A COMPUTER PROGRAM
FOR
PREDICTION OF TIDES

THESIS

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By

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Discimus hinc tandem qua causa argentea Phoebe
Passibus haud aequis graditur; cur subdita nulli
Hactenus astronomo numerorum fraena recuset:
Cur remeant nodi, curque auges progrediuntur.
Discimus & quantis refluum vaga Cynthia pontum
Viribus impellit, fessis dum fluctibus ulvam
Deserit, ac nautis suspectas nudat arenas;
Alternis vicibus suprema ad littora pulsans.

At last we learn the reason why the silver Moon
Travels with unequal steps; and why she
Has refused to be subdued and has been subject to
The numbers of no astronomer until now.
And why the nodes regress, and why the apsides progress.
We learn also how the wandering Moon sets the tides into motion,
Whereby the surf now deserts the kelp along the shore,
Exposing shoals of sand suspected by sailors;
And then alternately drives the waves high upon the beach.

from the "Ode to Newton" by Edmond Halley, published in the
preface to the Principia by Isaac Newton (1687)

translated by Donald Olson (1992)
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The author is also grateful to Ed Wallner (Wayland, Massachusetts) for assistance in obtaining harmonic constants for Boston and other ports. Certain parts of the present computer program are derived following the theoretical methods advocated by Wallner, in cases for which the author judges them to be superior to the methods employed by the NOS.
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I. INTRODUCTION
The ocean tides have been associated with the motion of the moon for a long time. The history of tidal observations has been traced back at least to Posidonius (135 B.C. - 51 B.C.): "Posidonius says that the movement of the ocean observes a regular series like a heavenly body, there being a daily, monthly, and yearly movement according to the influence of the moon."²

The moon's apparent influence upon the tides was explained in many ways in the centuries between Posidonius and Isaac Newton. It was Newton who, in his Principia, first provided a scientific basis for understanding the connection between the moon and the tides.³ In order to begin to apply his theory of gravitation to an analysis of the tides he made the simplifying assumption that the surface of the earth was entirely covered by water, that is, there were no land masses (Figure 1). To avoid the complications of tidal currents, he assumed that the earth and moon, moving with their correct relative speeds, yet moved with indefinite slowness when compared to the ocean. Therefore, the ocean remained always in equilibrium and the tidal bulges remained beneath the moon. This theory is known as the Equilibrium Theory of Tides.⁴
II. INTRODUCTION TO CELESTIAL MOTIONS WHICH AFFECT THE TIDES

As the height of the tide depends on the positions of the moon and sun it is required to describe those positions. The moon has an elliptical orbit about the earth, as does the earth about the sun. From a more convenient viewpoint, the geocentric point of view, the sun appears to orbit the earth in an elliptical orbit. A geocentric coordinate system defines the origin to be the center of the earth. For strictly positional astronomy, coordinate systems are used which project the orbital positions of bodies onto the so-called celestial sphere, which is considered to be at an infinite distance from the origin.

One such coordinate system, the equatorial coordinate system (Figure 2), is defined as:

...[an] astronomical coordinate system in which the fundamental reference circle is the celestial equator and the zero point is the vernal equinox, \( \Upsilon \). The coordinates are right ascension, \( \alpha \), and declination, \( \delta \), which are measured along directions equivalent to those of terrestrial longitude and latitude, respectively.
LUNAR ORBIT

The orbit of the moon about the earth is an ellipse having a very nearly constant eccentricity of 0.054900489\textsuperscript{9}. The mean distance of the moon from the earth is given by the semi-major axis (384,400 \times 10^3\text{ m})\textsuperscript{10}. However, since the orbit is an ellipse the moon-earth distance varies during each monthly orbit from a minimum (perigee) of about 360,000 km to a maximum (apogee) of about 410,000 km. The tide generating force due to gravity depends on distance and thus on where the moon is relative to perigee.

The position of lunar perigee relative to the vernal equinox changes with time. It has long been known that the apsides have a progressive motion.\textsuperscript{11} Apsides are the two points in an orbit that lie closest to (periapsis) and farthest from (apoapsis) the center of gravitational attraction. The prefixes peri- and apo- are used in describing the apsides of specific orbits. The line of apsides is the straight line connecting the two apsides and is the major axis of an elliptical orbit.\textsuperscript{12}

The eccentricity of the lunar orbit, while being very nearly constant, does alternately increase and decrease, being a maximum when the sun passes the moon’s line of apsides and a minimum when the sun is at right angles to it.\textsuperscript{13} This leads to a perturbation of the lunar orbit called ejection.
SOLAR ORBIT

The sun's orbit about the earth has an eccentricity of 0.01675104 and a semi-major axis (mean distance) of 149,597,870 km. The small eccentricity of this orbit makes for a small fractional variation in sun-earth distance. However, it must be taken into account for precision in the tide calculation.

The projection on the celestial sphere of the apparent orbit of the sun, called the ecliptic, is not co-planar with the celestial equator, but is at an inclination of about 23.5° relative to it. The two points at which the orbit of a celestial body intersects a reference plane, usually the plane of the ecliptic or celestial equator, are called nodes. When the body, such as the sun, moon, or a planet, crosses the reference plane from south to north it passes through the ascending node; crossing from north to south the body passes through its descending node. The line joining these two nodes is the line of nodes.

LUNAR ORBIT RELATED TO SOLAR ORBIT (ECLIPTIC)

The lunar orbit has an almost constant inclination, $i$, of 5.1453964° relative to the ecliptic. (Figure 4)

Therefore, during a draconic month (which is the time interval of mean length 27.2212 days between two successive passages of the moon through the ascending node) the moon's latitude ranges from about +5° to -5°.
The line of nodes of the moon's orbit has a westward motion of about 19° per year. Therefore, in approximately 18.6 years the lunar line of nodes regresses completely one time around the ecliptic. At some point in this 18.6 year cycle the ascending node passes through the vernal equinox and then the moon's obliquity, its orbital inclination relative to the celestial equator, is a maximum. So, the range of declinations of the moon is maximized, having extreme values of +28° ( +23° + 5° ) and -28° ( -23° - 5° ). Approximately 9.3 years later, the node passes the autumnal equinox and the range of declinations of the moon is minimized, having extreme values of +18° ( +23° - 5° ) and -18° ( -23° + 5° ). This variation in the obliquity of the moon's orbit introduces an
important inequality in the tidal movement which must be taken into account.\textsuperscript{19}

**ANOTHER METHOD OF DESCRIBING POSITION**

The local hour angle of a celestial body is the angle measured westwards along the celestial equator from an observer’s meridian to the hour circle of the celestial body. It is usually expressed in hours, minutes, and seconds from 0h to 24h. It is thus measured in the same units but in the opposite direction to right ascension which it can replace in the equatorial coordinate system. The angle measured eastwards along the equator from the meridian is sometimes called the meridian angle. Due to the daily apparent rotation of the celestial sphere, a star’s hour angle increases continuously from 0h at the meridian; after six hours the hour angle is 6h and after 24 hours the star again crosses the meridian.\textsuperscript{20}

**III. HISTORY OF TIDE THEORIES AND PREDICTION**

Before the 1860s tide prediction was done with simple methods involving lunitidal intervals. The mean high-water lunitidal interval is defined as the average time interval between the transit of the moon across the meridian of the place and the following high water.\textsuperscript{21} It is an easy matter to estimate the time of moon’s passage across the meridian.
Then, a crude estimate of the time of high tide can be done by setting

\[(\text{time of high water}) = (\text{time of lunar transit}) + (\text{lunitidal interval}).\]

Such methods were in use in the 1700s and 1800s. Popular almanacs of the day listed the time of lunar transit ("southing of the moon") and estimates of the time of high water.\(^2\) This crude technique was only roughly accurate, since it did not take into account effects of lunar distance (perigee and apogee), lunar declination (moon north or south of the celestial equator), solar distance (perihelion or aphelion), and solar declination (sun north or south of the celestial equator).

HARMONIC ANALYSIS

The principles of harmonic analysis of the tides were actually first developed in 1773 by Pierre Simon Laplace and were published in his *Mécanique Celeste*. However, no widespread practical use was made at that time. Some studies were made of high and low waters, to compare theory with observation, but there was no attempt at general tide prediction or production of tide tables.\(^3\)

William Thomson (Lord Kelvin) devised a method of harmonic analysis about 1867. The theory was further developed by G. H. Darwin and J. C. Adams and took a final
form by 1883. The terminology developed then is still in use today with regard to the naming of the tide components, their theoretical forms, and the various quantities used in the analysis: node factors, equilibrium arguments, phases, and speeds. These quantities will be described in detail below for the harmonics used in the present program.

The first tide-prediction machine was constructed under the direction of William Thomson (Lord Kelvin) in 1873. This machine employed 10 harmonics and produced an analog curve of tide height versus time.

The United States Coast and Geodetic Survey built a tide-prediction machine in 1882. This analog machine was designed by William Ferrel and accounted for 19 harmonics. This mechanical device had dials from which times of high and low water could be read off, but no tide curve of height versus time was produced. The Ferrel machine was used for all Tide Tables produced from 1885 through 1911.24

An improved mechanical tide-prediction machine, employing 37 harmonics, was developed by R. A. Harris and E. G. Fischer of the U. S. Coast and Geodetic Survey in 1910. This machine produced an analog graph of tide height versus time and also could give figures for high and low waters. The machine stopped when the pen reached an extreme, and an operator could manually read the time of high or low water from dials on the side of the machine. An intricate gear system was required to mechanically sum 37 harmonics, each with a different
amplitude, frequency, and phase angle. The Harris-Fischer machine produced all tide predictions in the United States for the period 1912 until 1965. It has been stated that, during World War II, armed guards were placed in force around the machine, since it was irreplaceable to the war effort. Manual summing of 37 harmonics, with pencil and paper and logarithm tables, was prohibitively time-consuming.

The first computer programs for tide prediction were developed in 1956. These codes, in machine language, were developed for the IBM 701, then for the IBM 704, and finally for the IBM 7094 mainframe computers. An improved computer program was developed in 1966 in the FORTRAN IV language for the IBM 7094 mainframe. Computer methods have been used for tide prediction in the Tide Tables from 1966 to the present.

IV. HARMONIC ANALYSIS AND HARMONIC PREDICTION OF THE TIDES

In the harmonic analysis method, the complicated behavior of the actual tide, due to the complicated motions of the moon and sun, is regarded as being made up of a combination of simple tides, each of which is due to a hypothetical tide-producing body which moves at a constant speed in a circular orbit in the plane of the equator.

In Newton’s equilibrium tide, the tidal bulges follow the moon. Thus, to a first approximation, the principal hypothetical body moves along the equator at a constant rate which follows the actual moon. The daily motion of the actual
moon is a combination of the rotation of the earth and the orbital motion of the moon around the earth. This results in a "tidal day" of 24 hours 50 minutes. This gives a period of 12 hours 25 minutes for the principal hypothetical body since there are two tidal bulges. The tide due to this hypothetical body is a simple tide having only one high water and one low water per period. The range of this tide as well as the time interval between the meridian passage of this body and high water vary from place to place and can only be determined from actual tidal measurements.

However, actual tidal measurements are a combination of all the different tides due to different hypothetical bodies. For instance, since tides are a gravitational effect there must be a hypothetical body which makes a tide reflecting the elliptical nature of the orbit of the moon, that is reflecting the varying distance between the earth and moon. This body would cause a high water when the moon is at perigee, and a low water when the moon is at apogee. When the moon is at perigee, the high water would be in phase with the high water of the tide due to the principal hypothetical body. Likewise, when the moon is at apogee the low water would be in phase with the high water of the tide due to the principal hypothetical body. Therefore the so-called elliptic hypothetical body must have a period such that in an anomalistic month, the time interval from perigee to perigee of mean length 27.55 days, it loses one complete revolution on
the principle hypothetical body.

Since 12 hours 25 minutes = 12.42 hours, then in 27.55 days or 661 hours the principal body will complete $661 \div 12.42 = 53.25$ revolutions. So the elliptic body must complete 52.25 revolutions in 661 hours which means its period must be 12.65 hours or 12 hours 39 minutes.$^{30}$

Since these are hypothetical bodies, it is not required to calculate the tide-producing forces due to these bodies, or to explain how these bodies could maintain stable orbits, but only to consider the constituent effect upon the tide due to each hypothetical body. This is called harmonic analysis, an explanation of which is beyond the scope of this paper, but the results of which will be used for tide prediction.

For the harmonic prediction of the tide it is necessary to calculate the sum of the constituent tides. The tide height at any time is given by the superposition of 37 constituents which represent periodic variations in the relative positions of the sun, moon, and earth.$^{31}$
Thus, the height of the tide at any time is represented by the equation

\[ h = Z_0 + \sum_{n=1}^{27} f_n H_n \cos (V_n + u_n - \kappa_n) \]

where

- \( h \) = height of the tide at time \( t \)
- \( Z_0 \) = mean height of water level above local chart datum
- \( f_n \) = node factor in the 18.6 year lunar nodal cycle
- \( H_n \) = mean amplitude of the constituent \( n \)
- \( V_n \) = argument of the constituent \( n \)
- \( u_n \) = argument of the constituent \( n \)
- \( \kappa_n \) = epoch (phase) of the constituent \( n \)

Certain quantities \((f_n, V_n, u_n)\) are determined astronomically and are therefore the same for every port and place in the world. These variables depend only on the position of the sun and the moon. The effect of the declination of the moon upon the tide is accounted for by the nodal factors \(f_n\) of the constituents. The factor \(f_n\) depends upon the longitude of the moon's ascending node and is introduced in order to reduce the mean amplitude of the constituent, \(H_n\), to the true amplitude on the date used for the tide prediction.\(^{32}\) \(V_n\) and \(u_n\) are arguments which taken together give the theoretical phase of the constituent in the equilibrium tide. \(V_n\) is an angular quantity which involves multiples of the hour angle of the mean sun, the mean longitudes of the moon and sun, and the mean longitude of the
lunar or solar perigee. \( u_n \) is an angle which depends upon the longitude of the moon’s ascending node.\(^{33}\) The equations for such quantities involve only astronomical variables.

