

Findings of the Dynamic Geometry Project

at Texas State University

2009-2016

Edited by: Edwin Dickey, Zhonghong Jiang,
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Preface

The Dynamic Geometry in Classrooms Project was funded to investigate the efficacy of the “Dynamic Geometry Approach” on students’ geometry learning over the course of a full-year for two consecutive years using a repeated randomized controlled experiment design. The project was supported by the National Science Foundation under the Discovery Research K-12 program. September 1, 2009, marked the project’s beginning and its work concluded on August 31, 2016. This monograph, the Findings of the Dynamic Geometry Project at Texas State University, serves as a compilation of and repository for the many reports, refereed publications, and other products developed by the project team. It is intended to serve as a single source for important documents critical to the project and as a resource for future researchers who seek to investigate how the dynamic geometry software and instructional approaches might positively impact the teaching and learning of geometry in secondary schools.

The monograph is organized in five sections. The first addressing the research conducted to document the impact and efficacy of the dynamic geometry approach. The second section includes reports of studies that address the implementation of the approach. Section three includes publications on how geometry and technology content knowledge were assessed. Section four provides resources developed by the project and disseminated in peer-reviewed journals that teachers can use to promote the dynamic geometry approach to learning. The fifth and final section includes the different cognitive and affective instruments used by the project researchers as well as protocols and instruments used as part of project participants’ observations and interviews.

The project team was led by Principal Investigator Dr. Zhonghong Jiang, Professor of Mathematics Education at Texas State University. Project co-Principal Investigators and Senior Personnel included Drs. Gilbert Cuevas, Alejandra Sorto, Samuel Obara, Sharon Strickland, Alexander White, and Selina Vasquez-Mireles, all faculty members at Texas State University. Doctoral students supported by and contributing to the project included Ewelina McBroom (graduated in 2013), Shawnda Smith (graduated in May 2016), Alana Rossenwasser, and Brittany Webre. Master Teachers who contributed significantly to the project launch and the implementation of the two important experimental studies from 2011 to 2013 included Linda Gann, Mark Bell, Janie Love, Lisa Villalon, Isabel Fears, Ruby Valent, Lori Robinson, and Lori Callis. The project Advisory Board included Drs. James Wilson and John Olive of the University of Georgia, Dr. Richard Lehrer of Vanderbilt University, and Dr. Larry Hatfield of the University of Wyoming. The project External Evaluator was Dr. Ed Dickey of the University of South Carolina.

The project benefited greatly from the contributions of the mathematics teachers representing the school districts in Central Texas who participated in the professional development, implemented the project materials over two years, provided project team members with access to their students and classrooms, and obliged the research mission by providing the enormous amount of data required to support the project research findings.

Drs. Dickey, Jiang, and White, assisted by Ph. D student Brittany Webre, conceived of this monograph as an appropriate method of compiling and documenting the project’s work and impact as well as editing the full volume. They acknowledge and express gratitude to all the contributors mentioned above as well as the support from the Chair of the Department of Mathematics at Texas State University, Dr. Susan Morey, and her former chairs Drs. Nate Dean and Stanley Waymen.



Section 1

Impact of the Dynamic Geometry Approach on Student Achievement

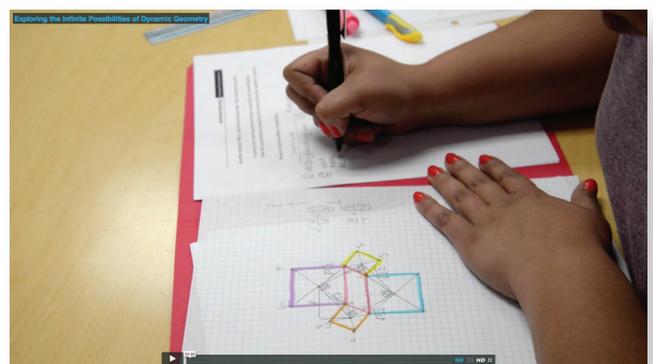
The Dynamic Geometry in Classrooms project was funded to investigate the efficacy of the “Dynamic Geometry Approach” on students’ geometry learning. The papers in this section provide evidence and documentation of how the approach had a positive impact on student achievement and attitudes based on a repeated randomized controlled experiment for two consecutive years. The experiment included a control group of students (over 600 each year) taught by teachers (over 30 each year) using what was called a “business-as-usual” approach and a treatment group over comparable size in which teachers used the dynamic geometry approach with computer software. Participating teachers were randomly assigned to the two groups.

The first paper, “The Efficacy of the Dynamic Geometry Approach,” provides a comprehensive presentation and analysis of the experiment and the significant finding that students in the treatment group that employed the dynamic geometry approach with technology demonstrated both achievement and affective gains over those in the control group.

The second paper, “The Effect of Dynamic Geometry Approach on Achievement and Conjecture Ability” reports on how the study tests that hypothesis by comparing student learning in classrooms during the second experimental year. Student learning is assessed by a geometry test and other tests. Data for answering the research questions of the study are analyzed by appropriate Hierarchical Linear Model (HLM) methods. Three different levels of Geometry courses were considered for the analysis: Regular, Honors School and Advanced Honors. For students attending Regular Geometry, the experimental group outperformed the control group in geometry performance and conjecturing ability.

The third paper, “A Dynamic Geometry-Centered Professional Development Program and Its Impact,” provides details on the professional development work and analyses of impact as presented as part of the North American Chapter of the International Group for the Psychology of Mathematics Education. It includes classroom observation data, the teachers' responses to the implementation questionnaires and findings revealing that most teachers in the Dynamic Geometry (DG) group were faithful to the DG instructional approach. Teachers in the DG group scored higher on a conjecturing-proving test than did teachers in the control group. The students of teachers in the DG group scored significantly higher than the students of teachers in the control group on a geometry achievement test.

The fourth paper, “Randomized Controlled Trials on the Dynamic Geometry Approach,” appeared in the *Journal of Mathematics Education at Teachers College*, is included here with permission. It provides details on the experimental study and data analysis for the first year of the experiment. HLM models showed that the treatment group significantly outperformed the control group in geometry achievement. While the effect of the DG treatment is of moderate size for all participating students the largest effect size occurs with students in Regular Geometry classes.



The final document in this section is a poster prepared for the project’s contribution to the National Science Foundation 2015 Video Showcase and available at <http://resourcecenters2015.videohall.com/presentations/570>.

The poster and video were developed by doctoral students supported by the project.

THE EFFICACY OF THE DYNAMIC GEOMETRY APPROACH

Jiang, Z., White, A., Strickland, S., Dickey, E., Rosenwasser, A.

ABSTRACT

The efficacy of the Dynamic Geometry (DG) approach was investigated in an experimental study to determine the impact of effective use of dynamic software on student achievement in high school geometry as compared to classes not using computer tools. The study tested the conjecture that use of the DG approach to engage student in constructing, investigating, conjecturing, and proving would result in better geometry learning. Students in 64 classrooms, each randomly assigned to the experimental (DG) or control group were tested before and after the instructional intervention to measure changes in geometry content learning, conjecturing and proving ability, and beliefs about the nature of geometry. Data analyzed using hierarchical linear modeling provided evidence that students in the DG group significantly outperformed their counterparts in the control group as measured by a valid, reliable geometry achievement test. Furthermore, students in the DG group developed more positive attitudes toward geometry exploration and the making of conjectures compare to the control group students.

1 INTRODUCTION

National education organizations place a heavy emphasis on the importance of geometric reasoning and using geometric models in problem solving (National Council of Teachers of Mathematics, 2009; National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010). In keeping with these calls, we designed and conducted a study to investigate the impact of an approach to high school geometry that utilizes dynamic geometry (DG) software and supporting instructional materials to supplement ordinary instructional practices in Central Texas. The rationale of this investigation derives from the urgent need for improving students' geometry learning.

Although geometric reasoning is an important human ability indispensable for students at all levels, the challenges of learning and teaching geometry continue to be a major problem nationally. U.S. students' geometry achievement at all grade levels is low (Battista, 2007; Senk, 1985). At least 40% of the students struggled with state test items consisting of applications of the Pythagorean theorem, reasoning about geometrical ideas, transformations (usually in coordinate grid settings), spatial visualization, and angles in polygons (Dick & Burrill, 2009). In addition, U.S. students' measures on international tests of achievement tend to be at the lowest level in geometry. Geometric insight among the students is absent even after completing a formal course in geometry (Gonzales et al., 2008; Mullis et al., 2004).

Scholars have attributed the poor performance to how geometry is taught in schools, as instruction tends to focus more on the product than the process and fails to help children understand mathematics as a logical system (Vincent, 2005). Geometry courses, as typically taught, do not help children develop an understanding of content but rather encourage memorization of definitions and theorems (Liu & Manouchehri, 2012). Clearly, there is a crucial need to fundamentally change this situation so as to improve geometry teaching and learning in our classrooms.

The effective use of technology offers a viable solution to this problem. Research supports the educational value of the instructional use of computers for three reasons. First, students spend more time on task, and therefore learn faster. Second, students work at their own levels and rate. Third, materials can incorporate good practices of instructional and learning theory (Moursand, 1999). Dynamic geometry (DG) is an active, exploratory study of geometry carried out with the aid of interactive computer software, which has been available since the early 1990's. The most widely used DG software packages include The Geometers' Sketchpad (GSP) (Jackiw, 2009), Cabri Geometry (Laborde & Bellemain, 2005) and Geogebra (Hohenwarter, 2001). As noted by the authors of the Common Core State Standards for Mathematics (CCSSM), "Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena" (NGA and CCSSO 2010, p.73). With its main features such as dragging and measuring, DG software allows students to engage in both constructive and deductive geometry (Schoenfeld, 1983) as they build, test and verify conjectures using easily constructible models.

With growing usage of DG software (based on the sales data of Key Curriculum Press, approximately 60% of high schools in the U.S. have GSP4 or GSP5 licenses), increasing numbers of research studies have examined its impact on learning. However, very few studies have used experimental or quasi-experimental designs (Dixon, 1997; Myers, 2009). Most have examined the impact of the use of DG software (e.g., Baccaglioni-Frank and Mariotti, 2010; Choi-Koh, 1999; Gerretson 2004; Hannafin et al. 2001; Hollebrands 2007; Vincent, 2005) using qualitative methods.

Collectively, this body of research has revealed several important findings. First, there is evidence that when dynamic geometry technology is used effectively, students' learning of mathematics is impacted positively. Second, DG technology can be used as a cognitive tool in a manner that facilitates students' explorations and investigations; promotes conjecturing, verifying, explaining, and logical reasoning; and enhances learners' conceptual understanding of geometry concepts. Despite these impressive results, nearly all of the studies were small in scale and either used exploratory, phenomenological methods or conducted comparisons without employing a true experimental design.

While the exploratory phenomenological studies are important for providing detailed accounts of students' learning and their learning process, it is timely for the educational research community to use modern statistical methodologies to document the findings capable of impacting governmental policies, informing school leaders on curriculum, and

meeting the societal expectations (American Statistical Association, 2007). To determine the effectiveness of DG on learning more broadly, quantitative comparative studies are needed that include experimental designs using random assignments to treatment and control conditions for valid comparisons. There is currently an absence of studies using experimental designs that examine the impact of a DG approach (to be described in detail in the next section) at a large scale. The research reported here is such a study. This article expands on an earlier report (Jiang et. al., 2011) of preliminary findings from the first year of implementing the randomized controlled trial experiments and provides a comprehensive report and conclusions that document the efficacy of the DG approach. First, a description of the research design of the study is provided followed by the presentation of the result from the quantitative analysis of student measures. A discussion of how the evidence supports the DG approach for improving high school student learning in geometry is provided at the end.

2 THE DYNAMIC GEOMETRY APPROACH

The instructional approach of using dynamic geometry software in this study is referred to as the dynamic geometry approach. To guide the creation and selection of DG activities to be used in the teachers' professional development and their geometry classrooms, the research team that included the authors as well as university mathematics education faculty members and school district master teachers formulated the following operational definition of this approach based on the research studies cited above:

As learning activities emphasized by the DG approach, students are expected to:

- Construct dynamic geometric objects with DG tools
- Construct dynamic representations of problem situations with DG tools
- Perform actions (drag, measure, transform, and/or animate) on the constructed objects/situations
- Observe variants and invariants related to the characteristics of objects under different actions
- Investigate mathematical relationships and/or solutions in multiple ways
- Formulate conjectures
- Test conjectures
- Receive immediate feedback from the software
- Think mathematically and prove (or disprove) their conjectures

Teaching activities that are consistent with the DG approach require teachers to:

- Facilitate the student use of DG software
- Help students construct mathematical ideas through active explorations and investigations
- Present prepared DG environments for students to explore mathematical relationships
- Encourage productive struggle through which students persist in solving problems with complicated constructions and explore related concepts
- Facilitate students' argumentations by asking "why" questions – prompting students to furnish justifications for their statements and checking the validity of their justifications (Vincent, 2005)
- Extend students' explorations by asking "what if" questions

3 THEORETICAL FRAMEWORK AND RESEARCH QUESTIONS

While a growing body of literature supports the notion that students interacting with the Geometer's Sketchpad (GSP) and other dynamic environments can aid in their learning of mathematics, and particularly geometry, it is not always clear how that learning happens or what role the teacher plays in the learning process. And although the "how" aspect of that learning is not the primary focus of this study, we nevertheless, sought a framework that might guide the professional development activities for teachers and help us understand how an ongoing learning context that uses DG frequently might reasonably improve student achievement. Therefore, an integrative framework (Olive & Makar, 2009) drawing from Constructivism, Instrumentation Theory and Semiotic Mediation was used to guide the study.

Central to Instrumentation Theory is the process of *Instrumental Genesis* – How a tool changes from an artifact to an instrument in the hands of a user, and how both the tool and user are transformed in the process (Olive, 2011). Within this framework, as students and teachers interact with and within DG environments these interactions with the tool (GSP) influence the next acts by each person, and continue in an interplay between the tool and user. For example, as students "drag" a geometric object and observe outcomes from that act, the user (student or teacher) adjusts her or his thinking, which in turn influences the next interaction with the tool. Since GSP allows users to adjust their sketches and the relationships within them, users are transforming the tool, their use of the tool, and their thinking.

Vygotsky (1978) first introduced the notion of *semiotic mediation*. According to this construct, cognitive functioning is closely linked to the use of signs and tools, and affected by it. Vygotsky placed an emphasis on language as a mediator of learning. In our study, language as classroom discourse (teachers and students discussing shared tasks and their mathematics learning) is considered an important component of the DG learning process. We also consider GSP to be a semiotic mediator in addition to discourse, following Drijvers et al. (2009). Students and teachers, using GSP and language as mediators, transform their thinking about the tasks and the mathematics embedded within them.

Olive and Makar (2009) focus on the mathematical knowledge and practices that may result from access to digital technologies. They put forward a new tetrahedral model that integrates aspects of instrumentation theory and the notion of semiotic mediation. "This new model illustrates how interactions among the didactical variables: student, teacher, task and technology (that form the vertices of the tetrahedron) create a space within which new mathematical knowledge and practices may emerge" (Olive, 2011, p.3). The aspect of the study described here, though primarily concerned with knowledge as measured by student test scores, considers the larger context in which this learning occurs as important. This framework informed the design of a professional development summer institute that preceded the experimental phase of the study and semester follow-up sessions as well as the sorts of materials and district level support provided to the teachers in the study.

Four research questions were addressed in this study but this article focuses on the first two:

1. How do students in the experimental DG condition perform in comparison with students in the control condition on measures of a geometry achievement test?
2. How does the DG intervention affect student beliefs about the nature of geometry?
3. How does the fidelity and intensity with which the instructors implement the DG approach in their classrooms relate to student learning?
4. What characterizes the learning communities in the experimental and control groups?

As the first efficacy study on the DG approach at a moderately large scale in the nation, the answers to these research questions contribute to the knowledge base of whether or not (and how) the DG approach can effectively help our geometry teachers develop sound content and pedagogical knowledge and enhance students' geometry learning.

4 RESEARCH DESIGN

The participants in this study included samples of geometry teachers and their students at high schools mainly in five Central Texas school districts. Based on a power analysis to determine the optimal sample size, 60 teachers were needed for the study. Taking attrition into consideration, 76 geometry teachers were selected from those who applied to participate in the project with support from their principals.

The research study followed a mixed method, randomized cluster design, with the teacher or the teacher's class of students as the unit of randomization. The 76 teachers selected were randomly assigned in equal numbers to an experimental group (the DG group) and a control group (a non-technology group). Although we expected the teachers who taught more than one Geometry class to utilize a consistent pedagogical approach, only one "target" class per teacher was randomly selected to participate in the study. Therefore, each teacher was represented in the study with measurements from only one classroom of students, and the classroom and teacher unit of analysis overlap, yielding a design where the students are nested within teachers/classrooms, which are nested within schools.

4.1 The DG Instructional Materials

In this study, the DG software used was The Geometer's Sketchpad (GSP). To help users take full advantage of the power of GSP, we developed a specific collection of DG instructional materials, entitled *Recommended GSP Activities for Your Geometry Classroom*, mainly based on a large set of well-designed DG learning activities in books (such as *Exploring Geometry with The Geometer's Sketchpad*) and on line (such as *Sketchpad Lesson Link*) published by Key Curriculum Press. The collection of materials also drew from the related learning activities created by numerous mathematics teachers, educators, and researchers (including ourselves) through their published articles in classroom journals such as *Mathematics Teacher*. We revised materials based on project team reviews that assured the critical features of the DG approach were incorporated. While doing so, close attention was paid to the consistency with CCSSM and Texas geometry curriculum standards. All the

materials were compiled within an electronic lesson management system administered by the project with access provided to all participating teachers.

4.2 Professional Development

Teachers' professional development (PD) was an important component of the study as without PD training, "teachers often fail to implement new approaches faithfully" (Clements, Sarama, Spitler, Lange, & Wolfe, 2011, p. 133). The teachers in the experimental group participated in 54 hours of PD, starting with a context-relevant week-long institute in the summer before the start of the school year in which they were to implement the DG approach, followed by six Saturday sessions during the school year.

The nature of each PD session was interactive and emphasized participants' active involvement and conceptual understanding of mathematics. Important geometric concepts, processes, and relationships were presented or revisited through challenging problem situations, which were explored with GSP as a tool. Teachers learned GSP skills in the process of using them to tackle the problems. They came to learn first hand, as learners of mathematics, how GSP encourages mathematical investigations by allowing users to manipulate their geometric constructions to answer "why" and "what if" questions, by allowing them to backtrack easily to try different approaches, and by giving them visual feedback that encourages self-assessment. They also discussed pedagogical and technological implications of using DG in their classrooms. Time was allocated for the teachers to share their work, present ideas to the group, and generally debrief together. These also included new task activities aligned to school and district curriculum guides so that teachers plan when to implement DG units.

Changing one's beliefs about and practices of teaching in the way that the DG approach requires is not a trivial process and was an important, planned purpose of the PD sessions. To address these teacher affective issues, a support group consisting of three members of the research team (two senior staff members and a doctoral student) was formed. These three team members concentrated on providing constant support to the teachers in their implementation of the DG treatment. They regularly visited the participating schools to provide timely on-site assistance to the teachers; encouraged the participating teachers to use phone and/or email communications with the support group for asking questions, expressing concerns, and/or requesting help; and made significant efforts to build stronger collaborative relationships between the university and the participating schools and districts so that the administration would be supportive. Incentives including stipends and instructional tools/supplies were also provided to leverage teachers' faithful implementation of the DG treatment.

The control group was characterized as the "business-as-usual" group. The teachers in this group taught geometry as they had always taught the course. The control group teachers also participated in professional development both in summer and during the school year addressing the same mathematical content included in the DG group PD but in a manner that did not employ technology. For this group the conjecturing and proof methods important to

the DG group were emphasized but without the use of dynamic geometry software. The PD facilitators for this group used an approach that most teachers were already using to conduct their training: mainly lecturing, heavily relying on the textbook, and working mostly with more traditional tools, manipulatives, and paper-pencil activities. The amount of instructional time spent on this regular workshop was the same as that for the DG group PD training. The purpose of holding this non-DG workshop was to address a confounding variable. With this comparable amount of meaningful PD, if differences appeared on the project's measures between the experimental and control groups, we would be able to rule out the possibility that the PD activities accounted for them rather than the interactive DG learning environment. To induce the control group teachers to emphasize conjecturing and proof consistent with the PD offered, they received incentives equivalent to those for the teachers in the DG group.

4.3 Measures

The independent variable central to this study was the treatment group (experimental group using the DG approach or control group not using the DG approach). The dependent variables serving as outcomes in this study included what teachers learned from the professional development, students' performance on measures of geometry knowledge and skills, and their beliefs about geometry. Changes in outcome measures were captured using a teacher survey instrument and teacher observational data as well as student belief survey and pre- and post-test scores on geometry tests. This article focuses on the student cognitive and affective measures.

Student-level measures: Student learning was assessed by a geometry knowledge test and a measure of student beliefs about the nature of geometry. The geometry knowledge test consisted of a pre- and a post-test. The pre-test was an established, published and publicly available measure of high school student geometry achievement called the Entering Geometry Test (ENT). Developed by Usiskin (1982) and others at the University of Chicago the ENT is a valid multiple-choice geometry test assessing students' geometric knowledge before entering a full-year high school geometry course. ENT consists of 20 multiple-choice items, and its reliability has been established at $\alpha = .77$. As a post-test, the project research team developed an Exiting Geometry Test (XGT) using items released from the California Standards Test: Geometry (CSTG). Validity of the CSTG items was established by review committees consisting of mathematics content experts, teachers and administrators that ensured each item appropriately measured California academic content standards in Geometry (California Department of Education, 2009). With attention to Texas geometry standards and participating school district geometry curricula, the research team selected 30 multiple-choice items from the CSTG for a pilot version of the XGT. The pilot XGT was administered to four non-project classes at a participating high school. Five items from the pilot XGT were removed based on psychometric analyses documenting weak performance in item difficulty and discrimination as well as from feedback provided by high school geometry experts working as project master teachers. Reliability for the final 25-item XGT was established at $\alpha = .875$. A factor analysis was performed and indicated that the XGT functioned as a one-dimensional measurement instrument. Item Response Theory (IRT)

scoring analyses were used to ensure the XGT's construct validity. Students' 'abilities' and item parameters from the scored test were examined allowing the research team to determine that the XGT items collectively provided a range of performance that holistically represented a well-functioning instrument. Because the test result data adhered to a three-parameter logistic IRT model, the team concluded that the XGT validly measured the intended high school geometry construct.

To measure student beliefs about geometry, the research team developed a student belief questionnaire by adapting the mathematics version of the Views about Sciences Survey (VAMS) (Halloun & Hestenes, 1996). The development of this instrument included several phases: an extensive review of the literature generating a set of items and compiling them in a preliminary form of the instrument, experts' review and feedback, pilot testing, and substantive revisions based on statistical analyses, ultimately yielding a final 18-item version of the instrument. Responses to each of the items used a Likert-type 5-point scale of Strongly Agree, Agree, Don't Know, Disagree, or Strongly Disagree quantified respectively as 5, 4, 3, 2, or 1 for data analysis purposes. Students with high motivation to do exploring, conjecturing, and/or proving were expected to agree or strongly agree with the positively worded statement of the item (hence score as a 4 or 5). A higher total score represented a more positive attitude towards the exploring, conjecturing, and/or proving activities. The results of the instrument reliability analysis generated a Cronbach's $\alpha = .853$ on the survey administered before the treatment began and $.796$ after the treatment providing evidence of reliability. Comparable to the geometry content post-test (XGT), a factor analysis of the responses to the belief instrument identified a uni-dimensional scale.

Teacher level measures: Results from data gathered about teachers are not the focus of this report, but provide valuable information regarding the student data. The critical features of the DG approach include using dynamic visualization to foster students' conjecturing spirit, students' habits of testing conjectures, focusing on relationships, and explaining what is observed, students' disposition and abilities to reason logically as well as students' conjecturing-investigating-proving learning style within explorational problem situations. Two measures of implementation fidelity (the DG Implementation Questionnaire [DGIQ] and the Geometry Teaching Observation Protocol [GTOP]) were developed as a means of assessing the teachers' adherence to the DG approach. The validity and reliability of the two instruments were established through the projects' advisory board carefully reviewing the instruments and providing feedback for revision, and the research team's use of psychometric analyses on the measures, such as an IRT analysis on the DGIQ and the GTOP. Results of these analyses were presented at a national research meeting for feedback from other studies/researchers (Sorto et al., 2014).

The final version of the DGIQ contains six multiple-choice items and ten open-response questions. A typical multiple-choice item is: "How many times per week did the students work in a computer lab/classroom using GSP software?" (Response possibilities are *None, One time, Two times, and More than two times* for specific weeks.) An example of an open-response item is, "Please describe how the use of DG tools has helped you improve your

understanding of students (via formative assessment of students' learning). Provide explicit examples." The advisory board for this study commended the requests for specific examples, indicating: "These are often very revealing and in fact may provide a lot of insight about how a teacher is thinking about his/her instruction."

The DGIQ was administered to the teachers in the experimental group. A different version of the questionnaire was administered to the control group teachers to examine how they teach geometry without using technology.

The research team adapted the *Reformed Teaching Observation Protocol* (Sawada et al., 2002) to develop the GTOP in a manner that appropriately addressed and assessed implementation of the DG approach. The main part of GTOP consisted of 25 items organized into four sections: (1) *Description of intended dynamic geometry lesson*, (2) *Description of implemented dynamic geometry lesson*, (3) *Assessment of quality of teaching*, and (4) *Assessment of engagement and discourse*. An example from the *Implemented Lesson* section is: "The activities in the lesson are designed to move students along the following trajectory (or part of it): from initial conjecture, to investigation, to more thoughtful conjecture, to verification and proof." (Score possibilities were evaluated on a 5-point scale from 0 = *not observed* to 4 = *very descriptive*.) The project's advisory board reviewed the GTOP providing comments for revision and improvement. One board member commented: "The decomposition of teaching into specific practices with dynamic software or with emulating those of dynamic software (e.g., a teacher could ask the class to consider what would happen if a quality of a figure etc. were to change) is very good."

Researchers observed both the experimental and control group teachers using the GTOP. GTOP scores provided comparative data about the strategies and teaching styles observed for teachers in the different groups. To obtain the GTOP scores of the teachers in both groups, we organized an observer group formed by all members of the research team except the three members in the support group. (The support group was not involved in classroom research observations to avoid another confounding variable that had serious implications for the integrity of the research design.) The observers were first trained. During the DG (and non-DG) implementation, they observed 16 participating teachers' classes. There were at least two observers for each class observed, and each teacher was observed at least three times during the school year. A comprehensive profile of the teacher's practices was compiled from the data gathered from the observations over time. The observers individually scored each observed class. Discussions among observers took place if there were significant scoring discrepancies. Rescoring occurred if consensus could not be reached. The extensive time and effort required to document accurately the observed implementation was necessary. Relying solely on teachers' self report would not sufficiently prove implementation fidelity. Using the GTOP approach, we were able to determine to what extent the teachers in both groups adhered to the assigned instructional approach.

5 DATA ANALYSIS

The principal method for quantitative data analysis involved fitting a two-level hierarchical linear model to the data using HLM V6.02 software (Raudenbush, 2004). At level 1, the student level, the analysis focused on student achievement (XGT) data. At level 2, the regression coefficients produced at level 1 were used to estimate teacher/classroom level intercept and linear slope coefficients. All outcomes were reported using the estimates and inferential statistics obtained from the HLM software, as well as measures of practical effect sizes (Bloom, Richburg-Hayes, & Black, 2007). The first two research questions on cognitive (geometry achievement) and affective (beliefs about geometry) elements were answered using the regression coefficients associated with students' treatment vs. comparison membership as a predictor of post-test scores, with relevant covariates included at the student level (e.g., geometry pre-test achievement scores) and the teacher/classroom level to account for outcome variance at their respective levels.

Qualitative data analysis followed accepted principles that span qualitative research paradigms (Bogdan & Biklen, 1992; Ely & Anzul, 1991; Knuth, 2002; Strauss and Corbin, 1998; Tesch, 1990) to answer the fourth research question on the characteristics of the learning communities. The quantitative and the qualitative data analyses mentioned above, as a whole, answered the third research question that related to implementation fidelity.

The remainder of the paper will report the quantitative analysis results and answer the first two research questions.

6 RESULTS

As stated earlier, the two research questions addressed in this report were:

1. How do students in the experimental condition perform in comparison with students in the control condition on measures of a geometry achievement test?
2. How does the DG intervention affect student beliefs about the nature of geometry?

The researchers used two-level hierarchical linear models (HLM) to examine the impact of the DG approach on overall student achievement. The models were conducted using student pre-test (ENT) scores as a covariate. Once the significant predictors of overall achievement were identified, performance on each individual item was investigated in order to better understand the possible effect of the DG treatment on student learning. We used mixed logistic regression with the same predictors used in the HLM to estimate the impact of DG for each item.

The sample of classrooms studied included three different levels of Geometry: Regular, Pre-AP and Middle School (middle school students taking high school credit Pre-AP Geometry). Since the classroom expectation and performance of the students at each of these levels was often very different, the factor *Class Level* was included in each model. Additionally, the years of classroom experience of the teachers in the sample varied greatly, ranging from 0 to

35 years. Given the emphasis of technology in the study, the possible effect of teaching experience was unclear. A more experienced teacher may have greater command of the classroom but be less able to implement the technology whereas several newer teachers reported having some GSP experience as a component of their undergraduate training. For this reason, the covariate *Years Exp* (number of years of classroom experience) was included in the models.

During the summer PD sessions mentioned above, the participating teachers completed a demographic survey that included information about years of teaching experience, the class level of the teacher, and gender. From our initial teacher sample (76 participants), six teachers dropped because of family/health concerns or change in job status. An additional six teachers were dropped because they provided incomplete post-test data. Therefore, 64 teachers submitted complete post-test data for analysis in the study, of which 33 were in the experimental group (DG group), and 31 were in the control group.

6.1 Students' Geometry Knowledge Performance - HLM Model 1

In development of Model 1 (summarized below) full factorial designs were explored and insignificant interactions were discarded. We now discuss the final version of Model 1.

The HLM Model 1 equations are:

(Level 1: Student)

$$XGT_{ij} = \beta_{0j} + \beta_{1j}(ENT - \overline{ENT}) + \varepsilon_{ij}$$

(Level 2: Classroom)

$$\beta_{0j} = \gamma_{00} + \gamma_{01}DG_j + \gamma_{02}PreAP_j + \gamma_{03}MS_j + \gamma_{04}YrsExp_j + \gamma_{05}(YrsExp * PreAP)_j + \gamma_{06}(YrsExp * MS)_j + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + u_{1j}$$

where i represents student i , j represents classroom j , ε and u are independent normally distributed error terms. The dependent variable, XGT_{ij} , is the score on the Exiting Geometry Test of student i in classroom j . The level 1 β coefficients are random; β_{0j} is the intercept and β_{1j} is the slope of the ENT pre-test scores from students in classroom j . The γ coefficients are fixed; γ_{00} is the expected score of a student with an average pre-test score from a Regular Geometry control group classroom with a teacher with no experience; γ_{01} is the DG treatment effect; γ_{02} is effect of the Pre-AP classroom (compared to the Regular classroom); γ_{03} is the effect of the Middle School (MS) classroom (compared to the Regular classroom); γ_{04} is slope of Years of Experience of the teacher; γ_{05} and γ_{06} are interaction terms between Years of Experience and Pre-AP and MS levels of the classroom, respectively; and γ_{10} is the mean pre-test slope aggregated across all classrooms.

Model 1 examines the effect of the DG intervention when taking into account Entering Geometry Test (*ENT*) centered about the overall mean as well as *Class Level* and *Years Exp*. The results of this model (shown in Table 1) indicate that the DG effect was strongly

significant ($p = .003$). Table 2 shows the summary statistics for each level of class. Comparing the means, the DG group students scored higher than the control group students in each level of Geometry and the effect size was substantially larger at the Regular Geometry level ($ES = .41$).

As expected, ENT is a significant predictor of student performance on XGT ($p = .000$). However, even controlling for this pre-test, compared with Regular students, on average Pre-AP students scored 7.5 points higher ($p = .005$) and Middle School students scored 19.2 points higher ($p = .001$). The effect of teaching experience depended on the level of the course. It had a negative effect on the achievement of the students in Regular Geometry classes, but was positive for Pre-AP students. The effect of experience in the Middle School group was not significantly different from Regular classes. Interpreting the coefficients, we see that an increase in 10 years of experience raised the scores 4.3 points for the Pre-AP group and decreased the scores by 3.3 points for the Regular group.

Table 1 HLM results with pre-test as a covariate

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx d.f.	p-value
Intercept: γ_{00}	52.41	1.607	32.609	52	.000
DG Effect γ_{01}	5.277	1.653	3.193	46	.003
Level					
Pre-AP: γ_{02}	7.493	2.568	2.918	46	.005
M. School: γ_{03}	19.22	5.684	13.46	78	.001
YearsExp_ γ_{04}	-.3291	.1549	-2.124	57	.038
Level*Years Exp:					
Pre-AP * Years Exp: γ_{05}	.7641	.2215	3.450	46	.001
M. School*Years Exp_ γ_{06}	2.000	1.733	1.154	75	.252
ENT100 Centered_ γ_{10}	.4266	.0317	13.46	831	.000

Note. XGT is the response variable.

Table 2: Summary Statistics for Years of Experience, Exiting Geometry Test (XGT) and Entering Geometry Test (ENT)

	DG			Control			ES
	N	M	SD	N	M	SD	
Years of Experience							
Overall	33	7.00	7.18	31	6.48	8.29	.07
Class Level							
Regular	20	6.44	7.89	19	5.63	6.72	.11
Pre-AP	12	8.30	6.25	8	10.75	11.94	-.28
Middle School	1	4.00	NA	4	2.00	2.16	NA
XGT							
Overall	463	62.93	19.48	411	59.96	20.06	.15
Class Level							
Regular	255	54.39	18.08	214	47.59	14.85	.41
Pre-AP	193	72.27	15.55	156	69.84	15.28	.16
Middle School	15	88.27	7.00	41	86.97	10.08	.14
ENT							
Overall	463	67.41	16.76	411	66.30	18.96	.06
Class Level							
Regular	255	60.96	16.68	214	55.96	17.20	.30
Pre-AP	193	75.16	13.15	156	74.49	13.16	.05
Middle School	15	77.33	12.94	41	89.14	8.36	-1.2

Note. Includes only data for subsample with matching pre- and post-test results

To further compare the DG effect to other variables in the model of equations, a sequence of models was generated and analyzed. These models were very similar to Model 1 above but contained fewer variables. Table 3 shows the variables contained in each model. Note Model 1F, is the same Model 1 analyzed above. To compare the nested models, the size of the error variance at the student level and classroom level were computed. By comparing the relative size of the variances, we assessed the effect size for each of the variables. This process is comparable to examining the change in R^2 for standard regression models. The results, summarized in Table 3, show the large differences in the post-treatment scores by level of the course. Starting with just a random effect for the classroom, we see that the initial within classroom variance is 206 and the between classroom variance is 200. This translates to an intra-class correlation of nearly 0.5. We then analyzed a sequence of comparable models adding one variable at each step. Note that models 1E and 1F include the interaction between teaching experience and the level of the class. Inclusion of the pre-test score explained about 15% (170 compared to 206) of the student level variance and over 50% of the between classroom variance. This indicates that there were large differences between the students' knowledge at the beginning of the course as measured by the pre-test between the different classrooms involved in the studies. These differences might be a result of variable quality of the schools involved in the study or tied to tracking practices of students at different levels of the course. The comparison between models 1B (with just pre-test) and 1C (with just level of the class) indicates that tracking of the students into different levels of the course had a major impact. Finally, examination of the results models 1E and 1F, indicate that years of

experience of the teacher and the use of the DG approach in the classroom had similar impact size. After accounting for the level of the course, the pre-test score and years of experience, the DG effect explains about 19% of the remaining variance (6 out of 31) between classrooms.

Table 3 Comparable Models

Model	Int.	ENT	Level	Years Exp	DG	Student Level Error (σ^2)	Classroom Level Error (τ^2)
1A	X					206	200
1B	X	X				170	96
1C	X		X			206	46
1D	X	X	X			170	38
1E	X	X	X	X		170	31
1F	X	X	X	X	X	170	25

6.2 Students' Geometry Knowledge Performance - Mixed Logistic Regression Model 2

To better understand the nature of the impact of the DG approach on student learning, each item of the *XGT* was analyzed separately. In this case, the outcome for each item is dichotomous (1 = *Correct*, 0 = *Incorrect*), and the same predictors as HLM Model 1 were used. A HLM logistic model was used to estimate the log-odds of a correct response.

The HLM Logistics Model 2 equations are:

(Level 1: Student)

$$\log\left(\frac{p_{ij}}{1-p_{ij}}\right) = \beta_{0j} + \beta_{1j}(ENT - \overline{ENT})$$

(Level 2: Classroom)

$$\beta_{0j} = \gamma_{00} + \gamma_{01}DG_j + \gamma_{02}PreAP_j + \gamma_{03}MS_j + \gamma_{04}YrsExp_j + \gamma_{05}(YrsExp * PreAP)_j + \gamma_{06}(YrsExp * MS)_j + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + u_{1j}$$

where p_{ij} represents the probability that student i in classroom j gets the item correct ($1-p_{ij}$) is the probability that student i in classroom j gets the item incorrect, the ratio $\frac{p_{ij}}{1-p_{ij}}$ converts probabilities to odds, which range over the non-negative Real numbers; by computing the log of odds (logit), we map probabilities over all Real numbers; and then by computing the exp (log odds), we return to the odds ratio), u_{0j} and u_{1j} are normally distributed error terms. As in model 1, the level 1 β coefficients are random; β_{0j} is the intercept and β_{1j} is the slope of the centered ENT pretest scores in classroom j . The γ coefficients are fixed effects on the log-odds of getting the item correct; where i represents student i , j represents classroom j , ε and u are independent normally distributed error terms. γ_{00} is the intercept; γ_{01} is the DG treatment effect; γ_{02} is effect of the Pre-AP classroom (compared to the Regular classroom);

γ_{03} is the effect of the Middle School classroom (compared to the Regular classroom); γ_{04} is slope of Years of Experience of the teacher; γ_{05} and γ_{06} are interaction terms between Years of Experience and the level of the classroom; and γ_{10} is the slope of the pre-test scores aggregated overall classrooms. As an example, the results for the item 1 model are shown in Table 4.

