Expression for Energy Losses in the Ball

Equation 14 in the main text sums up all energy losses occurring in the ball during and after impact. From experimental observations (to appear elsewhere) the following approximation for η(t) can be obtained:

\[ \eta(t) = \tilde{\eta}_b \sin \left( \frac{\pi t}{T_b} \right) \quad \text{for } 0 \leq t_b \leq T, \tag{A19} \]

where \( t_b \) is a point in time between 0 and \( T \), and \( \tilde{\eta}_b = \tilde{\eta}_b(J) \) denotes the maximum of the ball indentation \( \eta_b \) at a given impulse \( J \). Note that \( f_s(t) \) is not symmetric about \( T/2 \), the reason being the change in sign of the damping force \( \beta \eta_b \tilde{\eta}_b \) of the ball during impact, and the asymmetry between compression and expansion phase. The exact form of the function \( g(\eta_b) \) is not known. On the grounds of present experimental evidence, a likely candidate function for \( g(\eta_b) \) is given by

\[ g(\eta_b) = (24.9 \eta_b)^4 + (0.465 \eta_b)^2 (1 + \eta_b), \tag{A20} \]

where \( \sigma' = 0.03 \) m denotes the ball radius \( \sigma \) reduced by the shell thickness of 3 mm. It must, however, be emphasized that more appropriate approximations for \( g(\eta_b) \) could possibly be obtained in the future when more experimental results become available on the energy losses in tennis balls. When Equation A20 and Equations 14-17 of the main text are used, the expression for \( E_b(T) \) is found to be

\[ E_b(T) = L_b \left( 0.089 \tilde{x}_b \right)^2 + (0.662 \tilde{x}_b)^4 + (0.552 \tilde{x}_b)^4 / T \]

\[ + k_s \left( \exp(c_s \eta(T)) - 1/c_s - \eta(T) \right) + 0.44 \eta(T) \tilde{x}_b(T), \tag{A21} \]

where \( \tilde{x}_b \) is the normalized maximum indentation defined by \( \tilde{x}_b = \eta_b(v)/\sigma' \). Because the values of \( \tilde{x}_b, \eta_b(T) \), and \( \eta_b(T) \) at impact end time \( T \) are all unknown, it is impossible to decompose Equation A21 into its three components. However, experimental relations \( E_b(T,J) \) can be obtained and are discussed in the main text. It should be noted that an experimental function \( E_b(T,J) \) is valid only for given ball properties \( k_s \) and \( c_s \) which, in turn, depend on similar string properties \( k_s(\xi, \zeta), c_s(\xi, \zeta) \). In other words, the experimentally determined values \( k_s \) and \( c_s \) appearing in Equation 16 are themselves functions of string properties so that, more precisely, one should write \( E_b = E_s k_s(\xi, \zeta), c_s(\xi, \zeta) \), \( c_s k_s(\xi, \zeta), c_s(\xi, \zeta) \), \( \tilde{f}_s(t) \); that is, the dynamic behavior of a tennis ball during impact also depends on string properties characteristic for the impact point \( S(\xi, \zeta) \) at the string membrane and, possibly, also on the form of the impulsive force function \( f_s(t) \).
A study was needed that tested the effect of string tension on rebound accuracy using oversized rackets because studies have shown significantly greater ball rebound speeds for oversized rackets compared to smaller racket heads (Elliott, Blanksby, & Ellis, 1980; Groppel, Shin, Thomas, & Welk, 1987). Impact conditions would also be more realistic if off-center impacts were studied because research has shown that most impacts occur off-center (Knudson, 1991b; Ohmichi, Miyashita, & Mizumo, 1979; Plagenhoef, 1979) and off-center impacts have been observed to have nonsignificant and significant reductions in ball rebound velocity compared to central impacts (Elliott, 1982a, 1982b; Elliott et al., 1980; Ellis et al., 1978). Controlled impact conditions are needed to make comparisons across string tensions because the intraindividual variation of hand-held rackets is quite large (Hatze, 1992; Knudson & White, 1989).

The purpose of this study was to examine the effect of three string tensions and two impact locations on ball rebound accuracy in a vertical plane for an oversized racket. If ball rebound accuracy data from different racket head size and impact location conditions supported the hypothesis of greater rebound accuracy for lower string tensions (Knudson, 1991a), the choice of string tension to maximize accuracy in flat shots like volleys would be more clear.