Other variables \((Z_o, H_n, \kappa_n)\) are port-dependent. They are determined by empirical observation of tide levels in the port. In principle, observations of the water level are made hourly for as long a period as is practicable. For the highest accuracy, measurements should extend for at least one 18.6 year nodal cycle. Within this period, all significant astronomical modifications of tides will occur.\(^{34}\)

Therefore, nearly all possible configurations of the moon have been sampled if observations are made for the full 18.6 years. Such a situation is considered optimal for estimating the station constants for a port.

In practice, observations often are restricted to shorter time periods. There are methods for deriving estimates of the station constants (reference level \(Z_o\), amplitudes \(H_n\), and phases \(\kappa_n\)) from observations over time periods of: 14, 15, 29, 58, 87, 105, 134, 163, 192, 221, 250, 279, 297, 326, 355, and 369 days. These periods are related to astronomical periods. For example, the shortest periods (14 or 15 days) are such that tide ranges at a new moon or a full moon, and at a perigee or an apogee will be observed.

During World War II, the station constants for Tarawa (Gilbert Islands) given in Appendix B were derived from a 134-day series of observations beginning on December 1, 1943.
At other more established ports, of course, observations over the full 18.6-year period have been performed.

V. ASTRONOMICAL FACTORS

Any tidal constituent must have a frequency which is simply derived from six fundamental frequencies, which are the inverses of the following periods:

1 mean solar day
   (period of the earth’s rotation relative to the mean sun, an abstract reference point that follows a circular orbit around the celestial equator moving eastward at a constant speed and completes one circuit in the same time, one tropical year, as the apparent sun takes to orbit the ecliptic.)

1 tropical month
   (period of the moon’s orbital motion around the earth)

1 tropical year
   (period of the earth’s orbital motion around the sun)

8.85 years
   (period for the progression of the lunar line of apsides)

18.6 years
   (period for the regression of the lunar line of nodes)

20900 years
   (period for the progression of the line of apsides of the earth’s orbit around the sun)

Every tide component has a characteristic frequency and
period, which must be a simple combination of these six basic
frequencies.

To make this more precise, the equations will be given
for the six fundamental astronomical variables. These
equations are taken from Jean Meeus' *Astronomical Formulae for
Calculators.*

The mean longitude of the sun can be given by

\[ h = 279.69668 + 36000.7692 T + 0.0003025 T^2 \]

where \( T \) here represents the number of Julian centuries
reckoned from 12h ET on December 31, 1899, that is:

\[ T = \frac{JD - 2415020.0}{36525} \]

where JD is the Julian date, the number of days and fractions
of a day that have elapsed since noon Greenwich Mean Time on
Jan 1, 4713 BC. (The consecutive numbering of days makes the
system independent of the length of month or year and the JD
is thus used to calculate the frequency of occurrence or the
periodicity of phenomena over long periods. The system was
devised in 1582 by the French scholar Joseph Scalinger.)
The mean longitude of the solar perigee can be given by

\[ p_1 = h - M , \]

where \( h \) is the mean longitude of the sun (given above) and \( M \) is the sun's mean anomaly which can be given by

\[ M = 358.47583 + 35999.04975 T - 0.00015 T^2 - 0.0000033 T^3 . \]

The mean longitude of the moon can be given by

\[ s = 270.434164 + 481267.8831 T - 0.001133 T^2 + 0.0000019 T^3 . \]

The mean longitude of the lunar perigee, derived in analogy to the mean longitude of the solar perigee, can be given by

\[ p = 334.329556 + 4069.034 T - 0.010325 T^2 - 0.0000125 T^3 . \]

The longitude of the lunar ascending node can be given by

\[ N = 259.183275 - 1934.142 T + 0.002078 T^2 + 0.0000022 T^3 . \]

In addition to the primary variables listed above, several additional quantities are needed.

One of these is the obliquity of the ecliptic, \( \omega \), which can be given by

\[ \omega = 23.452294 - 0.0130125 T - 0.00000164 T^2 + 0.000000503 T^3 . \]
Certain intermediate angles must be defined to generate the correct node factors \( f_n \) and the arguments \( V_n \) and \( u_n \). The equations are from USCGS Special Publication No. 98. Using the standard notation, we define the following angles: \( \nu \), the right ascension of the intersection, and \( \xi \), the longitude in orbit of the intersection, are given by

\[
\nu = AX - AY
\]
\[
\xi = N - AX - AY
\]

\[
AX = \tan^{-1} \frac{\cos\left(\omega - i\right)}{2} \tan\frac{N}{2}
\]

where

\[
AY = \tan^{-1} \frac{\sin\left(\omega - i\right)}{2} \tan\frac{N}{2}
\]

\( N = \) the mean longitude of the lunar node

The obliquity of the lunar orbit, \( I \), can be found from the equation

\[
\cos I = \cos i \cos \omega - \sin i \sin \omega \cos N
\]

\( P \), the mean longitude of the lunar perigee as measured from the intersection, is simply

\[
P = p - \xi
\]
A term, \( \nu' \), which is in the argument of the \( K_1 \) constituent, can be given by

\[
\nu' = \tan^{-1} \frac{\sin 2I \sin v}{\sin 2I \cos v + 0.3347}
\]

and \( \nu'' \), which is a term in the argument of the \( K_2 \) constituent, can be given by

\[
\nu'' = \frac{1}{2} \tan^{-1} \frac{\sin^2 I \sin 2v}{\sin^2 I \cos 2v + 0.0727}
\]
The $L_2$ constituent includes two intermediate variables, an amplitude factor, $R_a$, and a term in the argument, $R$, which can be given by

$$R_a = \frac{1}{\sqrt{1 - 12 \tan^2 \frac{I}{2} \cos 2P + 36 \tan^4 \frac{I}{2}}}$$

$$R = \tan^{-1} \left[ \frac{\sin 2P}{\left( \frac{1}{6 \tan^2 \frac{I}{2}} \right) - \cos 2P} \right].$$

The $M_1$ constituent also includes two intermediate variables, an amplitude factor, $Q_a$, and a term in the argument, $Q_u$, which can be given by

$$Q_a = \frac{1}{\sqrt{\frac{1}{4} + \frac{3 \cos I \cos 2P}{2 \cos^2 \frac{I}{2}} + \frac{9 \cos^2 I}{4 \cos^4 \frac{I}{2}}}.$$
Related to $Q_u$ is another intermediate variable, $Q$, such that

$$Q_u + Q = P$$

where

$$Q = \tan^{-1} \left[ \frac{5 \cos I - 1}{7 \cos I + 1} \right] \tan P.$$

These angles will be incorporated into the equations of the astronomical factors for the harmonics used in the computer program. The following section tabulates and describes these harmonics.

VI. THE 37 HARMONICS USED IN TIDE PREDICTION

In theory, an infinite number of terms should be retained in the cosine expansion of the tide height versus time. In practice, only a finite number are retained. For most ports, it is standard in the system developed by the U. S. Coast and Geodetic Survey to tabulate 37 harmonics.

The 37 harmonics used in the USCGS system are listed on the "Form C&GS-444" for the port being studied. For the present study, copies of this form were obtained from the National Ocean Survey for the ports of Boston (Massachusetts), Los Angeles (California), San Diego (California), San Francisco (California), and Tarawa (Gilbert Islands).

Copies of the "Form C&GS-444" for Boston and several other ports will be given in Appendix B.
PROPERTIES OF THE 37 HARMONICS USED IN TIDE PREDICTION

The purpose of the next section is to explain in more detail each of the 37 harmonics used in the computer tide prediction program. For each of the harmonics, eight properties will be given, as follows.

1. NUMBER

The number of the constituent will be given, as an integer from 1 to 37. This number corresponds to the position on the rows of "Form C&GS-444," which contains 37 rows from #1, the $M_2$ tide, to #37, the $(MS)_4$ tide.

2. COMMON NAME

These well-known common names, which were originally assigned in the 1800s, are as given on "Form C&GS-444." The name can indicate properties of the tide, for example the $M_2$ tide is the principal lunar semidiurnal (2 high waters per day) tide. The $S_2$ tide is the principal solar semidiurnal tide.

3. NAME IN SPECIAL PUBLICATION NO. 98

These names are taken from Table 2 and Table 2a of Special Publication No. 98. In this system, tides are assigned to the A-series, the B-series, or to the "shallow-water constituent" series.

4. COMMENTS

For each of the 37 tide components, comments are provided. Definitions presented are those given in the
standard Tide and Current Glossary published by the National Ocean Survey.

Some tides are related to the moon (lunar tides), some to the sun (solar tides), some are combinations (lunisolar tides), or they can have other causes (shallow-water overtides and meteorological tides). A constituent tide can be classified based on its speed. It can be diurnal (one high water and one low water per tidal day), or semidiurnal (two high waters and two low waters per tidal day), or terdiurnal (three high waters and three low waters per day), or it can have a different period.

Comments will also indicate the physical significance of the component. For example, some tides express the effects of the varying lunar distance, as the moon moves through perigee and apogee. Other tides represent declinational effects, as the moon or sun moves north or south of the celestial equator.

Indicated in the comments will be the "shallow-water constituents." These result from the fact that when a wave moves into shallow water near shore, the wave loses its simple harmonic form. Some of these components are called "overtides" by analogy with overtones in acoustics. These components have speeds which are exact multiples of one of the elementary constituents. For example, the principal lunar and solar tides are $M_2$ and $S_2$. The lunar overtones are called $M_4$, $M_6$, and $M_8$, while the solar overtones are denoted by $S_4$, $S_6$, and $S_8$. Their arguments and speeds are exact multiples of those
of the fundamental tides.

Also worth mentioning are the "meteorological" tides. These include the tides called Sa (solar annual), Ssa (solar semi-annual), and Si (solar diurnal), with periods corresponding to the tropical year, the half tropical year, and the solar day. These constituents appear in the cosine series expansion of the tide height versus time. However, their effects cannot be separated from the general seasonal effects of the weather, seasonal variations in temperature, barometric pressure, direction and force of the wind, and other meteorological effects. All such effects with long time-scales are allowed for in the Sa and Ssa constituents. The Si tide (solar diurnal) also includes some meteorological effects, particularly in places where periodic land and sea breezes occur in different directions during the day and the night.

5. NODE FACTORS

For each tide component, the node factors are taken from the equations given in the text of Special Publication No. 98. The node factors are slowly changing periodic functions which modify the amplitudes of the constituents which depend on the longitude of the moon's node.\(^{38}\)

Note that the exact formulae from Special Publication No. 98 are used in the present work and in the computer program given here. This represents one of the significant improvements of the present program, compared to the computer
programs used by the NOS for tide prediction.

In the NOS computer programs, the node factors are evaluated only once per year. The same node factor is used for every day of that year. The NOS programs were based on methods that pre-date the 1950s and the computer age and much of the NOS programming retains the philosophy of "look-up tables." In the early 1900s, the node factors were tabulated for the middle of every year from 1850 to 1999. These same tables are still used in the modern NOS program.

In the program developed here, the node factors are computed from the exact formulae, 1440 times per day. Even on a personal computer this computation runs very quickly, with an entire day's tide curve produced in less than 15 seconds on a PC 386-class computer.

In summary, our program computes node factors from exact formulae and makes no use of the simplified tables in Special Publication No. 98.

6. EQUILIBRIUM ARGUMENTS $u$

The equations for the arguments $u$ are taken from the text of Special Publication No. 98 and from Table 2 and Table 2a in Special Publication No. 98. Again, our program makes use of the exact formulae and makes no use of simplified look-up tables.

7. EQUILIBRIUM ARGUMENTS $v$

The equations for the arguments $v$ are also taken from the text of Special Publication No. 98 and from Table 2 and Table
8. SPEED

The constituent speeds are taken from Table 2 and Table 2a in Special Publication No. 98.

Any tides with speeds equal or nearly equal to 15 degrees per hour are considered "diurnal" in type. Such harmonics describe tidal bulges which travel around the earth at the cited speed and cause one high water and one low water per day.

Those tides with speeds equal or nearly equal to 30 degrees per hour are considered "semidiurnal" in type. Such harmonics describe tidal bulges which travel around the earth at the cited speed and cause two high waters and two low waters per day.

Any tides with speeds equal or nearly equal to 45 degrees per hour are considered "terdiurnal" in type. Such harmonics describe tidal bulges which travel around the earth at the cited speed and cause three high waters and three low waters per day.
HARMONIC CONSTITUENTS
Name: M₂ tide

Designation in USCGS Special Publication No. 98: A₃₉

The M₂ tide is the principal lunar semidiurnal constituent. It represents the rotation of the earth with respect to the moon.

Equilibrium arguments:

\[ V(1) = 2T - 2s + 2h \]
\[ u(1) = 2\xi - 2\nu \]

Speed:

\[ a(1) = 28.9841042 \, \text{o/hour} \quad \text{(semidiurnal)} \]

Node factor:

\[ f(1) = \frac{\cos^4 \frac{I}{2}}{0.9154} \]
Name: $S_2$ tide

Designation in USCGS Special Publication No. 98: $B_{39}$

The $S_2$ tide is the principal solar semidiurnal constituent. It represents the rotation of the earth with respect to the sun.

Equilibrium arguments:

\[ V(2) = 2T \]
\[ u(2) = 0 \]

Speed:

\[ a(2) = 30.0000000 \, ^\circ/\text{hour} \quad (\text{semidiurnal}) \]

Node factor:

\[ f(2) = 1 \]
constituent #3 of 37

Name: $N_2$ tide

Designation in USCGS Special Publication No. 98: $A_{40}$

The $N_2$ tide is the larger lunar elliptic semidiurnal constituent. This constituent, along with $L_2$ (#33 of 37), modulates the amplitude and frequency of $M_2$ for the effect of variation in the moon's orbital speed due to its elliptical orbit.

