional preference usually exhibits attributes from one learning theory over the others, earning 8 total points for one learning theory.

**Basic Mathematics course lesson plans’ performances.** The Basic Mathematics curriculum incorporates five lessons directed towards functions—*Intro to Linear Functions, Graphing Quadratic Functions, Graphing Polynomial Functions, Rational Functions,* and *Radical Functions.* These lesson plans received scores based on the criteria set by the lesson plan rubric. The collective results are displayed in Table 10. Since the Model Developmental Mathematics Program (MDMP) students were taught using these function lesson plans, the values for each part provided a benchmark to compare with the MDMP participants’ scores.

**Table 10**  
*Scores and Totals of MDMP Function Lesson Plans*

<table>
<thead>
<tr>
<th>MDMP LPs</th>
<th>Part 1</th>
<th>Part 2</th>
<th>Part 3</th>
<th>Part 4</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intro to Linear Functions</td>
<td>15</td>
<td>19</td>
<td>14</td>
<td>7</td>
<td>55</td>
</tr>
<tr>
<td>Graphing Quadratic Functions</td>
<td>15</td>
<td>18</td>
<td>17</td>
<td>7</td>
<td>57</td>
</tr>
<tr>
<td>Graphing Polynomial Functions</td>
<td>14</td>
<td>17</td>
<td>13</td>
<td>7</td>
<td>51</td>
</tr>
<tr>
<td>Rational Functions</td>
<td>15</td>
<td>15</td>
<td>12</td>
<td>7</td>
<td>49</td>
</tr>
<tr>
<td>Radical Functions</td>
<td>15</td>
<td>20</td>
<td>13</td>
<td>8</td>
<td>56</td>
</tr>
</tbody>
</table>

The first section of the rubric was used to assess the lesson plans structures, the Basic Mathematics lesson plans’ (BMLP) scores are considerably higher than the MDMP track participants’ scores, because the Basic Mathematics lesson plans were developed
with a research-based template that met multiple criteria on the rubric (see Figure 6). The Basic Mathematics lesson plans’ scores were also higher than the MDMP track participants’ scores based on the second section of the rubric (four characteristics of teacher quality) due to the Basic Mathematics lesson plans’ authors’ profound education and teaching experiences that surpasses the education and experiences of the MDMP track participants. The professionals who wrote the Basic Mathematics lesson plans had at least a bachelor’s degree in Mathematics or Mathematics Education and were pursuing advanced degrees, so their pedagogical, content, and pedagogical content knowledge were more developed than those of the undergraduate participants.

Figure 6. The Lesson Plan Scores for Structure and Teacher Quality were Represented as a Box Plot for MDMP Track Participants and Basic Mathematics Lessons. This figure illustrates the difference in the minimum, maximum, and quartile values received on structure and teacher quality sections of the lesson plan rubric for the MDMP track participants’ lesson plans and the Basic Mathematics course lesson plans.
The Basic Mathematics lesson plans’ scores are higher than the MDMP track participants’ scores based on the criteria indicated in learning theories and standards subsections of the rubric; however, the differences in the scores are not as significant as they are on the first two sections. The Basic Mathematics lesson plans’ scored higher than the MDMP participants’ lesson plans on the third section (learning theories) due to the emphasis of all three learning theories in each lesson plan. The MDMP track participants’ scores on the learning theories section ranged from zero to six with approximately 78.5% of the participants scoring from zero to two, whereas the Basic Mathematics scores ranged from 3 to 7 (see Table 11). The Basic Mathematics lesson plan scores are high on the fourth component (standards and policies) compared to the MDMP track participants’ lesson plan scores, because the template used to develop the Basic Mathematics lesson plans required the author(s) to indicate the specific state standards. The participants may have performed better if a template was given to them or if they had formal training on state and national standards; these are two tools the Basic Mathematics’ authors have at their disposal.
Figure 7. The Lesson Plan Scores for Learning Theories and Standards were Represented as a Box Plot for MDMP Track Participants and Basic Mathematics Lessons. This figure illustrates the difference in the minimum, maximum, and quartile values received on the learning theories and standards subsections of the lesson plan rubric for the MDMP track participants’ lesson plans and the Basic Mathematics course lesson plans.

There are three main learning theories—constructivism, cognitivism, and behaviorism. The Basic Mathematics lesson plans were assessed with criteria from each learning theory; these scores are displayed in Table 11. The five Basic Mathematics function lesson plans scores listed in Table 11 are Intro to Linear Functions, Graphing Quadratic Functions, Graphing Polynomial Functions, Rational Functions, and Radical Functions. Each learning theory approach was emphasized in each of the lesson plans at different levels of concentration. Constructivist approaches had the most influence on the lessons Graphing Quadratic Functions, Graphing Polynomial Functions, and Radical Functions; however, Graphing Quadratic Functions has identical scores in the
behaviorism and constructivism subcategories. These results differed from the MDMP track participants’ lesson plans; they emphasized behaviorist approaches instead of constructivist approaches. The differences in the scores distribution may be due to the type of instruction used in the College Algebra course.

Table 11

**Basic Mathematics Lesson Plans - Learning Theory Scores**

<table>
<thead>
<tr>
<th>Lesson plan</th>
<th>Constructivism</th>
<th>Behaviorism</th>
<th>Cognitivism</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intro to Linear</td>
<td>4</td>
<td>6*</td>
<td>4</td>
</tr>
<tr>
<td>Functions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Graphing Quadratic</td>
<td>6*</td>
<td>6*</td>
<td>5</td>
</tr>
<tr>
<td>Functions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Graphing Polynomial</td>
<td>7*</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Functions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rational Functions</td>
<td>4</td>
<td>3</td>
<td>5*</td>
</tr>
<tr>
<td>Functions</td>
<td>6*</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

*Note: * participants’ highest scores.

**Model Developmental Mathematics Program survey analysis.** Besides comparing the Model Developmental Mathematics Program (MDMP) participants’ lesson plans to the lesson plans used to teach the MDMP, two surveys were distributed and completed by the MDMP students enrolled in the Spring 2012 semester. One survey was developed for the Basic Mathematics course and one for the Pre-Algebra course. The results of these MDMP surveys were used to validate the use of standards-based best practices in the classroom as well as determine if the students identify the different instructional techniques used in the classroom compared to other teachers’ instructional methods and their disposition towards teaching mathematics.

**Responses from the Pre-Algebra students.** The content taught in the Model Developmental Mathematics Program (MDMP) courses foreshadows functions, but to understand the students in the Pre-Algebra course interpretation of functions they were
asked, “What aspect of functions has been taught to you in the developmental mathematics course?” Eleven students responded with the answer of input and output as well as the vertical line test. Seventeen students used graphing calculators to represent equations in the classroom and eighteen students recalled transformations. Lastly, three participants did not recognize any of the answer choices selected by the other participants.

The MDMP students were asked about their feelings towards hands-on activities, in particular algebra tiles. A majority of the respondents felt algebra tiles were helpful (29 out of 39), but 15 of the 29 students did not feel the algebra tiles were helpful to them. In addition, 11 students thought the algebra tiles were childish and did not see the purpose of using them, and two students felt that the curriculum should be taught by lecture only.

The Pre-Algebra students were asked what aspects of the course helped with developing strategies to solve multi-step problems. Thirty-two students responded with writing down step-by-step procedures to help with multi-step problems. The next highest response for this question (24) was that taking notes helped with learning multi-step problems. Last, five replied that developing pictorial diagrams helped them with learning multi-step problems.

Based on the responses to question four about the curriculum structure, 32 students believed the curriculum was developed by blending geometry, measurement, probability, statistics, and algebra. According to 12 students, these lessons were developed to start with expressions to equations to functions. Only one student replied that the whole curriculum concentrated on equations.
The last question on the Pre-Algebra survey asked students about their feelings towards the transformation lesson, when students were divided into small groups and were given one transform: reflection, rotation, or translation, then moved to a new group and everyone taught each other about their specific transformation. Seven students said the method was inappropriate and teachers should teach the curriculum. This method was ineffective for nine students; they said they did not learn the content using this method. Discussing content with other students made 10 students realize that they could teach others mathematics. Seventeen MDMP students liked this method of teaching and thought more topics should be taught using cooperative learning groups.

Responses from the Basic Mathematics students. Basic Mathematics, Math 1311, students were given a 5-question survey on their disposition of the class content and instruction. The Basic Mathematics students were asked how linear functions were taught in their class. Two hundred four students replied different representations—tables, graphs, and algebraic symbols—were used to describe functions. There were 41 students who replied that functions were discussed in groups; whereas, 81 participants said that their instructors just lectured and they wrote down notes. The last multiple-choice answer option addressed the use of graphing calculators to teach functions and there were 133 students with this response.

The Basic Mathematics students were also asked about their disposition about using algebra tiles—manipulatives—in the classroom. Fifty-nine students replied algebra tiles were too childish and they did not see the purpose. One hundred eleven students felt like algebra tiles were helpful, but not for them, and 99 felt they were beneficial and more lessons should have hands-on activities. Thirteen students favored lecture-based
instruction more than the algebra tiles and 17 students indicated they were not taught using algebra tiles.

The Math 1311 students were also asked about the types of strategies they would use to solve multi-step problems. Two hundred twenty-two students replied they would write down each step as a class. Pictorial diagrams were helpful for 61 students, one hundred eighteen students preferred note taking, and five mentioned no strategies were taught.

The students enrolled in the Math 1311 course, were asked about the developed curriculum for the course. According to 139 students, the Basic Mathematics course was a blend of geometry, measurement, probability, statistics, and algebra. Eighteen stated that the course only concentrated on equations. One hundred twenty-five students described the content as progressing from expressions to equations to functions. Unfortunately, 28 students could not see any connections in the curriculum.

The fifth question asked Basic Mathematics students how they felt about the type of instruction used to teach rational equations. The lesson was taught using jigsaw cooperative learning groups. Twenty-four students felt this form of instruction was inappropriate and teachers should use didactic discourse in the classroom to teach content. The jigsaw was used to teach forty-one students rational equations, but the students mentioned they did not learn the content with this method. The students who participated in the cooperative learning activity, 78 students, realized they could teach mathematics to others. One hundred sixteen students liked the instructional method, and they wanted more topics taught with this type of instruction. The jigsaw was not used to teach 43 students rational equations.
The Model Developmental Mathematics Program (MDMP) at Texas State is a departmental mathematics course, meaning all the MDMP students were taught the same content, using the same standards-based best practices, and assessed with the same instruments. These surveys were first used to verify the courses were taught with the appropriate methods, and second to distinguish if the MDMP students realize they are taught with different types of instructional methods and their disposition for these methods.

**Participants’ lesson plan performances.** The participants developed a lesson plan on functions to be analyzed by a lesson plan rubric to determine the participants’ pedagogical content knowledge (PCK). The Model Developmental Mathematics Program (MDMP) track participants’ scores were compared to the College Algebra track participants’ scores as well as the two different majors—elementary and middle school. Table 12 was compiled from all the participants’ lesson plan scores based on the lesson plan rubric. Table 12 lists each participants’ mathematics track—MDMP and College Algebra—and their scores of each subcategory.
Table 12

*Participants’ Lesson Plan Scores and Totals by Track*

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>College</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>4</td>
<td>15*</td>
<td>8*</td>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>22</td>
<td>8**</td>
<td>15*</td>
<td>9*</td>
<td>3</td>
<td>35</td>
</tr>
<tr>
<td>23</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>24</td>
<td>10*</td>
<td>17*</td>
<td>12*</td>
<td>5*</td>
<td>44</td>
</tr>
<tr>
<td>25</td>
<td>6</td>
<td>8</td>
<td>4</td>
<td>6*</td>
<td>24</td>
</tr>
<tr>
<td>26</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>29</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>35</td>
<td>7*</td>
<td>8</td>
<td>7**</td>
<td>2</td>
<td>24</td>
</tr>
<tr>
<td>MDMP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>30</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>17</td>
</tr>
<tr>
<td>31</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>32</td>
<td>1</td>
<td>6</td>
<td>3</td>
<td>4**</td>
<td>14</td>
</tr>
<tr>
<td>33</td>
<td>8*</td>
<td>11*</td>
<td>7**</td>
<td>4**</td>
<td>30</td>
</tr>
<tr>
<td>34</td>
<td>6</td>
<td>10*</td>
<td>5</td>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>36</td>
<td>9**</td>
<td>12**</td>
<td>7**</td>
<td>4**</td>
<td>32</td>
</tr>
<tr>
<td>37</td>
<td>9**</td>
<td>10*</td>
<td>5</td>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>39</td>
<td>3</td>
<td>8</td>
<td>5</td>
<td>3</td>
<td>19</td>
</tr>
<tr>
<td>40</td>
<td>6</td>
<td>8</td>
<td>4</td>
<td>1</td>
<td>19</td>
</tr>
<tr>
<td>42</td>
<td>6</td>
<td>6</td>
<td>6**</td>
<td>4**</td>
<td>22</td>
</tr>
<tr>
<td>43</td>
<td>8*</td>
<td>12*</td>
<td>11*</td>
<td>5*</td>
<td>36</td>
</tr>
<tr>
<td>44</td>
<td>8*</td>
<td>12*</td>
<td>7**</td>
<td>6*</td>
<td>33</td>
</tr>
<tr>
<td>45</td>
<td>1</td>
<td>7</td>
<td>4</td>
<td>2</td>
<td>14</td>
</tr>
</tbody>
</table>

*Note:* * top quartile scores. ** top 50% scores. The values in the parentheses under “Part 1,” “Part 2,” “Part 3,” and “Part 4” are the total possible points.

The first section of the lesson plan rubric (lesson plan structure) was divided into five subsections: objectives, introduction, body, conclusion, and assessments. Two five-number summaries were calculated to compare the MDMP track and the College Algebra track participants’ overall lesson-plan structure scores (see Figure 8). The MDMP track participants’ five-number summary (Min. = 1, Q1 = 2.25, Med. = 6, Q3 = 8, Max. = 9) scores are similar to the College Algebra track participants’ scores (Min. = 2, Q1 = 3, Med. = 5, Q3 = 7.25, Max. = 10). Although the scores are similar, participants on the MDMP track have a higher median score (6) compared to the College Algebra track.
participants (5); however, the highest score was achieved by a College Algebra track participant (10). The distribution of the data set for MDMP track participants below the median is wider than the other quartiles due to the MDMP participants’ varying degrees of low performance on lesson plan structuring.

![Figure 8](image)

**Figure 8.** College Algebra and MDMP Tracks Participants’ Performance on Lesson Plan Structure. This figure illustrates the difference in the minimum, maximum, and quartile values received on structure of the lesson plan rubric for the College Algebra track and the MDMP track participants.

The second portion of the lesson plan rubric (four characteristics of teacher quality) was divided into the subcategories: pedagogical knowledge, content knowledge, pedagogical content knowledge, and disposition. Again, 2 five-number summaries were calculated from the set of participants’ scores to compare the groups’ knowledge and disposition. The distribution of MDMP track participants’ scores is minimum is 2, $Q_1$ is
6, median is 8, \( Q_3 \) is 10.75, and maximum is 12, whereas the five-number summary for
the College Algebra track is minimum is 4, \( Q_1 \) is 4.75, median is 8, \( Q_3 \) is 15, and
maximum is 17 (see Figure 9). The median for both groups is the same (8), but the
College Algebra track participants have a wider score distribution above the median (9).
The MDMP track participants have a concentrated spread above the median (4). The
distribution of scores below the median for College Algebra track participants is between
2 and 8, and the distribution for the MDMP track participants is between 4 and 8. The
range above the median is more dispersed for the College Algebra track participants,
whereas the concentration of MDMP participants’ scores below the median has a wider
range.

The third section of the lesson plan rubric was comprised of three learning theory
categories—constructivism, behaviorism, and cognitivism—each weighted equally. The
five-number summary calculated from the distribution of learning theory scores for the
MDMP track participants is minimum is 2, \( Q_1 \) is 4, median is 5, \( Q_3 \) is 6.75, and maximum
is 11, whereas the five-number summary for the College Algebra track is minimum is 1,
\( Q_1 \) is 3.5, median is 5.5, \( Q_3 \) is 8.25, and maximum is 12 (see Figure 10). Through
observation of the score distribution, there appears to be no difference between the
MDMP track scores and the College Algebra track scores, but the range for the College
Algebra track participants’ scores (range = 11) has a wider dispersion than the MDMP
track participants (range = 9). For both groups, the scores above the median have a
greater distribution than the sets in the lower two quartiles.
The values listed in Table 13 are the MDMP track participants’ lesson plan learning theory scores; the criteria used for scoring the lesson plans were divided into three subcategories: constructivism, behaviorism, and cognitivism. Based on the participants’ scores, the behaviorist approach—possibly unbeknownst to the participants— influenced their lesson plans more than the other two learning theories. In total, nine participants earned high scores in behaviorism; however, six of the participants scored an identical value for one of the other learning theories—two participants have identical high scores in constructivism and behaviorism and four have identical scores in cognitivism and behaviorism (see Figure 11). Besides the number of participants earning
high scores on behaviorism, the differences in the score distribution appears to be substantial.

Figure 10. College Algebra and MDMP Tracks Participants Use of learning Theory Approaches Scores. This figure illustrates the difference in the minimum, maximum, and quartile values received on the learning theory section of the lesson plan rubric for the College Algebra track and the MDMP track participants.

The participants could earn at most 8 points for each subcategory—constructivism, behaviorism, and cognitivism—since each subcategory has four measureable components. The MDMP track participants’ scores ranged from 0 to 6, with only 21% of all their scores being above a 2. The behaviorism subsection of the rubric had the greatest number of scores 2 or better, 12 out of 14 MDMP track participants; whereas 7 out of 14 MDMP track participants scored a 2 or better on the constructivism
subgroup and 6 out of 14 MDMP track participants scored a 2 or better on the cognitivism subcategory. The distribution of the participants’ scores for each subcategory varied with constructivism having the widest range (6), while the range of the other two learning theories is three. This verifies that the behaviorist approach was used more often by participants (12 out of 14 above a 1), but the constructivist approach was used at higher levels by a small portion of the participants (highest score is 6).

Table 13

MDMP Track Participants’ Learning Theory Scores

<table>
<thead>
<tr>
<th>Participant</th>
<th>Constructivism</th>
<th>Behaviorism</th>
<th>Cognitivism</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>1</td>
<td>2*</td>
<td>2*</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
<td>2*</td>
<td>1</td>
</tr>
<tr>
<td>31</td>
<td>0</td>
<td>1*</td>
<td>1*</td>
</tr>
<tr>
<td>32</td>
<td>2*</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>33</td>
<td>2</td>
<td>4*</td>
<td>1</td>
</tr>
<tr>
<td>34</td>
<td>2*</td>
<td>2*</td>
<td>1</td>
</tr>
<tr>
<td>36</td>
<td>2</td>
<td>2</td>
<td>3*</td>
</tr>
<tr>
<td>37</td>
<td>0</td>
<td>2</td>
<td>3*</td>
</tr>
<tr>
<td>39</td>
<td>1</td>
<td>2*</td>
<td>2*</td>
</tr>
<tr>
<td>40</td>
<td>0</td>
<td>3*</td>
<td>1</td>
</tr>
<tr>
<td>42</td>
<td>3*</td>
<td>3*</td>
<td>0</td>
</tr>
<tr>
<td>43</td>
<td>6*</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>44</td>
<td>4*</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>45</td>
<td>0</td>
<td>2*</td>
<td>2*</td>
</tr>
</tbody>
</table>

*Note: * participants’ highest score(s) on the type(s) of learning theory approaches used in their lesson plan.
Figure 11. Number of Participants with High Scores in Each Learning Theory Approach. The Venn diagram displays the number of MDMP track participants with high score(s) in those particular learning theory approaches. The total number of MDMP track participants is 14.

The fourth section of the lesson plan rubric was used to score participants’ adherence to state and national standards and policies. This section was divided into three subsections—content standards, process standards and mathematical proficiencies, and pedagogical standards. The content standards and process standards and mathematical proficiencies are two points each but the pedagogical standards has four components for a total of 8 points.

The five-number summary indicating the MDMP track participants’ alignment to the state and national standards is minimum is 1, Q₁ is 3, median is 3, Q₃ is 4, and maximum is 6, but the five-number summary for the College Algebra track is minimum and Q₁ are both 2, the median is 3, Q₃ is 3.5, and has a maximum score of 6 (see Figure 12). Similarities between the group scores include the same median and maximum score
values. The MDMP track participants have higher values for the first and third quartiles compared to the College Algebra track participants. Based on the five-number summary comparison between the MDMP and College Algebra track participants, the MDMP track participants have a higher percentage of participants (75%) earn a 3 or higher on the standards and policies section compared to the College Algebra participants (50%).

![Box plot comparing College Algebra and MDMP track participants' scores on lesson plan rubric sections](image.jpg)

*Figure 12.* College Algebra and MDMP Track Participants’ Scores on Aligning Lesson Plans to Standards. This figure illustrates the difference in the minimum, maximum, and quartile values received on standards section of the lesson plan rubric for the College Algebra track and the MDMP track participants.

**Interviews.** The participants were interviewed after completing the measures of the *Early Indicators of Effective Teachers* instrument. There were five questions asked of all participants (see Appendix F); the responses were coded by examining main themes
that emerged after the question was asked to gauge participants’ pedagogical and pedagogical content knowledge and disposition. In addition, participants’ were asked individualized questions—not coded—based on their lesson plans to further determine their pedagogical and pedagogical content knowledge.

To resolve participants’ disposition, they were first asked why they want to be a teacher. Question one asked participants why they wanted to become teachers; from their responses, six themes emerged (see Figure 13). The first theme observed with the greatest difference was the participants’ responses to external influences—factors in the classroom and at home including teachers’ and parents’ perspectives on teaching. In total, thirteen participants identified wanting to be a teacher because their parents were involved with education, they had positive and negative experiences with teachers, or they had already been involved in education. From the sample of participants, eight identified their love of children or their desire to help children as the reason they chose to become teachers. The remaining themes were distributed as follows: seven said they wanted to be positive role models and that teaching was a noble and rewarding profession, five enjoyed teaching, and four wanted to be coaches or needed the experience for another career path.
Figure 13. Distribution of Responses to “Why do you want to become a teacher?”

The participants were asked to describe their teaching experiences and education levels—question two; their responses varied from no experience to having an Associate’s Degree in Child Development (see Figure 14). There were three participants with no teaching experiences, four participants’ teaching experiences were limited to in-class peer tutoring or assisting friends and family with schoolwork, and six participants took part in high school teacher preparation programs. The majority of participants who were in a high school teacher preparation program, five out of six, visited local elementary and middle schools weekly where they assisted teachers, tutored and taught students, and developed lesson plans. Of the six participants who partook in high school preparation programs, one visited an elementary school only one day when he monitored students as they worked on assigned coursework.

Only three College Algebra track participants had taken pedagogy or pedagogical
content courses—none of these courses were related to mathematics. Overall, half of the participants had teaching experiences stemming from employment or volunteer work at schools, churches, after school programs, and daycares. These participants developed impressions of what it means to be a teacher by observing teachers’ classroom methods, developing their own teaching methods, and preparing curriculum and lesson plans.

Figure 14. Participants’ Various Teaching Experiences.

To understand why a particular grade level was chosen for the lesson plan activity, the participants were asked the third question—why did you choose [insert grade level] grade. If the grade level was not indicated on the lesson plan, the participant was asked what was the intended grade level and then why was it chosen. The grade levels were then mapped to the Texas Essential Knowledge and Skills (TEKS) to verify that the lesson plans were age and grade appropriate (see Appendix J). According to the TEKS, grades K–5 are elementary and grades 6–8 are middle school, but elementary and middle
school teachers graduating from Texas State (2010) earn a Bachelor of Science degree in either EC–6 or 4–8 (NCTM, 2011a). The TEKS division of elementary and middle school grades, K–5 and 6–8, was used to identify the appropriateness of the lesson plans’ content and pedagogy.

Approximately 45.4% of the eleven participants who focused on elementary school grade lesson plans met the TEKS requirements for grades K–5. From the 7 middle school lesson plans that were developed, 5 accurately reflected the TEKS requisites (∼ 71.4%). The indicated grade levels for lesson plans above grade 5 were more accurately aligned to the standards than the elementary lesson plans. In addition, 9 of the 10 participants who intentionally or unintentionally aligned their lesson plans to the ninth-grade TEKS standards were MDMP track participants.

The participants’ responses to question four, “What inspired you to write the lesson plan you chose?” were coded based on the influences they cited. According to the participants, influences such as prior classroom experiences, personal cognitive experiences, and/or the assessment taken for this study affected the development of their lesson plans. Thirteen participants acknowledged that the teaching methods they experienced in their courses and the types of instruction they used when helping peers inspired their lesson plans. Personal cognitive experiences, such as the participants’ own understanding of the content and learning processes, influenced nine participants’ lesson plan development; these participants wanted to make their lessons interesting, interactive, and clear so students who struggle with mathematics could comprehend the content. There were two participants who mentioned that their ideas were taken from particular questions on the modified MKT test, whereas five participants did not understand the
concept of functions and either wrote vague lesson plans or developed lesson plans on a
different mathematical topic. From the five students who did not understand functions,
four were on the MDMP track. The one College Algebra participant who struggled with
functions was enrolled in Math 1315 for a second time, and she developed a lesson plan
on rational expressions, not functions.

Figure 15. Participants’ Inspiration for Their Lesson Plans.

According to Willoughby (1990), functions are a mathematical topic that can be
taught at any grade level; therefore, the participants were asked to complete a lesson plan
on functions. Furthermore, during the interviews the participants were asked when they
learned functions to see if their experiences were reflected in the grade level they chose.
The responses ranged from elementary school to college (see Figure 16). There were five
MDMP track participants and two College Algebra track participants who were taught
functions at the elementary and middle school grade levels. From these seven
participants, six indicated they did not fully understand functions at the lower grade levels, so they relearned or refreshed their memory in high school and/or college courses. The other participant who mentioned learning functions in the lower grades, participant 40, did not indicate learning functions in college, but she did mention she was only capable of remembering information for tests. Participant 40 also scored low on the modified MKT test.

![Figure 16. Participants’ Self-Report of When They Learned Functions.](image)

Each participant was asked question nine, “Did you reflect on the lesson plan” after writing it and “What would you change about the lesson plan?” To accommodate each of the participants, the *Early Indicators of Effective Teachers* instrument and interviews were scheduled on an individual basis. After completing the instrument, the first eight participants had at least a week to reflect on their lesson plans before their interviews, but the last fourteen participants were interviewed immediately after writing
their lesson plans because of time constraints—the semester was ending and the participants had final exams.

During the interviews, approximately 60% of the participants said they felt their lesson plans were lacking content and clear explanations on the intended instructional methods. Participant 30 was the only person who felt their lesson plan was too complex for the grade level they initially selected. Of the participants who had time to reflect on their lesson plans, six of the eight mentioned they would add more content and details; similarly, seven of the fourteen who did not have time to reflect mentioned they would add more content and explanations. The gap between interview times did not have a significant effect on the participants’ responses.

![Figure 17. Changes Participants Would Make After Lesson Plan Reflections.](image-url)
Case study analyses. To better understand the Model Developmental Mathematics Program (MDMP) track participants’ knowledge and disposition, four case studies were conducted. The participants were chosen based on their performances on the Early Indicators of Effective Teachers, which consisted of two instruments—a modified Mathematical Knowledge for Teaching (MKT) test and a lesson plan activity.

To determine which participants warranted further study, their modified MKT test scores and their four scores on the lesson plan assessments—one for each part—were listed on a sheet of paper. Then, five-number summaries for each set of scores were calculated. Each of the scores was colored based on the corresponding quartile; for instance, scores falling in the first quartile were colored pink. The chosen participants scored either low on the modified MKT test and low on the lesson plan, low on the modified MKT test and high on the lesson plan, high on the modified MKT test and low on the lesson plans, or scored high on both assessments.

For each case study presented in this section, the participants’ demographic information, the reason they were selected, their background in education, and their responses to selected interview questions are detailed. A survey given to all College Algebra students in the Fall 2011 and Spring 2012 semesters—used to identify students who met the criteria of this study—was used to collect the participants’ demographic information. Each of the participants’ modified MKT test standardized scores and their lesson plan scores were compiled in order to select individuals with differing proficiency levels. The details of the participants’ experiences and beliefs were gathered through one-on-one interviews in which the specific questions listed in the protocol (see Appendix F) were discussed; these interviews determined factors that influenced the
types of knowledge and disposition gained by MDMP track participants. In addition, case study analyses were performed on four MDMP track participants’ responses to the interview questions and performance on the assessments to understand the impact of specific factors of the MDMP curriculum and content.

All of the participants selected for case study analysis were preservice teachers at Texas State University-San Marcos (Texas State). At the time of the study, all the participants were enrolled in a College Algebra course and had taken at least one developmental mathematics course at Texas State; all the participants chosen for analysis were freshman, besides Lacy who was a sophomore.

**Case 1: Maria.** Maria (pseudonym), participant 30, was a nineteen-year-old Hispanic female seeking a Bachelor of Science degree in Interdisciplinary Studies with a concentration in grades 4–8 Generalist. She completed the modified MKT (Mathematical Knowledge for Teaching) test and scored in the lower quartile (standardized score of –1.84026). Upon finishing the modified MKT test, Maria developed a lesson plan on functions, and she ranked in the lower half of the participants on lesson plan structure (score 5) and scored in the lower 25% on the other three sections (scores 5, 4, and 3). Maria was chosen based on her results—low modified MKT and lesson plan scores—so they could further be investigated.

Maria chose to become a teacher “to make a difference like [her] teachers did in [her] life” (see Appendix K). She was involved in a high school peer program called Peer Assisted Leadership (PAL). After applying to the program, she was selected, from a pool of applicants at her high school to participate. PAL members were assigned to local elementary schools where they assisted teachers two to three times a week. Maria helped
teachers at various elementary grade levels by reading books to students, helping students
with spelling and reading, as well as tutoring 3rd and 4th graders for the mathematics
portion of the TEKS test. On occasion, Maria was responsible for taking students to
recess while the teachers were indisposed. According to Maria, these experiences were
enjoyable and helped her make the decision to become a teacher.

Positive classroom experiences alone are not enough to become a quality teacher;
preservice teachers must become proficient in the content knowledge specified in state
and national standards—including functions. While Maria recalled being taught
functions in her developmental mathematics and College Algebra courses, she still lacked
an understanding of functions. When asked if she knew the definition of a function, she
replied, “No, kind of” and that she did not care. She could recall $f$ of $x$ [$f(x)$], the form to
indicate the input and output values of a function, and that a line is a type of function, but
when asked to develop examples, Maria gave general responses. For example:

Interviewer: So what part of functions do you think they would understand?

Because you said you would show them examples and give them
problems. So what do you think, how much detail would you go into?


Like a skeleton kind of. Just a brief kind of explanation.

Interviewer: You said you would go into different kinds of functions, so what
different kind of functions were you referring to here?

Maria: I don’t know to be honest.

These responses combined with Maria’s statements, “I’m not a big fan” of mathematics,
“I don’t care,” and “I struggle with math,” indicated she would be an incompetent
mathematics teacher (see Appendix K). She lacked content and pedagogical content
to knowledge and had a poor disposition towards mathematics.

Maria’s difficulty with mathematics content affected the quality of her lesson plan. According to Maria, she wrote her lesson plan for the fourth grade level because she thought students in grades K to 3 would not understand the basic concepts of functions. Due to her weak content knowledge on functions, the structure of her lesson plan was based on the teaching methods of her mathematics instructors. Maria stated:

Other teachers kind of structure [their lesson plans by] first they kind of get what it is, then they will give you like an example, and then you have to try it. Then like now they will go over the example with you, and then go into more detail about it and then give you like an assignment on it. Then go over the assignment with you so you can have like a homework assignment for that night. Well that’s what my teachers would do. I’m just basing it off of what my teacher did (see Appendix K).

Maria’s lesson plan on functions (see Appendix L) was generalized. It was more of a lesson plan outline than a complete lesson plan. Her lesson plan included four steps. The first step was to explain the concept of functions, show an example, and allow the students to work independently on one example. Next, according to Maria, the teacher should describe in detail the different types of functions. The last two steps required the teacher to assign a worksheet to be completed and reviewed before the end of class. This lesson plan structure modeled the behaviorist theory for teaching.

Maria’s score on the modified MKT test ranked low when compared to the scores of the other participants. When individual test item responses were investigated, 37 of
the 56 items had a response option of “I’m not sure”; Maria was among the highest responding with this option. She chose “I’m not sure” the 3rd most number of times—on nearly 30% of the 37 problems. Based on the number of times Maria selected “I’m not sure,” her low modified MKT test scores, and her unconfident interview responses, Maria lacked the content knowledge, pedagogical content knowledge, and confidence needed to be a quality teacher.

Maria’s lesson plan also scored among the lowest of the participants’ scores. Her lesson plan was limited to the type of instruction used, such as modeling—the teacher models the desired behavior—and worksheets comprised of practice problems. The modeling process is a technique used in the MDMP lesson plans on functions, but the MDMP lessons also included group work and discovery-based learning which was not present in Maria’s lesson plan.

**Case 2: Carlos.** Carlos (pseudonym), participant 31, was a nineteen-year-old Hispanic male seeking a Bachelor of Science degree in Interdisciplinary Studies with a concentration in grades 4–8 Mathematics. He completed the modified Mathematical Knowledge for Teaching (MKT) test and ranked in the upper quartile (standardized score of –0.99212). Upon finishing the modified MKT test, Carlos completed a lesson plan on functions on which he scored in the lower quartile on all four sections (scores 2, 2, 2, and 1). Carlos was chosen for this case study analysis because of his high performance on the modified MKT and low performance on the lesson plan activity.

Carlos’s inspiration to be a teacher stemmed from his love of sports. Carlos wanted to be a “football, basketball, baseball, [and] track” coach because he participated in these sports in high school (see Appendix K). He liked mathematics and teaching, but
his real desire was to become a coach.

His experience with teaching in high school was limited to visiting a local elementary school for one day, where he watched students work while the elementary school teachers “went out to lunch” (see Appendix K). The teachers instructed him to “maintain” the elementary students in the classroom to do their assigned work. On another occasion, Carlos helped his friend with mathematics because he felt confident in his ability to explain mathematics. Compared to the other participants, his teaching experience was limited. Undoubtedly, this contributed to his low score on the lesson plan activity.

For Carlos, mathematics content was “interesting.” According to Carlos, even though he did not always make “perfect scores” on assignments, he “still feel[s] like [he is] good at it.” He just “need[s] to have someone explain [it] to [him] real good in order for [him] to get it real good.” He believed a “good teacher equals a good student” (see Appendix K). Carlos recalled past experiences with his ninth grade teacher who taught him functions. Coincidentally, he wrote a lesson plan for the same level, ninth grade.

After reflecting on his lesson plan, Carlos thought he created a general lesson plan similar to how his instructors taught him functions. He said this about his instructor: “[He] talks about what he is going to teach, and then he does it on the board and does examples and like shows us how to work through it. And that’s what, well that is what I was trying to say in the paper but I don’t think I said it pretty good” (see Appendix K). There is a slight disconnect between what he noticed in his mathematics class and what he wrote in his own lesson plan. After discussing the structure and content of his lesson plan, Carlos mentioned that in retrospect, he would have added details to his explanations
about teaching the content and aiding struggling students. Carlos was trying to write a
lesson plan on how he was taught in the classroom, which corresponds to research on
teaching how you have been taught.

Like Maria, Carlos gave general examples and descriptions of the mathematical
content covered in his lesson plan. When asked to elaborate on specific components in
his lesson plan, Carlos offered vague responses, as seen in the following excerpt from the
interview.

Interviewer: So you didn’t really put any specific details about functions in here,
what specific topic on functions you can teach to freshman?

Carlos: Just the basics. Just the basics of functions, and then from there whatever
else they need to learn or just take another step ahead or something.

Interviewer: So what kind of examples would you give them?

Carlos: First real easy ones, in order for them to get it, then from there just, as
soon as they start understanding it more then start going to harder and
harder more difficult.

Interviewer: Do you think you can give me an example of a question you might
ask about functions?

Carlos: I would probably just like teach them about a table oh is this a function or
whatever. Like do you think this is a function. And from there start the
lesson or something. I mean they should at least know what a function is
or you know or at least the beginning of a function (see Appendix K).

Carlos’s inability to articulate exactly how he would teach functions may be due to his
lack of pedagogical content knowledge. Despite the lack of details in his lesson plan, it
did progress from simple to complex concepts that resembled constructivist-teaching methods that are used in the MDMP.