**Method**

Three identical Prince oversized tennis rackets were strung alternately with Prince Synthetic Gut (nylon) strings (16 gauge) at three string tensions (200, 267, and 334 N). String tensions represented the range of typical tensions and corresponded to the more common English units of 45, 60, and 75 lb, respectively. Each racket was marked with 3M reflective tape strips and a round wooden bead (15 mm in diameter) that was covered with reflective tape and glued to the racket (Figure 1). Strips (5 mm wide) of tape marked the tip of the racket head and the center of the throat, while the reflective bead marked the top, lateral geometric center of the racket head. The rackets were secured in a horizontal position to a wooden arm by three C clamps. Nine new tennis balls were bounce tested and projected from a distance of 2.29 m by a Lobster ball machine stabilized with weights and elastic cords. Impact locations on the racket face were central and 0.08 m superior to the geometric center of the racket face. Mean (±SD) preimpact ball velocities were 28.9 ± 1.9 m·s⁻¹ at an angle of 25.4 ± 1.8° to the horizontal. The balls had negligible rotation prior to impact (<10 rad·s⁻¹).

Ten central impacts for each condition were filmed at 200 fps by a Photonic 1PL camera placed 7.92 m from the center of the racket face. Camera speed was established by internal timing lights set at 100 Hz. A mirror was placed at a 45° angle to the racket to record impact location in the longitudinal direction of the racket face. Data were digitized on a Numonics 1224 digitizer interfaced to a Zenith 128 computer. Trials were digitized for ±3 frames before and after impact for the following points: tip (lateral geometric center) of the racket head, tip of the racket head, center of the ball, mirror image of the tip of the racket, mirror image of ball position, and the mirror image of the throat of the racket.

Kinematic data were not smoothed because the greater accuracy of a small field size limited the number of images of the ball in the field of view (Hudson, 1990). Also, there were no smoothing procedures that handled very small data sets, eliminated the effect of adjacent points on the smoothed data, and were unaffected by boundary effects (D'Amico & Ferrigno, 1992; Smith, 1989; Woltring, 1985; Wood, 1982). To minimize errors in angle and velocity calculations two analysis procedures were used. First, position data for each trial were established by averaging three digitizations by one skilled operator (Groppel, Shin, Spotts, & Hill, 1987; Looney, Smith, & Srinivasan, 1990). Second, the angles and velocities of ball approach and rebound were determined from the mean of two finite difference calculations between the three samples before and after impact. The racket angle at impact was calculated from the frame prior to impact to avoid detection of motion of the frame caused by impact.

The vertical plane angles of approach (θ) and rebound (Φ) relative to the horizontal were calculated to monitor the consistency of the impact conditions (Figure 1). So rebound accuracy across trials could be compared, an accuracy ratio (AR) representative of a tennis volley was calculated as the angle of rebound divided by the angle of approach, both measured relative to an axis normal to the racket face. An accurate rebound in a vertical plane was defined as an angle of rebound close to normal to the racket face, since the study simulated a primarily flat shot, like a volley, with a target at right angles to the racket. An AR of unity corresponded to equal angles of approach and rebound, so the smaller the AR, the more accurate the ball rebound. Data were analyzed by a two-way ANOVA (2 x 3—Location × String Tension) with statistical significance accepted at the .01 level of probability.
**Results**

Impact locations measured from film were consistent. Across all trials, the mean impact location was $0.8 \pm 1.8$ cm proximal to the geometric center of the racket face. Mean impact locations in the vertical dimension were $0.08 \pm 0.9$ cm above geometric center for central impacts, and $0.80 \pm 1.8$ cm above for the off-center trials. Due to timing variations in the projection of the ball by the ball machine, not all conditions had 10 observations.

Mean and mean–standard deviation accuracy ratios (AR) for all conditions are plotted in Figure 2. A smaller AR corresponds to greater accuracy or a rebound more in line with normal to the racket face. The unequal frequencies and different variances (Figure 2) of the AR data required the variances in the data to be equalized with a square-root transformation prior to ANOVA analysis (Kirk, 1982). The two-way ANOVA revealed significant main effects for impact location, $F(1, 44) = 84.2, p < .01$; string tension, $F(2, 44) = 11.8, p < .01$; and the interaction of string tension and location, $F(1, 44) = 6.2, p < .01$.