Table 4 Mixed Logistic Regression results for item 1 of the XGT with pre-test ENT as a covariate

Fixed Effect	γ : Coefficient	Exp (γ)	p-value
Intercept: γ_{00}	-0.043	0.958	.758
DG Effect: γ_{01}	0.310	1.363	.162
Level			
Pre-AP: γ_{02}	0.542	1.719	.114
Middle School: γ_{03}	1.371	3.939	.156
Years of Experience	0.028	1.028	.188
Level*Years of Experience			
Pre-AP: γ_{04}	-0.016	0.984	.596
Middle School: γ_{05}	0.245	1.278	.485
ENT (Mean Centered): γ_{10}	0.014	1.014	.005

Note. XGT Item1 (1= Correct, 0 = Incorrect) is the response variable. Logit is the link function.

This method of analysis uses a linear model to estimate the log-odds of a correct response. A positive γ coefficient indicates that the predictor has a positive effect on the performance when controlling for the other predictors in the model. To facilitate interpretation, $Exp(\gamma)$, which represents the effect on the odds of a correct response is also shown. For example, for item 1 of the XGT, the odds of a student in the DG group correctly answering item 1 are 1.363 times that of a student in the control group who had the same pre-test score when both students are in the same level of class with a teacher with same number of years of experience.

A summary of the results for all 25 items on the XGT post-test is presented in Table 5. The first column contains a brief description of geometry concept addressed by the item. Five of the items are discussed in more detail below. The next two columns provide the percentage of students in each group that answered the item correctly. The values in the final two columns were computed using the mixed logistic models used to compute the data in Table 4. The *p-value* column represents the statistical significance of the *DG* effect in the model, and $Exp(\gamma)$ represents the effect of DG on the odds of a correct response.

Table 5 Summary statistics of the results for all 25 items on the XGT post-test

Item Description	DG (%)	Control (%)	p-value	Exp(γ)
1. Congruent Triangles	66	57	0.162	1.36
2. Congruent Triangles	86	81	0.410	1.20
3. Triangle Inequality	48	43	0.123	1.34
4. Congruent Triangles	82	76	0.229	1.25
5. Similar Triangles	71	66	0.410	1.20
6. Similar Triangles	56	51	0.986	1.00
7. Properties of a Parallelogram	89	86	0.656	1.11
8. Point on a Circle	55	46	0.013	1.65
9. Area of Figure Composed of Right Triangles	65	63	0.786	1.07
10. Area of a Circle	64	58	0.496	1.15
11. Volume Using a Net	59	51	0.015	1.71
12. Area of Figure (Rectangle – Right Triangles)	62	60	0.538	1.12
13. Area of Trapezoid	55	50	0.357	1.26
14. Exterior Angles of a Triangle	59	51	0.065	1.38
15. Exterior Angles of a Regular Hexagon	48	41	0.088	1.35
16. Pythagorean Theorem	63	54	0.490	1.21
17. Construct Bisector of Angle	33	29	0.095	1.58
18. Pythagorean Theorem	59	55	0.692	1.08
19. Angles Inscribed in a Circle	68	67	0.520	1.11
20. Transformation (Rotation)	61	60	0.988	0.98
21. Transformation (Translation)	79	73	0.383	1.26
22. Properties of a Trapezoid	49	44	0.561	1.13
23. Similar Triangles	47	38	0.079	1.40
24. Parallel Lines with Transversal	54	47	0.086	1.36
25. Parallel Lines with Transversal (Proof)	50	46	0.932	1.02

Note that the odds of a correct response were greater for the DG group (than the control group) for every item except items 6, 20, and 25. For those items, Exp(γ) was equal or very close to 1 (1.00, .98 and 1.02, respectively), suggesting equal odds. Although the impact of the DG treatment on performance appeared to be very broad with increases in student performance on 22 of the 25 items, it is informative to examine items where that increases were greatest. Since the analysis on items of the *XGT* is exploratory in nature, the researchers focused on seven of the items (8, 11, 14, 15, 17, 23, and 24) where the Exp(γ) was greater than (or equal to) 1.35 and significantly larger than 1 at the $\alpha = .10$ level.

Analysis indicated that the seven items, provided in Figure 1, with a significant DG effect address a broad spectrum of geometry topics: circles (Item 8), volumes (Item 11), exterior angles (item 14), hexagons (Item 15), perpendicular bisector construction (Item 17), similar triangles (Item 23), and parallel lines with a transversal (Item 24). The researchers noted that the nature or structure of each of the seven items are quite different as well. In Items 11, 14, 17, and 24 a figure was included, while in Items 8, 15, and 23 had no figure included. Item 14 is a standard computation of the measure of an exterior angle. Item 17 relates to a compass

and straightedge construction, but is one of a few simple construction exercises introduced in class. Item 8 involves the definition of a circle. Item 15 involves the properties of a regular hexagon. In item 11, students need to visualize the three-dimensional box formed by a two-dimensional net before calculating its volume. In Item 23, students need to know the requirements for two triangles to be similar and properties of some special triangles. In Item 24 students need to understand the conditions for two lines to be parallel and the relationships of the angles formed by two parallel lines with a transversal. These further indicate that the impacts observed in the data analysis do not appear to be connected to any one particular task or topic. Rather, working with dynamic geometry provides for a general and overall improvement of geometry knowledge.

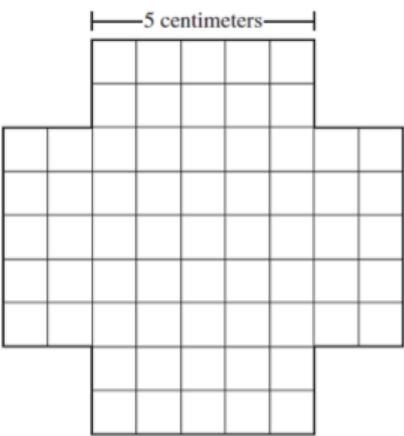
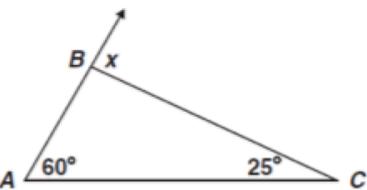
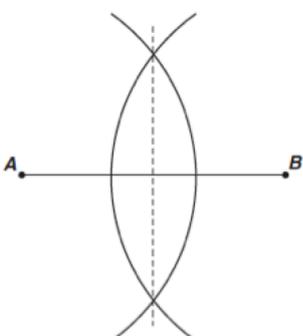
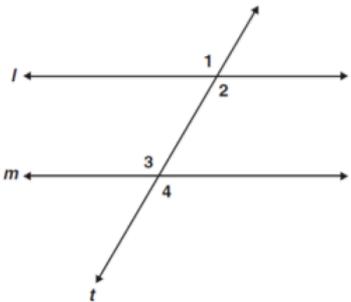
<p>11. The four sides of this figure will be folded up and taped to make an open box.</p>  <p style="text-align: center;">← 5 centimeters →</p> <p>What will be the volume of the box?</p> <p>A 50 cm³ B 75 cm³ C 100 cm³ D 125 cm³</p> <p>14. What is $m\angle x$?</p>  <p>A 35° B 60° C 85° D 95°</p>	<p>17. What geometric construction is shown in the diagram below?</p>  <p>A an angle bisector B a line parallel to a given line C an angle congruent to a given angle D a perpendicular bisector of a segment</p> <p>24. In the diagram below, $\angle 1 \cong \angle 4$.</p>  <p>Which of the following conclusions does <i>not</i> have to be true?</p> <p>A $\angle 3$ and $\angle 4$ are supplementary angles. B Line l is parallel to line m. C $\angle 1 \cong \angle 3$ D $\angle 2 \cong \angle 3$</p>
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Fig. 1a Four of the Seven items that included figures from the XGT geometry post-test

<p>8. The diameter of a circle is 12 meters. If point P is in the same plane as the circle, and is 6 meters from the center of the circle, which <i>best</i> describes the location of point P?</p> <p>A Point P must be on the circle.</p> <p>B Point P must be inside the circle.</p> <p>C Point P may be either outside the circle or on the circle.</p> <p>D Point P may be either inside the circle or on the circle.</p>	<p>15. What is the measure of an exterior angle of a regular hexagon?</p> <p>A 30°</p> <p>B 60°</p> <p>C 120°</p> <p>D 180°</p> <p>23. Which triangles must be similar?</p> <p>A two obtuse triangles</p> <p>B two scalene triangles with congruent bases</p> <p>C two right triangles</p> <p>D two isosceles triangles with congruent vertex angles</p>
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Fig. 1b Three of Seven items without figures from the XGT geometry post-test

6.3 Students' Beliefs about Geometry - HLM Model 3

Hierarchical linear models were also used to investigate the impact of the project on the beliefs of the students about geometry. As described in the student-measures section earlier, students' responses to the belief questionnaire identified a single factor. The implementation of the DG approach in the participating schools emphasized using GSP to explore geometric constructions to form and verify conjectures. For this reason, we posited that students from classes taught by DG teachers would agree more strongly than control group students with items 3–11, which deal specifically with construction and conjectures. Therefore, for the purposes of this study, results were reported for the instrument as a whole, *Beliefs about Geometry*, and two subscales *Beliefs about Construction and Conjectures* and *Beliefs about Proof and Logic*. Table 6 displays the mean and standard deviation for each item, both subscales and the overall scale. The maximum score for any item is 5.0 and the minimum is 1.0.

Table 6 Summary statistics by item on the Student Belief Questionnaire

Beliefs about Geometry Items	Pre		Post	
	M	SD	M	SD
Beliefs about Proof and Logic Items	3.98	0.48	4.11	0.52
1. Geometry involves logical thinking.	4.25	0.73	4.32	0.75
2. Logical arguments are a major part of geometry.	3.38	0.91	3.59	0.97
12. Proofs are used to justify geometric ideas.	3.87	0.81	4.08	0.83
13. Proofs involve explorations of geometric situations.	3.71	0.80	3.89	0.86
14. Knowing definitions is important in geometric proof.	4.21	0.84	4.28	0.85
15. Understanding what a theorem says helps to prove it.	4.06	0.82	4.31	0.82
16. Whenever I study the proof of a theorem I try to understand each step and reason for it.	3.92	0.85	4.02	0.90
17. In geometry I can use true statements to justify each statement I make.	3.98	0.82	4.10	0.82
18. In geometry I can use facts, rules, definitions, &/or properties to reach a logical conclusion.	4.32	0.78	4.38	0.78

Beliefs about Geometry Items	Pre		Post	
	M	SD	M	SD
Beliefs about Construction and Conjecture Items	3.66	0.43	3.81	0.50
3. A geometric construction is a drawing used to show a geometric idea.	3.73	0.77	3.96	0.76
4. I can use a computer application to create a geometric construction.	3.79	0.91	3.98	0.96
5. Geometric constructions allow me to explore geometric relationships.	3.74	0.79	3.93	0.87
6. Making conjectures (i.e., educated guesses) is a good habit of geometric learning.	3.25	1.11	3.40	1.15
7. Measurement can help me make conjectures about geometric relationships.	3.85	0.79	4.00	0.82
8. Investigating examples is important when making a conjecture.	3.83	0.83	3.98	0.84
9. If I find a counterexample then the conjecture is false.	3.26	0.86	3.48	0.96
10. Technological tools are particularly useful informing conjectures.	3.52	0.85	3.64	0.91
11. Technological tools can help me test conjectures.	3.76	0.81	3.86	0.87
Beliefs about Geometry Overall	3.80	0.40	3.96	0.45

The analyses on the student gain scores on the belief instrument (not displayed here) showed significant, positive differences between pre- and post-treatment administrations during the school year at each level (Regular, Pre-AP, or Middle School) for the overall Beliefs about Geometry, Beliefs about Construction and Conjectures, and Beliefs about Proof and Logic. HLM models 3A, 3B and 3C (collectively called Model 3) were used to investigate the impact of the DG treatment on beliefs.

The HLM Model 3 equations are:

(Level 1: Student)

$$BAG_{ij} = \beta_{0j} + \beta_{1j}ENT + \beta_{2j}BAG_Pre_{ij} + \varepsilon_{ij}$$

(Level 2: Classroom)

$$\beta_{0j} = \gamma_{00} + \gamma_{01}DG_j + \gamma_{02}PreAP_j + \gamma_{03}MS_j + \gamma_{04}YrsExp_j + \gamma_{05}(YrsExp * PreAP)_j + \gamma_{06}(YrsExp * MS)_j + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + u_{1j}, \quad \beta_{2j} = \gamma_{20} + u_{2j}$$

where i represents student i , j represents classroom j , ε and u are independent normally distributed error terms. Three versions of the model were analyzed each with a different dependent variable but identical predictors. The dependent variable was the Beliefs about Geometry Scale score for model 3A, the Beliefs about Construction and Conjecture score for model 3B and the Beliefs about Proof and Logic score for model 3C. The level 1 β coefficients are random; β_{0j} is the intercept, β_{1j} is the slope of the content pretest (ENT) scores in classroom j and β_{2j} is the slope of the Beliefs about Geometry scores administered before the treatment in classroom j . The γ coefficients are fixed; γ_{00} is the intercept aggregated over all classrooms; γ_{01} is the DG treatment effect; γ_{02} is effect of the Regular Geometry classroom; γ_{03} is the effect of the Pre-AP classroom; γ_{04} is slope of Years of Experience of the teacher in the Regular classroom; γ_{05} is slope of Years of Experience of the teacher in the Pre-AP classroom; γ_{06} is slope of Years of Experience of the teacher in the Middle School classroom; and γ_{10} is the slope of the ENT pre-test scores aggregated over all classrooms and γ_{20} is the slope of the Beliefs about Geometry scores administered before the

treatment and aggregated over all classrooms. Tables 7-9 show the HLM analysis results on the students' scores, administered after the treatment, for the overall *Beliefs about Geometry* scale and the two subscales:

Table 7 HLM Model 3A for post-treatment *Beliefs about Geometry* scale

Fixed Effect	Coefficient	Standard Error	Approx d.f.	T-Ratio	p-value
Intercept: γ_{00}	3.921	0.034	53.8	115.9	0.000
DG: γ_{01}	0.037	0.034	43.9	1.11	0.274
Level					
Pre-AP: γ_{02}	0.090	0.052	44.1	1.72	0.092
Middle School: γ_{03}	0.098	0.123	91.1	0.80	0.426
Years Teaching: γ_{04}	-0.003	0.003	55.2	-0.94	0.350
Level*Years Teaching					
Pre-AP: γ_{05}	0.000	0.045	41.6	0.02	0.987
Middle School: γ_{06}	0.011	0.038	95.0	0.30	0.767
ENT: γ_{10}	0.002	0.001	792.3	2.96	0.003
Beliefs about Geometry Pre-test: γ_{20}	0.414	0.035	850.8	11.92	0.000

Table 8 HLM Model 3B for post-treatment *Beliefs about Construction and Conjecture* scale

Fixed Effect	Coefficient	Standard Error	Approx d.f.	T-Ratio	p-value
Intercept: γ_{00}	3.740	0.040	52.0	92.5	0.000
DG: γ_{01}	0.096	0.041	43.6	2.37	0.022
Level					
Pre-AP: γ_{02}	0.033	0.063	43.9	0.53	0.600
Middle School: γ_{03}	-.018	0.063	85.9	-.12	0.902
Years Teaching: γ_{04}	-0.001	0.004	49.8	-0.29	0.775
Level*Years Teaching					
Pre-AP: γ_{05}	-0.001	0.005	40.1	-.21	0.832
Middle School: γ_{06}	0.019	0.045	89.9	0.423	0.673
ENT: γ_{10}	0.003	0.001	825.1	3.45	0.001
Beliefs about Geometry Pre-test: γ_{20}	0.422	0.040	872.7	10.5	0.000

Table 9 HLM Model 3C for post-treatment *Beliefs about Proof and Logic* scale

Fixed Effect	Coefficient	Standard Error	Approx d.f.	T-Ratio	p-value
Intercept: γ_{00}	4.115	0.040	55.4	104.15	0.000
DG: γ_{01}	-0.043	0.040	46.0	-1.08	0.288
Level					
Pre-AP: γ_{02}	0.135	0.144	93.0	0.94	0.351
Middle School: γ_{03}	0.134	0.061	46.5	2.18	0.035
Years Teaching: γ_{04}	-0.005	0.004	53.4	-1.28	0.206
Level*Years Teaching					
Pre-AP: γ_{05}	0.019	0.044	98.7	0.42	0.678
Middle School: γ_{06}	0.001	0.005	42.3	0.28	0.778
ENT: γ_{10}	0.002	0.001	813.1	1.98	0.048
Beliefs about Geometry Pre-test: γ_{20}	0.415	0.041	873.7	10.14	0.000

From these three tables, we can see that students in the DG classes scored significantly higher after the DG approach treatment (i.e., developed more positive attitudes towards the exploring and conjecturing activities) than those in the control classes on the *Beliefs about Construction and Conjecture* subscale ($p = 0.022$). However, there was no significant difference between the scores of the students in the two treatment groups on the overall *Beliefs about Geometry* scale ($p = 0.274$), or on the *Beliefs about Proof and Logic* subscale ($p = 0.288$) administered after the treatment. These findings were consistent with our hypothesis that the DG approach significantly impacts students' beliefs about construction and conjecture, and supported by the qualitative data that we collected (e.g., classroom observation data and teachers' responses to the DG Implementation Questionnaires). Through the implementation of the DG approach, the teachers in the experimental group increased their levels of conjecturing about geometric relationships, and changed how they presented geometry concepts to their students. As evidenced by the gains in post-test scores, their students were able to develop and retain understanding of geometric relationships through their explorations of geometric constructions and conjecturing, and also apply what they have learned in new contexts. As evidenced by the gains on the *Beliefs about Construction and Conjecture* subscale, students' beliefs about geometry changed to include stronger views about how conjecturing and the construction process are important. However, even though the researchers made tremendous efforts in facilitating teachers' and students' proving activities, the teachers and their students' attitudes towards logical reasoning and proving were not as positive as those towards exploring and conjecturing. The teachers reported to the researchers that their school districts' geometry curriculum de-emphasized the importance of proofs, and that they were expected by their principals to spend a large amount of instructional time on the preparation and administration of state standardized tests that include very little emphasis on proof.

7 DISCUSSION

The HLM models taking pretest, class level, and teaching experience into account provided evidence that students taught by teachers in the Dynamic Geometry group scored significantly higher than those taught by the control group teachers in geometry achievement as measured by the Exiting Geometry Test, and on *Beliefs about Construction and Conjecture*. Given that teachers were randomly assigned to the two groups and that both groups received comparable professional development of the same duration on the same topics and with the same cognitive load (differing only in whether DG technology was used or not), the results from this study provide empirically derived evidence that supports the efficacy of the DG approach and that the DG approach had a positive impact on high school geometry student learning – it accounted for a significant portion of the variance in geometry achievement observed between the treatment groups in the study.

One “unusual” result of the HLM analysis of the data is the effect of teaching experience on student achievement on the geometry post-test – teachers with more years of experience had a positive impact on achievement of students in the Pre-AP level classes and a negative

impact for students in the Regular level classes. This finding was considered “unusual” because we generally assume that experience promotes effectiveness. However, “Over 40 years of teacher productivity research suggests that the simple assumption that ‘more is better’ requires greater nuance; experience effects are complex and depend on a number of factors ... Studies have also documented some evidence that effectiveness *declines* after some point, particularly among high school teachers” (Rice, 2010, pp.1-2). In fact, evidence suggests that the most experienced (25+ years) high school mathematics teachers may be less effective than their less experienced counterparts (Ladd, 2008). So, further research is necessary to fully understand why the negative impact of teaching experience occurred for the Regular level classes. Our initial explanation relates to the expectations of teachers at the two different levels. Based on years of interaction with middle and high school teachers, the research team members have noticed the tendency for experienced teachers to have more rigid beliefs about students’ abilities to achieve. Pre-AP classes were composed of mostly middle to high achieving students, and hence teachers had higher expectations for those students. Meanwhile, since students in Regular classes had a record of low to middle achievement, teachers likely had lower expectations for those students. In contrast, as the research team has also noticed, novice teachers were more willing to believe that all students could learn. Therefore, we speculate based on these findings that teacher expectations for students play a critical role in student achievement. This might play a larger role in DG environments where teachers generally need to place a degree of trust in their students’ thinking—that learners can and will be able to notice patterns, conjecture relationships, and build proofs or reasonable justifications for these activities. These may not be absent in non-DG classrooms but the technology component (with resulting management and time issues) on top of the inquiry activities might result in less adherence to discovery-related approaches to learning and teaching.

The specific classroom tasks used by the DG teachers (as indicated on teachers’ responses to the DG Implementation Questionnaire and informed by classroom observations—but not described in this paper) included primarily activities involving figures and other geometric images, “dragging” these objects, exploring changes, and conjecturing about relationships. This study provided evidence that for the DG group, learning transcended specific tasks that made use of images or diagrams dynamically. Students taught by DG group teachers scored higher than students from the control group on non-pictorial/image-based and completely static tasks from a wide range of topics. This may be related to the fact that working with DG software increases students’ engagement with geometry, making the topic more exciting or interesting. Alternatively, a different explanation involves the student’s ability to make sense of basic geometric objects. When given the chance to dynamically interact with a geometry concept, students may be better able to internalize the basic definitions and theorems. This is aligned with the semiotic mediation framework. As DG students engage in the DG tasks, they are mediating their understandings of not just the tasks but also the larger mathematics contexts. As Drijvers et al. (2010) write, “In this respect, any artifact may be considered both from the individual point of view — for instance, the pupil coping with a task and acting with a tool to accomplish it — and from the social point of view — for instance, the corpus of shared

meanings recognizable by the community of experts, mathematicians or mathematics teachers. From a socio-cultural perspective, the tension between these two points of view is the motor of the teaching-learning process centered in the use of an artifact” (pp. 116-117). The specific tasks are of importance but so is the larger discourse of the classroom through which these activities are mediated. Thereby as students engage in DG environments and tasks, their learning of geometry as a whole, and not just that related to the specific tasks, is activated.

The first two research questions of the project, as described above, were essentially answered. To thoroughly address all research questions of the study and to better understand issues such as the effect of teaching experience on geometry achievement, while continuing to use the HLM models to analyze quantitative data, we will conduct and report in-depth qualitative analysis on the participating teachers’ responses to the DG Implementation Questionnaire, and the data collected in the classroom observations and student and teacher interviews. Guided by the integrative framework represented by the new tetrahedral model (Olive & Makar, 2009), in further data analysis processes, we will focus mainly on the interactions among teachers, tasks (conjecturing/proving tasks presented to the participating teachers or students), technology (DG tools), and students, so as to develop new insights related to the ways of thinking and communicating that are characteristic of the DG environments.

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THE EFFECT OF DYNAMIC GEOMETRY APPROACH ON GEOMETRY ACHIEVEMENT AND CONJECTURE ABILITY

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The reported study compares effects of the dynamic geometry (DG) approach with standard instruction that does not make use of computer tools. The basic hypothesis of the study is that the DG approach results in better geometry learning for many students. The study tests that hypothesis by comparing student learning in classrooms randomly assigned to treatment and control groups. Student learning is assessed by a geometry test and other tests. Data for answering the research questions of the study are analyzed by appropriate Hierarchical Linear Model (HLM) methods. Three different levels of Geometry courses were considered for the analysis: Regular, Honors School and Advanced Honors. For students attending Regular Geometry, the experimental group outperformed the control group in geometry performance and conjecture ability.

INTRODUCTION

The term *Dynamic Geometry (DG)* refers to active geometric explorations carried out with interactive computer software such as the Geometer's Sketchpad (Jackiw, 2009), which has been available since the early 1990's. The approach emphasizes the use of the software to model, conjecture, and test conjectures. Along with the widespread use of DG software, many related research studies have been conducted. A relatively small group of researchers (e.g., Dixon, 1997; Gerretson, 2004; Myers, 2009) used experimental or quasi-experimental designs in their studies. Most of the studies (e.g., Choi-Koh 1999; Vincent, 2005; Hollebrands, 2007; Baccaglioni-Frank and Mariotti, 2010) used qualitative research methods. Collectively, this body of work has revealed important findings: If dynamic geometry technology is used effectively, it can have a significant impact on students' learning. When used as a cognitive tool, it can: facilitate students' exploration and investigation activities; promote their conjecturing, verifying, explaining, and logical reasoning spirits and abilities; and enhance their conceptual understanding of important geometric ideas. However, almost all of the studies were either exploratory phenomenological studies, or comparative studies that did not employ a true experimental design and/or were conducted on a small scale. This paper reports results from the second year of a large scale two-year study funded by NSF comparing effects of the DG approach with standard approach in high school geometry classrooms.

THE DYNAMIC GEOMETRY (DG) APPROACH

The instructional approach of using dynamic geometry software in this study is referred to as the dynamic geometry (DG) approach. To guide the creation and selection of DG activities to be used in the teachers' professional development and their geometry classrooms, based on the research studies cited above, we (the research team) came up with the following operational definition of this approach:

As learning activities emphasized by the DG approach, students are expected to:

- Construct dynamic geometric objects with DG tools
- Construct dynamic representations of problem situations with DG tools
- Perform actions (drag, measure, transform, and/or animate) on the constructed objects/situations
- Observe variants and invariants related to the characteristics of objects under different actions
- Investigate mathematical relationships and/or solutions in multiple ways
- Formulate conjectures
- Test conjectures
- Receive immediate feedback from the software
- Think mathematically and prove (or disprove) their conjectures

As teaching activities recommended by the DG approach, teachers are expected to:

- Facilitate the student use of DG software
- Help students construct mathematical ideas through active explorations and investigations
- Present prepared DG environments for students to explore mathematical relationships when the constructions of the DG environments are too complicated for students or when the constructions themselves are not the focus of the related explorations
- Facilitate students' argumentations by asking "why" questions – prompting students to furnish justifications for their statements and checking the validity of their justifications
- Extend students' explorations by asking "what if" questions

RESEARCH QUESTIONS AND DESIGN

The research questions of the study include:

- 1) How do students taught in a DG oriented instructional environment perform in comparison with students in the control condition?
- 2) How is students' learning related to the emphasis placed on conjecturing in instruction?

The population from which the participants of this study were sampled is geometry teachers and their students from high schools in Central Texas. The research study follows a multi-site randomized cluster design, with teachers as the unit of randomization. Initially 76 geometry teachers were selected. The 76 teachers were randomly assigned to the Experimental Group (DG) and the Control Group. Due to attrition and change of class assignments, only 45 of the original 76 teachers participated in the both years of data collection. In addition, 3 new teachers participated in only the second year. All the schools involved in the study group their geometry students based on past achievement in mathematics. The three levels were *Regular*, *Honors* (10th graders), *Advanced Honors* (9th and below). The level was not taken into account in the random placement, but was accounted for in the analysis after teachers were assigned classes by their schools. In the first year, one class for each teacher was randomly selected to participate. In the second year of the study, in order to maintain the sample size given attrition of teachers, two classes per teacher were randomly chosen for some teachers. Therefore, in year two of the study each teacher is represented with

measurements from one or two classrooms of students.

Teachers in both treatment and control groups received relevant but separate professional development. A weeklong summer institute was offered to the participating teachers prior to the first year, followed by 6 half-day Saturday PD sessions during each school year. The PD was planned and implemented collaboratively by project staff that included university faculty members and school-based master teachers. The project team and master teachers served as partner facilitators for all PD sessions. The teachers in the experimental (DG) group focused on developing their conceptual understanding of mathematics using the DG software as a tool. They experienced how DG environments encourage mathematical investigations by allowing users to manipulate their geometric constructions to answer "why" and "what if" questions, by allowing them to backtrack easily to try different approaches, and by giving them visual feedback that encourages self-assessment. The PD workshops in the control group addressed the same mathematical content as the DG group but without the use of technology. The PD sessions for the control group utilized patty paper, compass and straight edge. Previously reported results indicate that the PD had a positive effect on teachers' conjecturing and proving capabilities (Jiang, et al. , 2015a) and that DG group teachers were able to develop a new learning style where they used the software to generate and test conjectures leading more quickly to proofs (Jiang, et al. 2015b).

Instrumentation

There were significant changes in the research context between year 1 and year 2 of the study. The state implemented a new testing policy which had a large impact on Geometry. In year 1 of the study, the majority of students took a 10th Grade exam whose content included a significant portion of algebra and number skills. In year 2, students in *Advanced Honors* took an End of Course exam specific to geometry. In response to this, the team used its own instruments to assess student learning.

Geometry Content Knowledge: To measure learning that occurred during the year, each student took an entering and exiting test on geometry. The Entering Geometry Test (ENT) was developed by Usiskin (1982) and his research team at University of Chicago to test for knowledge that students should have prior to taking high school geometry. ENT consists of 20 multiple choice items and has a reliability of $\alpha=.77$. The Exiting Geometry Test (XGT), which was developed by the project team for year 1 of the study (Jiang et al., 2011), measures content students should learn during the course in high school geometry. XGT has 25 multiple-choice items with a reliability of $\alpha=.88$.

Beliefs about Geometry: Adapting work on beliefs about problem solving (Kloosterman, 1992), about mathematics (Halloun et al., 1996), about the nature of proof (Christou, 2004), and the nature of dynamic geometry (Hoyles, & Jones, 1998), a scale consisting of 9 likert-type items about student beliefs about the role of Conjecture and Construction in geometry ($\alpha=.72$) was also developed by the research team. A higher total score represents a more positive attitude towards the exploring, conjecturing, and construction activities. Students completed an identical instrument at the beginning and end of the school year.

Conjecture and Proof Skills: Adapting frameworks suggested by Koedinger (1998) and Harel and Sowder (2007), the team created a Conjecture Proof Test (CPT) with 20 multiple-choice items ($\alpha = .70$) which assesses the students ability to generate and investigate Conjectures as well as to argue logically. For example, to measure the ability of students to generate conjectures, students must determine in given context, if a statement is trivial, unclear, easily determined to be false or in fact a good conjecture. One type of item used to assess their ability to investigate a conjecture, students are asked to choose among four sets of geometrical objects, which set would be the best to test a given conjecture.

Implementation: The Dynamic Geometry Implementation Questionnaire (DGIQ) was adapted from a teacher questionnaire developed by the University of Chicago researchers (Dr. Jeanne Century and her colleagues) for an NSF-funded project and based on the critical features of the DG approach. The final version of the DGIQ consisting of 31 multiple-choice items and ten open-response questions was administered to the teachers in the DG group six times across the school year. For the multiple choice questions, teachers were asked how frequently they used the computer to demonstrate geometrical concepts, students used the computers, students engaged in each step of the conjecture-proof process, students used each dynamic feature of the DG software, etc. An appropriately modified version of the questionnaire (CIQ) consisting of 31 multiple-choice items and 5 open-response questions was administered to the control group teachers (also six times) to examine how they teach geometry without using dynamic technology. To address research question 2, an Implementation Score averaging the 4 items of DGIQ and CIQ related to student engagement in creating and testing conjectures across the 6 reports of the year was computed. A higher implementation score indicates that the teachers reported that students were engaged in conjecturing a higher percentage of the time. Given that the expectations for Regular Geometry did not include proof, proof related activities were not included in the score.

DATA ANALYSIS

Results from initial data analysis on the geometry pretest and posttest from the first year of data collection indicated that the DG treatment had a small but significant positive effect on student achievement in geometry. Notably the effect was largest with students in the Regular Geometry track (Jiang et al., 2011). In this paper, we present analysis for year 2.

The principal method of data analysis involves fitting a two-level hierarchical linear model to the data. This multilevel approach enables us to address the two research questions while taking into account the nested structure of the data (i.e. students nested within teachers' classrooms).

Results

Two-level hierarchical linear models (HLM) were employed to assess the impact of the use of the DG approach on student learning. The original sample of classrooms studied included three different levels of Geometry: Regular, Honors School and Advanced Honors. Since the classroom expectations and preparation of the students in each of these levels is very different, the three levels were analyzed separately. The results shown below refer to the Regular level only which had 15 teachers with 543 students in the DG group, and 16 teachers with 525 students in the control group. Results from year 1 indicated that the largest treatment effect occurred in the Regular level (Jiang et

al., 2011). In year 2, the results were similar; the treatment did not have a significant effect in the two Honors levels.

HLM Results

In development of the HLM models summarized below, full factorial designs were explored and insignificant interactions were discarded. Only the final model is discussed.

The HLM Model equations are:

(Level 1: Student)

$$Y_{ij} = \beta_{0j} + \beta_{1j}ENT_j + \beta_{2j} BConPre_j + \beta_{3j}BConPost_j + E_{ij}$$

(Level 2: Classroom)

$$\beta_{0j} = y_{00} + y_{01}DG_j + y_{02}IScore_j + u_{0j}$$

$$\beta_{1j} = y_{10} + u_{1j}$$

$$\beta_{2j} = y_{20} + u_{2j}$$

$$\beta_{3j} = y_{30} + u_{3j}$$

i represents student i , j represents classroom j , E and u are independent normally distributed error terms. The dependent variable, Y_{ij} , is the score on the Exiting Geometry Test of student i in classroom j for Model 1 and is the Conjecture Proof Test for model 2. The level 1 β coefficients are random; β_{0j} is the intercept, β_{1j} is the slope of the ENT pretest in classroom j and β_{2j} and β_{3j} are the slopes of the Beliefs scale for the pre and post assessment, respectively. The γ coefficients are fixed; y_{00} is the expected score of a student with an average pretest score from a control classroom with an average IScore; y_{01} is the DG treatment effect; y_{02} is average slope of the IScore; y_{10} , y_{20} and y_{30} are the slopes, aggregated overall classrooms, of the ENT, BCon-Pre and BCon-Post, respectively.

Model 1, shown in Table 1, examines the effect on performance on the Exiting Geometry Test (XGT) of the DG intervention when taking into account the Entering Geometry Test (ENT), Implementation Score (IScore) and the pre and post beliefs about conjecture (BCon). The results indicate that the only strongly significant predictor of XGT was ENT. However, both the DG effect (p-value = .084) and the post beliefs about conjecture (p-value = .080) were weakly significant. The IScore which measure the amount of conjecturing the students were engaged was not significant. The DG group scored 8.5% higher on average than the control group when controlling for the other variables.

Table 1: HLM Results for XGT and CPT

Effect	Model 1 XGT			Model 2 CPT		
	Coefficient	Std. Error	p-value	Coefficient	Std. Error	p-value
DG Effect	8.47	4.65	.084	8.45	2.88	.009
BCon-Pre	-1.46	1.75	.403	-1.22	1.49	.411
BCon-Post	2.46	1.40	.080	3.65	1.23	.003
IScore	0.17	0.13	.204	0.18	0.08	.038
ENT	0.27	0.04	.000	0.20	0.04	.000

Model 2 shows the results for an analogous model for predicting the Conjecture and Proof Test. In this case, the DG effect (p-value = .009), the post score for BCon (p-value = .003), IScore (p-value = .038) and ENT (p-value = .000) are all significant at the 0.05 level. On average, the DG group scored 8.45 % higher on the CPT. The IScore is recorded on a scale of 0 – 100, where 100 indicates that teachers reported that students were engaging in conjecturing in each class throughout the year and 0 indicates that the students did not conjecture at any time through the year. Hence, interpreting the slope of the IScore, we see that a 10% increase in the amount of conjecturing in the classroom results in 1.8% increase in the CPT score.

DISCUSSION

The HLM model 1 showed that for students in the Regular Level, the Dynamic Geometry group outperformed the Control group in geometry achievement in year 2 of the study. This was consistent with the results from year 1, although in the second year $0.05 < p\text{-value} < 0.10$. Model 2 shows that the treatment effect was more significant (p-value = .009) for the performance on the Conjecture-Proving test. This is consistent with the DG approach’s emphasis on using the software to model, conjecture and test conjectures. Given that teachers were randomly assigned to the two groups and that both groups received professional development of the same duration on similar topics, the results from this study provide strong evidence to support the finding that the DG approach did make a difference – it did cause the improved geometry achievement observed in the study. This has implications for educational policies regarding the use of technological tools to learn geometry in high school classrooms. Evidence from this study suggests that the dynamic geometry approach is an alternative to teach the subject for those students that are often neglected because of previous performances in lower grades. Hence, addressing issues of equal opportunity to high-level mathematical practices such as conjecturing and proving.

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A DYNAMIC GEOMETRY-CENTERED TEACHER PROFESSIONAL DEVELOPMENT PROGRAM AND ITS IMPACT

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This study investigated the impact of a dynamic geometry (DG)-centered teacher professional development program on high school geometry teachers' content knowledge and their students' geometry learning. 64 geometry teachers were randomly assigned to an experimental (DG) group and a control group. Both groups received appropriate and relevant professional development. Classroom observation data and the teachers' responses to the implementation questionnaires revealed that most teachers in the DG group were faithful to the DG instructional approach. Teachers in the DG group scored higher on a conjecturing-proving test than did teachers in the control group. The students of teachers in the DG group scored significantly higher than the students of teachers in the control group on a geometry achievement test.

Keywords: Teacher Education-Inservice (Professional Development); *Technology*

Introduction

Dynamic geometry (DG) refers to an active, exploratory study of geometry carried out with the aid of interactive computer software available since the early 1990's that allows for learner knowledge construction and exploration. The most widely used current DG software packages include the Geometers' Sketchpad (Jackiw, 2001), Cabri Geometry (Laborde & Bellmain, 2005) and Geogebra (Hohenwarter, 2001) as well as variations that are applications within handheld graphing calculators or applets on web sites. DG environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena (CCSSI, 2010). With distinguishing features of dragging and measuring, DG software can be used to help students engage in both constructive and deductive geometry (Schoenfeld, 1983) as they build, test and verify conjectures using easily constructible models.

In a funded four-year research project, we conducted repeated randomized control trials to investigate the efficacy of an approach to teaching high school geometry that utilizes DG software as a supplement to regular instructional practices. Our basic hypothesis was that the use of DG software and DG teaching methods that engage students in constructing mathematical ideas through experimentation, observation, data recording, conjecturing, conjecture testing, and proving would result in improved geometry learning experiences for most students. The use of DG software and teaching methods was referred to as the DG approach in the project. The DG software used by the project was the Geometers' Sketchpad (GSP).

In this paper, we report the results from the second year of the project on teacher content knowledge and student achievement. We investigated the impact of the professional development of teachers and their students' geometry achievement in the DG group. The study built upon related research studies on mathematics teachers' professional development (e.g., Carpenter et al., 1989), including those concentrating on technology-centered (and especially DG-centered) professional development (e.g., Meng & Sam, 2011).