Even though Carlos did not give specific descriptions of functions in his lesson plan (see Appendix L), he ranked in the top quartile for answering the function items correctly on the modified MKT. He was also among the participants who selected “I’m not sure” the fewest number of times, on approximately 5% of the 37 questions. His content knowledge met the requirements for a quality teacher, but his pedagogical content knowledge and pedagogy were limited. For example, in his lesson plan, he claimed, “I would have my freshman in high school pay attention when going over functions in class.” When asked how he would get the students to pay attention, he responded, “By getting their attention towards me, like teachers, so of course you are going to be like ‘oh hey listen up’ or something.” Even though Carlos possessed adequate content knowledge, his limited pedagogical training and lack of pedagogy would hinder his ability to teach mathematics content.

**Case 3: Isabella.** Isabella (pseudonym), participant 33, was an eighteen-year-old Hispanic female seeking a Bachelor of Science degree in Interdisciplinary Studies with a concentration on grades 4–8 Generalist. She scored in the lower quartile on the modified Mathematical Knowledge for Teaching (MKT) test (standardized score of –1.64409). Upon finishing the modified MKT test, Isabella completed a lesson plan on functions, on which she scored in the top 50% on teacher quality, learning theories, and standards and policies sections (scores 10, 7, and 4) and in the top 25% on lesson plan structure (score 10). Isabella’s scores were further investigated to determine what factors may have contributed to her low modified MKT test score and high lesson plan scores.
Isabella wanted to be a teacher because she liked kids and helping others. She felt that she could be a “good role model” for students. For her, satisfaction would come from “just knowing the fact that [she will] have such an impact on a child’s life” so they can have a better future, such as a “continued education,” to possibly “become a teacher as well or something bigger like a doctor.” She wanted her students to have a “good life” (see Appendix K). Her motivation for becoming a teacher also resulted from her primary and secondary grade teachers’ acknowledgment of her dedication and perseverance for going to college and pursuing a career in education.

This participant had less training than Maria or Carlos. She did not participate in any high school teacher preparation programs or tutor any peers in an informal setting. The inspiration for her lesson plan was a result of her experiences as a student. She said her mathematics instructors’ teaching methods influenced the development of her lesson plan on functions. According to Isabella, her instructors explained every step needed to solve problems, and their lesson plans usually started with “a definition, some examples, go over it, and then usually give them a quiz the next day.”

Isabella’s knowledge of how to develop a lesson was limited because she had not yet enrolled in pedagogy courses. She stated that college is where she should be “taught to teach” (see Appendix K). Even though Isabella’s experiences were limited, she was observant in her classrooms and was able to develop a quality lesson plan on functions based on her experiences as a student.

Isabella recalled learning functions as early as ninth grade. She was also taught functions in her college mathematics courses—developmental mathematics and College Algebra. She chose to write her lesson plan for the 9th grade because that was the first
grade she could recall learning functions (see Appendix L). Her lesson plan had more
detail on functions than Carlos’s or Maria’s. It included defining a function first, then
showing examples such as $f(x) = x^2$. Then the students would be required to work on
practice problems together as a group and then independently. Isabella noted the need for
an in-class assessment and a quiz the following day. During Isabella’s interview, she
mentioned teaching with several methods: “Just give [the students] options [to] see which
[ones] work for them.” This was due to her personal preference of being given options to
choose from when given mathematics problems.

At the time of the interview, Isabella did not have the content knowledge to
competently teach mathematics, but her dedication and pedagogy showed promise that
she could one day become a quality teacher. Isabella avowed that if she did not learn
what was needed in her college courses, she would “do [her] research” to become an
expert. Unfortunately, Isabella had less confidence in mathematics content compared to
other subject matter, like social studies, because she was not interested in the course.
According to Isabella, she “pays attention” in her history courses because she is
interested in the subject and already feels knowledgeable enough in that area to teach it.

Her weakness in mathematics may be due in part to her lack of confidence. Out
of all the participants, Isabella chose the “I’m not sure” option on the modified MKT test
the second highest number of times. She marked approximately 32% of the 37 items
with “I’m not sure.” Isabella also noted in her interview that she was “not a strong math
student” and had low confidence in her ability to write a lesson plan. She mentioned that
she was “unsure how to write a lesson plan” and had “no clue how to write one.” Even
though she lacked confidence, her lesson plan scored among the top 50% of participants.
Isabella was not competent enough in the content and pedagogical content knowledge needed to effectively teach mathematics.

Case 4: Lacy. Lacy (pseudonym), participant 43, was a nineteen-year-old White, non-Hispanic, female seeking a Bachelor of Science degree in Interdisciplinary Studies with a concentration in Early Childhood (EC) through Grade 6 English as a Second Language (ESL) Generalist. She was further investigated because she outperformed most of the participants on both the modified Mathematical Knowledge for Teaching (MKT) test and the lesson plan activity. She ranked in the upper 25% of the participants on the modified MKT test (standardized score of –0.90196). Lacy ranked in the upper quartiles on the lesson plan activity, outperforming at least 50% of the participants, on parts 1, 2, and 4 (scores 7, 10, and 4) and 75% on part 3 (score 10).

According to Lacy, she had always been around education because her mother was a preschool and now a kindergarten teacher. She was often at school seeing different aspects of teaching. Lacy was always around kids—babysitting her “whole life.” She claimed that “kids have been a huge part of [her] life and [she] just love[s] working with them.” She had no experience with teaching other than tutoring peers during the 2012 Spring semester. She said, “math was never” her subject, but this was the “first semester” she was “ever … good at math” and it was a “huge shock” to her (see Appendix K). Other than tutoring, Lacy’s inspiration for developing her lesson plan on functions was her lesson plan writing experience in a music and art pedagogy college course.

In her lesson plan, Lacy wanted to incorporate a small project that integrated different subject areas, such as mathematics and art. Her project integrated mathematics,
English, and art. The lesson described a booklet activity, where the students were to develop a booklet containing different types of functions represented multiple ways—words, graphs, and tables. She felt that fourth grade students were the ideal group for her lesson plan; she believed older students would not be motivated to participate in the activity.

Lacy’s lesson plan was structured similarly to Isabella’s. First, the teacher was to start by “giving the definition of a function in both words and drawing out examples,” and then the teacher would demonstrate different types of functions (see Appendix K). One difference between the lesson plans was that Lacy used a linear function, while Isabella used a quadratic function. In addition, Lacy’s lesson plan incorporated group work and discussion, while Isabella’s lesson plan focused on completing multiple practice problems and then reiterating the steps. Lacy’s experiences in a pedagogy course and art and music courses aided her development of a successful lesson plan.

Lacy’s mathematics confidence level was high due to her success in College Algebra. Like Carlos, Lacy only marked “I’m not sure” on approximately 5% of the MKT items and performed in the upper quartiles on the overall assessment, which demonstrated her mathematical content proficiency. Even though Lacy was successful with the content, she still had a poor disposition towards mathematics due to her past experiences in mathematics. Her disposition towards mathematics was changing at the time of the interview, which could be due to her acceptance of a facilitator position for a mathematics program Summer I of 2012. Lacy’s content knowledge and disposition was apparent in her high MKT scores and in her interview responses, but Lacy also demonstrated pedagogical and pedagogical content knowledge in her lesson plan on
functions. She used age appropriate mathematical examples and definitions. She incorporated activities and discussions which are techniques used by quality teachers.

The data collected, participants’ MKT test scores, lesson plan scores, interviews, and MDMP students’ survey responses, and presented in Chapter 4 was used to determine if the MDMP provided elementary and middle school preservice teachers with an environment that was conducive for learning not only mathematical content but pedagogy. Chapter 5 presents the conclusions that were determined based on the collected data as well as provide limitations and further research ideas.
CHAPTER V

DISCUSSION

The goal of this study was to determine the effects of standards-based best practices used in the Model Developmental Mathematics Program (MDMP) courses on preservice teachers’ knowledge and disposition. Elementary and middle school preservice teachers were selected, because the practices in the MDMP courses reflect those recommended for grades K–12, a majority of preservice teachers seeking a degree in grades K–8 are weak in mathematics, and according to the authors of Remediation Higher Education’s Bridge to Nowhere, the preservice teachers in the MDMP are less likely to graduate from college (Complete College America [CCA], 2012). Once the preservice teachers graduate, they are expected to transfer their own content, pedagogical, and pedagogical content knowledge and disposition to the next generation of teachers.

This chapter presents the results of this study in a nonstatistical and theoretical manner. The results presented in Chapter 4 will be further explored and expounded upon by drawing connections between the literature review and the outcomes of the analyses performed on the collected data.

Summary of Results

For this study, twenty-two elementary and middle school preservice teachers’ knowledge and disposition were assessed and compared. Each participant completed a modified Mathematical Knowledge for Teaching (MKT) test, a written lesson plan task
on functions, and an interview. Quantitative analyses—two-way between groups analysis of variance (ANOVA) tests—were used to compare the participants’ 56-item multiple-choice MKT test scores for each subscale of items to determine the preservice teachers’ content knowledge (CK), common content knowledge (CCK), specialized content knowledge (SCK), and knowledge of functions.

The first ANOVA test compared mean standardized scores of the College Algebra track and the Model Developmental Mathematics Program (MDMP) track participants for the overall content test. There was a statistical significant difference between the groups’ mean scores; the College Algebra students outperformed the MDMP students on the overall MKT test. In addition, the College Algebra participants outperformed the MDMP participants on the CCK subscale of the modified MKT test, but the difference in the participants’ scores on the SCK subscale and function subscale on the modified MKT tests were not statistically significant.

The participants were also required to develop a lesson plan and complete an interview for qualitative analysis. The lesson plans were evaluated using a rubric designed for this study. In addition, the MDMP lesson plans on function were evaluated with the same rubric to establish a benchmark. The participants’ scores were lower than the MDMP lesson plans’ scores. The MDMP scores were higher because the lesson plans have been evolving over the last 10 years to maintain alignment to state and national standards, and because qualified professionals developed the lesson plans. On the other hand, the MDMP participants’ low scores may be due to the time lapse between their enrollment in the MDMP and collecting data for this study. To compensate for the duration of time between completion of the MDMP and enrollment in College Algebra, a
survey was given to the MDMP students to verify they were aware of the pedagogy and not just learning content. The students enrolled in the MDMP at the time of the study were aware of the different instructional techniques and its benefits in the classroom.

To determine early indicators of effective teachers in the MDMP participants, their lesson plan scores were compared to the College Algebra participants’ lesson plan scores. The distribution of lesson plan scores, given by the five-number summary, for each participant was illustrated with box-plots to aid in comparing participants’ scores. The MDMP participants’ scores were similar to the College Algebra participants’ scores on all sections of the lesson plan rubric, but the College Algebra participants’ scores were higher than the MDMP participants’ scores, besides the last section, standards and policies. The items were also further evaluated to determine the number of participants that selected “I’m not sure” on 37 of the 56 test items as well as evaluating specific test questions where a majority of the participants selected the same wrong solution.

Upon completion of the lesson plan writing activity, an interview was conducted with each participant. The participants interviews were coded and main themes were identified about the participants’ lesson plans, experiences, and disposition towards mathematics. The analyses performed on this mixed methods study were used to answer three research questions.

Question Results

Research question 1. RQ 1 was identifying any significant differences in Mathematical Knowledge for Teaching (MKT) scores for elementary and middle school preservice teachers who completed the Model Developmental Mathematics Program (MDMP) at Texas State university-San Marcos (Texas State) compared to those who
The overall content knowledge (CK) and common content knowledge (CCK) scores were significantly different; the College Algebra participants outperformed the MDMP students on both the content and common content knowledge test items. The difference of mean scores for content knowledge was 0.67636 standard deviations and for CCK was 0.92433 standard deviations. The specialized content knowledge (SCK) and function subscales had no significance difference between the two group means. This indicates the means were approximately equal. The College Algebra participants outperformed the MDMP participants on the SCK subscale. The mean difference between the two groups was 0.4315, less than half a standard deviation difference. The modified MKT test had 37 questions with the answer choice “I’m not sure.” Majority, 9, of the top ten participants choosing this option the most were MDMP track participants. Investigating the distribution of these “I’m not sure” answer options, 15 of the 28 SCK items had this response, whereas 22 of the 28 CCK items had this response.

The difference in MDMP and College Algebra participants’ CK scale and CCK subscale scores may be due to the MDMP participants’ lack of content knowledge because they were not college ready when they enrolled in College Algebra or they may have anxiety towards mathematics testing. Several participants mentioned in the interview that mathematics was not their favorite or best subject. However, based on the distribution of participants selecting “I’m not sure” and their performances on the modified MKT, one may assume the MDMP students do not lack knowledge but confidence. When given fewer items with the response “I’m not sure,” the students may perform better or equivalent to its comparison group.
In theory, the MDMP goals are to develop college-ready students with standards-based best practices supported by well-trained instructors. Based on the results of the MKT test, students lack confidence in mathematics or perform poorly due to lack of content knowledge. Other contributions to the participants’ performances on the MKT test may be due to MDMP instructors not respecting the written lesson plan, meaning they did not incorporate the activities or switched sequences of the lesson plans.

**Research question 2.** The Model Developmental Mathematics Program (MDMP) preservice teachers in this study were influenced by their different experiences as students, peers, and observers in and out of the classroom. RQ 2 asks what early indicators of effective teachers identified in preservice teachers at Texas State who completed the MDMP course(s) compared to those who started in College Algebra using the lesson plan rubric and interview analyses. The rubric scored lesson plans based on four areas: lesson plan structure, four characteristics of teacher quality, learning theory approaches, and alignment to standards and policies. In short, the College Algebra students outperformed the MDMP students on the written lesson plan activity, but the difference in the scores were minimal.

The MDMP curriculum is well developed and exhibits all the traits specified in the lesson plan rubric as well as being taught by trained professionals. The Basic Mathematics function lesson plans were evaluated with the rubric designed in this study; the scores obtained from the evaluation were used as ideal benchmark standards for the MDMP participants’ scores. The MDMP participants’ scores did not meet the ideal benchmark score even though they were exposed to these lesson plans. In the categories learning theory and standards and policies, the MDMP participants scoring the highest
were approximately equal to the minimum scores obtained by the Basic Mathematics lesson plans but no other scores were comparable.

The MDMP track participants’ lesson plan scores were also compared to the College Algebra track participants’ lesson plan scores. The first section scored using the lesson plan rubric was the lesson plan structure. The students that prominently displayed some type of outline or structure to their lesson plan learned lesson plan writing in a previous course or a pedagogy course that they were enrolled in at the time of the study. Preservice teachers knowing how to structure a lesson plan is an indicator of quality teaching. Preservice teachers without formal training lacked the ability to structure a sound lesson plan. The experiences the students received in the MDMP classroom did not impact the structure of the lesson plan but affected the lesson plan content, structure of the main teaching script of the lesson plan, and methods used to teach the content of their lesson plans. These results indicate that it is essential to combine lessons that promote pedagogy and content.

The results of the second section of the rubric, the four characteristics of teacher quality, explained the content knowledge, pedagogical knowledge, pedagogical content knowledge, and disposition of the preservice teachers. The College Algebra participants and the MDMP participants’ scores had the same median, but the MDMP participants’ scores below the median had a larger distribution while the College Algebra participants’ scores above the median had a wider distribution compared to the opposing group. Only a few participants received some type of formal training on developing lesson plans, preservice teachers without formal training had lesson plans that lacked content and unique instruction. Therefore, the lack of complexity in the lesson plans may be due to
the preservice teachers’ ignorance on developing lesson plans and not their knowledge of the content or pedagogy. This was also represented in the interviews; the participants were asked about their lesson plans, and a majority of the participants said they would redo or add content to their lesson plans to make the lesson plans clear for the reader. Besides knowledge, disposition was also explored. During the interviews, students were asked why they wanted to become a teacher and others, more specifically, mathematics teachers. The mathematics teachers, who performed well on the modified MKT test, enjoyed the subject matter and wanted to help others learn mathematics. Motivation and interest in the subject area can be suggested as a key element for developing teacher quality.

The third component of the lesson plan rubric assessed participants’ use of the three learning theory approaches. The type of instruction used more prominently in the Basic Mathematics function lesson plans was a constructivist approach; whereas, the MDMP track participants concentrated more on a behaviorist approach. All the Basic Mathematics function lesson plans had attributes from each learning theory, but not all of the MDMP participants’ lesson plans had characteristics of each approach. The differences in the instructional methods used may be contributed to participants’ College Algebra class, since College Algebra courses are primarily taught with didactic discourse. As explained by Quinn (1997), changing ones views takes time and one developmental mathematics course may not have changed their pedagogical instincts—teacher-centered approach to a student-centered approach. The Spring 2012 MDMP students were surveyed; the students revealed they were aware of the various instructional methods used in the courses but the MDMP students enrolled in College Algebra did not use these
techniques when developing their lesson plans. This may be due to the duration of time between the completion of the MDMP course(s) and the study.

The last area scored using the lesson plan rubric was the adherence of the lesson plans to state and national standards. Even though the maximum score of the College Algebra participants was higher than the MDMP participants’ maximum score, the MDMP participants’ scores for the first and third quartiles were higher than the College Algebra scores. This may be due to the grade level chosen by the MDMP participants. From the 14 MDMP participants, nine participants consciously or unconsciously developed a ninth grade lesson plan. Only three participants indicated their lesson plan addressed ninth grade standards, whereas five of the other six participants selected grades lower than ninth grade. The alignment of the lesson plan to the ninth grade standards may be due to the participants’ recollection of being taught functions in the MDMP course(s), since the content and process standards in the MDMP are similar to those taught in ninth grade Algebra I. The number of ninth grade lessons developed by MDMP participants, nine, is significantly higher than those developed by the College Algebra participants, one. From the interviews, all of the participants besides one student recalled learning functions in high school or college, so there exists a strong correlation between the content taught in the MDMP and the preservice teachers’ conceptual understanding of mathematics content, Curriculum Renewal Across the First Two Years (CRAFTY) first recommendation.

The participants performing well on the lesson plans were also the participants who performed exceptional on the modified MKT test. There may exist a correlation between test scores and lesson plan scores. Observing the high achieving MDMP and
College Algebra participants, meaning they performed one standard deviation below the mean or higher on the modified MKT test, these students were enrolled in pedagogy courses as well as College Algebra at the time of the study. Supporting Shulman’s (1986) idea of pedagogical content knowledge, all the participants that were enrolled in both courses were able to integrate what they learned to develop their function lesson plans. This supports the idea of integrating both content and pedagogy in one content course to aid in the development of pedagogy and pedagogical content knowledge.

**Research question 3.** The results of the Model Developmental Mathematics Program (MDMP) participants’ performances on the *Early Indicators of Effective Teachers* have shown some contributing factors to developing effective teachers. The results of the MDMP survey and the four case studies were investigated to identify the MDMP track participants’ experiences and impressions in the classroom, especially the MDMP that contributed to their performances on the *Early Indicators of Effective Teachers*.

The MDMP track participants took either Basic Mathematics or Pre-Algebra and Basic Mathematics before enrolling in College Algebra. The instructors of these courses use similar instructional methods but the mathematical content differs. Since the MDMP track participants have been removed from the MDMP environment for as little as a semester, the MDMP students enrolled at the time of the study were surveyed about the content and instructional techniques used in the MDMP courses.

Both courses are taught using technology and manipulatives, which are tools that are very useful when developing conceptual understanding of the content. The participants were asked questions about particular lessons that incorporated various
instructional methods such as algebra. Majority of the students (Pre-Algebra: 29 out of 42 respondents and Basic Mathematics: 210 out of 299) utilizing algebra tiles realized hands-on activities were important, even though not all of the respondents favored using manipulatives to learn. The MDMP students also preferred using graphing calculators, algorithms, and cooperative learning groups to learn mathematics, but these methods were sparsely used in the MDMP track participants’ lesson plans.

Another contributing factor to teacher quality is disposition. According to Quinn (1997), it takes time to change a person’s disposition towards teaching methods, which was noticed in the types of instructional approaches used by the MDMP participants. This may be due to the MDMP preservice teachers’ College Algebra instructors’ teaching methods. The more content courses that incorporate various instructional techniques, standards-based best practices and hands-on activities, will help preservice teachers develop pedagogy and pedagogical content knowledge that are attributes of quality teachers.

To further investigate the MDMP participants’ experiences and impressions from the program, four participants having varying levels of results on the *Early Indicators of Effective Teachers* instrument were chosen for case study analyses. Lacy and Isabella performed well on the lesson plan activity, whereas Lacy and Carlos performed well on the modified MKT test. Maria was chosen because she performed poorly on both the modified MKT test and the lesson plan rubric. Participants, Lacy and Isabella, performing well on the lesson plan rubric was due to their organization of the lesson plan—definitions, examples, discussions, activities—which resembles similar structure to the MDMP curriculum. Lacy and Isabella did not directly report learning this from the
MDMP courses, but stated they used their “mathematics instructors’ teaching methods,” meaning their classroom experiences influenced their organization and the content used in their lesson plans. Lacy and Isabella’s lesson plans included pedagogical techniques used in the MDMP such as multiple representations, group work, and discussion.

Carlos and Lacy performed well on the modified Mathematical knowledge for Teaching (MKT) test, which may be due to their high confidence in mathematics. Carlos mentioned he liked mathematics and that it was a subject matter of interest for him. Lacy’s positive disposition towards mathematics was contributed to her recent success in mathematics. She said that mathematics was never her favorite subject but she was understanding the content and helping other students in class—contributing to her content, pedagogical knowledge, and pedagogical content knowledge and positive disposition. Carlos and Lacy also had more confidence on the modified MKT test than Maria and Isabella, which was evidenced in the number of times they selected “I’m not sure” on the modified MKT test. The positive disposition towards mathematics may be a contributing factor to their success in becoming a quality teacher.

**Conclusions**

These results and existing literature that espouses the effects of standards-based best practices on preservice teachers’ content, pedagogical, and pedagogical content knowledge and disposition have shown correlation between positive disposition towards mathematics and attributes of quality teachers. In addition, the Model Developmental Mathematics Program (MDMP) track participants’ poor results on the modified Mathematical Knowledge for Teaching (MKT) test may not be due to a lack of knowledge but a lack of mathematical confidence as evident by the low performing
participants—Maria and Isabella—choosing “I'm not sure” on more than 20% of the 37 items and the high performing participants—Carlos and Lacy—choosing “I’m not sure” on approximately 5% of the 37 questions.

According to Cavallo (2010), disposition towards teaching and mathematics is a major contributing factor to developing quality teachers. The standards-based best practices used by the MDMP instructors may contribute to positive disposition in mathematics. The MDMP students responded on the survey that due to cooperative learning that discovered they could teach others the concepts taught in class, which was also mentioned by Lacy in her interview. The cooperative learning groups provided the MDMP students with an environment to discuss mathematics and maybe inspiring them to become teachers.

**Limitations**

A limitation of this study is the number of participants in this study. Since the participation was voluntary and the administration of the instrument took approximately 1.5 hours, many students were disinterested or simply unable to spare the time to participate in this study. In addition, the limited number of participants, categorized in one of five different majors, were regrouped into one of two categories—elementary and middle school preservice teachers—prior to analysis, which lessened the number of factors contributing to participants’ scores on the *Early Indicators of Effective Teachers* instrument.

There are a limited number of preservice teachers who had taken the College Algebra track at Texas State. The participation between the groups was disparate, because the number of student on the Model Developmental Mathematics Program
(MDMP) track were not equal to the number of respondents on the College Algebra track.

Students in developmental mathematics probably came into the program with poor disposition towards mathematics. As stated in the literature review, it takes time to change a person’s disposition; it cannot be fixed in one or two semesters. In general, people who have a history of being unsuccessful in a course do not engage themselves in that class, so students required to enroll in the MDMP courses probably lack mathematical proficiency and motivation to succeed in the course.

Another limitation is the restriction on the type of audience intended to complete the assessments. The Early Indicators of Effective Teachers instrument was developed specifically to assay elementary and middle school preservice teachers, but a modified form of the Early Indicators of Effective Teachers instrument can be used by a multitude of institutions to create curriculum that consistently evaluates the quality of their preservice teachers. Universities and preservice teacher training programs with similar content to the MDMP can use this instrument to examine the differences in preservice teachers’ MKT scores and lesson plan activity. The results, standardized scores on the modified MKT test, can further determine preservice teachers’ levels of common content knowledge and specialized content knowledge in mathematics. In addition, the lesson plan rubric can be adapted to classify preservice teachers’ pedagogical and pedagogical content knowledge and disposition across different content areas.

Suggestions

Based on the data gathered and analysis completed in this study, five suggestions were compiled. First, standards-based best practices should remain in the developmental
mathematics courses. The Model Developmental Mathematics Program (MDMP) track participants and College Algebra track participants’ mean specialized content knowledge (SCK) scores on the Mathematical Knowledge for Teaching (MKT) tests were less than half a standard deviation in difference; this difference was not significant. There was a significant difference between participants’ overall content knowledge scores and common content knowledge scores but the MDMP track participants’ SCK may be due to the standards-based best practices used in the classroom.

Second, a support piece should be added to the developmental mathematics program to boost student confidence. The MDMP track participants performed poorly on the content knowledge and common content knowledge items on the MKT test compared to the College Algebra participants. When investigating individual test items, it was determined that the MDMP participants who earned a low score on the exam were also the same participants that chose “I’m not sure” more often than the College Algebra participants. The participants’ confidence in selecting an answer choice other than “I’m not sure” represents their lack in confidence for the content, not necessarily their capability of answering the question.

Third, there should be a College Algebra course specifically designed for preservice teachers. The idea of developing credit-bearing mathematics courses for preservice teachers is supported by Curriculum Renewal Across First Two Years. This would provide preservice teachers with an advantage of learning mathematical knowledge for teaching, because the preservice teachers are learning with standards-based best practices, the consecutive courses are taught with the same instructional techniques, and skills needed by teachers can specifically be addressed in the courses.
Fourth, the idea of having consecutive mathematics courses introduces the possibility of having mathematics courses as a block course. Students sometimes take breaks between consecutive courses, which may result in the student performing poorly in the latter course(s). A block course would allow students to continuously proceed through the recommended courses without interruptions as well as the courses could be designed specifically for their major.

Lastly, these suggestions do not have to be restricted to mathematics. Preservice teachers must acquire credits for English, science, history, and mathematics. Incorporating standards-based best practices in all content courses would provide preservice teachers with various modes of instructing for these specific content areas. These suggestions should allow preservice teachers to become better quality teachers because of the classroom experiences they are having as students.

**Recommendations for Future Research**

Standards-based best practices should be implemented across various disciplines. To measure success of implementation, the lesson plan rubric can be modified to help assess curriculum for other subject areas as well as be used to assess preservice teachers’ pedagogical, content, and pedagogical content knowledge and disposition in these other areas of study.

To improve this study, alternative comparison groups could be established. For instance, compare MDMP participants to developmental mathematics students from other colleges and universities. The students should have the same knowledge and disposition before proceeding into the study. Another form of comparison would be the participants’ pre- and post-test evaluations. Participants’ pre- and post-test scores would show their
personal gains as well as help identify strengths and weaknesses of the program.

According to compiled research, preservice teachers can improve their knowledge and disposition due to exposure of practices similar to those used in the MDMP; this study should be recreated on a larger scale with more participants to determine the effectiveness of the MDMP. The United States is falling behind other countries in graduating math, science, technology, and engineering professionals, so developing students who are interested in these fields is a priority. High-quality teachers are in demand to teach mathematics and science to the changing student population, meaning they are also required to learn new types of instruction and develop curriculum conducive for STEM professions. The difficulty with producing high-quality teachers is that they start college underprepared and they have poor disposition towards mathematics. It is ideal to build a mathematical foundation for preservice teachers, such as the foundation built by the MDMP, as well as incorporate K – 12 mathematical pedagogy to aid with the development of high-quality elementary and middle school preservice teachers.
## APPENDIX A

### SAMPLE DEMOGRAPHICS

<table>
<thead>
<tr>
<th>Participant</th>
<th>Sex</th>
<th>Age</th>
<th>Classification</th>
<th>Ethnicity</th>
<th>Major</th>
<th>First Math Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>F</td>
<td>18</td>
<td>Freshman</td>
<td>African American White, Non-Hispanic</td>
<td>K-12 Generalist</td>
<td>1315</td>
</tr>
<tr>
<td>22</td>
<td>M</td>
<td>18</td>
<td>Freshman</td>
<td>White, Non-Hispanic</td>
<td>4-8 Math/Science</td>
<td>1315</td>
</tr>
<tr>
<td>23</td>
<td>F</td>
<td>19</td>
<td>Sophomore</td>
<td>Hispanic</td>
<td>4-8 Generalist</td>
<td>1315</td>
</tr>
<tr>
<td>24</td>
<td>F</td>
<td>18</td>
<td>Sophomore</td>
<td>White, Non-Hispanic</td>
<td>EC-6 Generalist</td>
<td>1315</td>
</tr>
<tr>
<td>25</td>
<td>F</td>
<td>25</td>
<td>Junior</td>
<td>White, Non-Hispanic</td>
<td>EC-6 ESL Generalist</td>
<td>1315</td>
</tr>
<tr>
<td>26</td>
<td>F</td>
<td>30</td>
<td>Sophomore</td>
<td>White, Non-Hispanic</td>
<td>EC-6 ESL Generalist</td>
<td>1315</td>
</tr>
<tr>
<td>28</td>
<td>F</td>
<td>20</td>
<td>Sophomore</td>
<td>Hispanic</td>
<td>K-6 Generalist</td>
<td>1311</td>
</tr>
<tr>
<td>29</td>
<td>F</td>
<td>24</td>
<td>Sophomore</td>
<td>Hispanic</td>
<td>4-8 Math/Science</td>
<td>1315</td>
</tr>
<tr>
<td>30</td>
<td>F</td>
<td>19</td>
<td>Freshman</td>
<td>Hispanic</td>
<td>4-8 Generalist</td>
<td>1311</td>
</tr>
<tr>
<td>31</td>
<td>M</td>
<td>19</td>
<td>Freshman</td>
<td>Hispanic</td>
<td>4-8 Math</td>
<td>1311</td>
</tr>
<tr>
<td>32</td>
<td>F</td>
<td>20</td>
<td>Sophomore</td>
<td>Hispanic</td>
<td>K-6 Bilingual Generalist</td>
<td>1311</td>
</tr>
<tr>
<td>33</td>
<td>F</td>
<td>18</td>
<td>Freshman</td>
<td>Hispanic</td>
<td>4-8 Generalist</td>
<td>1311</td>
</tr>
<tr>
<td>34</td>
<td>F</td>
<td>18</td>
<td>Freshman</td>
<td>Hispanic</td>
<td>K-6 ESL Generalist</td>
<td>1300</td>
</tr>
<tr>
<td>35</td>
<td>F</td>
<td>19</td>
<td>Sophomore</td>
<td>White, Non-Hispanic</td>
<td>K-6 ESL Generalist</td>
<td>1315</td>
</tr>
<tr>
<td>36</td>
<td>F</td>
<td>21</td>
<td>Senior</td>
<td>White, Non-Hispanic</td>
<td>K-6 Generalist</td>
<td>1311</td>
</tr>
<tr>
<td>37</td>
<td>F</td>
<td>18</td>
<td>Sophomore</td>
<td>Hispanic</td>
<td>K-6 Bilingual</td>
<td>1311</td>
</tr>
<tr>
<td>39</td>
<td>F</td>
<td>20</td>
<td>Sophomore</td>
<td>White, Non-Hispanic</td>
<td>4-8 Generalist</td>
<td>1311</td>
</tr>
<tr>
<td>Participant</td>
<td>Sex</td>
<td>Age</td>
<td>Classification</td>
<td>Ethnicity</td>
<td>Major</td>
<td>First Math Class</td>
</tr>
<tr>
<td>-------------</td>
<td>-----</td>
<td>-----</td>
<td>----------------</td>
<td>---------------------------</td>
<td>---------------------------</td>
<td>------------------</td>
</tr>
<tr>
<td>40</td>
<td>F</td>
<td>18</td>
<td>Freshman</td>
<td>Hispanic</td>
<td>K-6 Generalist</td>
<td>1311</td>
</tr>
<tr>
<td>42</td>
<td>F</td>
<td>19</td>
<td>Sophomore</td>
<td>White, Non-Hispanic</td>
<td>K-6</td>
<td>1311</td>
</tr>
<tr>
<td>43</td>
<td>F</td>
<td>19</td>
<td>Sophomore</td>
<td>White, Non-Hispanic</td>
<td>EC-6</td>
<td>1300</td>
</tr>
<tr>
<td>44</td>
<td>M</td>
<td>25</td>
<td>Sophomore</td>
<td>Hispanic</td>
<td>K-6 Bilingual Generalist</td>
<td>1300</td>
</tr>
<tr>
<td>45</td>
<td>M</td>
<td>34</td>
<td>Junior</td>
<td>White, Non-Hispanic</td>
<td>4-8 Generalist</td>
<td>1311</td>
</tr>
</tbody>
</table>
APPENDIX B

THEORETICAL FRAMEWORK

The theoretical framework on the following page identifies the dependent and independent variables used to develop the lesson plan rubric.
Lesson Plan Rubric
(Compilation of Rubrics Used)

4 Characteristics of Teachers’ knowledge

CK
- Specialized Language
- Content Examples

PK
- Linear/ Chronological
- Prior Knowledge

PCK
- Level Appropriate Explanations
- Appropriate use of Representations
- Symbols with Care
- Openness

Constructivism
- CRA Model
- Activities/ Manipulatives
- Cooperative Learning Groups
- Discussion/ Reflection

Behaviorism
- Stimulus
- Modeling
- Neg/Pos Reinforcement
- Constructive Practice

Cognitivism
- Scaffolding
- Algorithmic Writing
- Memorization/
- Association
- Organizational Frameworks

Disposition
- Reflection
- Level of Thinking
- Awareness
- Openness

Structure of Lesson Plan

Body
- Anticipatory Set
- Discussion

Objectives
- Clear Objectives
- Alignment to Standards

Conclusion
- Summary
- Revisit Anticipatory Set

Introduction
- Anticipatory Set
- Discussion

State Standards/ Policies
- TX CCRS
- TEKS

Assessments
- Questioning Techniques
- Independent Work
- HW Questions
- Connections between Lesson Plans

Standards/ Policy
APPENDIX C

INDIVIDUAL LESSON PLAN RUBRIC AND COMPILED LESSON PLAN RUBRIC

The four-section rubric in this section was used to evaluate the MDMP and participants’ lesson plans. There were two versions of the rubric developed: an individual lesson plan rubric and a compiled lesson plan rubric. The individual lesson plan rubric can be used for analyzing one person’s rubric. The compiled lesson plan rubric was designed to display multiple lesson plans results for comparison.
# Part 1: Structure of Lesson Plan

<table>
<thead>
<tr>
<th>Lesson Plan Elements &amp; Descriptions</th>
<th>Overall Grading Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objectives</strong></td>
<td></td>
</tr>
<tr>
<td>Clear Objectives – the objectives are clear to students &amp; other instructors</td>
<td>0 = not observed, 1 = objective(s) stated but not clear, 2 = clear objective(s) stated</td>
</tr>
<tr>
<td>Alignment Standards – the objectives should be aligned to current standards used by school for the appropriate grade level</td>
<td>0 = lesson not aligned to standard(s), 1 = lesson not aligned to correct standard(s), 2 = lesson is aligned to correct standard(s)</td>
</tr>
<tr>
<td><strong>Introduction</strong></td>
<td></td>
</tr>
<tr>
<td>Anticipatory set/Activity – an open-ended question or activity that may not be accessed without the lesson</td>
<td>0 = no introduction, 1 = questions or activities inappropriate, 2 = has anticipatory set or activity that correlates to the lesson</td>
</tr>
<tr>
<td>Discussion – discussion of anticipatory set/activity to assess students current understanding of the content</td>
<td>0 = no introduction, 1 = discussion is not in-depth, looking for surface answers, 2 = has in-depth discussion for anticipatory set/activity</td>
</tr>
<tr>
<td><strong>Body</strong></td>
<td></td>
</tr>
<tr>
<td>Script – the lesson plan reads as a script so anyone can interpret the lesson</td>
<td>0 = no description of the lp, 1 = lp is bulleted, 2 = lp is scripted</td>
</tr>
<tr>
<td>Meet Objectives – the content addressed in the body of the lesson plan aligns to stated objectives</td>
<td>0 = does not meet objective(s), 1 = meets some objective(s), 2 = meets all objective(s)</td>
</tr>
<tr>
<td><strong>Conclusion</strong></td>
<td></td>
</tr>
<tr>
<td>Summary – a wrap-up of the lesson plan; extensions; clarifications of misconceptions; Q &amp; A period</td>
<td>0 = not observed, 1 = observed but no connection to lp, 2 = observed and strong connection to lp</td>
</tr>
<tr>
<td>Revisit Anticipatory Set – connect the lesson to the initial question; answer the initial question</td>
<td>0 = not observed, 1 = observed somewhat, 2 = observed easily</td>
</tr>
<tr>
<td><strong>Assessments</strong></td>
<td></td>
</tr>
<tr>
<td>Questioning Techniques – probing questions that instigate discussion</td>
<td>0 = not observed, 1 = questions do not instigate discussion, 2 = probing questions that instigate discussion</td>
</tr>
<tr>
<td>Independent Work – worksheets, in-class work</td>
<td>0 = not observed, 1 = questions aligned to some objectives, 2 = questions aligned to all objectives</td>
</tr>
<tr>
<td>HW Questions – aligned to objectives</td>
<td>0 = not observed, 1 = questions aligned to some objectives, 2 = questions aligned to all objectives</td>
</tr>
<tr>
<td>Connections b/t lps – this lesson connects to prior and future lessons</td>
<td>0 = not observed, 1 = connections not clear, 2 = clear connections</td>
</tr>
</tbody>
</table>
# Part 1: Structure of Lesson Plan