Equilibrium arguments:

$$V(3) = 2T - 3s + 2h + p$$
$$u(3) = 2\xi - 2\nu$$

Speed:

$$a(3) = 28.4397295 \, ^\circ/\text{hour} \quad \text{(semidiurnal)}$$

Node factor:

$$f(3) = f(1)$$
The K₁ tide is the lunisolar diurnal constituent. This constituent, with O₁ (#6 of 37), expresses the effect of the moon’s declination; they account for the diurnal inequality, which is the difference in height of the two high waters or of the two low waters of each day. In extreme cases, they can change tides which are normally semidiurnal (two high waters and two low waters each day) into diurnal tides (one high water and one low water each day). Also, K₁, taken together with P₁ (#30 of 37), expresses the effect of the sun’s declination.

Equilibrium arguments:

\[ V(4) = T + h - 90° \]
\[ u(4) = -\nu' \]

Speed:

\[ a(4) = 15.0410686 \, °/\text{hour} \quad \text{(diurnal)} \]

Node factor:

\[ f(4) = \frac{C_1}{\langle C_1 \rangle} = \frac{\sqrt{A^2 + 2AB \cos \nu + B^2}}{(\cdot5 + 75e^2) \sin(2\omega)(1 - 1.5 \sin^2 i) + B} \]

where

\[ A = (\cdot5 + 75e^2) \sin 2I \]
\[ B = (\cdot5 + 75e^2_1) S' \sin(2\omega) \]
\[ e = 0.054900489 \quad \text{(eccentricity of moon's orbit)} \]
\[ e_1 = 0.01675104 \quad \text{(eccentricity of sun-earth orbit)} \]
\[ S' = 0.4602 \]
Name: $M_4$ tide

Designation in USCGS Special Publication No. 98: Shallow Water Constituent

The $M_4$ tide is a shallow water overtide of the principal lunar constituent $M_2$. Therefore, its speed is exactly twice as fast and its node factor is the square of the node factor of the $M_2$ tide. The equilibrium arguments are also exactly twice those of the $M_2$ tide.

Equilibrium arguments:

$$V(5) = 4T - 4s + 4h$$
$$u(5) = 4\xi - 4\nu$$

Speed:

$$a(5) = 57.9682084 \degree/hour$$

Node factor:

$$f(5) = [f(1)]^2$$
Name: $O_1$ tide

Designation in USCGS Special Publication No. 98: $A_{14}$

The $O_1$ tide is a lunar diurnal constituent. Along with $K_1$ (#4 of 37) it expresses the effects of the moon's declination. They account for the diurnal inequality.

Equilibrium arguments:

$$V(6) = T - 2s + h + 90^\circ$$

$$u(6) = 2\xi - \nu$$

Speed:

$$a(6) = 13.9430356 \, ^\circ/\text{hour} \quad (\text{diurnal})$$

Node factor:

$$f(6) = \frac{\sin I \cos^2 I}{2} \frac{I}{0.3800}$$
The $M_6$ tide is a shallow water overtide of the principal lunar constituent $M_2$. Therefore, its speed is exactly three times as fast and its node factor is the cube of the node factor of the $M_2$ tide. The equilibrium arguments are also exactly three times those of the $M_2$ tide.

**Equilibrium arguments:**

\[
V(7) = 6T - 6s + 6h \\
u(7) = 6\xi - 6\nu
\]

**Speed:**

\[a(7) = 86.9523127 ^\circ/\text{hour}\]

**Node factor:**

\[f(7) = [f(1)]^3\]
Name: \((MK)_3\) tide

Designation in USCGS Special Publication No. 98: Shallow Water Constituent

The \((MK)_3\) tide is a terdiurnal compound tide formulated by \(M_2 + K_1\). Its arguments \(V\) and \(u\) are the sums of the arguments of the \(M_2\) and \(K_1\) constituents. Note that its node factor is the product of the node factors of \(M_2\) and \(K_1\).

Equilibrium arguments:

\[
V(8) = 3T - 2s + 3h - 90^\circ
\]

\[
u(8) = 2\xi - 2\nu - \nu'
\]

Speed:

\[
a(8) = 44.0251729 ^\circ/\text{hour} \quad \text{(terdiurnal)}
\]

Node factor:

\[
f(8) = f(1) \times f(4)
\]
Name: \( S_4 \) tide

Designation in USCGS Special Publication No. 98: Shallow Water Constituent

\( S_4 \) is a shallow water overtide of the principal solar semidiurnal constituent. Its equilibrium arguments and its speed are exactly twice those of the \( S_2 \) constituent.

Equilibrium arguments:

\[
\begin{align*}
V(9) &= 4T \\
u(9) &= 0
\end{align*}
\]

Speed:

\[
a(9) = 60.0000000 \degree/\text{hour}
\]

Node factor:

\[
f(9) = 1
\]
Name: \((MN)_4\) tide

**Designation in USCGS Special Publication No. 98:** Shallow Water Constituent

\((MN)_4\) is a quarter diurnal compound tide originating from \(M_2 + N_2\).

**Equilibrium arguments:**

\[
V(10) = 4T - 5s + 4h + p \\
u(10) = 4\xi - 4\nu
\]

**Speed:**

\[
a(10) = 57.4238337 \, {^\circ}/\text{hour}
\]

**Node factor:**

\[
f(10) = [f(1)]^2
\]
Name: \( \nu_2 \) tide

Designation in USCGS Special Publication No. 98: \( \lambda_{43} \)

\( \nu_2 \) is the larger lunar evectional constituent. With \( \lambda_2 \) (#16 of 37), \( \mu_2 \) (#13 of 37), and \( (S_2) \), this constituent modulates the amplitude and frequency of the principal lunar constituent, \( M_2 \), to account for the variation in solar attraction of the moon. This attraction results in a slightly pear-shaped lunar ellipse and a difference in lunar orbital speed between motion toward and away from the sun. Although \( (S_2) \) has the same speed as \( S_2 \), its amplitude is extremely small.

Equilibrium arguments:

\[
V(11) = 2T - 3s + 4h - p
\]
\[
u(11) = 2\xi - 2\nu
\]

Speed:

\[
a(11) = 28.5125831 \degree/\text{hour} \quad \text{(semidiurnal)}
\]

Node factor:

\[
f(11) = f(1)
\]
Name: S\(_6\) tide

Designation in USCGS Special Publication No. 98: Shallow
Water Constituent

S\(_6\) is a shallow water overtide of the principal solar constituent, S\(_2\)' thus its equilibrium arguments and its speed are three times those of S\(_2\).

Equilibrium arguments:

\[ V(12) = 6T \]
\[ u(12) = 0 \]

Speed:

\[ a(12) = 90.0000000 \, ^\circ/\text{hour} \]

Node factor:

\[ f(12) = 1 \]
Name: $\mu_2$ tide

Designation in USCGS Special Publication No. 98: $A_{45}$

The $\mu_2$ tide is a variational constituent. Along with $\lambda_2$ and $\nu_2$ it modulates $M_2$ for the effects of evection.

Equilibrium arguments:

\[
V(13) = 2T - 4s + 4h
\]
\[
u(13) = 2\xi - 2\nu
\]

Speed:

\[
a(13) = 27.9682084 \, ^\circ/\text{hour} \quad (\text{semidiurnal})
\]

Node factor:

\[
f(13) = f(1)
\]
Name: (2N)_2 tide

Designation in USCGS Special Publication No. 98: A_{42}

The (2N)_2 tide is a lunar elliptic semidiurnal constituent. It is a second-order constituent.

Equilibrium arguments:

\[ V(14) = 2T - 4s + 2h + 2p \]
\[ u(14) = 2\xi - 2\nu \]

Speed:

\[ a(14) = 27.8953548 \, ^\circ/\text{hour} \quad (\text{semidiurnal}) \]

Node factor:

\[ f(14) = f(1) \]
Name: (00) tide

Designation in USCGS Special Publication No. 98: $A_{31}$

(00), is a lunar diurnal constituent. It is a second-order constituent.

Equilibrium arguments:

$$V(15) = T - 2s + h - 90^\circ$$
$$u(15) = -2\xi - \nu$$

Speed:

$$a(15) = 16.1391017 \, ^\circ/\text{hour} \quad (\text{diurnal})$$

Node factor:

$$f(15) = \frac{\sin I \sin^2 \frac{I}{2}}{0.0164}$$
**Name:** $\lambda_2$ tide

**Designation in USCGS Special Publication No. 98:** $A_{44}$

The $\lambda_2$ tide is the smaller lunar evectional constituent.

**Equilibrium arguments:**

\[
V(16) = 2T - s + p + 180^\circ \\
u(16) = 2\xi - 2\nu
\]

**Speed:**

\[
a(16) = 29.4556253 \, ^\circ/\text{hour} \quad (\text{semidiurnal})
\]

**Node factor:**

\[
f(16) = f(1)
\]
Name: $S_1$ tide

Designation in USCGS Special Publication No. 98: $B_{71}$

The $S_1$ tide is a solar diurnal constituent.

Equilibrium arguments:

$$V(17) = T$$
$$u(17) = 0$$

Speed:

$$a(17) = 15.0000000 \, ^\circ/\text{hour} \quad (\text{diurnal})$$

Node factor:

$$f(17) = 1$$
Name: \( M_1 \) tide

Designation in USCGS Special Publication No. 98: \( A_{16} \) and \( A_{23} \)

The \( M_1 \) tide is the smaller lunar elliptic diurnal constituent. This constituent, with \( J_1 \) (#19 of 37), modulates the amplitude of the declinational \( K_1 \) for the effect of the moon's elliptical orbit. A slightly slower constituent, designated \( (M_1) \), with \( Q_1 \) (#26 of 37), modulates the amplitude and frequency of the declinational \( O_1 \) for the same effect.

Equilibrium arguments:

\[
V(18) = T - s + h + p - 90^\circ
\]
\[
u(18) = -\nu - Q_u
\]

Speed:
\[
a(18) = 14.4966939^\circ/\text{hour} \quad (\text{diurnal})
\]

Node factor:
\[
f(18) = \frac{f(6)}{Q_a}
\]

where
\[
Q_a = \frac{1}{\sqrt{\frac{1}{4} + \frac{3\cos I \cos 2P}{2} + \frac{9\cos^2 I}{4\cos^4 I}}}
\]

and \( P = p - \xi \)
Name: $J_1$ tide

Designation in USCGS Special Publication No. 98: $A_{24}$

The $J_1$ tide is the smaller lunar elliptic diurnal constituent. This constituent, with $M_1$, modulates the amplitude of the declinational $K_1$ for the effect of the moon's elliptical orbit.

Equilibrium arguments:

\[ V(19) = T + s + h - p - 90^\circ \]
\[ u(19) = -\nu \]

Speed:

\[ a(19) = 15.5854433 \, ^\circ/\text{hour} \quad (\text{diurnal}) \]

Node factor:

\[ f(19) = \frac{\sin 2I}{0.7214} \]
Name: Mm tide

Designation in USCGS Special Publication No. 98: $A_2$

The Mm tide is the lunar monthly constituent. It expresses the effect of irregularities in the moon's rate of change of distance and speed in orbit.

Equilibrium arguments:

$$V(20) = s - p$$
$$u(20) = 0$$

Speed:

$$a(20) = 0.5443747 \, ^\circ/\text{hour}$$

Node factor:

$$f(20) = \frac{2}{3} \cdot \frac{-\sin^2 I}{0.5021}$$
Name: Ssa tide

Designation in USCGS Special Publication No. 98: $B_6$

The Ssa tide is the solar semiannual constituent. This constituent, along with the Sa constituent (#22 of 37), accounts for the nonuniform changes in the sun's declination and distance. In actuality, they mostly reflect yearly meteorological variations influencing sea level.

Equilibrium arguments:

\[ V(21) = 2h \]
\[ u(21) = 0 \]

Speed:

\[ a(21) = 0.0821373 \degree/\text{hour} \]

Node factor:

\[ f(21) = 1 \]
Name: Sa tide

Designation in USCGS Special Publication No. 98: B₆₄

The Sa tide is the solar annual constituent. (see #21 of 37, the Ssa constituent)

Equilibrium arguments:

\[ V(22) = h \]
\[ u(22) = 0 \]

Speed:

\[ a(22) = 0.0410686 \, ^\circ/\text{hour} \]

Node factor:

\[ f(22) = 1 \]
Name: MSf tide

Designation in USCGS Special Publication No. 98: A₃

The MSf tide is the lunisolar synodic fortnightly constituent.

Equilibrium arguments:

\[ V(23) = 2s - 2h \]
\[ u(23) = 0 \]

Speed:

\[ a(23) = 1.0158958 \, ^\circ/\text{hour} \]

Node factor:

\[ f(23) = f(20) \]
Name: Mf tide

Designation in USCGS Special Publication No. 98: \( \lambda_6 \)

The Mf tide is the lunar fortnightly constituent. This constituent expresses the effect of departure from a sinusoidal declinational motion.

Equilibrium arguments:

\[ V(24) = 2s \]
\[ u(24) = -2\xi \]

Speed:

\[ a(24) = 1.0980331 \, ^\circ/\text{hour} \]

Node factor:

\[ f(24) = \frac{\sin^2 \frac{I}{2}}{0.1578} \]
Name: $\rho_1$ tide

Designation in USCGS Special Publication No. 98: $A_{18}$

The $\rho_1$ tide is the larger lunar evectional diurnal constituent.

Equilibrium arguments:

$$V(25) = T - 3s + 3h - p + 90^\circ$$
$$u(25) = 2\xi - \nu$$

Speed:

$$a(25) = 13.4715145 \degree/\text{hour} \quad (\text{diurnal})$$

Node factor:

$$f(25) = f(6)$$
Name: Q₁ tide

Designation in USCGS Special Publication No. 98: A₁₅

The Q₁ tide is the larger lunar elliptic diurnal constituent. (see the M₁ constituent, #18 of 37)

Equilibrium arguments:

\[ V(26) = T - 3s + h + p + 90° \]
\[ u(26) = 2\xi - \nu \]

Speed:

\[ a(26) = 13.3986609°/\text{hour} \quad (\text{diurnal}) \]

Node factor:

\[ f(26) = f(6) \]
Name: T₂ tide

Designation in USCGS Special Publication No. 98: B₄₀

The T₂ tide is the larger solar elliptic constituent. This constituent, along with R₂ (#28 of 37), modulates the amplitude and frequency of S₂ for the effect of variation in the earth's orbital speed due to its elliptical orbit.