Theoretical Framework and Research Questions

An integrative framework (Olive & Makar, 2009) drawing from Constructivism, Instrumentation Theory and Semiotic Mediation was used to guide the study. Within this framework, as teachers and students interact in DG environments, their interactions with the DG technology tool influence the next act by each person, and continue in an interplay between the tool and user. As a user (teacher or student) "drags" an object and observes outcomes from that act, the user adjusts her or his thinking, which in turn influences the next interaction with the tool. Because DG technology allows users to adjust their geometry sketches and the relationships within them, users are transforming the tool, their use of the tool, and their thinking.

This study addresses the following research questions:

- Did teachers in the experimental (DG) group develop stronger conjecturing and proving abilities than did teachers in the control group?
- How well did the teachers implement the DG approach with fidelity in their classrooms?
- Did the students of teachers in the experimental group over a full school year achieve significantly higher scores on a geometry test than did the students of teachers in the control group?

Method

The participants in the study were sampled from the geometry teachers at high schools and some middle schools in Central Texas school districts. The study followed a randomized cluster design, randomly assigning 64 teachers to either an experimental group or a control group receiving relevant professional development, implementing the instructional approaches respectively assigned to them, helping the project staff in administering the pre- and post-tests of the participating students, and participating in other data collection activities of the project.

Professional Development and the DG Treatment

In order to effectively implement the DG approach in their classrooms, teachers must first master the approach. Without professional development, "teachers often fail to implement new approaches faithfully" (Clements et al., 2011, p. 133). So teachers' professional development (PD) was a critical component of the project. For our PD to be effective, it had to be sustained, rigorous, and relevant to participating teachers, with substantial support from their school districts. Based on these guiding ideas, a weeklong summer institute was offered to the participating teachers in the DG group, followed by 6 half-day Saturday PD sessions during the school year. The PD was planned and implemented collaboratively by project staff that included mathematics and education university faculty members and school-based master teachers selected based on their success as mathematics teachers and their experience with DG software. The project team and master teachers served as partner facilitators for all PD sessions.

The teachers in the experimental (DG) group were actively involved in each PD session and focused on developing their conceptual understanding of mathematics using the DG software as a tool. They worked on challenging problems and developed important geometric concepts, processes, and relationships while building DG skills and teaching methods. They experienced how DG environments encourage mathematical investigations by allowing users to manipulate their geometric constructions to answer "why" and "what if" questions, by allowing them to backtrack easily to try different approaches, and by giving them visual feedback that encourages self-assessment.

Typically, each activity in a PD session consisted of the following instructional events: 1) Presenting a task (exploring concepts/relationships or solving a problem) to the teachers; 2) Requesting teachers to use DG tools to construct the related geometric object or problem situation

(with help if necessary) or providing them with a prepared DG environment; 3) Asking teachers what conjecture(s) they can make based on their initial observation; 4) Requesting teachers to use dragging, measuring, and multiple, linked representations to experiment with the constructed or provided DG environment, and observe what characteristics change and what remain the same; 5) Asking teachers to further make and test conjecture(s); 6) Reminding teachers to redo events #4 and #5 in a new aspect or at a higher level, as appropriate; 7) Asking teachers to summarize and reflect on what they have conjectured; and 8) Helping teachers develop explanations to prove or disprove their conjecture(s).

In each PD session, teachers either worked individually at a computer or in small groups. In either case, PD facilitators encouraged teachers to share ideas and help each other. The facilitators circulated, observed (to monitor the progress) asked questions, and provided necessary assistance. They also initiated whole group discussions as needed.

In terms of content, the summer PD sessions concentrated on important and commonly taught topics of high school geometry: triangle congruence and similarity, properties of special quadrilaterals, properties of circles, and geometric transformations. School year follow-up PD aligned with the course scope and sequence determined by the participating school districts.

The PD facilitators modeled what teachers were expected to do with their students in geometry investigations. To help teachers change their instructional practices, their engagement of students, and how they facilitated student learning, mathematical explorations were always followed by discussions on questions such as “How will you teach this content using DG software?” and “How will you lead your students in conjecturing and proving using DG software?” The PD facilitators valued teachers learning from each other and sharing ideas and also sought to provide opportunities to apply new teaching skills. Therefore, teachers were encouraged to present their insights on and experiences with the DG approach and to describe problems they might have experienced or anticipated with other teachers offering suggestions to address the concern. Teachers also prepared lesson plans that they shared with the entire group.

The Control Group

The teachers in the control group taught geometry as they had done before. They also participated in a PD workshop that addressed the same mathematical content as the DG group but without the use of technology. The PD sessions for the control group utilized teaching methods with which teachers were already familiar. The PD facilitators lectured and involved teachers in activity-based instruction. Participants engaged in problem solving without using technology tools. They spent the same amount of time in PD training as the teachers in the DG group. The control group PD was included in the research design to control the variables tied to professional development and ensure both groups experienced sustained, rigorous, and relevant development in high school geometry teaching. Since all teachers participated in PD sessions and all were presented with the same mathematics content, any differences measured between the two groups would be attributed to the presence (or lack of) the interactive DG learning environment (since it was the only instructional difference between the two groups).

Measures and Data Analysis

Measures

A measure of teachers’ conjecturing-proving knowledge. A conjecturing-proving test was developed by the project team to measure teacher knowledge. As a result of a thorough literature review, geometry construct development, item construction, Advisory Board members’ review, and several pilot tests with resulting revisions, a test consisting of 26 multiple-choice items and 2 free-

response proofs was produced. The test was administered to the teacher participants as both a pre- and post-test at the PD summer institute.

Teachers' implementation fidelity and classroom observations. The DG approach involves intensive use of dynamic software in classroom teaching to facilitate students' geometric learning. The critical features of the DG approach include using the dynamic visualization to foster students' conjecturing spirit, their habit of focusing on relationships and explaining what is observed, and their logical reasoning desire and abilities. To capture these critical features of the DG approach, two measures of implementation fidelity (the DG Implementation Questionnaire [DGIQ] and the Dynamic Geometry Observation Protocol [DGOP]) were developed. The DGIQ was adapted from a teacher questionnaire developed by the University of Chicago researchers (Dr. Jeanne Century and her colleagues) in an NSF-funded project, based on the critical features of the DG approach. The final version of the DGIQ consisting of six multiple-choice items and ten open-response questions was administered to the teachers in the experimental group six times across the school year. A different version of the questionnaire was administered to the control group teachers (also six times) to examine how they teach geometry without using dynamic technology.

The DGOP was developed to address the critical features of the DG approach. It was adapted from the *Reformed Teaching Observation Protocol* (Sawada et al., 2002). The final version of the DGOP consisted of 25 items with a 4-point Likert response scale from *Never Occurred* to *Very Descriptive* addressing four different aspects: (1) *Description of intended dynamic geometry lesson*, (2) *Description of implemented dynamic geometry lesson*, (3) *Assessment of quality of teaching*, and (4) *Assessment of engagement and discourse*. For the control group, an observation protocol (CGOP) was developed by removing from the DGOP items related to the implementation of DG software functions such as dragging and dynamic measuring. The DGOP or CGOP was administered in 16 Geometry classrooms (8 selected from each group). Each classroom was visited by two observers. Each selected teacher was observed four or five times across the school year.

Student level measures. Two instruments were used for measuring students' geometry knowledge and skills: (for the pre-test) Entering Geometry Test (ENT) used by Usiskin (1981) and his colleagues at University of Chicago; and (for the post-test) Exiting Geometry Test (XGT). The XGT was developed by selecting items from California Standards Tests – Geometry. The final version for XGT had 25 multiple-choice items. (See Jiang et al., 2011 for the details of the two tests.)

All research instruments mentioned above, except the student geometry pre-test, were developed by the project team. For all project-developed measures, the Cronbach's Alpha statistical values were within the acceptable ranges for reliability (e.g., reliability was calculated with Cronbach's alphas of 0.957 and 0.952 for the DGOP and CGOP, respectively.) Item Response Theory (IRT) scoring routines were applied to the DGIQ and students' post-test data providing evidence for the instruments' construct validity.

Data Analysis

Two-level hierarchical linear modeling (HLM), other statistical methods, and the constant comparison method (Glaser & Strauss, 1967) were employed to analyse the quantitative and qualitative data.

Results

Findings about Teacher Content Knowledge from the Conjecturing-Proving Test

The participating teachers completed the conjecturing-proving test at the beginning and end of the summer PD institute. A statistic for teacher content knowledge as measured by the instrument was calculated by adding the number of correct multiple-choice responses with points from free-

response items. Average scores were 20.49 on the pre-test and 21.86 on the post-test with an average gain of 1.37. A paired-sample t-test showed that this gain was statistically significant ($p = .003$). These results show that the PD had a positive effect on teachers' conjecturing and proving capabilities. The teachers in the experimental group showed a greater average gain (1.56) than the teachers in the control group (1.18); however, this difference was not statistically significant ($p = .670$).

Findings about DG Approach Teaching from the Classroom Observations

Table 1 provides the results of the DGOP administration measuring the levels of fidelity of the dynamic approach implementation in the DG group. If we focus our attention to the mean scores (with a maximum score of 4) for the DG group, we observe that the three aspects with the highest scores were Good Lesson Design, Use of DG Features, and Teachers' Knowledge. The data provides evidence that the teachers in the DG group demonstrated an intention to implement the DG approach and to some extent they demonstrated knowledge about how to integrate the dynamic approach to teaching geometry. Overall, teachers in the DG group were implementing the DG approach at a moderate level (2.28). In part, this moderate level of implementation was explained by the challenges reported during the school year such as the inaccessibility of computer labs in the first several weeks and the pressure to spend time preparing for the state required tests. However, the majority of the classrooms observed can be described as being faithful to the DG teaching approach.

Table 1: Comparison between DG and Control Groups

Aspect	Sub-aspect	Mean DG	Mean Control	<i>p</i> -value
Intended Dynamic Lesson	Good lesson design	2.81	1.85	.032*
	Use of dynamic features	2.75	0.70	.000*
Implementation	Actions beyond use of software	2.06	1.33	.095
Quality of Teaching	Cognitive demand	2.30	1.78	.113
	Teachers' knowledge	2.89	2.84	.924
	Conjecture/Proof	1.93	1.40	.206
Engagement and Discourse		2.37	2.29	.735
Overall DGOP		2.28	1.68	.088

Comparing the two groups, Table 1 also shows the mean values of the CGOP and the *p*-values assessing the significance of the treatment effect computed using a mixed effect ANOVA. Results confirm the efficacy of the DG treatment by showing significant differences in the two aspects related to the intention to implement a dynamic lesson. As a whole, lessons in the DG group had a significantly better design aligned with the DG teaching approach, moving students from initial conjecture, to investigation, to more thoughtful conjecture, to verification and ultimately to proof. Further, lessons in the control group did not use dynamic features in teaching geometry. With respect to the other aspects of the DGOP (or CGOP), the two groups did not differ significantly, however all the DG ratings were higher than those of the control group. Note that most of those aspects assessed elements of the lesson that were not related to the use of dynamic features.

Findings about DG Approach Teaching from the Implementation Questionnaire

The purpose of the DGIQ was to assess the DG group teachers' effectiveness and comfort in using GSP in teaching geometry. Also, the questionnaire results provided the frequency of teacher and student use of GSP. Figure 1 shows how the teachers rated themselves on their effectiveness and comfort in using GSP. Out of 31 teachers who completed the questionnaire, 29% felt that they were

at the high level of effectiveness, 61% at the middle level, and 10% at the low level. However, the majority of the teachers (97%) felt very comfortable or somewhat comfortable in using GSP in teaching. Overall, the teachers felt more comfortable than effective in using GSP with only one teacher not feeling comfortable in using GSP in teaching of geometry.

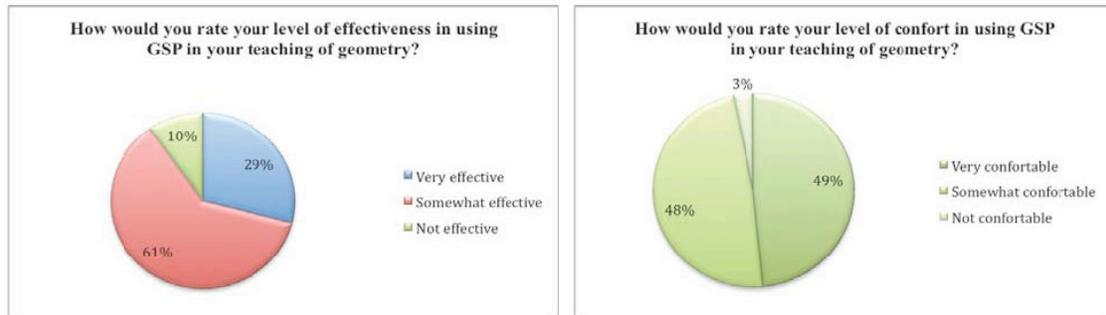


Figure 1: Effectiveness in Using GSP and Level of Comfort in Using GSP

Figure 2 shows average teacher and student use of GSP throughout the school year for those in the DG group. The “average teacher use of GSP” represents the average number of times per week the teacher used the demonstration computer in his/her classroom to do GSP presentations and demonstrations. The “average student use of GSP” represents the average number of times per week students worked in a computer lab doing hands-on explorations with GSP. Out of the 31 teachers who completed the questionnaire, 77% of them used GSP at least one time per week and 38% at least two times per week. However, the student use was lower, with 61% of them using GSP at least one time per week, and only 10% two times per week.

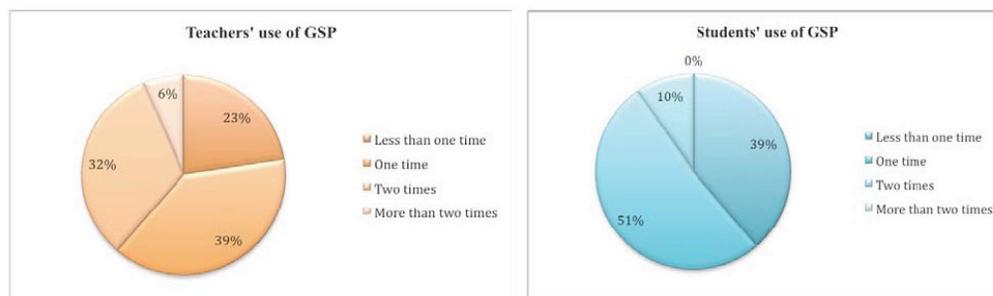


Figure 2: Average Teacher Use of GSP and Average Student Use of GSP per Week

Therefore, in terms of “taking students to the computer lab to do hands-on activities with GSP,” the teachers’ implementation of the DG approach was at the medium level of intensity. This finding is consistent with data from the classroom observations. However, almost all teachers were positive or enthusiastic in using GSP in geometry teaching. Again, considering the challenges that the teachers experienced during the school year, data supported the conclusion that most of the teachers implemented the DG approach faithfully.

Even though some teachers in the DG group might have not felt as effective in using GSP because of the students’ limited use of GSP (one time or less than one time per week), some of them might still be considered very effective if we focus on the ways they used the DG approach. One teacher provided such an example. He felt “somewhat effective” and his students used GSP on average one time per week, but his classroom observations showed very effective use of GSP. During one of the observations, his students were exploring the midsegments of a triangle and their goal was

to come up with as many conjectures as possible. Students completed the constructions on their own, made initial conjectures based on their observations, used measurements to confirm their conjectures, and wrote their final conjectures. The teacher circulated among students and provided guiding questions when needed. One student made many measurements but no conjectures. The teacher asked this student, “Do you notice any relationships? What conjectures can you make?” These questions helped the student focus on the objective of the lesson and form conjectures based on the measurements and observations. During the lesson, students also engaged in conversations with one another to discuss their observations and conjectures. Students were actively involved in their learning and the teacher took on the role of a guide by prompting his students through questioning. This lesson not only showed effective use of GSP, but also addressed higher-level thinking.

Findings about Student Achievement from the Geometry Test

Two-level hierarchical linear modeling (HLM) was employed to model the impact of the use of the DG approach on overall student geometry achievement measured by the student post-test (XGT). The model was analysed using student pre-test (ENT) scores as a covariate. The sample of classrooms studied included three different levels of Geometry: Regular, Pre-AP and Middle School (middle school students taking Pre-AP Geometry). Since the classroom expectation and quality of the students in each of these levels were very different, the factor *Class Level* was included in the model. Additionally, the covariate *Years Exp* (number of years of classroom experience) was included in the model. The results of the model indicated that the DG effect was strongly significant ($p = .002$). Comparing the means, the DG group was higher than the control group in each level of Geometry and the effect size (.45) was substantially larger at the Regular Geometry level. (See Jiang et al., 2011 for the details of the HLM analysis results.)

Using the integrative framework (Olive & Makar, 2009) as a lens, further quantitative and qualitative data analysis on the impact of the DG professional development is ongoing.

Discussion

The HLM model taking pretest, class level, and teaching experience into account provided evidence that the students of DG group teachers scored significantly higher than the students of control group teachers on the Exiting Geometry Test. Given that teachers were randomly assigned to the two groups and both groups received comparable sustained, rigorous, and relevant professional development on the same geometry topics, the results of this study provide evidence to support the finding that the DG professional development positively impacted the students’ geometry achievement. Both DG and control group teachers demonstrated significant gains on the Conjecturing-Proving Test through the one-week summer PD institute. This result suggests that both the PD sessions designed for the DG group and those designed for the control group had an effect on teachers’ conjecturing and proving ability. Although the DG and control teachers did not differ significantly on their mean gain scores, the DG teachers’ mean gain score was 32% higher than that of the control teachers. Classroom observation data revealed that lesson plans that the DG group teachers prepared were designed significantly better than the control group teachers’ lessons by facilitating students’ conjecturing and proving abilities. The teachers’ DGOP ratings (overall and in each sub-scale) were consistently higher for the DG group although most of the differences were not statistically significant. In summary, the results of this study suggest that the DG professional development offered to the participating teachers had a significant positive effect on the teachers’ mathematics conjecturing-proving content knowledge and their ability to implement a dynamic geometry approach to teaching. The teachers, in turn, helped their students achieve better geometry learning.

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Randomized Control Trials on the Dynamic Geometry Approach*

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The project reported here is conducting repeated randomized control trials of an approach to high school geometry that utilizes Dynamic Geometry (DG) software to supplement ordinary instructional practices. It compares effects of that intervention with standard instruction that does not make use of computer drawing/exploration tools. The basic hypothesis of the study is that use of DG software to engage students in constructing mathematical ideas through active investigations results in better geometry learning for most students. The study tests that hypothesis by assessing student learning in classrooms randomly assigned to treatment and control groups. The project is currently in its second year, and has just completed its first implementation of the DG approach, related data collection, and some initial data analysis. HLM models showed that the treatment group significantly outperformed the control group in geometry achievement. While the effect of the DG treatment is of moderate size for all participating students the largest effect size occurs with students in Regular Geometry classes.

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Keywords: Dynamic Geometry, Assessment, HLM models.

Dynamic Geometry (DG) represents geometry explorations performed with interactive computer software such as the Geometer's Sketchpad (Jackiw, 2009) or Cabri Geometry (Texas Instruments, 1994), which has been available since the early 1990's. As Goldenberg & Cuoco (1998) point out, "The term *dynamic geometry*...has quickly entered the literature as a generic term due to its aptness at characterizing the feature that distinguished DG from other geometry software: the continuous real-time transformation often called 'dragging.' This feature allows users, after a construction is made, to move certain elements of a drawing freely and to observe other elements respond dynamically to the altered conditions" (p. 351). Teachers use DG software to help students construct mathematical ideas through active explorations and investigations such as dragging, measuring, observing, conjecturing, conjecture testing, reasoning, and proving. A four-year research project has been funded by the National Science Foundation to conduct repeated randomized control trials of this instructional approach to high school geometry, which is referred to as the DG approach here. This article describes the project and reports the initial findings from its year-2 study.

Dynamic Geometry Related Research

Along with the widespread use of DG software, many related research studies have been conducted. A relatively

small group of researchers (e.g., Dixon, 1997; Gerretson, 2004; Myers, 2009) used experimental or quasi-experimental designs in their studies. Results obtained from the statistical analyses of these researchers suggested that students experiencing the dynamic geometry instructional environment significantly outperformed students experiencing a traditional environment on content measures of the concepts and skills taught during the experiments.

Most of the studies used qualitative research methods. Choi-Koh (1999) used a clinical interview procedure to examine a secondary school student's development of geometric thought. She identified four learning stages—intuitive, analytical, inductive, and deductive learning in terms of symbol, signal, and "implicatory" properties; and found that the use of active visualization with the dynamic software facilitated the progress from symbol to signal and then to implicatory character. In an exploratory, phenomenological study, Hannafin, Burruss, & Little (2001) noted two overarching themes: issues of power and learning. The teacher had difficulty relinquishing control of the learning environment, while students enjoyed their new freedom, worked hard, and expressed greater interest in the mathematics content. Through a constructivist teaching experiment, Jiang (2002) discovered that as they explored geometry problems with DG software, preservice teachers developed a new learning style—exploring problem situations through a learning process characterized by initial conjecture—investigation—more

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thoughtful conjecture—verification (or proof)—proof (or verification).

Vincent (2005) found that the motivating context and the dynamic visualization afforded by the DG learning environment fostered conjecturing and intense argumentation; and that the teacher's intervention was important in fostering the students' augmentations. Accascina and Rogora (2006) claimed that the use of DG software facilitated students' understanding by helping them create good concept images of three-dimensional objects. Hollebrands (2007) identified different purposes for which high school Honor Geometry students used dragging and measures, and found that these purposes seemed to be influenced by students' mathematical understandings, the types of abstractions they made, and the different strategies they used. Sinclair et al (2009) argued that as the very nature of dynamic mathematical representations, continuity and continuous change occurring over time offer different opportunities for students' narrative thinking, which the static diagrams and pictures are unable to provide. Building on the work of Arzarello, Olivero and other researchers, and as part of a more general qualitative study aimed at investigating cognitive processes during conjecture-generation in a DG environment, Baccaglioni-Frank and Mariotti (2010) presented "a model describing some cognitive processes that can occur during the production of conjectures in dynamic geometry and that seem to be related to the use of specific dragging modalities" (p. 225) and used it to analyze students' explorations of open problems.

According to the research studies described above, if DG software is used effectively, it can make a significant difference to students' learning; when used as a cognitive tool, it can facilitate students' exploration and investigation activities, promote their conjecturing, verifying, explaining, and logical reasoning skills and abilities, and enhance their conceptual understanding of important geometric ideas. However, almost all of the studies were either exploratory phenomenological studies that involved a small number of participants, or comparative studies that were conducted during a relatively short period of time (ranging from a week to less than one semester).

The qualitative studies are important since they can reveal students' actual, detailed learning processes. "In fact, good quantitative studies generally require a qualitative rationale" (American Statistical Association, 2007, p. 43). However, there is a need for education research designs using modern statistical methodologies "if the quality of education research is to meet the requirements that government policies and societal expectations are placing upon it" (American Statistical Association, p. 44). If one wants to find the convincing efficacy of DG, quantitative comparative studies are necessary (Schneider et al, 2007). To make sure that the changes observed in a dependent variable are caused by

the intervention rather than by some extraneous factors, true experimental designs with randomized assignment to treatment and comparison conditions should be used whenever and wherever possible.

Van Hiele (1986) mentioned that it took nearly two years of continual education to have the students experience the intrinsic value of deduction. Research found that the use of DG software can save instructional time (Gray, 2008), but to find significant development of students' geometric thought such as having moved from one geometric thinking level up to the next higher level, a relatively long term of instruction (such as a full school year) is very much needed.

Based on these considerations, most (if not all) of the studies mentioned above need careful replication and amplification (Jones, 2005). DG software, though very widely used, has not been rigorously evaluated. The need for achieving a more thorough understanding of the power of DG software is clear.

A Four-Year Research Project Funded by NSF

The primary goal of this project is to investigate the efficacy of the DG approach on students' geometry learning over the course of a full school year. Based on the idea that effects of innovations like dynamic geometry are often greater in the second year of use than in the initial getting-acquainted first year, data collection opportunities are provided in two consecutive implementations of the dynamic geometry treatment. Thus, the general plan for the four-year project is as follows: Year 1: Preparation (All research instruments, recruitment of participants, professional development training and resource materials, etc.); Year 2: The first implementation of the DG treatment, and related data collection and initial data analysis; Year 3: The second implementation of the DG treatment, and related data collection and continued data analysis; and Year 4: Careful and detailed data analysis and reporting. The project is presently in progress during its year 2.

Theoretical Framework

The theoretical foundation of the DG approach and the theoretical framework of this research project consist of the constructivist perspective and the van Hiele model.

The constructivist perspective suggests that knowledge cannot be passively transmitted from one individual to another but is actively constructed by the learners themselves (Steffe & Cobb, 1988). The traditional approach to geometry instruction is teacher centered and based on definitions, theorems, and proofs, with little attention to whether students understand teacher's lecture. In contrast, the DG approach to geometry instruction is based on students' experimentation, observation, data

recording, conjecturing, and proving. As Olive (1998) indicates, “Such an approach would give students the opportunity to engage in mathematics as mathematicians, not merely as passive recipients of someone else’s mathematics knowledge. From a constructivist point of view, this is the only way children can learn mathematics” (p. 399).

Van Hiele (1986) postulated that students progressed through a sequence of five discrete thought levels in geometric reasoning. The five levels are: 1. *Recognition*, 2. *Analysis*, 3. *Order*, 4. *Deduction*, and 5. *Rigor*. According to the van Hiele theory, the main reason the traditional geometry curriculum fails is that it is presented at a higher level than those of the students (de Villiers, 1999). The DG learning environment is a suitable environment in which students can explore geometry at their geometric thinking levels. Teachers can prepare activities that match students’ current van Hiele levels so that students can continue their explorations with little help from their teachers and make the transition to the next higher level.

Research Questions

The project seeks to answer the following research questions: 1) How do students taught in a DG oriented instructional environment perform in comparison with students in the control condition on measures of a geometry test and a conjecturing-proving test? 2) How does the DG intervention affect student beliefs about the nature of geometry and their beliefs about the nature of mathematics in general? 3) How does the DG intervention contribute to narrowing the achievement gap between students receiving free or reduced price lunch and other students? 4) How is students’ learning related to the fidelity and intensity with which the teachers implement the DG approach in their classrooms? and 5) What characterizes the different learning communities in the experimental and control classes?

Sample

The population from which the participants of this efficacy trial were sampled is the geometry teachers and their students at high schools in Central Texas School Districts in which 50% or more of the students are eligible for free or reduced price lunch. The rationale for us to focus on this group of students is twofold: 1) It is very important to study how economically disadvantaged students learn mathematics, especially in Central Texas where there are high percentages of this group of students; 2) According to Riordan and Noyce (2001), the two strongest predictors of school performance are the baseline mean school score on the previous statewide test and percent of students receiving free or reduced price lunch.

For determining the sample size, a power analysis was conducted. We used the Optimal Design software (Spybrooke, Raudenbush, Liu, & Congdon, 2006) in order to determine the optimal number of classrooms to be included in the study. Having set appropriate parameters we ran the Optimal Design algorithm, which indicated that the optimal number of classrooms would be 30 for one treatment group, and 60 in total. Taking a 20% attrition rate into consideration, 76 classes ($76 - 76 \cdot 20\% = 60$) were used for the study. With help from the school districts in Central Texas, 76 geometry teachers were selected from those who applied to the project with support from their principals.

Research Design

The research study follows a mixed methods, multi-site randomized cluster design. Random assignment was used, with teachers as the unit of randomization. The 76 teachers selected were randomly assigned to two groups—the experimental group and the control group. For schools where the selected teachers teach more than one class, only one class per teacher was randomly selected to participate in the study. Therefore each teacher is represented in the study with measurements from only one classroom of students, and the classroom and teacher unit of analysis overlap, yielding the design where the students are nested within teachers/classrooms, which are nested within schools.

In this project, the DG software used is mainly the Geometer’s Sketchpad (GSP). For some 3-D activities, the Cabri 3-D software is used with GSP. To help users take full advantage of the power of GSP, we provided them with many well-designed GSP-based learning activities. We chose the activities that most fit Texas geometry curriculum from sources such as GSP curriculum modules published by the Key Curriculum Press and learning activities that the project staff members have created during their long careers in using GSP with high school mathematics teachers and students.

The teachers randomly assigned to the experimental group have been participating in a GSP workshop, of which the main part was conducted in the summer of project year 1. In the workshop, these teachers have opportunities to become familiar with GSP, use GSP to investigate problem situations, and explore ways in which they could use GSP with their students. The follow-up sessions of the summer institute consist of six Saturday half-day sessions during the 2010-2011 school year, a whole-day session in summer 2011, and three more Saturday half-day sessions during the 2011-2012 school year.

The control group is a “business-as-usual” group. The teachers in this group teach as before. Specifically they teach in the paper-and-pencil manner, involving use of manipulatives, compass, protractor, and ruler. They also participate in a workshop, in which the same mathematical

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content taught in the GSP workshop is introduced to them, in a non-GSP environment.

Measures and Data Collection

Student-level Measures

The instruments used to assess students' geometry learning during project year 2 included a pretest—Entering Geometry Test (ENT) used by Usiskin (1982) and his research team at University of Chicago, and a geometry posttest (XGT), which was developed by the project team through selecting questions from released items of the California Standards Test: Geometry (CSTG).

The reason of using ENT for the pretest is that this test has been used by numerous studies on students' geometry learning over the past 29 years, and has been considered as a good and easy-to-administer multiple-choice geometry test to assess students' geometric background before entering a full-year high school geometry course. ENT consists of 20 multiple choice items each with 4 possible responses and has a reliability of $\alpha=.77$.

Released items from CSTG were chosen for XGT because all CSTG questions have been evaluated by committees of content experts, including teachers and administrators, to ensure their appropriateness for measuring the California academic content standards in Geometry. In addition to their content validity, all items were reviewed and approved to ensure their adherence to the principles of fairness and to ensure no bias exists with respect to characteristics such as gender, ethnicity, and language (California Department of Education, 2009).

When developing XGT, the project team worked carefully in selecting items from CSTG that are closely aligned with Texas geometry standards and the geometry curricula of the participating school districts. Thirty items were chosen and pilot-tested in the non-project classes at a participating high school. Based on the pilot-test results and the feedback of the master teachers (who are high school geometry curriculum and instruction experts working for the project), five items were removed. The final version for XGT has 25 multiple-choice items. Based upon the pilot results, the instrument has high reliability ($\alpha=.875$). Factor analysis provided strong evidence that XGT corresponded to uni-dimensional scale. Item Response Theory (IRT) scoring routines were applied to the scored posttest to generate examinee 'abilities' and item parameters, which allowed us to determine that collectively the items included on the posttest provided a range of performance that holistically represented a well-functioning instrument. The adherence of the data to the three-parameter logistic IRT model provided some evidence for the assessment's construct validity.

For all other measures to be described below, psychometric properties were also examined, and their

Cronbach's Alpha statistical values are within the acceptable ranges for reliability. More psychometric analyses were examined for some of the instruments and provided evidence supporting the validity of each.

Student-level measures also included a Conjecturing-Proving Test and a student belief questionnaire. Both were developed by the project team. The Conjecturing-Proving Test was used as a pilot test at the end of 2010-2011 school year, and the data will be analyzed mainly for establishing the validity and reliability of the measure. The student belief questionnaire was developed to measure student beliefs about the nature of geometry and their beliefs about the nature of mathematics in general, and administered with the pretest and the posttest. It was adapted from the mathematics version of the Views about Sciences Survey (VAMS) (Halloun, 1996). The purpose of VAMS is to assess student views about knowing and learning mathematics and assess their relation to student understanding. Because VAMS measures students' views of mathematics more generally, the project team paid close attention to incorporating the critical features of the DG approach in the adaptation process.

Teacher-level Measures

To determine how to capture the critical features of the DG approach, we have designed measures of fidelity of implementation—both a DG implementation questionnaire and a classroom observation instrument. The DG implementation questionnaire was adapted from a teacher questionnaire developed by the University of Chicago researchers (Dr. Jeanne Century and her colleagues) in an NSF funded project. We made significant changes and additions to address the extent to which the DG approach is implemented by the teachers. The current version of the questionnaire consists of items designed to gather information on aspects such as how many times per week the students worked in a computer lab with GSP installed on each computer; what features of GSP were used over the past month; and how the use of GSP has influenced the way the teacher plan and implement instruction. This questionnaire has been administered with the teachers in the experimental group six times during project year 2. An equivalent but different version of the questionnaire has been administered with the control group teachers to examine the degree to which they faithfully implement the business-as-usual approach.

The classroom observation instrument—the Geometry Teaching Observation Protocol (GTOP) has been developed by adapting the Reformed Teaching Observation Protocol (Piburn et al, 2000) based on the critical features of the DG approach. GTOP has been used for both groups of teachers. The scores provide data to compare the teachers' teaching styles and strategies. During project year 2, we have observed classes of eight teachers in the experimental group and eight in the control

group. These teachers were chosen to ensure diversity in their gender, level of class (Regular, Pre-AP, or Middle School) and years of teaching experience. There are at least two observers for observing each class, and each teacher has been observed four or five times. The four or five observations were evenly distributed throughout the entire school year. The purpose is to get as comprehensive a picture of the teacher's teaching practice as possible.

To probe more deeply into the teachers' and students' thinking processes, and to gather evidence about the range and variability of students' development of the most important abilities that the DG approach fosters, this study uses in-depth interviews of selected students and teachers to collect qualitative data. Interview protocols have been designed and used for the interviews. During project year 2, eight students and six teachers were selected for the interviews, from a diverse set of most engaged students and teachers involved in the project. We have completed three interviews for each of the interviewees except for one student (two interviews) and one teacher (one interview). All interviews were videotaped and the taped footage will be transcribed.

All instruments mentioned above (with necessary changes informed by the implementation feedback) will be used again during project year 3.

Data Analysis

The project team has completed the first implementation of the DG approach and related data collection. Some initial data analysis (the analysis on the geometry pretest and posttest data and the psychometric analysis on the project developed instruments) has been conducted. More thorough analysis of the collected data is still on going and will be conducted during project year 3. The analysis of the geometry pretest and posttest data is reported below.

Two-level hierarchical linear modeling (HLM) was employed to model the impact of the use of the Dynamic Geometry approach on student achievement while taking into account the nested structure of the data (i.e. students

nested within teachers' classrooms). The models were analyzed once with student pretest (ENT) scores included as a covariate and once without the pretest scores. The rationale for excluding the ENT scores is twofold. First of all, teachers were randomly assigned to each of the treatment groups, so pretest control is not required to determine accurate estimates of the treatment effect. In fact, a separate analysis of the pretest, not presented here, showed no significant difference ($p = .724$) between the two treatment groups. Secondly, the subsample of students with matched pretest and posttest scores is smaller than the sample of students taking just the posttest. This is due to a variety of factors including the fact that some students were absent the day of the administration of either pretest or posttest and that some students changed classes during the school year. Additionally, in order to maintain privacy, each student was issued a special project ID only known to the student and their teacher. In some instances, students and/or teachers made errors in using the ID making the match impossible. The reduction of the sample was not uniform across the different types of students and classes and may introduce bias into the results.

Results

The sample of classrooms studied included three different levels of Geometry: Regular, Pre-AP and Middle School (middle school students taking Pre-AP Geometry). Since the classroom expectations and quality of the students in each of these levels are very different, the factor *Class Level* was included in each model. Additionally, the years of classroom experience of the teachers in the sample varied a lot, ranging from 0 years all the way up to 35 years. For this reason, the covariate *Years Exp* (number of years of classroom experience) was included in the models.

During the project year 1 professional development workshop, the participating teachers completed a demographic survey that included information about years of teaching experience, the level of the class chosen and gender. From our initial teacher sample (76 participants), six teachers did not compete project year 2 mainly due to either

Table 1. Summary Statistics for Years of Experience of Teachers by Treatment and Level

	DG			Control		
	<i>n</i>	<i>M</i>	<i>SD</i>	<i>n</i>	<i>M</i>	<i>SD</i>
Overall	33	7.00	7.18	31	6.48	8.29
<i>Class Level</i>						
Regular	20	6.44	7.89	19	5.63	6.72
Pre-AP	12	8.30	6.25	8	10.75	11.94
Middle School	1	4.00	NA	4	2.00	2.16

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Table 2. Model 1: HLM Results without Pretest as a Covariate

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx d.f.	p-value
Intercept	90.60	6.323	14.328	88	.000
DG Effect	7.435	1.763	4.217	51	.000
Level					
Regular	-35.10	6.282	-5.587	88	.000
Pre-AP	-21.15	6.566	-3.221	83	.002
Level*Years Exp					
Regular*Years Exp	-.3241	.1587	-2.042	57	.046
Pre-AP * Years Exp	.4785	.1763	2.714	46	.009
M. School*Years Exp	.6477	1.947	0.333	91	.740

Note. XGT is the response variable.

family/health or job displacement reasons. An additional six teachers submitted incomplete posttest data or failed to submit the data. Therefore, 64 teachers submitted complete posttest data for analysis in the study. Among them, 33 are in the experimental group (DG group), and 31 are in the control group.

Table 1 shows the summary statistics for years of experience of these teachers by *Treatment* and *Class Level*. For each of the two HLM models described in Table 1, full factorial designs were explored and insignificant interactions were discarded. The final models are discussed below.

HLM Results without Pretest as a Covariate

Model 1, shown in Table 2, examines the differences in student outcomes between the DG group and the control group while accounting for years of teaching experience and level of the class. The DG group significantly outperformed the control group ($p = .000$, $ES = .3327$). As expected, level of the class was highly significant as well ($p = .000$). Due to the coding of variables, the intercept reflects the Middle School group performance. Examining the coefficients in Table 2 and the mean values in Table 3, we see that the Middle School group substantially

outperformed the high school Pre-AP students, who in turn outperformed the students in Regular Geometry. In particular, controlling for experience of the teacher and treatment group, the Pre-AP students scored 21.15 points lower than the Middle School students, while the Regular students scored a full 35.1 points lower than the Middle School group. Though not indicated in Table 2, the difference between the Pre-AP and Regular students is statistically significant ($p = .000$). Interestingly, the effect of years of experience differed by level of the class as well. Experience had a positive effect on the two higher performing groups, but had a negative effect on the achievement of the students in Regular Geometry classes. However, the effect of experience in the middle school group was not significant and the size of the coefficients in all groups is somewhat small. For the Regular group an increase of 10 years of experience corresponded to a drop of 3.2 points on the XGT, while for the Pre-AP group a similar change in experience was associated with a 4.8 point increase. Compare this to the 7.4 point increase due to the DG effect. Note the main effect for *Years Exp* is not shown in Table 2. A model including this effect was considered, but it was insignificant.