<table>
<thead>
<tr>
<th>Lesson Plan Elements</th>
<th>Topic</th>
<th>Value</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objectives</strong></td>
<td>Clear Objective(s)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Alignment Standards</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Introduction</strong></td>
<td>Anticipatory set/Activity</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Discussion</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Body</strong></td>
<td>Script</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Meet Objective(s)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Conclusion</strong></td>
<td>Summary</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Revisit Anticipatory set</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Assessments</strong></td>
<td>Questioning Techniques</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Independent Work</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>HW Questions</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Connections b/t lessons</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Name</td>
<td>Objectives</td>
<td>Introduction</td>
<td>Body</td>
</tr>
<tr>
<td>------</td>
<td>------------</td>
<td>--------------</td>
<td>------</td>
</tr>
<tr>
<td></td>
<td>Clear Obj.</td>
<td>Alignment</td>
<td>Anticipatory Set/Activity</td>
</tr>
<tr>
<td></td>
<td>Standards</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comments:
Part 2: Four Characteristics of Teacher Quality

<table>
<thead>
<tr>
<th>Lesson Plan Elements &amp; Descriptions</th>
<th>Pedagogical Knowledge</th>
<th>Overall Grading Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Prior Knowledge</strong> – content aligns to prior classes and grades</td>
<td>0 = not observed, 1 = not connected to all content, 2 = content is connected to prior lp</td>
<td></td>
</tr>
<tr>
<td><strong>Linear/Chronological LP</strong> – information presented in logical/chronological order within the lesson plan</td>
<td>0 = not observed, 1 = not all components in chronological order, 2 = all components in chronological order</td>
<td></td>
</tr>
</tbody>
</table>

**Content Knowledge**

| Specialized Language – terms used correctly, no mnemonics to describe processes without correct interpretation explanation (i.e. ‘Substitution” instead of “plug-in”) | 0 = not observed, 1 = not all terms are correct, 2 = all terms correct | |
| Content Examples – examples appropriately reflect content | 0 = not observed, 1 = appropriate examples mentioned but not provided, 2 = examples appropriate for lesson | |

**Pedagogical-Content Knowledge**

| Level Appropriate Explanations – explanations are at the level of the student, so the students are able to understand | 0 = not observed, 1 = not all explanations at students level, 2 = all explanations at students level | |
| Appropriate use of Representations – demonstrations, examples, and activities appropriate for content | 0 = not observed, 1 = some demos, examples, activities are appropriate, 2 = all demos, examples, activities are appropriate | |
| Symbols with Care – using symbols appropriately, example: when to use X or • for multiplication | 0 = not used, 1 = used inappropriately, 2 = used appropriately | |

**Disposition**

<p>| Reflection – the instructor reflected on the lesson, its content, its appropriateness, constructive criticism | 0 = not observed, 1 = somewhat observed, 2 = instructor reflecting constantly | |
| Level of Thinking – think like the student, understand the appropriateness of the lesson for the student | 0 = not observed, 1 = lesson somewhat at students level of thinking, 2 = lesson at students level of thinking | |
| Awareness – awareness of student capabilities and limitations | 0 = not observed, 1 = lesson may not meet all students’ needs, does not fully have the students in mind when developed, 2 = lesson developed to accommodate students and understand student needs | |
| Openness – do not prefer one thing over another, open to use various teaching methods and tools | 0 = not observed, 1 = use some various teaching techniques, 2 = uses various teaching techniques in the class, open to explain in multiple ways | |</p>
<table>
<thead>
<tr>
<th>Lesson Plan Elements</th>
<th>Topic</th>
<th>Value</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pedagogy Knowledge</strong></td>
<td>Prior knowledge</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Linear/Chronological Lesson</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Content Knowledge</strong></td>
<td>Specialized language</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Content Examples</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Pedagogical-Content Knowledge</strong></td>
<td>Level Appropriate Explanations</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Appropriate Use of Representation</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Symbols w/ care</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Disposition</strong></td>
<td>Reflection</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Level of thinking</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Awareness</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Openness</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Part 2: Four Characteristics of Teacher Quality

<table>
<thead>
<tr>
<th>Name</th>
<th>Pedagogy Knowledge</th>
<th>Content Knowledge</th>
<th>Pedagogical-Content Knowledge</th>
<th>Disposition</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prior knowledge</td>
<td>Linear/Chronologic Language</td>
<td>Specialized Language</td>
<td>Content Examples</td>
<td>Level Appropriate Explanations</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Comments:**

__________________________________________________________________________________________
## Part 3: Learning Theories

<table>
<thead>
<tr>
<th>Lesson Plan Elements &amp; Descriptions</th>
<th>Overall Grading Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Constructivism</strong></td>
<td></td>
</tr>
<tr>
<td>CRA – instruction moves from concrete to representational to abstract</td>
<td>ymen = not observed, 1 = only two steps used, 2 = model used properly</td>
</tr>
<tr>
<td>Activities/Manipulatives – activities and/or manipulatives used in the classroom to teach content</td>
<td>ymen = not observed, 1 = manipulatives used inappropriately, 2 = manipulatives used appropriately</td>
</tr>
<tr>
<td>Cooperative Learning Groups – used a cooperative learning group: jigsaw, think-pair-share, RoundRobin brainstorming, team pair solo, etc.</td>
<td>ymen = not observed, 1 = group work, 2 = cooperative learning groups</td>
</tr>
<tr>
<td>Discussion/Reflection – in-class discussing and reflecting on content</td>
<td>ymen = not observed, 1 = discussion, 2 = discuss and reflect</td>
</tr>
<tr>
<td><strong>Behaviorism</strong></td>
<td></td>
</tr>
<tr>
<td>Stimulus – external influences that effect activities</td>
<td>ymen = not observed, 1 = stimulus that does effect learning, 2 = stimulus that effect learning</td>
</tr>
<tr>
<td>Modeling – model expected outcome</td>
<td>ymen = not observed, 1 = model with error, 2 = model accurately</td>
</tr>
<tr>
<td>Neg/Pos Reinforcement – negative and positive reinforcement to affect student behaviors</td>
<td>ymen = not observed, 1 = reinforcement distributive ineffectively, 2 = reinforcement distributive effectively</td>
</tr>
<tr>
<td>Constructive Practice – worksheets, extra practice problems</td>
<td>ymen = not observed, 1 = practice not related to content, 2 = practice related to content</td>
</tr>
<tr>
<td><strong>Cognitivism</strong></td>
<td></td>
</tr>
<tr>
<td>Scaffolding – the content builds on itself as well as builds on prior knowledge</td>
<td>ymen = no order, 1 = somewhat ordered to build on prior knowledge, 2 = sequenced such that content builds on itself</td>
</tr>
<tr>
<td>Algorithmic Writing – writing algorithms to answer specific questions (step by step process)</td>
<td>ymen = not observed, 1 = simple algorithms, 2 = detailed algorithms</td>
</tr>
<tr>
<td>Memorization/Association – mnemonics, acronyms or a concept associated with a diagram (i.e. associate vertical line test to the definition of a function)</td>
<td>ymen = not observed, 1 = teacher develops memorization task, 2 = student develops memorization task</td>
</tr>
<tr>
<td>Organizational Framework – flow charts, tables, pictorial representations</td>
<td>ymen = not observed, 1 = teacher develops organizational framework, 2 = student develops organizational framework</td>
</tr>
<tr>
<td>Lesson Plan Elements</td>
<td>Topic</td>
</tr>
<tr>
<td>----------------------</td>
<td>----------------------------</td>
</tr>
<tr>
<td><strong>Constructivism</strong></td>
<td>CRA</td>
</tr>
<tr>
<td></td>
<td>Activities/Manipulatives</td>
</tr>
<tr>
<td></td>
<td>Cooperative Learning Groups</td>
</tr>
<tr>
<td></td>
<td>Discussion/Reflection</td>
</tr>
<tr>
<td><strong>Behaviorism</strong></td>
<td>Stimulus</td>
</tr>
<tr>
<td></td>
<td>Modeling</td>
</tr>
<tr>
<td></td>
<td>Neg/Pos Reinforcement</td>
</tr>
<tr>
<td></td>
<td>Constructive Practice</td>
</tr>
<tr>
<td><strong>Cognitivism</strong></td>
<td>Scaffolding</td>
</tr>
<tr>
<td></td>
<td>Algorithmic Writing</td>
</tr>
<tr>
<td></td>
<td>Memorization/Association</td>
</tr>
<tr>
<td></td>
<td>Organizational Framework</td>
</tr>
</tbody>
</table>
### Part 3: Learning Theories

<table>
<thead>
<tr>
<th>Name</th>
<th>Constructivism</th>
<th>Behaviorism</th>
<th>Cognitivism</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CRA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Activities/</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>manipulatives</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Coop. learning</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>groups</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Discussion</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>/Reflection</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Stimulus</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Modeling</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Neg/Pos</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Reinforcement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Constructive</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Practice</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Scaffold-ing</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Alg. Writing</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mem./Associ</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Org. Framework</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Comments:**

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________
### Part 4: Standards/Policies

<table>
<thead>
<tr>
<th>Lesson Plan Elements &amp; Descriptions</th>
<th>Overall Grading Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Content Standards</strong></td>
<td></td>
</tr>
<tr>
<td>Algebra; Numerical; Measurement; Geometry</td>
<td>0 = not observed, 1 = one content covered, 2 = connections between multiple content areas</td>
</tr>
<tr>
<td>Prob/Stats; Functions</td>
<td></td>
</tr>
<tr>
<td><strong>Process Standards/Mathematical Proficiencies</strong></td>
<td></td>
</tr>
<tr>
<td>Problem Solving – the ability to analyze, formulate, and determine a solution to a problem (i.e. strategies used for application problems)</td>
<td></td>
</tr>
<tr>
<td>Modeling – model problems with graphical or pictorial representations as well as through actions</td>
<td></td>
</tr>
<tr>
<td>Reasoning – understanding why; does the solution make sense (logical reasoning)</td>
<td>0 = not observed, 1 = two or less process standard used, 2 = more than two multiple process standards used in instruction</td>
</tr>
<tr>
<td>Connections – connections of mathematics to real world, within the math content, and across disciplines</td>
<td></td>
</tr>
<tr>
<td>Communication – discussion between peers and between peers and instructors as well as the use of mathematical terminology</td>
<td></td>
</tr>
<tr>
<td>Representations – using multiple representations to represent one concept</td>
<td></td>
</tr>
<tr>
<td>Conceptual Understanding – understanding the whole and not just pieces of the whole</td>
<td></td>
</tr>
<tr>
<td><strong>Pedagogical Standards</strong></td>
<td></td>
</tr>
<tr>
<td>Connections Math and Real World – connections to mathematics and the real-world; application problems, real-world activities</td>
<td>0 = not observed, 1 = somewhat of a connection, 2 = a solid connection between mathematics and real-world problems</td>
</tr>
<tr>
<td>Technology – use of technology as a learning tool</td>
<td>0 = not observed, 1 = used in the classroom but not for learning, 2 = observed in the classroom to promote learning</td>
</tr>
<tr>
<td>Manipulatives – use of manipulatives as a learning tool</td>
<td>0 = not observed, 1 = used in the classroom but not for learning, 2 = observed in the classroom to promote learning</td>
</tr>
<tr>
<td>Group Learning – discussion within and outside of class for learning</td>
<td>0 = not observed, 1 = used in the classroom but not for learning, 2 = observed in the classroom to promote learning</td>
</tr>
</tbody>
</table>
## Part 4: Standards/Policy

<table>
<thead>
<tr>
<th>Lesson Plan Elements</th>
<th>Topic</th>
<th>Value</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Content Standards</strong></td>
<td>Algebraic</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Numeric</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Measurement</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Geometry</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Probability/Statistics</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Functions</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Process Standards/Mathematical Proficiencies</strong></td>
<td>Problem Solving</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Modeling</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Reasoning</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Connections</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Communications</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Representations</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Conceptual Understanding</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Pedagogical Standards</strong></td>
<td>Conn. Math &amp; Real World</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tech</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Manipulatives</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Group Learning</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Name</td>
<td>Content Standards</td>
<td>Process Standards/ Mathematical Proficiencies</td>
<td>Pedagogical Standards</td>
</tr>
<tr>
<td>------</td>
<td>-------------------</td>
<td>-----------------------------------------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comments:
APPENDIX D

ITEM DISTRIBUTION OF THE MODIFIED MATHEMATICAL KNOWLEDGE FOR TEACHING TEST

<table>
<thead>
<tr>
<th>Mathematical Domain</th>
<th>Tasks of Teaching</th>
<th>Common Content Knowledge (CCK)</th>
<th>Specialized Content Knowledge (SCK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical</td>
<td></td>
<td>2005 A 9</td>
<td>2005 A 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PV 2006 2</td>
<td>2005 B 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2005 A 6 (a-c)</td>
<td>2005 B 6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2005 A 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Rational No. 2008 A 15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No. of Items: 5</td>
<td>No. of Items: 5</td>
</tr>
<tr>
<td>Expressions</td>
<td></td>
<td>2007 A 3 (a-d)</td>
<td>2005 22 (a-d)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2005 B 15 (a-d)</td>
<td>2005 26 (a-d)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No. of Items: 8</td>
<td>No. of Items: 8</td>
</tr>
<tr>
<td>Equations</td>
<td></td>
<td>2007 A 27</td>
<td>2005 A 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2007 B 30 (a-f)</td>
<td>2007 A 6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2007 A 35</td>
<td>2007 B 11</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2007 B 28</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2007 A 31 (a-c)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No. of Items: 7</td>
<td>No. of Items: 7</td>
</tr>
<tr>
<td>Functions</td>
<td></td>
<td>2007 B 35</td>
<td>2007 B 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2005 B 13</td>
<td>2007 B 9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2005 B 21 (a-e)</td>
<td>2007 A 15</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2005 A 16 (a-d)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No. of Items: 7</td>
<td>No. of Items: 7</td>
</tr>
</tbody>
</table>
## APPENDIX E

QUESTIONS CHOSEN FOR MODIFIED MATHEMATICAL KNOWLEDGE FOR TEACHING TEST

<table>
<thead>
<tr>
<th>Problem Number</th>
<th>Problem Originate</th>
<th>Slope</th>
<th>Difficulty</th>
<th>CCK/SCK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2005 A 8</td>
<td>0.538</td>
<td>0.531</td>
<td>SCK</td>
</tr>
<tr>
<td>2</td>
<td>2005 A 9</td>
<td>0.660</td>
<td>–0.377</td>
<td>CCK</td>
</tr>
<tr>
<td>3</td>
<td>2005 B 3</td>
<td>0.692</td>
<td>–0.311</td>
<td>SCK</td>
</tr>
<tr>
<td>4</td>
<td>P.V. 2006 2</td>
<td>0.775</td>
<td>–0.840</td>
<td>CCK</td>
</tr>
<tr>
<td>5</td>
<td>2005 B 6</td>
<td>0.564</td>
<td>1.309</td>
<td>SCK</td>
</tr>
<tr>
<td>6</td>
<td>2005 A 4</td>
<td>0.437</td>
<td>–0.957</td>
<td>SCK</td>
</tr>
<tr>
<td>7</td>
<td>2005 A 6a</td>
<td>0.555</td>
<td>0.799</td>
<td>CCK</td>
</tr>
<tr>
<td>8</td>
<td>2005 A 6b</td>
<td>0.548</td>
<td>–1.683</td>
<td>CCK</td>
</tr>
<tr>
<td>9</td>
<td>2005 A 6c</td>
<td>0.532</td>
<td>–2.386</td>
<td>CCK</td>
</tr>
<tr>
<td>10</td>
<td>Rat. No. 2008 A 15</td>
<td>0.762</td>
<td>–2.170</td>
<td>SCK</td>
</tr>
</tbody>
</table>

**Numerical Reasoning**

<table>
<thead>
<tr>
<th>Problem Number</th>
<th>Problem Originate</th>
<th>Slope</th>
<th>Difficulty</th>
<th>CCK/SCK</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>2007A Info 3a</td>
<td>0.347</td>
<td>1.864</td>
<td>CCK</td>
</tr>
<tr>
<td>12</td>
<td>2007A Info 3b</td>
<td>0.717</td>
<td>–0.678</td>
<td>CCK</td>
</tr>
<tr>
<td>13</td>
<td>2007A Info 3c</td>
<td>0.875</td>
<td>–1.156</td>
<td>CCK</td>
</tr>
<tr>
<td>14</td>
<td>2007A Info 3d</td>
<td>0.485</td>
<td>–0.740</td>
<td>CCK</td>
</tr>
<tr>
<td>15</td>
<td>2005 Info 22a</td>
<td>0.673</td>
<td>–2.26</td>
<td>SCK</td>
</tr>
<tr>
<td>16</td>
<td>2005 Info 22b</td>
<td>0.92</td>
<td>–1.8</td>
<td>SCK</td>
</tr>
<tr>
<td>17</td>
<td>2005 Info 22c</td>
<td>0.446</td>
<td>–0.495</td>
<td>SCK</td>
</tr>
<tr>
<td>18</td>
<td>2005 Info 22d</td>
<td>0.782</td>
<td>–1.9</td>
<td>SCK</td>
</tr>
<tr>
<td>19</td>
<td>2005 Info 26a</td>
<td>1.4</td>
<td>–1.19</td>
<td>CCK</td>
</tr>
<tr>
<td>20</td>
<td>2005 Info 26b</td>
<td>0.88</td>
<td>–0.62</td>
<td>CCK</td>
</tr>
<tr>
<td>21</td>
<td>2005 Info 26c</td>
<td>1.44</td>
<td>–1.46</td>
<td>CCK</td>
</tr>
<tr>
<td>22</td>
<td>2005 Info 26d</td>
<td>1.078</td>
<td>–1.17</td>
<td>CCK</td>
</tr>
<tr>
<td>23</td>
<td>2005 B 15a</td>
<td>1.163</td>
<td>–0.807</td>
<td>SCK</td>
</tr>
<tr>
<td>24</td>
<td>2005 B 15b</td>
<td>0.877</td>
<td>–0.854</td>
<td>SCK</td>
</tr>
<tr>
<td>25</td>
<td>2005 B 15c</td>
<td>1.20</td>
<td>–0.578</td>
<td>SCK</td>
</tr>
<tr>
<td>26</td>
<td>2005 B 15d</td>
<td>1.130</td>
<td>–1.060</td>
<td>SCK</td>
</tr>
</tbody>
</table>

**Expressions**
Table Continued

<table>
<thead>
<tr>
<th>Equations</th>
<th>27</th>
<th>27</th>
<th>27</th>
<th>27</th>
<th>27</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>2007 A 6</td>
<td>0.715</td>
<td>–1.406</td>
<td>SCK</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>2007 B 11</td>
<td>0.949</td>
<td>–0.573</td>
<td>SCK</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>2007 B 28</td>
<td>0.955</td>
<td>–1.153</td>
<td>SCK</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>2007 A 27</td>
<td>1.082</td>
<td>–0.812</td>
<td>CCK</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>2007B Info 30a</td>
<td>0.751</td>
<td>0.507</td>
<td>CCK</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>2007B Info 30b</td>
<td>0.811</td>
<td>–1.010</td>
<td>CCK</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>2007B Info 30c</td>
<td>0.872</td>
<td>–0.222</td>
<td>CCK</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>2007B Info 30d</td>
<td>1.072</td>
<td>–1.121</td>
<td>CCK</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>2007B Info 30e</td>
<td>0.943</td>
<td>–0.638</td>
<td>CCK</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>2007B Info 30f</td>
<td>1.173</td>
<td>–0.697</td>
<td>CCK</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>2007A 35</td>
<td>0.946</td>
<td>0.467</td>
<td>CCK</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>2007 A 31a</td>
<td>0.7</td>
<td>–1.053</td>
<td>SCK</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>2007 A 31b</td>
<td>0.744</td>
<td>–0.854</td>
<td>SCK</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>2007 A 31c</td>
<td>0.908</td>
<td>–1.121</td>
<td>SCK</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>2007 B 1</td>
<td>1.03</td>
<td>–0.97</td>
<td>SCK</td>
<td></td>
</tr>
</tbody>
</table>

| Functions |
|-----------|----|----|----|----|
| 43 | 2007 B 4 | 0.331 | 2.039 | SCK |
| 44 | 2007 B 9 | 0.63 | –0.35 | SCK |
| 45 | 2007 B 35 | 0.546 | 0.391 | CCK |
| 46 | 2005 B 13 | 0.45 | –2.153 | CCK |
| 47 | 2007 A 15 | 0.598 | 0.114 | SCK |
| 48 | 2005 B 21a | 1.395 | –1.772 | CCK |
| 49 | 2005 B 21b | 1.532 | –1.66 | CCK |
| 50 | 2005 B 21c | 1.073 | –1.48 | CCK |
| 51 | 2005 B 21d | 0.878 | –0.67 | CCK |
| 52 | 2005 B 21e | 0.963 | –1.94 | CCK |
| 53 | 2005 A 16a | 0.777 | 0.409 | SCK |
| 54 | 2005 A 16b | 1.102 | –0.678 | SCK |
| 55 | 2005 A 16c | 0.652 | –0.293 | SCK |
| 56 | 2005 A 16d | 0.598 | 0.974 | SCK |
APPENDIX F

INTERVIEW PROTOCOL

Interview Questions

1. Why do you want to be a teacher?

2. Do you have any experience with any aspect of teaching? (Extra-curricular: tutoring, Ready-set-teach, etc.)

3. Why did you choose ____ grade?

4. What inspired this lesson plan about functions? (Experience, etc.)

5. Where did you gain the knowledge of functions?

6. Outline of lesson plan is:
   a. Why did you structure your lesson plan in this manner?

7. What is the students’ knowledge before learning this lesson plan on functions?

8. Read questions from lesson plan top to bottom.

9. Having this experience of writing a lesson plan, did you reflect on the lesson plan? What would you change about the lesson plan?
APPENDIX G

DEMOGRAPHIC SURVEYS

G.1 Demographic Survey with Quiz

G.2 Demographic Survey without Quiz
Dear Participant,

My name is Lindsey Gerber and I am a doctoral student at Texas State University-San Marcos working on my dissertation, “Effects of a Model Developmental Mathematics Program on Elementary and Middle School Preservice Teachers.” The purpose for this survey is to find students meeting the criteria to participate in my study. Students meeting the criteria will take two different assessments and participate in an interview.

The first instrument is a 56-item multiple-choice assessment that includes mathematical questions for elementary and middle school teachers. The assessment will not take more than one hour. Next, participants will be asked to submit a writing sample of a lesson plan.

Door prizes will be given after assessments have been completed, including classroom games, mathematical manipulatives and more. After all activities have been completed an interview appointment must be made. Please allow approximately 15 to 30 minutes for the interview. When participants complete all required parts of the instrument they will receive a free quiz grade in their current mathematics course.

This study has received Institutional Review Board (IRB) exemption from the Office of Research Compliance at Texas State University-San Marcos. Federal regulations describe six categories of research that may qualify for exempt status. This study is categorized as

(1) Research conducted in established or commonly accepted educational settings, involving normal educational practices such as (ii) research on effectiveness of or the comparison among instructional techniques curricula, or classroom management methods.

For more information about exemption contact the Office of Research Compliance at 512-245-2314 and use the exemption approval number EXP2011M9626.

If you meet the requirements and would like to participate in the study, please provide your contact information. I appreciate your consideration on being a part of my research study.

_________________________________________________________

e-mail

_________________________________________________________

phone number

Sincerely,

Lindsey Gerber
Texas State University-San Marcos
Doctoral Teaching Assistant
Last Name ____________ First Name ____________ ID# ____________

1. Gender (Circle ONE of the following): Male or Female

2. Age: ________________

3. Classification (Circle ONE of the following):
   - Freshman
   - Sophomore
   - Junior
   - Senior
   - Other: __________

4. Ethnicity (Circle ONE of the following):
   - White, non-Hispanic
   - African-American
   - Hispanic
   - Asian-Pacific Islander
   - American Indian/Alaskan
   - International
   - Other: ________________

5. Did you attend a community college before coming to Texas State University-San Marcos? (Circle ONE of the following.) Yes or No

   If yes, what mathematics course(s) were taken at the community college? When? What grade was received?
   - Mathematics course: ________________, when: ________________, grade __________
   - Mathematics course: ________________, when: ________________, grade __________
   - Mathematics course: ________________, when: ________________, grade __________
6. Participant’s **first** mathematics course at **Texas State University – San Marcos**

(Circle **ONE** of the following):

- MATH 1300
- MATH 1311
- MATH 1315

7. Are you currently interested in pursuing teacher certification? (Circle) **Yes** or **No**

a. If it is one of the options listed below, circle the appropriate certification:

- BS, major in Interdisciplinary Studies (**Early Childhood through Grade 6**)
  - **ESL Generalist**
  - **Bilingual Generalist**

- BS, major in Interdisciplinary Studies (**Grades 4 – 8**)
  - **Generalist**
  - **Mathematics**
  - **Mathematics/Science Composite**

  **Other:** ______________________

  Certification not listed, place here: ________________________________

  List any minors (if possible): ________________________________

b. If **not** seeking teacher certification, list major: _________________

  minor (if applicable): _________________________________
G.2 Demographic Survey without Quiz

[Date]

Dear Participant,

My name is Lindsey Gerber and I am a doctoral student at Texas State University-San Marcos working on my dissertation, “Effects of a Model Developmental Mathematics Program on Elementary and Middle School Preservice Teachers.” The purpose for this survey is to find students meeting the criteria to participate in my study. Students meeting the criteria will take two different assessments and participate in an interview.

The first instrument is a 56-item multiple-choice assessment that includes mathematical questions for elementary and middle school teachers. The assessment will not take more than one hour. Next, participants will be asked to submit a writing sample of a lesson plan.

Door prizes will be given after assessments have been completed, including classroom games, mathematical manipulatives and more. After all activities have been completed an interview appointment must be made. Please allow approximately 15 to 30 minutes for the interview. When participants complete all required parts of the instrument they will receive a $20.00 gift card to HEB, Walmart, Target, or Starplex.

This study has received Institutional Review Board (IRB) exemption from the Office of Research Compliance at Texas State University-San Marcos. Federal regulations describe six categories of research that may qualify for exempt status. This study is categorized as

(1) Research conducted in established or commonly accepted educational settings, involving normal educational practices such as (ii) research on effectiveness of or the comparison among instructional techniques curricula, or classroom management methods.

For more information about exemption contact the Office of Research Compliance at 512-245-2314 and use the exemption approval number EXP2011M9626.

If you meet the requirements and would like to participate in the study, please provide your contact information. I appreciate your consideration on being a part of my research study.

____________________________________
e-mail

____________________________________
phone number

Sincerely,
Lindsey Gerber
Texas State University-San Marcos
Doctoral Teaching Assistant
Last Name ____________ First Name ____________  ID# ____________

1. Gender (Circle **ONE** of the following):  **Male**  or  **Female**

2. Age: ____________

3. Classification (Circle **ONE** of the following):
   - Freshman
   - Sophomore
   - Junior
   - Senior
   - Other: ____________

4. Ethnicity (Circle **ONE** of the following):
   - White, non-Hispanic
   - African-American
   - Hispanic
   - Asian-Pacific Islander
   - American Indian/Alaskan
   - International
   - Other: ____________

5. Did you attend a community college **before** coming to Texas State University-San Marcos? (Circle **ONE** of the following.)  **Yes**  or  **No**
   
   If yes, what mathematics course(s) were taken at the community college? When? What grade was received?
   
   Mathematics course: ____________, when: ____________, grade __________
   Mathematics course: ____________, when: ____________, grade __________
   Mathematics course: ____________, when: ____________, grade __________

   Turn Over →
6. Participant’s **first** mathematics course at Texas State University – San Marcos  
   (Circle **ONE** of the following):  
   - MATH 1300  
   - MATH 1311  
   - MATH 1315  

7. Are you currently interested in pursuing teacher certification? (Circle) **Yes** or **No**  
   a. If it is one of the options listed below, circle the appropriate certification:  
      - BS, major in Interdisciplinary Studies (Early Childhood through Grade 6)  
      - ESL Generalist  
      - Bilingual Generalist  
      - BS, major in Interdisciplinary Studies (Grades 4–8)  
      - Generalist  
      - Mathematics  
      - Mathematics/Science Composite  
      - Other: ______________________  
      - Certification not listed, place here: _________________________________  
      - List any minors (if possible):______________________________________  

   b. If **not** seeking teacher certification, list major: ________________  
      minor (if applicable): ________________
APPENDIX H

FIVE-QUESTION SURVEY

H.1 1300 Five-Question Survey

H.2 1311 Five-Question Survey
Dear Participants,

This is a short survey to identify the developmental mathematics classroom environment and types of instruction. The results from the study will remain absolutely confidential. It has received permission from the Internal Review Board (IRB) approval. The number is EXP2011M9626. In completing this survey you give your consent to participate in this study.

Thank you,
Lindsey Gerber
Answer the following multiple choice questions by circling your response. If more than one applies, circle multiple responses.

1. What aspect of functions has been taught to you in the developmental mathematics course?
   a. The idea of input and output.
   b. Vertical Line test.
   c. Using graphing calculators to represent equations.
   d. Transformation on the coordinate plane.
   e. No aspect of functions was foreshadowed.

2. When learning about polynomials, first you constructed operations with algebra tiles, next you drew a picture that represented the algebra tiles, and last you worked algebraic problems. How do you feel about this form of teaching?
   a. Algebra tiles were too childish and I didn't see the purpose.
   b. Algebra tiles were helpful to some but not for me.
   c. Algebra tiles were helpful and more lessons should have hands-on activities.
   d. Lessons should be taught with lecture only.
   e. The instructor did not teach using this method.

3. When solving and simplifying mathematical problems multiple steps are taken, what are strategies you learned from the developmental mathematics courses that can help you with multi-step problems?
   a. Writing down step-by-step procedures as a class.
   b. Developing pictorial diagrams.
   c. Taking notes.
   d. No strategies were taught.

4. How is the curriculum built?
   a. Blending geometry, measurement, probability, statistics, and algebra.
   b. Concentrating on equations only.
   c. The lessons are developed to start with expressions to equations to functions.
   d. I cannot see any connections.

5. When learning different transformations you were divided into small groups and were given one form: reflection, rotation, or translation. After learning the given transformation, you were moved to a new group and everyone shared their transformation. How did you feel about this type of instruction?
   a. It’s inappropriate, teachers should teach the curriculum.
   b. I didn’t learn the content.
   c. Discussing content with other students made me realize that I could teach others math.
   d. I liked this method of teaching and more topics should be taught this way.
   e. The instructor did not teach using this method.
Dear Participants,

This is a short survey to identify the developmental mathematics classroom environment and types of instruction. The results from the study will remain absolutely confidential. It has received permission from the Internal Review Board (IRB) approval. The number is EXP2011M9626. In completing this survey you give your consent to participate in this study.

Thank you,
Lindsey Gerber
Answer the following multiple choice questions by circling your response. If more than one applies, circle multiple responses.

1. How was linear functions taught to you in the developmental mathematics course?
   a. With different representations—tables, graphs, and algebraic symbols.
   b. Discussed in groups.
   c. The instructor just lectured and you wrote down notes.
   d. Graphing calculators were used to represent functions.

2. When learning about polynomials, first you constructed operations with algebra tiles, next you drew a picture that represented the algebra tiles, and last you worked algebraic problems. How do you feel about this form of teaching?
   a. Algebra tiles were too childish and I didn’t see the purpose.
   b. Algebra tiles were helpful to some but not for me.
   c. Algebra tiles were helpful and more lessons should have hands-on activities.
   d. Lessons should be taught with lecture only.
   e. The instructor did not teach using this method.

3. When solving and simplifying mathematical problems multiple steps are taken, what are strategies you learned from the developmental mathematics courses that can help you with multi-step problems?
   a. Writing down step-by-step procedures as a class.
   b. Developing pictorial diagrams.
   c. Taking notes.
   d. No strategies were taught.

4. How is the curriculum built?
   a. Blending geometry, measurement, probability, statistics, and algebra.
   b. Concentrating on equations only.
   c. The lessons are developed to start with expressions to equations to functions.
   d. I cannot see any connections.