Equilibrium arguments:

\[ V(27) = 2T - h + p₁ \]
\[ u(27) = 0 \]

Speed:

\[ a(27) = 29.9589333 \degree/\text{hour} \quad (\text{semdiurnal}) \]

Node factor:

\[ f(27) = 1 \]
constituent #28 of 37

Name: R_2 tide

Designation in USCGS Special Publication No. 98: B_{41}

The R_2 tide is the smaller solar elliptic constituent. (see the T_2 constituent, #27 of 37)

Equilibrium arguments:

\[ V(28) = 2T + h - p_i + 180^\circ \]
\[ u(28) = 0 \]

Speed:

\[ a(28) = 30.0410667^\circ/\text{hour} \quad (\text{semidiurnal}) \]

Node factor:

\[ f(28) = 1 \]
The (2Q), tide is a lunar elliptic, second-order, diurnal constituent.

Equilibrium arguments:

\[ V(29) = T - 4s + h + 2p + 90° \]
\[ u(29) = 2\xi - \nu \]

Speed:

\[ a(29) = 12.8542862 °/\text{hour} \quad (\text{diurnal}) \]

Node factor:

\[ f(29) = f(6) \]
Name: $P_1$ tide

Designation in USCGS Special Publication No. 98: $B_{14}$

The $P_1$ tide is a solar diurnal constituent. (see the $K_1$ constituent, #4 of 37)

Equilibrium arguments:

$$V(30) = T - h + 90^\circ$$

$$u(30) = 0$$

Speed:

$$a(30) = 14.9589314 \, ^\circ/\text{hour} \quad \text{(diurnal)}$$

Node factor:

$$f(30) = 1$$
Name: \((2SM)_{2}\) tide

Designation in USCGS Special Publication No. 98: Shallow Water Constituent

The \((2SM)_{2}\) tide is a compound semidiurnal constituent. Its origin is given by the formula \(2S_{2} - M_{2}\). Note that \(V(31) = 2V(2) - V(1)\) and \(u(31) = 2u(2) - u(1)\)

Equilibrium arguments:

\[
V(31) = 2T + 2s - 2h
\]
\[
u(31) = -2\xi + 2\nu
\]

Speed:

\[
a(31) = 31.0158958 \, ^{\circ}/\text{hour} \quad (\text{semidiurnal})
\]

Node factor:

\[f(31) = f(1)\]
Name:  \( M_3 \) tide

Designation in USCGS Special Publication No. 98:  \( A_{32} \)

The \( M_3 \) tide is a lunar terdiurnal constituent.

Equilibrium arguments:

\[
V(32) = 3T - 3s + 3h \\
u(32) = 3\xi - 3\nu
\]

Speed:

\[
a(32) = 43.4761563 \, ^\circ/\text{hour} \quad \text{(terdiurnal)}
\]

Node factor:

\[
f(32) = [f(1)]^{3/2} = \frac{\cos^6 \frac{T}{2}}{0.8758}
\]
The \( L_2 \) tide is the smaller lunar elliptic semidiurnal constituent. This constituent, along with constituent \( N_2 \) ( #3 of 37 ), modulates the amplitude and frequency of \( M_2 \) for the effect of variation in the moon's orbital speed due to its elliptical orbit.

Equilibrium arguments:

\[
V(33) = 2T - s + 2h - p + 180^\circ
\]

\[
u(33) = 2\xi - 2\nu - R
\]

Speed:

\[
a(33) = 29.5284789 \, ^\circ/\text{hour} \quad (\text{semidiurnal})
\]

Node factor:

\[
f(33) = \frac{f(1)}{R_a}
\]

where

\[
R_a = \frac{1}{\sqrt{1 - 12 \tan^2 \frac{I}{2} \cos 2P + 36 \tan^4 \frac{I}{2}}}
\]

and \( P = p - \xi \)
Name: (2MK), tide

Designation in USCGS Special Publication No. 98: Shallow Water Constituent

The (2MK), tide is a shallow water terdiurnal compound constituent. It is compounded according to the formula $2M_2 - K_1$.

Equilibrium arguments:

\[ V(34) = 3T - 4s + 3h + 90^\circ \]
\[ u(34) = 4\xi - 4\nu + \nu' \]

Speed:

\[ a(34) = 42.9271398^\circ/\text{hour} \quad (\text{terdiurnal}) \]

Node factor:

\[ f(34) = [f(1)]^2 \times f(4) \]
constituent #35 of 37

Name: K₂ tide

Designation in USCGS Special Publication No. 98: A₄₇ and B₄₇

The K₂ tide is a lunisolar semidiurnal constituent. This constituent modulates the amplitude and frequency of M₂ and S₂ for the declinational effect of the moon and sun respectively.

Equilibrium arguments:

\[ V(35) = 2T + 2h \]
\[ u(35) = -2 \nu'' \]

Speed:
\[ a(35) = 30.0821373 \, ^\circ/hour \quad (\text{semidiurnal}) \]

Node factor:

\[ f(35) = \frac{C_2}{<C_2>} = \frac{\sqrt{A^2 + 2AB \cos 2\nu + B^2}}{(.5 + .75e^2) \sin^2 \omega (1 - 1.5 \sin^2 i) + B} \]

where

\[ A = (.5 + .75e^2) \sin^2 I \]
\[ B = (.5 + .75e_1^2) S' \sin^2 \omega \]
\[ e = 0.054900489 \quad (\text{eccentricity of moon's orbit}) \]
\[ e_1 = 0.01675104 \quad (\text{eccentricity of sun-earth orbit}) \]
\[ S' = 0.4602 \]
constituent #36 of 37

Name: M₈ tide

Designation in USCGS Special Publication No. 98: Shallow Water Constituent

The M₈ tide is a shallow water overtide of the principal lunar semidiurnal constituent, M₂. Its equilibrium arguments and its speed are four times those of M₂.

Equilibrium arguments:

\[ V(36) = 8T - 8s + 8h \]
\[ u(36) = 8\xi - 8\nu \]

Speed:

\[ a(36) = 115.9364169 \, ^\circ/\text{hour} \]

Node factor:

\[ f(36) = [f(1)]^4 \]
constituent #37 of 37

Name: \((MS)_4\) tide

Designation in USCGS Special Publication No. 98: Shallow Water Constituent

The \((MS)_4\) tide is a quarter diurnal shallow water compound constituent. Its origin is given by \(M_2 + S_2\).

Equilibrium arguments:

\[
V(37) = 4T - 2s + 2h
\]
\[
u(37) = 2\xi - 2\nu
\]

Speed:

\[
a(37) = 58.9841042 \, ^\circ/\text{hour}
\]

Node factor:

\[
f(37) = f(1)
\]
VII. COMPUTER PROGRAM

The computer program used in this thesis was written in the language of Microsoft Quickbasic 4.5. The code was developed to run on IBM-PC and compatible machines.

Quickbasic 4.5 automatically compiles before running the source code.

The compiled code runs efficiently, producing an entire tide curve for a day in a few seconds on a 386 machine with math coprocessor. The tide height is calculated for every minute of the day, producing $24 \times 60 = 1440$ points on a day’s graph.

The complete program listing is given in Appendix A.

VIII. CHECKING THE COMPUTER PROGRAM

The expected accuracy is plus or minus 5 minutes, or better, in the time of high or low water. The expected accuracy in tide height is plus or minus about 0.2 foot, formally. However, the actual tide height on any given day can differ from prediction because of the effects of wind, barometric pressure, and other meteorological variables.

The program was checked against the published tide tables of the NOS, as shown in the following table. Note that the present program evaluates astronomical factors 1440 times per day, while the NOS program evaluates certain factors (in particular, the nodal factors) only once per year, and uses
those values for the entire year. Therefore, the present program should be as accurate, or more accurate, than the NOS program. Slight differences (plus or minus a few minutes, and plus or minus a few tenths of a foot) in the following table are attributable to the different algorithms.

**COMPARISON OF PROGRAM PREDICTIONS WITH NOS TIDE TABLE**
(for Boston, Massachusetts)

<table>
<thead>
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<th>Date</th>
<th>Thesis Program Time</th>
<th>Thesis Program Height</th>
<th>NOS Tide Table Time</th>
<th>NOS Tide Table Height</th>
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<td>hr min</td>
<td>feet</td>
<td>hr min</td>
<td>feet</td>
</tr>
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<td>1.233</td>
<td>01:59</td>
<td>1.2</td>
</tr>
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<td>08:22</td>
<td>10.048</td>
<td>08:22</td>
<td>10.1</td>
</tr>
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<td></td>
<td>14:50</td>
<td>0.355</td>
<td>14:50</td>
<td>0.3</td>
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<td>21:02</td>
<td>8.6</td>
</tr>
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<td>02:50</td>
<td>1.2</td>
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<td>03:36</td>
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IX. APPLICATION

The motivation for the present program was to produce tide calculations for use in several other projects at SWTSU.

Results from this program have already been used to calculate the time of high and low waters in Boston Harbor on April 18 and 19, 1775. April 18 was the night on which Paul Revere began his famous ride, and April 19 was the date of the battles at Lexington and Concord, which began the American Revolution.

Graphs of the Boston tides on those days and a pre-print of a published article which included results from this program are included in Appendix D. The article appeared as "Paul Revere's Midnight Ride" in the April 1992 issue of Sky & Telescope magazine.39
X. CONCLUSION

A computer program was written based on the theoretical foundation delineated in Special Publication No. 98. The program's precision was enhanced by application of the techniques of Wallner. Careful verification of tide calculations from the program indicate that tide predictions can be extended, without appreciable accumulation of error, over a time interval of 2000 years.

The use of harmonic analysis in tide prediction is an excellent example of using astronomical theory and mathematical physics to produce results of interest.
APPENDIX A

Program Listing: TIDES

Microsoft QUICKBASIC 4.5
for IBM PC (or compatible)
TIDES

based on
MANUAL OF HARMONIC ANALYSIS AND PREDICTION OF TIDES
Special Publication No. 98
Coast and Geodetic Survey
Revised (1940) edition

DEFDBL A-Z
DIM AMPL(37), KAPPA(37), NAMECONST$(37)
DIM F(37), V(37), U(37)
SCREEN 9: CLS
PI = 4# * ATN(1#)
TWOPI = 2# * PI

100 'READ IN PORT AND STATION CONSTANTS
READ CITYNAME$
READ LONGITUDEDEG, LONGITUDEMIN, LONGITUDEEW$
READ LATITUDEDEG, LATITUDEMIN, LATITUDENS$
READ TIMEMERIDIAN, TIMEMERIDIANEW$
READ Z0, HTMIN, HTMAX
FOR J = 1 TO 37
READ AX, NAMECONST$(J), AMPL(J), KAPPA(J)
KAPPA(J) = KAPPA(J) * PI / 180#
LONGITUDE = LONGITUDEDEG + LONGITUDEMIN / 60
IF LONGITUDEEW$ = "E" THEN LONGITUDE = -LONGITUDE
IF TIMEMERIDIAN$ = "E" THEN TIMEMERIDIAN = -TIMEMERIDIAN
NEXT J

' INCREASE CONSTITUENT AMPLITUDES FOR 1987 TIDE TABLES
' (EXCEPT Sa and Ssa)
FOR J = 1 TO 37
IF J <> 21 AND J <> 22 THEN AMPL(J) = AMPL(J) * 1.024
NEXT J

200 'INPUT Y/M/D FROM KEYBOARD
GOSUB 10000 ' CALENDAR TO JD
JDUTOHUT = JD
TAPPROX = (Y - 1900) / 100
DELT = (.411 + 1.205311 * TAPPROX + .511 * TAPPROX * TAPPROX) / 1440#
JDETOHUT = JDUTOHUT + DELT

300 'ASTRONOMICAL DATA
JDET = JDETOHUT
T = (JDET - 2415020#) / 36525#
GOSUB 20000 'COMPUTE S, H, P, PI, N
GOSUB 30000 'COMPUTE N, I, NU, XI, ..., Q

CLS
PRINT "ASTRONOMICAL DATA"
PRINT MO; "/", DA; "/", IY;
PRINT TAB(50); USING "JDUT "###.########"; JDUTOHUT
PRINT USING "TIME T = "###.########"; T;
PRINT TAB(50); USING "JDET "###.########"; JDETOHUT
PRINT USING "MEAN LONG. OF MOON (s) "###.#######"; S * 180# / PI
PRINT USING "MEAN LONG. OF SUN (h) "###.#######"; H * 180# / PI
PRINT USING "MEAN LONG. OF LUNAR PERIGEE (p) "###.#######"; P * 180# / PI
LOCATE 24, 1: PRINT "HARD COPY ASTRONOMICAL DATA ? (Y/N)";
WHILE INKEY$ <> "": WEND
340 HCADS$ = INKEY$
IF HCADS$ = "" THEN GOTO 340
IF HCADS$ = "Y" OR HCADS$ = "y" THEN
LPRINT "ASTRONOMICAL DATA"
LPRINT MO: "/"; DA; "/"; IY;
LPRINT TAB(50); USING "JDUT 180#/#180#"; JDUT0HUT
LPRINT "TIME T = " 180#/#180#"; T;
LPRINT TAB(50); USING "JDET 180#/#180#"; JDET0HUT
LPRINT USING " MEAN LONG. OF MOON (s) 180#/#180#"; S * 180# / PI
LPRINT USING " MEAN LONG. OF SUN (h) 180#/#180#"; H * 180# / PI
LPRINT USING " MEAN LONG. OF LUNAR PERIGEE (p1) 180#/#180#"; P1 * 180# / PI
LPRINT USING " MEAN LONG. OF LUNAR ASC. NODE (N) 180#/#180#"; N * 180# / PI
LPRINT USING " OBLIQUITY OF LUNAR ORBIT (I) 180#/#180#"; I * 180# / PI
LPRINT USING " OBLIQUITY OF ECLIPTIC (OMEGA) 180#/#180#"; OMEGA* 180# / PI
LPRINT USING " INCLINATION OF LUNAR ORBIT (INCL) 180#/#180#"; INCL * 180# / PI
LPRINT USING " RT. ASCENSION OF INTERSECTION (NU) 180#/#180#"; NU * 180# / PI
LPRINT USING " LONG. IN ORBIT OF INTERSECTION (XI) 180#/#180#"; XI * 180# / PI
LPRINT USING " TERM IN ARGUMENT OF K1 CONSTITUENT (NUP) 180#/#180#"; NUP * 180# / PI
LPRINT USING " TERM IN ARGUMENT OF K2 CONSTITUENT (NUPP) 180#/#180#"; NUPP * 180# / PI
LPRINT USING " MEAN LONG. LUNAR PERIGEE FROM INT. (CP) 180#/#180#"; CP * 180# / PI
LPRINT USING " FACTOR IN AMPL. OF L2 CONSTITUENT (RA) 180#/#180#"; RA
LPRINT USING " TERM IN ARGUMENT OF L2 CONSTITUENT (R) 180#/#180#"; R * 180# / PI
LPRINT USING " FACTOR IN AMPL. OF M1 CONSTITUENT (QA) 180#/#180#"; QA
LPRINT USING " TERM IN ARGUMENT OF M1 CONSTITUENT (QU) 180#/#180#"; QU * 180# / PI
LPRINT USING " TERM IN ARGUMENT OF M1 CONSTITUENT (Q) 180#/#180#"; Q * 180# / PI
LPRINT
LPRINT " GOTO 330"
END IF