Table 3 shows the summary statistics for each level of

Table 3. Summary Statistics for XGT by Treatment and Level

	DG			Control			ES
	n	M	SD	n	M	SD	
Overall	617	62.44	19.06	604	55.91	20.19	.33
Level of Class							
Regular	357	54.38	17.64	366	45.54	15.10	.55
Pre-AP	244	72.56	16.09	191	68.27	15.50	.27
Middle School	16	88.25	7.01	47	86.38	10.10	.19

Note. Sample includes all Post Test data.

class separately. The DG group outperformed the control group in each level, but the effect size was largest for the students in the Regular Geometry classes.

HLM Results with Pretest as a Covariate

Model 2, shown in Table 4, examines the effect of the DG intervention when taking into account Entering Geometry Test (*ENT*) as well as *Class Level* and *Years Exp.* To simplify interpretation of the other coefficients, *ENT* was centered by subtracting the overall mean. Due to complications with matching student codes, including *ENT* in the model reduces the student sample. Comparing the sample sizes listed in Tables 3 and 5, we see that loss in sample was largest for the Regular Geometry classes. At that level the sample dropped 30% from 723 to 508. Comparing the treatment groups, DG dropped 19% and the control group dropped by 27%. Given the results shown in Table 3 above, this sample reduction would be expected to diminish the size of the DG effect, since the DG had the largest effect for the students in the Regular Geometry group. Table 5 shows the summary statistics for each level of class in the smaller matched subsample. Comparing the means, we see the following pattern again: the DG group outperformed the control group in each level of Geometry and the effect was substantially larger at the Regular Geometry level. However, the effect sizes were smaller for each of the levels of Geometry than in the full sample reported in Table 3. As with Model 1, the results for Model 2 indicate the DG effect was strongly significant ($p = .002$). As expected, including *ENT* in the model reduced the size of the *Class Level* effect on student performance on *XGT*. However, even controlling for the pretest, compared with Middle School students, on average Pre-AP students scored 13.2 points lower ($p = .049$) and Regular students scored 20.1 points lower ($p = .004$). Consistent with the results from Model 1, teaching experience had a positive effect on the two higher performing groups, but had a negative effect on the achievement of the students in Regular Geometry classes. Once again, the effect of experience in the Middle School group was not significant. The size of the coefficient in the Middle School group was much larger in Model 2 than Model 1, but in neither case was it significant. For the other two groups, the coefficients were similar in value between the two models. In Model 2, an increase in 10 years of experience raised the scores 4.5 points for the Pre-AP group and decreased the scores by 4.1 points for the Regular group.

Discussion

Effect of the DG Treatment

Both data analysis models (HLM without pretest as a covariate and HLM with pretest as a covariate) showed

that the Dynamic Geometry group significantly outperformed the Control group in geometry achievement. This project used random assignment to form the treatment and control groups. The teachers in the control group also attended a professional development workshop. The amount of instructional time spent on this regular workshop was the same as that for the GSP workshop offered to the DG group teachers. The purpose of holding this workshop was to address a confounding variable. With this comparable amount of professional development, if differences appear on the project's measures between the treatment and control groups, we are able to rule out the possibility that the professional development activities can account for them rather than the DG learning environment. This true control group, in addition to random assignment, provides strong evidence to support the finding that the DG approach did make a difference—it did cause the improved geometry achievement observed in the study. In the first efficacy study on the DG approach at a moderately large scale in the nation, this finding is a noteworthy contribution to the field of mathematics education.

While the effect of the DG treatment is significant and of moderate size for all participating students, the largest effect size occurs with Regular Geometry students. There are many factors that could contribute to this. First, dynamic constructions offer stronger visualization than static drawings (Laborde, 1998). As bringing a strong visual component to mathematics is key to understanding for all students (Archavi, 2003; Clements & Battista, 1992), it is indispensable for students challenged by language learning and cognitive issues (Reimer & Moyer, 2005; Key Curriculum Press, 2009). Students in Regular classes are those of low to average academic abilities and are more likely (than Pre-AP students) from the group of special education (learning disabilities), at risk, or economically disadvantaged students. They require more visual and more innovative activities to help them develop their conceptual understanding and mathematical reasoning ability. Therefore, they could benefit more by the stronger visualization brought by DG tools than those in Pre-AP classes. Secondly, in comparison to Pre-AP students who are better prepared and more motivated, Regular students need more motivation and engagement to spark their learning interest. DG software is helpful to teachers as they design environments and contexts that address the motivational needs of the students. For instance, the transformations available in GSP and its animation feature, as well as the ease of using buttons make the software a wonderful tool to design and implement various engaging environments for learning mathematics. The DG engagement effect would be greater to Regular students than to Pre-AP students. In addition, from a state curriculum standpoint, Pre-AP classes work on some logic and proofs whereas Regular classes do not. Since the DG approach focuses on conjecturing, reasoning, and proving, Regular DG students are exposed

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Table 4. Model 2: HLM Results with Pretest as a Covariate

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx d.f.	p-value
Intercept	79.05	6.119	12.919	28	.000
DG Effect	5.62	1.678	3.352	43	.002
Level					
Regular	-20.10	6.119	-3.284	21	.004
Pre-AP	-13.23	6.293	-2.102	20	.049
Level*Years Exp					
Regular*Years Exp	-.4137	.1516	-2.729	53	.009
Pre-AP * Years Exp	.4451	.1617	2.753	22	.012
M. School*Years Exp	1.811	1.868	0.969	20	.344
ENT (Mean Centered)	.4114	.0366	11.237	47	.000

Note. XGT is the response variable.

to these important activities and abilities, and would do much better than Regular “control” students who have not seen these. In Pre-AP classes, the students see proofs whether they are in the DG group or not, so the difference would be smaller.

Effect of Teaching Experience

One unusual result of the HLM analysis of the data is the effect of the teaching experience on student achievement on the geometry posttest. In both models discussed above, greater experience of the teacher had a positive impact on achievement for the Pre-AP level and a negative impact for the Regular level. Further research is necessary to fully understand why this occurred. One possible explanation is the expectations of teachers for the two different levels. Based on years of interaction with middle school and high school teachers, the project researchers have noticed the tendency for experienced teachers to have very different and very rigid beliefs about the ability of students to achieve. Pre-AP classes are composed of mostly middle to high achieving students,

and hence teachers have high expectations of what those students can learn. Meanwhile, since students in Regular classes have a record of low to middle achievement, teachers have very low expectations of what students can learn. It is not uncommon to hear an experienced teacher react to some innovative teaching activity by saying, “Well this might work with my Pre-AP students, but my Regular kids won’t get it. With the Regular kids we need to focus on the basics.” Our experience has been that more novice teachers are willing to believe that all students can learn.

Future Work

With the results described above, we have partially answered the first research question of the project. As many researchers (e.g., Artigue, 2000) have pointed out, the issue is not only which is best, but also how is the DG approach different—what are the epistemic differences? Because of this, the project has included a strong qualitative component. We will further analyze the data (both quantitative and qualitative) that have been collected. During the second implementation of the DG approach

Table 5. Summary Statistics for XGT by Treatment and Level

	DG			Control			
	<i>n</i>	<i>M</i>	<i>SD</i>	<i>n</i>	<i>M</i>	<i>SD</i>	<i>ES</i>
Overall	501	62.36	19.26	438	59.12	20.40	.16
Level of Class							
Regular	276	54.19	17.64	232	46.81	15.10	.45
Pre-AP	210	71.26	16.09	163	69.28	15.50	.13
Middle School	15	88.27	7.01	43	87.07	10.10	.13

Note. Includes only posttest data for subsample with matching pretest results

that will be conducted in project year 3, we will continue to focus on collecting high-quality quantitative and qualitative data and analyzing the data.

To thoroughly address the first research question and answer the second research question, the principal method of data analysis will continue to involve fitting two- and three-level Hierarchical Linear Models to the data. This multilevel approach also enables us to address research question 3 and examine the potential treatment effect with respect to the ethnic, socio-economic, and linguistic characteristics of the students and the demographic composition of schools.

Qualitative data analysis will use the constant comparative approach (Glaser & Strauss, 1967; Grove, 1988) to answer research question 5. Constant comparison involves analyzing and interpreting data during and after data collection. By systematically analyzing data during its collection, the researchers can make appropriate adjustments to look for evidence that conflicts with emerging theories, as well as evidence that might support those theories. This process reduces the likelihood that the researchers' theories are based on personal biases.

The quantitative data analysis and the qualitative data analysis reported above, as a whole, will answer research question 4 that relates to implementation fidelity.

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Exploring the Infinite Possibilities of Dynamic Geometry

A Video Presentation of the DG Project

Dr. Zhonghong Jiang Dr. Gilbert Cuevas Dr. Alex White Dr. M. Alejandra Sorto Dr. Sharon Strickland Dr. Samuel Obara Brittany Webre Shawnda Smith

NSF Award #: 0918744

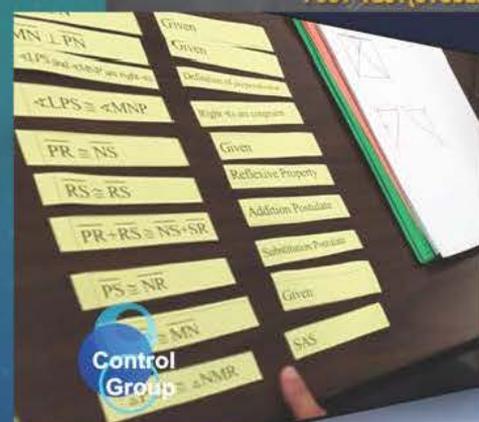
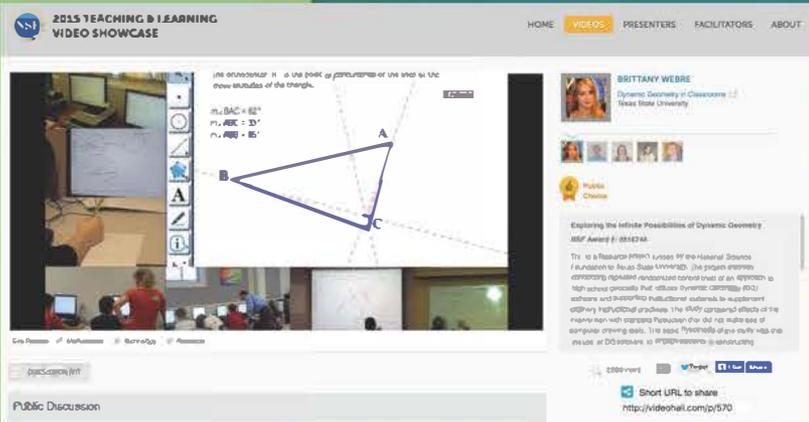
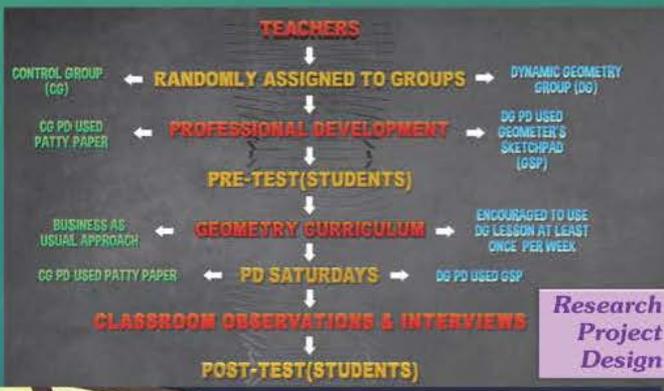
This is a research project funded by the National Science Foundation to Texas State University

The project involved conducting repeated randomized control trials of an approach to high school geometry that utilizes Dynamic Geometry (DG) software and supporting instructional materials to supplement ordinary instructional practices.

The study compared effects of the intervention with standard instruction that did not make use of computer drawing tools.

THE FRAMING QUESTION THAT GUIDED THE PROJECT & RESEARCH:

Can dynamic geometry software improve achievement, reasoning, and engagement?



Professional Development Activities

One of the goals of the PD was to help teachers develop a new learning style that included:

- 1) Conducting problem solving through a learning process characterized by constructing the problem situation
- 2) Investigation/exploration
- 3) Making conjecture(s)
- 4) Testing conjecture(s)
- 5) Proving conjecture(s)

About the Event

The NSF 2015 Teaching and Learning Video Showcase: Improving Science, Math, Engineering, and Computer Science Education event will be held online May 11-15, 2015.

GOALS: The event will showcase cutting-edge NSF-funded work to improve teaching and learning and will allow colleagues affiliated with MSPnet, CADRE, CIRCL, CAISE, STELAR, CS10K community, and ARC to view, discuss, and comment on each others' work. It will also allow each project to disseminate their work to the public at large, helping NSF achieve its goal of broad dissemination of innovative work.

PRESENTERS: Projects from these resource center communities were invited to present their work by creating a short (~3 minute) video that showcases their intervention, innovation, and/or research. Videos will address potential impact, promise, and challenges.



About the Project:

The basic hypothesis of the study was that the use of DG software to engage students in constructing mathematical ideas through experimentation, observation, data recording, conjecturing, conjecture testing, and proof results in better geometry learning for most students.

The study tested that hypothesis by assessing student learning in 64 classrooms randomly assigned to treatment and control groups. Student learning was assessed by a geometry standardized test, a conjecturing-proving test, and a measure of student beliefs about the nature of geometry and mathematics in general.

Teachers in both treatment and control groups received relevant professional development, and were provided with supplemental resource materials for teaching geometry. Fidelity of implementation for the experimental treatment was monitored carefully.

Data obtained for answering the research questions of the study were analyzed by appropriate HLM models and qualitative methods. Results have provided strong evidence about the effectiveness of the DG approach in high school teaching; evidence that can inform school decisions about innovation in that core high school mathematics course.



TEXAS STATE UNIVERSITY PRESENTS WITH THE SUPPORT FROM NSF

A DYNAMIC GEOMETRY RESEARCH PROJECT BY ASSOCIATION WITH GEOMETR'S SKETCHPAD

THE PhD RESEARCHER AND PI: BRITTANY WEBRE & SHAWNDA SMITH

IMPACT OF DYNAMIC GEOMETRY IN CLASSROOMS

PRINCIPAL INVESTIGATOR: DR. ZHONGHONG JIANG

DR. GILBERT CUEVAS DR. ALEX WHITE DR. ALEJANDRA SORTO

DR. SAMUEL OBARA DR. SHARON STRICKLAND PATRICK SMITH

BRITTANY WEBRE BEARLY JACOBSON'S GAMES ANSYS CLASS BY GALE

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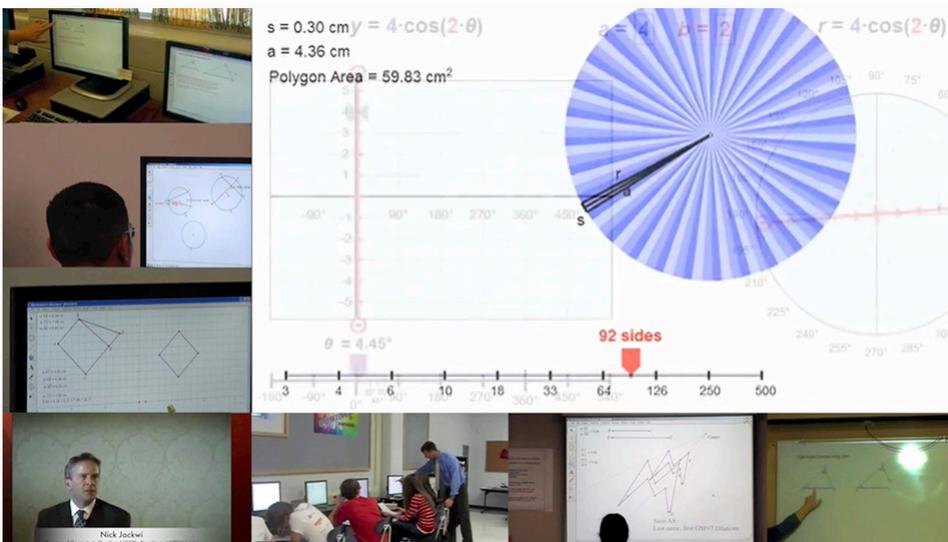


Section 2

Implementation of the Dynamic Geometry Approach in Secondary School Classrooms

The Dynamic Geometry in Classrooms project was funded to investigate the efficacy of the “Dynamic Geometry Approach” on students’ geometry learning and required particular attention to the implementation of the dynamic geometry approach. The seven papers and documents in this section provide information and research results on various aspects of implementation and the professional development activities used in the project.

The first paper, “Difference in Self-Reporting Implementation of Instructional Strategies Using a Dynamic Geometry Approach,” examines which self-reported implementation of instructional strategies using a dynamic geometry approach led to students making conjectures, testing conjectures, and eventually proving their conjectures. The data collection included a self-reported questionnaire given to all of the project’s participating high school geometry teachers collecting both quantitative and qualitative data. The results of the predictive linear model with proving conjectures as a response variable indicate that teachers who spent class time instructing students to work individually on these three conjecture tasks reveal that students were less likely to get prove their own conjecture when working alone regardless of the level of geometry class. Making conjectures and testing conjectures were statistically significant and positively correlated with teachers who implemented class discussions. Furthermore, instruction that prompted group work had the largest positive correlation with Regular Geometry students proving conjectures.



The second paper, “Proving Activities in High School Geometry: A Comparison among Dynamic and Non-Dynamic Geometry Classrooms,” examines the self-reported proof activities of two groups of teachers, those using dynamic geometry software and those not using it. Results indicate that the teachers' self-reported proof activities across the year were at roughly the same rate with the dynamic geometry teachers reporting more (but not statistically significant) proof activities than the teachers not using the software in their classrooms.

The third paper, “Teachers’ Perspectives of the Implementation of a Dynamic Geometry Instructional Approach and Its Relationship to Student Achievement,” qualitatively examines teachers’ perspectives on the implementation of a dynamic geometric approach (DG). The participating teachers were involved in summer professional development to implement a DG approach in their respective classrooms. The students in the study demonstrated that computer usage improved as teachers became more familiar with the software. On the other hand, teachers



Section 2

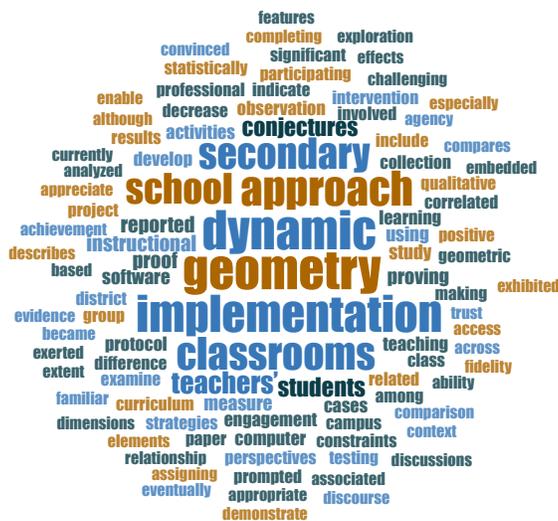
Implementation of the Dynamic Geometry Approach in Secondary School Classrooms

and students did not develop nor appreciate the conjecturing often associated with dynamic geometric software. Investigating and conjecturing seemed to take a lot of time and would hamper their progress in completing the objectives of the topic. Other issues related to the implementation process involved lack of access to the computer lab, lack of properly functioning computers, and lack of the appropriate software on the campus computers. The study also indicates that because proofs were not part of the curriculum, teachers were less interested in engaging students in learning proofs. Teachers were also less willing to teach proofs because they themselves struggled with proofs.

The fourth paper, “Curriculum, Student Engagement & Learning to Trust: Two Cases of Dynamic Geometry Implementation,” contains a qualitative examination of two particular cases involving participant teachers in the experimental study. In both cases the teachers’ students exhibited learning gains although each teacher came to settle at two very different approaches to implementation. The authors speculate that the teachers’ journeys and outcomes are largely related to the resources and constraints embedded in their teaching context, specifically related to campus and district features as well in terms of how much agency each exerted in challenging district and campus constraints.

The fifth paper, “An Observation Protocol Measuring Secondary Teachers’ Implementation of Dynamic Geometry Approach,” was published in the research journal *Mathematics Teaching* and appears here with permission. The paper reports on the examination of the extent to which teachers implement a dynamic approach to teaching geometry in secondary classrooms using a 25-item observation protocol (included in Section 5 of this monograph) based on four dimensions which include planned and implemented dynamic geometry approach elements, quality of instruction, and engagement and discourse. The reliability and validity of the observation protocol instrument is established.

The final two documents in this section are reports on the 2010 and 2011 professional development workshops planned and provided by project leaders and master teachers. The agenda outlining the activities of each workshop as well as the report of the project external evaluator who attended both workshops are included.



DIFFERENCE IN SELF-REPORTING IMPLEMENTATION OF INSTRUCTIONAL STRATEGIES USING A DYNAMIC GEOMETRY APPROACH

Brittany Webre, Shawnda Smith, Gilbert Cuevas

Texas State University

ABSTRACT

This study compares which self-reported implementation of instructional strategies using a dynamic geometry approach leads to students making conjectures, testing conjectures, and eventually proving their conjectures. The data collection includes a self-reported questionnaire given to all of the project's participating high school geometry teachers collecting both quantitative and qualitative data. The data collection of this study is currently being analyzed. The results of the predictive linear model with proving conjectures as a response variable indicate that teachers who spent class time instructing students to work individually on these three conjecture tasks reveal that students were less likely to get prove their own conjecture when working alone regardless of the level of geometry class. Making conjectures and testing conjectures were statistically significant and positively correlated with teachers who implemented class discussions. Furthermore, instruction that prompted group work had the largest positive correlation with Regular Geometry students proving conjectures.

INTRODUCTION

Geometry is a high school graduation requirement in the United States. It is important that students possess the ability to reason geometrically, or spatially, both in and outside the classroom; the issue of learning and teaching geometry continues to be a major problem nationally. U.S. students' geometry achievement is low (Battista, 2007). In order to better investigate this issue, we conducted a four-year research study, *Dynamic Geometry (DG) in Classrooms*, funded by a National Science Foundation grant. This project developed a curriculum that uses Dynamic Geometry software, Geometer's Sketchpad (GSP), to engage students in developing mathematical ideas through experimentation, observation, formulation of conjectures, testing of conjectures, and proving in the Geometry classroom. This project assessed student learning in 64 classrooms randomly assigned to experimental (DG) and control groups (no technology). The teachers of both groups were required to complete an Implementation Questionnaire multiple times throughout the year. This questionnaire asked teachers to report on instructional strategies and the frequency of students' time spent making, testing, and proving conjectures. This paper will report on the following research question: Which self-reported instructional strategies promoted students' conjecturing in the geometry classroom?

LITERATURE REVIEW

Dynamic Geometry

In this study, teachers were divided into two groups, the Dynamic Geometry group and the control group. The Dynamic Geometry group taught their geometry course using GSP software. Educational software, such as GSP, can assist in developing students' understanding of mathematical concepts and increase their reasoning skills (CBMS, 2001). The ability for students to take advantage of dynamic features such as dragging, measuring, and observing what changes and what stays the same, leads to the understanding of "the universality of theorems in a way that goes far beyond typical paper and pencil explorations," (CBMS, 2001, p. 132). After several years of research into the use of technology in the classroom, teachers became the key elements in the complexity of the learning situations in the technology classrooms (Laborde et. al., 2006, p. 294). Vincent (2005) found that the DG motivating context and the dynamic visualization fostered conjecturing and intense argumentation; and that the teacher's intervention was an important feature of the students' augmentations—prompting the students to furnish justifications for their statements and checking the validity of their justifications. Herbst and Brach (2006) argue that classroom tasks that demand high levels of cognitive activity from students require teachers to ensure engagement of the student.

Teacher Self-Reports of Implementation of Instructional Practices

In this study, teachers were asked to give descriptions of the ways they had implemented instructional strategies addressing student explorations of geometric concepts, the facilitation of conjecturing, and the approaches to geometric proof. Although teacher self-reports are frequently employed when researching the implementation of instructional strategies, a question frequently surfaces: How accurate are self-reported data collected through surveys? Cook and Campbell (1979) raise three threats to the validity of self-reports: (a) subjects tend to report what the experimenters expect to see, (b) the reports may reflect the subjects' own abilities, or opinions, (c) the subjects' inaccurate recall of past behaviors. Some researchers have argued that self-report data is of questionable validity, while others (Chan, 2009) point to studies of self-reported psychological constructs, which have obtained construct validity. Teacher self-report measures have been shown to be reliable and valid "when the reports are retroactive, completed more than once, and focusing on specific behaviors during a brief period (Kozol & Burns, 1986, p. 208). Reddy, Fabiano, and Peters (2015) report internal consistency and reliability between measures of teacher self-reports of general instructional and behavioral management strategies.

FRAMEWORK

This paper uses an adapted a version of Van Hiele's Model of Geometry Learning for the foundation of the study's theoretical framework. This modified version has only four phases that include: 1) Geometry teacher chooses an instructional method on the process of proof, 2) During this instructional method, the student is prompted to make a conjecture about the present problem situation, 3) Students are encourage to test their conjecture, and 4) Students are directed to prove or disprove that conjecture.

Purpose of Study

The purpose of this study was to compare the teacher's self-reported instructional strategies along with the approximate percentage of class time students spent making conjectures, testing their conjectures, and proving conjectures. Because this study reported here was just one part of a larger four-year dynamic geometry research project, broader context from the larger project may be illuminating. The teacher's choice of instructional strategy was a variable that was not controlled for in the Hierarchical Linear Modeling of the larger study; this model showed students' geometry achievement scores in the classes taught by the DG teachers (the experimental group) was significantly higher than students whose teachers were in the control group, with a large effect size for the students in the Regular Geometry classes. Therefore, this study examines the differences in the teacher's choice of instructional strategy for the regular level geometry class versus the honors (Pre-AP) geometry class.

What differences in the teachers' self-reported implementation questionnaire's instructional strategies promoted geometry students making conjectures, testing conjectures, and proving conjectures?

Significance of Study

The overall research project's study confirmed the hypothesis that the use of DG technology to engage students in constructing mathematical ideas through experimentation, observation, data recording, conjecturing, conjecture testing, and proof results in better geometry learning for urban high school students. This study is analyzing only the questionnaires to determine whether there exists a relationship between the teacher's choice of instructional strategy and time that students spent on conjectures, testing their own conjectures, and proving their conjectures. Because many high school students, particularly those in regular level geometry class, are not accustomed to doing mathematical proofs, it is a time consuming process, especially when seeing it and learning it for the first time. The goal is to find which instructional strategies are ideal to use and help promote students' developing and proving their own conjectures.

METHODOLOGY

Population and Sampling

The study was conducted at a central Texas university in partnership with three central Texas school districts. The population sampled included geometry teachers who volunteered to participate in the research project. The research study followed a mixed method, randomized cluster design, with the teacher or the teacher's class of students as the unit of randomization. The project team members began randomly assigning the 64 high school geometry teachers into two equally-sized groups, one for the experimental treatment group (the DG group) and the other the control group (commonly referred to as the *'business as usual'* or non-technology approach).

Instrumentation

The DG Teacher self-report implementation questionnaire contains six multiple-choice items and ten open-response questions. This study’s questions asked teachers how often did they use the following instructional strategies: class discussions, individual work, group work, teacher demonstrations, student demonstrations, and teacher interacting with students; other questions prompted teachers to approximate the percentage of class time that students formed conjectures, tested their conjectures, and proved their conjectures. Jiang (2015), the project’s principal investigator, recently published the results on the reliability and validity of using this questionnaire as an instrument. He analyzed the data using each time point of the study (5-6 week intervals), and found that the Cronbach’s Alpha range from 0.85 to 0.95. As for the validity, “The results reveal that in terms of the level of effectiveness in using GSP in teaching geometry, from those teachers who completed the questionnaire (total of 31), 29% of the teachers were at the high level, 61% of the teachers were at the middle level, and 10% of the teachers were at the low level,” (Jiang, 2015, p. 3).

The DGTQ instructional method question asked, “When reflecting on your teaching, how often did you use the following formats during the past 5-6 weeks: Class discussion, Individual work, Small group work, Teacher demonstration, Student interaction with you (as the teacher), and Student demonstration?” The response items were coded using a Likert scale shown in Table 1 below. The research team made the decision of using this coding theme where zero representing the average response, so that positive value responses scores would stand out and making it easier to reveal which items were implemented most frequently. The next item on the questionnaire asked the participating DG teachers, ‘What percent of your students did the following (Form conjectures, Test conjectures, Prove or disprove their conjectures) during the past 5-6 weeks?’ These responses were coded as follows:

Instructional Strategies Response Scale		Percentage of Class Time that Students did Conjecture Tasks	
Response Choices	Codes	Response Choices	Codes
I have not used this	-2	None	0
Rarely	-1	1 – 25%	12.5%
Every few sessions	0	26 – 50%	37.5%
Most class sessions	1	51 – 75%	62.5%
Nearly all class sessions	2	76 – 100%	87.5%

Table 1: Coding of Questionnaire’s Response Choices

Results

This study's data collection began with the original 66 questionnaire responses from teachers who participated in the DG project over this two-year period, but preliminary data analysis revealed that five teacher data points were labeled as outliers after utilizing the Mahalanobis distance outlier test. Table 2 below explains the grouping of the remaining teacher data points.

Project Year	Number of PreAP teachers	Number of Regular teachers
Year 2	11	15
Year 3	15	20
Total	26	35

Table 2: DG (Treatment) Geometry Teachers separated by class level

After removing outliers, the researchers explored the potential relationships between the six different instructional methods and the three different conjecture activities by calculating the Pearson r correlation coefficient among the 18 different interactions on aggregate data, followed by class level, and then year of the project.

Class discussion was the only instructional strategy with a statistically significant correlation to the conjectures tasks when analyzing all class levels together as a whole. This method of discourse was positively correlated with both making conjectures ($r = 0.288^*$) and testing conjectures ($r = 0.289^*$). The Regular level geometry classes revealed 15 out of 18 positive associations between instructional methods and conjecture activities. There was a statistically significant positive correlation between Regular teachers' practice of class discussion and students making geometry conjectures ($r = 0.360^*$) as well as testing their conjectures ($r = 0.454^{**}$). Additionally, teachers of Regular geometry classes who spent class time allowing students to work in groups was positively associated with students proving their conjectures ($r = 0.388^*$). However, teachers who spend class time assigning students to work individually was negative correlated with students proving or disproving their conjectures with a Pearson $r = -0.384^*$. For the PreAP classes, the relationship between each of the six different instructional strategies and three conjecture tasks were predominately negative correlated with one another on 16 of the 18 interactions over both years. For example, making conjectures had statistically significant negative correlations with both the use of teachers' demonstrations ($r = -0.412^*$) and assigning individual work ($r = -0.434^*$). Testing conjectures had a negative association with the use of individual work. Furthermore, when taking the project year into account, more statistically significant correlations are revealed, see Table 3 below. Even though the project year was not a statistically significant on its own on the aggregate data set, but it did have an interaction effect on the Regular level geometry class. As seen in Table 3, the Regular classes in Year 2 reported 14 out of 18 negative correlations between methods and conjecture tasks; Then in Year 3 of the project, the Regular level classes dramatically increased the percentage of time that students spent on making, testing, and proving/disproving their conjectures which in turn revealed 16 out of 18 positive interactions with three of the correlations being statistically significant.

	PreAP Year 2			PreAP Year 3			Regular Year 2			Regular Year 3		
	MC	TC	PC	MC	TC	PC	MC	TC	PC	MC	TC	PC
CD	0.170	-0.201	-0.205	-0.254	-0.243	-0.344	-0.147	-0.050	-0.050	0.378	0.487*	-0.040
IW	-0.190	-0.113	-0.523	-0.661**	-0.701**	-0.376	-0.053	-0.306	-0.426	0.080	0.204	-0.365
GW	-0.426	-0.207	-0.024	0.007	-0.063	-0.171	-0.318	0.061	-0.082	0.152	0.304	0.585**
TD	-0.146	-0.278	-0.414	-0.588*	-0.286	-0.772**	0.035	-0.289	-0.596*	0.211	0.376	0.362
SD	-0.002	-0.044	0.224	-0.117	0.076	-0.358	0.094	0.079	-0.069	0.271	0.222	0.482*
IT	0.438	0.219	0.128	-0.610*	-0.444	-0.871	-0.440	-0.329	-0.129	0.041	0.227	0.145

*. Correlation is significant at the 0.05 level (2-tailed). **. Correlation is significant at the 0.01 level (2-tailed).
 CD = Class Discussion, IW = Individual Work, GW = Group Work, TD = Teacher Demo, ST = Student Demo, IT = 1-on-1 interaction with teacher M= Make T=Test P=Prove

Table 3: Correlations of Instructional Methods and Conjecture Task Sorted on Class Level & Year

Next, the ANOVA results was examined to explore the difference in means across the PreAP and Regular level classes. There was a significant effect of the independent variable, class level, on the following dependent variables: individual work [$F(1,59) = 5.076, p = .028$], making conjectures [$F(1,59) = 50.987, p = .000$], testing conjectures [$F(1,59) = 74.715, p = .000$], and forming conjectures [$F(1,59) = 34.734, p = .000$]. There was not a significant effect on the remaining dependent variables: class discussion [$F(1,59) = 2.820, p = .098$], group work [$F(1,59) = 5.076, p = .401$], teacher demo [$F(1,59) = 5.076, p = .096$], student demo [$F(1,59) = 5.076, p = .296$], and 1-on-1 interaction with teacher [$F(1,59) = 5.076, p = .065$].

The researchers then use these findings to determine which would be the best predictors of students making, testing, and proving their conjectures. The linear regression models for making conjectures and testing conjectures only had the class level as a statistically significant predictor and did not have any instructional methods as statistically significant predictors at the $\alpha = 0.05$ level. The linear model with the response variable as proving conjectures revealed that the predictor instructional method of individual work negatively affected the percentage of time that students engage in proving/disproving their conjectures. This model had a statistically significant model intercept coefficient of $\beta_0 = 36.14$, $\beta_1 = 32.53$ (representing the effect of the PreAP class level), and $\beta_2 = -11.832$ representing the effect ($\alpha = 0.06$) of using individual work as the method of class instruction.

For the linear model with making conjectures as the response variables, statistically significant model intercept coefficient of $\beta_0 = 45.05$ represents the predicted percentage of time that the Regular class level students spend making conjectures, then the next coefficient $\beta_1 = 28.08$ represents the predicted additional time that PreAP students spend on making their own conjectures. There this model predicts that Regular Geometry students are predicted to spend 45% of class time to making their own geometric conjectures versus the PreAP students who spend about 73% of their class time on forming conjectures. The model's next predictor was Class Discussions with a coefficient of 5%, but this coefficient's significance was .087 which was outside our target of $\alpha = 0.05$. The testing conjectures linear model is nearly identical to making conjectures instead with $\beta_0 = 39.04$, and $\beta_1 = 31.39$. The proving conjectures model was unique in that it was the only regression model than included a statistically significant instructional method in the model.

DISCUSSION

The objective of this study was to investigate which instructional strategies are ideal to use and help promote students' developing and proving their own conjectures. The linear model with proving conjectures as the response variable predicted that teachers who spend class time assigning individual work to geometry students will likely decrease those students' time spent trying to prove or disprove their own conjecture by 11% regardless the level of the class. This finding aligns with statements of demanding high levels cognitive activity of Herbst and Brach (2006), the process of developing and proving/disproving a geometric conjecture requires increased level of critical thinking and problem solving. For many American high school Geometry students, this is their first encounter with the cumbersome cognitive proof process and if a geometry teacher assigns these students to work on these conjecture activities individually, as seen in the above correlations and data results then not many students will be able to complete both the first two tasks of making and testing their conjecture, and less likely to reach the prove/disprove activity regardless of their class level.

For this current study, we only analyzed the data from the treatment group due the positive effect on the DG Regular class level students' achievement on the standardized state Geometry test. The researchers want to explore what teaching methods were implemented that contributed to the treatment group students' achievement gains.

Even though the results did not have a statistically significant instructional method that predicted students being able to make and test their conjectures, the correlation revealed some ideas of what doesn't work. In future studies, beginning with knowing what does not work helps one find what does work – just like a student trying to prove their conjecture, sometimes you just need a test out some counterexamples first.

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PROVING ACTIVITIES IN HIGH SCHOOL GEOMETRY: A COMPARISON AMONG DYNAMIC AND NON-DYNAMIC GEOMETRY CLASSROOMS

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ABSTRACT

Some researchers have suggested that dynamic geometry software can inhibit the likeliness of taking an exploration and conjecture to the proving/justifying stage because the ability to measure and otherwise be convinced of the truth of a conjecture might decrease the perceived need for more formal proof. This paper examines the self-reported proof activities of two groups of teachers, those using dynamic geometry software and those not using it. Results indicate that the teachers' self-reported proof activities across the year at roughly the same rate with the dynamic geometry teachers reporting slightly more (but not statistically significant) proof activities than the teachers not using the software in their classrooms.

INTRODUCTION

Educators have been enthusiastic about the potential of using dynamic geometry software environments in schools for quite a long time. The ability to drag, measure, investigate, transform, and animate has captured the interest of teachers and students as well. While most people find the idea of using such software for investigating conjectures and verifying them as promising, there remains concern that the easy ability to measure and “see” results of explorations has brought up concern that dynamic geometry software may hinder the desire for users to obtain a proof, as well as possibly contribute to a general sense that proof in geometry is no longer needed or valued. This study aims to address this issue by examining the self-reported proving descriptions of two groups of geometry teachers, using or not using Geometer’s Sketchpad (Jackiw, 2001). The primary research question is whether there is a difference in proving between the two groups, with special attention to addressing the hypothesis that teachers using dynamic geometry (DG) might give less attention to proofs.

DG and Proving

For professional mathematicians, proving a claim via deduction is the final say on a mathematical observation, elevating it from conjecture to theorem. The absence of a proof does not imply the mathematical observation is false—just not yet proven. Because of these epistemological preferences of the field, teaching proof in mathematics classes for high school students (and often more relaxed versions of justification are encouraged in lower grades) has been repeatedly called for (e.g. NCTM 1989, 2000), particularly given that proof was on the decline in high schools for many years.