5. When learning how to solve rational equations you were divided into small groups and were given one method of solving to learn. After learning the given method, you were moved to a new group and everyone shared their method of solving. How did you feel about this type of instruction?
   a. It’s inappropriate, teachers should teach the curriculum.
   b. I didn’t learn the content.
   c. Discussing content with other students made me realize that I could teach others math.
   d. I liked this method of teaching and more topics should be taught this way.
   e. The instructor did not teach using this method.
## APPENDIX I

### ALIGNMENT CHART

<table>
<thead>
<tr>
<th>Topics</th>
<th>Objectives</th>
<th>Gen Cat</th>
<th>THECB</th>
<th>CCRS</th>
<th>THEA</th>
<th>MKT</th>
<th>1315</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting</td>
<td>Students will be able to:</td>
<td>Basic</td>
<td>DM</td>
<td>V.A.1, VIII.A.1 -5</td>
<td>1a, 10a</td>
<td>N A</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>1. draw a tree diagram and determine the number of outcomes.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. determine when to use the Fundamental Counting Principle and use it accurately.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. determine when to use the addition rule and use it accurately.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4. apply counting techniques to real world problems.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Permutations and Combinations</td>
<td>Students will be able to:</td>
<td>Basic</td>
<td>DM</td>
<td>1a, 10b</td>
<td>N A</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>1. define a permutation.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. evaluate factorials.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. define a combination.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4. differentiate between a permutation and a combination.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-Variable Linear Expression</td>
<td>Students will be able to:</td>
<td>Beginning/Intermediate</td>
<td>DM</td>
<td>II.A.1, II.B.1</td>
<td>10b</td>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>1. simplifying Expressions by combining like terms.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. evaluating expressions by substitution.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-Variable Linear Equation</td>
<td>Students will be able to:</td>
<td>Beginning/Intermediate</td>
<td>DM</td>
<td>II.C.1</td>
<td>4a, 10b</td>
<td>31</td>
<td>34</td>
</tr>
<tr>
<td>Statistics: Types of Data, and Data Collection</td>
<td>Students will be able to:</td>
<td>Basic</td>
<td>DM</td>
<td>VI.B.1, VI.C.4</td>
<td>1a, 10b</td>
<td>N</td>
<td>NA</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1. differentiate between categorical versus quantitative data.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. distinguish among different types of data collection.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. distinguish between different types of bias in data collection.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Five Number Summary</td>
<td>Students will be able to:</td>
<td>Basic</td>
<td>DM</td>
<td>VI.B.3, VI.C.3</td>
<td>1a,b,c, 2a,d, 10b</td>
<td>N</td>
<td>NA</td>
</tr>
<tr>
<td>1. find the five number summary from a set of raw data.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. construct a box and whiskers plot using the five number summary.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. find the measures of central tendency from raw data.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear Inequalities in 1-Variable</td>
<td>Students will be able to:</td>
<td>Beginning/Intermediate</td>
<td>IA</td>
<td>II.C.1</td>
<td>3b, 10a</td>
<td>6, 32, 33</td>
<td>D</td>
</tr>
<tr>
<td>1. graph linear inequalities in 1 variable.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. solve linear inequalities in 1 variable.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rectangular Coordinate Plane</td>
<td>Students will be able to:</td>
<td>Beginning</td>
<td>DM</td>
<td>II.D.2, VI.C.2, IX.A.1, IX.B.1, IX.C.2</td>
<td>10b</td>
<td>46</td>
<td>P</td>
</tr>
<tr>
<td>1. determine components of a rectangular coordinate plane.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. plot points on a coordinate plane.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. determine coordinates on a Cartesian Plane.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. graph a scatter plot.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intro to Linear Equations in 2- Variable</td>
<td>Beginning/ Intermediate</td>
<td>IA</td>
<td>II.B.1, II.C.1, II.D.1-2</td>
<td>3a, 4b, 10b</td>
<td>11 - 14, 39 - 41</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>----------------------------------------</td>
<td>-------------------------</td>
<td>----</td>
<td>--------------------------</td>
<td>-------------</td>
<td>------------------</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>Students will be able to:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. determine if a point is a solution to the linear equation in two variable.</td>
<td>Beginning/ Intermediate</td>
<td>IA</td>
<td>II.B.1, II.C.1, II.D.1-2</td>
<td>3a, 4b, 10b</td>
<td>11 - 14, 39 - 41</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>2. find missing coordinate of an ordered pair solution given one coordinate.</td>
<td>Beginning/ Intermediate</td>
<td>IA</td>
<td>II.B.1, II.C.1, II.D.1-2</td>
<td>3a, 4b, 10b</td>
<td>11 - 14, 39 - 41</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>3. use graphical representation to determine the definition of a line.</td>
<td>Beginning/ Intermediate</td>
<td>IA</td>
<td>II.B.1, II.C.1, II.D.1-2</td>
<td>3a, 4b, 10b</td>
<td>11 - 14, 39 - 41</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>4. graph linear equation using tabular method.</td>
<td>Beginning/ Intermediate</td>
<td>IA</td>
<td>II.B.1, II.C.1, II.D.1-2</td>
<td>3a, 4b, 10b</td>
<td>11 - 14, 39 - 41</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>Intercepts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Students will be able to:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. find intercepts of a line from examining a graph.</td>
<td>Beginning/ Intermediate</td>
<td>IA</td>
<td>II.B.1, II.C.1, II.D.1-2</td>
<td>3a, 4b, 10b</td>
<td>11 - 14, 39 - 41</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>2. find intercepts of a linear equation in two variables algebraically using substitution.</td>
<td>Beginning/ Intermediate</td>
<td>IA</td>
<td>II.B.1, II.C.1, II.D.1-2</td>
<td>3a, 4b, 10b</td>
<td>11 - 14, 39 - 41</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>Graphing and Slope</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Students will be able to:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. tell the difference between a positive and a negative slope.</td>
<td>Beginning/ Intermediate</td>
<td>IA</td>
<td>II.B.1, II.C.1, II.D.1-2</td>
<td>3a, 4b, 10b</td>
<td>11 - 14, 39 - 41</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>2. define slope and determine the slope given two points.</td>
<td>Beginning/ Intermediate</td>
<td>IA</td>
<td>II.B.1, II.C.1, II.D.1-2</td>
<td>3a, 4b, 10b</td>
<td>11 - 14, 39 - 41</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>3. graph a linear equation given in any form; y-intercept, standard, etc.</td>
<td>Beginning/ Intermediate</td>
<td>IA</td>
<td>II.B.1, II.C.1, II.D.1-2</td>
<td>3a, 4b, 10b</td>
<td>11 - 14, 39 - 41</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>4. solve a real life application using the slope formula.</td>
<td>Beginning/ Intermediate</td>
<td>IA</td>
<td>II.B.1, II.C.1, II.D.1-2</td>
<td>3a, 4b, 10b</td>
<td>11 - 14, 39 - 41</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>Table Continued</td>
<td>Students will be able to:</td>
<td>Beginning/Intermediate</td>
<td>IA</td>
<td>II.D.1</td>
<td>3a,c, 4b, 10a</td>
<td>28</td>
<td>D</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------------------------</td>
<td>------------------------</td>
<td>----</td>
<td>--------</td>
<td>----------------</td>
<td>----</td>
<td>----</td>
</tr>
</tbody>
</table>
| Horizontal and Vertical Line | 1. graph a vertical line and know its equation  
2. graph a horizontal line and know its equation  
3. determine the difference between the equations of the vertical and horizontal lines. | | | | | | |
| Graphing and Finding Equations | Students will be able to: | | | | | | |
| | 1. distinguish the different traits of parallel and perpendicular lines.  
2. find the equation of a line when: the slope and the y-intercept is given, when the slope and a point is given, and when two points on the line are given.  
3. understand the introduction of graphs of functions | | | | | | |
| Correlation | Students will be able to: | | | | | | |
| | 1. determine whether data is positively or negatively correlated.  
2. distinguish between strong versus weak correlation of the data.  
3. find the line of best fit of a given set of data points.  
4. predict new values based on the line of best fit. | | | | | | |
| Intro to Systems of Equations | Students will be able to: | | | | | | |
| | 1. define systems of equations.  
2. solve a system of equations either graphically or by the substitution method. | | | | | | |
<p>| Table Continued |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| <strong>Solving Systems (Addition Method)</strong> | <strong>Students will be able to:</strong> | <strong>Beginning/Intermediate</strong> | <strong>IA</strong> | <strong>II.B.1, II.C.1, II.D.2, VI.B.2</strong> | <strong>1a,b,c, 4e, 5b, 10b</strong> | <strong>29, 47, D</strong> |
| <strong>Intro to Linear Functions</strong> | <strong>Students will be able to:</strong> | <strong>Intermediate</strong> | <strong>IA</strong> | <strong>VII.A.2, VII.B.2</strong> | <strong>3e, 6a, 10b</strong> | <strong>43, D</strong> |
| <strong>Exponent Rules</strong> | <strong>Students will be able to:</strong> | <strong>Beginning</strong> | <strong>IA</strong> | <strong>II.D.1</strong> | <strong>10a</strong> | <strong>23, P</strong> |
| <strong>Intro to Polynomials</strong> | <strong>Students will be able to:</strong> | <strong>Beginning</strong> | <strong>IA</strong> | <strong>II.B.1</strong> | <strong>6b, 8a, 10b</strong> | <strong>15, 18, P</strong> |
| Table Continued |
|------------------|------------------|------------------|------------------|
| Intro to factoring | Students will be able to: 1. define factoring through factoring monomials by using factor trees. 2. identify and factor polynomials with a greatest common factor. 3. factor by the grouping technique. 4. organize their thoughts in a factor flow chart. | Beginning/Intermediate | IA | II.B.1 | 6a, 10b | 23 | 26 | 38 |
| ac-Methods | Students will be able to: 1. factor a quadratic expression, if it is factorable. 2. organize their thoughts in a factor flow chart. | Beginning/Intermediate | IA | II.B.1, IX.B.1, IX.C.2 | 6a, 10b | 15 | 18 | 23 | 26 | 38 |
| More Quadratic Factoring | Students will be able to: 1. factor quadratics using “Difference of Squares” formula. 2. factor using “Sums of Cubes” and “Difference of Cubes” formulas. | Beginning/Intermediate | IA | II.B.1 | 6a, 10b | N | A |
| Solving Quadratic Equations using Factoring | Students will be able to: 1. determine the definition of a quadratic equation. 2. use the zero product principle and factoring in solving equations. 3. develop an algorithm for solving quadratic equations by factoring. 4. solve quadratics using factoring. | Beginning/Intermediate | IA | II.C.1 | 7c, 10a | 37 | 38 |</p>
<table>
<thead>
<tr>
<th>Table Continued</th>
<th>Students will be able to:</th>
<th>Intermediate</th>
<th>IA</th>
<th>II.C.2</th>
<th>7c, 10b</th>
<th>37</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve by completing the square</td>
<td>1. use a graphical representation to complete the square.</td>
<td>Intermediate</td>
<td>IA</td>
<td>II.C.2</td>
<td>7c, 10b</td>
<td>37</td>
<td>P</td>
</tr>
<tr>
<td></td>
<td>2. review how to solve a quadratic equation by using the Completing the Square technique.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simplifying radicals and Complex numbers</td>
<td>Students will be able to:</td>
<td>Beginning/</td>
<td>DM/IA</td>
<td>I.C.1</td>
<td>6d, 10b</td>
<td>5</td>
<td>P</td>
</tr>
<tr>
<td></td>
<td>1. simplify basic radical expressions.</td>
<td>Intermediate</td>
<td>IA</td>
<td>II.C.2</td>
<td>7c, 10b</td>
<td>10</td>
<td>P</td>
</tr>
<tr>
<td></td>
<td>2. find basic Complex numbers.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Derive the quadratic formula</td>
<td>Students will be able to:</td>
<td>Intermediate</td>
<td>IA</td>
<td>II.C.2</td>
<td>7c, 10b</td>
<td>10</td>
<td>P</td>
</tr>
<tr>
<td></td>
<td>1. understand that the Quadratic formula was derived by utilizing the completing the</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>square methodology for solving quadratic equations.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. solve quadratic equations by using the quadratic formula.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Graph Quadratic Function</td>
<td>Students will be able to:</td>
<td>Intermediate</td>
<td>IA</td>
<td>II.D.1−2</td>
<td>3e, 6e, 7a, 10b</td>
<td>53−56</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td>1. find a parabola by using the definition of a conic section.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. find the vertex and x- and y- intercepts of the graph of a quadratic function.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. graph quadratic functions based on the vertex and intercepts.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4. determine if the graph represents a function.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5. determine if the graph is a one-to-one function and investigate why it is one-to-one.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table Continued

<table>
<thead>
<tr>
<th>Polynomial Functions</th>
<th>Students will be able to:</th>
<th>Intermediate</th>
<th>IA</th>
<th>III.B.1</th>
<th>3e, 6b, 7a,d, 10a</th>
<th>N</th>
<th>A</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1. describe characteristics of short run behavior of a polynomial function.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. describe characteristics of long run behavior of a polynomial function.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Empirical and Theoretical Probability</th>
<th>Students will be able to:</th>
<th>Basic</th>
<th>DM</th>
<th>V.B.1</th>
<th>1a,b,c, 2b, 10a</th>
<th>4</th>
<th>NA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1. conduct an experiment to understand the definition of empirical probability</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. understand what theoretical probability is and build the appropriate probabilities for the sample space</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent and Independent Probability</th>
<th>Students will be able to:</th>
<th>Basic</th>
<th>DM</th>
<th>V.A.1, VIII.A.1 -5</th>
<th>1a,b,c,2b, 10a</th>
<th>4</th>
<th>NA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1. find the probability of an independent event</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. find the probability of a dependent event</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. distinguish the difference between dependent and independent events</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Multiply And Divide Rational Expressions</th>
<th>Students will be able to:</th>
<th>Beginning/ Intermediate</th>
<th>IA</th>
<th>II.A.1, II.B.1</th>
<th>6c, 10b</th>
<th>1, 2, 3, 10</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1. define ‘rational expression’</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. reduce/simplify rational expressions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. multiply rational expressions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4. divide rational expressions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Table Continued</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Add and Subtract Rational Expressions</strong></td>
<td>Students will be able to:</td>
<td>Beginning/Intermediate</td>
<td>IA</td>
<td>II.A.1, II.B.1</td>
<td>6c, 10b</td>
<td>7, 8, 9</td>
<td>P</td>
</tr>
<tr>
<td>1. find the least common multiple of algebraic expressions in groups.</td>
<td>2. find the least common denominator of rational expressions in groups.</td>
<td>3. add rational expressions in groups.</td>
<td>4. subtract rational expressions in groups.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Solving Rational Equations &amp; Rational Functions</strong></td>
<td>Students will be able to:</td>
<td>Beginning/Intermediate</td>
<td>IA</td>
<td>II.C.1,II.D.1,II.D.2</td>
<td>3e, 6c, 10b</td>
<td>27, 30, 35, 42</td>
<td>P/D</td>
</tr>
<tr>
<td>1. compare different methods for solving rational equations in groups.</td>
<td>2. solve rational equations individually.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Normal Distribution</strong></td>
<td>Students will be able to:</td>
<td>Basic</td>
<td>DM</td>
<td>VI.B.3, VIII.A.1</td>
<td>1e, 10b</td>
<td>N</td>
<td>NA</td>
</tr>
<tr>
<td>1. calculate the standard deviation.</td>
<td>2. define normal distribution.</td>
<td>3. apply the bell curve to real world applications.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Intro to Radicals</strong></td>
<td>Students will be able to:</td>
<td>Beginning/Intermediate</td>
<td>IA</td>
<td>I.C.1</td>
<td>6d, 10b</td>
<td>5</td>
<td>P</td>
</tr>
<tr>
<td>1. define square roots individually.</td>
<td>2. define radical expressions individually.</td>
<td>3. simplify large powers of complex numbers.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Product and Quotient Rule with Radical Expressions</strong></td>
<td>Students will be able to:</td>
<td>Beginning/Intermediate</td>
<td>IA</td>
<td>I.B.1</td>
<td>6d, 10b</td>
<td>N</td>
<td>P</td>
</tr>
<tr>
<td>1. simplify radical expressions.</td>
<td>2. multiply radical expressions.</td>
<td>3. divide radical expressions.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table Continued

<table>
<thead>
<tr>
<th>Add and Subtract Radical Expressions</th>
<th>Students will be able to: 1. simplify radical expressions individually. 2. add radical expressions in groups. 3. subtract radical expressions in groups.</th>
<th>Beginning/Intermediate</th>
<th>IA</th>
<th>I.C.1</th>
<th>6d, 10b</th>
<th>NA</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solving Radical Equations</td>
<td>Students will be able to: 1. solve radical equations in groups.</td>
<td>Beginning/Intermediate</td>
<td>IA</td>
<td>II.C.1</td>
<td>10b</td>
<td>NA</td>
<td>P</td>
</tr>
<tr>
<td>Radical Functions</td>
<td>Students will be able to: 1. perform basic transformations of the parent function of a radical equation. 2. compare and contrast different transformations to determine what different parts of an equation do to the graphical representation.</td>
<td>Intermediate</td>
<td>IA</td>
<td>II.D.2</td>
<td>3e, 6e, 10a</td>
<td>NA</td>
<td>D</td>
</tr>
</tbody>
</table>
## APPENDIX J

### ALIGNMENT OF PARTICIPANTS’ LESSON PLANS TO THE TEKS

<table>
<thead>
<tr>
<th>Participant</th>
<th>Lesson Plan Grade Level</th>
<th>TEKS Objective(s)</th>
<th>Meet Standards</th>
</tr>
</thead>
</table>
| 21          | 8th grade               | • Compare & Contrast proportional & non-prop. Linear relationship  
• Generate differ rep of numerical relationships | Yes |
| 22          | 7th grade and up        | • Concrete & pictorial models to solve equ. & use symbols to record actions  
• Compare & Contrast proportional & non-prop. Linear relationship  
• Generate differ rep of numerical relationships | Yes |
| 23          | 8th grade               | • Compare & Contrast proportional & non-prop. Linear relationship  
• Generate differ rep of numerical relationships | Yes |
| 24          | 7th grade               | • Concrete & pictorial models to solve equ. & use symbols to record actions | No, 8th–9th grade |
| 25          | 1st–2nd grade           | • Skip count  
• Patterning Add/Sub | Yes |
| 26          | 5th–6th grade           | • Identifying patterns w/ graphical organizers (lists, table, chart, & diagrams)  
• 1-variable linear relationships | No, 12th grade level |
| 28          | 7th–8th grade           | • Compare & Contrast proportional & non-prop. Linear relationship  
• Generate differ rep of numerical relationships | No, 9th grade |
<table>
<thead>
<tr>
<th></th>
<th>Grade</th>
<th>Concepts</th>
<th>Notes</th>
</tr>
</thead>
</table>
| 29 | 3rd grade  | - Patterning Mult.  
                  - Lists, tables, & charts to express & identify patterns | No, 5th grade concept and not a function |
| 30 | 4th grade  | - Patterning Mult/Divide  
                  - Organization structures to analyze & describe data | No, 9th grade & algebra 2 |
| 31 | 9th grade  | - Foundations for functions: define, characteristics  
                  - Linear functions  
                  - Quadratic function | Yes, but vague |
| 32 | 12th grade (Pre-Cal) | - Define functions  
                  - Know characteristics of all functions  
                  - Transformation of functions | No, 9th grade |
| 33 | 9th grade  | - Foundations for functions: define, characteristics  
                  - Linear functions  
                  - Quadratic function | Yes |
| 34 | 5th–6th graders | - Identifying patterns w/ graphical organizers (lists, table, chart, & diagrams)  
                  - 1-variable linear relationships | Yes |
| 35 | 5th grade  | - Find patterns with graphic organizers | Yes |
| 36 | 5th grade  | - Find patterns with graphic organizers | Yes |
| 37 | 9th grade  | - Foundations for functions: define, characteristics  
                  - Linear functions  
                  - Quadratic function | Yes |
| 39 | Elementary | - Patterning Add/Sub/ Mult/Divide  
                  - Organization structures to analyze & describe data | No, 9th grade |
| 40 | 7th grade  | - Concrete & pictorial models to solve equ. & use symbols to record actions | Yes |
| 42 | ‘Little kids’ | - Patterning Add/Sub/ Mult/Divide  
                  - Organization structures to analyze & describe data | No, 9th grade + |
| 43 | 4th grade  | - Patterning Mult/Divide  
                  - Organization structures to analyze & describe data | No, 9th grade |
<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 44 | Kindergarten | • 1 – 1 correspondence  
• Patterns to make predictions  
(Add) | Yes |
| 45 | 8th grade | • Compare & Contrast  
proportional & non-prop.  
Linear relationship  
• Generate differ rep of  
numerical relationships | Yes, but only for basic functions. LP is not specific on the types. |
APPENDIX K

INTERVIEWS

K.1: Participant 21
K.2: Participant 22
K.2: Participant 23
K.2: Participant 24
K.2: Participant 25
K.2: Participant 26
K.2: Participant 28
K.2: Participant 29
K.2: Participant 30
K.2: Participant 31
K.2: Participant 32
K.2: Participant 33
K.2: Participant 34
K.2: Participant 35
K.2: Participant 36
K.2: Participant 37
K.2: Participant 39
K.2: Participant 40
K.2: Participant 42
K.2: Participant 43
K.2: Participant 44
K.2: Participant 45
LG: Here let me give you a copy of what you wrote down, so you can look over it.

P 21: Okay.

LG: Okay let me bring up my questions here. Alright so the first question is umm, why do you want to be a teacher?

P 21: Why do I want to be a teacher?

LG: Yea!

P 21: Okay! I want to be a teacher mainly because I’m not really going to be like a … I’m going to be a coach, so I have to do both coaching and teaching. But I want to do math because I know that people don’t like math, a lot, and I don’t understand why because I think it is a ball. I think it is so fun. And it is like, I want them to understand that math is not just something that they can blow off and that you use it in your everyday life, whether you’re going shopping or doing stuff like anything really like in your daily life. You really use it more than you think you do and I kind of want them to see that, and kind of do better because I know America is not really the best at math. So far other countries are beating us a little bit and I don’t like that I am competitive. So I kind of want to help that.

LG: That coaching coming out in you.

P 21: Yea, yep, yep yep.

LG: Oh!...ummm

P 21: Oh! My Mom was a math teacher too. So that kind of … a little bit. She’s a 6th
grade math teacher.

LG: 6th grade?

P 21: Yea. She always made me do math.

LG: So do you have any experience with teaching?

P 21: No but I’m about to. Umm. This Christmas I’m about to do some math tutoring for a umm, I guess a family friend. If that is what you want to call it. She is a teacher at the preschool I told you my sister goes to and she was there when my other sister went there. And she has a grandson who needs tutoring in like 3rd grade and he needs math tutoring. I’m going to do that and I’m going to do math tutoring next semester for 4–8 grade.

LG: How are you getting set up with this tutoring?

P 21: Uh which one?

LG: Just both of them. Well ones a family.

P 21: Yea ones a family. The family one I just gave her my number because we live down the street from each other and the other one is through Job for Cats. I saw it was on there, so it was on there. It’s still showing for a long time. Like the application deadline is through the middle of January.

LG: Nice. Job for cats. I’m going to write that down maybe I might be needing a job. So umm, this lesson plan here you chose 8th grade, why did you choose 8th grade?

P 21: I figured by that time you kind of, you have been introduced to functions, by now, so hopefully you know maybe kind of not the whole definition of what a function can do but you understand it is basically a line and it’s points connected so whether each and I think it is more comprehensible. I can explain better to them
than I could to a 6th grade level, it’s like “oh my gosh” but I don’t really have any experience right now so I know that… my mom tells me 6th grade is hard to teach. I’m like oohhhhh. And I went up there one day and they kind of they didn’t look very smart. I was like oohhh. I want smart people. I don’t want to be too hard on myself. I think that is a good grade.

LG: You want to do older students?

P 21: Yea. They’re more, not willing but they, that is kind of where you got to get them because if you don’t get them in like that area, 6th–8th, they’re either going to hate math or they already know how they feel about math and they just don’t want to do it. They want to get it over with. They don’t want to do math again.

LG: I see that with my students now in college.

LG: So what inspired your lesson on functions?

P 21: What inspired mine? I know that, I know a couple of teachers. Like you have had different experiences with teachers some like over explain it and make it more confusing than it actually is. And um I know that was my Precal teacher. I was like I don’t understand what your saying. Precal is not algebra but my god it was like, if I just did this by myself without listening to you I think I can understand it. Your talking and you are making it not in layman’s terms. Your trying to make sure everybody understands it, you are making it so it’s like broad and general. But I need focus more attentive right here so that is what I think that’s a better way of teaching it. If I can just… I just lost my train of thought. Now my math college algebra teacher he doesn’t really teach us it’s kind of like online teaching. It’s college algebra, it’s like 9th grade alg. But I mean I can pretty much do it by
myself. You go online and it shows you how to do it, the video, step by step. And it actually lets you figure out the problem by filling in the blanks, telling you your right or wrong and what you need to fix it basically.

LG: That was high school?

P 21: No this was now. College Algebra. It’s like a program or something. ..umm…it’s like my Pearsonlab.com and then you got to buy like uh .

LG: My math lab

P 21: Yea, yea, my math lab thing. It’s pretty, it’s pretty self-explanatory.

LG: So you learn through that program, not really in class?

P 21: Yea. We don’t really learn from the teacher. I kind of learn from myself. I’m more independent though. …_____.a problem.

LG: No no no. people learn different ways.

P 21: Yea people learn different ways.

LG: So where did you gain your knowledge about functions?

P 21: My knowledge?

LG: How did you learn about functions?

P 21: Umm. I think it was 9th grade. My math teacher kind of honed in on what a function was and I mean the easiest way I can tell you is a line. If there are two points on the same line it is not a function. It can’t be, I think it is \( y = \), yea you can’t have 2 \( y \)’s so if it got 2 \( y \)’s on this line it is not a function and so I kind of... I don’t even know if I did this right. I don’t even know if this is how you do a lesson plan. But I was like this is how I would understand how it, so hopefully you would understand what I’m talking about. I’m sure through educational
understand a better way to present this.

LG: All I want to see is a rough sketch on how you would lay it out.

P 21: Cool, cool!

LG: So the outline that you did is you decided to introduce the definition, then you did examples, then you made the students work and then you gave homework. Is this how you usually….

P 21: Yea that is usually how we did. But yea, most of the time nobody ever asked a question so it’s just like here give a question and then… or they give you homework and nobody every did it. I’m sorry I’m not going to say they were bad kids and I’m a good kid, but I did my homework by myself and I didn’t cheat or I didn’t ask questions. If I had a question I would go up and ask the teacher because I need you to a, you know, talk to me. And teachers really don’t do interactive stuff anymore. It kind of like stopped after 8th grade. Um you know like racing wars games. I really like that because I am competitive. So I thought it was you know interactive and fun. But no one else thinks that it is fun. So um whatever. I mean yea I just kind of talked to them and they really didn’t like go around the room, I had some teachers do that, like do you have a question like because you don’t want to be like “Hey do you have a questions?” (Stern) and nobody ever raises their hand but if you kind of go up to them and say “hey do you have a question? (soft) I see you are not writing you not typing, whatever, you know, let me help you.” You know.

LG: Yea I do that all the time. Um … So um before a student sees this lesson, what do you expect them to already known before they see this?
P 21: Like in 8th grade?

LG: Before like maybe the lesson or two lessons before, like what do you expect the students to know before you go in and go straight to the vertical line test. Like do they know anything about functions?

P 21: I’m hoping that.

LG: Like what level do you think they are already at before you get to this lesson plan?

P 21: This lesson? Basic math skills you know, how to plot a point on a line, what is parentheses 3 comma 2 parentheses where is that on a line? Can you draw uh a square so I don’t know, x and y axis, and can you do that by yourself because a lot of people really don’t know how to do that and it surprises me even to this day. You know kind of, can you do that and then we can start there we can build up on.

LG: Okay so like plotting points. How about umm graphing the actual equation? Should they know how to do that already?

P 21: Yea I mean. You got the coloring books at Ihop where you had to connect 32 to 31, can we kind of do that. Don’t crawl for me now just go around.

LG: Okay so (giggle). So in your definition of a function you wrote, an equation that does not have, you wrote, similar and multiple solutions, what did that mean? Were they the same thing or you wrote multiple solutions then you wrote similar above it.

P 21: I probably meant to put an arrow between “have” and ‘multiple” so “have similar multiple solutions.”

LG: Okay.
P 21: So like 3 2 and then 1 2. I could have explained more.

LG: And then you wrote “x = y, but cannot have two y’s,” what do you mean by that?

LG: Umm good question. (read to herself) Maybe I meant to put “there could only be one x and one y but there cannot be similar y’s. umm, so I think I’m scatter brained a little bit here, umm that is a function right, x = y?

LG: x = y is a function…. and in your case here what do you mean x and a y?

P 21: Uuhh I mean the points on a graph. The point…The first number is x and the second number is y. They can be, it can be any number you want it to be but the y in this column cannot equal another , the same one.

LG: Okay! I think I know what you meant by it but I was just kind of curious because I didn’t know if you meant an x can equal a y like you could have a 3 3.

P 21: Oh no that is not what I meant. Oh you can do that yea but…

LG: But you cannot have two y’s. You couldn’t have a 3 3 and a 2 3 or something.

P 21: Yea you can’t.

LG: And then you wrote “to check answer or graph, graph equation and use vertical line test” And what answer are you referring to when you say “to check answer?”

P 21: You may, I mean your answer may not be the right answer, so any answer is an answer but is it the right answer. This is where I want you to kind of check yourself instead of..

LG: But what are you referring to as an answer? Like are you referring to like a graph, a function, a point. I mean what’s an answer?

P 21: Your graph, your graph. I want you to immediately draw the graph because I know that visualization is pretty important it helps you understand better so
whenever you are working it out on your piece of paper I want you to always check your answer, always make sure that it, it makes sense in your head because that is how I always like… if I don’t understand a word you’re saying but if you can get it and you can think independently by yourself and check your own answer. And I think you will do well.

LG: Okay! And you wrote “define vertical line”

P 21: Oh that was for me. I said define for them what a vertical line test is.

LG: So what is your definition of a vertical line test?

P 21: A vertical line test umm. When you draw the graph and you, however you drawn it or whatever it may or may not be right but if you can draw like just a regular line down the middle of the graph and you see two points that met or touched on the same line then it is not a function.

LG: And then umm. Now these graphs that you have here would this be given to the students or the students suppose to develop these?

P 21: Oh I would draw them on the board for them,

LG: So you would give them these graphs?

P 21: Yea I would give it for them.

LG: Would you ever give them the equation and ask if an equation is a function?

P 21: Yea

LG: Is that or would you think that would be before or after this?

P 21: That would be before to kind of develop what we are talking about show it visually then come back to how we got from here to herewith this line to this graph. People don’t know how to make a line into a graph obviously to make a
function. Like $y$ equals blah blah blah but if it is like $y + 2 = 5x$ then they have to bring it all back and then make it into a line.

LG: So you are saying give them the equation, they have to put it in slope-intercept form if it is a line and they have to connect the dots. Now all of these here like you have a parabola and a radical function, and a circle and a cubic function it looks like umm, would you consider these like lines?

P 21: Which ones?

LG: All of them. Could you, you’ve been talking about making, drawing lines and these are graphs but I wouldn’t consider these graphs lines.

P 21: Do you just mean like a line like that is it or a function? Like I probably should have started with just a line but I was kind of shelling…I was probably moving too fast on a couple of them.

LG: It’s fine, it’s fine. I mean that’s what I do I have multiple graphs and if the students don’t know what they are. I was just curious what type of equations you would give them if would start off with just lines or are they …

P 21: Yea lines are the basic ones, you must do a line before you get to a circle, a circle is just a line turned around so it’s kind of …so you should start with that first.

LG: Um Now the ask questions part, you said “use examples on the board and overhead” are you referring to the previous ones you were talking about.

P 21: Yea Yea! Like if I write the line, like you said, on the board, can you draw that and do that and show me how you got there.

LG: In umm… now , and then you wrote “use games, such as racing to the board, make class interactive and fun , Umm, would this help them learn the concept or
is this just for motivation?

P 21: Both I think. It is good to have motivation to want to learn and if you can’t like, you can be there and not, just get it over with. Are you going to remember it, like beyond this, if you know what I am saying, to help you with you use this basic math further on and like precal and all this stuff. Can you remember it? Not just for this, to get the answer and to get it over with. Can you repeatedly do the same thing over and over again.

LG: Okay so you are talking about that extra practice?

P 21: Yea that extra practice. I know that when I was involved in the actual learning process I learned better than just sitting there and the teacher is just talking to me and I thinking about a lot of things I could be doing.

LG: Having experienced this little activity here, writing the lesson plan, would you have changed anything with the lesson plan? Did you go back and reflect afterwards?

P 21: I probably thought like I was like man …because I don’t really know what I was supposed to do so I just did what I thought was just a step plan not like a full out lesson plan, this is how I’m going to do it and I’m going to be the best teacher in the….I try to make things really simple for people, you don’t want to make it very complicated for them, who needs that? We have other things to do, so I just, I’m sure I can do better but I need practice so it definitely helped me realize that I need practice.

LG: Well would you have changed umm the way you umm the way the structure is in the lesson plan or any of the concepts or do you think that you could give this to
somebody and they could teach it well?

P 21: I think it is a pretty good skeleton for what you should do introduce the lesson, how it applies to life because I hate when my teachers don’t tell me why I’m doing this. I just, tell me why I am doing this crazy stuff. Tell me why. And then using ___?___ like crazy like 5 billion shells, no can we do like 500 Twinkies or something like that do you really want to divide by 2—between her and her friend like stuff like that.

LG: So some kind of like real-world applications?

P 21: Real-world applications. Yes. Then give some examples. If you have questions come talk to me. We can do this right here or you can come to my desk. I think that is… if you really don’t understand I’ll take the time . Come after class or whatever. Then I’ll give the homework out. I’ll probably, I loved when my teachers did this, give you like two problems to do or not two problems, but like help you with two problems. Others are like do the homework then go. No come back because your homework is completely different than what you just talked about and umm, keep it pretty open so we can all …because you may have a question that somebody else has a question about and they didn’t understand it so I’m not going to call you out or anything but ___?____. That question is probably confusing so let me tell you how to do this but not really give the answer away.

___?___

LG: They can figure it out.

P 21: Yea. Figure it out without me telling you.

LG: Okay, so umm. So do you have anything else to add?
P 21: Add to the conversation or the lesson plan?

LG: Yea

P 21: No it is really fun. Definitely. Definitely hit me like oh man this is ___?__.

Teaching is no joke anymore, everybody, it’s hard. It’s hard. But I think it will be worth it.

LG: You don’t realize you are going to graduate and you are going to start teaching and have homework.

P 21: Oh yea, my mom, she grades papers all day. But it’s cool. I think it is rewarding.

Even if it is not in your face rewarding. Like thank you teacher, get an apple every day or whenever, not physical gifts like ___?__ , but seeing people get better. It’s pretty cool.
K.2: Participant 22

Date: 11/28/2011
Time: 11:35am
Duration: 15:21

LG: Okay. So here is your lesson plan you wrote so you have a copy of it. Refresh your memory from all the turkey you have eaten over Thanksgiving. Um alright so umm. Just some background information so, um first of all what made you decide to become a teacher?

P 22: Well um one of the main reasons why I wanted to become a teacher was because um I had some bad experiences with teachers and it was because I didn’t feel like I was taught right and I helped some students like my fellow classmates in like math class and I ended up explaining it better to them than the teacher did. And so I was like their in-class tutor to a certain extent and I kind of want to become a teacher to go and rather have another student in the class explain it to them be the one that does it right the first time.

LG: Was this in like high school experience or college experience or both or what?

P 22: About the end of Junior high through high school. So a while. But another reason why I want to go into education is because I also want to become a youth and family coordinator for church and I felt like education was probably the best major to pick to help me prepare for that. If I ended up going down that road but also like I can get into education a lot better than other things. Like I looked at engineering and stuff ‘cause I’m good with numbers, for the most part, and it didn’t really keep my interest. Because my dad knows a few engineers and he says they love to take things a part and put them back together just to see how
things work and I’m not really like that. So I figured that was probably not the best route to go so the next best thing I thought was teaching and I could see myself getting into that. So,…

LG: Do you have any experience with any kind of aspect of teaching?

P 22: Outside of helping people in class, no.

LG: No tutoring, no extracurricular programs?

P 22: Non mam

LG: So the lesson plan you wrote, you wrote for 7th grade and up, why did you decide to choose that age group?

P 22: Um because I feel like functions and stuff like this is more for that grade level and up. But I also feel that, I feel that any grade up to 6th grade wouldn’t understand it to begin with, no matter how much I try to explain it. I feel like they might not understand it. Whereas once you get into 7th grade and up they begin to comprehend math and things a little bit easier and so that’s the main reason why I chose it, it’s just because I didn’t feel like the material would fit for that age group below it.

LG: Okay. So umm, what inspired your lesson on functions? Like why did you choose to do it with these topics and in this order?

P 22: Umm. Because it’s … I feel like this is the best way to approach it. Because umm like one way that one of my teachers explained it to me whenever I was first introduced to functions, I thought this was one of the best ways to put it. Was they used a vending machine as the example. Like one that has all of the snacks and stuff and you press the number and you press a certain letter and like you get that
only. That is the only thing you get from it. And so I felt like that was a really good example for that. And one of the first things that I feel should be start off with would be the fact that there is one x and one y and then after that I decided that the next easiest thing to go over would be looking at them in ordered pairs. Because as long as you keep the numbers in front of them, for the most part, I feel like they can understand it better, you are just giving it to them in a different format. And I feel more like if you put a graph in front of them first they will probably freak out and not want to do it. They would probably be like oh my god what is this. And so I started off with a box because it is more organized looking and then I just kind of went through it and what looks neatest to what looks the ugliest to mess with. So they would have an easier time going from, once they understand this, ordered pairs, then this is a piece of cake and it is not as ugly looking to them. And so that is kind of why I went through it that way. Just kind of like what is neatest to what is ugliest.

LG: Ok. Never heard it put that way but no but no that’s good. I liked the order that you did it. Umm, so a where did you gain your knowledge of functions?

P 22: Umm my probably one good teacher I really had in high school. Ummm it was my algebra 2 teacher. She was very, very patient in the way she taught. It was very slow and methodical the way she went through it. And so, the first time she went through it I completely understood functions. That was the first time I feel like I was introduced to it in the way I understood it. I’d been introduced to it back in about 7th - 8th grade-ish but I didn’t really understand it, it was still too difficult for me to understand because umm my teacher, he went he like ran
through it fairly quickly and she went through it much slower, much easier paced at a level where, she went through it saying “you understand this part?” and once everyone in the class pretty much understood it, it’s like we are going to this part now. Everyone understands it. Move on and so she was very methodical on how she went through it and she went through it much in the same way that this is kind of. Uhh maybe a little bit different but she is probably the first person who taught it in the way that I understood it. And I relearned it again earlier this semester from professor ________ my teacher and he taught it in a very good way also, he likes to use letters a lot more than I would prefer though. He says x, y a lot and use other letters like h and z but it doesn’t work as well for me. I feel like she taught it much easier. But I still understood it so I mean. Yea!

LG: So in the outline of your lesson, you kind of did like little chunks of things like concept and example, concept and example, and concept and example, Um so this is from, is this from your experience in the classroom, how most teachers teach you?

P 22: Umm, somewhat yea. Um some teachers they like to go through it all then do bigger chunks than I am, I believe because they rather than giving concept-example, I feel like they just group it altogether really quick because they are on a time schedule and have to speed through it most of the time and I understand that to a certain extent because that is how it is you are on a time schedule and if you don’t meet your time and get through it in that time, your probably not going to get through it again another time. Need to have time to get through the other things you need to do. I felt like this would be an easy simple way to get through
the lesson within that amount of time in a way that they would still understand it. Because most teachers, they kind of rush through it because they are like oh no I don’t have enough time and like they will get through it and they will have 20 minutes left of class and be like here is some work. And it’s just like yeeaahh.

LG: How long do you think this lesson would take you? You said within the limit. What time were you thinking?

P 22: I feel like I could get it done in within a high school or junior high timed class which is about 45 minutes to 50 minutes maybe so.

LG: Now you represented these in three different ways using a table, using a relation, and using a graph. Um why did you choose these and are there other forms that you could have chosen?