500 'NODE FACTORS AND ARGUMENTS
THA = PI - LONGITUDE * PI / 180#
GOSUB 40000 ' COMPUTE NODE FACTORS F(J), ARGUMENTS U(J)
GOSUB 50000 ' COMPUTE ARGUMENTS V(J)
CLS

LL = 1: UL = 13
PRINT "NODE FACTORS AND ARGUMENTS"
PRINT CITYNAME$; " ";
PRINT LONGITUDEDEG; CHR$(248); LONGITUDEMIN; ""; LONGITUDEDEWS; " ";
PRINT LATITUDEDEG; CHR$(248); LATITUDEMIN; ""; LATITUDENS
PRINT MO; "; DA; "; IY; " Oh UT"
PRINT USING "JDUT #######.####"; JDUT0HUT
PRINT USING "JDET #######.####"; JDETOHUT
PRINT USING "LONG. OF LUNAR ASC. NODE (N) ####### "; N * 180# / PI
PRINT
PRINT " J CONSTIT F AMPL F*AMPL V U V+U KAPPA V+U-K" FOR J = LL TO UL
PRINT USING "## "; J;
PRINT NAMECONSTIT$(J);
PRINT USING "####.####"; F(J);
PRINT USING "##.##"; AMPL(J);
PRINT USING "##.##"; F(J) * AMPL(J);
PRINT USING "##.##"; V(J) * 180 / PI;
PRINT USING "##.##"; U(J) * 180 / PI;
VPLUSU = V(J) + U(J)
VPLUSU = VPLUSU - TWOPI * INT(VPLUSU / TWOPI)
PRINT USING "##.##"; VPLUSU * 180 / PI;
PRINT USING "##.##"; KAPPA(J) * 180 / PI;
VUKAPPA = V(J) + U(J) - KAPPA(J)
VUKAPPA = VUKAPPA - TWOPI * INT(VUKAPPA / TWOPI)
PRINT USING "##.##"; VUKAPPA * 180 / PI
NEXT J
WHILE INKEY$ <> "": WEND
PRINT: PRINT "HIT SPACE BAR TO CONTINUE"
WHILE INKEY$ <> CHR$(32): WEND
IF UL = 13 THEN LL = 14: UL = 26: GOTO 550
IF UL = 26 THEN LL = 27: UL = 37: GOTO 550

LOCATE 23, 1: PRINT "HARD COPY NODE FACTORS AND ARGUMENTS? (Y/N)"
WHILE INKEY$ <> "": WEND

HCNFAS = INKEY$
IF HCNFAS = "" THEN GOTO 570
IF HCNFAS = "Y" OR HCNFAS = "y" THEN
PRINT "NODE FACTORS AND ARGUMENTS"
PRINT CITYNAME$; " ";
PRINT LONGITUDEDEG; CHR$(248); LONGITUDEMIN; ""; LONGITUDEDEWS; " ";
PRINT LATITUDEDEG; CHR$(248); LATITUDEMIN; ""; LATITUDENS
PRINT MO; "; DA; "; IY; " Oh UT"
PRINT USING "JDUT #######.####"; JDUT0HUT
PRINT USING "JDET #######.####"; JDETOHUT
PRINT USING "LONG. OF LUNAR ASC. NODE (N) ####### "; N * 180# / PI
PRINT
PRINT " J CONSTIT F AMPL F*AMPL V U V+U KAPPA V+U-K" FOR J = 1 TO 37
PRINT USING "## "; J;
PRINT NAMECONSTIT$(J);
PRINT USING "##.##"; F(J);
PRINT USING "##.##"; AMPL(J);
PRINT USING "##.##"; F(J) * AMPL(J);
PRINT USING "##.##"; V(J) * 180 / PI;
LPRINT USING "##H#.H#"; U(J) * 180 / PI;
VPLUSU = V(U(J) + U(J))
VPLUSU = VPLUSU - TWOP * INT(VPLUSU / TWOPI)
LPRINT USING "###.###"; VPLUSU * 180 / PI;
LPRINT USING "##H#.H#"; VPLUSU = VPLUSU - TWOP * INT(VPLUSU) / TWOPI
VUKAPPA = V(U(J) + U(J) - KAPPA(J))
VUKAPPA = VUKAPPA - TWOP * INT(VUKAPPA / TWOPI)
LPRINT USING "###.###"; VUKAPPA * 180 / PI
NEXT J
LPRINT CHR$(12)
GOTO 560
END IF

'MENU FOR OUTPUT OF TIDE HEIGHTS
CLS : HCHILO$ = "N"; HCTHVSTS$ = "N"
LOCATE 15, 1
PRINT "SPACEBAR.....GRAPH ONLY"
PRINT : PRINT " 1 ....GRAPH & HARD COPY HIGH AND LOW WATERS"
PRINT : PRINT " 2 ....GRAPH & HARD COPY TIDE HEIGHT vs. TIME"
PRINT " AND HIGH AND LOW WATERS"
PRINT : PRINT " 3 ...ANOTHER DATE"
WHILE INKEY$ < > "": WEND
A$ = INKEY$
IF A$ = CHR$(32) THEN GOTO 700
IF A$ = "1" THEN HCHILO$ = "Y": GOTO 680
IF A$ = "2" THEN HCTHVSTS$ = "Y": GOTO 650
IF A$ = "3" THEN
CLS
LOCATE 15, 1
PRINT "LAST DATE WAS: ";
PRINT MO; "/"; DA; "/"; IY
PRINT
GOTO 200
END IF
GOTO 620
PRINT : INPUT "TIME STEP (in minutes) = "; TIMESTEP
'HEADER FOR HARD COPY OUTPUT (either 1 or 2)
LPRINT : LPRINT MO; "/"; DA; "/"; IY;
LPRINT CITYNAME$; " ";
LPRINT "TIME MERIDIAN "; TIMEMERIDIAN: CHR$(248); TIMEMERIDIANEW$; " ";
LPRINT " (ZO "; LPRINT USING "H#.UN"; ZO; IF)"
'DRAW FRAME AND LABELS
CLS
LINE (XL, YT)-(XR, YB), \, B
FOR J = 0 TO 24
XP = XL + (XR - XL) * (J / 24)
LINE (XP, YB)-(XP, YB - LT): LINE (XP, YT)-(XP, YT + LT)
IF J / 6 = INT(J / 6) THEN
AX = 2 * LT
LINE (XP, YB)-(XP, YB - AX): LINE (XP, YT)-(XP, YT + AX)
XC = 80 * (XL / 640) + (J / 24) * (XR - XL) * (80 / 640)
YC = 25 * (YB / 350)
LOCATE YC + 2, XC - 1
PRINT J; "h";
END IF
NEXT J
FOR J = HTMIN TO HTMAX
YP = YB - (YB - YT) * (J - HTMIN) / (HTMAX - HTMIN)
LINE (XL, YP)-(XL + LT, YP): LINE (XR, YP)-(XR - LT, YP)
IF (J / 5) = INT(J / 5) THEN
AX = 2 * LT
LINE (XL, YP)-(XL + AX, YP): LINE (XR, YP)-(XR - AX, YP)
XC = 80 * XL / 640
LOCATE YC + .5, XC - 5
PRINT J;
END IF
NEXT J
LOCATE 1, 24
PRINT MO; "/"; DA; "/"; IY;
LOCATE 21, 1
PRINT CITYNAMES; ";
PRINT LONGITUDEDEG; CHR$(248); LONGITUDEMIN; "": LONGITUDEEWS; " ";
PRINT LATITUDEDEG; CHR$(248); LATITUDEMIN; "": LATITUDEW$;
PRINT " (TIME MERIDIAN ", TIMEMERIDIAN; CHR$(248); TIMEMERIDIANEWS; ")";
PRINT " (Z0; : PRINT USING "##.##", Z0; : PRINT ")"
'HIGH AND LOW WATERS TO SCREEN
AY = .5 + .5 * (YT + YB) * 25 / 350
AX = XL * 80 / 640
LOCATE AY, .2 * AX
PRINT ";
AY = YT * 25 / 350
LOCATE AY + 2, EXTRCOL
PRINT "HIGH AND LOW WATERS"
LOCATE AY + 4, EXTRCOL
PRINT ";
1000 'TIDE HEIGHT VS. TIME
HTPP = 0: HTP = 0
LL = -1: UL = 1441
FLAGMAX = Timestep
FLAG = FLAGMAX + LL
JDUTOHLLOCAL = JDUTOHUT + TIMEMERIDIAN / 360#
FOR MINUTEOFDAY = LL TO UL STEP 1
JDUT = JDUTOHLLOCAL + MINUTEOFDAY / 1440#
JDET = JDUT + DELT
T = (JDUT - 24150200) / 36525#
GOSUB 20000 'GIVEN: TIME T COMPUTE: S, H, P, P1, N
THA = PI + (MINUTEOFDAY * PI / 720#) + (TIMEMERIDIAN - LONGITUDE) * PI / 180#
GOSUB 50000 'GIVEN: THA,S,H,P,P1 COMPUTE: V(J)
'COMPUTE TIDE HEIGHT
HT = Z0
FOR J = 1 TO 37
HT = HT + F(J) * AMPL(J) * COS(V(J) + U(J) - KAPPA(J))
NEXT J
'PLOT POINT ON GRAPH
XP = XL + (XR - XL) * (MINUTEOFDAY / 1440#)
YP = YB - (YB - YT) * (HT - HTMIN) / (HTMAX - HTMIN)
PSET (XP, YP)

1500 'CHECK FOR HIGH OR LOW WATER
IF MINUTEOFDAY < 1 THEN 1600
EXTRS = ""
IF (HTP - HTPP) * (HT - HTP) < 0 AND (HT - HTP) < 0 THEN EXTRS = "HIGH WATER": GOTO
1550 IF (HTP - HTPP) * (HT - HTP) < 0 AND (HT - HTP) > 0 THEN EXTRS = "LOW WATER": GOTO
1550 GOTO 1600
1550 HOUREXTR = INT(.0001# + (MINUTEOFDAY - 1#) / 60#)
MINEXTR = MINUTEOFDAY - 60# * HOUREXTR - 1
HTEXTR = HTP
HRS = RIGHTS(STRS(HOUREXTR), 2)
IF HOUREXTR < 10 THEN HRS = "0" + RIGHTS(HRS, 1)
MNS = RIGHTS(STRS(MINEXTR), 2)
IF MINEXTR < 10 THEN MNS = "0" + RIGHTS(MNS, 1)
Y = CSRLIN: LOCATE Y + 2, EXTRCOL
PRINT HRS; ":"; MNS;
PRINT USING "###.###": HTEXTR;
LPRINT TIDE HEIGHT AND/OR HIGH AND LOW WATERS
IF HCHILO$ = "Y" OR HCTHVST$ = "Y" THEN
LPRINT HRS; ":"; MNS;
LPRINT USING "###.###": HTEXTR;
LPRINT " <-- ": EXTRS
END IF
1600 IF FLAG = FLAGMAX AND HCTHVST$ = "Y" THEN
HOUR = INT(.0001# + (MINUTEOFDAY) / 60#)
MIN = MINUTEOFDAY - 60# * HOUR
HRS = RIGHTS(STRS(HOUR), 2)
IF HOUR < 10 THEN HRS = "0" + RIGHTS(HRS, 1)
MNS = RIGHTS(STRS(MIN), 2)
IF MIN < 10 THEN MNS = "0" + RIGHTS(MNS, 1)
LPRINT HRS; ":"; MNS;
LPRINT USING "###.###": HT
FLAG = 0
END IF
1980 HTPP = HTP: HTP = HT
FLAG = FLAG + 1
1990 NEXT MINUTEOFDAY
LOCATE 23, 1, 0
WHILE INKEY$ <> "": WEND
2000 A$ = INKEY$
IF A$ = CHR$(32) THEN GOTO 2100
GOTO 2000
2100 PRINT "Press D for another date or ESC to quit";
2200 A$ = INKEY$
IF A$ = CHR$(27) THEN GOTO 9000
IF A$ = "D" THEN
CLS
LOCATE 15, 1
PRINT "LAST DATE WAS: ";
PRINT MO; "/"; DA; "/"; IY
PRINT
GOTO 200
END IF
GOTO 2200