Dynamic geometry software such as Geometer’s Sketchpad offer students and teachers new ways to engage with the content. Students can drag a single instance of a triangle, for example, by grasping a vertex (or side) and watch as that triangle becomes new ones. Selecting a different vertex, the action is

repeatable and an entire class of triangles can be explored in a few swoops of the mouse. Such power to “see” the geometry in action can bring exciting! The software allows users to measure angles and sum them thereby allowing those simple drags of the mouse and subsequent formation of a myriad of triangles to produce very clear evidence, for example, of the sum of the interior triangles being 180 degrees. These measurement capabilities combined with the power to check/verify an observation for an entire classes of a geometric object have been argued as possibly contributing to the decline in the perceived need for a proof to establish the truth of the claim (Hoyles and Jones, 1998; Laborde, 2000; Onir, 2008). Others such as deVillier (2003) suggest that there are multiple functions of proof, especially in pedagogic environments, that go beyond the verification function. The claims that DG software may lessen the need for proof as critics suggest or that DG software may increase uses of proof outside verification has not been explored in large scale projects. This work seeks to contribute to this knowledge by asking whether teachers using DG software incorporate fewer proof-related activities than peers not using the software.

Background and Method

This work is situated within the context of a larger NSF-funded project, The Dynamic Geometry Project. In that work we collected data from 70 teachers in a large urban center in the southwest United States. These teachers were randomly assigned to two groups—one for “business as usual”, the Control group, where teachers committed to NOT using DG technology in their high school geometry classes, and the Experimental group (DG group) where teachers were asked to use Geometer’s Sketchpad twice a week with their students. The data reported in this paper includes 35 teachers in both the Control and Experimental sections for a total of 70 teachers. The results from this paper are from Year 1 of the teachers’ work in the project. Both groups were offered a week-long summer professional development institute focusing on an approach to geometry teaching whereby students explore, form conjectures, and then prove their conjectures. The primary difference between the PD for each group of teachers was the nature of the materials used in the PD and encouraged for use in the classroom. The Control group learned to use hands-on materials such as patty paper, compass, straightedge, protractor, and Mira, among others. The DG group, by contrast, focused exclusively on developing the teachers’ Geometer’s Sketchpad skills and building a repertoire of activities for their classroom. In addition to the week-long institute, teachers also participated in “just in time” day-long sessions during the academic year with content related to their district’s curriculum schedule.

The data for the larger project included a host of student test scores, task-based interviews, select classroom fidelity-related observations, and teacher surveys. This paper reports on the results of the teachers’ surveys which were filed at the end of each 6-week grading period. In particular, this paper focuses on the survey question asking teachers to describe what activities from their classroom related to proof and justification over the previous 6-weeks. Each teacher was asked to fill this out a total of six times although not each teacher completed each of the six.

To analyze these short-answer responses, the author reviewed each teacher’s submitted answer to the proofs-related survey question and looked for themes. One obvious theme, for example, was that not many teachers reported doing proof-based activities in their classrooms, regardless of whether they were in the Control or DG group. The next theme that emerged was related to the degree of specificity of response. From this observation, the author created a coding scheme, described as follows.

0--The teacher reported doing no proof activities during that reporting period, or the teacher left that question blank when otherwise filling out other questions, indicating an implied answer that no proof activities were conducted.

1—The teacher reported something that was not obviously a geometry context or not obviously a proof. For example, a few teachers answered that they had spent time teaching and justifying the steps for solving an algebraic equation. This was not in a geometry context (in as much as the teacher did not offer a geometry relationship in their response) and so received a score of 1. Another example of an activity reported that scored a 1 was when a teacher reported “proving” that an equation held true for a select set of values by calculator verification. This is a case of empirical verification and was deemed not a proof or moving towards some deductive rationale. For the purposes of this work, reference made to formal, paragraph, 2-column, or flow chart proofs were counted as deductive whereas statements about verifying numerical findings via calculator or “visual proofs” were not.

2—The teacher reported a single specific geometric or proving context. Many of these examples arose during presumed units on angle measure and length relationships within figures. For example, “vertical angles are equal” was taken to mean that a class lesson involved proving that vertical angles are equal. These codes were usually applied to specific yet terse statements such as the example given.

3—The teacher reported more than one specific result proven (as in code 2 above) and/or reported a more general statement of the activities that implied more than one proof-related lesson. “Triangle Congruence Theorems” was included as a level 3 as the implication is that lesson(s) were directed towards the full typical set of such theorems although the teacher did not specifically write them out. Another type of statement that was coded as level 3 was when a teacher wrote more fully about the proof goals and activities whether or not the teacher included a specific content topic for the reporting period. For example, “We have just started to use formal proofs. We have worked on forming conjectures using patty paper. We have written paragraph proofs & are beginning to use the 2 column proofs. I have laminated the forms so they can be used repeatedly + they can easily erase + correct.” The implication is that the class has done this over an extended period of time across multiple lessons during the reporting period.

Overall, the limitations related to teacher specificity. It was often hard to determine what a teacher meant and so there may be more level 1 codes than would reflect the teachers’ intentions. But, in general, these reflect the range of responses offered in the data. And, as is often a limitation in survey data, we cannot know the extent to which the teachers’ under or over-reported. Under-reporting seems particularly possible for the DG group teachers as many of them seemed to only report on proof activities involving Geometer’s Sketchpad activities. For example, this teacher (and others similarly) wrote, “The Geometers Sketchpad 5 has not been installed to our campus computers. District and school personnel have been the main road blocks. Hopefully this week, we can resolve these issues.” Although the teacher may have done no proof activities at all, it was hard to determine this given that the teacher qualifies the statement with the comment on the installation delay at the school site. Quite a few teachers in the DG group made such qualifications on their survey response, suggesting the potential for under-reporting for this group.

Results

The teachers’ reports primarily revealed no significant difference between teachers’ proof-related activities depending on the use of Geometer’s Sketchpad or not. Table 1 summarizes the results per report as well as two measures that reflect the year as a whole. The first of these, Whole Year, reports on the average score across all six report. The second, Mid-Year, drop both the first and the last reports from the average because many of the teachers in the study remarked that the start of the year (and hence report 1) focuses on establishing routines, reviewing previous material, and learning vocabulary; and the end of the

year (and hence report 6) focuses on the state test preparation and so they do not a lot time to new material, nor to proving. Notice that the n-values differ by report because some teachers did not submit one or more of the six reports across the year. When a teacher did not submit a report, their data was dropped for the overall average measure, further decreasing the n-value and thus limiting the ability to statistically distinguish between the groups.

Table 1: Average Proof-related activities score by report and group type

	<i>Control</i>			<i>DG</i>		
	Mean	N	S.D.	Mean	N	S.D.
<i>Report 1</i>	.48	(n=29)	.83	.67	(n=24)	1.09
<i>Report 2</i>	1.81	(n=31)	1.08	2.07	(n=27)	1.21
<i>Report 3</i>	1.45	(n=29)	1.24	2.04*	(n=27)	1.26
<i>Report 4</i>	1.04	(n=25)	1.24	1.17	(n=28)	1.33
<i>Report 5</i>	1.03	(n=29)	1.24	.90	(n=31)	1.19
<i>Report 6</i>	.66	(n=29)	1.04	.69	(n=29)	1.11
<i>Whole Year Average</i>	1.13	(n=18)	.76	1.28	(n=13)	.86
<i>Mid-Year Average</i>	1.36	(n=22)	.80	1.73	(n=21)	.93

(excludes first & last reports) *p-value = .042

Overall we can see that the DG group mean scores are higher than the Control group for all but one of the reports. In Report 5 the Control group’s mean was higher than that of the DG group by .13. Only one of these is significant, though, Report 3, where the DG outscores the Control group by .59 which is roughly half a point difference, from about a 1.5 for Control to about a 2 for DG. The Whole Year and Mid-Year Averages also show the DG mean to be higher, but again not at significant levels.

Below are Figures 1 and 2 which are boxplot of the summary data for each the Whole Year and Mid-Year Averages broken apart by DG and Control, respectively. We can see that the DG group has higher median scores as well.

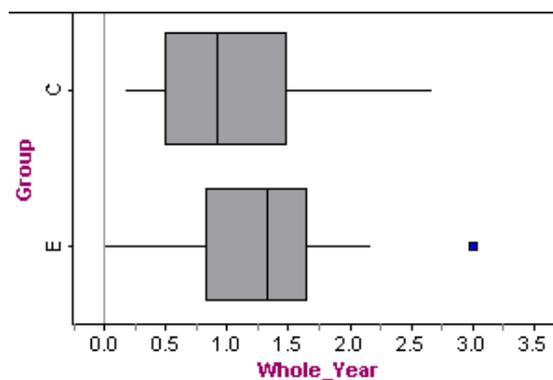


Figure 1: Whole Year Average by group type; Control and Experimental (DG)

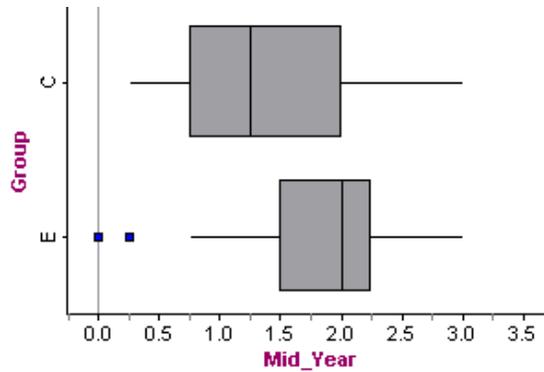


Figure 2: Mid-Year Averages by group type; Control and Experimental (DG)

Discussion

The teachers in the study, overall—joining both DG and Control, did not report many proof-related activities. This result is not new, as many have known of the decline in proving in high school geometry, a result despite calls for such inclusion in the curriculum by many (NCTM, XX). The DG Project encouraged a cycle of exploration, conjecturing, and proving for both groups. The mean for both groups across the whole year were 1.13 and 1.28 (Control and DG, respectively). This corresponds to average reports slightly better than a score of 1, which implies the comments were usually non-geometry or were difficult to see the “proof” in what the teachers wrote. Yet, although neither group reported many lessons focused on geometric proof, it does not seem to be the case that teachers and classrooms using DG were any less likely to engage students in proof-related activities compared to non-DG teachers’ classrooms. In fact, their means and medians were slightly higher on all except one report where DG was lower and one where DG was higher and statistically significant. Given also the indications that DG teachers may have under-reported their total proof activities (by only sharing activities of proving that were also done using Geometer’s Sketchpad), these DG numbers might have been higher. The data available, though, can only conclude that DG teachers report including proof-based lessons no less than non-DG teachers. This itself is worthy of acknowledging. Geometer’s Sketchpad, and other similar software, contains features that could serve to “verify” conjectures via non-deductive techniques. Students and teachers might be inclined to stop an activity once a basic level of faith in the veracity of a conjecture were determined through one or many of the software’s capabilities for measurement, transformations that “map” figures onto others, animations that reveal a non-variant element, and so forth. These concerns did not seem to bear out fully here in this work.

Though hard to determine via easy quantifiable measures, a theme that emerged and might be worthy of further investigation was the generally more specific nature of the DG group’s responses. More teachers in this group used specific language around content to describe their activities. Teachers in both groups reported single theorem responses, but more of the DG teachers included further details. For example, a teacher in the Control group might merely write, “properties of quadrilaterals” whereas the DG teacher would say “Properties of quadrilaterals...the diagonals of the parallelogram bisect one another...they [students] used parallel lines and congruent triangles to show the diagonals bisected each other.” Both would have scored a 2 because both imply a collection of theorems, but the latter one offers detail about the technique used as well as that students were the actors. The less detailed reports, in general,

by the Control group teachers does not necessarily mean they also did not use the same technique or that students were not also doing the work (in contrast to a teacher providing direct instruction, perhaps). All we can conclude for now is that they elected not to be more descriptive in their responses. This might be because the Control group felt the lessons were more routine and obvious and therefore did not need explication; the DG group may have felt the need to communicate possibly because they perceived the lessons as non-standard.

While it did seem to be the case that DG teachers' reports were more detailed, overall, across the groups there seemed to be a lack of shared language about what was a proof or what to include in their responses. Many reports were given a score of 1 because the responses were hard to pinpoint just what was done. They reported something after all—presumably something the teachers felt addressed the question asked on the survey. Grossman (2009) suggests that teachers need a common language for practice and this study relates in as much as there did seem to be trouble articulating activities and practices related to proving. The possible differences in descriptive language suggested above between the two groups may be related to the software offering a shared set of vocabulary for actions via the vocabulary inherent to the software's menus—drag, transform, etc. Chazan (1993) found that most students do not know the difference between an empirical and deductive argument. The responses by some teachers in this work suggest they also struggle to distinguish or at least to distinguish between the word “proof” as was used in the survey question and between more inductive techniques. Recall that one of the exemplars for the coding scheme level 1 offered previously involved a teacher reporting on calculator (numerical) verification as a proof-related activity. Though not the most common answer, it was not an unusual response to see empirical evidence offered as a proof-related activity. Similarly, a small group of teachers also made statements suggesting that paragraph proofs are not “formal” with the implication that 2-column proofs “counted” and others such as paragraph or flow chart proofs did not.

Finally, to reiterate, teachers in the study did not report using proof-related activities often—in either group, yet the slight advantage—though not significant—went to the DG group. This study set out to address the concern from the literature on the potential for DG software to deemphasize or discourage the step from inductive verification to deductive—what we typically refer to as proving. This was not the case in terms of reported activities. DG teachers did not report less proof-related activities and there may be reason to suspect they did more than they shared in the surveys. Nevertheless, the field certainly has more to learn about the nature of these activities as well as more work to be done helping teachers incorporate more deduction in their classrooms as well as more to be done in helping develop a shared language around proof and justification practices in their classrooms.

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TEACHERS' PERSPECTIVES OF THE IMPLEMENTATION OF A DYNAMIC GEOMETRY INSTRUCTIONAL APPROACH AND ITS RELATIONSHIP TO STUDENT ACHIEVEMENT

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ABSTRACT

This qualitative study describes teachers' perspectives on the implementation of a dynamic geometric approach (DG). The participating teachers were involved in summer professional development for three years to enable them to implement a DG approach in their respective classrooms. The students in the study demonstrate that computer usage among students improved as teachers became more familiar with the software. On the other hand, teachers and students did not develop nor appreciate the conjecturing often associated with dynamic geometric software. Investigating and conjecturing seemed to take a lot of time and would hamper their progress in completing the objectives of the topic. Other issues related to the implementation process involved lack of access to the computer lab, lack of properly functioning computers, and lack of the appropriate software on the campus computers. The study also indicates that because proofs were not part of the curriculum, teachers were less interested in engaging students in learning proofs. Teachers were also less willing to teach proofs because they themselves struggle with proofs.

INTRODUCTION

The National Council of Teachers of Mathematics documents (National Council of Teachers of Mathematics, 2000) promote dynamic geometry software that creates an environment for exploring, conjecturing, and proving mathematical ideas. Computer software tools (dragging, measuring, etc.) that are dynamic in nature provide a means to achieve such an environment by observing what happens when one drags and measures figures. These tools have been discussed among mathematics educators and have generated great interest and enthusiasm (Oner, 2008). This study investigates high school geometry teachers' perspectives and instructional strategies when they implemented a dynamic geometry approach through the use of these tools to engage students' active mathematics learning.

With dynamic software, there is a possibility that the ways students view proofs are likely to change as the dynamic features bring meaning to students. With that said, technology alone cannot be the magic bullet that transforms what goes on in the classroom while students learn mathematics, "rather [it is] the teacher's decision about how, when, and where to use technology that determines whether its use will enhance or hinder students' understanding of mathematics" (Hollebrands & Zbiek, 2004, p. 259).

Although there is minimal agreement among researchers in mathematics education on how and when to use technology, there is a consensus that using technology effectively can help students learn mathematics in a more meaningful way (Reed, Drijvers, & Kirschner, 2010). Additionally, "effective teachers maximize the potential of technology to develop students' understanding, stimulate their

interest, and increase their proficiency in mathematics” (National Council of Teachers of Mathematics, 2008, p. 1). On the other hand, there is limited know-how among teachers about the extent to which dynamic geometry can be integrated into classroom instruction (Oner, 2008). Hollebrands and Smith (2009) have pointed out that dynamic software “has been shown to have a positive impact on students’ understanding of geometry and reasoning ability, the implementation of these tools and the nature of the tasks selected by teachers are crucial factors in determining their effects on students’ learning” (p. 231). Although researchers and policy makers support the implementation of a dynamic geometry (DG) approach in K-12 instruction, teachers are responsible for deciding how and when to use it in geometry instruction (Liu, 2011).

LITERATURE REVIEW

Dynamic Geometry Software

In the 1980s, Judah Schwartz and Michal Yerushalmy developed what was perhaps the first dynamic software application in geometry that allowed users to explore the logical consequences of actions on geometric objects on the computer screen. Their software, the Geometric Supposer, was the forerunner to what we see today as Cabri (Laborde & Bellemain, 2005) and The Geometer's Sketchpad (GSP) (Jackiw, 2001). Cabri and GSP are very similar in many aspects, although some commands are a bit different. For the purpose of this study, GSP was selected as the dynamic software to be implemented. The primary characteristic of the dynamic software is its click-and-drag feature. This feature enables the user to manipulate the object dynamically, measure, and make calculations instantaneously whereas paper and pencil methods are static in nature (Goldenberg, Scher, & Feurzeig, 2008), laborious, and prone to error. The dragging process, in which the user can change the location of points, lines, or entire figures, is a tool that helps students determine if the construction is made correctly; and this activity can initiate mathematical discussions that may lead to deductive proof (Mariotti, 2000). According to Mariotti, dragging includes different phases: the first phase is to “play around” with the constructed geometric figure to have a sense of how it behaves. The second step is finding any patterns and making conjectures and, finally, the third phase is to learn from the second stage to come up with more explorations (Mariotti, 2000). Despite disagreement based on Euclidean geometry about measurement, the dynamic geometry measurement tool is very vital as it works in conjunction with dragging. “Distance, lengths, perimeters, area, and angles of constructed figures can be measured and the measurements change ‘continuously’ in the process of dragging the geometry figure” (Olivero & Robitti, 2007, p. 137).

The Role of Conjecture Using Dynamic Software

In order to teach formal proofs in geometry, attention needs to be paid to conjecturing and inductive reasoning as a step towards deductive proof (NCTM, 2009). The process of conjecturing may lead to formal and informal explanations as to why given conjectures are valid, and this can lead to a more formal deductive proof. In that process, students are given the opportunity to explore dynamic geometric shapes to come up with conjectures on their own without the help of a teacher or textbook. This kind of instruction is recommended by the National Council of Teachers of Mathematics (NCTM)

because students need to learn mathematics conceptually rather than as a process of memorization of facts. Additionally, the NCTM (National Council of Teachers of Mathematics, 2000) outlines four essential elements of reasoning and sense making with geometry:

- Conjecturing about geometric objects. Analyzing configurations and reasoning inductively about relationships to formulate conjectures.
- Construction and evaluation of geometric arguments. Developing and evaluating deductive arguments (both formal and informal) about figures and their properties that help make sense of geometric situations.
- Multiple geometric approaches. Analyzing mathematical situations by using transformations, synthetic approaches, and coordinate systems.
- Geometric connections and modeling. Using geometric ideas, including spatial visualization, in other areas of mathematics, other disciplines, and in a real-world situation. (pp. 55-56).

Research by de Villiers (1992) reported that the environment provided for students in the process of conjecturing improves the quality of conjectures they come up with and the conviction of those conjectures. One aspect of dynamic software that is important is that it makes it easier for a student to test conjectures. This means that it has the potential of not only encouraging students to explore and develop conjectures but also to prove those conjectures.

Implementation of Technology

For the teachers to implement the intended technology designed to foster reasoning, a supportive environment for integrating technology into their instruction is critical. District and school leaders need to work hand in hand with teachers to provide them with resources and professional development opportunities. Having teachers supported in that way is essential to successful implementation of technology integration (Bebell, 2003).

Having easy access to technology for both teachers and students is essential. A number of studies indicate that lack of access to technology resources is one of the main factors that hinder technology integration. In many schools, computers are located in a separate room from the classroom and the teacher has to make prior arrangement to meet students in that room. This can be a scheduling challenge for teachers wanting to use the computer lab (Zhao, Pugh, Sheldon, & Byers, 2002). Research continues to show that when students had direct access to computers, they performed significantly better than those students who had access only to the computer lab (Mann, Shakeshaft, Becker, & Kottkamp, 1999). On the other hand, research also indicates that teachers who had such access in their classroom rather than in the computer lab had better technology integration and better instructional skills. Further, regarding the limited access to technology, a study by (Norris, Sullivan, Poirot, & Soloway, 2003) notes that: “When two-thirds of teachers report having no more than one computer for an entire classroom of students, it is unsurprising to discover that 44% of respondents report that they use computers in curricular activities less than 15 minutes per week” (p. 20).

Other studies demonstrate that access alone cannot lead to the use of the intended technology. In a study by Kleiner and Lewis (2003) with 92% of teachers having access to technology, only 14% of teachers made the technology an integral part of their instruction.

Research Questions

The study reported in this article built upon the research literature discussed above. Although other researchers have examined the impact of using various dynamic geometry tools on teachers' instruction and students' learning (Isiksal & Askar, 2005; Laborde, 2000); very few of the studies have investigated how teachers perceived the process of engaging their students in exploring, conjecturing, and proving geometric ideas using dynamic software in real classroom settings. This article addresses the following research questions:

1. How did teachers report guiding students through the process of exploring, conjecturing, and proving?
2. What factors did teachers identify as facilitating or hindering the DG implementation process?
3. How did teachers feel about the process?

THEORETICAL FRAMEWORK

The theoretical perspectives that inform this research are sociocultural theories of learning that involve secondary school teachers and students. As Goos (2008) puts it,

Sociocultural theories view learning as the product of interactions between people and with material and representational tools offered by the learning environment. Because it acknowledges the complex, dynamic and contextualized nature of learning in a social situation, this perspective can provide rich insights into conditions affecting the innovative use of technology in school mathematics" (p. 174).

The framework for this study is based on the work of Vygotsky (1986) in which he argues that social interaction develops higher functions. Vygotsky notes that just studying one individual alone cannot tell much about a child's development without exercising external world. "Through participation in activities that require cognitive and communicative functions, children are drawn into the use of these functions in ways that nurture and 'scaffold' them" (pp. 6-7).

Sociocultural theories of learning include concepts that are associated with learning such as scaffolding, reciprocal teaching, and collaborative learning – each of which highlights the social interactions that promote learning. In this case, we are looking at the interaction between the teacher, technology, and students in the teaching-learning process. The assumption is that the result of scaffolding, reciprocal teaching, and collaborative learning will influence the way the teacher envisions the implementation process in his or her classroom. As earlier noted, factors in consideration in this study include learning technology and its dynamic aspects, learning proofs, and the interaction between teacher, students, and technology to enable the learning process.

METHOD

The present study derives from a quantitative study assessing the efficient use of DG software (The Geometer's Sketchpad-GSP) to engage students in constructing mathematical ideas through exploring, conjecturing, and proving results. This article reports on the qualitative component of a quantitative study implemented in 64 classrooms in a large school district in Texas.

Data Collection

This study examined teacher implementation and was qualitative in nature, focusing on the 36 high school teachers in the DG group. The teachers completed the DG Implementation Questionnaire six times during the school year while the study was being conducted. The participants had teaching experience ranging from 2 years to 20 years of teaching. They participated in one-day monthly professional development and in this professional development project, they undertook mathematical activities using the DG approach that involved exploration, conjecturing and proving. The meetings were held in one of the school's computer rooms.

During the monthly professional development meetings, the participating 36 teachers reflected on and answered the same four questions at the early stage of the DG implementation (a few months into the study), in the middle of the implementation, and finally at the end of the implementation. These questions served as a guide for them to reflect upon and write about their experiences with the implementation:

1. Describe how you engaged students in doing experimentation (constructing, dragging, measuring, animating, transforming, observing, etc.) with DG tools.
2. Describe how you engaged students in forming and testing conjectures with DG tools.
3. Describe any instances when you have led students through the process of proof.
4. Describe any challenges you have experienced and any concerns you might have regarding the use of DG tools.

Data Analysis

The research questions explored in this study were:

1. How did teachers report guiding students through the process of exploring, conjecturing, and proving?
2. What factors did teachers identify as facilitating or hindering the DG implementation process?
3. How did teachers feel about the process?

To analyze this data, the constant comparison method served as the data analysis process (Glaser & Struss, 1967; Strauss & Corbin, 1990). The analysis is based on the notion that as teachers interacted with students, professional development personnel and the DG software can influence how teachers make sense of the DG approach to teaching mathematics. In this analysis, we focused on how the teachers using the DG approach handled exploring, conjecturing and proving and how that influenced the teachers' instructional approaches. Based on the teachers' responses to the DG Implementation Questionnaire, four categories were examined: exploring, conjecturing, proving, and challenges regarding the use of DG tools. Using inductive reasoning, the relationship between categories was established. Constant comparative approach provided the means with which to analyze the participants' perspective in order to integrate those perspectives in the context of the social interaction dynamics in the study.

STUDY FINDINGS

Exploration

Data collected from teachers was based on the question: How did you engage your students in doing experimentation (constructing, dragging, measuring, animating, transforming, observing, etc.) with DG tools. The initial responses to the question indicated that students were slowly learning how to navigate through the software. As one of the teachers pointed out:

At first some of the students were trying to construct their way until I came by and used the dragging tool, and they realized that they needed to construct the proper way. I also have them show me, as I monitor, what they did to get the measure of the segments, angles, etc... We have not done the transforming yet.

Another teacher noted that students were told that one of the requirements for their figures was that they pass the drag test to show that the figures didn't fall apart. The teacher gave the students step-by-step instructions to follow while using GSP, and she said, "every once in a while, there were leading questions for the students to answer to make them think about their next conjecture. I would walk around and ensure that they were dragging and measuring things accurately."

However, another teacher related her experiences this way:

This six weeks we focused solely on linear equations and system of equations. I was unaware that there was [sic] lessons on Key Curriculum [websites] dealing with this [DG] material therefore we did not spend very much time in the lab. My students did work with angle relationships the first week and verified the relationships that exist when parallel lines are cut by a transversal. I have learned that if I go quickly through the prep work for the investigation for the whole class while they have the paper but not the computer, they have more success once at the computers.

Another teacher reported a more ambitious approach at the start:

I used Geometry Sketchpad for instructor and student demonstrations for all applicable lessons. In particular, transformations, tessellations, angle pairs, triangle angle relationships, and triangle special segments were all introduced, explored, and extended with Geometry Sketchpad. In the classroom, three computers were available for teacher and student constructions, measuring, transforming, etc. Beginning 27 Sept, the lab was available for use on Mon and Wed for the project group. Students used DG curriculum lessons or teacher-developed lessons that corresponded with curriculum to construct geometric figures, drag and test constructions, measure angles and sides, etc.

At the middle of the project, teachers' responses showed that they were engaging students in geometric explorations such as parallel lines cut by a transversal as well as constructions of some specific quadrilaterals by using the dragging and measuring tools more frequently. They were also beginning to use the transformation tools. At this point, teachers and students were learning more about GSP tools and how those tools can be used to enhance learning. Although there were some animations, it was most often a result of playing with the software and nothing much related to what they were learning. One of the teachers noted:

Some students did animate - but only to play after they finished the construction. Students who didn't do the constructions correctly were not able to come to the correct conclusions when testing by dragging.

Another teacher reported, "The students are using the dragging and measuring more. I have students take turns showing the class how to find angles and label them. I also started having a student at the

demonstration computer leading the class.” That teacher also randomly picked students to demonstrate activities and then had students work in pairs, in measuring, dragging, and observing the changes. Some students used transformations and tried animation, but for the most part, they just used measuring, dragging and observation techniques.

Another teacher noted a significant challenge in that students were not engaging with the program and not very willing to do anything when the teacher was not standing next to them. "I don't know if this is delayed because of my absence at the beginning of the year; however, I am concentrating right now on students' developing conjecturing skills. I am going to give them small things to do on the sketchpad to encourage enhanced engagement." After some months, the teacher noted that:

For the two weeks that we did DG, I had a student at my computer, and he would start the steps. The student would stop at a point, and the class would discuss the properties. Then I would choose another student to continue with the DG steps and repeat the process till we got the assignment done. The class would tell the computer student that it was right or wrong, and then drag, measure, and observe. When we did rectangles and rhombuses, I had students measure the side lengths, they also had to drag their rectangles and rhombuses to make sure they kept their shape. When they created their hexagons, they were also asked to drag their polygon. When the students did the tumbling blocks, they got practice in transforming. The students also like to do animations.

As teachers indicated, some students at this mid-way period of the implementation were doing some animations. When reporting how they engaged students in experiencing the DG environment, one teacher noted:

I created "activities" that lead students to a discovery. For example, I asked students to draw a pair of parallel lines cut by a transversal. I then asked them to measure all of the angles and compare the measures of the alternate interior, alternate exterior, corresponding and consecutive interior angles.

Students were then asked to make a conjecture about the angle relationships, test their conjecture by moving the transversal so that the angle measures changed and compare their findings with those of their peers. In most of the GSP activities "[that] we have done I have structured them so that they have steps." Teachers frequently said "steps," in reference to their belief that having activities with steps to follow will help students learn mathematics. Some of the teachers indicated that when students were given such activities, it made class instruction easier because students could follow the script.

Additionally, the teachers discussed the effects of changing the dimensions of a solid that offered opportunities for the students to drag, measure and compare the measurements of the nets and then the surface area and the second day the volume. It was very visual and aided the students to see the solid change and to see the immediate effects of the changes in the dimensions. At this point of the implementation process, there was a sense that teachers were doing dragging, measuring, and conjecturing but with minimal use of animations.

Towards the end the study, the teachers were again asked the question about how they engaged students in doing experimentation (constructing, dragging, measuring, animating, transforming, observing, etc.) with DG tools. One teacher noted that:

Students constructed polygons and transformed them. Choosing their colors was engaging to students. Animating was exciting to them and seemed like a video game to some of the students. Students were able to discuss their observations. They were also engaged in being able to come up and use sliders to change the visualization of three-dimensional figures.

The teachers reiterated that students enjoyed the ease of measuring ratios, by just clicking on segments and measuring ratios. Transforming and animating the size or shapes of polygons was exciting to them and every student was engaged. Some mentioned the animating aspect reminded them of video games, which provoked discussions about the creation of video games and computer software. Mathematical discourse ensued as students discussed their conjectures, based on their observations. Another engaging aspect of GSP was the emphasis on the visualization of three-dimensional figures and the use sliders to change perspectives. Another teacher pointed out:

"When I use GSP as a demonstration tool I try to create animations that demonstrate the properties that we will be working on in the lab. This gives the students some understanding of the property and if they finish early they like to try to create their animation that is similar to what was shown".

For the couple of weeks before the end of the study, the students were working with translations, rotations, and reflections. They constructed shapes using the Construct tool for parallel lines, perpendicular lines, and other objects. They then measured the lengths and angles of the various shapes. Finally, they performed the indicated transformations and measured the resulting figures. They also dragged the different shapes to see that they would remain the same. Students constructed parallelograms, rectangles, rhombuses, and squares. They did well with written directions for the first three shapes. They drag tested and checked the properties and conjectures for these shapes. When it came to the squares, some students did very well in constructing them. Others constructed a shape and claimed it was a square but then found errors during drag testing and checking. Their classmates helped them correct the errors. At the end of the process all of the students were conversant about the related constructions and animations, and they enjoyed them, as one of the teachers observed:

Put up an animated picture on the screen that was cool and told them that at the end of their exploration today they would have the skill to make one of these of their own. It did get them excited to go through the exploration, but I felt sad that they didn't have time to work more on the exploration.

Conjecturing

Teachers were asked to describe how they engaged students in forming and testing conjectures with DG tools. At the beginning of the semester, teachers' responses to the question indicated that little was going on in the classroom. Learning how to navigate technology was a big challenge, especially constructing the figures that were needed. They even had a hard time figuring out what the term conjecturing means and how to use the DG tools to come up with conjectures. As one teacher noted:

The first days in the computer lab were a major learning experience for all of my students. In forming and testing, the students are weak. I've noticed the students are concentrating on getting thru the directions and trying to get the pictures to look the same. We are getting thru the conjectures together and testing them. But unless I stop the lesson to point certain steps and "dragging," the students won't pay any attention to it.

It was evident that teachers engaged students in conjecturing by engaging them to write the conjecture after the experiment, discuss the conjecture as a class, discuss counterexamples, and finally come up with something that works by refining the original work. The teachers would ask them to say something they noticed based on their sketches and measurements. On the same note, another teacher pointed out, "this was a challenge my students are so used to 'giving the right answer' that they are unwilling to risk. I support and praise EVERY observation stated even 'It made a segment' or

something very obvious." This teacher continued noting that she has started having students make some conjecture for the situation and marking their paper that they did make an attempt. Then they tested their conjectures and revised as needed. "Without the step of marking the papers many students wouldn't try to make a conjecture, they waited for info to be given to them." She noted that students constructed geometric figures, measured angles, and sides, and formed conjectures from lesson materials or teacher prompts but getting students to think at this level was very difficult. Students were reluctant to volunteer their conjectures.

At the middle of the implementation of the DG approach, the teachers reported that the students seemed to be adjusting to the concept of conjecturing. The students could verbalize their conjectures, but they had a hard time writing them. They tested their conjectures with the measure and drag techniques. One teacher noted that when using "Sliding Bases" the students concluded that with a parallelogram, as long as the base and height don't change, the area would be the same. When using "Areas of Polygons and Circles" they had to explore the relationship between the formulas for finding the area of a regular polygon and the area of a circle. On that activity, one teacher noted that:

I tell them that we are exploring and encourage them to be unique and not to worry too much about the way they express their conjecture. Later on, as they become more comfortable and have learned by example, I will require greater quality of response. We are currently exploring as a whole class with one student driver. Conjecturing has been encouraged by my refusal to answer my questions. Then once students begin sharing ideas, we are as a whole class investigating them and refining them. There was one-on-one interaction between the students and myself. Students also talked among themselves when forming and testing conjectures.

The teachers reported that students formed conjectures about triangles including exterior angles, interior angles, congruent triangles, similar triangles and special segments; and, for example, the students tested their conjectures by dragging the vertices of the triangle to see if calculated sums remained constant. They tested conjectures about congruent and similar triangles by measuring angles and side lengths. Then they used the GSP calculator to set up proportions for similar triangles to verify corresponding sides and angles.

Another teacher asked questions that helped direct the students to a conclusion based on the GSP sketch:

I request that students drag/measure and then explain to me what is happening. I also ask them to tell me why. I continue to remind students that there is no wrong conjecture, and they need not worry about looking stupid. Any thought of why or how something is happening is a good one, and the goal is that they think. Usually when students get a conjecture wrong, I have them go back and take them test-by-test through their lab. This way I can talk them through and insert leading questions that will help them in conjecturing

The teacher used hard copies of DG curriculum worksheets that were from Key Curriculum LessonLink websites or other published GSP lessons to ask students about their observations, to form their conjectures, and to test those conjectures using drag tests, measurements, and/or additional constructions. Some pre-made constructions had sliders like "Smoothing the Sides" to assist students in visualizing the connections between sides and interior angles. More advanced students received one-on-one questions regarding their constructions and conjectures to prompt further high-level thinking. Through a targeted lesson activity, the students were led through a series of questions that helped them

form and test their conjectures. There was an indication by teachers that initial exposure requires no class discussion about the topic, just thought and answer time before involving the whole class.

Towards the end of the project, the teachers were asked again to describe how they engaged students in forming and testing conjectures with DG tools. The teachers' responses were similar to those at the middle stage of the implementation, but at this point, students were comfortable with GSP and particularly the animation tool. As noted by one teacher:

We did the parallelogram together and in doing it we formed conjectures. Then the students came up to the demonstration computer and tested the conjectures. I encourage them to take the activity as an experiment. They are researchers trying to observe and discover what is happening as they apply techniques and solve the problem. I asked students questions after they constructed, while they were dynamically dragging and observing measures. As a class, we also discussed different conjectures and could visually display as the student GSP technician used the geometer's sketchpad on the Promethean board.

By using the transformation features of GSP, the students were able to see how the shapes kept unchanged under different transformations except dilation. They were able to make conjectures that certain transformations are isometries, and others are not. While doing the teacher demonstrations, the teacher discussed with students what was happening, what changed and what remained the same. They talked about what one thought was true, and then investigated the situation. Students formed conjectures about the relationships between scale factors, perimeters, and areas. Visual representations of transformations and measurements provided strong feedback on whether these conjectures were correct and helped students remember the relationships. The teacher noted that when doing the square, the students knew the properties they were taught in middle school: "When we were going over them I had students present their squares, and we tested the conjectures and double checked the middle school properties."

Questioning techniques were used to engage students while they were dynamically dragging and observing measures of their constructions. Whole class discussions were held to discuss the different conjectures students formed. Students would go to the entire class and present their findings. As teachers progressed in the conjecturing process, it was clear that students had a sense of conjecturing and what role it can play. The students as well as the teachers saw the benefits of the DG tools in helping the student build conjectures and test them.

Proving

Teachers were also asked to describe any instances when they had led students through the process of proof. One of the teachers reported:

This past week, we gave them a problem with 2 triangles made with 2 parallel lines connected by 2 transversals that intersected in the middle such that the 3 sides were all congruent to their corresponding side of the opposing triangle. I then led them through the proof of showing the angles congruent by definition of vertical angles & Alternate interior angles of parallel lines. (We haven't covered the other versions triangle congruence yet only CPCTC.)

Another teacher stated:

We did verbal proofs of the four special angle types when lines are parallel. Using corresponding angles, vertical and linear pairs. I also had students verbally talk us through how to prove lines are parallel. I include

proofs with the notes of the lesson and guide the students to 80% of the time. I would love to have more time. The students are still not comfortable with writing proofs. They want it to be perfect, so they are hesitant about figuring out what the next step is.

In the process of proofs, one teacher noted that:

We continue to do proofs using the 2-column method and filling in some blanks. My students are not yet able to complete an entire proof on their own without a lot of assistances. I have led students to a proof of the angles formed by transversals and the internal angles of a triangle. I modeled the angles of the transversals for alternate interior and same side interior and let the students complete the proofs for alternate exterior and same side exterior. We just started triangle congruence. We will be generating proofs within the next week.

At this point, it became apparent that a number of teachers were not sure what constituted a proof as opposed to testing a conjecture. By dragging and animating GSP, some teachers thought that constituted a proof.