P 22: Umm I chose these ways because these are the primary ways that students will see them for instance like on taks test or staar, or whatever it is called now. That’s, these are the three ways main ways they will end up seeing it on that and so I felt like these are the ways that it should be taught. I do think that there are other ways it could be taught probably but what they are I don’t really know. These are three main ways that I really know of. But any other way that there is I don’t feel they really need to deal with when it comes to state testing or anything like that or I mean… Until they get into higher level math, I don’t feel like they truly need to know any of the other ways so there might be.

LG: What did you expect them to know before they learned this lesson?

P 22: Umm I don’t know. I kind of wrote it in a way so that like they could start off knowing Algebra 1 math and still understand it, maybe less than that even. I kind
of start off with ground zero with this I felt because, I mean, as long as they understood like basic graphs, I feel like they would be fine. Like graphing and coordinating that is all they would truly need to know for them to understand this lesson plan.

LG: Okay so you would show them a graph, but how about an actual equations? Would you make them draw out equations or use equations?

P 22: Yeah, I would uh I would that would be part of what I did with the graph area would be to have them understand equations with it. Um I feel like maybe I would be for the next class or whatever I had with the students. Um that way they understood this basically because if you give them $x + y = \text{whatever number}$ they wouldn’t understand, even if you just went through all of this with them. Uh you could teach it to them in an easier way like in a whole separate class from this one. Yea but I would go over that one point or another.

LG: You did write on here that I would then tell them a process on how to determine if their graph is a function or not this is known as the vertical line test. You didn’t define the vertical line test on here but what do you, what is the vertical line test to you?

P 22: Oh um for whatever the graph is going, if they can draw a line straight down on any point along the x-axis, just straight up and down if it doesn’t touch the line more than once then it’s a function. So…

LG: You didn’t know you were going to be quizzed did you?

P 22: No I did not. (Giggle.) I’m ready.

LG: So having experience writing this lesson plan, would you have changed anything?
P 22: Umm. Like you said about uh the actual equations add that in here. Umm like I said it would have been for another class time, not within that one class period maybe the next class day or so, but I would have definitely added that to this. I feel like that would be the only other thing on here. Like within functions, I feel like that would be the only other thing besides what I already have in the lesson that they need to know for their next levels of math that they would go through that has to deal with functions or anything like that.

LG: How about any activities?

P 22: Umm. I honestly don’t know. Umm. Math teachers don’t normally do a lot of activities these days umm, but if I could come up with one I would definitely do that because I feel like…Usually whenever I had like little activities in math class, I was like oh my gosh we get to play today, this is the best. And it was always kind of like a pick me up for math and I feel like math does need more activities and stuff but ____ todays schools they kind of rush through the topics to make sure the students have these topics by the end of the time before their test because that’s basically all that matters. I mean to a school ___that is truly what matters what they get on those tests but if I came up with an activity for them I would totally do it because I feel like math does need more activities and more hands on stuff kind of like science to a certain extent

LG: So have you, you didn’t take developmental math here did you?

P 22: No I did not. This is, College Algebra is my first math class here.

LG: And you didn’t take anything at a community college?

P 22: Nope. I’m a freshman.
K.3: Participant 23

Date: 11/29/2011
Time: 1:16pm
Duration: 10:29

LG: Why do you want to be a teacher?

P 23: I’ve always wanted to be, as corny as it sounds, be a role model. I know growing up I had some good teachers and some bad teachers and each one of them kind of shaped me in a way like the bad ones, well not the bad ones but the ones that weren’t so great, they were still memorable. Even though they were… you know.

LG: Like what not to do?

P 23: Yea and so you know they kind of and I had a homeroom teacher that kind of brought me out of my shell but the really nice ones, it’s always good to connect to the teacher, so it is not only like, cause I know I’m able to explain things in a number of ways until someone understands it. But just even more on like being that role model to them because. I mean when you are a teacher, some kids growing up that’s when you are shaping them. Into the person you are going to be so.

LG: Okay so, do you have any experience with teaching?

P 23: Um I do. I was in a program that was called Ready Set Teach in high school and I would assist a teacher in elementary school. I had 3rd graders, and I also did middle school 8th grade English.

LG: Um so you wrote me a couple of things down for a lesson plan. I will let you have a copy to look at so you can remember what you wrote. First of all what does “TLW” stand for?
P 23: The learner will.

LG: Oh the learner will. Is this kind of, how did you base your lesson plan? What did you base that from?

P 23: This is how kind of like how we would write them in that ready set teach course. It was just kind of like what we, the objective of the day and what we wanted them to learn overall.

LG: What inspired you to write this?

P 23: I guess I just thought this was pretty much what they needed to know about functions. Like I don’t know, I guess this covers it all.

LG: Do you think if you would have given this to a substitute or another teacher they would know what you wanted the students to do?

P 23: If they were familiar with the subject yes but if they were just completely, just there to sub and just watch the kids then I guess probably not.

LG: What do you think would need to be in a lesson plan for somebody who wasn’t a regular teacher?

P 23: Um I would probably refer them to a page in the book that explains it more thoroughly that way I wouldn’t have to write anything down.

LG: How did you gain your knowledge about functions?

P 23: Well I learned in high school, I think, and I took a college course, algebra in high school, after the first semester and I dropped it. And then well here in college I kind of to refresh my memory on it yea.

LG: Did you take a developmental math course here at Texas State or did you just go straight into College Algebra?
P 23: I went straight into college algebra

LG: College Algebra, did you take any courses at a community college? Math…

P 23: Math courses no.

LG: So in here you wrote, sorry what did you say TLW …

P 23: The learner will

LG: So they will be able to identify functions given a table and a graph. So how
would they identify functions?

P 23: They would be able to tell if a certain table or graph is a function or not.

Obviously with a graph it would be like…

LG: What would the definition of the function that they would be using to identify
within one of those two models?

P 23: Um I knew it, that for every, oh hang on give me a second, it’s for every x there is
only one y. so in a graph if there is a number 2 as the x then there can only be…
something like that. Well I know with the graph it is the vertical line test.

LG: Okay so the vertical line test for the graph and for the table identifying  for every
x there is only one y. So you couldn’t have two of the same x’s mapping to two
different y’s?

P 23: Yea

LG: Now you solve equations and graph to identify whether it is a function or not. So
what type of equations would you be giving?

P 23: Quadratic equations cause the….which one of those? I forgot what they were
called. Um they were like the parent functions.

LG: Ok. The parent functions. So what um what equations, would all the equations be
functions? If you are just doing the parent functions.

P 23: No because you can manipulate the equation to where it like um flip it across the x-axis like um for example the absolute value when reflected over the uh, I want to say y or you can rotate it or something so it wouldn’t pass the vertical line test.

LG: What do you think the students known before they came into this lesson?

P 23: About functions?

LG: About anything. Like maybe what would be the lesson before this?

P 23: Ummm! Maybe I guess just how to solve equations.

LG: Solving equations. That would be very important for them to know. Ok. How about the definition of a function, would that be introduced in this lesson or a previous lesson? I mean is this like the very beginning of functions this is how you would introduce the topic?

P 23: Uh yes. I think just uh solving equations would be first that way they can just focus on that and get that going and then I can introduce what a functions is and then apply what they learned before.

LG: So having done this little exercise would you have changed anything that you wrote in the lesson plan?

P 23: Um probably since you just brought it up, about referring to the book for just in case it was a substitute, so the students wouldn’t get confused.

LG: I did forget to mention you wrote this for 8th grade? Why did you choose that grade?

P 23: Umm I guess maybe it’s just personal because I remember being really good at math in 7th grade and at 8th grade I kind of figured that they would be able to
grasp this concept to. It’s not too hard.

LG: Is that the grade level you want to teach at?

P 23: I really did like 8th grade, when we did the ready, set, teach. I thought I wasn’t going to, I was like man they got attitude but they were really good, I liked them.

LG: You said only English though? 8th grade English? And what was the other class you taught?

P 23: It was an elementary third grade GE so it was all subjects.
K.4: Participant 24

Date: 4/10/2012
Time: 11:06am
Duration: 20:01

LG: What courses were dual credit?

P 24: Political science and English. All my English.

LG: None of it was math though.

P 24: No

LG: Ok that is the subject I am interested in. So first of all, why do you want to be a teacher?

P 24: I like working with kids, they are a lot funner than adults. And um I don’t know, I think it is like a noble job, you know, teaching a young kid and I think it has a lot of influence on their young lives.

LG: And you have chosen EC–6th?

P 24: Uh huh

LG: So you enjoy, you want to teach the younger population?

P 24: Yes, because I just got out of high school last year and I can say that I don’t think I can handle high schoolers yet.

LG: Do you have any experience with teaching or any aspect of education?

P 24: Last semester I took the teaching poetry to children class so I actually got to go into the classroom and teach poetry to 1st graders. And that would be my only experience besides that I would play teacher with my sister.

LG: It’s okay I use to do the same thing. What did you have to do to prepare you for this poetry…?
P 24: I had to make lesson plans, choose poetry from certain books or on our own. And you definitely had to make lesson plans that was a big part of our grade, umm I had to get props if I needed them and at the end we had to compile and read all of our children’s poetry and had to decipher their handwriting and then had to scan everything and make a book compiled with all of our poetry and pictures and all kinds of stuff.

LG: And what class is this?

P 24: Teaching poetry to children, it’s an honors class.

LG: Humm Interesting. Was that in the English department or literature, education?

P 24: Yea I think it’s education, I think. it was instead of a reading class.

LG: So in your lesson plan you chose 7th grade. I know that is not in the band you have chosen for your degree, so why did you choose 7th grade?

P 24: Because I think functions is are a little complicated, for like… maybe 6th graders or 5th graders could to do it, but I have a real hard time gauging right now like what age group does what. So I guess I just need to look over my TEKS some more because I don’t know, I consider functions kind of like higher math, maybe like 5th or 6th grade, 7th probably.

LG: What inspired the lesson plan about functions?

P 24: I try to keep things as simple as possible, because I have a horrible time with math like it just doesn’t click. So I like tried to have a lot of visuals. And um just try to explain it in a nice straight forward way to get a nice basis for all the other math concepts that come after functions.

LG: What do you mean by visuals?
P 24: Like in here I said I wanted to put lots of like different graphs, with like crazy loops in it and everything, like the graphs they usually haven’t seen because they are probably only going to get to see the parent functions and quadratic maybe so I wanted some crazy graphs. Maybe because I don’t know, I think it would be visually stimulating and have kids interact like have them draw the line test and have everyone look at it and have like examples of “T” tables on the board so they can visually see how every x can only have 1 individual y.

LG: Okay um. Where did you gain your knowledge of functions? When did you learn functions?

P 24: Um I don’t really know what grade it was. I know we talked about it all throughout high school kind of touching up on it. So I’m sure I learned it sometime in middle school.

LG: So where did you gain, so did you already have this in your knowledge before you came to college?

P 24: Yes I did.

LG: Is this how somebody represented it to you?

P 24: Probably, I know we did a lot about the whole, we did lots like the examples of the T table like this is a Function, this is a function. And we did the line test. It probably, would be 8th grade. I’m not really sure. But I know like in high school, even in the senior year we had to go over it in calculus and pre-cal. Touch on it.

LG: What was your high level of math in high school?

P 24: Calculus. I passed but I hated it, I got a 1 on the AP test because I did do ____.

LG: So, you didn’t take developmental math here?
P 24:  Nu uh

LG:  And you didn’t take it anywhere else you started straight into college algebra?

P 24:  uh huh

LG:  I wrote some notes down that I want to ask you about for your lesson plan.

[LOOK AT LESSON PLAN.] Right here you mention ‘they follow along in the textbook.’ Do you think textbooks are essential for teaching?

P 24:  I don’t think they should be the main point of teaching. I think they should be there as a guide because sometimes they have really handy like summaries of stuff and … so I think textbooks are supplemental they are not like the main. People shouldn’t just say read these pages in a textbook and then go from there. I think the teaching should be the main point, the textbook is just to help.

LG:  What is this? ‘Explain that for every x value there can be only one unique y value.’

P 24:  Basically point out the table point out like how this one is, the x value 1 can have 0 but that one can also be 1 and that can’t be every x value …every x value here must have a unique y value, it can’t be the same or else it is not a function I’m somewhat confusing myself. I forgot… for every x value there can be only one y value. Like (1, 0) multiple times because it is just the same point over and over again but (1, 1) and (1, 0) would make it not a function. Because they would be stacked up on each other.

LG:  So this right here is the definition of a functions?

P 24:  Off the top of my head yes.

LG:  I think it is a good definition. Right here it says using the graphs mentioned
above, which graphs were you referring to?

P 24: The graphs up here. When I show them all kinds of graphs with loops and all sorts of shapes, they probably haven’t seen before. And after we talk about functions have them go to the same graphs and do the vertical line test on them.

LG: Ok. So I know you mentioned about, you like to do pictorial…pictures and tables and so right here you do, you say group discussion. So do you think group discussion is essential for the classroom?

P 24: I think so to like conclude the lesson, it’s very important because people might students might have different views that we need to get out there so we can interact in and I think it is a good dynamic to have all the students talking about something we just learned and throwing their ideas out there. Because it’s kind of like, I don’t want to be the kind of person that just tells and tells and tells. I want them to think of their own ideas and collaborate as like a group. I think that is important.

LG: Oh up here you said uhhh you said “ask for defining characteristics of each,” what do you mean by characteristics?

P 24: It’s kind of like patterns, like a lot of them have curves and some of them the slopes are really crazy, or really low, slopes increasing very fast or very slow. And I plan on like maybe graph like some of them and like drawings on a graph and stuff and just like looking at the similarities, the differences between all the graphs.

LG: Do you think slope and steepness and stuff like that does it matter when you are talking about functions?
P 24: Not necessarily um I mean I guess not, but If something was going like this then and then the slope drops off completely then that wouldn’t be a function, so yea I guess it is kind of important. Sometimes it is hard to tell in graphs, if it goes straight down or there is a slight slope to it, so I mean it is a good thing to notice the slope of any graphs, but not necessary when we are talking about functions.

LG: Prior to this lesson what do you think the students are coming in already knowing?

P 24: Probably already talked about functions, what the parent function is, how to find slope and just those kind of basics, like what are ordered pairs, how to plot points, how to make their own graphs and then I guess it goes to functions from there.

LG: Why did you structure your lesson plan in this way? With this engagement and then next, next, next…

P 24: There is this 5 E model that we are learning in general science, and I couldn’t think of any of the other E’s. I know one of them is like elaborate and there is like engage and other one. And if I would have had that model here, I probably would have used that but I guess it is kind of a guided and just so I have like, ok there is this section of the lesson plan that and then we are moving to this section and this section and this section.

LG: It’s sequential in how you would teach it?

P 24: Uh huh

LG: Ok do u think a lesson plan should have some type of outline to them?

P 24: I think so especially like in elementary education just because it gets so out of control, you have to constantly be talking about behavior and I know my 1st grade
class, when I first approached children they were off the wall crazy. I think it is good because you can get flustered and kind of forget where you are so it is good to have a good outline but it is also good to kind of have it malleable, you know you can change it at a moment’s notice and like sometimes I try to like put like,… like if a discussion goes a different way what else I can lead to with this lesson. I think it is good because I know I can, everything can just go out of my head real fast and it will keep me on track.

LG: So it is kind of an outline for class?

P 24: Uh huh.

LG: I don’t know if you thought about this lesson plan after you wrote it. Would you change anything about it? I mean did you go and do any research or think about it or anything afterwards?

P 24: You know I thought about it a little afterwards. Thinking like did I choose the right grade, did they learn it like in 7th grade. I would probably add more explanations because right now it is vague. And I’d probably like, research functions a little more and like what a textbook would say and try and like look at that a little bit more and like incorporate that into the lesson, because right now…it is really short too. This lesson would not take that long. So probably would have to add a lot of other stuff into it.

LG: You what you need to elaborate on …

P 24: Yea, probably need to elaborate on what a function is and why. Certain points on a graph are like what… certain lines are functions or not and I wrote like demonstrate the vertical line test but in the future I would probably elaborate on
this, like the steps I would take to demonstrate it.

LG: What kind of steps would you take to demonstrate?

P 24: I mean it is a very simple thing so uh probably just get two different graphs, one’s a function and one is not and just show them like how you can put it anywhere on the graph that is a function and there will not be two points on the vertical line and then go onto the next graph which is a not a function and show how certain part of the graphs it passes but if anywhere on that graph at all there are two points on the same vertical line then it is not a function and in the lesson plan I probably would like, obviously this one I wouldn’t really be able to unless I sketch them out but I would have like pictures of the graph on that I would be using on my lesson plan. I would have everything already planned out.

LG: So your subject interest is science and history. Now being first through sometimes third or first through second, sometimes you have to teach all subjects. So do you feel confident that you could teach all subjects?

P 24: I’ll feel confident that by the end of college I’ll be able to, right now it scares me a little but um and what’s good is I plan on getting a real good solid knowledge of like fundamental like math and English, like the fundamental rules that will set up the rest of their…it’s a little scary but...

LG: Where do you think you are going to learn some of those fundamental rules?

P 24: Hopefully in my education core block they will go over some of that and if not then just doing it on my own, because like it scares me. Cause like it’s a lot of responsibility to kind of set up because I know that some of my, it probably wasn’t in elementary school but further on I didn’t get a good foundation of
knowledge on certain things especially in math and I’m struggling with it now and I don’t ever want to do that to a kid. So I’m going to take it upon myself to really get all that stuff down so I’m not misleading them or anything.

LG: But if you were suppose to, if you were aloud to teach just the content area it would be one of these two?

P 24: Uh huh. And obviously history probably wouldn’t be huge part of first two or three grades.

LG: I’m sure there is a little in there.

P 24: A little

LG: I think that is about it. Do you have anything you want to tell me?

P 24: I just really don’t like math.

LG: It’s not everybody’s forte but alright.
LG: First, is there any particular age you are interested in?

P 25: I would say between Kindergarten and 2nd grade.

LG: Is there a subject matter you enjoy more?

P 25: Art and English

LG: Art and English so not math huh?

P 25: No, I mean it is actually fun to teach kids but I feel more confident in art and English.

LG: Ok. First, why do you want to be a teacher?

P 25: I’ve wanted to be a teacher literally forever. I don’t know. Like my mom was always involved with my elementary school. So I was always around there, around the teachers and the conference rooms. I helped them set their rooms over the summer. And so I’ve always liked it or been comfortable with it. And then I started working at a preschool and it’s just really rewarding teaching kids. Really rewarding and I feel like I’m really good at it and people always tell me I am good at it and really creative so, so I feel that it is a good area for me to be in. Because I really like it, and I don’t get bored of it and it’s not like I’m doing something I’m bad at, so I can’t mess it up.

LG: Well that’s good. That’s good.

P 25: For the sake of the kids, I feel bad when they have bad teachers you know. Because there is enough of them out there.
LG: That is what I want to help with in the future. Do you have experience with the aspect of teaching?

P 25: Like Lesson plans and carrying them out or…?

LG: Yea that or even teaching or tutoring or any programs that you have been in.

P 25: I’ve taught preschool for 6 years, we had to write lesson plans and I was the lead teacher so I had to get my certification and all that stuff. And now I do my internships in elementary schools. So it’s kind of like the same thing but I had a lot more responsibility when I was at the preschool. And that was kids from ages like 2 to 6.

LG: So you have gone through and have taken education courses?

P 25: I got my CDA, my Child Development Associates and I got that at ACC and now I’m just going into my field blocks in my teaching degree.

LG: What are somethings that you taught in the daycare?

P 25: Uh, all kinds of things. Well most of it was… The school I worked at was a high scope school, I don’t know if you know much about the different kinds, but in high scope schools it’s all about learning through play. So we would set up experiences so they would learn through it. So like you would set up sorting kind of experiences and then we would take from that and make bar graphs, we would sort by color, we would mix paint to make colors. Everything was like integrated in one thing so we could cover all the subject areas. Do you know what I mean? So it is introducing the kids to all subject areas but they just feel like they are playing. They don’t realize they are learning so much. So it kind of covered all basis. We had a very detailed list in high scope, there is a very detailed list about
social development, art, music, math concepts, time and all those things and every
lesson plan u right has to touch base on all those things every single day.

LG: Oh really so you had to cover all subject matter?

P 25: Yea it was very intense but when you realize that they are learning a lot of those
things through play, it becomes easier to find ways to teach them.

LG: I think that is cool. Um. Did you have any experience before you went into the
preschool?

P 25: I started in the preschool when I was a senior in high school so I mean not really.
I mean just babysitting and stuff like that but not in teaching.

LG: So this is your lesson plan right here, if you want to look over it just a little bit so
you can remember since I know it has been a week. So is this usually like the
lesson plans you did in preschool?

P 25: No actually I have never done one like that before. That just… I was trying to
think how it could be simpler but I want to teach 1st–2nd grade so I tried to push it
a little more to that age group. You know what I mean?

LG: Uh huh. So what does your other lesson plans look like compared to this one?

P 25: Well when I was in the preschool they were, it was like giving them an activity,
like you sorting color bears, like that’s something really simple that they have as
an option all the time. And when they sort them they are counting, and then they
move on to having, oh you have 3 bears over here and 4 bears here and it was
integrating into adding and subtracting. And it just kind of built as the kid, you
know, per kid, not all kids were on the same page. But if a kid was particularly
interested in that kind of stuff then you can build off of it then you really start
getting into mathematics. But for the purpose of this I kind of tried to think of something more complex for the age groups that I would be teaching. A little more complex but it is still pretty simple.

LG: So what inspired the m&m lesson?

P 25: I liked and I still to this day like graphs and graphing and stuff like that. I know whenever I have worked with kids in the past that if you have something that they are interested in and they are building up to something then they stay interested and really learn from it. So I figured with m&ms it is hands on and it is something they like and they are building up to it. Like they have a problem and they are figuring out the amounts the functions are, and they are putting it into a graph. And it’s progressing more and more so it’s not like $2 + 2 = 4$. It’s something that like progresses so they can keep doing it throughout the day, I mean week, build off of each thing.

LG: Where did you gain your knowledge of functions?

P 25: High school I would say. I mean because I’m 25 and I haven’t taken math since high school until I came here. Here it’s like really refreshing my memory of things that I learned in high school. It’s been so long.

LG: So you just started in college algebra here?

P 25: I put it off for a long time. I tried taking it once before with a different professor here but I felt like he just assumed everyone knew what they were suppose to be doing and he didn’t realize that some of us haven’t taken math in so long and we are not all freshman. And I just couldn’t keep up and he just assumed I knew all of these things but the teacher I have now he is really good at, if you don’t
remember it he will show in the book where you can go back and prereq kind of stuff or he will help you understand it. And so that is why, I am doing really good in that class now. Surprisingly, I surprise myself.

LG: So now why did you structure your lesson plan this way? Where you gave a whole bunch of information then…?

P 25: I am supper organized and I just wanted, I guess, whoever was looking at it to see what I was, like to see the steps the kids would be going through and what it would probably look like.

LG: What would the students’ knowledge be prior to taking this lesson? Prior to you teaching this?

P 25: Well they would know how to multiply simple equations, like I gave them numbers that I feel like when I was in that, well 3rd grade I knew memorized, but when I was in first and second grade I was learning stuff like that and I don’t if it was because of the school or my mom pushing multiplication but I remember knowing simple things like that. And the kids that I know in first grade, like the ones I nanny, she’s in 1st grade, she knows like simple ones like that, she knows the set ones up to about 100. So I was like that is something they should know. And they should be able to feel confident about and as they go into the next part this should help them understand it more because they have that base knowledge of multiplying.

LG: So why did you choose to start off with a story?

P 25: Oh like a story to go along with this?

LG: Uh huh.
P 25: I thought about it and then I was like umm I wanted to keep it simple just for this. Like the story is really. Like I know whenever I get problems like this, if they are word problems, I can understand it better. Like I can write my own function and write out $x = \text{this}$ and $m = \text{this}$ when I have a story. When it is just given to me it kind of like, I don’t know, I don’t feel like it is easier when it is just given to me because I feel like I am missing something. I know I should be doing something that I am not. I don’t know if that makes sense. I feel like if it was a simple problem and I get to the end then I think it that it was too easy.

LG: _____ connect.

P 25: Yea like sometimes when there is a simple problem I feel like when I get to the end I’m like maybe I did this wrong because it was too easy. But I feel like if I have a small little story line then I’m pulling out the information myself and I know how to use it. I don’t know that’s just me.

LG: Interesting. I know a lot of people opposite of that. They are like just give me the information, give me the formula, so I can just plug in numbers. So that is interesting. So you have this equation that you wrote down here, is there kind of any restrictions to it?

P 25: I think it’s pretty simple. I think, I don’t expect there to be restrictions to it.

LG: Ok, so like right here you only did a couple of points and I think this is based off of your data here, would it make sense to connect those dots?

P 25: Yea! I mean I guess they were just seeing, okay, the only restrictions, if I understand what you are saying, is however many problems are on the homework that night, according to the story. You know. And so I feel like if I were to give
them an exact amount of how many problems were going to be on the homework because you don’t think on the homework they are going to assign like 100 problems. But I didn’t know if I should just for the example should I put in number problems, because I think that would change depending on how much the kids knew and how high they could go in multiplying. So depending on the class, or the group of kids.

LG: Because from what I understand if they get one question right they get 3 m&m and if they get 3 questions right they get 9, so what about 2, are you going to enforce that? Would you expect them to do all of them?

P 25: Yea I probably would.

LG: And what about half questions? Do they get half questions?

P 25: Yea that’s what I started to think about, and I was like the only thing I was wondering was if I should tell them, like put each of them in groups, divide the class into groups and have them say each student did 3 problems and your student did 6 problems and the third group did 9 so each person can figure out and they can come together and make the chart, the graph together but I don’t know I didn’t. I just kind of kept it simple and open.

LG: And you feel that 1st and 2nd graders can do this lesson?

P 25: I think so. Knowing some of the kids that I nanny, they’re in first grade, and I feel like they could do this. And other kids. And I am not sure I think it depends on the class that is why I put 1st to 2nd because I know some 1st graders that can do it and I feel like 2nd graders could definitely do it.

LG: Having made this lesson plan did you reflect on it and if so would you make any
changes?

P 25: I did reflect on it and I was thinking about like I said the grouping them and bringing them together, so it would be a small group kind of project. And I figured if it was just individuals they wouldn’t have as much fun with it. That is about all I could think of. And I did think about if they chose different numbers besides the threes and stuff like that. But I just kept it simple like this because I think it would depend on my group of kids. If I knew what they were capable of.

LG: Another thing is, I didn’t…over here in your table, here you wrote 3x and M, you said they were = so that means this should be the same as this. Yea this should have been just x but I put 3x just because recently like my professor sometimes he will write that down to remind you, you’re doing the x times 3. So I thought maybe with little kids they would appreciate that but I did…it should just be x but I thought maybe for the purpose of the age to do it like that because when he does it like that, he only does it sometimes, but when he does it I’m like okay that helps me to remember if I would’ve wrote it that way. In high school I wasn’t taught to write it that way.

LG: Well is there anything else you would like to add?

P 25: No not that I can think of. I kept it really simple.

LG: I liked your lesson I thought it was really good.
LG: Okay so first of all why do you want to be a teacher?

P 26: Because I love sharing my knowledge of what I have been taught and what I have experienced to kids because they just want to absorb it. And so it is enjoyable to teach them stuff or show them things because they are so excited and want to learn. To try and tell an adult like..Ahh ok. So I love the passion that the kids have. So that is what I want to do, I want to share with them.

LG: Do you have any experience with the aspect of teaching?

P 26: Yes I have worked at many daycares. I helped my mother when she had an after school program and so I helped her do fun games and learning activities there and I also volunteer at my daughter’s school. And help the teachers with different things like that not only field trips but also things in the classroom. You know if they need someone to come in and assist with certain activities or games I can go in there and help them with that. Only because I am a parent volunteer. I haven’t started my teaching stuff yet.

LG: So did you actually have to prepare anything or was it more like assisting?

P 26: Assisting.

LG: More assisting. So you just followed the teachers instructions?

P 26: Right.

LG: So you did choose between grades K through 6, I see on here, so what grade would you want to teach?
P 26: I want 2nd grade.

LG: Why 2nd grade?

P 26: They’re still at the age where they’re eager. And they don’t quite have the attitude as they do when they get older and the attitude with me … I can teach older but I prefer the younger because they are more able to learn because they still want to. The older they get the less they want to.

LG: I know what you mean. What inspired your lesson on functions?

P 26: That is the only kind of thing I could remember somewhat. When I’m actually doing the functions in class I can do it. I like that part. So on the test where I didn’t know anything on the test, I knew those answers. You know, systems of functions, you know things like that I can actually throws those things out fairly well. That is why I wanted to use that.

LG: What grade level was your lesson aimed towards?

P 26: It was going to have to be probably 5th or 6th because that might be something they can do then. Because there is no way it could be taught to a younger kid.

LG: Just so we can remember. [pull out her lesson plan]

P 26: Oh yea I remember.

LG: So does that mean you reflected on what you wrote?

P 26: Yea I thought about it and gone man that was horrible.

LG: Why do you think it is horrible?

P 26: Because it’s what I remember but that doesn’t necessarily mean that is how it should be taught. I would have to look at the instruction we were given and see how I would reword it. Since I didn't have that readily available, I kind of just
pulled things out of my head.

LG: Would that have affected your lesson plan that you had chosen?

P 26: I kind of just put down what I remember, I kind of how I would describe how it works. That is what I put down.

LG: Would you include any activities or something, would you expand on this? Or is it something you would just show the students?

P 26: I think you need to have some kind of background on something before you show them. Because if you just throw something at them, like a lot of times gets happens to us, then we don’t really understand why you did this to get this in the first place. You know. What is this we have to do to this, you know why is this so I think I would have to show the back ground to this first and then get into the systems of functions. I would have to show the functions themselves and how to work them and how to distribute them and all those kind of things. And then I would have to throw in the systems of functions because you have to put this one into this one so I would actually have to back up, not necessarily expand but back up and then teach this.

LG: So are you expecting that is what they need before you teach this lesson? Like would those be prior lessons or all taught on the same day?

P 26: No those would be prior lessons, so they could practice and do the homework on how to do the simplified versions the you start making it more complicated the next class period. I think it would be way too much to do in one day.

LG: Where did you gain your knowledge of functions? I know you said this pretty much came from the class that you took.
P 26: In class.
LG: Now is it just the college class.

P 26: I have never heard of it before, like the functions, the system of functions, I’ve never seen before. So I actually did learn that in class. A lot of people know it, the people behind me always spit out formulas and things and stuff and I have never seen that before and if I did it was all of 11, 13 years ago so for me it’s difficult because if I did learn it, I don’t remember but I can atleast say that I have seen that but I don’t know how to do it but I’ve seen it. But in this I’ve never seen it before.

LG: So why did you structure your lesson plan in this way? So like the first sentence here, I kind of see it as an objective, is that what you planned for it to be?

P 26: Yes, you have to state an objective on what you plan the student to learn from the lesson. And so that is what I wanted them to learn from the lesson.

LG: Where did you get that from? You like why did you word it that way. Lol.

P 26: Because when you have to make a lesson plan in my theater class and my music, it’s how to teach theater and incorporate music in the classroom. In those classes you have to state your objective, what the students are suppose to learn from your lesson. You have to state it clearly on the lesson plan. So I kind of figured that was my way of stating the objective.

LG: So you have had a little bit of lesson plan writing?

P 26: Yes

LG: What do you want to teach when you, because I know a lot of K–2 is all subjects but if there were a couple of subject that you were specific in wanting to teach,
what would those be?

P 26: I would either choose music or science because I like those two. Math not so much.

LG: I don’t see how you can do science without math? Lol.

P 26: When you are talking about little kids you are doing the fun version not the upper grade levels where you have to do this math in order to understand this and this. No.no.no.no.no.

LG: Like Bill Ni the science guy kind of math?

P 26: Yea there you go. Because they’re going to understand a little bit of math by 2\textsuperscript{nd} grade, my daughter is doing it. And they understand more now a days than I think I did.

LG: Oh no we are moving it ..

P 26: So those are the two I prefer.

LG: You said this was going to be for 5–6\textsuperscript{th} grade?

P 26: Yea I don’t see a younger grade understanding it.

LG: So right here in this lesson you start off with having three functions and compositing with three functions. Did you start with three functions or would you start with gradually build up to three?

P 26: That would be like in the review, like on the day before, you know like I was saying you had to back up a little bit, rewind to make sure they understand all of this. You would show them the regular function and see how it works and you put 2 together and then you say today we are going to take all of these and throw them all together in a big jumble and then we are going to figure it all out. So that
is kind of how it was thrown at me. Yea we learned this last class period now we are doing this. So I guess that is pretty much what happened to me. And it makes sense whenever you understand how to place it, substitute for the variable, I mean it makes sense if you think of it as a variable, as a number. Like this whole thing right here let’s pretend it is a number throw it in there for now and then work it out. I think that is how I would describe it. Does that make sense at all?

LG: Yea Yea no I, I mean a lot of times people refer to functions as having an input and output and that is kind of like how you are explaining it, you are going to take this and put it into this function. That’s going to be the input.

P 26: Okay, yea, yea, yea.

LG: So right after here you wrote ‘the student will learn how to replace the f of x with the given value of x. now you said they will learn this, so how would they learn this?

P 26: We will have taught it before but now they will understand take this variable, this whole thing as one and stick it for x. That’s what they would probably learn in the one before but I didn’t write it that way.

LG: Is there any kind of instruction you would use for this to describe it?

P 26: Practice. Practice. Practice. You know they can see me do it, but then they would actually have to know, understand the concept of it, they should already, but without practice it is still going to look Greek. So I can describe it all day long but they are going to actually have to do it. Then we would probably go over any questions next class period.

LG: So it sounds more like you give examples for them to look at. For them to grasp
the concept through examples and then do some guided practice and independent practice?

P 26: And like have them help me work through a couple of problems after I showed them. You know be like ‘what’s next?’ you know. See if they get their answer and if they don’t it would give me an idea of why. You know then I would be like well if they are trying to give me this answer then why are they getting that. And I can explain where they are going wrong.

LG: You said you reflected on this or thought about it, so what would you change with it?

P 26: Oh let’s see. I thought about it afterwards like oh that was horrible. But at the time I was like aaaaahhh. I would probably reword it completely as far as like ‘in prior class period they would have learned how to do this and now we will take it another step further.” And use this for this, you know substitute, and then take that final thing and substitute it in for the next thing so then we will have a final answer. I would word it differently.

LG: So that is pretty much all I have. Do you have anything else you want to add?

P 26: No
LG: So you wrote down that you wanted to teach 4 through 8th graders. What subject would you like to teach?

P 28: Let’s see. I haven’t chosen a subject yet, so I am not sure.

LG: Do you choose a subject in middle school?

P 28: I have no idea if you do they didn’t tell me when I went in because they, I wanted to be an elementary teacher but they said that I wouldn’t have as great as a job opportunity as if I would have been a middle school, at the middle school area. So I chose to do the middle school area.

LG: Okay so you haven’t quite picked a subject?

P 28: No

LG: So what do you feel is your strongest that you think you would be able to teach.?

P 28: Like academic wise?

LG: Any of the topics.

P 28: Music was my first choice because I really wanted to become a music teacher, but I guess if I could choose something else I don’t know, history would be something that I like.

LG: Any particular grade level? I know you were just kind of thrown in 4–8th but…

P 28: I would really like to teach middle school, 6th through 8th or somewhere in there, maybe 7th I guess.

LG: First question is why do you want to become a teacher?
I think it has to do with my parents. My dad, at least, my mom wanted to be a teacher but she never finished getting her certification, but my dad did and I just saw how much he loved it, especially working with the little ones, so that’s kind of where, why I wanted to become an elementary teachers at first because that is what he was, elementary music teacher. Well he was an all level music teacher but elementary was his favorite.

LG: I remember elementary school music. It was fun. It was one of my favorite classes. Do you have any experience with the aspect of teaching?

P28: I don’t think so.

LG: You’ve never taken any classes that dealt with teaching, you never tutored anyone, or…

P28: Oh, I’ve helped out my cousin. He is in second grade right now, like with reading and stuff. Like helping out with homework but I haven’t like taught anybody or tutored anybody. I did work at a daycare for a summer but I just helped them … reading stories that is all I was doing.

LG: You haven’t gotten taken any of your classes that deals with teaching?

P28: No, not yet I wanted to try and take all the basics out of the way before I started doing all of that.

LG: That’s very smart. How about with your dad and his teaching?

P 28: No help.

LG: You never went to school with him?

P28: I went to his classes like when he use to come here for, Texas State, to take classes. I came here to some of his classes but I was little so I’m not sure if I
would remember anything.

LG: Okay. So for your lesson plan let me open this up so you can remember what you wrote right here. So you picked middle school is there any particular grade that you thought this would associate with better?

P28: Probably seventh or eighth. I was trying to remember when I first learned functions or at least something that was going into that.