Treatment–Contrast Interactions were calculated since the significant interaction precluded valid post hoc comparisons between means (Kirk, 1982). Treatment–Contrast Interactions demonstrated that the interaction between impact location and the difference in string tension from 200 to 334 N was significant, $F(2, 44) = 5.75, p < .01$. In other words, the change in AR (slope in Figure 2) across string tensions from 200 to 334 N was different for central versus off-center impacts. Off-center impacts showed a greater loss in accuracy as string tension was increased.

**Discussion**

Results of the present study supported previous research demonstrating that the increased elasticity of lower string tensions tends to improve rebound accuracy in static tennis impacts (Knudson, 1991a). The oversized racket strung with 200 N of tension had the most accurate rebounds (AR = $1.11 \pm 0.09$) in central impacts. This was more accurate than a study of central impacts on a midsized racket (AR = $1.50 \pm 0.16$) strung with nylon at a slightly higher tension of 222 N (Knudson, 1991a). It appears that oversized rackets may provide more accurate rebounds because the oversized racket strung at 267 N was more accurate (AR = $1.16 \pm 0.1$) than the previous study (Knudson, 1991a) of a midsized racket in the same string tension and impact location conditions (AR = $1.52 \pm 0.2$). Similar to the previous study of rebound accuracy (Knudson, 1991a), the present study found that the typical standard deviation for mean ARs was 0.1, which corresponded to a variance of $\pm 2.5^\circ$ in rebound angle for these impact conditions.

How these results relate to players attempting to impart large amounts of spin is not clear. Stroking a tennis ball with a slightly open/closed racket face and using a steep racket path less aligned with the target, in order to generate spin, may place different demands on the strings and the frame. Future studies should utilize high-speed videography (2000 Hz) to make higher sampling rates and larger samples cost-effective. Data are needed in a variety of stroking conditions to establish whether it is the effect of string elasticity or the combined effect of dwell time and racket displacement that has the greatest influence on ball rebound accuracy when string tension is varied.

It was concluded that string tension, impact location, and the interaction of tension and location significantly affect ball rebound angle in static tennis impacts. Results supported previous research (Knudson, 1991a) demonstrating that higher string tensions have less accurate rebounds in a vertical plane. Off-center impacts showed less accurate vertical plane rebounds and a greater loss of accuracy as string tension increased. The oversized rackets in this study created more accurate rebounds in central impacts than midsized rackets in similar string tension and impact conditions.

**References**


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A Panning Videographic Technique to Obtain Selected Kinematic Characteristics of the Strides in Sprint Hurdling

John W. Chow

This paper describes a panning videographic technique for measuring stride lengths and horizontal velocities of strides over an entire hurdle race. The technique requires that tapes of alternate black and white sections be placed at the inside border of the inside lane and the outside border of the outside lane of a track as spatial reference and that a vertical reference be videotaped when it is erected at different locations within the track. The stride lengths and horizontal velocities obtained with the panning technique were compared with the corresponding values that were obtained with conventional stationary camera techniques. The results indicate that if three panning cameras are used to cover the entire hurdle race, the average absolute errors in stride length and horizontal velocity are 0.07 m and 0.15 m/s, respectively. Such errors are considered acceptable for some applications. Certain properties of the panning technique are discussed.

A single-camera panning videographic technique for obtaining the stride lengths of a runner during a 100-m race was proposed by Chow (1987) several years ago. The technique requires that paired background markers be placed at 10-m intervals on opposite sides of the eight-lane track. The known locations of the markers are used as spatial reference for determining the stride lengths.

The technique was later adapted by Hay and Koh (1988) to obtain the stride lengths during the approach run of the long and triple jumps. Instead of using background markers, Hay and Koh (1988) painted parallel dashed lines (6 cm wide) on each side of the runway. These lines were placed at 1.80-m intervals, and the known locations of these lines relative to the takeoff board aided in determining the horizontal distance from the toe of the support foot to the front edge of the takeoff board for each support phase of the approach.

The technique has been further developed so that the horizontal velocity of the center of gravity (CG) during the flight phase of a stride can be determined.

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