In the middle of the implementation, one of the teachers noted:

Proving exterior angle measure of any polygon the students constructed three different polygons with different sides one was pentagon, the other was a triangle and the last was an octagon using the measuring tool they found the measure of each exterior angle and made the conjecture that it was always 360 even when dragging the vertex of the polygon. Students were led through the proof for Sum of the Angles of a Polygon - both interior and exterior. They were also shown a proof that justified how to determine which angle measures would work to create a regular polygon.

The teacher also noted that students proved the sum of the interior angles of a triangle with prompts and assistance from the teacher. They also proved the exterior angle theorem using the sum of the interior angle theorem and the linear pair theorem. In addition, they used the triangle congruence theorems and postulates to prove triangles were congruent. The teacher reported that they did the process of proof with paper and pencil on angles that were congruent based on the theorems of pairs of angles that were either congruent or supplementary. This was not a formal proof, but more of a paragraph proof of how/why the angles were equal or supplementary. The teacher and students went through the process of proof with the midpoint quadrilateral lab proving why they were always parallelograms.

A number of teachers noted that they haven't had proofs they have just tested conjectures. "I teach the very low-level students and we are still building on getting them to make connections. I have stopped doing proofs in this class since they are doing TAKS (Texas Assessment of Knowledge and Skills) and no proofs are required." Teachers noted that they have constantly lead the students through proofs and encourage them to attempt proofs on their own. In general, several of the teachers at this point of implementation did minimal proofs.

Towards the end of the implementation, teachers noted that they attempted to ask students to prove the sum of the interior angles of a triangle using parallel lines and transversal and angle addition postulate as one of them responded:

As a whole class, we made proofs of similar figures and their corresponding sides ratios. The student GSP technicians were able to use the mathematical notation to write two-column proof with the input of fellow students and teacher prompts. Through proof, students were able to create rules for the effects of scale factor on linear measures and area.

One teacher noted that students were asked to prove the relationship of opposite angles in parallelograms, rectangles, squares, rhombuses and kites by showing that they were congruent. This was done using the properties of the special quadrilateral and congruent triangles. Students were, reportedly, still struggling with completing the proofs on their own:

Students had to prove similar figures using the GSP curriculum. This proof was then discussed as a whole class and brought to a consensus. The students were able to use the mathematical notation to write either paragraph or two-column proofs using GSP. Help was given with the input of other classmates and teacher prompts. Students were asked to prove that the opposite angles of most special Quads were congruent. This was accomplished by using the properties of the parallelogram and congruent triangles.

Many of the teachers noted that they did not do any proofs simply because it was not required in the curriculum and also because they needed to spend more time on algebra to prepare students for the state exams.

Challenges of the use of DG tools

At the beginning of the implementation process, teachers were asked to describe any challenges they have experienced and any concerns they might have regarding the use of DG tools. Lack of or limited lab accessibility was a challenge as noted by at least one of the teachers:

The only challenges I see are getting lab time and trying to fit more lab time in with the new curriculum without getting too far behind. In my experience, I have found that students cannot use the GSP activities as individual lessons. They are great as either an engagement or enrichment piece. The availability of the computer lab, and getting the notebooks in the classroom can be a plus in implementing the software. We don't seem to have enough time in one period and sometimes I only have the one day reserved. (Sometimes I don't feel like I can get to every student to make sure they are understanding everything).

Other challenges that the teachers pointed out included some students' inability to log on to a computer because they forgot their passwords and computer malfunctioning. In that respect, teachers also noted that even with correct student passwords, computers took too long to boot up — hence interfering with instructional time. In some cases, it took the technology person a long time to install DG software in the lab, but school principals were very cooperative. Teachers pointed out that varying level of computer literacy was a challenge. For their lower level students, teachers consistently noted that DG was a great demonstration tool, but the learning curve for these students was steep and often got in the way of the learning process as they struggled with the software itself and had little time to focus on the mathematics. "TIME TIME TIME I don't have the time to wait for the students to feel forced into coming up with their own ideas through exploration. I REALLY wish I did."

In the middle of the implementation process, the teachers noted that lab accessibility continued to be a challenge. Many responses were similar to the following one:

I would love to be in a lab every day so that students could explore and use DG for all topics - getting labs is hard - we already monopolize them every Monday for the Geometry classes. Right now, it is just a matter of fitting in lab times.

Classroom management was noted to be a problem in that students did not follow verbal or written instructions. Computers had to be hand-delivered to students and then collected 5 minutes prior to the end of class to prevent loss or further damage to computers. As a result, there was less than 45 minutes

for computer startup and shutdown and classroom explorations/investigations. If other activities were needed, there was not enough time to complete most assignments in one class period, and the curriculum does not allow further exploration to achieve desired results. District technology networks were also noted to be an issue:

Over the last couple of weeks, the district network has been VERY slow, and the students are having trouble logging on. Once they do get logged on, again, I am having trouble getting them to take what we are doing seriously. They don't want to explore for answers they want to be given answers. Unfortunately, with the time restrictions, I can't really wait them out like I would want to.

Time was often noted to be a big issue in incorporating GSP lessons with the required curriculum. "I have been so tight on keeping up with the calendar that I have not been able to use the lab the way I should. The biggest challenge has been that the EOC scope and sequence did not lend itself to using DG very much this time."

Using GSP effectively was a challenge because creating excellent investigations and learning to write investigations clearly takes some practice, and anticipating false results, or diagnosing them when they occur, can be a daunting task. As one teacher noted:

One concern I came across was working with the Triangle Inequality lab. Students were able to construct and measure to find that the sum of the lengths of two sides of a triangle were apparently equal to the length of the third side. I realize this isn't possible, but due to rounding, it appeared so and confused a few students.

Towards the end of the implementation, there was still a sense that time was a major setback in the process. As one teacher noted:

I have totally bought into using the DG tools and exploration to introduce the topics. Overall I love DG. However, I had started the parallelogram section with the intent of continuing with area and such. But then our Geometry calendar got redone and I couldn't get back on track with the DG. There is a need for more time to implement the fullness of students' creating their own proofs to their own constructions independently. Time is always a factor plus I have to follow the curriculum that the district provides. This could be done if students had access anytime to the program, such as a classroom set of laptops or a computer lab for the mathematics department. The more time we have in the lab, the less challenges we see, as students are more comfortable with the DG tools.

The teachers, in general, reported that they enjoy using the DG tools. But, unfortunately, they also noted that the content they need to teach and the time they have to teach that content do not give them the luxury of fully utilizing the DG technology. "I would like to have one-half day to ensure students have the necessary DG skills to explore the next day. Then allow them more freedom to play with the sketch and see what properties they can find."

Teachers also noted that the software, curriculum, and what is expected by the school and the district often get in the way in the implementation process. "We need to give the same quizzes and tests. You cannot add this exploration into the current curriculum. It is wonderful and does, I believe, improve learning, but the focus of the curriculum must change to benefit really the students." Some teachers noted that they have been directed by the administration to begin reviewing for TAKS (Texas Assessment of Knowledge and Skills) beginning in February.

Challenges include NOT having a classroom set of laptops or a computer lab specifically for the mathematics department. Also time factors and the need for students to be able to write proofs independently, instead of doing so dependent on the whole class to write proofs collaboratively.

Some teachers noted that they had increasingly become attracted to the concept of having students use the DG tools to familiarize them with new concepts and explore ideas before presenting them to class. While these teachers encountered some challenges, the students also encountered a number of issues while learning the DG tools. Many teachers were able to work around these problems, but there were remaining issues that were beyond the teachers' control and remain unresolved. A major concern voiced by principals was that this kind of technology may take away instruction time and therefore not improve test scores and then the principal catches hell and loses his/her job. Furthermore, there was a constant discussion between the principals and the lead teachers about the contribution of the software and its impact on test scores.

DISCUSSION

The introduction of dynamic geometry software provides an avenue of changing how geometry has been taught for a long time. Geometry in the past has been taught with paper and pencil. The DG tools provide opportunities for students to investigate properties of geometric figures intuitively and deductively (Olive, 1998) by exploring, conjecturing, and proving. Research findings from other studies indicate "students instructed in geometry with computers often score significantly higher than those having just classroom instruction" (Clements, 2003, p. 156). In the data collection and analysis process during the DG implementation, the theoretical framework (Bennison & Goos, 2010; Goos, 2008) highlighted the complexities of implementing the technology. Their framework emphasizes that integrating technology is complex and that context plays a significant role in technology integration. The concept of the zone of proximal development factors highlighted in the social-cultural framework such as access, student behavior, knowledge of the teacher and students, colleague administrative support, etc. promotes the zone of free movement.

Findings from the study indicated that most teachers made minimal use of computers but as time went on, many teachers were increasing their computer usage. In responding to how teachers guided students through the process of exploring, conjecturing, and proving, the data indicated that teachers might tell students exactly what to explore or students might choose what to explore, but they needed to know how to construct the figure correctly. There was an indication that students used the dragging utility in their process of exploration in various ways — some students performed random dragging with the aim of making sure the figure was correctly constructed. Others were dragging as a way to "play" with the figure without any aim in mind. In such cases, there was an indication that some teachers used questioning techniques to make the students stay more focused. Another form of dragging was dragging for patterns. This level of investigation was not reported much until the middle and end of the implementation process. Students were not only dragging for a pattern; they were also making use of the measuring tools of DG. There was a general consensus that students were involved in constructing, dragging and measuring, but the animation was done minimally. Additionally, students used

transformations more towards the end of the implementation process. The data suggest that students did not know what conjecture meant but when the concept was explained and explored; they knew what it was and even used it in the mathematical investigation.

In terms of proofs, there was a sense that students thought that testing conjectures was a proof — that dragging several times and coming with the same result was adequate for proof. As Clements (2003) points out, for a proof effort to be considered a mathematical proof, it must have both inductive and deductive reasoning. Other studies indicate that the concept of proof is lacking not only for students but also for the teachers (Pandiscio, 2002). A number of students in those studies did not see the need for proving the conjecture because there was overwhelming evidence by the dynamic geometry software that it was always true.

It should be noted that dynamic software is there to improve but not to replace deductive proof (de Villiers, 1999). To help students hone their deductive proof skills, teachers need to be encouraged to ask the “why” questions (de Villiers, 1999). Using dynamic geometry software can convince both students and teachers that a given investigation is true but does not explain why it is true. In general, most students seldom did proofs. There was a sense that students thought that testing conjectures was a proof in itself and that just the use of the DG software was a valid proof. Teachers also noted that because proving was not a curriculum requirement, neither the master teacher nor the principal encouraged exploring that concept. The alignment between the curriculum and the state assessment greatly impacted the value placed on proofs and what was covered (or not) during instruction. The exclusion of proofs from the curriculum has adversely affected the teaching of proofs and their importance. As Hanna (1995) notes proof exclusion is not a good strategy, “those who would insist upon the total exclusion of formal methods, however, run the risk of creating a curriculum unreflective of the richness of current mathematical practice. In doing so, they would also deny to teachers and students accepted methods of justification” (p. 46).

During the implementation process, many teachers faced computer related obstacles such as lack of access, lack of properly functioning computers, missing software, and slow computers. A key issue that was noted by teachers was the extra time required to teach with the use of technology and the feeling that this extra time could be better utilized to cover the syllabus in a traditional non-technology method. A final concern raised was that the school administration encouraged the teachers to use the time for geometry lesson to teach algebra instead, as geometry is not a part of the State test. Both of these issues hampered the teaching of geometry and the use of technology during the instructional process.

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CURRICULUM, STUDENT ENGAGEMENT & LEARNING TO TRUST: TWO CASES OF DYNAMIC GEOMETRY IMPLEMENTATION

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Abstract:

The paper reports on two cases of teachers' dynamic geometry implementation during a multi-year project that included student learning measures, classroom observations, teachers' self-reports, and interviews. In both cases the teachers' students exhibited learning gains although each teacher came to settle at two very different approaches to implementation. We speculate that their journeys and outcomes are largely related to the resources and constraints embedded in their teaching context, specifically related to campus and district features as well in terms of how much agency each exerted in challenging district and campus constraints.

Keywords:

Dynamic geometry, geometry instruction, professional development, conjecturing, proving, student engagement

Introduction

This is the story of two high school geometry teachers who each took a different path for implementing dynamic geometry in their classrooms. The story is told through two years of mixed methods research and supporting data embedded in an NSF-funded, large project exploring students' outcomes in randomly assigned classrooms to either use or not use dynamic geometry software (Author). Through their journeys we hope to illuminate the successes they faced in terms of student growth, the shifts they made in implementing dynamic geometry into their practice, and their interpretations of their own agency related to the types of classrooms and districts they taught in as well as their how they were situated personally in their school contexts.

As mentioned, this paper arose from a larger research project attempting to account for student learning gains that might be attributable to the use of dynamic geometry (DG) software, specifically the Geometer's Sketchpad (GSP) (Jackiw 2009) in this case. More details will be provided in the methods section, but largely the project worked with several school districts in and around a large urban city in the southwestern United States. More than 60 teachers were randomly assigned into either the DG or non-DG (control) groups and used (or did not use) GSP for two years. Those teachers' students were given an entering geometry exam based on material a student would have been expected to have known as entering high school geometry students, representing up to typical 8th grade material. These were used as measures to verify that students in the DG and non-DG groups were similarly prepared. Then at the end of each year the students took a second, exiting geometry exam based on material they would have been expected to know by the end of a high school geometry course. These exiting scores were then compared across DG and non-DG classrooms and determined that students in DG environments outperformed those not in DG environments (see Author for detailed results). In that work [Author] and colleagues found that the section level (for example regular and advanced track) as well as the teachers' years of experience were important variables. Teachers with fewer years experience who were teaching Regular sections of geometry (to mostly 10th grade students who were between 15-16 years old, typically) had students who performed better on their posttests; and teachers with more years of experience who were teaching in Advanced sections (to mostly 9th grade students who were 13-14 years old) had students who performed better on their posttests.

Given the above findings relating the track of the class section and the teachers' years of experience, we set out to learn more about teachers' implementation across these variables. Along with student achievement data, the project had teachers keep and submit self-reports of their teaching activities, and subsets of teachers were observed for fidelity purposes. We use a case study approach to answer our research question:

1. How do teachers with differing years' experience and teaching in different tracks implement the DG approach?
2. How do their differing contexts play a role in how they implement the DG approach?

The two teachers in this case study reflect two successful implementations of DG (that match the conditions found in the quantitative aspect of the project) but came to do so in very different ways. Maria, who was a new teacher working in a Regular section, jumped in full throttle. Her teaching reflected the project's goals (as will be described) right from the start but began to confront obstacles that challenged her resolve and ultimately led to a modified approach as she tried to balance the constraints of her teaching context with her goals for students. Carla, a veteran teacher and her department's chair who taught in an Advanced track section, took a more hesitant route and slowly built up across two years to implementing as the project had intended. She found her teaching context far more supportive with fewer barriers than did Maria. We will return to the details of their implementation and contexts in upcoming sections.

Background Literature:

Education researchers have been interested in the possible power of DG software such as *GSP* and *Cabri Geometry* (Laborde & Bellemain 2005) for quite some time. Dixon (1997) reported that students experiencing DG instruction significantly outperformed students experiencing a traditional environment on content related to geometric reflections, rotations, and two-dimensional visualization. Findings from Almeqdadi (2000) and Myers (2009) revealed that students in a DG-group compared to students in a traditional setting had significant benefits and DG software held potential to close gender and SES achievement gaps. In a study conducted by Hannafin, Burruss, & Little (2001), findings centered around two main themes: issues of power and learning. While the teacher had difficulty relinquishing control of the learning environment, students liked the new freedom encouraged by the DG instruction, worked hard, and expressed greater interest in the subject material. Vincent (2005) found that the dynamic visualization offered by the DG software motivated and fostered conjecturing and intense argumentation; and that the teacher's intervention—prompting the students to furnish valid justifications for their statements—was an important feature of the students' argumentations. Sinclair et al. (2009) argued that “the very nature of dynamic mathematical representations—being intrinsically temporal, occurring over time—offer very different opportunities for narrative thinking than do static diagrams and pictures traditionally available to learners” (p.441).

The most prominent feature of DG involves “dragging”, or the ability to take a representative construction of some geometric object or objects in relation to one another and “drag” points and lines around the screen as students observe variant and invariant behaviors, measures, and so forth (Goldenberg & Cuoco 1998). This allows students to notice patterns, generate conjectures, and quickly test those conjectures for a wide example-space (Kaput 1992). There have been a wide variety of reactions, though, to using DG in proof-based activities with some researchers claiming that the powerful “drag” and measurement features can leave students reasonably secure in their conjectures without feeling the need to advance to a proof stage (Oner 2009); others, including deVillier (1998 2003) argue that experiencing geometric relationships inductively does not eliminate the need for a proof so much as bring forth more ways to think about the role of proving in mathematics and mathematics education, such as to build explanations of phenomena as well as confirm the truth of the observed relationships. Jones (2000) suggested that “the students' explanations can evolve from imprecise, 'everyday' expressions, through reasoning that is overly mediated by the software

environment, to mathematical explanations of the geometric situation ... This latter stage ... should help to provide a foundation on which to build further notions of deductive reasoning in mathematics" (p. 55). Hollebrands (2007) identified different purposes for which students used dragging and measuring when exploring in GSP environments. These purposes appeared to be influenced by students' mathematical understandings that were reflected in how they conducted reasoning with physical representations, the types of abstractions (empirical abstractions, pseudo-empirical abstractions, or reflective abstractions) they made, and the reactive or proactive strategies they employed.

As Kasten and Sinclair (2009) report, teachers find GSP to be "the most valuable software for students" (from Becker, Ravitz and Wong 1999) but "[l]ittle is known about what teachers find "valuable" about *Sketchpad*, or how teachers choose, on their own accord, over the course of the school year, to implement it." Although randomly assigned to the treatment (DG) group, the two teachers in our study did volunteer with the understanding that they *might* be in the DG group and because they did use it over two years of instruction, we believe this case study could illuminate how high school geometry teachers might implement DG, to complement Kasten and Sinclair's study of elementary grade teachers.

Project Philosophy

The Dynamic Geometry Project, from which this research stems, promoted an approach to geometry instruction commonly advocated for in reform/technology enhanced (such as with DG) classrooms (e.g. NCTM 2000; CCSSI 2010). The project included a three-part framework for students' experiences in geometry along with professional development for the project teachers, encouraging them to use DG twice a week and to include three processes often mentioned in DG-related literature—Exploration, Conjecturing, and Proving—in each DG lesson (although some lessons might take more than one day to complete the cycle). Largely, these practices are considered "reform" in as much as they deemphasize rote learning of theorems and their application in simple problem exercises and favor a more in-depth problem solving approach to curriculum. This is, of course, not to suggest that there is no room for repetition, but rather that students develop a better conceptual map of important advanced mathematics, and geometry specifically, by engaging in the exploration-conjecturing-proving (ECP) cycle. The exploration aspect of the cycle promotes student engagement and curiosity about geometric phenomena. The conjecturing phase attempts to focus students' explorations on specific observations and patterns and to articulate these observations as precise statements involving an "if/then" structure. The final proving phase seeks to introduce and strengthen classic deductive reasoning through careful consideration of the construction "givens" and resulting related network of relationships between the objects from the exploration phase.

Finally, it might be helpful to understand the sorts of professional development (PD) and lesson resources made available to the project teachers. All teachers received PD appropriate to their group, where the primary distinction was that the control teachers' sessions were absent the technology component and instead focused on tools such as patty paper, for example. Both groups had the same number of days and hours spaced approximately the same throughout the two years. Because this paper focuses on two of the DG teachers, we will describe their PD in more detail. All DG teachers had an

initial full-week summer institute, six Saturday follow-up PD sessions during the first school year, a one-day workshop the second summer, and four Saturday PD sessions during the second school year. In these PD meetings teachers worked through problem sets and activities they might elect to use with their students. As teachers became more familiar with DG (particularly in the Saturday follow-up PD sessions), they began to lead the sessions, sharing activities they had created or walking other teachers through some of the lessons they had found or adapted from other sources. They were also given a document prepared by the team with a collection of classroom activities and tasks that were adapted from available resources, pulled from a variety of published research outlets, and available online. They were also provided with access to the *Sketchpad Lesson Link* published by Key Curriculum Press. Finally, the teachers operated an online sharing site, just for themselves, where they also shared materials amongst one another.

Methodology and Data

Teachers' Contexts

Both teachers worked in schools in districts in and around a cluster of large metropolitan cities in the southwestern United States. Maria taught in a district that was a suburb of one of the urban areas and Carla taught in a district between the urban areas that was rapidly changing from rural to suburban as both cities expanded toward her town. Maria was a second year teacher and had prior knowledge of GSP coming into the project and was working in a classroom with enough computers that she did not need to take her students to a campus lab. She taught regular track courses, which means that students enrolled in her DG course were primarily 10th graders (15-16 years old) and had average to lower past state achievement test scores. Carla was a 22-year veteran teacher and also served as her department's chair, but did not express knowledge of GSP entering the project. Carla did not have computers in her classroom and used the computer lab a few doors down the hall from her classroom. She taught an advanced track course of mostly 9th grade students (14-15 years old) who had scored higher than average on past state achievement tests.

Data Sources

We have already discussed some aspects of the data sources in the overview of the project, but we will further describe these now. In addition to each teacher's students taking the Entering and Exiting Geometry tests for Years 1 and 2 of the project, we also collected qualitative data—self reports, classroom observations, and exit- interviews.

Each teacher in the DG group was asked to submit monthly reports of their DG activities from the start of the year through mid-spring when the state exam schedule began. Each of the two case study teachers completed each report by responding to a Dynamic Geometry Implementation Questionnaire (Authors) for both years. The questionnaire asked teachers to record the number of days spent with students using GSP each week, requesting the names/sources/content of the activities used, how the students had engaged in activities related to exploring, conjecturing, and proving, what evidence teachers felt they had of student learning via formative assessment, what challenges they faced, and other such implementation questions as “Describe your comfort level (level of ease) of the use of DG technology.”

Both Maria and Carla were among a subset of teachers who were also observed during the project. For each teacher selected for observations during a school year, two project team members visited the classroom up to 5 times with the average closer to 3 visits each. While there, each research team member took notes, later filled out a Geometry Teaching Observation Protocol (GTOP, Authors), modeled after the Reform Teaching Observation Protocol; (Piburn & Sawada 2002) and then met with their observation partner to come to consensus about the ratings. The GTOP included items scaled from 0 to 4 with 0 meaning a feature was absent and 4 meaning it was quite prominent in the lesson. The instrument included indicators for aspects of the lesson that were specific to GSP software such as whether students used the various tools of dragging, measuring, transforming and so forth as well as features less specific to the software and more specific to what we call the DG approach. These included items about exploring, conjecturing, and proving. Other items related to how engaged students were, whether they worked together, whether the task was cognitively demanding, and other engaging practices. A high average score indicated that the students did much of the work, explored, conjectured, proved, and otherwise participated in a student-centered inquiry style lesson. A low average score documented that the students did not do much of the work on the computer (might have been given pre-made sketches, for example) and/or that the teacher did most of the thinking for the students such as coming around to each student telling them what to notice or write down. Maria and Carla were observed five and four times, respectively, during Year 1 (i.e., the first year of DG implementation) of the study. Maria was observed again twice in Year 2 before requesting a break from observations. Carla was not observed in Year 2 (i.e., the second year of DG implementation) due to scheduling conflicts between the time that her DG course was being offered and between the researchers' own teaching schedule. At the time we were deciding which teachers to observe in Year 2 we did not know that Carla would later be an excellent candidate for such a case study as this. After consideration of the self-report data during analysis, we later contacted Carla and learned that she was still using the DG approach in her classes, even though the data collection portion of the DG project had ended. She, nevertheless, granted permission to observe her classroom once again (what would have been late spring of Year 3 had the project continued). To summarize, Maria had five observations in Year 1 and two in Year 2. Carla had four observations in Year 1 and one in follow-up (hypothetical Year 3 had the project included a 3rd year of implementation).

Finally, both Carla and Maria were contacted at the end of the year following the project's completion (what would have been Year 3) to get a sense of if and how they were still using the DG approach with students as well as their general impressions.

Analysis

The analysis phase consisted of reading Carla and Maria's Year 1 self-reports in their entirety. During this phase the first author looked specifically at the self-reports related to the number (and content of) each activity during each report, the challenges faced and their comments about student learning. A general description of each teacher's implementation was crafted. Next, the observation report ratings were examined against the self-reports. This repeated again for Year 2's data. The first author took the emerging narratives to a team meeting where the whole group reviewed her claims and confirmed her general analysis.

Overall, the analysis phase consisted of examining the frequency of student interactions with GSP, the nature of each teachers' lessons with respect to exploring, conjecturing, and proving, and comments by each teacher related to student learning and struggles with implementation (all based on the monthly self-report questionnaires). The observation reports were used to confirm what the teachers were saying in their self-reports. For example, if in the self-reports, a teacher described her practice as following a particular pattern, the observation data was compared with these patterns. The exit interviews were also used to triangulate the teachers' overall Year 1 and Year 2 data and to see if the teacher's overall impression of their two-year journey seemed consistent with the analysis of the other data. It is on these areas that the upcoming sections will focus.

The Cases

Maria—Prevailing via Compromise over Student and Curriculum Resistance

Maria's reports, and observations described a teacher who strongly believed that students need to be learning important thought processes rather than only facts, believed the DG approach can facilitate this, but struggled with student engagement with critical thinking, a strict district curriculum calendar, state testing disruptions, and colleagues that did not share her views.

Maria began the first year rating herself as Low on implementation and attributed the rating to the slow start of getting the software installed on her computers, but also introduced a theme that would continue—student interest in exploration versus their desire to take notes passively. As the months progressed she continued to describe episodes of student resistance, not to using technology per se, but resistance to the thought processes required to use GSP as a discovery and conjecturing tool.

I am not having a lot of success having the students test their conjectures. We cannot really discuss their ideas at the computers because I don't seem to get their attention. So, we come back to our desks to discuss ideas and I wish they were still at their computers to test and drag. I think next time I am going to have students come to the front and test our ideas as we discuss them. Again time is never enough. (November, Year 1)

[The project] has clearly shown me how students really don't learn very much from a lecture. They have no real understanding of the situation. That is apparent in their explorations. However, lecture and exploration together (sometimes in that order - sometimes not) seems to improve connection.... Today students came back from spring break and asked what we were doing today. I said lecturing. They said good - that's easier. I took that as a good sign that their explorations are requiring more thinking than just taking down notes. (March, Year 1)

One thing that was striking was her perseverance to continue to include exploration and conjecturing throughout the weeks despite the general sense that students would prefer she simply lecture. We could see from the first report that she believed exploration to be an important goal for them. Through the continued reports she described modifications to get them more comfortable with this. She praised efforts even when she felt they were not very insightful. She provided direct instruction on how to explore, she transitioned between computer and "class" time in an attempt to focus their attention on the conjectures, and she required written summaries for submission as a grade.

She also experimented with quizzes on the computer where students used GSP as a tool, had students present their work to each other, shared screen shots of student work with the class, combined lecture and GSP exploration, and set up the explorations as mini-mysteries to be solved.

There was a strong sense of frustration with her students throughout the reports. She mentioned successes but in a back-hand way—that the students acknowledge that lecture was easier was taken by her to mean that exploration on GSP was working—it was making them think. At one point in her November report she noted: “98% of them really enjoy working with it. 40% of them effectively use it to test, drag and really learn about the concepts.” Overall, it seemed she did not get resistance to using the computers but to the mental activity DG requires. She attributed much of this to their “training” or past experiences in mathematics classes. She felt that they had a false sense of what it means to do mathematics and that it was a struggle to keep them going on this particular path where mathematics is based on understanding the relationships between figures and not “1 right answer”. By the last report (roughly 27 weeks into the school year) not much had changed. She continued to name the same primary struggles and note the same concerns about student engagement. This case was similar to what Skemp (1976) pointed out in his famous paper “Relational Understanding and Instrumental Understanding”: a teacher was trying to teach relationally, but students had been taught instrumentally in the past and preferred to “learn” instrumentally.

In addition to the comments about student engagement and learning, themes of not having enough time given the curriculum pacing and the required formal state-related testing cycles emerged. This realization seemed to coincide with both the timing of the various benchmark, practice, and actual tests for the state as well as with her more frequent uses of the software. There was a dilemma between how often she felt the class needed to be using the software, how intense they should use it, and when to find the time to do so.

.... I am feeling frustrated with the speed of the curriculum that is on the calendar. I don't feel like the students have time to process an exploration, think about what is happening and propose then test conjectures. (November, Year 1)

Since the last session I have only stressed about how to include DG in a difficult schedule of testing. (January, Year 1)

She felt that the exploration and conjecturing processes were important to her students’ learning but also struggled to orchestrate lessons where, within 50 minutes, students could engage with GSP enough to build conjectures, have them also share those conjectures, possibly return back to their explorations, and complete a class discussion/consensus on the main ideas. She expressed frustration that were she to take longer then she might run the risk of falling behind on the district’s strict daily calendar of topics.

During Year 2 Maria’s same themes emerged with a new set of students. Overall, student engagement was hard to enlist but seemed to improve as she implemented some new strategies, and again, the district’s calendar seemed at odds with her goals for frequency and depth of student engagement with GSP. Almost every aspect reported from Year 1 repeated, but more intensely. Maria’s comments about students and the calendar became more urgent and she seemed more exhausted by it all.

In Year 1 Maria reported that about 41% of the reported weeks included a GSP activity. In the second year this dropped to 29.6% of the weeks involving GSP work. She did comment in the last few months of Year 2 that she had switched to using the software as a teacher-led demonstration tool (via a shared projector) more so than as student-led activities and it is unclear whether she counted these in her reports and if not, this might account for the sharp decrease from Year 1 to Year 2.

Maria may not have used GSP on many days or with many topics, but on those days observed she scored very well on the GTOP observation instrument. Maria was observed five times in Year 1 and twice in Year 2. As was described earlier in the instrument description, a high score in general meant that the students were actively engaged with the software, explored, shared, conjectured, and built toward rationale(s) if not an exact proof of the conjecture(s). A low score generally implied that the teacher, rather than the students, did much of the intellectual work and/or the lesson task was not particularly inquiry based. Maria scored higher than anyone else in the project (DG or Control) with an overall average score of 3.06 across Year 1 from the following scores in order of observation: 2.76, 2.88, 3.22, 3.06, and 3.36. Her Year 2 observations were also very high at 3.04 and 3.88. Recall that after two observations she requested that the team no longer observe her class, citing a general need for less distraction from the research team.

Figures 1 and 2 below show how Maria's Year 1 and Year 2 students clustered on scores for the Entering and Exiting tests. The Low category, for example, describes what percentage of her students scored in the lowest quartile compared to all students in the regular track classrooms across the project. The Mid cluster represents those scoring in the two middle quartiles, and the Upper is the percentage of her students scoring in the upper quartile.

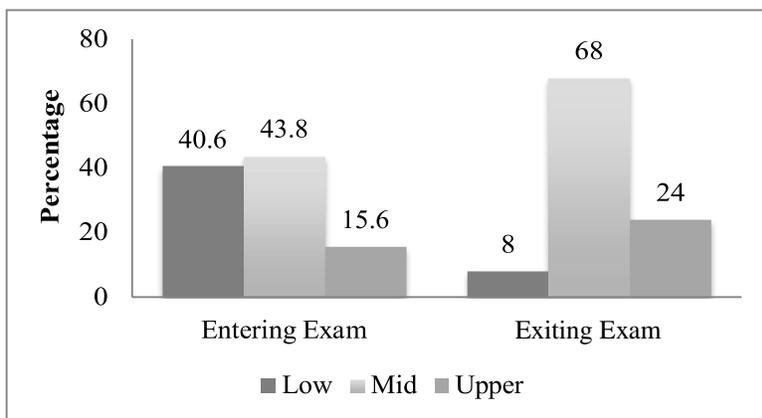


Figure 1: Year 1 Percentage of Maria's Students in the Lowest, Mid, and Upper Quartiles on the Entering (8th grade) Geometry Exam and Exiting (10th grade) Geometry Exam, Relative to Regular Track Students

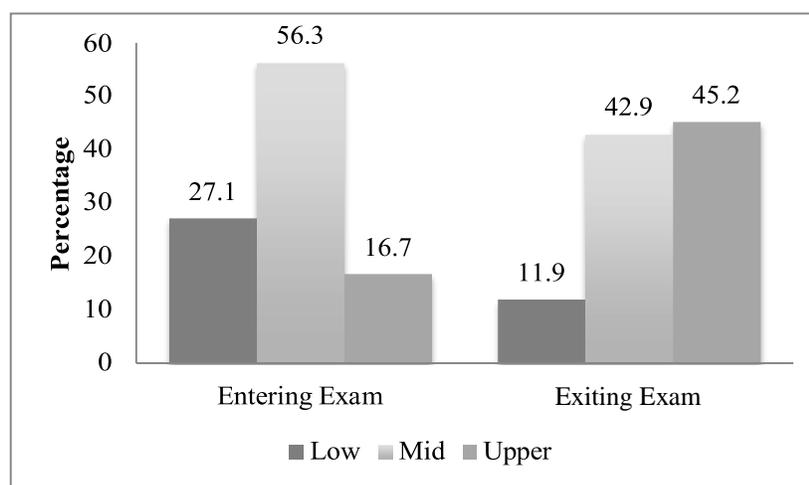


Figure 2: Year 2 Percentage of Maria’s Students in the Lowest, Mid, and Upper Quartiles on the Entering (8th grade) Geometry Exam and Exiting (10th grade) Geometry Exam, Respective to Regular Track Students

Despite her challenges with student engagement and curriculum/timing conflicts, Maria, across the two years, made consistent reference to believing that the software helped students learn. She expressed amazement that students seemed to get so little out of lectures, that they did not seem to “truly understand” what they were doing and why. She stated that the visual aspects of the software really helped and explained this is why she made the change to teacher-led demonstrations with the software—so they could at least “see” the results even if she felt she could not take the class time to have them explore on their own.

In communication with Maria the following school year, after the formal end of the large-scale data collection of the project, she was asked if and how she was using the software with her students now that she was no longer “required” to do so. She stated that she’d been moved into a new classroom, no longer having the computers in her room, but she continued to use GSP as a demonstration tool frequently. She also noted that the room change had moved her closer to more colleagues in the math department and that this, and increased state testing demands, had actually made her feel more pressure to conform to the district calendar. She stated holding her colleagues in high regard as good teachers but they were not a group open to change.

Overall Maria was new to teaching at the start of the project and very willing to try new things with her students and had an excellent knowledge of the software program itself but struggled to implement it in weekly lessons where students were doing the constructing and exploring work. She found ways to streamline the process by changing grading incentives, giving motivating introductory remarks, and reassuring students that risk-taking was good for their learning. Ultimately though, she found implementing two GSP lab lessons a week impossible, given the district’s tight calendar of topics and testing interruptions. To accommodate for this limited use of GSP she shifted to a more teacher-led approach, hoping to capitalize on student interest in the visual aspects of the animations she crafted while engaging them in whole class discussions of conjectures. Her story is not new, sadly, or unique.

There are many tales of new teachers tackling new teaching practices and meeting resistance along the way. Trying to engage students in new ways of learning is rarely easy or straightforward and

is not aided by inflexible curriculum or colleagues who do not share the same vision. Despite her struggles Maria's students did excel. At the start of Year 2 she had about 27% of her students testing in the lowest category on the Entering Geometry test and improved this to only 11.9% in that category by the end of the year. And while she began with 16.7% of students testing in the upper category, she ended with 45.2%. Her strategies of compromise seemed to be working in terms of her professional relationships with her colleagues and for her students' learning. Sometimes not quite good enough is still quite good indeed.

Carla—Learning to Trust Students

Carla, we shall see, was also committed to using GSP but had supports Maria did not, such as a principal who reserved the lab for her once a week for the whole year. But her implementation did not match the project's overall intent at first. She was more likely to use GSP in the lab as a verification tool after first teaching the lessons in the classroom. During Year 2 this changed and she began to trust her students to do the explorations on their own without first having been "taught" the material. Both her commitment and her fidelity of implementation grew as the project continued.

Carla did not mention many difficulties implementing DG in her class. She pointed out from time to time that students struggled with a particular activity but that overall they were using it, enjoying it, and also learning. In the October Year 1 report she noticed difficulties for students that do not construct, but rather draw their figures using the free-hand tools and how their resulting geometric relationships and measures did not work. This problem was quickly resolved in that it did not get mentioned again across her two years of reports. The most consistent thing she noted that could be termed a challenge was her own willingness to let go of teacher-led instructional habits and these comments repeated through Year 1.

My lack of wanting to change the routine is still holding me back a little. I need to trust my students more and let them learn by experimenting with GSP and all it can do.... I feel trapped in my old ways but would really like to be more tech savvy and let students lead in the learning more as they really like it! (November Year 1)

I use DG as a demonstration almost every time I introduce new material to the students. I have also begun to let students explore concepts on intro day to see if they can come up with the theorems using DG on their own. This has been the most rewarding. They remember the facts if they discover them! (March, Year 1)

Overall in Year 1, Carla used DG multiple times a week and reserved every Monday for going to the lab. But students did not always explore an idea first in GSP. Her usage was primarily as a verification tool after they had a classroom lesson on the material, despite her usage of the term "explore" to describe that work. That was misleading and in contrast to how the project, and she too by March, used the term "explore". In October of Year 1, Carla mentioned that she was starting to look for lessons where students are introduced to the material via DG rather than afterward, but her comments from November through February of that same year indicated hesitance to use these types of lessons often. In March she commented that she was letting students explore on "intro" day, meaning that on their first day of a new topic students are just now being allowed to explore the idea fresh, without having had teacher-led direction on the concepts previously. This represented a big change in Carla's DG

implementation. In previous remarks when she used the term “explore” it more closely resembled visual and measurement-related verification of a theorem rather than an exploration that leads to conjectures that might become a theorem. This latter, March, usage was much more in line with the project’s philosophy—to allow students the freedom to notice patterns and relationships through discovery activities, build conjectures from those and then hopefully prove or provide justifications for those relationships. Her earlier work was backwards in a way—she presented students with a theorem and examples in class and then in the lab had students construct those relationships and observe that they are in fact true.

In the first year, Carla reported using GSP for about 62% of the weeks. Overall, she used DG activities far more frequently than Maria, but her observations report scores from Year 1 were rather low. Recall that a low score generally implied that the students were not discovering or were being led by the teacher to a large extent. Her scores were: 1.12, 1.29, 1.41, and 2. By contrast recall that Maria had scores between 2.7 and 3.36 in Year 1. So although Carla used DG materials more frequently, when observed, those lessons were not well-aligned with the project’s vision. This was supported by her self-reports of her own growing comfort with the instructional method of letting students actually explore with the program rather than only verify known results. Her GTOPI scores steadily increased as her reports likewise included increased comments related to changing her instructional style.