LG: You don’t remember when you first learned functions?

P28: No to me math was all a jumble. I mean I was good then I had bad years and then I had good years, so yea.

LG: Are you in the good or the bad at the moment?

P28: Now I’m in the middle

LG: So maybe around seventh or eighth grade. What inspired your lesson about functions?

P28: I’m not sure actually what inspired it. I was kind of told to write it.

LG: That is true.

P28: I guess I tried to remember when I first learned functions. I tried to remember that and remember how like what defined a function or/and how did you figure out if it was function or if it wasn’t. Like the basic, easy ones that they, you know your teachers told you how to remember. So it wouldn’t be too complicated.

LG: And so, right here I have here where did you gain the knowledge of functions? And you said you don’t remember but you do have that knowledge in your head. So where do you think it came from?

P28: I guess my algebra class in high school. I’m not sure or the eighth grade going
into high school. They talked a little bit about it. I remember bits and pieces but…

LG: How about any of your college classes?

P28: The first class ever. The first math class I took here they talked about it.

LG: 1311?

P28: I think so.

LG: The developmental math class. The one right before the one you are in now.

P28: Yes. They talked about it a lot.

LG: So how did they present it to you?

P28: Very easy. Like it was kind of like a re-, how do you say it, remembering what I was told before because I didn’t take four years of math in high school. So it was kind of like a

LG: Recall

P 28: Recall of all the things I learned back in high school and like once they showed me how, it was like the function was, I was instantly, I started remembering it more and more, faster and faster.

LG: Okay

P 28: They gave a little more detail, I think about it which helped. For the most part I started to recall all of it.

LG: Why did you structure your lesson this way?

P28: I was trying to remember how teachers did it in high school and elementary, I mean, middle school and how they would ask you ok “what’s, what a function was?”’, you know, and “what is this?” and they would ask, you know, the students
to say what they thought it was. And then they would then go and write their own definition. The definition you had to write down and remember. And then they would start giving you examples of what it was and you would have to try to figure out which one were and weren’t and then they would make you solve it or write your own version of a function.

LG: Okay. So what do you think the students know before they were taught your lesson?

P28: Like if I was in a classroom you mean?

LG: Yea, like if this was a lesson that you were going to teach, what do you think they would already know before coming into this class?

P28: I think they would know that it was a… I think graph a function.

LG: Okay, graph.

P 28: Graphing. I think they know that it probably be a pair, like set pairs or something. I’m not sure exactly they know a whole lot, detail-wise but. I think they would know first of all it was like some kind of graph or something or could be a table of numbers.

LG: Okay. I did look through yours and I do have a couple of questions. Some of these we already kind of gone over with. So you wrote that “a function of the correspondence between set pairs to another set of pairs,” what does this mean to you?

P28: Where the set pairs line up on the graph and how they correspond with... I’m just repeating exactly what I wrote on there. How they correspond to the other set like what makes them… I didn’t think this was going to be so tough…[giggle]. I’m
not sure that kind of like something I remembered hearing my teacher say. I have
to actually see in order to tell you.

LG: Well here why don’t you draw it on here. It’s going to record what you write so
just go for it. So what do you mean by a function is a correspondence between set
pairs to another set of pairs [Using hand gestures, moving in shape of “C”].

P 28: [She draws a picture: look at livescribe notes.] I’m not even sure if this is right.
When they cross, do I need to draw a line?

LG: I’m just letting you go with it. You’re telling me. You’re telling me.

P28: Well I mean, if I remember correctly. Like what I had learned. opps…if I
remember from my math classes how they relate to each other, where they

LG: Okay!

P 28: Where they stand on the graph, where they do make a cross or whether they don’t
cross. I think I’m confusing what I am learning right now in class too though.

LG: What are y’all learning right now?

P28: Like the three, to put ahh linear equations with three variables; we just finished
linear equation with two variables.

LG: Oh! Systems of equations.

P28: Yea! System of equations, and I’m confusing them. I’m sorry.

LG: That’s fine. So after you give the definition of a function, it says then give
examples of what are functions and then you have two bullets you go “- x and y
do not repeat” and “- try vertical test to see if function or not,” so are those two
separate types of examples or is that all encompassed in one, like what kind of
example, would represent this?
P28: I just remember the vertical test like uh like they can see and come back. They
don’t like, how do you say umm, it doesn’t come back against the line again.
Like if it loops back around like if it crosses the vertical line again then that is not
a function. Or something. And I remember being told something about x and y,
one of them or both of them I can’t remember exactly, but they can’t repeat or
then it is either going to be a vertical line or a horizontal line and that’s not really
uh function. If I remember correctly.

LG: So it says how, try the vertical line to see if function or not. So how would you
demonstrate this?

P 28: Um I guess make a vertical line or either make a vertical line or use a piece of
paper and see if they will cross. Do the line. ____ told us how to use it.

LG: So you would actually give them pictures and ask them to determine…?

P 28: Yea. Give them a picture of a graph or something and show, see if at all do they
ever cross it back over again.

LG: But then it says then give examples and ask which are functions. It that different
from the previous?

P 28: Well I guess I meant like just continuing on from it. Seeing if they can tell now
which ones are functions and which ones aren’t or given or maybe they wanted to
draw a function or not draw it and see, okay maybe say yes this is a function, no
that is not.

LG: Okay. Your last part says, then solve for the ones that are functions. What do you
mean by solve?

P 28: I guess if I had given them, like uh I guess that should have been like more added
into there, like if I had showed them how to also figure out the equations, what a function was. And then given like equations along with the graph then they could try and figure it out. I never stated that anywhere in there but if I had that is what I meant by that. Does that make sense?

LG: Yea, yea. So having this conversation with me, I’m not for sure, did you re…. did you think about this lesson plan after you wrote it?

P 28: I did and I kept thinking about okay I didn’t make myself clear I am not even sure if I was even right when I was saying what a function was. Because I know sometimes my math can get jumbled up into one thing and I’ll just assume. Okay I remember what a graph, I remember this and then I start putting other ones in and I can’t. That is what is making me struggle in math right now. Is I keep putting in different math solving, I am trying to solve math equations that I know are not suppose to be solved that way.

LG: Well would you change this lesson at all?

P 28: Maybe a little bit, maybe. Being more specific on what I was talking about and my examples are, like how to tell if it is a function or not a function and as far as this last part maybe that could have been a different lesson plan. The last part, the solve for the __, because I never stated in my lesson plan how to even solve a function or anything so that is kind of ___.

LG: I think that is pretty much it. I don’t have any more questions on the lesson. Do you have anything else you want to tell me?

P 28: No, besides I don’t really know if this is even what a function really is.

LG: Well I think if you would of wrote, a function is a correspondence between set
pairs such that the x maps to one y or something. For a second I thought it was a
definition of a relation which is just a set of numbers or a set of points but then
you said between pairs and pairs and I was like well…so

P 28: Ok, I’m so sorry about that then.

LG: No it is ok.
LG: It’s not your first time taking 1315, but you didn’t have to take any developmental mathematics?

P29: nuhuh!

LG: And you want to do math/science composite?

P29: uhhuh!

LG: For 6th grade. You’re a dancer. Would you want to teach that on the side too?

P29: uhhuh, definitely.

LG: You don’t want to stop that. I know what you mean. Okay, so. Your lesson plan…let me, just so you can recall what you wrote last week. Here. [Hand original copy of lesson plan.] I will let you read over that quickly.

P29: Okay, I’m ready.

LG: So first, why do you want to be a teacher?

P29: Because I know I had ummm…difficulty learning, you know certain subjects and what not. And I feel that I know how difficult it is to learn a subject. The subject that I found most difficult would be math and history, so I would want to go back and teach those two things to the students, because I found it hard to learn those two things and I find that having a teacher that had difficulty in this subject, you know, there more able to connect with you on a different level.

LG: Okay. Do you think if you had difficulty learning it you would have a good understanding to explain it to them too?
P29: uhhuh! Yea!

LG: Alright. That’s awesome. Do you have any experience with any aspect of teaching?

P29: I’ve worked with children for… I’ve been working since I was 17, and I’ve worked with children the whole time and that’s the field I’ve stayed in. And I do after school programs, I do teach them a lot of different activities and a lot of different games and things like that.

LG: So, is your activities and games like educational activities and games?

P29: There more active, outside just sports and things like that. The more of um… I don’t really think that it is much of a difference of course I have to give them certain instructions and directions in order to do certain… things in the game, so I… it isn’t educational but it’s a different form of education. It’s not book educational but it is expanding your mind in the different activities.

LG: Okay! Besides this have you taken any courses or done any tutoring or anything like that?

P29: Just my job with Extended Care that I worked for. They just do a lot of trainings and things like that.

LG: You said Extended Care.

P29: Yes.

LG: For lesson plan. I don’t think there was a grade level. What grade level would you think that your lesson plan would be more specific to?

P29: Maybe third grade.

LG: Third grade. What inspired your lesson?
P29: I guess the actual test I took. It has a lot of different fractions and things of that sort.

LG: Now, I did mention to write a lesson plan on functions and you did on adding rational expressions. Is there a reason why you did that?

P29: To be honest I don’t understand the difference between, you know, the functions and this factoring.

LG: Okay! Where did you gain your knowledge of adding rationals?

P29: In grade school. I don’t know.

LG: So have you not had any experience in college.

P29: No just that 1315 math I’m taking actually.

LG: And they never discussed anything about…

P29: I guess very briefly at the beginning of the semester but not, they don’t touch too much on it.

LG: Not really in depth?

P29: Nuhhuh

LG: Why did you outline your lesson in that way?

P29: What do you mean?

LG: How you have an example then you explained it. What made you structure it that way?.........Instead of like maybe putting it the opposite way around or having several examples or anything like. What made you do it that way?

P29: I don’t know. I just find it easier to learn. If someone is given the direct information on how to do something instead of different examples on how to get one answer. I chose to do the actual number problem for the reason that it is math
and you work with a lot of numbers. So you are giving a visual understanding of how the numbers work together and then giving a brief writing examples will be helpful.

LG: Okay! What do you think your students know before they come in to learn that. I mean what did you teach them the day before or two days before that…to give them a foundation to learn that?

P29: The understanding of fractions… and how fractions work.

LG: Now you state in there that…right here you wrote… “and how to find the that you take the bottom numbers and see where they connect.” What do you mean by “see where they connect?” Because you use the word “connect” quite a few times.

P29: I should have been more specific. See where they combine…see where they overlap each other.

LG: What is an example of “overlap each other?”

P29: To find a number within another number.

LG: Okay. Over here you write, at the bottom, “to simplify you multiply the numbers out and cross out common pairs”, so what do you mean by “multiply the numbers out” and by “cross out”?

P29: That’s where I mean, where the um… where you find common numbers inside of one of them.

LG: Okay! And what do you mean by “cross out”?

P29: Find the pairs and eliminate them. They eliminate eachother.

LG: You said “if you” I think you meant can’t “simplify the ending factor leave it as
is. To simplify you multiply the numbers out and cross out common pairs. I
would take the class from step to step and explain why.” So, just your
background, why does that work? Why can you do that, multiply them out and
cross out common pairs?

P29: Because most of the time when one number is constantly, constantly used in a
problem, you are able to, you are able to eliminate them because of it’s just too
consistent.

LG: And you wrote down “I would not give them other options to do one problem. I
would pick the simplest way to do it and stick with that all the way through the
section.” Now why would you do that?

P29: Because personally I find it difficult to have, to use so many ways to get one
thing. You know. If you are able to get one thing with a certain (how do I say
that), if there’s multiples of way to get one answer then I feel that you shouldn’t
have to overwhelm them with all the options in doing so. If all the options come
to the same answer, why wouldn’t you just be able to give them that one.

LG: Okay. So having experience writing this lesson plan, did you reflect on it at all?

P29: What do you mean?

LG: Did you think about it after you wrote it, like when you left?

P29: Yes, definitely. I feel that I could of explained myself a bit better. And I
probably would of worked out the problems and instead of working out the
problem and writing a note. Like I did. I would have probably worked out the
problem and at each step that I did write a little tally mark to what I was doing.
And that is where I said “I would take the class from step to step and explain
why.” I think that is what I would have done. I would have taken the numbers and put into more ... larger...took of most of the paper, you know, and have them see visually and explain to them step-by-step why, while I was doing it.

LG: Okay. Well that’s all I have. Do you have anything else you would like to say?

P29: No
K.9: Participant 30

Date: 4/18/2012
Time: 11:33am
Duration: 11:20

LG: I just have a few questions for you about your lesson plan as well as your perspective on being a teacher. So um first question, why do you want to be a teacher?

P 30: Well because I want to make that difference like my teachers did in my life. Like I’ll always remember my kindergarten teachers and my 1st, 2nd, 3rd grade teachers. So, I want to make that difference.

LG: Not your high school teachers?

P 30: No they were more friends than teachers to me. My high school teachers were anyway.

LG: ok well that is good. And why did you decide to choose elementary school?

P 30: Because they are cute. That and the making the difference part too. And I was in PALs so. It’s like we would go to the elementary schools and like sit with a particular group of students and help them with subject that day. So I liked doing that and they were like my kindergarten and my first grade kids.

LG: Well I guess that brings me to #2, [haha] do you have any experience with the aspect of teaching?

P 30: Oh just that PALs program.

LG: So what is PALs?

P 30: We have to get accepted by, like we have this questionnaire thing and we have to get accepted into doing it and then every week, three days out of the week we go
to an elementary school in our home town. We only have six so there are two
different classes and we get three each, so we go to the elementary schools and
you get paired with someone and you go to that one classroom like that particular
grade and the counselor assigns you the grade so we just like help them read or if
they are having trouble reading then we will help them read the words out and
stuff or like if… one time they were having a spelling test and we have to help
them, like a pre spelling test kind of thing, and stuff like that. Just to help them.

LG: So would you call that like a tutoring aspect or?

P 30: Kind of yea. I guess in a way but sometimes it would be like we are going to take
them to recess today or … it was just kind of like a break for the teachers kind of.
But I enjoyed it so…it was kind of like tutoring for like the 4th grade kids and 3rd
graders, like they have to take the TAKS test and stuff so sometimes it was like a
tutoring thing. Like they’d ask how to do math and stuff like how would you
figure out that problem, like how would you divide, stuff like that we would have
to teach them.

LG: Okay so you’ve had experience teaching like all different types of subjects?

P 30: Uh huh!

LG: There’s one in particular?

P 30: Nahuh

LG: So if, I know some of the younger elementary schools it’s, you have to teach all
subjects so what subjects would you like to teach?

P 30: Probably reading. I like reading. So math not so much. I’m not a big fan. But I
mean I liked reading them books and stuff. Especially the kindergarten kids like
they would just listen and they would like, “oh” and like so verbal and cute. I
don’t know. I thought it was fun. They were so adorable.

LG: So this lesson that you wrote you chose 4th graders?

P 30: Yea because I think teaching math to 4th graders would be easier than teaching
math to kindergarten kids because I mean what would a kindergarten person have
to learn about functions. The 4th grader might have to like to get the basic
knowledge of it maybe.

LG: You didn’t go into too much detail about a function but I was just curious on your
knowledge. Umm do you know what a function is?

P 30: No, kind of but if you were to tell me then I would be like oh yea I remember that.
haha. But it’s math, like I don’t care.

LG: So you don’t remember any aspect of functions?

P 30: Well I know like f(x) is a function, right? Like it’s a line that’s a function, stuff
like that. I remember that kind of stuff. But ..

LG: So what part of functions do you think they would understand? Because you said
you would show them examples and give them problems. So what do you think,
how much detail would you go into?

P 30: Not so much. Not like hard stuff. Just simple. Not too in depth on it. Like a
skeleton kind of. Just a brief kind of explanation.

LG: You said you would go into different kinds of functions, so what different kind of
functions were you referring to here?

P 30: I don’t know to be honest.

LG: So the format here, the way you structured this lesson, why did you structure it
this way?

P 30: Because this is how I’ve seen people, like other teachers, kind of structure it first
they kind of get what it is, then they will give you like an example, and then you
have to try it. Then like now they will go over the example with you, and then go
into more detail about it and then give you like an assignment on it. Then go over
the assignment with you so you can have like a homework assignment for that
night. Well that’s what my teachers would do. I’m just basing it off of what my
teacher did.

LG: So that is what inspired your lesson?

P 30: Yea.

LG: So you know you were telling me a few things about functions about uh knowing
a line is a functions and f of x, where did you gain that knowledge from?

P 30: In college

LG: What particular class?

P 30: My algebra class that I am in now.

LG: Did you only take algebra here or did you take another class?

P 30: I’ve only taken algebra here. I mean in high school too but it was one of those
blow off classes. The teacher didn’t really care.

LG: That’s good to hear. (Sarcasm)

P 30: It is true.

LG: So before the students come in to learn this lesson on functions, what do you think
they will already know before they come in here?

P 30: Probably how to solve something or something. I don’t know. What should they
know before that? What did I know before that? I guess how to uh I guess how to like what a graph is, how to set up a table and stuff like that maybe like that kind of stuff. I’m not sure.

LG: There is no wrong answer.

P 30: Ok.

LG: So here you wrote go over the worksheet as a class and discuss why they got them wrong.

P30: ‘If they got them wrong’ is what I should have put there. Because I know that it will be, I think it would be kind of hard to give them something just after learning it that first day. Because I mean I know I use to have trouble like if a teacher just taught us an assignment then we had, taught us something and then we had something to do, I would forget everything. I don’t know how to do it. So it would be like going over it slower I guess, just step by step by step. To show them like this is the first part and this is the second part and this is the third part and this is your answer. This is kind of like what I think. Stuff like that.

LG: So for somebody that struggles with math, what do you think would be a good way to teach them?

P 30: Just for me, because I struggle with math, I just like when they go slower and they explain why, not just like, okay well this is it. But why? Where did you get that from? Well I would tell them that this is how and explain why this is this from that.

LG: Okay. Now I know you have only had a few minutes. I’ve given people usually like a week to think about this, did you think about this after you wrote it?
P 30:  Yea.
LG:  Do you think you would change anything about it?
P 30:  Uh huh, I would probably change the last part. Just not they got them wrong but if they got them wrong, kind of. And just explain to them this is why and maybe not go, maybe know what a function is before I try and teach it.
LG:  That would be good.
P 30:  And then instead of like maybe, um going into detail just kind of like a brief kind of thing. Like not too much detail. I’m pretty sure they are not going to need that much detail on functions in fourth grade. So just like, so they can be like oh yea I like remember learning something about that before, just not in depth.
LG:  So you are kind of giving them a foreshadowing?
P 30:  uh huh.
LG:  Well I think that is all I had. Do you have anything you would like to add?
P 30:  No.
LG:  Again, what was the name of the group that you were in in high school?
P 30:  Pals, P-A-L-S.
LG:  And do you have to be a senior to do that class?
P 30:  That is a junior and a senior and I did it both times.
LG:  Okay, Did it help you anything with um how you would teach?
P 30:  That is the reason why I wanted to be a teacher first of all. I wanted to be a speech pathologist but then I changed my mind real quick after I got into that class. It was just like a whole different, I didn’t know, I knew I was a little kid once but just the way they were and how I saw their teachers like with them I like wanted
to do that. Like I want them to. Like the way I remember my teachers. Like I
remember learning my ABCs cause of my teacher and like having to count cause
of my teachers and stuff like that. So I don’t know. That is why I wanted to be a
teacher.

LG: What grade do you want to teach?

P 30: Kinder and first. Like that’s like when they are really young because they get out
of hand when they are older. My third graders are crazy like when we take them
to recess they like get too rowdy for me. Um I don’t know they will teach you
how to whistle and everything. I was like oh no. It’s not for me, not this grade.

LG: I guess you have to be really patient when they get to that age.
LG: So first of all why do you want to be a teacher?

P 31: I want to be a teacher because I like maths and would love to go into coaching too. Be able to coach a sport.

LG: Ok so um what kind of sport do you like to coach?

P 31: Football, basketball, baseball, track…

LG: Oh so a little bit of everything huh?

P 31: Yea that is what I did in high school so.

LG: Is there any other reasons besides liking content and wanting to coach?

P 31: Teaching school, also, not just coaching, you know.

LG: Do you have any experience teaching?

P 31: Nah, I mean I have, I had to take a field day at an elementary school, and after they let the person in the classroom take care of their kids and um like just to have them there. And I did a pretty good job I guess.

LG: Did you get to teach them anything?

P 31: No I didn’t get to teach them anything but I told them like just be have and like stuff. It was cool. It was fun.

LG: Did you get to play any games or anything?

P 31: Nah! They were just sitting down and like I didn’t know what to do. I didn’t know. Like it’s cause the teachers went out to lunch like they had a field day or whatever. So they put us in classroom like me and my friends and they like just
told us to maintain them there, like if they did their work they do their work but if they didn’t it’s okay. I was like ok.

LG: Have you ever tutored anyone?

P 31: Well I helped one of my friends before in math and that is about it.

LG: So whenever you. So is this your first semester to take math here at Texas State?

P 31: I took one last semester but it was like the intro to math or something.

LG: So the one right underneath college algebra, 1311?

P 31: Yes mam

LG: Ok. So why do you enjoy math so much?

P 31: I just like it. I find it interesting. But I also think a good professor equals to a good student. Or a good teacher = a good student.

LG: Okay. I’m guessing you do well in math?

P 31: Sometimes

LG: Even though you don’t make all perfect scores, you still feel…?

P 31: I still feel like I am good at it. I just need to have someone explain to me real good in order for me to get it real good.

LG: So you choose freshman?

P 31: Like I didn’t know what level so I just chose freshman.

LG: Okay so you feel like people that younger might not understand functions?

P 31: Well no. It’s not that they won’t understand it. You just need to learn how to explain it to them in order for them to get it.

LG: What inspired your lesson on functions?

P 31: Well what do you mean?
LG: Why did you decide to do, to set it up this way, where you start the lesson with …?

P 31: Like I just start it and ask them questions. To see if they get it and what they don’t understand or anything like that. And then just try to help them out afterwards, I mean explain the lesson.

LG: So what, is this how you usually see classes taught or?

P 31: Uhhuh, like the way it works?

LG: Start the lesson, yea like the way it works?

P 31: Well no the way my professor does it, like this semester, he talks about what he is going to teach. And then he does it on the board and does examples and like shows us how to work through it. And that’s what, well that is what I was trying to say in the paper but I don’t think I said it pretty good.

LG: There is no wrong or right answer here. So where do you think you gained the knowledge of functions?

P 31: Like here or back in the old days?

LG: Whenever.

P 31: Back in the old days in high school.

LG: So you didn’t really put any specific details about functions, in here, what specific topic on functions you can teach to freshman?

P 31: Just the basics. Just the basics of functions, and then from there whatever else they need to learn or just take another step ahead or something.

LG: So what kind of examples would you give them?

P 31: First real easy ones. In order for them to get it, then from there just, as soon as
they start understanding it more then start going to harder and harder more
difficult.

LG: Do you think you can give me an example of a question you might ask about
functions?

P 31: I would probably just like teach them about a table oh is this a function or
whatever. Like do you think this is a function. And from there start the lesson or
something. I mean they should at least know what a function is or you know or at
least the beginning of a function.

LG: So before this lesson they should already have…?

P 31: At least a little knowledge of what it is.

LG: What else do you think they should know before they are taught this lesson?

P 31: Like the x’s and y’s stuff like that.

LG: So why did you choose freshman again?

P 31: Because I didn’t know what grades, I mean, I didn’t know a specific grade so I
chose freshman.

LG: Okay um. So you said I would have my freshman in high school pay attention.
How would you make them pay attention?

P 31: By getting their attention towards me, like teachers, so of course you are going to
be like “oh hey listen up” or something.

LG: Okay. Um. So it says just start on the lesson. What would maybe be like the first
thing you do on the lesson?

P 31: Probably just ask them like, if they know what a function is. Probably on the
board give a little example about it and let them figure it out or something. And if
they don’t know or if they do, I’ll just start from there. It just depends on what
they know.

LG: Ok so you trying to assess their knowledge. So you wrote the next day you would
help them understand better.

P 31: Yea the next day go over problems and try to figure out who needs help or like
what other people don’t understand it, who does understand it. And from there
just go.

LG: So how would you clarify if they didn’t understand from the way you taught the
prior day. How would you clarify?

P 31: I would take my time and go with each student, whoever needed help, like take
my time and go desk to desk or something. And then if they needed help I would
help them and try and make them understand better. Like person to person or
something.

LG: So like more on an individual basis?

P 31: Yes ma’am.

LG: Umm did you think about this lesson after you wrote it?


LG: No you didn’t think of it.

P 31: I didn’t think if the lesson or anything like that.

LG: Umm do you think you would change it after we’ve talked?

P 31: Maybe, more than likely.

LG: What do you think you might add to it or take away?

P 31: I would probably add more detail to like me explaining everything or me put more
detail on what I am going to try to do to help them.

LG: Ok. Well um I don’t have any more questions, so do you have anything else you would like to add?

P 31: No I’m good.
LG: So first, why do you want to be a teacher?

P 32: I think it’s for me like throughout my whole life I always thought about what I would change like in the classroom, what I would do different if I was a teacher. I think every class I would ask myself that, even in college. I like it.

LG: What do you find is your answer?

P 32: Well, just like different types of like I guess like forms of like I would be different, like teach them in a different way or I don’t know. I also like the teacher’s schedule.

LG: It nice, especially if you had kids from what I understand. I don’t know.

P 32: Yea and like the summer vacation. And also it would give me the opportunity to be a high school soccer coach. That is like one of my goals.

LG: So I’m guessing you play now or you played in high school?

P 32: I played in high school.

LG: That’s cool. Do you have any experience with teaching?

P 32: Not really. I was in this organization called TAFE in high school. Like it’s about teaching um other, younger schools, like elementary schools and read to kids. Or we would go to like events, and dance in front of little kids and teach them a dance or nursery rhymes. And I guess that’s far…

LG: So nothing too formal?

P 32: No not yet.
LG: Did you ever tutor anyone?

P 32: No

LG: What was the main purpose of this organization?

P 32: If knowing you wanted to be a teacher or not.

LG: It’s just introducing you to the idea of teaching?

P 32: Yea. We would shadow teachers sometimes.

LG: What did that all entail to write lessons or you were part of anything?

P 32: No we were just like a student aid during our elective hours. We would grade paper sometimes.

LG: So you wrote a lesson on functions. Here so you can skim over it to see what you wrote. So why did you choose 12th grade?

P 32: Because that is like the last um… I guess I like put myself in like the position of like. How would I don’t know. Like the last class that I remember. Then I can um like. How can I explain it. Like put myself in like the scenario of like high school. Being like a high school teacher. You know what I mean. I don’t know if that makes sense. Because I want to be a high school teacher. That is like the last memory I had that I could like compare being a student versus being a teacher. And I had a really good math teacher.

LG: What math class were you in in twelfth grade?

P 32: Pre-cal

LG: Do you think people learn functions before 12th grade?

P 32: I don’t think so. I remember learning it in 12th grade but not before.

LG: What inspired you to write your lesson plan on functions?
P 32: Well right now we learned, I’m in college algebra, and we recently learned about that. And my teacher he was pretty good at teaching functions. It wasn’t really hands on. Like compared to my lesson plan I mean. Because you know how college is just like lecture. Versus high school where it was more like, they would like provide a calculator for us like a graph to dry erase.

LG: On here it said you took the developmental mathematics course here at Texas State, so um what kind of experiences did you have in there with functions? Did you have any?

P 32: Umm I don’t remember. Yea I don’t remember.

LG: Did you do any kind of activities in that class?

P 32: Not really no. it was just like lecture, homework, online quizzes and tests.

LG: Ok, so. Here you wrote you would definitely do graphs and or grids to look at visuals. What’s the difference between graphs and grids?

P 32: Well um like grid um like the grid it’s like a grid paper like the notebook. And the graph, well like um when I mention a graph it’s like on a calculator or like in high school we had like the dry erase, dry erase grid I mean graphs. And those are very helpful.

LG: Um so you said you would use those as visuals to show what functions look like. So what does a function look like? What kind of graphs would you actually give them?

P 32: Parabola, square root function, cube functions, common functions

LG: The common functions. You also put, you also incorporated word problems um easy enough for graph but difficult to solve. So um first of all what type of word
problems would you do? And um why do you say easy to graph but hard to solve? What do you mean the difference between those two?

P 32: Well like hard to solve the equation, like hard to actually get like uh equation, cause I know he did that in my 1315 class. Where we would have like word problems like if someone works x amount of hours and gets paid like y amount of money or whatever. And then we would have to make the equation and then solve the equation. And easy enough to make a graph to like understand the answer at the end, like see…

LG: So would they do the equation first then the graph?

P 32: Yes

LG: Um what type of word problems would you use?

P 32: Like real life, like realistic word problems.

LG: Do you have any real world problems in your class, currently, or in any of your of your college classes.

P 32: Yea we are having word problems right now. We are learning about matrices. We have word problems about, I know we had that was about, like if someone paid at a store with $1.75 with nothing but coins, how many nickels did they have and how many dimes. And stuff like that.

LG: Mkay. Would you use very similar problems or would your word problems be different?

P 32: Probably a little different.

LG: What do you mean? What’s the difference you would think?

P 32: Probably something that would be able to describe like, like if I, like if the answer
was to be like uh like the square root function, that’s like the line, I already forgot how they look. Something that would like describe like my graph, like if the answer would be like increasing and something that would be decreasing and like I said job and the hours of, how many hours of work versus how much money you would get. And stuff like that.

LG: So here you wrote another thing I would also do is give my twelfth grade class equations and have them use candies or small circular objects, so they could form their answers on the grid so what do the candies or the small circles represent?

P 32: Umm nothing it would be like instead of drawing them. We did that in my twelfth grade class and like the whole class enjoyed it. Like it wasn’t like just boring like, like in the calculator stuff, so it is kind of like, like a more hands on umm problem. I think that is like earlier easier to understand.

LG: What would they do with the candies though?

P 32: Like graph.

LG: Like you put them all in a row or something?

P 32: Yea

LG: And what do you mean form their answers?

P 32: Form the graph.

LG: So you also wrote, so they would do this to also understand what functions are and what functions are not, so what would be the difference between the functions and the things that are not functions?

P 32: Well like passing the vertical test and the horizontal line test.

LG: So what, what inspired you to come up with this type of lesson?
P 32: Um give the teachers that don’t have these kinds of stuff it works.

LG: It has worked for you. And you like, what aspects of this worked for you?

P 32: Um like having an actual graph and graphing it myself. Like you having a graph and an equation like instead of like a calculator and an equation, you know what I mean. I guess I understood it more when I didn’t have a calculator because I didn’t rely on one. Because I had to learn how to graph.

LG: Where did you gain the knowledge of functions?

P 32: In my, the last time I remember is my 12th grade class and a lot in 1315.

LG: So in 1311 they didn’t talk about functions?

P 32: Umm I don’t remember. I know there was a lot of equations. We did like linear equations and stuff like that. We did.

LG: Before you teach this lesson to the class, what do you think they need to know before they came in?

P 32: They would probably know how to solve equations.

LG: Anything else?

P 32: Probably would know how to put uh, like um form an equation from the word problem.

LG: So did you reflect on your experience, like after you wrote this did you go back and think about what you wrote?

P 32: Right now I did.

LG: Um do you think you would change anything about it?

P 32: Umm I would probably be more specific like um like I would have probably answered like all of the questions that weren’t very clear like explaining more like
what a grid was um being more specific on about how they would use the candy.

Um maybe also stating that many of these examples like I used them on my

previous math teachers. And that they would helpful for me.

LG: Alright that is all I have. Do you have anything you would like to add?

P 32: No
K.12: Participant 33

Date: 4/19/2012  
Time: 9:41am  
Duration: 13:01

LG: Why do you want to be a teacher?

P 33: I want to be a teacher because I like kids, I like helping. Just knowing the fact that I’ll have such an impact on a child’s life like possibly take them on to a continued education and possible become a teacher as well or something bigger like a doctor or something like, have a good life I guess you know. Take them to where, it’s like you are getting them started to like become something bigger so it’s like a good thing to see them come to, pass you by it’s like you are a little part of their life and you would be such a good role model and a good impact. I don’t know. I think that is like the satisfaction knowing that I taught that kid and they are in college now. Like I’ve talked to some of my teachers before and they’re like happy when I tell them that I’m in college and their like “oh wow” not a lot of people would like do it and stuff like that. That is one of the main reasons.

LG: Have you had any experience with the teaching aspect?

P 33: No no I haven’t.

LG: Have you not taken any classes, no tutoring?

P 33: No

LG: So in your lesson plan you chose 9th grade. Why did you choose 9th grade for this?

P 33: I chose, I don’t know. I mean I would communicate better with older kids. I mean I like little kids too but I just think it would be better to like teach older, I
like not older, they are not that old but either way. They are young but they’re…

LG: What grade level did you put for certification?

P 33: Uh I think I put down 9th and that is what you want to do 9 -12th.

LG: Um I know 9–12th is more made discipline specific, so what subject would you like to teach?

P 33: Well possibly, not math, like a science or social studies, something like that.

LG: Not math?

P 33: I’m not a strong math student.

LG: What inspired your lesson?

P 33: I don’t know just uh remembering the math course that I took just, that is how we usually do it. We would… since I’ve started school.

LG: Which math course are you remembering it from?

P 33: The one I remember is the one from 9th grade.

LG: Did you start off here in developmental math or in college algebra?

P 33: Developmental math, 1311.

LG: Did that have any impact on your lesson plan?

P 33: I’m sure it did because huh. I had professor and he explained like every step so yea it did.

LG: Did it help you with your college algebra class?

P 33: Yes definitely.

LG: So in 1315, the course you are in now. Did ya’ll talk a lot about functions?

P 33: Yea we started off with, they are still in there I think. I mean we have so much stuff that I, it’s kind of hard to remember but I’m sure we touched functions.
LG: So I know we talked about hearing it in 9th grade and a little bit in developmental math and college algebra, but where did you think you actually gained your knowledge of functions?

P 33: Honestly I don’t know.

LG: Why did you structure your lesson in this manner?

P 33: Well I don’t how to do a lesson plan yet, but I just went along on how I seen teachers teach. I guess that is the big one.

LG: So usually they start with like a definition, some examples, go over it, and then usually give them a quiz the next day.

P 33: Uh huh. Yea

LG: Umm before students come in and learn this topic. What are you already expecting them to know?

P 33: Uh well it depends. Um of course like division like their maths like multiplication like the ones you are suppose to know by the grade level I am sure if I was a teacher at the time I’d probably know what to expect from them but right now I can’t really say.

LG: Okay um. So let me see here. What’s this part over here um on the side?

P 33: I wrote that like just little notes.

LG: Oh so you kind of outlined it first?

P 33: Yea.

LG: Ok. ok. I see that now. So you wrote give definition, set of outputs. Is that your definition for function?

P 33: I think, it is sort of like a set of output and inputs and stuff like that. So yes.
LG: So what do you mean introduce the material properly? Is there a certain way to only…?

P 33: I mean, I would think, if uhh, if I like knew exactly how I should explain how what a function was I would try to see um how other people teach it as well and remember how I was taught it about it and um just look up like the, I would just show them like my way and I’ll show them how like the ___ teach them and stuff like that. That is what ___ means just combine all the elements to where the kids learn it using several methods. Just give them options. See which works for them. Because I know if you give me options and I see which one is the easier one for me. If it works for them then it is good.

LG: Okay um. You just mentioned uh if you knew how to teach functions, where do you think you are going to learn that?

P 33: Uh I don’t know.

LG: Are you expecting it to come from a class?

P 33: I mean I think I will do my research as well, I mean if I was majoring in, if I was going to be a math teacher. I think I would have to uh I would probably know this already. You know but um yea I think it is from the classroom. I mean that is how teachers are. I mean we’re taught to teach. You know sort of.

LG: So um if you go into science or social studies, um do you feel like you have that knowledge already or do you think you still knowl… you know, you need to grow the actual content?

P 33: Um I am a big social studies, um I really like that one, I think that is the one I would want because I’m a history buff. I like history. I think it is because I really
like it and I know a lot of it. Um like in class I’m always paying attention because I really like it and so I know a lot of it already and of course I will have to learn more because I would like to teach them as well, like just be familiar with the material but I think I would be exposed to that already.

LG: Now here you said I would demonstrate what a function looks like and then says give practice examples. What do the practice examples look like based on…?

P 33: Oh I don’t know I was oh like show them how the problem is and like put numbers in, instead, you know just like plug in and ask them to try and solve it.

LG: So here it says you give out a worksheet, is your class based on this worksheet or what is the worksheet used for?

P 33: I would think it was like, my math teacher now, she gives us like worksheets and we go through them it’s like our notes at the same time, so. There are problems given out and we work with them, we work on them together and then she gives us certain questions to where we can try on our own and then we go over them again and um you get like more practice on them and so ya.