9000 END

10000 'CALENDAR TO JD
PRINT "INPUT CALENDAR DATE"
WHILE INKEY$ <> "": WEND
PRINT : INPUT "YEAR = "; A$
IY = VAL(A$)
IF IY < INT(IY) THEN 10000
PRINT : INPUT "MONTH (1-12) = "; A$: MO = VAL(A$)
IF MO < INT(MO) THEN 10000
PRINT : INPUT "DAY = "; A$: DA = VAL(A$)
IF DA < INT(DA) THEN 10000
D = 0 ' ZERO HOURS, MINUTES, AND SECONDS
IF MO = 1 OR MO = 2 THEN Y = IY - 1: M = MO + 12
IF MO > 2 THEN Y = IY: M = MO
C = 0
IF Y < 0 THEN C = -.75
A = INT(.001 + Y / 100#)
B = 2# - A + INT(.001 + A / 4#)
IF IY < 1582 THEN B = 0
IF IY = 1582 AND MO < 10 THEN B = 0
IF IY = 1582 AND MO = 10 AND DA < 10 THEN B = 0
AX = 365.25# * Y + C
IF AX >= 0 THEN JD = INT(AX + .01#)
IF AX < 0 THEN JD = INT(AX - .01#)
JD = JD + INT(30.6001# * (M + 1))
JD = JD + DA + D + 1720994.5# + B
WHILE INKEY$ <> "": WEND
RETURN

20000 'GIVEN: TIME T  COMPUTE: S, H, P, PI, N
  (ASTRONOMICAL FACTORS)
S = 270.434164# + 481267.8831# * T - .001133# * T * T + .0000019# * T * T * T
S = S * PI / 180#
S = S - TWOPI * INT(S / TWOPI)
H = 279.69668# + 36000.76892# * T + .0003025# * T * T
H = H * PI / 180#
H = H - TWOPI * INT(H / TWOPI)
P = 334.329556# + 4069.034# * T - .010325# * T * T - .0000125# * T * T * T
P = P * PI / 180#
P = P - TWOPI * INT(P / TWOPI)
M = 358.47583# + 35999.04975# * T - .001133# * T * T - .0000019# * T * T * T
M = M * PI / 180#
M = M - TWOPI * INT(M / TWOPI)
P1 = 1 - H - M
P1 = P1 - TWOPI * INT(P1 / TWOPI)
N = 259.183275# - 1934.142# * T + .002078# * T * T + .0000022# * T * T * T
N = N * PI / 180#
N = N - TWOPI * INT(N / TWOPI)
RETURN

30000 'GIVEN: TIME T  COMPUTE: N, I, NU, XI, ..., Q
  (ASTRONOMICAL FACTORS)
N = 259.183275 - 1934.142# * T + .002078# * T * T + .0000022# * T * T * T
N = N * PI / 180#
N = N - TWOPI * INT(N / TWOPI)
OMEGA = 23.452294# - .0130125# * T - .000000503# * T * T * T
INCL = 5.1453964#
INCL = INCL * PI / 180#
INCL = INCL - TWOPI * INT(N / TWOPI)
OMEGA = OMEGA + PI * INT(N / TWOPI)

\[ \begin{align*}
N & = N \times \pi / 180 \\
N & = N - 2\pi \times \text{INT}(N / 2\pi) \\
\text{OMEGA} & = 23.452294 - 0.0130125 \times T - 0.000000503 \times T \times T \times T \\
\text{INCL} & = 5.1453964 \\
\text{INCL} & = \text{INCL} \times \pi / 180 \\
\text{INCL} & = \text{INCL} - 2\pi \times \text{INT}(N / 2\pi) \\
\text{OMEGA} & = \text{OMEGA} + \pi \times \text{INT}(N / 2\pi) \\
\end{align*} \]

\[ \begin{align*}
\text{CP} & = \text{PI} / 180 \\
\text{P} & = \text{CP} - \text{TWOPI} \times \text{INT}(\text{CP} / \text{TWOPI}) \\
\text{RA} & = 1 / \text{SQR}(1 - 12 \times \text{TAN}(\text{II2} / 2) \times \cos(\text{INCL} / 2) + 36 \times \text{TAN}(\text{II2} / 2)^2 - \cos(2\times \text{CP})) \\
\text{QA} & = 1 / \text{SQR}(0.25 + 1.5 \times \text{SIN}(\text{I}) \times \cos(\text{OMEGA} / 2) + 2.25 \times \text{SIN}(\text{OMEGA} / 2)^2 / \cos(\text{OMEGA} / 2)^4) \\
\text{QU} & = \text{ATN}(\text{SIN}(\text{CP}) / (\text{SIN}(\text{I} / 2) \times \text{COS}(\text{OMEGA} / 2) + \cos(2\times \text{CP}))) \\
\text{Q} & = \text{ATN}(0.5 \times \text{SIN}(\text{I}) - 1 \times \text{TAN}(\text{CP}) / (1 \times \text{COS}(\text{I} + 1)) \\
\text{IF} \cos(\text{CP}) < 0 \text{ THEN } \text{Q} = \text{Q} + \pi \\
\text{Q} & = \text{Q} - 2\pi \times \text{INT}(\text{Q} / 2\pi) \\
\text{RETURN} \\
\end{align*} \]
F(21) = 1#
F(22) = 1#
F(23) = F(20)
F(24) = SIN(I) / (SIN(OMEGA) * 2 * COS(INCL / 2#) * 4)
F(25) = F(6)
F(26) = F(6)
F(27) = 1#
F(28) = 1#
F(29) = F(6)
F(30) = 1#
F(31) = F(1)
F(32) = F(1) * 1.5
F(33) = F(1) / RA
F(34) = F(1) * F(1) * F(4)
SPRIME = .4602#: ECCM = .05490489#: ECCS = .01675104#
AK2 = (.5# + .75# * ECCM * 2) * SIN(I) * 2
BK2 = (.5# + .75# * ECCS * 2) * SPRIME * SIN(OMEGA) * 2
CK2 = SQR(AK2 * 2 + 2# * AK2 * BK2 * COS(2# * NU) + BK2 * 2)
AVGCK2 = (.5# + .75# * ECCM') * SIN(OMEGA) * 2 * (1# - 1.5# * SIN(INCL) * 2) + BK2
F(35) = CK2 / AVGCK2
F(36) = F(1) * 4
F(37) = F(1)
U(1) = 2# * XI - 2# * NU
U(2) = 0#
U(3) = 2# * XI - 2# * NU
U(4) = -NU
U(5) = 4# * XI - 4# * NU
U(6) = 2# * XI - NU
U(7) = 6# * XI - 6# * NU
U(8) = 2# * XI - 2# * NU - NUP
U(9) = 0#
U(10) = 4# * XI - 4# * NU
U(11) = 2# * XI - 2# * NU
U(12) = 0#
U(13) = 2# * XI - 2# * NU
U(14) = 2# * XI - 2# * NU
U(15) = -2# * XI - NU
U(16) = 2# * XI - 2# * NU
U(17) = 0#
U(18) = -NU - QU
U(19) = -NU
U(20) = 0#
U(21) = 0#
U(22) = 0#
U(23) = 0#
U(24) = -2# * XI
U(25) = 2# * XI - NU
U(26) = 2# * XI - NU
U(27) = 0#
U(28) = 0#
U(29) = 2# * XI - NU
U(30) = 0#
U(31) = -2# * XI + 2# * NU
U(32) = 3# * XI - 3# * NU
U(33) = 2# * XI - 2# * NU - R
U(34) = 4# * XI - 4# * NU + NUP

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U(35) = -2 * NUPP
U(36) = 8 * XI - 8 * NU
U(37) = 2 * XI - 2 * NU
RETURN

5000 ' GIVEN: THA,S,H,P,P1 COMPUTE: ARGUMENTS V(J)
V(1) = 2 * THA - 2 * S + 2 * H
V(2) = 2 * THA
V(3) = 2 * THA - 3 * S + 2 * H + P
V(4) = THA + H - PI / 2#
V(5) = 4 * THA - 4 * S + 4 * H
V(6) = THA - 2 * S + H + PI / 2#
V(7) = 6 * THA - 6 * S + 6 * H
V(8) = 3 * THA - 2 * S + 3 * H - PI / 2#
V(9) = 4 * THA
V(10) = 4 * THA - 5 * S + 4 * H + P
V(11) = 2 * THA - 3 * S + 4 * H - P
V(12) = 6 * THA
V(13) = 2 * THA - 4 * S + 4 * H
V(14) = 2 * THA - 4 * S + 2 * H + 2 * P
V(15) = THA + 2 * S + H - PI / 2#
V(16) = 2 * THA - S + P + PI
V(17) = THA
V(18) = THA - S + H + P - PI / 2#
V(19) = THA + S + H - P - PI / 2#
V(20) = S - P
V(21) = 2 * H
V(22) = H
V(23) = 2 * S - 2 * H
V(24) = 2 * S
V(25) = THA - 3 * S + 3 * H - P + PI / 2#
V(26) = THA - 3 * S + H + P + PI / 2#
V(27) = 2 * THA - H + P1
V(28) = 2 * THA + H - P1 + P1
V(29) = THA - 4 * S + H + 2 * P + PI / 2#
V(30) = THA - H + PI / 2#
V(31) = 2 * THA + 2 * S - 2 * H
V(32) = 3 * THA - 3 * S + 3 * H
V(33) = 2 * THA - S + 2 * H - P + P1
V(34) = 3 * THA - 4 * S + 3 * H + PI / 2#
V(35) = 2 * THA + 2 * H
V(36) = 8 * THA - 8 * S + 8 * H
V(37) = 4 * THA - 2 * S + 2 * H
FOR J = 1 TO 37
V(J) = V(J) - TWOPI * INT(V(J) / TWOPI)
NEXT J
RETURN

60000 DATA "BOSTON"
DATA 71,3,"W"
DATA 42,21.3,"N"
DATA 75,"W"
DATA 5,22,-4,14
DATA 1,"M2",4.470,327.6
DATA 2,"S2",.704,3.8
DATA 3,"N2",",.984,295.5
| DATA | 4 | "K1" | .480,134.8 |
| DATA | 5 | "M4" | .072,101.0 |
| DATA | 6 | "O1" | .380,115.3 |
| DATA | 7 | "M6" | .110,219.0 |
| DATA | 8 | "(MK)2" | .019,14.4 |
| DATA | 9 | "S4" | .0,0 |
| DATA | 10 | "(MN)4" | .033,90.6 |
| DATA | 11 | "NU2" | .217,304.9 |
| DATA | 12 | "S6" | .0,0 |
| DATA | 13 | "M2" | .036,258.6 |
| DATA | 14 | "(2N)2" | .133,275.2 |
| DATA | 15 | "(OO)1" | .016,154.3 |
| DATA | 16 | "LMDBA2" | .068,4.3 |
| DATA | 17 | "S1" | .0,0 |
| DATA | 18 | "M1" | .027,125.0 |
| DATA | 19 | "J1" | .031,140.3 |
| DATA | 20 | "Mm" | .0,0 |
| DATA | 21 | "Ssa" | .054,96.2 |
| DATA | 22 | "Sa" | .085,142.6 |
| DATA | 23 | "MSf" | .0,0 |
| DATA | 24 | "Mf" | .0,0 |
| DATA | 25 | "Rho1" | .014,106.9 |
| DATA | 26 | "Q1" | .067,95.4 |
| DATA | 27 | "T2" | .063,337.1 |
| DATA | 28 | "R2" | .006,3.8 |
| DATA | 29 | "(2Q)1" | .010,95.8 |
| DATA | 30 | "Pi" | .153,133.1 |
| DATA | 31 | "(2SM)2" | .0,0 |
| DATA | 32 | "M3" | .0,0 |
| DATA | 33 | "L2" | .190,18.8 |
| DATA | 34 | "(2MK)2" | .023,354.0 |
| DATA | 35 | "K2" | .202,5.9 |
| DATA | 36 | "M8" | .0.019,29.9 |
| DATA | 37 | "(MS)4" | .029,148.6 |
APPENDIX B

STATION SHEETS

BOSTON (MASSACHUSETTS)
AND
OTHER PORTS
### Tides

#### Standard Harmonic Constants for Prediction

**Station**: 844-3970, Boston, MA  
**Lat**: 42° 21' 3"N  
**Long**: 71° 36' 41.4"W  
**Long**: 71° 36' 41.4"W

<table>
<thead>
<tr>
<th>Component</th>
<th>$f$ Amplitude</th>
<th>$\kappa$ Epoch</th>
<th>$\pm A$</th>
<th>$B_{180^\circ}$</th>
<th>$C_{360^\circ}$</th>
<th>$-D$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_2$</td>
<td>4.472</td>
<td>3.274</td>
<td>-0.7</td>
<td>4.577</td>
<td>-324.8</td>
<td></td>
<td>Time meridian: 75° 45' 53.52</td>
</tr>
<tr>
<td>$S_2$</td>
<td>0.704</td>
<td>2.8</td>
<td>-3.94</td>
<td>321.4</td>
<td>-355.4</td>
<td></td>
<td>Extreme range: 14 ft.</td>
</tr>
<tr>
<td>$N_2$</td>
<td>0.574</td>
<td>2.9</td>
<td>-3.94</td>
<td>321.4</td>
<td>-355.4</td>
<td></td>
<td>Dist.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_1$</td>
<td>0.480</td>
<td>1.348</td>
<td>-1.3</td>
<td>1.080</td>
<td>-295.4</td>
<td></td>
<td>Marigram gear.</td>
</tr>
<tr>
<td>$M_4$</td>
<td>0.072</td>
<td>1.010</td>
<td>-5.6</td>
<td>1.021</td>
<td>-295.4</td>
<td></td>
<td>Marigram scale.</td>
</tr>
<tr>
<td>$Q_1$</td>
<td>0.380</td>
<td>1.153</td>
<td>-1.3</td>
<td>3.389</td>
<td>-116.4</td>
<td></td>
<td>$z = 4.28$ (4.28-78) ft.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Permanent current.</td>
</tr>
<tr>
<td>$M_6$</td>
<td>0.110</td>
<td>2.190</td>
<td>-8.5</td>
<td>1.313</td>
<td>-2.105</td>
<td></td>
<td>The DATUM is a plane. ft.</td>
</tr>
<tr>
<td>$(M_{20})$</td>
<td>0.099</td>
<td>2.190</td>
<td>-7.0</td>
<td>1.099</td>
<td>-2.105</td>
<td></td>
<td>below mean (low-water-spring)</td>
</tr>
<tr>
<td>$S_6$</td>
<td>0.033</td>
<td>2.984</td>
<td>-8.1</td>
<td>0.984</td>
<td>-2.737</td>
<td></td>
<td>lower-low water.</td>
</tr>
<tr>
<td>$M_8$</td>
<td>-0.227</td>
<td>3.049</td>
<td>-5.8</td>
<td>2.022</td>
<td>-3.049</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_10$</td>
<td>0.034</td>
<td>2.086</td>
<td>2.3</td>
<td>0.37</td>
<td>-2.669</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(2N)$</td>
<td>0.133</td>
<td>2.352</td>
<td>2.6</td>
<td>0.134</td>
<td>-2.776</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(2N)$</td>
<td>0.133</td>
<td>2.352</td>
<td>2.6</td>
<td>0.134</td>
<td>-2.776</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$O_1$</td>
<td>0.054</td>
<td>2.031</td>
<td>-5.4</td>
<td>0.054</td>
<td>-2.958</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$O_2$</td>
<td>0.054</td>
<td>2.031</td>
<td>-5.4</td>
<td>0.054</td>
<td>-2.958</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$O_3$</td>
<td>0.010</td>
<td>1.956</td>
<td>6.8</td>
<td>0.101</td>
<td>-10.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_1$</td>
<td>0.153</td>
<td>1.331</td>
<td>-3.7</td>
<td>0.157</td>
<td>-12.94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(GSH)_1$</td>
<td>0.005</td>
<td>1.905</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_1$</td>
<td>0.190</td>
<td>1.88</td>
<td>-5.5</td>
<td>1.953</td>
<td>-13.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(2MK)_1$</td>
<td>0.023</td>
<td>3.54</td>
<td>-1.3</td>
<td>0.241</td>
<td>-25.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_1$</td>
<td>0.019</td>
<td>2.93</td>
<td>-3.9</td>
<td>2.970</td>
<td>-38.76</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_1$</td>
<td>0.019</td>
<td>2.93</td>
<td>-3.9</td>
<td>2.970</td>
<td>-38.76</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(MS)_1$</td>
<td>0.022</td>
<td>1.98</td>
<td>-10.7</td>
<td>0.253</td>
<td>-137.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