Whereas Maria reported conflicts with her students’ engagement and concern over the calendar—finding time to implement the full DG cycle she sees as necessary—Carla did not. Carla’s comments about students early on suggest that they were not skilled at constructing but they improved. She never mentioned district or state curricular conflict. On the other hand, Maria early on expressed a particular vision of using GSP that did not match Carla’s. Maria wanted students to explore the ideas using the technology first and then have summary discussions on the students’ conjectures and observations. The majority of Carla’s lessons were the other way around and likely avoided the time constraint issue as a result. Lecturing on the material followed by a lab activity to verify results takes less time than having students construct the learning themselves. Furthermore, being the department chair could have alleviated any concerns Carla might have faced about sanctions for getting off-calendar. And finally, we speculate that working with advanced track students, who Carla referred to as curious, interested, and on-task, might have made things move more quickly. Maria, by contrast, reported students that were not familiar with exploring and hesitant to try, preferring to be “told” answers. And finally, the structured lab days scheduled by her principal on her behalf may have provided strong motivation for Carla to take her students to the lab as often as she did.

Things changed for Carla in Year 2. At the end of Year 1 we saw that she was beginning to embrace the DG approach advocated for by the project team. By the end of the second year Carla was well on her way to implementing DG in the ways the project envisioned. She only reported conflict in terms of getting lab time—other teachers wanted to use the lab as well, not giving her the full access she preferred. Her reports indicated that she scheduled the lab every Monday for GSP activities and that she had altered her curriculum calendar so that these Mondays would be concept introduction and exploration days with the rest of the week focused on more traditional instruction leading off of these Mondays.

In Year 2 Carla reported that she used GSP for 70% of the weeks, an increase from 62% the year before.

Also, recall that in Year 1 her observation scores were low and that this was consistent with her self-reports; particularly that GSP was being used as a verification tool rather than as an exploration of new ideas. Given her statements from the Year 2 self-reports, which provided evidence that she had completely altered her teaching with GSP, we were eager to see if her observation reports captured this change. Sadly, we learned that she was not observed in Year 2 due to scheduling conflicts. With more than 60 teachers in the project and a limited research team we could not always match when a particular teacher taught a project class with our own teaching responsibilities. After realizing this, we then contacted Carla to see if she was still using DG activities even though her responsibilities to the research project had ended. We were delighted to hear that she was still using DG weekly (still every Monday) and grateful that she invited us to visit. We then visited her class one more time and the difference was immediately noticeable. Students were obviously unfamiliar with the topics (interior and exterior angle sums of polygons), worked through the constructions, investigations, and proving on their own with intermittent teacher support in the form of mini-discussions led by Carla spaced through the hour. These new observations matched her own self-reports and the two observers rated her at a 3.6 (Year 1 scores ranged between 1.12 and 2).

Figures 3 and 4 represent Carla's students' scores in Years 1 and 2, respectively, clustered by the percentage of students who scored in the lowest, middle two, and upper quartiles for all advanced track project students.

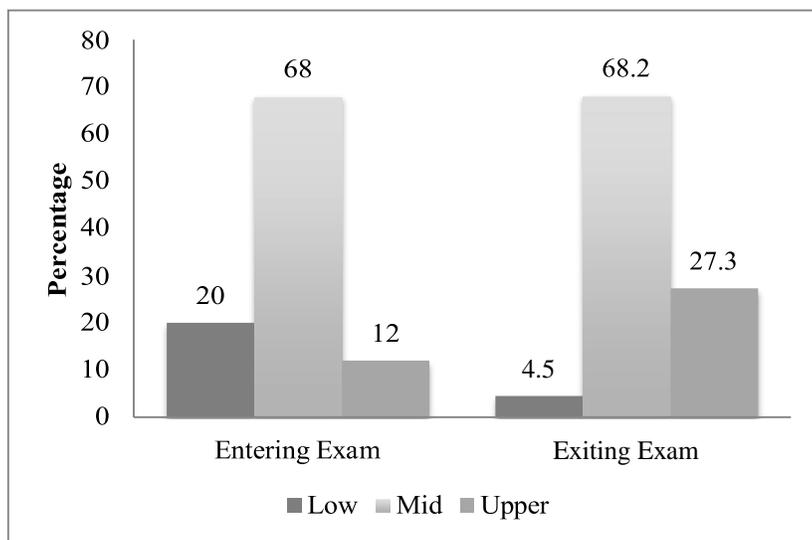


Figure 3: Year 1 Percentage of Carla's Students in the Lowest, Mid, and Upper Quartiles on the Entering (8th grade) Geometry Exam and Exiting (10th grade) Geometry Exam, Respective to Advanced Track Students

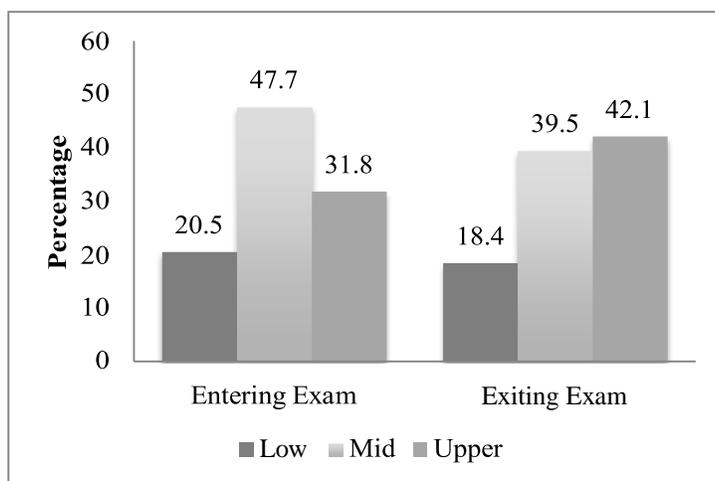


Figure 4: Year 2 Percentage of Carla's Students in the Lowest, Mid, and Upper Quartiles on the Entering (8th grade) Geometry Exam and Exiting (10th grade) Geometry Exam, Respective to Advanced Track Students

Finally, like Maria, we communicated with Carla after the project ended. She expressed that she was still using GSP (as our observation that spring confirmed) and credited her campus math department culture as supportive: a principal who reserved the lab for the geometry teachers, and her fellow teachers had also “bought into [DG].” She also noted project team PD as supporting her change in Year 1 to Year 2, particularly the PD time to share lesson ideas, the online file sharing system between project teachers, and the pre-made lessons available to use. In contrast to Maria, she found her overall school supportive rather than in conflict with her goals.

Regarding student engagement, Carla described how her students in Year 2 came up with their own conjectures: “I didn't have to present the theorems and the postulates. They could determine what was happening, and make a conjecture statement, not really prove yet, but they could make a really good statement about what was going to be true based on dragging, measuring, and exploring what happened in the drawing. I really like that”.

In summary, Carla's story was one of change once she felt comfortable with the technology and began to have faith that her students could conjecture and explore on their own before having seen the theorems. The only frustration (aside from her own comments about her hesitance to “let go”) Carla expressed related to students not constructing (drawing instead) and that at first they wanted to be told what they were supposed to notice. Neither seemed to be as persistent a problem as Maria had. Carla did not express any curriculum conflicts. She did, though, talk about changing her curriculum sequence in Year 2 so that Mondays could be used for introducing ideas that would develop in the class lectures later in the week. She did not seem to think that changing her curriculum calendar was out of the question the way Maria did. There are several possibilities for this. Carla was the department chair and had a supportive team, including her principal, whereas Maria talked on one occasion about her fellow teachers' goals not being aligned with her goals. Overall, there seemed to be differences in each teacher's sense of agency. Carla felt that she could make changes as needed to benefit her students whereas Maria felt more constrained by district policy.

In addition to her self-reports indicating a change in her approach we also saw changes in Carla's observation scores from a low of 1.12 up to a 3.6. Her students' exam score shifts were not as dramatic as Maria's students were, but a larger portion of her Year 2 students began already scoring in the upper quartile. Nevertheless, we discuss this further in the discussion section.

Discussion

The two teachers' journeys were quite different and so was how they reported their support systems and frustrations. We speculate that campus context supports and teacher agency played a strong role in these stories. At the end of Year 1 Maria enacted the DG project's goals more clearly than did Carla. Maria was working harder, it seemed, to overcome more obstacles—she was a new teacher, with less campus supports, seemingly difficult students to engage, and a diminished sense of agency for directing the curriculum calendar. Carla was doing the DG lessons more often than Maria but not at as high a level of fidelity at first. Her integration of DG was often as an add-on to her classroom lectures. As a result, Carla didn't need to deal with student disengagement to the same extent as Maria because she was not struggling to get them to construct the “aha's” for themselves—they had the results already and were verifying them at the computer. As the project's two years progressed, Carla adjusted her teaching greatly and felt that she could change her calendar as she made shifts in her practice.

In terms of the regular and advanced track contexts, we did not see anything from either context that specifically pointed to these environments as contributing to their practice. That is, neither teacher directly said something to the effect of “Because my students are in the X track, I'm dealing with Y issues.” But we did see much more frustration from Maria, who was in a regular track classroom. On the other hand, Maria was a new teacher working with a group of other teachers who did not share her goals. We cannot say how much of her difficulty was actually attributable to the class context vs. the overall non-supportive environment. Had she been working with a team of like-minded teachers on her campus, would she have framed these difficulties as a “typical” and “normal” thing to deal with rather than as a set of reasons that led to eventual decline in DG student-led work?

In Year 2 and in Year 3 follow-up we saw further hints that the isolation experienced by Maria was contributing to her change in usage. She began the year with high expectations but more quickly than before shifted away from the student-led activities. She maintained that the DG approach was needed and that even demos could create positive learning—through the visual “seeing” of the mathematics in action. She again attributed this change to student resistance and time-related challenges. The student resistance might have in part been related to her classroom context. We repeatedly heard teachers of regular track classrooms describe students as unmotivated. Were they truly less engaged? Possibly. But we also heard some of this from Carla in the advanced track class when she described the start of the semester as a struggle to get students thinking rather than waiting for her, the teacher, to declare the “point” to the activity. Did the advanced students adapt more quickly? Or did Carla's (perceived) freedom with curriculum and strong expectations by her principal allow her to “wait them out” longer? Maria expressed a desire to wait the students out, too, but felt that she didn't have this as an option. She had to keep on pace. Alternatively, Carla did not seem to

automatically assume that her advanced track students would explore or conjecture on their own. That trust developed over the first year and took root in Year 2.

At this point it might be helpful to frame Maria's and Carla's students' posttest scores within the larger project's findings. Overall, the project found that DG students outperformed non-DG students at significant levels but that the largest effect size, .45, was between regular track DG and regular track non-DG students (Author). Essentially, DG helps all students, but really helps regular track students. This seems to bear out in Maria's and Carla's students' scores. Both saw gains for their students each year but Maria's results were more dramatic, even in Year 2 when she shifted to more demo-based use of GSP. It is hard to say that the DG approach fully accounts for Maria's students' scores—maybe she's a really good teacher. Nevertheless, when examined alongside the overall project findings, there is reason to believe that Maria's story aligns well with the larger picture. Furthermore, Maria's own reports attested to her students benefiting from the processes and visual images that her DG lessons facilitated. And a good teacher certainly crafts lessons to match her students' learning needs and selects tools that aid and further those goals, as Maria did. None of this is intended to suggest that Carla is not as good a teacher. We think her reports and gains indicate the opposite. But it is the case that her students' score increases were not as striking, but again this matches well to the finding that the advanced track DG students outperform non-DG students as well, but with not as large gains.

Possibly a few targeted and in-depth DG lessons with more frequent teacher-led demos goes a long way for students who might not often be provided with opportunities to explore and experience visually interesting geometry. The opportunity to experience really thinking about geometry, being engaged in probing conversations about patterns, and formalizing these patterns into rationales and proofs seems to have mattered in how the teachers talked about their students' learning and in the students' scores. Both teachers created these opportunities for their students although their paths were different. Maybe the lesson here is that a specific dogmatic approach to “how to teach” is less important than providing the opportunity and access to engaging content and that DG offers a stimulating environment for discussions about relationships and geometric structure to unfold. And that this is of particular significance for students who may rarely be engaged in “doing” mathematics.

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**An Observation Protocol Measuring Secondary Teachers'
Implementation of Dynamic Geometry Approach***

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Abstract

Studies that measure the impact of an instructional intervention rely on measures of fidelity in the classroom, especially when the intervention involves the use of technology. To examine the extent to which teachers implement a dynamic approach to teaching geometry in secondary classrooms, we develop a 25-item observation protocol based on four dimensions which include planned and implemented dynamic geometry approach elements, quality of instruction, and engagement and discourse. We present evidence of reliability and validity of the instrument.

Keywords: Secondary; Dynamic geometry; Observation protocol; Teaching practices.

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**An Observation Protocol Measuring Secondary Teachers' Implementation of
Dynamic Geometry Approach**

1. Introduction

Since the publication of the U. S. Common Core State Standard for Mathematics (CCSSM), the nature of the teaching and learning of geometry in secondary schools has changed to explicitly include transformational geometry and the use of dynamic geometry environments to “provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

Research suggests that alternatives to traditional instructional approaches can be successful in moving students toward meaningful justification of ideas. “In these approaches, students worked cooperatively, making conjectures, resolving conflicts by presenting arguments and evidence, proving nonobvious statements, and formulating hypotheses to prove. Teachers attempted to involve students in the crucial elements of mathematical discovery and discourse” (Battista & Clements, 1995, p.50).

Dynamic geometry (DG) is an active, exploratory study of geometry carried out with the aid of interactive computer software such as the Geometers’ Sketchpad (GSP) (Jackiw, 2001). The instructional approach for using DG software to facilitate students’ learning is referred to as the DG approach in this paper. Many researchers have conducted studies on using the DG approach in geometry learning. In a study conducted by Hannafin, Burruss, & Little (2001), findings centered around two main themes: issues of power and learning. While the teacher had difficulty relinquishing control of the learning environment, students liked their new freedom encouraged by the DG instruction, worked hard, and expressed greater interest in the subject material. Vincent (2005) found that the dynamic visualization offered by the DG software motivated and fostered conjecturing and intense argumentation; and that the teacher’s intervention—prompting the students to furnish valid justifications for their statements—was an important feature of the students’ argumentations. Hollebrands (2007) identified different purposes for which students used dragging and measuring when exploring in DG environments. These purposes appeared to be influenced by students’ mathematical understandings that were reflected in how they conducted reasoning about physical representations, the types of abstractions (empirical abstractions, pseudo-empirical abstractions, or reflective abstractions) they made, and the reactive or proactive strategies they employed. Baccaglioni-Frank and Mariotti (2010) presented “a model describing some cognitive processes that can occur during the production of conjectures in dynamic geometry and that seem to be related to the use of specific dragging modalities” (p. 225) and used it to analyze students’ explorations of open problems. Thus, when used as a cognitive tool, DG technology can facilitate students’

exploration and investigation activities, promote their conjecturing, verifying, explaining, and logical reasoning abilities, and enhance their conceptual understanding of important geometric ideas.

A research project funded by the National Science Foundation (NSF) conducted repeated randomized control trials to investigate the efficacy of an approach to high school geometry that utilizes DG software to engage students in constructing mathematical ideas through experimentation, observation, data recording, conjecturing, conjecture testing, and proving. This approach was referred to as the DG approach in the project, and the DG software used by the project was GSP.

This paper describes an instrument developed by the DG project research team to measure the fidelity of DG implementation. The paper proceeds as follows. Section 2 presents background studies related to measuring teaching practices using observation protocols. Section 3 describes the development of the instrument, evidence of reliability and validity, while Section 4 concludes with a discussion.

2. Background

Many education researchers have observed school classrooms and measured their characteristics for different purposes (Shavelson, Webb and Burstein, 1986). The number of classroom observation instruments reviewed by Roshenshine and Furst (1973) and later Brophy and Good (1986) is almost as large as the number of studies reviewed – over 150. These particular instruments measured only behaviors related to teaching in general, such as pacing of instruction, classroom management, clarity and questioning the learners. More recently, observation protocols have been developed in the context of measuring teacher and teaching quality such as the Framework for Teaching (Danielson Group, 2011), the Local Systemic Change (LSC) Classroom Observation Protocol (Horizon Research, 2005) and the Classroom Assessment Scoring System (CLASS) (Pianta, La Paro, & Hamre, 2007). Although these protocols are commonly used, even in international settings, they are not designed to measure the teaching quality of a specific content domain. In response to the need to better explain the mathematical aspects as well as the mathematical pedagogy harnessed during lessons, mathematics educators have turned their attention to the development of more specific observation protocols and instruments. Two observation protocols with published validity information aligned with content specific teaching and practices are 1) the Reform Teaching Observation Protocol (RTOP) for mathematics and science (Sawada et al., 2002; Piburn et al., 2000) and 2) Mathematical Quality of Instruction (MQI) for mathematics (Hill, Charalambos, & Kraft, 2012; Hill et al., 2008; Learning Mathematics for Teaching Project, 2011). For the purpose of measuring a specific area within mathematics, Geometry, and a specific approach inherent by the content itself, the dynamic approach, we found the above protocols informative but not adequate. Next, we describe how we adapted the RTOP and

MQI elements to design a unique observation protocol to measure the extent to which the dynamic geometry approach is implemented in secondary geometry classrooms.

3. Measuring implementation of Dynamic Geometry approach with an observation protocol

3.1. Instrument Development

The purpose of the observation protocol in the Dynamic Geometry (DG) Project was to measure the *fidelity of implementation*, or the extent to which teachers implemented the DG approach in their classrooms. We began with the development of a blueprint for dynamic geometry teaching based on a review of empirical studies and key documents related to the teaching and learning of geometry with technological tools (e.g., Hollebrands, 2007; Baccaglini-Frank and Mariotti, 2010; NCTM, 2000), mathematics quality of instruction (Hill, et al. 2008, Stein, et al. 2000), and reformed teaching (Pibum & Sawada, 2002). Table 1 shows the four dimensions of the blueprint serving as the basis for the observation protocol. Our team developed most of the items, but some were adjusted from existing items developed by previous researchers. Each item used a 4-point Likert response scale borrowed from the *Reformed Teaching Observation Protocol* (RTOP) from never occurred (0) to very descriptive (4). The scale reflects the degree, to which the aspect was *characteristic* of the lesson observed, as opposed to the number of times the aspect occurred.

Using an initial protocol of 34 items, the research team observed and scored a set of videotaped lessons taught by geometry teachers. In addition, external experts were consulted for feedback on the items. The purpose of this process was to improve the description of the items and to write a training manual that could help raters achieve the desired level of reliability. The training consisted of detailed explanations of each aspect, use of a video lesson taught by a master teacher (a high school teacher with rich experience in geometry curriculum and instruction working for the DG project) to practice coding, discussion of disagreements, and clarification of terms. For a multi-year project exploring the effect of the DG approach on students' geometry learning, an instrument that could describe the fidelity of the DG implementation was necessary. During the first year of DG implementation, pairs of trained raters completed observations of the same class and met immediately afterwards to discuss codes. Both independent codes and reconciled codes were collected for analysis. In spring 2011 a total of 66 observations were processed (33 geometry lessons). The inter-rater reliability for this initial set of observations was 0.837. The external experts continued to improve the instrument after its first-year use in the classrooms. For example, the initial instrument had the simple descriptor "the lesson leads the class to drag", that was improved to a fuller description: "the lesson leads the class to drag for the purpose of determining whether observed quantities and/or relationships remain constant, are changing, or are otherwise

impacted by the action.” The final version of the DG Observation Protocol (DGOP) consists of 25 items organized in the same four dimensions described in Table 1 (see entire instrument in Appendix A).

Table 1

Dimensions of Dynamic Geometry Teaching Approach

Dimension 1: Intended Dynamic Geometry Approach

The lesson as planned has key features of the DG approach (e.g. tasks that involve the use of the dynamic moving and measuring functions of the software to do observations and explorations; activities move students from initial conjecture, to investigation, to more thoughtful conjecture and, to verification).

Dimension 2: Implemented Dynamic Geometry Approach

The lesson as implemented has key features of the DG approach. This includes the use of software functions (drag, transform, animate, measure, and construct) and actions beyond the use of software (observation, investigation of mathematical relationships in multiple ways, form and test conjectures, motivated to think mathematically, and prove their conjectures).

Dimension 3: Quality of Instruction (Teacher’s Role)

Teacher provides opportunities for students to engage in high levels of cognitive demand mathematical practices, teacher demonstrates knowledge for teaching geometry in the dynamic technological environment, and teacher guides the student through the process of conjecturing and proof process.

Dimension 4: Engagement and Discourse

Students share questions, hints, and progress reports with their neighbors. Students offer and request help from their peers when working on their own computer. Students engage in whole-class discussion when necessary.

3.2. *Evidence of Reliability*

During the second year of DG implementation, the final observation protocol was used in 25 geometry lessons. As before, two members of the research team observed and independently coded each lesson. The estimates of inter-rater reliability were calculated by a regression line of the observation of one observer on those of the other. Figure 1 shows the scatter plot of the data points with a correlation

coefficient of 0.901 and the proportion of variance explained (R -Squared) by the line is 0.812. These estimates of reliability are high and comparable with the RTOP instrument, which has a correlation coefficient of 0.98 and an R -Square of 0.954 (Sawada et al., 2002).

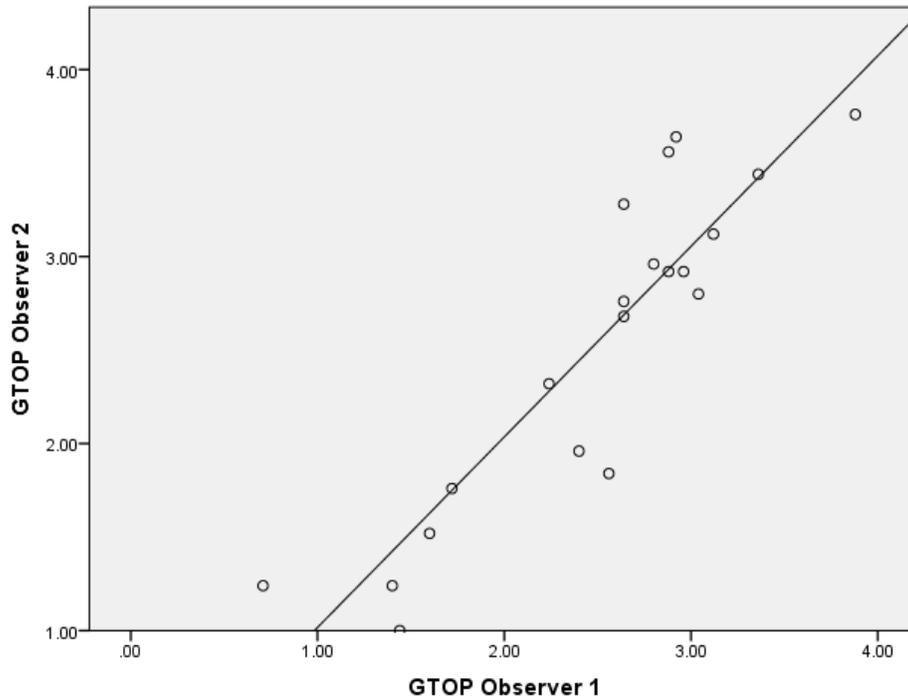


Figure 1. Estimate of inter-rater reliability of the observation protocol from observations in geometry classes. (R – Square = 0.812)

Further evidence of reliability of the observation protocol is presented in Table 2 where estimates were computed for each dimension or subscale and the instrument as a whole. Note that all the dimensions, except for the Intended DG Approach, have high estimates of reliability. Two things can be attributed to these lower estimates in this particular dimension. One possible explanation is the fact that the dimension was measured with only five items and the second possible explanation is that the items heavily depend on lesson plans and instructional materials available to the observers at the time of the observation. When these materials were not available, the observers were instructed to infer the intended goals and design of the lesson from the observed implemented lesson.

Table 2

Reliability Estimates for Dimensions of observation protocol

Dimensions		Correlation	R - Squared
Intended DG Approach		0.681	0.464
Implemented Approach	DG	0.911	0.830
Quality of Teaching		0.815	0.664
Engagement		0.825	0.681
Overall protocol	observation	0.901	0.812

3.3. Evidence of Face Validity

The face validity of the observation protocol draws on three major sources: (a) *Common Core State Standard for Mathematics*, and *Principles and Standards for School Mathematics* (NCTM, 2000); (b) *Reform Teaching Observation Protocol* (RTOP) for mathematics and science (Sawada et al., 2002; Piburn et al., 2000), and *Mathematical Quality of Instruction* (MQI) for mathematics (Hill, Charalambos, & Kraft, 2012; Hill et al., 2008; Learning Mathematics for Teaching Project, 2011); and (c) the operational definition of the dynamic geometry approach (Jiang et al., 2011). The detailed relationship between these documents and the observation protocol can be found in the reference manual and technical reports in Appendix B.

3.4. Evidence of Construct Validity

To test the hypothesis that *inquiry-oriented conjecturing and proving* is a powerful integrating force in the structure of the observation protocol, a correlational analysis was performed on the four dimensions or subscales. Each subscale will be used to predict the total score. High R-Squared values would support the hypothesis, offering strong support for the inquiry-base conjecturing and proving construct validity of the observation protocol (Table 3).

Table 3

Domains as Predictors of the Observation Protocol Total Score

Domains		Correlation	R – Squared
Intended DG Approach		0.842	0.709
Implemented Approach	DG	0.970	0.941
Quality of Teaching		0.897	0.805

Engagement

0.611

0.373

Except for the last domain of Engagement, all the domains are high predictors of the total observation protocol scores, providing strong evidence of the inquiry-oriented conjecture and proving validity of the observation protocol. The domain of Engagement relates to student and teachers practices around participation, discourse, and remediation. These practices, although important elements of the DG approach, are the most independent of the other domains in the sense that a lesson could score high on this dimension but low in the others or vice-versa. For example, teachers and students could be actively engaged in the class by discussing and helping each other about a topic that did not involve any of the intended or implemented goals of the lesson. In an inter-domain correlation analysis (not shown here) the only low correlation was between Engagement and Implemented DG Approach, providing evidence that these two domains are not highly associated.

To better understand construct validity of observation protocol we examined a two case studies of teachers, using their observation protocol scores, notes from observers that accompanied the observation protocol instrument, as well as the two teachers' own self-reports provided approximately monthly during each school year. The self-reports asked teachers to describe how they used DG to explore, conjecture, and prove in their classrooms, how/if they felt students were learning, and a variety of other questions aimed at issues of fidelity.

Maria scored very high on her observation protocol; outscoring all other DG teachers nearly every time. These scores should indicate that she is faithful to the DG approach. Her self-reports corroborated this information and added some nuance. Throughout the first year of the project she reported being very excited about using DG and thought the approach was helping her students learn. She described many lessons she had conducted or planned to conduct and these seemed in line with the observed lessons in terms of DG usage. As the first year drew to a close her self-reports indicated frustration with her curriculum calendar and how it conflicted with her desired use of DG for her students. For example, she preferred using two days per DG lesson where the first day was exploring and conjecturing and the second day wrapped up conjecturing and proving. She indicated that she felt pressured by the pace of her district's calendar to use DG less often and that by the end of the year she had been using DG more as a demonstration on the overhead projector than as an exploration activity with students at shared computer stations. The observations during the first year did not capture demonstration usage, we suspect, because many of her self-reports indicating this change happened after the conclusion of observations in her class for that year.

The second year our observations again ranked her highly in terms of observation protocol scores. Her self-reports again shifted away from beginning-year enthusiasm to more hesitancy to use the DG approach as often as she had in Year 1 and with a shift, again, to teacher-led demonstrations. Again, we do not have observer data that matches a demonstration approach to DG, as she described in her self-reports. We suspect this may be for two related reasons. Firstly, she never stated that she'd given up student-led DG lessons, just that she shifted away from using them as frequently. Secondly, it is likely that she planned her student-led lessons on days we observed. Our observations and her self-reports indicate that she can and did lead careful DG lessons with her students. The observation protocol captured these well. But the limited time to observe in her classroom did not allow for observation protocols to capture her changing approach to her lessons. The observation protocol seemed to be an indicator of her ability to lead a great lesson more so than a measure that she sustained this throughout the project. The observation protocol, then, seemed to measure her propensity and ability to do DG well rather than a measure that most or all of her lessons were of this high quality. We hypothesize that many of her later-year lessons, if we were able to observe more frequently, would reveal that her observation protocol scores would decrease in as much as she began using DG less often with students' leading the explorations, conjecturing, and proving. We further hypothesize that the observation protocol would still capture that distinction.

Carla, by contrast, scored low to medium on her observation protocol in Year 1 of the project (far below Maria). Her self-reports that year also corroborated those scores. She frequently mentioned ordering DG lessons differently than the project envisioned, going so far as to acknowledge fully in her reports that she was using the technology in "reverse" order. She taught concepts first in the classroom and then had students use GSP in the lab to verify results already learned rather than to explore as new content with the technology. Her low observation protocol scores aligned with her self-reported usage; that her lessons were not explorations of ideas but were verifications of already learned material. As the first year drew to a close her self-reports indicated that she wanted to try to do more exploring first with DG, but was not yet ready to fully change her teaching style.

Her Year 2 self-reports suggested that she underwent the change she had been considering the previous year. She began to reserve every Monday in the lab for students to explore and conjecture and then spent the rest of the week unpacking the ideas encountered in the Monday DG labs. Subsequent observations in her classroom, after her self-reported change, generated new observation protocol scores that were much higher than her Year 1 scores. This seemed to suggest that the observation protocol captured her range of lessons across both years.

In conclusion these two cases suggest the observation protocol can measure the overall fidelity to DG at least on the days observed. In Carla's case, she began the project not using DG as faithfully as the project envisioned and the observation protocol seemed to accurately measure that usage. The observation protocol was, though, able to measure her changes after Year 1, as she began to be more faithful to the project's intentions for

DG in the classroom. In both years her observation protocol scores matched her reported usage. Maria presents a more difficult case of fidelity. From the start Maria was quite skilled at teaching using the DG approach and her ability to do so did not decrease as her observation protocol scores revealed. What did seem to decrease, evidenced by her self-reports, was her frequency to use this style of DG lesson.

4. Discussion

In the continually-evolving world of technology, the teaching and learning of geometry in US secondary classrooms is becoming more dynamic in nature. By using a dynamic approach to teaching geometry, students are more exposed to more mathematical practices that facilitate the learning of processes such as exploring, conjecturing, and proving (Jiang et al, 2011). As this approach to teaching geometry is more-widely implemented in classrooms, either as part of curricular innovations or as future research projects, appropriately measuring the fidelity of DG implementation is necessary. Despite time and labor intensiveness of classroom observations, a measuring method that relies on observational data is needed to address the drawback of relying solely on teachers' self report when capturing implementation fidelity. This was why the DG Observation Protocol was developed and used in the project.

The DG Observation Protocol (DGOP) was built upon dynamic geometry research literature, measures of quality and reform teaching, and external experts' advice. The operationalization of the DG approach allows observers to measure the extent to which the DG approach is intended and implemented in the classroom. Our data show that with training, teams of observers using DGOP can reach high levels of inter-rater reliability. Further, quantitative and qualitative analyses of our observations provide evidence that the scores stemming from the observation protocol reflect the degree of fidelity of the implementation of the DG approach.

Some scholars may argue that certain aspects of teaching geometry in a dynamic environment are not captured in DGOP. A possible example may be, students' technological moves and discourses as they explore and conjecture during the lesson seem necessary to assess the level of engagement with the approach. However, with established validity and reliability this observation protocol has painted a clear picture of the DG approach that teachers implemented and provides a mechanism to quantifying that specification. It has the great potential to serve as a tool to further investigate the nature of the teaching and learning of geometry in dynamic environments.

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Appendix A

I. DESCRIPTION OF *INTENDED* DYNAMIC GEOMETRY LESSON

1. The lesson has appropriate objectives for the concept(s) being explored.
2. The lesson includes tasks that involve the use of the dynamic moving and dynamic measurement functions of the software to do observations and explorations.
3. The activities develop the notion of “figure” rather than “drawing” – attending to underlying relationships rather than particular of a specific drawing.
4. The activities in the lesson are designed to move students along the following trajectory (or part of it): from initial conjecture, to investigation, to more thoughtful conjecture, to verification and proof.
5. The activities are designed to so that students observe interesting mathematical phenomena and are motivated and challenged to understand why these phenomena occur and to explain them logically.

II. DESCRIPTION OF *IMPLEMENTED* DYNAMIC GEOMETRY LESSON

The lesson leads the class to

6. construct with limited explicit guidance by the teacher or other materials (i.e. handout w/ specific steps).
7. “drag” for the purpose of determining whether observed quantities and/or relationships remain constant, are changing, or are otherwise impacted by the action,
8. measure for the purpose of further exploring relationships and/or conjecturing, and/or disproving
9. observe the various quantities/qualities of their sketches, particularly relationships that might lead to conjectures or proofs
10. investigate mathematical relationships in multiple ways including using transformations and/or animations,
11. form conjectures based on students’ interactions with the software,
12. test conjectures using the software or other means (i.e. deductive reasoning),
13. take advantage of immediate feedback (as offered by the software or teacher)
14. reason inductively and/or deductively throughout
15. prove (or disprove) their conjectures.

III. ASSESSMENT OF *QUALITY* OF TEACHING

LEVEL OF COGNITIVE DEMAND

16. Students engage in recollection of facts, formulae, or definitions. (*Memorization*)
17. Students engage in performing algorithmic type of problems and have no connection to the underlying concept or meaning. (*Procedures without connections*)
18. Students engage on the use of procedures with the purpose of developing deeper levels of understanding concepts or ideas. (*Procedures with connections*)
19. Students engage in complex and nonalgorithmic thinking, students explore and investigate the nature of the concepts and relationships. (*Doing Mathematics*)

TEACHERS’ OBSERVED KNOWLEDGE

20. The teacher has a solid grasp of the geometry content at the level he or she is teaching. (*Grade level geometry knowledge*)
21. The teacher has knowledge of the use of instructional techniques specific to teaching geometry along with a deep understanding of the subject to appropriately integrate instruction with the concepts. (*Mathematical Pedagogical knowledge/knowledge for teaching*)
22. The teacher leads students to the appropriate geometric dynamic actions (drag, transform, animate, measure, and construct) according to the goals of the lesson. (*Dynamic Geometrical knowledge for teaching*)

IV. ASSESSMENT OF ENGAGEMENT AND DISCOURSE

23. Students are encouraged to share questions, hints, ideas, and/or progress with other students
24. Teacher circulates, observes (to monitor progress), asks questions, and provides necessary help as students work.
25. Teacher initiates class discussion when necessary.

Dynamic Geometry Professional Development Workshop

August 2-6, 2010



**Stevens High School
San Antonio, Texas**

External Evaluator Report Ed Dickey, Ph.D.

Approximately 80, mathematics teachers participated in 5 days of professional development activities related to the Dynamic Geometry project funded by NSF to Texas State University-San Marcos. Teachers were divided into two groups, randomly, and each group participated in a different, independent, professional development program consistent with the project's proposal. I participated in the programs beginning at 1 pm on August 4 and until the conclusion at noon on August 6, 2010. I rotated observation between the two groups throughout my visit.

Both groups completed a pre- and then a post-test related to mathematical content knowledge about geometry. Teachers in both groups also completed a questionnaire regarding their beliefs about geometry teaching and learning. Other demographic data was also collected that included teachers gender, experience, and current teaching assignments. Later analysis will examine the degree to which the two groups are comparable and consider threats to internal validity that might arise despite the random assignments.

One group participated in extensive and in-depth training on the use of Geometers Sketchpad (GSP) software for teaching high school geometry in a manner that addresses students' ability to make conjectures and prove theorems: the Dynamic Geometry

Approach. The other group participated in similar training addressing conjecturing and proof, but using dynamic methods such as paper-folding and constructions without using dynamic geometry software. Detailed agenda for each program are included in Appendix A (software group) and B (control group). Activities that I attend are indicated within the appendix as “Dickey Attended” within the Time column of the agenda.

The software group participated in structured experiences to build skills and knowledge specific to using the GSP software. These were designed to accommodate some participants who were novice users and others who were more experienced. Software group participants also worked with strategies and ideas for teaching conjecturing and proof using dynamic geometry software. This included geometry problems for the teacher participants to consider, work with, and even prove as well as discussion among participants and leader regarding the teaching of conjecturing and proof. Project PI Dr. Zhonghong Jiang, Dr. Samuel Obara, lead teacher Janie Love, and doctoral student Alana Rossenwasser planned and led significant segments of the software group workshop.

The control group had similar experiences but structured around the teaching of conjecturing and proof using methods that do not require dynamic geometry software. This group, like the other, participated in problem solving and discussion as well as structured experiences with strategies for developing students’ ability to generate conjectures and develop proofs using methods like paper folding, straight-edge rulers, compasses, isometric dot paper, web-based applets, and applications within TI-84 calculators related to transformations with matrices and conics. Videos of students discussing conjecturing were also used with this group. A Window on Geometry booklet was provided to all participants that included print versions of the materials used during the professional development sessions. Project co-PI Dr. Gilbert Cuevas, lead teachers Lisa Villalon and Lori Robinson, and doctoral student Ewelina McBroom planned and led significant segments of the control group workshop.

All participants signed a Consent Form related to photography, videotaping, and testing and consistent with research ethics and institution review of research practices. There were a total of 78 participants with 39 in the software group and 39 in the control. Professional responsibilities prevented some participants from attending different parts of the 5 day session but group leaders and project staff provided or will provide make-up experiences to ensure that all participants completed the planned training.

My observations of the software group led me to conclude that all participants completed the training with strong baseline knowledge of the GSP software as well as a clear understanding of what the dynamic geometry approach entailed. All participants in groups of 2 or 3 prepared a lesson plan that was presented to the entire group. From the presentations, it was clear to me as well as the project leaders that teachers were capable of designing and explaining lessons that required their students to make conjectures and engage in reasoning and proof through learning within a dynamic geometry software environment. Their presentations also included actually demonstrating their ability to use the GSP software and all participants, in my estimation, were functioning at an experience level that was beyond “novice” user. Some of the participants could be

characterized as “expert” users, but even those who described themselves as novices at the beginning of the training demonstrated competent and confident usage skills.

Participants in the control group demonstrated a very positive attitude to teaching geometry using a conjecturing and reasoning approach. They responded enthusiastically to activities that engage learners in inductive and deductive reasoning. I observed constructions using paper, compasses and straight-edge as well as paper folding activities. I heard participants express an eagerness to use the strategies learned during the workshop as part of their teaching.