LG: Okay. Um do you have that same kind of experience in the developmental math class?

P 33: Uh yes, yay.

LG: Okay. Do you get to do any hands on stuff in any of your classes?

P 33: Uh, no not really.

LG: Not really. Now you said to allow students to participate in class discussions. Do you feel like discussions are very important in the classroom?

P 33: Yea because I mean, if one person has a question and they see it, the answer like
oh, I had a similar question too. It just like bring them out and everything. And so it helps the teacher to know, like oh my students are struggling on this and stuff like that.

LG: Now your quiz the next day is that going to be your assessment like that is what you are going to look at to see if the students are struggling?

P 33: Yes it, the quiz will be just to see if they understand the material and if they don’t understand the material of course I would like go over it again. And of course go over the quiz to see if they were doing fine in it and if they have any questions as well. And then uh, I would see based on the test, like on the quiz I think I would offer help like you’re doing this wrong and just like a little bit different and stuff like that, so, yay.

LG: Okay, So having this experience writing a lesson plan, I know I didn’t give you a week like I have given some people, but did you reflect on this at all?

P 33: I mean I thought about it a little but I wasn’t too sure how to write it down or anything.

LG: Ok do you think you would change it after our talk at all?

P 33: A little but I still sort of unsure like how to write a lesson plan. I haven’t been exposed to that, having to write one and stuff like that. I have no clue how to write one

LG: Ok. Well I think that is all the questions I have so do you have anything you would like to add?

P 33: Oh no I am fine.
K.13: Participant 34

Date: 4/19/2012
Time: 11:40am
Duration: 12:34

LG: Why do you want to become a teacher?

P 34: I love kids, and I love helping people in general. At my church I am a somewhat teacher. We have 2 services and it’s a really, really big church and they have like their own building and I take care of the 2’s sometimes or the 3 year olds and we have a lesson plan and we have a snack time and recess time and that just gives me an inside how innocent they are and, you know, how like they are like sponges they take in what you say and it’s fun.

LG: Well you said you work at your church. Do you do any other, do you have any other type of experience with teaching?

P 34: Yes I’m a violin teacher in Austin. Um I work 5 hours a week and I teach 3rd, 4th and 5th graders and they are a handful.

LG: What do you have to do to prepare for your lessons?

P 34: I don’t really know because it’s kind of open like they tell me, the director tells me like what song they want to learn and so I’ll practice it on my own and when I got it down then I’ll teach them like different, measure by measure that way it is not like an overload for them. And then after like a week they have it all and then we rehearse as a group on Fridays.

LG: So you do individuals? Is that what you just said and then just mix them together?

P 34: No. The 3rd, 4th, and 5th graders, there are two classes. There is like the advanced and there is like the beginners. And the advanced there are like 4th and 5th and the
beginners there is like between 3rd graders and 4th graders, and so.

LG: So besides those two do you have any kind of tutoring or nothing else?

P 34: No, no.

LG: How long have you been doing the violin?

P 34: Playing myself, 7 years. Yea it’s…I love it.

LG: So on your lesson plan you didn’t really specify what grade levels do you see?

P 34: I don’t know even know what grade level. I don’t know like 5th 6th graders. I don’t remember elementary school that well.

LG: Is that what you wanted to do, is elementary school?

P 34: Yea. Either elementary school or high school English but I haven’t decided yet.

LG: There is a big jump right there. You do not want to do anything in the middle grades?

P 34: No that is like their worst stages. Like uh I know I was a big handful in middle school so I don’t want to like me going around so, Jesus.

LG: So um what grade in elementary do you think? Lower one or upper one?

P 34: No uh mid I would really like 2nd or 3rd grade. Um I did an internship my senior year at *** in Kyle and I was a third grade assistant and I loved it. I loved it so much. Because they are not like kindergarteners where you have to teach them like the bare minimum from scratch. But they still. I don’t know like I compared them to the 5th graders. I went to go watch my friend and her classroom she did 5th graders and 5th graders are so like rebellious and. I don’t know. But I really like the 3rd grade and 2nd grade.

LG: What inspired your lesson on functions?
P 34: Actually the packet. I could not like think of anything and then the packet helped me.

LG: There was a particular problem that helped you in there?

P 34: Yea it was like one about a teacher talking about how like functions only have one set and like …

LG: Where do you think you gained your knowledge of functions?

P 34: I don’t know. I’m not good at math at all. I am pretty sure it was in middle school. His name was Mr. ****, and he like went over somethings and he went over something and he was like y’all are like going into high school. And I’m like ok. I liked him.

LG: How about your class now? Do y’all talk about functions? So, You are in 1315 right?

P 34: Yea, I don’t know, we have been everywhere like literally. From like logs to like quadratic equations and I guess a little bit. I don’t know.

LG: Did you start here in college algebra or a developmental class?

P 34: I took, I guess, a developmental class with ****.

LG: Did y’all talk about functions in there?

P 34: Yea.

LG: Has any of your classes, or any of your experiences in your college classes have any influences on your lesson plans?

P 34: No. not really.

LG: Why did you structure your lesson plan in this manner?

P 34: Because I do not know how to do one. I just kind of like put all the information
there. I’ve never even seen a lesson plan even when I was assisting a 3rd grade
teacher. Like I never saw like a lesson. I had to do a lesson but like my teacher um
I was taking, I don’t know what class it was, but like my teach in high school she
helped me and um she didn’t really like tell, show me, she just gave it to me. She
is like do this, you know.

LG: And what did you have to do?

P 34: I had to like read a book about butterflies and like how they become a, like they
are caterpillars and then they come into cocoons and all that stuff so.

LG: And what was the moral of that lesson?

P 34: I think it was under science. I don’t know. They were going over. I don’t know
what they were going over. I don’t remember. She. I talked to the teacher and she
and wanted me to do that one.

LG: Before teaching this to your 5th and 6th graders, what do you think they should
already know how to do?

P 34: Probably like just basic math like of course adding, subtracting. I don’t really
know like what they are teaching now because I remember when I was in
elementary. I didn’t learn how to divide and multiply until I was like in 4th, 5th
grade. And then 6th grade is really hard because I struggle with math and um I’m
pretty sure they have to. Knowing basic multiplication and division, and fractions
and stuff so.

LG: Um…So…On the first lesson you said to review with students what a function
is—a number with only one pair—is that your definition of a function?

P 34: Yea I couldn’t remember the exact definition but yea.
LG: So what do you mean an number with only one pair?

P 34: Like you can’t have one number representing like 2, two of the numbers on this side, I don’t know how to explain it but I don’t remember like. It can’t be the same number right? On this, on the right side (looking at Table). It’s like if you have one you can’t like 1 and 2 because the same number right?

LG: Yea, um. And so you drew, is this your example you were talking about?

P 34: Uh huh.

LG: Can like two numbers over here (left side) can go to the same number over here (right side)?

P 34: I don’t think so. I don’t, I don’t think so.

LG: So you said go over some of the examples. Would the examples resemble what you have written here? Or what kind of examples would they be?

P 34: Yea. I was thinking like that, uh, going over like because in the packet it had different examples as well. It had like two numbers on the left side representing like five numbers on this side here (right) and having them say whether they are the correct function or not.

LG: Okay, you wrote let them work in groups to you find that’s productive to let them work together?

P 34: I think so because you can ask them, like ask your peers, like questions and get their feedback as well or how they interpret it too.

LG: So why did on your homework, you want them to develop the functions instead of you coming up with the functions and seeing if they are or not?

P 34: Yea um cause my sister does some stuff like that, like she will have them create
their own math problems and see if they are right or not, um, which I think is a good way for them to like really understand if they are doing it right instead of like going home with already functions and just like, like determining whether they are or not. I don’t know.

LG: So are they suppose to develop their questions and also give the answer to them or…?

P 34: No, like this one, like just put like numbers and like you know I don’t know how to say it. Like put the arrow like with one number like and the next number would be another number.

LG: So they are actually suppose to develop functions, they are not suppose to develop functions and non functions?

P 34: Yea

LG: It’s not going to be yes or no, they are just developing functions?

P 34: Yes

LG: Okay. Um having written this lesson plan and our talk, do you think you would change anything about it?

P 34: I don’t know. Probably like once I get more knowledge on how to write a lesson plan and more knowledge on math, but yea I probably, yea I would.

LG: You said more knowledge in math, so you have taken college algebra right now?

P 34: And the lower one. And like math is still really hard. There are like sectioned parts of like College Algebra that I do understand really well but then the next section is like something that I really struggle with and the next one I get it and the next one…It’s like I either know this section or I don’t. Um yea. I need to get
stronger in my math.

LG: Ok. So I really don’t have any more questions. Would you like to add anything?

P 34: No I am good.
K.14: Participant 35

Date: 4/22/2012
Time: 10:41am
Duration: 12:14

LG: First question, why do you want to be a teacher?

P 35: I’ve always loved kids and I’ve helped at vacation bible school and Sunday school classes at my church back home and I realized it is so exciting when a kid learns something and it’s so great to be a part of that. And I just want to be a part of that for the rest of my life to help them learn and grow into whoever they are going to become.

LG: Was anyone an inspiration to you?

P 35: Probably just people I worked with at my church and high school teachers who work with kids on a regular basis. Um, some teachers I had in first grade my teacher was very inspirational because she had been teaching for 30 years and you could really tell she loved the kids and loved incorporating activities to help them learn and I saw that more as I got older because my brother also had her as his teacher and once I got older I saw the way she treated all the kids. She was…

LG: Um, So did you pick that you wanted to teach PreK–6 grade?

P 35: Yes

LG: Is that one of the reasons why you decided to do younger grades?

P 35: Yea working with the kids that I did. I’ve worked with kids from pre K all the way through middle school and then of course I was around high school kids when I was in my youth group but I realize that I just like younger kids better. I just work with them better and get along with them better I guess. I’m thinking
that I probably want to teach somewhere between 2nd and 4th grade.

LG: Okay. Besides the Sunday school, do you have any other experiences with the aspect of teaching?

P 35: When I was in high school, the last 2 years, I was a student council class officer and we went and volunteered at an elementary school and I got to help a kindergarten teacher and I got to read to the kids and help them do some of their stuff. And that was fun and I also got to help with a 3rd grade class at the same school and then besides that I had nanny jobs where I have worked with kids and helped them do their homework.

LG: What grades did you help in the elementary?

P 35: Kindergarten and third grade.

LG: That is what I thought you said I was just making sure…There were no other programs at this school, it’s just this part of student council and this was your volunteer hours? Is that what it was?

P 35: Yes

LG: For the lesson plan, why did you decide to choose 5th grade?

P 35: I was kind of unsure about what grade they actually start introducing functions. just because. Although I did volunteer help, I never was around when they taught math. So I was thinking that it was probably higher level concept, you know besides just multiplying, adding you know all of that. And so I chose this because I thought that would be probably be the earliest they would introduce functions and it’s just a simple concept of what is a function, what makes something a function, not really getting into solving them and all that.
LG: What inspired your lesson on functions?

P 35: Um I just thought about getting down to the basics of it, you know what is it, what did they look like, trying to distinguish what a graph of one would look like and what isn’t a function and then the values that you, seeing a chart and seeing the x and y values, what would make it a function that way. Just pretty much the basics of it.

LG: Where do you think you gained the knowledge of functions?

P 35: I think mainly, I don’t have the best memory, but I just remember mostly in high school and uh this year in college algebra. I mean we obviously went into more detail with them but that’s what I mainly remember talking about them.

LG: Did you,… is ‘em…, did you take any other classes besides college algebra here?

P 35: Not yet

LG: So you didn’t have to take developmental math or anything. You started right into college algebra?

P 35: Yes

LG: Why did you structure your lesson this way?

P 35: I just know for me it’s easier for me to see something first to know what it looks like, what I will be working with so that is why the first thing that I thought would be good would be to just, you know if I was teaching maybe draw a picture or have it on the screen to show them pictures of graph to tell them this is what we are going to be looking at to familiarize them with what it is even. And then go into more of the numbers and what makes it look like that but I just know for me and math I need to see things before I can fully understand them.
LG: So what do you think they should know before they go into this class? Like what concepts should they have already learned? Could be like the day before, year before or something, what do you think they would need to already know?

P 35: Maybe just like a basic understanding of how different equations are structured and just how the variables work, of $x$ and $y$ on a graph. Because if they don’t know that then it will be hard for them to understand how you need one $y$ value for the $x$. If they don’t understand those concepts then it will be hard for them to understand that. But besides that I mean maybe just, to have seen graphs before, what different lines look on a graph but like I said I mean I kind of just went basics with the function. So it’s not like they need to know how to solve them or anything beforehand.

LG: So right here you wrote ‘I will go over what the graphs of different functions look like so the students can see what it will be doing.’ So what do you mean ‘so they can see what they will be doing?’ I know you kind of explained it a little bit earlier but what do you mean by this little phrase?

P 35: So once I would have find them some homework that says you know tell me if this is a function or not. They will be able to see right off the bat what they are going to have to be doing to understand it. So if I show them a picture of something and say this would be a function, this would not be a function. Later on when they see if they will understand what they will be doing to figure out if it is a function or not.

LG: Okay so do you mean it is more like uh like it is just starting them off, like the visualization of what you are trying?
P 35: Yes

LG: So you give some example here, so why is this one not a function?

P 35: Because if you did the vertical line test it will hit that vertical line twice.

LG: Ok. And over here you said this one is not a function. Why is this one not a function?

P 35: There are two values that are the same for the y.

LG: You mention charts here, what are charts?

P 35: I just meant like these, what do you call them, I guess tables would have been better.

LG: And you wrote “etc,” so what other types of representations do you think you would be talking about?

P 35: I think it would mainly just be the tables and graphs. Just a bunch of different examples.

LG: So having written this and us discussing it a little bit, do you think you would have changed anything about it?

P 35: I think maybe first I would kind of view the concept of x and y values, just to make sure that they understand that concept. Especially if it is something that they haven’t looked at in a while, just um kind of revisit that so that they can understand that before moving into, well this y value needs to be, correspond to this x, so that they know that before we go into that. I think that would probably be the only thing.

LG: Okay would you like to add anything else?

P 35: I don’t think so.
K.15: Participant 36

Date: 4/22/2012
Time: 11:34am
Duration: 21:09

LG: So first, why do you want to be a teacher?

P 36: I technically want to be a coach but that is a teacher so I kind want to learn, not learn but I want to be able to teach someone how I’m able to, … and visual. So it’s like some people since you just go by fast, I want to be that teacher that’s like oh make so she like made so much sense. I remember her from years to come and kind of help break down things. Like give back. Let me give someone an easier explanation then what I have ever gotten.

LG: What kind of coach do you want to be?

P 36: Track and field and cross country.

LG: I know coaches do have to teach a couple of courses, so what kind of courses do you want to teach?

P 36: My minor is going to be mass communications, so more kind of like your media, computer classes and elective classes per se.

LG: Do you have any experiences with the aspect of teaching?

P 36: Yes, I do. And a lot of my coaching classes are, well as far as my major we have teaching developmental classes, such as coaching track and field. I’ve done coaching gymnastics. I’ve also been a part of a summer league so I was like coaching then and um I taught dance classes, private lessons, I’ve taught swimming lessons to kids and kind of like developmental swimming techniques. I kind of have a lot of aspects I guess.
LG: What a sport?
P 36: Yea
LG: Was that your thing in high school?
P 36: Yes, I was constantly doing everything. My days would start out, even before
school started we had cross country and, then I was in school and then get out and
go to practice for running, then I would go to swim team practice then I would go
to dance and then I would come and work on my 4H animals. That was like my
Monday through Friday.
LG: So you were in 4H as well?
P 36: Yes
LG: What kind of animals do you have?
P 36: I showed steers and pigs and goats and we also had to do baking too on the side so
it was like … that was one of my moms, she was like, that was her, she was like if
we are going to do this that will be my input is the baking.
LG: I did 4H for…it wasn’t too long because I also did FFA. It was just too much
time.
P 36: Yea, FFA is a lot more time consuming, so that is why I didn’t so FFA in High
School was because I did outside dance, and outside everything else, outside
swimming, and everything so it was like in order to time consume everything else
4H is a lot less. It’s like meeting once a month. Ok you are here, here are a few
shows go to these, that is about it.
LG: Yea, well they were the only ones that did horses.
P 36: Yea, that is true.
MEL: They don’t do horses in FFA. For some weird reason.

P 36: Yea that’s true.

LG: Now you were saying that your mom teaches?

P 36: Yes she taught for 20 years, math and science and for the past three years she has done remedial math at the junior high level, 7th and 8th grade I think.

LG: Has she always been in the middle school grades?

P 36: Umm 5th, 6th she has a K–12 certification but her primary grade is 5th–8th more 5th–7th. She doesn’t necessarily prefer it but I mean that is what she can handle so that is where they put her.

LG: So did you ever get to go to school with her or see her work?

P 36: Oh yea. That is how it started out I wanted to be a teacher when I was younger because I was in love with my kindergarten teacher, she is a family friend and everything, and I learned so much so I was like I’m going to come back and be your student teacher and as I got older I watched my mom just get like more frustrated as years went on because I started understanding that maybe teaching is not all that fun and I watched her just get like frustrated but at the same time as I got older, now, like being where I am I liked looked at my mom and she gets frustrated and she’s gotten all the gray hairs from all the years and everything, but at the same time like I really I’m grateful and I’m kind of more inspired by the way she teaches because she teaches very much something like, we requested not to have each other in school but the way that she teaches is very understanding for any kind of students. It breaks it down very visual and she kind of like, since she is remedial math so she kind has to break it down so it is understandable and I
was just kind of like “wow” that kind of shows me something about the kind of person she is and the kind of teacher that takes pride in her job and wants to see the end results instead of just like through the testing and it’s just another day at school.

LG: So what kind of? How does she teach that it breaks it down?

P 36: Um she is very visual um I guess it is kind of the way she explains it. She is very like on the white board because whenever I would come home with homework and stuff, I even still Skype her still to this day with math homework, but it’s just like she will be like I don’t know she will be like I don’t know so she will put the problem out in like terms and I guess she knows how I understand things so she is able to be like this is how this goes and she kind of explains, step by step by step for me because that way, you can show me a problem, write it down and I can get it but them I’m not going to, give me another problem and I’m not going to be able to explain how I got it but she explains why this goes to this one and why it is equal to this. Since I’m able to understand it and explain it to other people.

LG: Does she use a lot of pictures or hands on stuff?

P 36: Hands on, she is very hands-on. Very hands on and with pictures but more hands on so than pictures.

LG: So, what grade level do you want to teach at?

P 36: I would necessarily be in the high school. I guess that is another thing that she showed me too that junior high is really intense.

LG: Hormones are starting to go through them?

P 36: Yea! And she text me the other day and she was like, we were talking about my
degree plan and she was like I’m getting too old to breaking up fights anymore.
And I was like, what kind of fights. It was a bad fight. Apparently there was like a
guy on top of a girl choking her and I was like mom, what are you around?

LG: Where does she teach?

P 36: In north Houston.

LG: So for your, oh what grade level was your lesson plan?

P 36: I guess I could do junior high, freshman I guess.

LG: Why did you choose that grade level?

P 36: Um I guess that is kind of like the grade level when you start getting introduced to
algebra. That is when I first remember 7th, 8th grade and the way that, that the
lesson plans I did the one to one and the junction function. I just remember that
being in middle school them writing that on the board. It was like the best way to
explain it because then these are just like the developmental steps that to carry on
to more advanced functions because you must know the basics 1-1, the input
output functions, I just felt like that was a good starting point at that age. Pretty
much that is where you get started with algebra.

LG: Ok, when did you learn the knowledge of functions?

P 36: Technically I guess I just learned it in junior high when I actually like learned it
learned it was probably freshman/sophomore year of high school.

LG: What kind of math courses have you taken here?

P 36: The one before algebra, then algebra.

LG: Did you learn anything about functions in those two?

P 36: Yes. It’s kind of more like you learned it, it’s an overview of the 1-1 and from
there it was more developed. Do you know what I mean?

LG: Through which class?

P 36: Uh, I mean of course we went over it in 1311, but I actually didn’t understand until 1315. I understood it, it just made more sense in 1315.

LG: So what inspired your lesson on functions?

P 36: The simpleness of it. That it’s easy to break down for pretty much any age. Uh I guess maybe even probably maybe a 6th grader could even understand the concept of 1-1 and the input/output.

LG: So why did you structure your class this way: the objective, warm up, lesson?

P 36: Well throughout the things I’ve always known that like you want a, when making a lesson plan you always have one main goal and that is kind of like your objective for the students to like learn like of course you have your lesson going into your, like you are going to teach this today, this tomorrow, but you have an objective for each day. For a week let’s say you want to work on linear functions, but day 1 you’re gonna wanna, your objective is gonna be how to use progress and to teaching them that, objective 2 would be the next day so it’s kind of like a normal lesson plan objective—explain how they are used. I should have done another explanation on that, what are functions, that should of gone in there but and a warm-up to start out like, you can’t just expect students to jump into something and them understand it so. A warm-up is kind of a basis start, a basic start. You kinda want to see where they are at, if they understand it and a warm up will be able to tell you if they get it or if they don’t get it. So that way if you know that by the warm up you can start your actual lesson and how to teach it to them.
and how they will. How they’ll be affected by it and if they will actually get it because the warm up you will be able to tell you know if you should explain it more, if you can explain it less if they understand it and then the cool down is kind of like, okay make sure they retain the information ask them simple questions like ok do you remember this from the warm up and now since we went through the lesson do you understand from the warm up that the lesson showed you this and this. And I mean it is the same way to like the lesson plan is the same way you would teach, you would coach, you always want to start out with the same kind of basics you can’t just expect someone to know something right away. Or how you want it to be taught. And how you want them to learn.

LG: What do you think students should know before they learn this?

P 36: What should they know before it?

LG: Like what kind of base knowledge should they have before they learn this?

P 36: That x and y coordinates. And how to look at a graph… And point coordinates, I guess too, so that way being able to look at it.

LG: Now your objective was explained how functions are used daily?

P 36: Daily/ in-life and I meant to do, I meant to add like explain how functions are used and explain functions.

LG: That’s okay. In here you don’t mention anything about it though.

P 36: Hmm well I meant in the sense like explain here, this would be like before you start your warm up explain what how they are used and explain like what they are then go into the like warm-up. Sometimes my brain thinks faster than I write, so it’s like I write and go back and like ummm. That is like mid-sentence in my brain
and I expect you to know what I mean.

LG: No it happens to me all the time. Okay so are they suppose to know what functions are, because you say ‘ask if they think these are functions?’

P 36: So they don’t already expect it. I mean would you, why not objective. okay you explain how functions are used kind of give a wide basis of what they are, really quick like a definition on the functions. And then given them that, and then in the warm-up, I’m not expecting them to know it. The warm-up is suppose to be how they think they can, kind of like a self get, solve and check, kind of scenario, that’s what I’m not saying the warm-up, I’m not expecting them to know it by all means. The warm-up is just to show me if they do and if they don’t, how I need to go on and explain the one to one and the output like the warm-up is the starting point for me to be able to see how much they know already.

LG: Now these examples you gave me you didn’t label if they were functions or not. So did you mix them up or what, or how would you do that? Which ones are functions and which ones aren’t?

P 36: Well yea that is what I am saying, I would give them this and be like is this a function set? is this a function set? is this a function set?

LG: Now I am asking you which ones are functions?

P 36: Yes, no, no

LG: Why are those no’s?

P 36: Because the y’s are repeating to the x. There are multiple, you know nevermind. They can’t be more like, there can’t be more right, there can’t be more than there can only be one, its one to one so only the x can go to one y.
LG: So ‘explain how the functions are 1 – 1 using coordinates,’ so are all functions 1-1?

P 36: No. can be. Can be/if they are.

LG: So what’s this function junction thing? Where did you learn that? What does it mean?

P 36: My teacher showed me that one, you know it was like whatever your x input can be whenever you put it into a scenario whatever you put, whatever the x input is it has to be the output, the output has to be x. so whatever. It’s kind of like the a = b, whatever a is has to still come out an equal, b has to equal a. Like whatever still happens in this middle which would be the kind of like your function, whatever happens in there still has to come out and equal x. That is just a quick little intro. I get how to explain it but it would be better if I had like an example to show you by putting x in.

LG: You mean something like that [drew picture on paper]. How to explain it?

P 36: If you’re like x is 3 put it in whatever…oh.no.no. You want it to equal three don’t you? Or whatever x is still has to come out as x even after this.

LG: So I see, what if x is equal to 3?

P 36: So then no matter what this is, it still has to come out and x equals 3 here. So whatever input for x you put here has to come out and the output has to equal x.

LG: Okay. So this here you had how they felt about learning functions?

P 36: So the kind of feedback you need as a teacher. Kind of like how they felt you taught your lesson like if they felt it was hard, they understood it. That is kind of like the developmental feedback, motor learning, you kind of have to like need
that feedback to know how to assess the next day and expect the lesson that you taught to see if they actually retain any of it by the way that you taught. And this review part is pretty much falls back into, it’s kind of just like a big cycle, it falls back into the warm-up. This time you can ask and be like so why is it the functions, tell me why this isn’t a function, so that it still pertains to everything back from the warm-up to the objectives to the lesson and it is a full understanding. And that way you can know if you can move forward or not of if you have to go back the next day. And go over this in more depth. For those who didn’t get it.

LG: Okay. Last thing, having written a lesson plan and us talking about it, would you change anything?

P 36: Definitely for sure. I mean course this was just like a quick review of but if I was actually going to do it for a teacher or something I would go through and change what I have here.

LG: How would you change that?

P 36: Be more, obviously the objectives be a little more descriptive on what I’m trying to say and the warm-up be a little, I would definitely go back through certain areas and be more descriptive. It was, the point I was trying to make as far as each, the lesson, the warm-up, whatnot.
K.16: Participant 37

Date: 4/25/2012
Time: 11:11am
Duration: 11:33

LG: Alright. So first question, why do you want to be a teacher?

P 37: Well I’ve wanted to be a teacher when I was little. And as I grew up and my teachers that I had in high school inspired me to be one.

LG: Were you like good at any topics?

Interrupted…

LG: Do you have any experience with the aspects of teaching?

P 37: Yea, in high school I took a class ready set teach, and they took us to elementary schools and, two days of the week, and we were suppose to sit there and teach and little kids and teach complete lessons for every subject.

LG: For every subject?

P 37: We were suppose to choose one, whether it was math, reading, whatever and create a lesson plan based on what they were doing in class.

LG: Which one did you do?

P 37: I did one about language arts, it was a poem lesson and I like teach them like shapes in the poem and like based on the shape they were suppose to make like a poem with it. Like create it. For 2nd graders.

LG: So you had to write the whole lesson plan out and teach it?

P 37: Uh huh.

LG: How in-depth was your lesson plan that you wrote?

P 37: It was not that long. I mean they were little kids and they were suppose to be
timed. It was about 20 minutes and plus they had to like cut out the shape and make the poem in time.

LG: So you were kind of connecting math and language arts? Those kind of shapes or like geometric shapes?

P 37: It was like any shape, like well it was a leaf, a cloud, a tree. Yea and when I was going to do a math one but when they were doing math I was in school at that time, I was in school at there school at a certain time a day only and during my class period. It was in language arts.

LG: So what grade do you want to teach?

P 37: Elementary, like either 1st or 2nd, or if I decide to switch to math like 8th grade.

LG: So first or second then if you switch to which?

P 37: A math major teach 8th grade or 9th grade.

LG: Are you good at math? Do you like it?

P 37: Yea I’m good at it like homework and quizzes and stuff like that is just when it is a test. It’s just like every other subject I get there and I’m like uuhhh. I’m going to do what I remember first. It’s like I freak out but yea.

LG: A little bit of anxiety?

P 37: Yea. Definitely math since 8th grade.

LG: So on your lesson plan you chose 9th grade. Why did you choose 9th grade?

P 37: Because the functions like back, I remember, I took it I remember back when I was a freshman, how the teacher thought of like functions.

LG: Do you think functions can be taught at a lower grade?

P 37: Yea probably like 8th grade. Like the sooner the better the students get to see,
I have more experience with it then.

LG: Do you talk about functions a lot in your class now in your college algebra class?

P 37: Yea that’s like what we’ve been doing lately, were like doing functions, solving like systems with substitution, elimination and all of that. And then we have jumped into matrices. Actually my test I took yesterday was over that.

LG: So what inspired your lesson? Is it just from your 9th grade class?

P 37: That and what I was doing earlier this semester.

LG: Did you take the developmental math class here—1311 or 1300?

P 37: 1311.

LG: Did y’all do any functions in that class?

P 37: Yea it was like basically a review from all of that functions from all ___ and all that.

LG: So based on these classes how did they teach you functions in the college courses that you’ve taken in the 1311 and the 1315?

P 37: They are pretty much the same. Like how to solve them and everything. It’s like the same stuff I’ve been doing since 9th grade.

LG: So I have where did you gain the knowledge of functions? But you kind of already said that. So from 9th grade on?

P 37: More like 8th grade because the teacher was telling us we would be doing that. It was way after the TAKS test and all that.

LG: Why did you structure your lesson this way, where you have objective, procedure, and stuff like that?

P 37: Because, when I was doing lesson plans for every other grade in high school, like
creating them and getting ready to teach class. They were like in that order but I completely forgot like the rest of it. Like what goes after. It was like the lesson plan, the grade, like the procedure. There was like something else but I blanked out about it.

LG: So before you teach this lesson to students, what do you think they should know prior to coming into this lesson?

P 37: They should know how to get, how to get like a function to be this way because you know how sometimes they are like switched like it’s $x + y = \text{some number}$. They have to like know how to like make it into $y = mx + b$ format. And how to find the solutions to the line. Ordered pairs and all that. And how to graph it as well.

LG: So how do students demonstrate knowledge? It says ‘students will demonstrate knowledge on how to find solutions to the functions. So what do you mean by demonstrating knowledge?

P 37: They will demonstrate how to find, how to locate a solution on the graph.

LG: So what are they given? So do you just like give them the function and they just find the solution or just part of the solution?

P 37: Yea you give them the function and kind of like when I said a different format and they kind of like solve it and make it into this put it into $y = mx + b$. Know how to find the slope of the line and then the $y$-intercept. And then how to plug in different values from the tab, from the graph to find out the like $y$ or the $x$. Like if they went to plug in 2 for $y$ they’d have to find $x$. If they plugged in 2 for $x$ they’d have to find $y$. 
LG: So when you say ‘the students are going to graph the function,’ what are they going to graph it based on?

P 37: Based on knowing the slope and the y-intercept. Like the ... of the spot. And for the slope they have to then graph it which is 5, (5, 0) right? Yea. And then they will go on from there right. Like if the slope is 2 they will go up 2 over 1.

LG: And you say student should be able to identify various points on the graph. Are their specific ones you want them to know?

P 37: Yea like where is, for example, like if I give them the line where is this number located. If it is on the line or not on the line.

LG: So how would you, so this is the procedures sounds like it would be more like how you would show them in class, would you have the students do any of their own work?

P 37: Yea.

LG: Like would you give them worksheets or work in groups?

P 37: Yea like a worksheet and then with the problems and the graph so I can graph it.

LG: Having experience writing this would you change anything about this lesson?

P 37: Yep probably if I looked over it like in the future and probably change some stuff. I mean when I take those classes that teach you how to make them a lesson plan it would probably add more and make it more clear.
LG: Why do you want to be a teacher?

P 39: I just never found anything else that I was really interested in. I liked, criminal justice is my major and I really like explaining what it is, defining things, teaching other people about criminal justice.

LG: So what grades do you teach with criminal justice?

P 39: I think just high school and college.

LG: I don’t know a lot about the criminal justice program, do y’all have a lot of math or statistics or anything like that?

P 39: Not at all.

LG: You don’t have to take a statistics course?

P 39: Just one that is it.

LG: Do you have any experience with teaching?

P 39: I do not

LG: So you never tutored anyone? No high school programs?

P 39: Nope

LG: Is your minor education?

P 39: No not yet I haven’t officially decided yet. I am still on the fence about it.

LG: So if you were teaching what grade would you want to teach?

P 39: I would definitely want to teach H.S.

LG: So for this you decided to pick elementary school for your lesson plan? Why did
you pick elementary school?

P 39: My knowledge of functions is not very broad so I figured that would be the easiest for me, plus elementary, I feel like is a more definition of what it is as opposed to in depth problems about it.

LG: How do you see functions? How do you define functions?

P 39: I probably couldn’t give you a good definition. I’m not good at math at all.

LG: What kind of pictures would you show?

P 39: Because you said show graph of functions? I thought pictures of almost like lines that you see like mountains or something, you know like the lines that you the lines that you would see.

LG: What inspired your lesson?

P 39: I don’t know. Like I said I’m not really into math. It’s actually really hard for me.

LG: Why did you outline it this way though, you decided to do definitions first then show pictures then give equations?

P 39: I definitely thought it would be easier showing the definition, telling you exactly what it is first and then showing you, I’m very much a visual learner so I thought that showing would help you be able to notice if it was or wasn’t a function.

LG: Do you know where you gained knowledge about functions?

P 39: Just in a normal high school math class.

LG: Have you talked about functions in your college algebra class?

P 39: I’m sure. I just. Functions are my least favorite in math my least favorite. (giggle) that’s one of the things that I really struggle with.
LG: Before teaching students functions, what do you think they should already know?

P 39: In a sense of.

LG: Like what kind of other content areas I mean, do you expect them to, like I mean
the lesson before this?

P 39: I would definitely expect them to have an understanding of how an equation
works. And how to work just a simple equation.

LG: What do you mean by working a simple equation, what do you mean by working?

P 39: Just be able to do simple addition, subtraction, multiplication, division.

LG: So you said ‘explain the things that are not function and why they are not for what
requirements they do not meet” so are you just telling the students this stuff right
here?

P 39: Yea just explaining you know what exactly, what requirements need to be met for
it to be a function or … if … or showing a picture that might kind of look like a
function but might not be a function and you know, explain to them why exactly it
is not a function.

LG: Now this one ‘working and involving,’ is this like a group thing, an individual
thing, classroom thing?

P 39: A worksheet kind of thing.

LG: Having done this now would you change anything. What would you do to
prepare to write a lesson plan?

P 39: I am not actually sure I haven’t taken kind of, you know, teaching classes yet so I
really don’t know how to formulate one at all. That’s just what’s at I like bullets
it’s just what seems easiest for me. I don’t feel like I need to explain, you know,
write down exactly what I am going to say or anything like that. Just need bullets to keep me on track.

LG: More like a list or an outline?

P 39: Yea
LG: So why do you want to be a teacher?

P 40: I have always felt that I am good for being a teacher. My mom is a teacher so I’ve kind of grown up around that and always been her little assistant so I’ve seen it. And I love it. And I love kids. So.

LG: What grade level?

P 40: Kinder to 2nd

LG: Are those the same grades your mom teach?

P 40: She use to be a reading specialist and then she did first grade and now she is doing fourth. So did 1st grade in there for a little while.

LG: Do you have any experience with the aspect of teaching?

P 40: Um I have had previous leadership positions on my dance teams. That has giving me a lot to do and just being around her and not like.

LG: No high school programs?

P 40: No not like a specific program. I’m gonna. I have a job and I’m going to be teaching next year for a dance class but not education wise.

LG: So with dance, are there any kind of, are you going to make up what you are going to have to teach them in dance? What kind of rules would you have?

P 40: What I had before, you know you would basically make like a lesson plan, you know, it was my senior year and I was like captain and my director was pretty new so I pretty much led the team. So pretty much I’m there, I making a
formation for a dance, choreograph the dance and then teach the dance. And then you have to clean the dance and of course perform it. So.

LG: So you decided to do 7th grade, why did you decide to choose 7th grade?

P 40: Huh because I would never want to teach past middle school and then I think I remember doing functions in middle school. You know like the basic, I did like a basic thing like the tables, then like going and putting it into number form and stuff.

LG: Why did you outline your lesson plan in this manner?

P 40: I didn’t know how to do it. It just seemed the most effective way for me to read it out.

LG: So what inspired the lesson plan?

P 40: Um the way. If I didn’t have like a lesson plan that is how I would go up and teach it.

LG: Now you just wrote down a ‘general definition of a function’? Do you know the definition of a function?

P 40: I mean there is the definition of a function like 2 pages in front. Lol.

LG: How would you explain functions?

P 40: I mean um. You are going to have, I just know the biggest rule because I mean, the subject really doesn’t make all that sense but I just know the biggest rule is that you can’t have it go to more than one number.

LG: Ok. So what type of example functions would you give them and non-functions?

P 40: Ones that are correct, that go to just one specific number kind of switch the up, switch up numbers and then make the ones not correct and make sure they
understand the difference.

LG: So most of them would be like a mapping where you have two circle and values in it connecting or any other type of examples you would give them?