*Sa and Sa from 20 year monthly MSL (1954-1984)*

**Compiled by**: G.F. 2/15/85  
**Verified by**: J.S. 2/1/85

---

82
<table>
<thead>
<tr>
<th>Component</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>1.688</td>
<td>2.70.1</td>
<td>+4.1</td>
<td>-27.48</td>
<td>Time meridian 120° W = -48 h.</td>
</tr>
<tr>
<td>S1</td>
<td>0.865</td>
<td>2.61.3</td>
<td>-3.5</td>
<td>+266.8</td>
<td>Extreme range 3.2 ft.</td>
</tr>
<tr>
<td>N1</td>
<td>0.392</td>
<td>2.45.9</td>
<td>+1.8</td>
<td>-257.9</td>
<td>Dial</td>
</tr>
<tr>
<td>K1</td>
<td>1.112</td>
<td>89.6</td>
<td>-2.1</td>
<td>+87.5</td>
<td>Marigram gear</td>
</tr>
<tr>
<td>M2</td>
<td>0.769</td>
<td>74.4</td>
<td>+6.7</td>
<td>-81.1</td>
<td>Marigram scale</td>
</tr>
<tr>
<td>Q1</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>Permanent current 2.8 m/s.</td>
</tr>
<tr>
<td>M4</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>The DATUM is a plane.</td>
</tr>
<tr>
<td>(M1K)</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>Below mean low water.</td>
</tr>
<tr>
<td>(MK1)</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>Low-water springs.</td>
</tr>
<tr>
<td>n</td>
<td>0.283</td>
<td>256.6</td>
<td>+8.4</td>
<td>-264.8</td>
<td>First used 1947 Tables.</td>
</tr>
<tr>
<td>s</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>(Modified 1966).</td>
</tr>
<tr>
<td>p</td>
<td>0.057</td>
<td>215.6</td>
<td>+12.8</td>
<td>-255.7</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>(2N)</td>
<td>0.527</td>
<td>227.9</td>
<td>+13.5</td>
<td>-239.4</td>
<td></td>
</tr>
<tr>
<td>(OOH)</td>
<td>-0.030</td>
<td>14.8</td>
<td>-4.8</td>
<td>+940</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>J</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>Mm</td>
<td>-0.065</td>
<td>97.7</td>
<td>-6.4</td>
<td>+95.8</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>Ssa</td>
<td>-0.047</td>
<td>234.0</td>
<td>+8.2</td>
<td>-273.3</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>Sa</td>
<td>0.273</td>
<td>185.9</td>
<td>-0.3</td>
<td>+182.6</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>MSf</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>Mf</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>n</td>
<td>0.027</td>
<td>67.9</td>
<td>+6.5</td>
<td>+78.4</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>Q</td>
<td>0.127</td>
<td>68.7</td>
<td>+11.1</td>
<td>+76.4</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>T</td>
<td>0.039</td>
<td>264.3</td>
<td>+3.1</td>
<td>-246.2</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>R</td>
<td>0.118</td>
<td>69.2</td>
<td>+6.4</td>
<td>+75.6</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>P</td>
<td>0.356</td>
<td>80.0</td>
<td>-1.4</td>
<td>+85.6</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>(2M1)</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>M2</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>L2</td>
<td>-0.030</td>
<td>253.0</td>
<td>+0.3</td>
<td>-254.3</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>(3M1K)</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>N</td>
<td>0.176</td>
<td>259.4</td>
<td>+4.1</td>
<td>+255.3</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>M4</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>1964 2 = 2.44.</td>
</tr>
<tr>
<td>(L2K)</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>1964 2 = 2.44.</td>
</tr>
</tbody>
</table>

Source of constants: From 3 - 369 day series (1932, 1938, 1940).
Sat. Sea from 20 years (1924-1944).

Compiled by: RCO 2-28-69
Verified by: J.
## TIDES and CURRENTS

**STANDARD HARMONIC CONSTANTS FOR PREDICTION**

**STATION:** San Diego, California  
**Lat.:** 32° 22' N.  
**Long.:** 117° 14' W.

<table>
<thead>
<tr>
<th>Component</th>
<th>M</th>
<th>portraits</th>
<th>ε</th>
<th>p</th>
<th>ε - p</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Remarks</th>
</tr>
</thead>
</table>
| M1       |   | 1.826   | 98.4 | 3.5 | 94.9  | + 2.7 | + | + | -2.71 | Time meridian LHA W. = + 8 h.
| S1       |   | 0.751   | 271.6| 6.7 | 264.9 | + 0.7 | + | + | 26.5 | Extreme range 2 ft. |
| N1       |   | 0.445   | 251.4| 6.8 | 244.6 | + 0.6 | + | + | 26.4 | Dusk |
| K1       |   | 1.600   | 93.4 | 3.2 | 1.155 | + 1.5 | + | + | 9.2  | Marigram gear |
| M4       |   | 0.695   | 78.5 | 5.7 | 72.7  | + 5.6 | + | + | 8.41 | Marigram scale 7.97 ft. |
| M2       |   | 0.014   | 26.9 | 6.2 | 0.088 | + 26.2 | + | + |       | Permanent current 2.97 ft. |
| S2       |   | 0.027   | 22.9 | 10.6| 0.033 | + 24.6 | + | + |       | The DATUM is a plane 0.5 ft. below mean lower low water. |
| N2       |   | 0.056   | 234.5| 11.2| 0.045 | + 245.6 | + | + |       | First used 1942. Charts modified 1946. |
| K2       |   | 0.055   | 188.2| 11.9| 0.032 | + 206.3 | + | + |       | Amplitudes of short period constituents increased 5%. |
| M3       |   | 0.013   | 222.2| 1.7 | 0.014 | + 223.9 | + | + |       | Amplitude from 1946 series only. |
| M4       |   | 0.055   | 222.2| 0.7 | 0.055 | + 222.2 | + | + |       | Amplitude from 1946 series only. |
| S4       |   | 0.117   | 6260| 9.3 | 0.106 | + 6269 | + | + |       | Amplitude from 1946 series only. |
| M5       |   | 0.043   | 2726| 10.9| 0.045 | + 2726 | + | + |       | Amplitude from 1946 series only. |
| M6       |   | 0.010   | 4724| 17.4| 0.024 | + 4741 | + | + |       | Amplitude from 1946 series only. |
| S6       |   | 0.176   | 6350| 14.3| 0.089 | + 6369 | + | + |       | Amplitude from 1946 series only. |
| M7       |   | 0.043   | 2726| 10.9| 0.045 | + 2726 | + | + |       | Amplitude from 1946 series only. |
| M8       |   | 0.010   | 4724| 17.4| 0.024 | + 4741 | + | + |       | Amplitude from 1946 series only. |
| M9       |   | 0.043   | 2726| 10.9| 0.045 | + 2726 | + | + |       | Amplitude from 1946 series only. |
| M10      |   | 0.010   | 4724| 17.4| 0.024 | + 4741 | + | + |       | Amplitude from 1946 series only. |

### Source of Constants

- From 2-369 day series (1932 and 1936)
- From 12 years monthly series (1927-1938)

Compiled by: Delle Date: 2-24-69  
Verified by: 

---

84
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Source of constants: NOS H.A. of 3,664 days of tide observations, begins 1911-01-01.

Compiled by: J.F. McC. 6/20/76
Verified by: J.F. K. 8/20/76.
### Tides

**Station: Tarawa, Gilbert Islands**

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<th>D</th>
<th>Remarks</th>
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<td>The DATUM is a plane 0.2 ft. above mean lower-low-water.</td>
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<td>(MN₁)</td>
<td>0.079</td>
<td>1.258</td>
<td>-3.7</td>
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**Source of Constants**: Analyses of 184-day series beginning on 1 Dec. 1943.

**Observations by U.S. Navy**

**Compiled by**: [Signature]

**Verified by**: [Signature]

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**Note**: The table provides harmonic constants for tidal predictions at Tarawa, Gilbert Islands. Each row represents a different harmonic constituent (e.g., M₁, S₁, N₁, etc.) with its amplitude, phase angle, and associated values used for tidal predictions. The remarks section details specific conditions or notes relevant to the tidal predictions, such as the time meridian and the permanent current. The datum is referenced to a plane 0.2 ft. above mean lower-low-water, and the constants were first used for predictions in 1943.
### TIDES: STANDARD HARMONIC CONSTANTS FOR PREDICTION

**Station**: Port Adelaide

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<th>H (Foot)</th>
<th>Epoch</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Remarks</th>
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<td>+.06</td>
<td>-6.80</td>
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<td>110.4</td>
<td>Time meridian 142° 30' = 9,500 h</td>
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<td>+6.72</td>
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<td>Migratory scale</td>
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The DATUM is a plane 100 ft. above mean sea level, and is approximately the datum of the Coast and Geodetic Survey charts.

\* Datum changed since the time of the Ten Years Survey in 1924.

Harmonic constants are from the harmonic analysis for year 1920, and are valid for year 1921.

Harmonic constants are from the harmonic analysis for year 1920, and are valid for year 1921.

APPENDIX C

SAMPLE OUTPUT FROM COMPUTER PROGRAM

WITH GRAPHICAL RESULTS
In this section are some examples of the output of the computer program. The output includes graphs of the daily tide curve with a listing of high and low waters, listings of astronomical data, node factors and arguments, and times and tide heights (with high and low waters indicated).

The examples are chosen to elucidate the effects on the tide of different astronomical configurations. Also included is a one-month tide curve. In addition, the responses of different ports to the same astronomical configuration are shown.

The last illustration of this appendix is taken from the NOS publication *Our Restless Tides*. It shows some of the individual main semidiurnal and diurnal tidal constituents which combine to form the complete tidal prediction.
Output From Computer Program

PLACE: Boston Harbor

DATE: January 19, 1992

COMMENTS: Note that an augmented tide range

\[ (+11.8) - (-1.6) = 13.4 \text{ feet} \]

occurs because both a full moon and a lunar perigee fall on this date.
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<td>16:37</td>
<td>-1.6</td>
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<tr>
<td>22:49</td>
<td>10.2</td>
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BOSTON 71°3'W 42°21.3'N (TIME MERIDIAN 75°W) (Z0 5.22)
ASTRONOMICAL DATA
1 / 19 / 1992
TIME T = 0.92047916

MEAN LONG. OF MOON (s) 107.4901
MEAN LONG. OF SUN (h) 297.6545
MEAN LONG. OF LUNAR PERIGEE (p) 119.7818
MEAN LONG. OF SOLAR PERIGEE (p1) 282.8037
LONG. OF LUNAR ASC. NODE (N) 278.8476

OBLIQUITY OF LUNAR ORBIT (I) 24.7298
OBLIQUITY OF ECLIPTIC (OMEGA) 23.4403
INCLINATION OF LUNAR ORBIT (INCL) 5.1454
RT. ASCENSION OF INTERSECTION (NU) -12.2295
LONG. IN ORBIT OF INTERSECTION (XI) -11.1726
TERM IN ARGUMENT OF K1 CONSTITUENT (NUP) -8.4979
TERM IN ARGUMENT OF K2 CONSTITUENT (NUPP) -8.6725
MEAN LONG. LUNAR PERIGEE FROM INT. (CP) 130.9544
FACTOR IN AMPL. OF L2 CONSTITUENT (RA) 0.9268
TERM IN ARGUMENT OF L2 CONSTITUENT (R) -15.3414
FACTOR IN AMPL. OF M1 CONSTITUENT (QA) 0.6921
TERM IN ARGUMENT OF M1 CONSTITUENT (QU) -20.0343
TERM IN ARGUMENT OF M1 CONSTITUENT (Q) 150.9888
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<th>F*AMPL</th>
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<th>U</th>
<th>V+U</th>
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Output From Computer Program

PLACE: Boston Harbor

DATE: December 1, 1992

COMMENTS: Note that a decreased tide range

\[ (+8.8) - (+1.8) = 7.0 \text{ feet} \]

occurs because a first-quarter moon and a lunar apogee fall near this date.
12 / 1 / 1992

HIGH AND LOW WATERS

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BOSTON 71° 3' W 42° 21.3' N (TIME MERIDIAN 75° W) (Z0 5.22)
### ASTRONOMICAL DATA

12 / 1 / 1992

**TIME T =** 0.92915815

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JDET 2448957.5013
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12 / 1 / 1992 Oh UT  
JDATE 2448957.500  
JDUT 2448957.5013  
LONG. OF LUNAR ASC. NODE (N) 262.0613

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Output From Computer Program

PLACE: Boston Harbor

DATE: January 1, 1992

COMMENTS: This, the first day of 1992, was run simply to check that the program was operating correctly. Good agreement with the published NOS tide tables for 1992 was obtained.
1 / 1 / 1992

HIGH AND LOW WATERS

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BOSTON 71° 3' W 42° 21.3' N (TIME MERIDIAN 75° W) (Z0 5.22)
ASTRONOMICAL DATA
1 / 1 / 1992

TIME T = 0.91998635

JDUT 2448622.5000
JDET 2448622.5013

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MEAN LONG. OF SOLAR PERIGEE (p1) 282.8028

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102
### Node Factors and Arguments

**Location:** Boston 71° 3' W 42° 21.3' N  
**Date:** 1/1/1992 0h UT  
**Calculations:** JDUT 2448622.5000, JDET 2448622.5013

**Long. of Lunar Asc. Node (N):** 279.8008

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1 / 1 / 1992 BOSTON (TIME MERIDIAN 75° W) (Z0 5.22)

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ONE-MONTH TIDE CURVE
for January 1992
at
BOSTON, MASSACHUSETTS
GRAPH OF INDIVIDUAL TIDAL CONSTITUENTS
APPENDIX D

"PAUL REVERE’S MIDNIGHT RIDE"
This appendix contains, in addition to a pre-print of a published article\textsuperscript{40}, two tide curve graphs illustrating the tide of Boston Harbor on April 18, 1775 and April 19, 1775.

Paul Revere crossed the harbor in a small rowboat at about 10:30 p.m. on April 18, 1775. Note on the graph that the tide was rising at this time. The historical significance of this rising tide is discussed in the article.

Also note, on the graph for April 19, 1775, that high tide occurred at about 1:26 a.m. At this time British troops were in the final stages of their disembarkation from small boats onto the marshes on the Cambridge side of Boston Harbor. The historical significance of this high water is explained in the article.
4 / 18 / 1775

HIGH AND LOW WATERS

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BOSTON 71° 3' W 42° 21.3' N (TIME MERIDIAN 75°W) (20 5.22)

4 / 19 / 1775

HIGH AND LOW WATERS

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BOSTON 71° 3' W 42° 21.3' N (TIME MERIDIAN 75°W) (20 5.22)
Henry Wadsworth Longfellow, in his Tales of a Wayside Inn (1863), began the landlord's tale with the stirring lines:

"Listen, my children, and you shall hear
Of the midnight ride of Paul Revere,
On the eighteenth of April, in Seventy-five;
Hardly a man is now alive
Who remembers that famous day and year."

The poem refers to the spring of 1775, when General Thomas Gage, British commander in Boston, received information that a considerable quantity of provincial military stores were concealed in Concord, a town about 18 miles west of Boston. On the night of April 18, Gage ordered a detachment of grenadiers and light infantry to proceed to Concord and destroy the depot. Boats began ferrying the troops from Boston to Cambridge across the tidal backwash of the Charles River, leading to the famous two-lantern signal ("two if by sea") from Old North Church. Paul Revere himself crossed the harbor sometime between 10 p.m. and 11 p.m. in a rowboat from Boston to Charlestown, from where he began his famous midnight ride towards Lexington and Concord.

It has long been recognized that Longfellow's poem has several historical errors. For example, the poem puts Revere on the wrong side of the harbor; actually, Revere was involved in sending the two-lantern signal and did not receive it. Also, the poem has Revere reach Concord; in reality, he was stopped just past Lexington, and it was another rider, Samuel Prescott, who reached Concord.
Longfellow's poem contains astronomical references; in particular, he mentions the Moon five times:

"...he...Silently rowed to the Charlestown shore,  
Just as the moon rose over the bay..."

"...The Somerset, British man-of-war;  
A phantom ship, with each mast and spar  
Across the moon like a prison bar..."

"...he paused to listen and look down  
A moment on the roofs of the town,  
And the moonlight flowing over all..."

"...A hurry of hoofs in a village street,  
A shape in the moonlight, a bulk in the dark..."

"...He saw the gilded weathercock  
Swim in the moonlight as he passed..."

Are these astronomical references accurate, or do they contain errors like the historical errors which Longfellow made? Specifically, was the Moon rising as Revere crossed the river? Was there bright moonlight throughout the midnight ride?

MOON ON APRIL 18-19, 1775

Following the methods of Jean Meeus' Astronomical Formulae for Calculators (Willmann-Bell, 1982), we obtained the following lunar phases:

April 15, 1775  Full Moon (22h UT)
April 22, 1775  Last quarter Moon (20h UT)

In Boston on the night of April 18, 1775, there was indeed a bright waning gibbous Moon, 87 percent sunlit. Moonrise occurred at about 9:37 p.m. Eastern Standard Time. This corresponds to 9:52 p.m. in local mean time for the meridian of Boston (71 degrees 3 minutes west longitude, or 4h 44m west of Greenwich) and to 9:53 p.m. in apparent solar time. Apparent solar time is obtained from local mean time by making a correction called the equation of
time, which was +1 minute on April 18 and varied between extreme values of about -15 minutes to +16 minutes over the course of 1775. Many astronomers are familiar with the equation of time as the difference between the time on a sundial (solar time) and the time on a clock (mean time). Regardless of how the time is expressed, it is clear that there was a bright Moon rising as Paul Revere crossed the harbor between 10 p.m. and 11 p.m.

We were able to locate several different colonial almanacs published in Boston for 1775, including those of Nathaniel Low, Isaac Bickerstaff, Isaiah Thomas, and Nathaniel Ames. Moonrise on April 18 was given as 9:45 p.m. (Low, Ames), 9:46 p.m. (Thomas), and 9:53 p.m. (Bickerstaff). These almanacs did not use standard time, as modern almanacs do, since the United States did not adopt standard time zones until 1883. But did the colonial almanacs use local mean time or apparent solar time? From a comparison of the almanac tables to computer calculations of sunrise for all 365 days of 1775, it became obvious that the colonial almanacs used apparent solar time.

PAUL REVERE'S ACCOUNTS

Paul Revere left three first-person accounts of the famous night. Two are in the Revere family papers; both describe how "the Moon shone bright."
The third account, a letter sent in 1798 to Jeremy Belknap of the Massachusetts Historical Society, contains the following often-quoted lines:

...went to the north part of the town, where I had kept a boat; two friends rowed me across Charles River, a little to the eastward where the Somerset man-of-war lay. It was then young flood, the ship was winding, and the moon was rising.

It seems hard to understand how Paul Revere could have passed the Somerset man-of-war without being detected. If he was to the east of the ship, why didn't the British see his rowboat silhouetted against the rising Moon?
The computer simulation provides the answer. The Moon that night had a southern declination (-18 degrees) which, combined with Boston's northern latitude (42 degrees 22 minutes north), caused the Moon to rise considerably south of east. When Revere crossed the harbor, roughly 45 minutes after moonrise, the Moon was low (altitude 6 degrees) in the southeast (azimuth 121 degrees). As shown in the accompanying map, a sentinel on the Somerset would have seen the Moon rising over the city of Boston, not over the open water of the bay, and therefore he would not have seen the rowboat against the rising Moon. This calculation helps to explain why Revere was successful in reaching his horse on the opposite shore!

TIDES

In addition to Revere's reference to a "young flood" as he was crossing the harbor, Longfellow's poem mentions tides three times, including a description of the ferrying of the soldiers on "the rising tide, like a bridge of boats." We can check these statements with computer programs based on harmonic analysis of the tides. Our program (S&T: November, 1987, page 526) calculates the tide curve as the sum of 37 periodic constituents, with amplitudes and phases tabulated as the "station constants" for the port.

Boston Harbor: Tide calculations
(times are apparent solar time)

April 18, 1775 1:14 p.m. high water
April 18, 1775 7:19 p.m. low water
April 19, 1775 1:26 a.m. high water

So, Revere's letter and Longfellow's poem are again both correct — there was a rising tide between 10 p.m. and 11 p.m. as Revere passed the Somerset.
Contemporary accounts also recorded the effects of the following high water which occurred early on April 19 and flooded the marshes on the Cambridge side of the river. A report by an anonymous British officer, recorded by Lieutenant Frederick Mackenzie of the Royal Welch Fusiliers, describes the events:

The Grenadier & Light Companies of the Regiments in Boston were ordered to assemble on the Beach near the Magazine at 10 o’Clock last night...embarked, and landed at Phipps’s farm. The boats then returned for the remainder, and it was near One o’Clock in the Morning before the whole were landed on the opposite shore...it was 2 o’Clock before they marched off. Their march across the marshes into the high road, was hasty and fatiguing, and they were obliged to wade, halfway up their thighs, through two Inlets, the tide being by that time, up. This should have been avoided if possible, as the troops had a long march to perform....

Lieutenant William Sutherland of the 38th Regiment described how he:

...embarked at the Magazine Guard & landed near Cambridge...here we remained for two long hours, partly waiting for the rest of the Detachment & for provisions, About 2 in the Morning on the 19th we marched...the Tide being in we were up to our Middles before we got into the road....

Colonial almanacs gave the time of morning high water, or "full sea", for April 19 as 2:34 a.m. (Low), 2:36 a.m. (Thomas), and 2:39 a.m. (Bickerstaff, Ames). All of these are somewhat later than the 1:26 a.m. computed here for that high water. Part of this difference could be explained by changes in the shoreline and the depths in Boston harbor through dredging and landfill.

However, there is reason to believe that our computed times may be fairly accurate, even when projected back to 1775, since we can show that a certain time constant for the port has not changed much in the last two centuries.

The "high water lunisolar interval" is defined as the time interval between the Moon's transit across the meridian of the place and the following high water. A related quantity, called "High Water Full & Change," is the average value of the high water lunisolar interval on the days of full Moon or
new Moon ("change"). We have located two series of navigation guides from the late 1700s; both state that HWF\&C equals 11h 30m for Boston harbor. For comparison, we computed the times of high waters and then subtracted the times of the preceding lunar transits on the days of the 25 new and full Moons in 1992; the average difference, or HWF\&C, is 11h 24m in 1992, in good agreement with the colonial value. This indicates that the harbor has not changed significantly, at least insofar as the times of high waters are concerned.

For the time of the high water on April 19, the colonial almanacs may be in error because of the relatively crude methods available then for tide prediction. The makers of colonial almanacs generally did not reveal their methods, but certain clues can be found in their tide tables. For example, on the full Moon day of April 15, 1775, our computer program gives a high water at 11:06 a.m., exactly the time listed in Nathaniel Low's almanac. However, Low gives the times of high waters as 8:06 on April 11th, 8:51 on the 12th, 9:36 on the 13th, 10:21 on the 14th, 11:06 on the 15th (full Moon), 11:58 on the 16th, 12:50 on the 17th, 1:42 on the 18th, and 2:34 on the 19th.

Inspection of the differences in these numbers shows that Low is simply giving the tides later by 45 minutes per day before the 15th and by 52 minutes per day after the 15th. Similar patterns are seen in his almanac throughout 1775, with the sudden discontinuities in the time difference generally occurring on the days of new Moon, first quarter, full Moon, and last quarter. This suggests that Low computed tide times in detail on only a few days each month, and he obtained the other times by simple "uniform differencing," equivalent to linear interpolation. The large errors possible in such crude methods were noted in 1802 in the first edition of The New American Practical Navigator by the famous navigation expert Nathaniel Bowditch, who criticized rules "in
which the tide is supposed to be uniformly retarded every day." Bowditch gave improved rules, but more accurate harmonic methods were not generally applied to ocean tides until the 1860s.

RELATION TO APRIL 18, 1992

On April 18, 1775, the Moon rose at about 9:37 p.m. EST, and 45 minutes later it stood 6 degrees above the southeastern horizon at azimuth 121 degrees. The Moon was in the waning gibbous phase and 87 percent sunlit.

On April 18, 1992, the Moon will rise at about 9:38 p.m. EDT, and 45 minutes later it will stand 6 degrees above the southeastern horizon at azimuth 128 degrees. The Moon will be in the waning gibbous phase and 96 percent sunlit. While the Moon in 1992 is somewhat closer to full, the expected difference in rise time is almost exactly cancelled by our use of daylight time.

If the sky is clear, the bright moonlight in 1992 will angle across Boston harbor, shine down the country roads, and illuminate Lexington Green and Concord Bridge, just as it did over two centuries ago, on the night before the first day of the American Revolution.
IV. APRIL hath 30 Days. 1775

BUT when the rage of winter's night is past,
(Kind heav'n's indulgences forever last.)
The vernal dawn of blooming spring appears,
And Sol's kind influence all nature cheers:
Refreshing showers prepare the earth for seed,
While pleasing verdure cloathes the fertile mead.

<table>
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<tr>
<th>First Quart. 7 Day 6½ Aft.</th>
<th>Last Quart 22 Day 2 Aft.</th>
<th>Full Moon 15 Day 5 Aft.</th>
<th>New Moon 29 Day 3 Aft.</th>
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This page for April is from "An Astronomical Diary; Or, Almanack For the Year of Christian Era, 1775," written by Nathaniel Low and published in Boston by John Kneeland.
Paul Revere passed east of the H.M.S. Somerset shortly after moonrise on April 18, 1775. However, sentinels on the man-of-war would not have seen the rowboat silhouetted against the rising Moon, since the Moon rose considerably south of east that night.
REFERENCES


4. Darwin, Tides and Kindred Phenomena, pp. 149 - 150


7. Danby, Celestial Mechanics, p. 10


19. Ibid.


25. Ibid., p. 9

26. Donald W. Olson, 1992, private communication


30. Ibid., p. 183


40. Ibid.