P 40: Yea. um if I knew more about this I could go on but I don’t.

LG: Well you mentioned down here function tables, so is that a function table you were talking about?

P 40: Uhhuh.

LG: Ok. Ok. What do you mean ‘you put function tables into other forms’?

P 40: Oh! So just kind of like going off if that is basic, whatever you know is next.

LG: Ok the continue section/chapter/lesson are you talking like same day, previous, I mean?

P 40: Probably the next day, 7th grade, yea I would do it the next day. After today.

LG: So section/chapter/lesson are you referring are all of those the same thing?

P 40: Um like section or chapter or lesson and so you are going to put it from a table into a to number form or problem form or whatever. Making it more complicated as it goes.

LG: What do you think students should know before they come into learning about functions? Like what basic principles do you think they should know before learning this?

P 40: Um I think they should know linear equations, right? And maybe slopes, just like a lot of graph stuff. Ok.

LG: Um having written an lesson plan, what would you have changed about it, or would you change anything about it?
P 40: I mean I haven’t learned how to do one yet, so.

LG: Most of the time I give people a week to think about it before I ask them. I was trying to rush the process this last week. So I know you really haven’t had too much time to think about it?

P 40: Yea I am still in my basic classes so …

LG: So what class are you in right now?

P 40: I’m in 1315

LG: Did you ever take 1311?

P 40: Yes

LG: Where do you think you understood the concept of functions? Has it been taught recently or? I know you said you think you learned it in 7th grade?

P 40: I feel like I’ve known about it for a long time, I just, I don’t know. I …usually with math I just usually focus on what I need to know for that…for those few weeks and then everything else and then I have to refer back to it to remember it.

LG: Do you think your experience in your current class different than the 1311 course?

P 40: Oh Yea!

LG: How is it different?

P 40: Bc I knew everything in 1311 already.

LG: Was that more of a review for you?

P 40: Yea basically I guess I’m not a good test taker and didn’t do good on my SATs so um I had to get through that class.

LG: So you did very well in the developmental math class?
P 40: Oh yea!

LG: Do you feel like it prepared you a little bit for the ..?

P 40: Um maybe there were a few things that I haven’t, I haven’t really known different ways how to do things but.

LG: In the college algebra class or the 1311 class?

P 40: In the 1311 class but I mean.

LG: Now I know you probably talked about functions in both classes right?

P 40: Uh huh Do you remember talking about functions in either class?

P 40: No, not specifically no. I think I have my math spiral with me lol!

LG: Ok let me ask you another questions, since you are wanting to teach K–2 and most of those are all subjects and you are going to need to know mathematics before you teach, right? So how would you prepare for those classes if you don’t feel that strong in teaching math?

P 40: Uh I mean I don’t remember… Do you have to teach functions in second grade?

[LG: some aspects] Some aspects of it. I don’t know. I mean usually the way, if I’m going to teach something I need to like review over it so this is like pulling me out of nowhere, so [giggle] I mean I would be obviously more prepared for that.
K.19: Participant 42

Date: 5/3/2012
Time: 11:22am
Duration: 9:56

LG: Why do you want to be a teacher?

P 42: Because I love kids. And I think I’m really good with kids and I think I’m really good at teaching little kids stuff.

LG: Do you have any experience with teaching?

P 42: I taught, I coached gymnastics like 3, 4, and 5 yr olds. Like 3 years so that teaching yea. Like getting their attention and stuff like that and my mom is a teacher. So like going up to her school like hiding underneath her desk. I would like pretend to be a teacher.

LG: What does she teach?

P 42: Well she doesn’t actually like teach a grade. She, my mom started her own job for elementary schools. She doesn’t like really have a college degree. She started her own job and it, um. She works in a really poor school, like a lot of the kids are from poverty. My mom is basically like a counselor but a counselor to the max like she is the person who reaches out to like churches to get like food or clothes or stuff like that. So she’s kind of, it like her own little job she started but now she started that business and there are two other schools that same worker have hired people.

LG: So what grade is your lesson do you think your lesson is aimed for?

P 42: Little kids because I want to teach little kids. So I did more of a game activity so they weren’t like bored with you know. Even then I probably wouldn’t be
teaching functions at little, when little kids.

LG: What inspired your lesson plan?

P 42: Um I wanted to do just a game or something fun. Not really boring, not worksheets, not you know. I feel like a lot. I’ve been trying to do my final for my music class and I find that a lot of the activities I find are really boring and really, like done over and over again. So I wanted to do something that they would have fun and run around and be laughing and stuff you know.

LG: Your music class you have a final project?

P 42: Yea and I like have to find a whole bunch of different lesson plans and it’s been really hard for me online to like find lesson plans like I don’t think are boring. So that is why I wanted to do a game or something. I feel like I haven’t really seen a lot.

LG: Have you had any success at adding stuff to the lesson plan and make them exciting for a chance?

P 42: Some stuff like the art ones yea and stuff like that but some of them you can’t get away from boring subjects.

LG: So do you think you can make a boring subject fun?

P 42: Yea with like really hard work. It’s just so hard because you have to have plenty of good ideas. You know, so it’s like. Right now I feel like on the computer you don’t really find any good ideas. You really have to like, you know find the root of something and you are like I will take that and add more. You know, like with just some imagination.

LG: Where did you gain your knowledge of functions?
P 42: From the FOCUS program.

LG: Did you not get any experience in high school?

P 42: Some but not as much as the FOCUS program

LG: So why did you structure your lesson plan like this?

P 42: Umm because it’s the grade I wanted to teach to me in easy and fun way to get their attention and get their minds thinking and their minds working.

LG: Why do you think you wrote it like ummm like in paragraph form?

P 42: Well yea it is like a game so I guess I could draw you pictures but. (giggle)

LG: Did you ever see your mom write a lesson plan? Did she every have to do that?

P 42: Yea, and the lesson plan, they are obviously not like that, like the ones for my music class, they have the TEKS and objectives you know.

LG: Why didn’t you think to that for this one?

P 42: I don’t know. I just it seemed a game would be okay, game/activity.

LG: So the other lesson plans are only structured for non-activity type lesson plans?

P 42: Um some of them are yea but some of them aren’t, no. But like, for instance, like in my, there is a science one I’m doing for my music class because we had to do, it’s for my like music class, we had to have all different subjects like for each day. Each day of the week, so some of my activities are game based you know like exercise in stuff you know and using your brain while you are doing that some of them are strictly like you we might have a book you read and them make like an art project from the book or something you know something like that.

LG: Before you teach a lesson over functions like this, what do you think the students already know before they come into class?
P 42: I think they should have already been introduced to, you know, because that game they are kind of like testing their knowledge on because they have to tell me the one I picked. They have to tell me what it is and then hopefully they are in the right corner and then they get to keep continue playing. But they should already gone over it in class like 2 days before that so they have some sort of idea.

LG: So explain the game to me, because for a second I thought the functions were on the ball and then it toss about…?

P 42: Yea and then I’m going to have functions labeled about the room and

LG: So like on the walls are like pictures and the actual name is on the ball of how does that work?

P 42: They would just run around while music is playing and then when the music stop and they would find a corner to run in fast. And then I’m gonna, you know, one that comes down on my finger and say ‘ok what kind of function is this? Look at it.’ And have everyone look at it and then you know and then um hopefully they guess right. And if they are in that right corner then they get to continue playing. So it is kind of more of a luck game not so much uh. If you are in that corner somehow you…

LG: So it is kind of like musical chairs where they all go to a corner with a function name or something and you draw out a function and they have to…?

P 42: And if they are in that corner they get to continue playing. And this is so they can like test.

LG: So if they continue playing does that mean no one else plays?

P 42: No I mean I think that I want them to be, cause I would play that game when I do
other stuff just like general corners, corner 1 corner 2 you know. And I would always like make sure all my kids got to play. Got to go back in and stuff.

LG: So it’s a group…more than one kid got to be in a corner?

P 42: Yea it’s a huge group thing. Sometime it’s just luck of who’s in that corner you know. I like doing that kind of stuff because I feel like that stuff is actually fun to little kids. You know. Like all my kids like hated when we would just go over stuff like when we would like just even though in gymnastics they would hate when we just sit there and be bored and be talking. When they would rather be like hands on playing something, doing something, you know. That’s why I want to do that.

LG: Do you think classrooms would be better?

P 42: Not classrooms, I mean I would have to take them like you know, somewhere. And I did that for ********, we had to go for like our health class, we had to go to ********, it’s like a preschool, and we had like a huge lesson plan like filled out and the beginning lesson plan it’s like we read and then there was a drawing aspect, where we had to draw about healthy food. And then there was an activity aspect, and where there I planned a caution rally took them out to the auditorium, lined them up, got them ready and had them in the lesson plan by playing teem in an activity or something like that or some kind of like fun game. You know.

LG: Do you think students learn better whenever they’re doing an activity or whenever you are just telling them the information?

P 42: I think way better with an activity. I mean it’s how, it’s better for me, I learn a lot better by having…and growing up I didn’t, I feel like I didn’t have activities as
many as I should have, I had a lot of worksheets and that is not learning a lot, you know.

LG: Before I’ve kind of given people a week to think about lessons but having just had this conversation or anything, do you think it would be, would you change your lesson plan?

P 42: I mean if you wanted to have an objective and all that stuff, yea but if just like an activity is just as …

LG: No its good. I like activities. That is how I love to run my classrooms?

P 42: I would rather have kids be learning and having fun and you know but still be able to pay attention. you know. Then something you know being so bored.
LG: Why do you want to be a teacher?

P 43: Well my mom has been a teacher since I was in kindergarten so I have always been up in the classroom, so after school I would still go to her classroom and kind of see everything. And I’ve babysat my whole life so kids have been a huge part of my life and I just love working with them.

LG: What does your mom teach?

P 43: She taught preschool for like 10 years and just moved to kindergarten.

LG: So r u wanting to teach what grades?

P 43: Well I think I want to do kindergarten but it varies like every day like sometimes I want to do 3rd grade but I think I do want to do kindergarten. To start out with.

LG: So if you were doing 3rd grade a lot of third grades have subject specific so what subjects would you teach?

P 43: See that is what I don’t know. That is why it changes but probably like language arts. Probably.

LG: Do you have any experience with teaching besides your mom being a teacher?

P 43: I mean no. I’ve never personally like gone into the classroom or anything like that but I will this summer because I am help with the FOCUS program.

LG: So have you done any tutoring or anything like that?

P 43: This last semester towards the end I started helping my classmates so that pretty much the extent of that but I mean math was never. This is the first semester I
have ever been good at math so it’s a huge shock to me.

LG: Well that is good. So for your lesson why did you choose fourth grade?

P 43: Just because I guess functions seem higher level to me, I was trying to pick something that, because this lesson plan isn’t, it wouldn’t be appropriate for 6th graders making little booklets but I think 4th grade they both kind of go together.

LG: I was reading about the booklets, what are the, what are the booklets for?

P 43: Like just because I am so use, I’m in a music and art class right now and we are doing so many lesson plans and I am so use to integrating 2 subjects into things so like I was doing math and kind of like an art project to help. Just put all their information together and it would just put different forms of that function on separate pages and kind of like decorate together or something like that.

LG: So each booklet would be one function or one type of function?

P 43: No like different forms of identity like in tabular form and on the graph and something like that. And put information about it.

LG: So you expect them to know all these forms?

P 43: Yea and this wouldn’t be in a day’s lesson plan. Just kind of jotting down ideas but you know kind of going over it throughout the week and then .

LG: What inspired your lesson? I know you mentioned the music class, anything else?

P 43: Yea I mean.

LG: How about your current math class?

P 43: Yea I mean functions is huge in the class we just took so I mean just starting out with the identity I guess is an easy way to go into everything else. And then just my other classes integrating different kind objects with it. Helps tie everything
together

LG: I know you said the math class you just currently finished has to do a lot with functions, but when do you think you first gained the knowledge of functions?

P 43: Um I honestly have no idea because I didn’t care about math at all so I never fully understood anything about it until this semester. So I mean, I’m sure it was all over the place in high school but I never grasped the, I didn’t care about math at all.

LG: So your current semester, that has been your first math class here?

P 43: No I failed developmental math twice freshman year then I got accepted into the FOCUS program so.

LG: Did you ever see any aspect of functions in the regular developmental math class?

P 43: Oh yea so I guess that has to do with it too.

LG: Why did you structure your lesson plan? I know you said you only put procedures and objectives? Why did you only put those and what other things are you missing that you would put in?

P 43: Well I’ve just I’ve been doing like so many lesson plans this semester and they have like whole like procedures, they have objective of both the subject you are integrating and like anticipatory set and stuff how the teacher goes about integration, extension to other subjects. Just all that but those are the two basics because procedures all about how you are gonna kind of do the entire lesson and the objectives is what they are suppose to be learning and how they are gaining from it. So I thought those were the two basics.

LG: Is this usually the order you have it in?
P 43: Yes those are the first 4 in order. Um and there is like 6 more after that.

LG: Before teaching this what do you think your students should know already before coming into this lesson?

P 43: Uummmm huh. This is a good question. I mean I guess they need the general knowledge of the function cause I said I would state the definition and hopefully they will have some previous knowledge on or at least heard about functions.

LG: What’s a definition and that would be it?

P 43: I really, I can’t remember what I knew in fourth grade to say what they should know but…

LG: Well I know you said you currently just started, you know functions have been a big part of, what did you know before going into functions That kind of helped you with the lesson?

P 43: Helped me out with functions in general. Um I guess just I mean really your basics, I mean we started out with absolute value and placement on the number line and stuff like that so basics like that and how absolute value is the distance, not exactly the opposite or whatever.

LG: So you said you would start of with the definition and you would draw out examples?

P 43: Yea like examples like they are going to be putting in their booklet. Give them some of the examples for them to go and do on their own. They would have to come up with. I don’t know just different numbers and the table or …

LG: Draw out do you mean drawing out the graph as well too or just tables?

P 43: Yea well just showing like kind of writing out the definition of a function, you
know how like each element in the first set can only go to one in the second set
and just kind of drawing that out for them.

LG: Okay. Drawing out the map?

P 43: Yes

LG: Okay, And now it says you would start with linear functions, the identity and the
constant. Do you feel those are the…?

P 43: Yea I just like for them, if we are just starting off with functions, that would be
kind of the easiest. I think, That’s I thought those a were the easiest. I could learn
off of those after understanding.

LG: Now you wrote, I would show all aspects of those functions,’ what aspects are
you talking about?

P 43: Like intercepts, increasing, decreasing, even, odd, neither, stuff like that. Domain
and range.

LG: After having a conversation with me, do you think you would change the lesson
plan at all?

P 43: I mean there is definitely there definitely needs to be some revising um. But I
definitely, after these classes I’ve tooken this semester I believe in using a project
to teach a specific lesson so I would always, I’m always going to be doing that
from here on out but.

LG: What do you mean by project? Like the booklet?

P 43: Yea like the booklet. yea because I feel like projects like that just tie up the
material and it gives them a way to huh um do things together and make it more
fun and so it’s not just like listening to the teacher and doing worksheets.
LG:  And you wrote here that they should present them but they could stay in the classroom?

P 43:  Yea after they like present them to the class, they would keep them on the book shelf so other people can go and looked at eachothers or…and this would be like go on with different types of functions like, not just these two so then probably like at the end of the … this whole lesson they would be a bunch of booklets for everyone, difference ones.

LG:  OK so usually would this be done all at one day?

P 43:  No No No. But I think this could not be done in one day because I would at least have to spend a couple of days going over the aspect of the functions stuff like that before they could even get into the project to be able to do it on their own.
K.21: Participant 44

Date: 5/3/2012
Time: 12:20pm
Duration: 10:23

LG: Why do you want to be a teacher?

P 44: Basically to help out other students that struggle in class, you know elementary. I’ve struggled really in all my life through school so I just want to try and return the favor back.

LG: What did you struggle with?

P 44: Math

LG: Was that the only subject?

P 44: Yes math was the only one I struggled with. I was placed in special education math about, I think, my freshman year of high school. Found out it was a big joke. I barely knew how, when I came to the focus program I barely knew how to multiply. So I made a big leap out of the focus program. I should be their poster child. It works. I mean I barely knew how to multiply and the special education program I thought it was kind of a joke. The coach that was doing special education program, we would just watch videos all day. I watched all these Happy Gilmore, I mean Adam Sandler movies and never had a real test or anything. I didn’t so all this stuff that I was taking in focus was my first time actually seeing, you know some of this algebra stuff so. Cause like when I was in special ed, we didn’t take any test, we watched movies everyday—Scary Movie or Happy Gilmore or whatever other Adam Sandler movie that was out there. So it was kind of weird doing it now.
LG: So are you doing well in your class?

P 44: I should, I will probably make a D in the class. That means I have to take college algebra again but at least I knocked out two of the dev. math classes that I had to take. I mean I could get a C, she says it all depends on personal growth. So I don’t know if I have grown enough to make a C but it happens so so I can’t do nothing about it.

LG: No but that sounds good that you think your succeed.

P 44: I succeeded I definitely I’m not walking away with, even though I didn’t technically pass because my degree needs a C or better. You know but at least I come out of the focus program knowing I can actually do a lot better than when I first took dev. math back in summer, this past summer.

LG: Do you have any experience with the aspect of teaching?

P 44: Um I taught guitar lessons, after school guitar lessons for ***** elementary in Austin TX. For about 2 yrs, so that is about the only experience I have with teaching. As far as like writing out TEKS or something like that so I came here because it was an after school guitar lessons. But I was working with the ***** elementary school children and it was whatever I said. I was teaching them things like Metallica and Black Sabbath you know really cool stuff not like Yanky Doodle. They liked it though and Master Puppets and stuff like that.

LG: How did you break down the music?

P 44: It depended on what type of student I had each year. Usually I had 5th grade boys and 5th grade boys got, I noticed, got bored trying to learn you know twinkle, twinkle little star so I would break it down to each grade level and I had about
give or take 20 kids in the room so I would have one, you know it is kind of hard to teach guitar lessons to that many kids, so I would have one advanced 5th grade boy or girl, whoever it was, and have them sit with the younger kids. And it would actually teach better that way. So I would have them break it down or something. If that makes sense to you.

LG: What grade level would you like to teach?

P 44: Kindergarten.

LG: Is that why you made your lesson plan …?

P 44: Yea I tried to make it to yea kindergarten.

LG: So what is the whole idea with the matts that are two different colors?

P 44: Well I was trying to think of a way to do this for kindergarten level, you the function whatever, to get them to understand the function thing. Um I was trying to say you know have them, I know it is really, you know basically a rough draft, I really didn’t put too much time to it I guess but have each student try to go, you know kind of like visually the definition of a function, or what I think is the definition of a function hopefully, I learned it, um you have two spots whatever and you assign some kids whatever to the mats whatever to the first one. And you do it well and pick only like 7 of them or something. And you get them back up or whatever and then you tell them, tell a couple of them to try and go to two different spots at the same time. Does that make sense? Two different …both mats whatever at the same time and then when they see that is impossible whatever to do and then you kind of go into a lesson that that’s pretty much the definition of a function. I think. Hopefully I am right.
LG: So your activity is the building of the definition?

P 44: Yes. Its kindergarten. I mean I don’t think they are going to get into too much college algebra.

LG: Nah I think it is good. Ok!ok!ok!

LG: What inspired your lesson?

P 44: I was trying to work around. It was a challenge to me see if I could do it for kindergarten level. I am sure there is a way better lesson plan I probably could of *** if I put a lot of thought into it. But it was just because I wanted to see if I could do it for kindergarten level that’s the reason why I did it for kindergarten. If I want to teach kindergarten hopefully, I can incorporate it somehow.

LG: So where did you gain your knowledge of functions?

P 44: FOCUS that was what was on every single test, ‘what is the definition of a function?’

LG: So it’s been burned into your brain?

P 44: Yes it’s been burned into my brain.

LG: So with you Happy Gilmore movies there was no instruction you didn’t learn?

P 44: No I mean, it was well it was ‘cause it was all these football players, no he was the baseball coach, all these baseball players and a bunch of football players, if you don’t pass you don’t play. So he made it as easy as possible for them to pass which is. I think we did have one test and it was like you know very, very simple multiplication i.e. 2 times 4and stuff like that. It was very very easy multiplication which I got an A on.

LG: How did that effect your TEKS, the test score on TAKS?
P 44: I didn’t take them I was exempt.

LG: You were? Were all the others students?

P 44: Yes they were in special education. They have a different degree you have to go you know like the lowest degree you can have to graduate or whatever and then they have a special education degree where you get exempt from certain things. So I was exempt.

LG: Why did you structure the lesson the way you did? How you have it in paragraph form? Why did you decide to do it that way?

P 44: Because I couldn’t remember the form of, we you know because all my other education teaching courses I’ve taken here at Texas State and at ACC they have a particular form and without me actually seeing it I couldn’t memorize it so I decided to kind of a rough copy of the idea behind it and what I was trying to go with, so. I thought it would be easier that way instead of me thinking about the TEKS that go underneath and the duration or whatever and all that good stuff. So you got the Rolla dash right there.

LG: So before students learn this, what do you think they should already know?

P 44: Um I was kinda going with the fact, my thinking was you know…let me think. I didn’t think of that question. You know maybe this could be the beginning of going into something, you know like going into some very basic math for the kindergarteners you know. because I know like in kindergarten it is how many cookies are in the jar type thing. So just kind of going into some very, very basic math of that kind mind set if that makes sense. So this could be like the beginning to leading into something.
LG: Ok so this is the beginning?

P 44: This is the beginning.

LG: So I know I have given other participants like a week to reflect on this to think about it, but after our little discussion do you think you would change this at all?

P 44: Um I would probably put it in like more professional look. Instead of just the paragraph. Are you talking about the actual lesson plan? Uuhh yea I would probably put a little more thought into like what kind of mats or not even have mats or have like maybe something else for them to identify with or something like that probably. Yea I would probably change it.
K.22: Participant 45

Date: 5/3/2012
Time: 1:19pm
Duration: 6:57

LG: Why do you want to be a teacher?

P 45: It was one thing I did in the military, be an instructor for certain things, so it is something that I enjoy. Plus it allows me to spend more time with my son.

LG: Having the same school hours?

P 45: Yea it helps, because he doesn’t live with me right now.

LG: Have you had any experience with teaching besides the military?

P 45: No just military.

LG: Just ordering people around?

P 45: Well I had to go through instructor training because I had small arms, safety, just different things I had to give classes on.

LG: So with those lesson plans did you have to come up with lessons or anything like that, did you have to go in order or something?

P 45: Yes.

LG: So what grade level is this for, do you think?

P 45: More of an 8th grade, I think. I don’t know. It’s been a long time.

LG: What inspired your lesson?

P 45: It because you wanted to go over functions and I feel you define it then go over the aspects of, you list all the common functions, then go over the aspect of each function you know, increasing, decreasing, domain, and range and explain what that is so they understand what they are looking at especially on a graph.
LG: Do you believe in showing more than one representation?

P 45: Yes.

LG: Where did you gain the knowledge of functions?

P 45: In the FOCUS group.

LG: Do you have any experiences from before? Did you take any classes before the FOCUS?

P 45: Math classes, yea I did 1311 two summers ago.

LG: Did y’all discuss it in there?

P 45: Not that I recall. It’s been such a long time since high school that I couldn’t honestly tell you about that.

LG: So I know you kind of explained to me earlier but why did you structure your lesson this way?

P 45: Because if I was teaching I would go by bullet points. I mean I would have all my, of course, PowerPoint or whatever is needed and I would go by that, but bullet points to for what order I will go into.

LG: And so why did you pick this order, defining before …?

P 45: Define and show examples and show and when you list them all show domain and range, show how, I probably should have ordered it better.

LG: So show these aspects for every single common function?

P 45: I would.

LG: Okay! So what do you think the students should know before learning this?

Before learning functions?

P 45: I don’t know.
LG: What do you think you knew before coming into this class?

P 45: I knew how to read a graph. Pick a point on a graph. How to plot a point on the graph. I know how to do that. But common functions a lot of those I didn’t know before going into this class. You know.

LG: I know I usually give people a week before I interview them and talk to them. I am just kind of like condensing down so just do you think you would change any of this after our discussion?

P 45: Yea I would probably, well if I had more time to prepare it, I would a little more detail and also I would make sure the order is correct so,

LG: What do you mean the order is correct?

P 45: Make sure as I am going down it, like as we are doing it for each function make sure the previous one will help you to the next one like you know the $x$ and $y$ intercepts would help with showing increasing, decreasing, and constant, you know. And make sure I have the right order and it flows well.
APPENDIX L

LESSON PLANS

L.1: Participant 21
L.2: Participant 22
L.2: Participant 23
L.2: Participant 24
L.2: Participant 25
L.2: Participant 26
L.2: Participant 28
L.2: Participant 29
L.2: Participant 30
L.2: Participant 31
L.2: Participant 32
L.2: Participant 33
L.2: Participant 34
L.2: Participant 35
L.2: Participant 36
L.2: Participant 37
L.2: Participant 39
L.2: Participant 40
L.2: Participant 42

L.2: Participant 43

L.2: Participant 44

L.2: Participant 45
Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

FUNCTIONS: 8th grade

1. **Define function**: An equation that does not have multiple solutions. An $x = y$, but cannot have 2 "y's".

2. **Use Examples**: Use examples on board/overhead. Make class interactive/fun.

3. **Ask Question**: Ask students to apply examples on board/overhead. Use games: racing games to board.

4. **Give Homework**: Give HW in class if time allotted & walk around & help answer individual questions.
L.2: Participant 22

Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

* I would start off by defining a function.

“A function is a set of numbers in which no x has more than one y.”

* I would then draw a table like in the example below and state again why it’s a function.

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>

“each x has only one y therefore it’s a function.”

* I would also draw up 3 more tables then ask the students to determine if they’re functions or not.
After I felt like the class understood how to determine if its a function by looking at a table. I'd draw up actual ordered pairs to show them the concept in that form.

(0,2)(1,4)(2,6),(3,8),(4,10)

I would then write up 3 more examples like above & have the students define them as functions or not again.

Last but not least I would show them a graph & explain to them the 3rd form of functions.

I would then tell them a process on how to determine if a graph is function or not. This is known as the vertical line test.
Finally I would do 3 examples again and ask them if they were functions or not.

I would then give them a handout of about 10 questions. It would have all 3 forms of functions I had gone over. They students would do the handout for homework and go through stating why or why not each problem was a function or not. We would go over the homework the next day of class and I'd answer any questions they had about it.

I believe this lesson plan could apply to 7th grade and up.
Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

8th grade:
Friday, November 18, 2011

TLW: Be able to identify functions given a table and a graph

TLW: Solve equations and graph to identify whether it is a function or not
Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

Functions Grade 7

Engagement: Show examples of graphs with loops and all sorts of shapes. Ask for defining characteristics of each.

Next: Give definition of a function, have them follow along in textbook:

give examples such as:

\[
\begin{array}{c|c|c}
\sqrt{x} & \sqrt{y} & \sqrt{x+y} \\
\hline
1 & 1 & 2 \\
2 & 2 & 2 \\
3 & 3 & 3 \\
\end{array}
\]

Explain that for every x value there can be only one unique y value.

Next: Using the graphs mentioned above, demonstrate the vertical line test. Have some students come to overhead or board to make the line and then have class say if it is right or wrong and why.

Next: To conclude the lesson have students notice any patterns among graphs used to see if some types of graphs are never or always functions and to have them explore why that is. Have group discussion and dismiss.
Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

This lesson would be used to teach a simple intro to functions by using tables and graphs. The students are told about a boy, Mikey, who is rewarded with 3 M+Ms for every 1 math problem he does on his homework that night. Some nights he is too tired to finish, and other nights he gets all the assigned problems done. So create a table showing how many M+Ms Mikey will get each night according on how many math problems he finished.

<table>
<thead>
<tr>
<th>3x</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>18</td>
</tr>
<tr>
<td>9</td>
<td>27</td>
</tr>
</tbody>
</table>

\[ 3x = M \]

\[ M = M+Ms \quad x = \# \text{ of math problems} \]
Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

Student will learn to use functions given to complete composite functions

Ex. if given $f(x) = 2x - 3$, $g(x) = \frac{5}{x}$, $h(x) = \sqrt{3}$

and the problem asks for $(f \circ g \circ h)$ the student will learn how to replace the $f(x)$ with the given value of $x$ in what sequence it is asked for.

This problems solution would be completed as $f(g(h(x))) = h(x) = \sqrt{3}$ $g(h) = \frac{5}{\sqrt{3}}$ $f(g(h)) = 2(\frac{5}{\sqrt{3}} - 3)$

Student would then solve using substitution if given the value of $x$. 
Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

- Grade level: Middle School

. What is a function?
  - A function is a correspondence between set pairs to another set of pairs.

. Then give examples of what are functions:
  . x and y do not repeat
  . Try vertical test to see if a function or not

. Then give examples and ask which are functions:

. Then solve for the ones that are functions.
In order to do this problem you have to find the **least common denominator**. And how to find that you take the bottom to numbers and see where they connect. $2$ connects to $8$ by $2 \times 4$ and $\frac{1}{8}$ connects to $8$ by $1 \times 8$. So the bottom numbers is $8$. And then you just add the top two numbers straight across. And because you can simplify this factor you may do so. If you can simplify the ending factor leave it as is. To simplify you multiple the numbers out and cross out common pairs. I would take the clear from step to step and explain why. I would not give them other options to do one problem. I would pick the simplest way to do it and stick with that way all the way through the section.
Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

4th grade:
1st: Explain what a function is with examples and let them try one problem
2nd: Go into detail about what different kinds of functions there are.
3rd: Assign a worksheet in class and help with any questions they have.
4th: Go over the worksheet together as a class and discuss why they got them wrong.
Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

I would have my freshman in high school pay attention when going over functions in class. Start on the lesson and then go over it and give problems. Then next day help them understand better.
L.11: Participant 32

Name: 32
Date:

Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

In my lesson plan for functions, I would definitely use graphs or/and grids to get a visual of what a function looks like. I would also incorporate word problems, easy enough to graph, but difficult to solve. Another thing I would also do is give my 12th grade class equations, and have them use candies, or small circular objects so that they could form their answers on their grid and understand what functions are, and what functions are not.
L.12 Participant 33

Name: 33

Date:

Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

The grade level the lesson plan is intended for is 9th grade.

First of all, I would have introduce the material properly. The way that I would introduce it is by explaining what a function is. [Define function] - Set of inputs and outputs.

Next would demonstrate what a function looks like. [f(x) = x^2]

Give practice examples (at least 5). Next ask students to try on their own. (If they have been given out a worksheet, notes)

Go over the given examples, ask if there are any questions. Allow students to participate in class discussions about certain material.

Once again - Go over what a function is. Show Step by Step gradually move on by showing harder examples (answer any questions)

Continue giving students opportunity to practice on example problems.

Review all by mixing up easy & hard problems.

Give out homework.

Next day give out quiz.
Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

- Review with students what a function is. A number with only one pair.

- Go over some examples and let them work in groups on a paper full of some type functions determining if it is a function or not.

- For hmw, have them come up with 8 functions on their own.
Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

First, I would go over what the graphs of different functions look like, so the students can see what they will be doing.

![Graphs of different functions]

Next, go over what makes something a function.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

Function

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

Not a function

Next, give students many examples to practice distinguishing what is and is not a function. Show graphs, charts, etc. examples → which are functions?

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Goal: For students to know whether or not something is a function.
What do you mean when you say
"Students can see what they will be doing"?

Why is \( y \leq 2 \) not a function?

\[
\begin{array}{c|ccc}
  x & 0 & 1 & 2 \\
  y & 3 & 4 & 1 \end{array}
\]

Not a function?

What are charts?

Any other type of example or just tables + graphs.
L.15: Participant 36

Name: 36

Date:

Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

Objective: Explain how functions are used in daily life.

Warm-up: Simple function questions to get their brain stimulated on function lesson. Ask if they think these are functions.

\[
\begin{array}{ccc}
1 & 2 & 3 \\
4 & 2 & 1 \\
2 & 3 & 4 \\
4 & 3 & 2
\end{array}
\]

Lesson: Explain how functions are one-to-one using coordinates x, y. Show the "function junction" that if you input x, the output should be x. Input x, output x.

- Show that after developing an understanding of 1 to 2 that a more advanced viewing would be \( f(a) = f(b) \)

Cool down: Wind the class down and wrap it up by asking how they felt learning functions. Review them real quick by asking certain one-to-one questions.
Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

1. Lesson Name = Functions
2. Grade = 9th

3. Objectives:
   - The student will learn how to graph linear functions and find slope and y-intercepts based on the form y = mx + b.
   - Students will demonstrate knowledge on how to find solutions to the function.

4. Procedure:
   - The student will be given the slope intercept form of the equation y = mx + b.
   - Based on the forms of the equation given, students will solve for y.
   - Students will then identify the y-intercept and the slope of the function y = mx + b, where m = slope (rise over run) and b = y-int.
   - They will then graph the function.
   - Students should be able to identify various points in the graph.
L.17: Participant 39

Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

Elementary

* Definition of a function
* Show graphs of functions
* Show pictures and let them decide if it is a function or not.
* Show equations and how to spot if it is a function
* Explain the things that are not functions and why they or not or what requirements they do not meet.

Work involving pictures as well as equations on if it is or is not a function
Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

7th grade
- definition of a function
- examples of functions/non functions
- practice
- have students make their own ex's
- homework over basic function tables
- put function table into other forms, to continue section/chapter/lesson
Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

I think a good lesson plan to teach about functions would be to have a ball with all different kinds of functions on it. I would do this exercise because it gets kids moving around and they are having fun while learning about math. I would label around the room all different functions, I would play some different functions, I would play some fun music and when I threw the ball in the air, I would pick an equation that the ball landed on. Students would have to tell me what function it is. Whoever is not in that specific labeled area of that function, must sit and watch the rest of the game while the other students continue playing.

This game/activity I think is a way better way to teach students about functions than worksheets are.
Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

**Title:** Linear functions

**Grade:** 4th grade

**Procedure:** I would begin this lesson by giving the definition of a function in both words and drawing out examples. Next, I would begin talking about linear functions, the identity and constant. I would show all aspects of those functions; graph, form, tabular, ordered pairs, etc. Once I felt as if the students were comfortable with this material, I would pair them up and have them make a booklet together about all the information over either the identity or constant.

**Objectives:** In math, the students will have a better understanding about functions, and specifically about the linear functions. The booklet they'll make in pairs will be a fun way to put together all the information, and it will also allow them to better their social skills while working together.

After the students have finished their function booklets, each pair of students can present theirs to the class, and afterwards the booklets will be kept in the classroom for students to look at whenever, and to see what the others did as well.
Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

Kindergarten

Have 2 mats possibly two different colors or something to distinguish them. You can pick 7 students or have them volunteer. Tell them which mat to go to. Tell them to get back up and this time tell a couple of them to go to both mats at once. Then when they see that is impossible, explain to them the concept of a function.
Section 2: Lesson Plan Construction

Lesson Plan

In the space provided below, please write a lesson plan on any aspect of FUNCTIONS. Be sure to indicate the grade level. Please write legibly.

- Functions
  1) Definition
     a) show examples, i.e. matching, sets

  2) List common functions
     a) show graphically

  3) Domain & Range
     a) define and work together

  4) "x" and "y" intercepts

  5) Increasing, decreasing, or constant

  6) Symmetry
     a) show symmetry of y-axis
     b) show symmetry about origin
REFERENCES


Texas Education Code, 30 TexReg, 1930 §111.22 (2006).


Texas Higher Education Coordinating Board (THECB) & Texas Education Agency (TEA). (2009). Texas college and career readiness standards. University of Texas at Austin: Austin, TX.


VITA

Lindsey Nicole Gerber was born in Plano, Texas, on February 18, 1983, the daughter of Penelope Jane Gerber and belated Peter John Gerber. After graduating from McKinney High school in 2001, she attended Tarleton State University in Stephenville, Texas where she received two degrees: Bachelor of Science in Mathematics in December 2005 and Master of Science in Mathematics in December 2007. During her undergraduate studies, Lindsey was a Supplemental Instruction leader for College Algebra and Pre-Calculus courses, and she graduated magna cum laude. While completing her Master’s degree, Lindsey was a graduate teaching assistant. She also taught Algebra 1, Geometry, Finance, and TAKS preparation courses at the charter school Erath Excels! Academy from 2006 – 2008. After graduating in 2007, Lindsey was an adjunct faculty member at Tarleton State University from January 2008 to May 2008; she taught College Algebra and Trigonometry. In August 2008, Lindsey entered the Doctoral Program for Mathematics Education at Texas State University-San Marcos, where she taught various mathematics courses as a Doctoral Teaching Assistant and developed curriculum for the developmental mathematics program and the FOCUS program—a grant funded program.

Permanent Email Address: lindsey.gerber18@yahoo.com

This dissertation was typed by Lindsey N. Gerber.