Teachers in the software group discussed the need to address computer lab reservations and software installation issues before the beginning of school. The project director with my assistance drafted a letter to be sent to the Principal at each participating school requesting assistance with ensuring that provided GSP software was installed on school computers and that priority be given to project teachers requiring computers labs. Teachers in the software group were encouraged to have an average of 2 computer lab experiences per week. It was clearly expressed that for the curriculum units that the project developed, the 2 computer labs per week experience by students is essential. Teachers were provided with print materials developed by Key Curriculum Press on planning lessons and managing student GSP work. Resources are and will be made available to teachers using the Texas State University TRACS collaborative learning environment available to participants online. Teachers had, even during the workshop, posted their presented lessons so that others would have access during the school year.

As a result of my observations during the workshop, I confidently conclude that the professional development experiences by both groups during the week of August 2-6 were entirely consistent with the Dynamic Geometry in Classrooms proposal. The project team was especially successful in building enthusiasm and support by teachers in both groups related to the project goals. Careful planning of meaningful mathematics experiences, demonstrated respect for teachers’ time and needs, and care to ensure the comfort and convenience of all participants helped to secure the cooperation and commitment critical to a project of this type. As proposed, professional development will continue during the school year, but at this time, participating teachers are well prepared to implement strategies that promote conjecturing and proof as part of high school geometry learning. Discerning whether the dynamic geometry approach has a significantly stronger effect on students’ ability to generate conjectures and proofs will be possible based on the parallel professional development offered to both participant groups.

APPENDIX A

Dynamic Geometry Group Professional Development Agenda

Day 1

Topic: The Tools of Geometry – Points, Lines & Angles

Time	Activity(ies)	Materials	Facilitator(s)
8:00 AM	Welcome and Introduction to the DG project	(both groups together)	Linda Gann
8:30 AM	Pre-tests	(then break into 2 groups)	Zhonghong
9:30 AM	Break	(after break, pair up novices with experienced GSP users)	Janie
9:45 AM	<ol style="list-style-type: none"> 1. Tutorial (especially for novices) 2. Exploration: Find the points that are equidistant from the two end points of a segment. 3. Making and testing your conjecture; 4. Exploration: The property of a perpendicular bisector. (Can you prove your conjectures?) 5. Getting to know GSP and new features of GSP 5. 	Computers; Handouts	Alana
11:15 AM	Lunch		
12:00 PM	<p>Group discussion on using GSP in the classroom: Classroom dynamics in a computer lab – introducing students to GSP5. Questions about GSP skills</p>	Handouts Possibly video clip	Janie
12:30 PM	Constructions (parallel lines, perpendicular lines, a segment that is congruent to a given segment, an angle that is congruent to a given angle, etc.) and related investigations	Computers; Handouts	Zhonghong
2:00 PM	Break		
2:15 PM	Exploring the interior/exterior angles of a triangle	Computers; Handouts	Alana
3:00 PM	Reflection question: What should a teacher do to help students learn more deeply by using GSP?	Note cards: participants respond individually, then in pairs or small groups, then summarized in large group	Alana

Day 2

Topic: Comparing Triangles – Congruence

Time	Activity(ies)	Materials	Facilitator(s)
8:00 AM	Construction: equilateral triangles and their centers Introduction to custom tools	Computers	Alana
8:30 AM	Challenging problem: <i>Let $\triangle ABC$ be an arbitrary triangle in the plane, and let $\triangle A'BC$, $\triangle AB'C$, $\triangle ABC'$ be equilateral triangles attached to the outside of $\triangle ABC$. Explore the situation and make your conjecture(s). Prove your conjecture(s).</i>	Computers; Handouts	Alana
9:30 AM	Break		
9:45 AM	Custom Tools: segment congruent to given segment; angle congruent to given angle Exploring triangle congruence postulates and using them to prove that segments or angles are congruent. (Do two and leave time for discussion)	Computers; Handouts	Zhonghong or Samuel
11:15 AM	Lunch		
12:00 PM	Group discussion on using GSP in the classroom: Introduction to LessonLink (first as student, then set up Key On Line account and see LessonLink as teacher)	Computers Handouts	Janie
1:00PM	Pair up participants for their presentations Present topics and give information about presentations	Computers; Handouts	Janie
1:45 PM	Break		
2:00 PM	Introduction to TRACS	Computers	Ewelina
2:15 PM	Revisit the problem(s)	Computers; Handouts	Alana
3:00 PM	Reflection question: What processes or strategies do students use to come up with their conjectures?	Note cards	Alana

Day 3

Topics: Comparing Triangles: Similarity Triangle Centers

Time	Activity(ies)	Materials	Facilitator(s)
8:00 AM	Details provided to participants on how data will be collected for the Dynamic Geometry Project (assessments, classroom visits, teacher reports, etc.)	Samples of required submissions by teachers	Zhonghong
8:30 AM	Challenging problem: $\triangle ABC$ is an <u>arbitrary triangle</u> . Points D , E , and F are respectively on sides \overline{BC} , \overline{CA} , and \overline{AB} . $BD = \frac{1}{3}BC$, $CE = \frac{1}{3}CA$, and $AF = \frac{1}{3}AB$. $\triangle PQR$ is formed by the construction of line segments \overline{AD} , \overline{BE} , and \overline{CF} . What is the relationship between $\triangle PQR$ and $\triangle ABC$?		Zhonghong
9:00 AM	Exploring triangle similarity postulates		Samuel
9:45 AM	Break		
10:00 AM	Using triangle similarity postulates to solve problems; Exploring the Fundamental Theorem of Similarity and triangle side-angle relationships and related problems	Computers; Handouts	Samuel
11:15 AM	Lunch		
12:00 PM	Group discussion on using GSP in the classroom: Facilitating students' conjecturing (List ways the teacher facilitates students' conjecturing in a computer lab setting and regular classroom setting)		Alana
12:30 PM Dickey Attended	Exploring the altitudes, medians, angle bisectors, and perpendicular bisectors of sides in triangles Finding "centers"	Copies of Lessons and Lesson Notes	Janie
2:00 PM	Break		
2:15 PM	GSP Project: Constructing the Euler Line and Nine-Point Circle	Computers; Handouts	Janie
3:00 PM Dickey Attended	Reflection questions: What does "conjecturing ability" mean in the high school geometry classroom?	Note cards	Janie

Day 4

Topics: Mathematical Reasoning and Lesson Planning

Time	Activity(ies)	Materials	Facilitator(s)
8:00 AM Dickey Attended	Revisit the challenging problem presented on Day 3. Explore ways of proof with GSP 5.	Computers; Handouts	Zhonghong or Samuel
8:30 AM	Find tangent to a circle from a point outside circle and justify (idea is to give teachers a problem they can “prove”)	Computers; Handouts	Zhonghong or Samuel
9:00 AM Dickey Attended	Discussion on “proof” (mathematical reasoning) in the high school geometry classroom Standards (TEKS and NCTM) relating to “proof”		Alana Janie
9:30 AM	Break		
9:45 AM Dickey Attended	Brief introduction to transformations	Computers; Handouts	Janie or Samuel
11:15 AM	Lunch		
12:00 PM	Action Buttons, sliders, hot text, etc. – tools for presentation sketches	Computers; Handouts	Zhonghong or Samuel
12:45 PM Dickey Attended	Lesson planning with dynamic geometry software	Computers; Handouts	Janie
1:30 PM	Break		
1:45 PM	Participants work on their presentations	Computers; Handouts	All
3:00 PM Dickey Attended	Reflection question: How should we take advantage of the powerful tools provided by dynamic geometry software in geometry teaching and learning?		Janie

Day 5

Topic: Quadrilaterals and Polygons

Time	Activity(ies)	Materials	Facilitator(s)
8:00 AM	Assignment of presentation groups to rooms, rehearsal time		All
8:15 AM Dickey Attended	Presentations (with time for discussion after each)		All
10:30 AM	Break		
10:45 AM	Post-tests		Zhonghong
11:30 PM Dickey Attended	Final Reflections: Getting Started with the Dynamic Geometry Project		All

APPENDIX B

Control Group Professional Development Agenda

DAY 1**TOPIC: The Tools of Geometry - Points, Lines & Angles**

Time	Activity(ies)	Description/Materials	Facilitator(s)
8:00 AM	<ul style="list-style-type: none"> Welcome/Introduction Overview of Research Project Goals of the PD 	<ul style="list-style-type: none"> Opening session – all participants present At the end of this session, participants will be divided into groups All participants will be given pre-tests 	Ms Gann Dr. Jiang
9:30 AM	BREAK		
10:15 AM	<ul style="list-style-type: none"> Geometry team building: Concentration <ol style="list-style-type: none"> Select a partner and face each other Partner A gets 30 seconds to study his/her partner (hair, jewelry, etc.) and then turns around Partner B then changes three things about himself/herself (switches watch to other arm, unties shoelaces, etc.) Partner A then turns back and identifies the 3 changes Partners swap roles. Partners celebrate 	<ul style="list-style-type: none"> Wrap-up: The message for this activity is to highlight how difficult it is in teaching to recognize changes and the nature of change in instructional practices. 	Ewelina Gil (wrap-up)
10:45 AM	<ul style="list-style-type: none"> What is geometry? <ul style="list-style-type: none"> Ask participants to write down their description of geometry as a field of studies. Share Lecturette – History and structure of geometry (5 min.) <i>Can you draw the figure?</i> This activity consists of two parts: First, participants will have to recreate geometric figures after they are be shown for a brief time. In the second part, participants will draw a figure after a verbal description is given. Object: Geometry is also about perception, language and communication 	<ul style="list-style-type: none"> 3 x 5 cards Powerpoint for lecturette Internet connection to Annenberg site for <i>Can you draw my figure?</i> 3 x 5 cards for <i>My Figure</i> activity See: http://www.learner.org/courses/learnin_gmath/geometry/session1/part_a/index.html	Gil
11:15 AM	<ul style="list-style-type: none"> Geometric constructions: 	<ul style="list-style-type: none"> Patty Paper 	Laurie

11:15 AM	<ul style="list-style-type: none"> • Geometric constructions: <ul style="list-style-type: none"> - Midpoint of a segment [compass] - Perpendicular to a segment [patty paper] - Bisector of an angle [compass/patty paper] • Informal proofs <ul style="list-style-type: none"> - Discussion of justifications the findings for each of the above constructions - Discussion of the nature of proof/logical justification for a conjecture: <i>Pieces of Proof</i> activity 	<ul style="list-style-type: none"> • Patty Paper • Compass and straight edge • Construction instructions • Different kinds of proofs handout • Note: Alternate between patty paper constructions and compass/straight edge 	Laurie Lisa Gil Ewelina
Noon	LUNCH		
1:00 PM	<ul style="list-style-type: none"> • Continue constructions 		
1:30 PM	<ul style="list-style-type: none"> • Parallel lines and transversals • Lecturette: Historical background (5 min.) • Construction • Proof/justification of conjecture 	<ul style="list-style-type: none"> • Powerpoint with Eratosthenes measure of the earth (See Musser, p. 240, 247) • Compass and straight edge 	Gil Ewelina
2:15 PM	<ul style="list-style-type: none"> • Application • Mind warmers: Problems • Follow the Falling Meteorite 	<ul style="list-style-type: none"> • Worksheet • Protractor, compass, straight edge • Colored pencils 	Laurie Lisa
??	BREAK		
3:00 PM	<ul style="list-style-type: none"> • Wrap-up and reflections <p>Participants will reflect and write down what they learned and what they would like to learn</p>	<ul style="list-style-type: none"> • 3 x 5 cards (2) • What did you learn? • What would like to learn? 	Lisa

DAY 2**TOPIC: Comparing Triangles - Congruence and Similarity**

Time	Activity	Description/Materials	Facilitator(s)
8:00 AM	<ul style="list-style-type: none">Wake up! Mind Warmer This problem requires participants to identify all the triangles in a picture. Sharing/Discussion of strategies used	<ul style="list-style-type: none">Use handout of figure. Participants may work in groups. Need to record strategy(ies) used to determine the number of triangles	
8:30 AM	<ul style="list-style-type: none">Proving triangles congruent- Did you know?	<ul style="list-style-type: none">Powerpoint with main points of the content	
8:45 AM	<ul style="list-style-type: none">Identifying what is needed to prove triangles congruent: Pieces of proof	<ul style="list-style-type: none">Follow the directions on the Illuminations activity.Proof kits, glue, blank paper	
10:00 AM	BREAK		
10:15 AM	<ul style="list-style-type: none">Congruent triangle construction- Justify your construction	<ul style="list-style-type: none">Participants will construct the incenter of a triangle, explain properties of the incenter and apply them to a new situation.	
11:00 AM	<ul style="list-style-type: none">Are they really congruent? Proofs	<ul style="list-style-type: none">In small groups, participants prove and share conjectures dealing with congruent triangles	
11:30 AM	<ul style="list-style-type: none">Similarity: Main Ideas	<ul style="list-style-type: none">In an exploratory activity, participants justify (or show why not) two triangles are similar	
Noon	LUNCH		

DAY 2 (cont'd.)

Time	Activity	Description/Materials	Facilitator
1:00 PM	<ul style="list-style-type: none">• Application of Similarity: George Washington's Nose	<ul style="list-style-type: none">• In small groups, participants use principles of similarity to determine the length of George Washington's nose carved on Mount Rushmore. Strategies for computing the length are shared•	
1:30 PM	<ul style="list-style-type: none">• Similarity: Proof Roulette	<ul style="list-style-type: none">• In small groups, participants prove and share conjectures dealing with similar triangles	
2:00 PM	<ul style="list-style-type: none">• Folding a Kite: Application of similarity	<ul style="list-style-type: none">• A hands-on activity where participants determine how to fold a kite to specific dimensions using properties of triangles and principles of similarity•	
2:30 PM	<ul style="list-style-type: none">• What did we learn that we could communicate to students? Applications and reflections. Group share	<ul style="list-style-type: none">• 3 x 5 cards. Participants highlight their learning from the activities and discuss ways to communicate content to their students.	

DAY 3 TOPIC: Geometric Movement: Transformations

Time	Activity	Description/Materials	Facilitator(s)
8:00 AM	<ul style="list-style-type: none"> Morning Brain Java: Four Tracks 	<ul style="list-style-type: none"> Participants solve a logic problem that introduces geometric transformations (rotational). One copy of Problem Handout for each participant. 	
8:30 AM	<ul style="list-style-type: none"> A VERY brief history of transformational geometry 	<ul style="list-style-type: none"> Overview of the development of transformational geometry. Powerpoint 	
8:40 AM	<ul style="list-style-type: none"> Transformations using patty paper 	<ul style="list-style-type: none"> Participants will perform translations, reflections, and rotations using patty paper Patty paper 	
9:15 AM	<ul style="list-style-type: none"> The basics of transformations: Writing directions for transformations 	<ul style="list-style-type: none"> Part 1: Determine the correct transformation from a description 	
9:30 AM	BREAK		
9:45 AM	<ul style="list-style-type: none"> The basics of transformations: Writing directions for transformations 	<ul style="list-style-type: none"> Part 2: Given three figures, participants will describe the steps that result in the required transformation 	
10:15 AM Dickey Attended	<ul style="list-style-type: none"> Symmetry: Basic exploration 	<ul style="list-style-type: none"> Participants examine a set of figures to determine the ones that have line symmetry. Based on their observations, they write a definition of line symmetry. Handout with figures 	
10:45 AM Dickey Attended	<ul style="list-style-type: none"> Findings centers of rotation: A video 	<ul style="list-style-type: none"> Participants watch a 9 minute video and discuss salient points 	
11:00 AM Dickey Attended	<ul style="list-style-type: none"> Rotation symmetry: Activity 	<ul style="list-style-type: none"> Using a given figure, participants create a figure with rotation symmetry. Handout and patty paper 	

DAY 3 (cont'd.)

11:30 AM	Transformations in the coordinate plane: Introduction	<ul style="list-style-type: none">• Participants determine the mapping for each of the transformations from examples shown on a Powerpoint. Powerpoint slides and handout	
Noon	LUNCH		
1:00 PM Dickey Attended	<ul style="list-style-type: none">• Applications of transformations in the coordinate plane	<ul style="list-style-type: none">• Participants solve problems involving transformations in the coordinate plane. They share and discuss solutions.	
1:30 PM	<ul style="list-style-type: none">• Composite transformations: Exploration	<ul style="list-style-type: none">• Participants explore composites of transformations and then create a “Guess my transformation” task. Graph paper.	
2:00 PM	<ul style="list-style-type: none">• Transformations using matrices: Introduction	<ul style="list-style-type: none">• Participants will follow the directions on the handout “Transformations Using Matrices.” Discussion of questions at the end of the session.• TI-84	
2:45 PM	<ul style="list-style-type: none">• What did we learn that we could communicate to students? Applications and reflections. Group share	<ul style="list-style-type: none">• 3 x 5 cards. Participants highlight their learning from the activities and discuss ways to communicate content to their students.	

DAY 4 Rise and

Time	Activity	Description/Materials	Facilitator(s)
8:00 AM	<ul style="list-style-type: none"> Rise and Shine! Puzzle 	<ul style="list-style-type: none"> Participants solve the “Move the Hole” puzzle. Patty paper, scissors. 	
8:15 AM	<ul style="list-style-type: none"> Math Fun Facts 	<ul style="list-style-type: none"> Rolling polygons. Overhead projector 	
9:00 AM	<ul style="list-style-type: none"> Video about polygon constructions. Discussion 	<ul style="list-style-type: none"> Projector Handout with discussion questions 	
9:15 AM Dickey Attended	<ul style="list-style-type: none"> Activity 1: Creating quadrilaterals by connecting midpoints 	<ul style="list-style-type: none"> Participants draw six quadrilaterals, connect midpoints of sides to create new quadrilaterals Metric rulers, paper, colored pencils 	Ewelina
	<ul style="list-style-type: none"> Activity 2: Finding the sums of the interior angles of a polygon 	<ul style="list-style-type: none"> Participants determine the sum of the interior angles of a polygon Rulers, unlined paper 	
BREAK			
Dickey Attended	<ul style="list-style-type: none"> Activity 3: Attributes of quadrilaterals 	<ul style="list-style-type: none"> This is an investigation of attributes of quadrilaterals Rulers, protractors, paper 	
Dickey Attended	<ul style="list-style-type: none"> Activity 4: Diagonals of quadrilaterals 	<ul style="list-style-type: none"> Participants examine properties of the diagonals of quadrilaterals Rulers, protractors 	
LUNCH			
Noon			
1:00 PM	<ul style="list-style-type: none"> Activity 5: The Big Quadrilateral Puzzle 	<ul style="list-style-type: none"> Participants discover missing properties of a given quadrilateral Worksheet 	

1:30 PM	<ul style="list-style-type: none"> To Infinity and Beyond! Polyhedra 	<ul style="list-style-type: none"> Group problem. Overhead and Powerpoint slide 	
2:00 PM Dickey Attended	<ul style="list-style-type: none"> Spatial Visualization: Explorations 	<ul style="list-style-type: none"> Two activities. First, participants explore isometric drawings of three dimensional figures built with cubes to determine if different perspectives represent the same figure. Second, given three figures composed of 7 cubes, participants determine surface area and volume and state conclusions based on observations. Cubes, handouts, transparencies. 	
3:00 PM	<ul style="list-style-type: none"> What did we learn that we could communicate to students? Applications and reflections. Group share 	<ul style="list-style-type: none"> 3 x 5 cards. Participants highlight their learning from the activities and discuss ways to communicate content to their students. 	

DAY 5**TOPIC: and Around We Go: Circles**

Time	Activity	Description/Materials	Facilitator(s)
8:00 AM	<ul style="list-style-type: none"> Get your brain in gear: Puzzle 	<ul style="list-style-type: none"> Show problem on the overhead. Have participants work in tables. Share solutions. 	
8:15 AM	<ul style="list-style-type: none"> Circles: Facts 	<ul style="list-style-type: none"> In 5 minutes, participants will list as many facts about circles as possible. Table groups will compete against each other. Poster paper, markers 	
8:30 AM	<ul style="list-style-type: none"> Circles: One more fact Nine-point circle 	<ul style="list-style-type: none"> Participants will construct a nine-point circle. Straight edge, compass. 	
9:00 AM	<ul style="list-style-type: none"> Activity 1: Inscribed polygons 	<ul style="list-style-type: none"> Participants will use paper folding to inscribe polygons in a circle Scissors, compass, protractors, two sheets of paper per participant Worksheet 	
	<ul style="list-style-type: none"> Activity 2: Tangents/Circles and lines 	<ul style="list-style-type: none"> Exploration of circles and tangent lines according to specific criteria Compasses, rulers, unlined paper Worksheet 	
9:45 AM	BREAK		
	<ul style="list-style-type: none"> It's Puzzle Time! The big circle puzzle 	<ul style="list-style-type: none"> This activity involves the application of certain circle theorems Worksheet 	
	<ul style="list-style-type: none"> The bigger circle puzzle 	<ul style="list-style-type: none"> This is a continuation of the previous activity Worksheet 	
	<ul style="list-style-type: none"> The biggest circle puzzle 	<ul style="list-style-type: none"> Continuation of the "Big circle puzzle" Worksheet 	

Dickey Attended	<ul style="list-style-type: none"> • Wrap-up: What did we learn that we could communicate to students? Applications and reflections. Group share 	<ul style="list-style-type: none"> • 3 x 5 cards 	
11:00 AM	<ul style="list-style-type: none"> • Posttests 		
Noon	END OF WORKSHOP		

Dynamic Geometry Professional Development Workshop

August 9, 2011

**South San Antonio High School
San Antonio, Texas**

**External Evaluator Report
Ed Dickey, Ph.D.**

Forty-four (44) mathematics teachers participated in a 1-day professional development workshop for the Dynamic Geometry project funded by the NSF DR K-12 program to Texas State University-San Marcos on August 9, 2011. The workshop began at 9 am at the South San Antonio High School Media Center where teachers were greeted by PI Zhonghong Jiang who provided a summary of project accomplishments from the 2010-2011 school year and plans for the 2011-2012 year. Besides outlining the project achievements from last year as well as preliminary results from the data analysis performed in summer, Dr. Jiang described in detail using a PowerPoint presentation the expectations for all participants for the coming school year including the policies for providing stipends.



Teachers also heard from Master Teacher Janie Love who explained the plan for the 3 professional development workshops to be held over the coming school year. Teachers were provided with 6 Saturday meeting dates and asked to select 4 dates for which they

would commit to attend. This ensured project leaders that teachers would be attending the required three sessions with the option of attending a fourth if desired. By the end of the workshop, meeting dates for September 17, October 29, November 19, and February 25 were confirmed and no teacher expressed a concern about attending at least 3 of the sessions. All participating teachers then completed a Conjecturing-Proving Test developed by the project team to assess teacher content knowledge at the beginning of the school year and to serve as pre-test measure in the research design of the project.

After a break following the full group presentation and pre-test administration, the Control Group of teachers (those who use a conjecturing and proof methodology without using the Geometer's Sketchpad software) moved to a breakout room next to the Media Center and the Treatment Group (those using the Dynamic Geometry methodology with software) moved to a computer lab adjacent to the Media Center. PI Jiang planned and organized the Treatment Group professional development session and Co-PI Gilbert Cuevas organized the Control Group session.

The treatment group built on the extensive and in-depth training on the use of Geometer's Sketchpad (GSP) software provided in August 2010 as well as the experiences and further professional development from the prior school year using GSP and the dynamic geometry approach to teaching in their classrooms. Ben Manchester, one of the teachers in the group, presented a series of explorations that he had developed and used with his own classes the previous school year. The activities included the development and use of sliders that Ben reported encouraged his students to make conjectures about the mathematical concepts that they were exploring and learning. Ben demonstrated then provided instruction so that the participating teachers could create and test explorations with sliders that involved slope, perimeter, area, and iteration. After these explorations, Dr. Jiang and doctoral student Alana Rossenwasser presented the "Pirate Problem" that required teachers to use geometry-based clues to locate where hidden treasure could be found and whether the geometry would allow finding of the treasure without knowing the initial starting point. Teachers worked in pairs to solve the problem and make a conjecture about whether the treasure could be found without the starting point. All were able to create sketches to solve the problem and nearly all made appropriate conjectures.





After a group lunch, the treatment group reconvened in the computer lab and finalized their conjectures about the Pirate program. At the end, some teachers presented their solutions and all concluded that the treasure could be found without knowing the starting point and provided a reasoned argument for why. Teachers then worked on an exploration that Dr. Jiang presented related to a square with a semicircle inserted. They generated different conjectures related to the construction that involved area of different triangles. Discussion of how to prove the conjectures followed.

The control group had similar experiences but structured around the teaching of conjecturing and proof using methods that do not require dynamic geometry software. This group began by exploring a “warm-up” activity that Dr. Cuevas presented from the www.archimedes-lab.org web site then participated in a discussion about different types of proof formats appropriate for high school geometry. The group then worked on the same Pirate problem explored by the treatment group. Teachers worked in groups of 4 or 5 to create a graphical representation of the problem situation and attempted to determine if the problem could be solved without knowing the initial condition. The groups offered some conjectures but I was unable to observe a group that actually solved the final question. Doing so without use of a dynamic geometry package proved to be quite difficult.



After lunch, the group explored problems obtained from the gogeometry.blogspot.com web site. Ewelina McBroom, a doctoral student on the project team, presented a series of geometry problems of increasing difficulty. Teachers in groups were able to solve the first two after discussion and reflection and their solutions constituted proofs. The third problem was identical to the one explored by the treatment group after lunch involving a square and interior semicircle. The control group was not able to generate any meaningful conjectures for this third problem because measuring area, critical to producing conjectures, was too challenging without dynamic geometry software.



The two groups reconvened in the Media Center where they were given packets that contained the pretests that they are to administer to their students within the first few weeks of the school year. Alana Rosenwasser, doctoral student on the project team,

provided detail explanations on administration and answered questions. Teachers asked questions about implementing the project strategies with geometry classes for the coming school year that will be held accountable to the new Texas End of Course (EoC) geometry test. The project leaders want to focus on classes comparable to last year's classes but welcome the addition of classes that will take the EoC test. Dr. Jiang promised to provide more information and guidance on this issue in the coming weeks.



Teachers asked to see, in advance, the items on the questionnaire that they will be expected to respond to throughout the year as this will allow them to reflect on the items and also to have prior knowledge of the scope and length of the questionnaire. Project leaders agreed that this was a good suggestion and that it would improve the quality of the data from the questionnaires.

Teachers were reminded of the resources available to them on the Texas State TRACS classroom management system and how the project focuses on 5 specific topics in geometry (not the entire geometry course). Teachers with difficulties regarding their teaching assignments for the coming year were counseled and project staff offered to assist with requesting geometry assignments for teachers who may have been reassigned to new teaching duties for the coming year.

The workshop adjourned at 3:30 pm. Project leaders met afterwards to de-brief on the workshop activities and to discuss plans for addressing the different issues that arose

during the day. Classes in Texas schools begin on August 22 allowing the team time to address the different teacher assignment questions and problems.

As a result of my observations during the workshop, I confidently conclude that the professional development experiences by both groups on August 9 were entirely consistent with the Dynamic Geometry in Classrooms proposal. The project team was especially successful in building enthusiasm and support by teachers in both groups related to the project goals. Careful planning of meaningful mathematics experiences, demonstrated respect for teachers' time and needs, and care to ensure the comfort and convenience of all participants helped to secure the cooperation and commitment critical to a project of this type. As proposed, professional development will continue during the coming school year, but at this time, participating teachers are well prepared to implement strategies that promote conjecturing and proof as part of high school geometry learning. Discerning whether the dynamic geometry approach has a significantly stronger effect on students' ability to generate conjectures and proofs will be possible in this second year of implementation based on the parallel professional development offered to both participant groups.



Section 3

Assessing Geometry Teacher Content Knowledge

The Dynamic Geometry in Classrooms project was funded to investigate the efficacy of the “Dynamic Geometry Approach” on students’ geometry learning requiring significant attention to the assessment of geometry teacher content knowledge. This knowledge included mathematical content knowledge as well as technological pedagogical content knowledge (TPACK). The papers in this section include four peer-reviewed publications and a doctoral dissertation.

The first paper, “Dynamic Approach to Teaching Geometry: A Study of Teachers' TPACK Development,” appeared as a chapter in the Handbook of Research on Transforming Mathematics Teacher Education in the Digital Age and is provided here with permission. The chapter documents the positive impact of the professional development for teachers’ Technological Pedagogical Content Knowledge (TPACK) development and their students’ achievement in geometry through the use of the dynamic geometry approach. The instruments used to develop and assess teachers’ TPACK included a Conjecturing- Proving Test, interviews and observation protocols. Participants’ TPACK levels were identified using a TPACK Development Levels Assessment Rubric. Findings show that teachers’ TPACK tended to remain within the three middle TPACK levels (accepting, adapting, and exploring). Recommendations and suggestions for future research are offered to those who implement school-based, mixed methods research studies involving technology.

The second paper, “Proposing a Model for TPACK Development: The Total PACKage Professional Development Model,” appeared in the Proceedings from the Society for Information Technology & Teacher Education International Conference 2014 and appears here with permission. The paper describes a professional development model for developing technological pedagogical content knowledge (TPACK). The model is based on a study of four mathematics teachers who participated in the project and used technology, specifically dynamic geometry software, in teaching high school level geometry. The model incorporates the ideas of virtual lesson study and TPACK in-practice. The model is cyclical and consists of seven stages: technological knowledge (TK) development, technological content knowledge (TCK) development, technological pedagogical knowledge (TPK) development, developing a technology-enhanced lesson, teaching, observing and reflecting.



The third paper, “Teaching Geometry with Technology: A Case Study of One Teacher’s Technological Pedagogical Content Knowledge,” also appeared in the Proceedings from the Society for Information Technology & Teacher Education International Conference but for 2013 and appears here with permission. A case study is presented that investigated how high school teachers developed and used their knowledge in teaching geometry



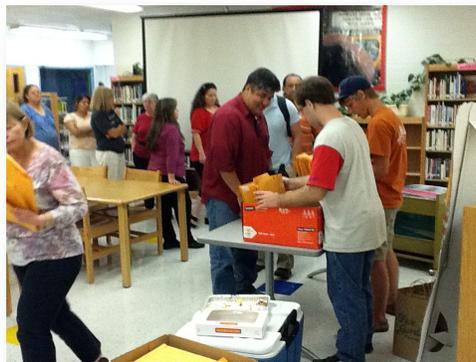
Section 3

Assessing Geometry Teacher Content Knowledge

with technology with particular attention to teachers' technological pedagogical content knowledge (TPACK) and their integration of dynamic geometry in the classroom instruction. This paper reports findings from one of the four cases. The sources of data included: an initial interview, observations, documents, a closing interview, a survey, implementation questionnaires, professional development attendance records and the researcher's log. Data analysis utilized the TPACK Development Model to describe the participant's dynamic geometry integration and to identify her TPACK development levels. The researcher was able to identify all TPACK development levels for the participant, which was an unexpected finding since the participant was an experienced teacher and long-term technology user.

The fourth paper, "Characteristics of Different Learning Environments in Geometry Classrooms," appeared in the Proceedings of the 41st Annual Meeting of the Research Council on Mathematics Learning. It reports on the analyses of the classroom environment using in-depth interviews of selected teachers to collect qualitative data to address the following research question: What characterize different learning environments in geometry classrooms? The main findings included: Since the experimental group teachers used dynamic geometry software to facilitate investigations, they were able to produce quality conjectures faster. However, as to proving, teachers varied considerably. Some could generate correct proofs, mostly for relatively simple geometric problems, some were able to work out parts of a proof but had difficulties to put the parts together, and others were very weak in proofs.

The fifth and final document in this section is the doctoral dissertation titled "Teaching with Dynamic Geometry Software: A Multiple Case Study of Teachers' Technological Pedagogical Content Knowledge" completed by Ewelina McBroom, a project staff member, in 2012 and under the direction of project PI, Dr. Zhonghong Jiang. The dissertation reported on a qualitative study that focused on how four teachers developed TPACK and how dynamic geometry was integrated into their instruction. The report included three unexpected findings. First, the participant with least teaching experience displayed the highest levels of TPACK. Second, the participant with most teaching experience with dynamic geometry showed the most inconsistency among the TPACK development levels, ranging from recognizing to exploring. Third, ongoing professional development and easy access to computers did not translate to frequent incorporation of dynamic geometry in teaching and learning. The participants claimed the curriculum and standardized testing to be the main barriers to increased technology use. Findings suggested that participants developed their TPACK through attending professional development workshops and implementing what they learned in the classroom instruction. Based on those findings, the dissertation included a professional development model designed for teachers interested in integrating dynamic geometry in the classroom instruction.



Handbook of Research on Transforming Mathematics Teacher Education in the Digital Age

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Chapter 20

Dynamic Approach to Teaching Geometry: A Study of Teachers' TPACK Development

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ABSTRACT

Secondary geometry teachers from several urban school districts participated in a two-year professional development focused on integrating dynamic geometry into teaching. The chapter documents the positive impact of the professional development for teachers' Technological Pedagogical Content Knowledge (TPACK) development and their students' achievement in geometry through the use of the dynamic geometry approach. Instruments used to develop and assess teachers' TPACK included a Conjecturing-Proving Test, interviews and observation protocols. Participants' TPACK levels were identified using a TPACK Development Levels Assessment Rubric. Findings show that teachers' TPACK tended to remain within the three middle TPACK levels (accepting, adapting, and exploring). Recommendations and suggestions for future research are offered to those who implement school-based, mixed methods research studies involving technology.

INTRODUCTION

High school geometry is a crucial subject for the 21st century. Dynamic geometry software provides teachers and learners with a valuable tool to construct knowledge and gain insights about geometric reasoning and proof. Research is needed on how to develop both teachers' and students' ability to make effective use of dynamic geometry software in school settings to meet the goals of Common Core State

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Standards Initiative (National Governor's Association Center for Best Practices and Council of Chief State School Officers [NGA & CCSSO], 2010) and other educational standards.

Geometry is an area in need of improvement in mathematics education. U.S. students' geometry achievement is low at all grade levels (Battista, 2007; Senk, 1985). Students entering high school have little knowledge or experience of geometric properties and relationships, often operating at the visual level of geometric thought and experiencing difficulties for tasks more than recognizing different geometric shapes (Fuys, Geddes, & Tischler, 1988; Scally, 1990). In the only states where researchers were easily able to find item analyses (such as Texas and Florida), at least 40% of the students struggled with state test items in topic areas including volume, applications of the Pythagorean Theorem, reasoning about geometrical ideas, transformations, spatial visualization, and angles in polygons (Dick & Burrill, 2009). Also, U.S. students' measures on international tests of achievement tend to be at the lowest level in geometry (Carnoy & Rothstein, 2013). Nearly all high-school teachers recognize that students rarely perceive a need for proof, and education researchers identify this result as a major issue in the teaching of geometry (de Villiers, 1999). Scholars have attributed the dilemma to two main factors: (1) "The foundation of most mathematics teachers in geometry is poor" (Adolphus, 2011, p. 143), and (2) geometry courses, as currently taught, do not help students develop an understanding of content but rather encourage memorization of definitions and theorems (Adolphus, 2011; Liu & Manouchehri, 2012).

Concurrently in the nation, the National Council of Teachers of Mathematics (NCTM) and the Common Core State Standards Initiative (CCSSI), with the goal of providing "an unprecedented opportunity for systemic improvement in mathematics education in the United States" (NCTM, 2014, p. 4), suggest "a foundation for the development of more rigorous, focused, and coherent mathematics curricula, instruction, and assessments that promote conceptual understanding and reasoning as well as skill fluency" (NCTM, 2014, p. 4). In high school geometry, for example, the Common Core State Standards require more precise definitions and more rigorous proofs, the concepts of congruence, similarity, and symmetry learned from the perspective of geometric transformation, and the definitions of sine, cosine, and tangent for acute angles founded with the Pythagorean Theorem in many real-world and theoretical situations (NGA & CCSSO, 2010). Also, both NCTM and CCSSI strongly suggest the integration of technology such as dynamic geometry software into teaching. NCTM (2014) issued the following technology statement: "For meaningful learning of mathematics, tools and technology must be indispensable features of the classroom" (p. 78). NGA and CCSSO (2010) emphasize, "Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena" (p. 74). Dynamic geometry software such as the Geometers' Sketchpad (GSP) helps develop students' understanding of mathematical concepts and increase their reasoning skills (Conference Board of the Mathematical Sciences [CBMS], 2001). One of the benefits of dynamic geometry software is its dragging feature, which enables students to see "the universality of theorems in a way that goes far beyond typical paper and pencil explorations" (CBMS, 2001, p. 132).

Along with the widespread use of dynamic geometry technology, many related research studies (Baccaglioni-Frank & Mariotti, 2010; Hannafin, Burruss, & Little, 2001; Hollebrands, 2007; Myers, 2009; Vincent, 2005) have been conducted and shown that if dynamic geometry technology is used effectively, it can make a significant difference in students' learning. When used as a cognitive tool, it can facilitate students' explorations and investigations, support their conjecture making and verification, promote their logical reasoning, and enhance their conceptual understanding of various geometric ideas. Further, it provides teachers a strong means to engage their students actively in learning while using technol-

Dynamic Approach to Teaching Geometry

ogy to solve meaningful and rich problems. Facing the challenge of geometry teaching and learning, a research-based solution is needed to help teachers transform their knowledge of dynamic geometry (geometric content knowledge of dynamic geometry, pedagogical knowledge, and pedagogical content knowledge) by integrating the dynamic geometry technology into their learning and teaching. This idea is consistent with NCTM and CCSSI technology statements. However, to meet their rigorous-curricula recommendations, the following difficulties need to be addressed: (1) Most teachers' geometry knowledge is weak, and (2) they typically have not learned geometry with dynamic geometry software and have not been prepared to teach using dynamic geometry approaches. These obstacles suggest the need for a well-designed teacher professional development program. This chapter reports on the results of a study using the technological pedagogical content knowledge (TPACK) construct (Mishra & Koehler, 2006; Niess, 2005) to investigate and assess secondary school geometry teachers' growth in their knowledge of teaching with dynamic geometry technologies.

BACKGROUND

Technological pedagogical content knowledge (referred to as TPACK) provided the research team with an appropriate conceptual framework for examining the development of teachers' ability to adopt and implement the dynamic geometry approach. This background supports how we developed our ideas for TPACK, assessing TPACK, the dynamic geometry approach, and professional development.

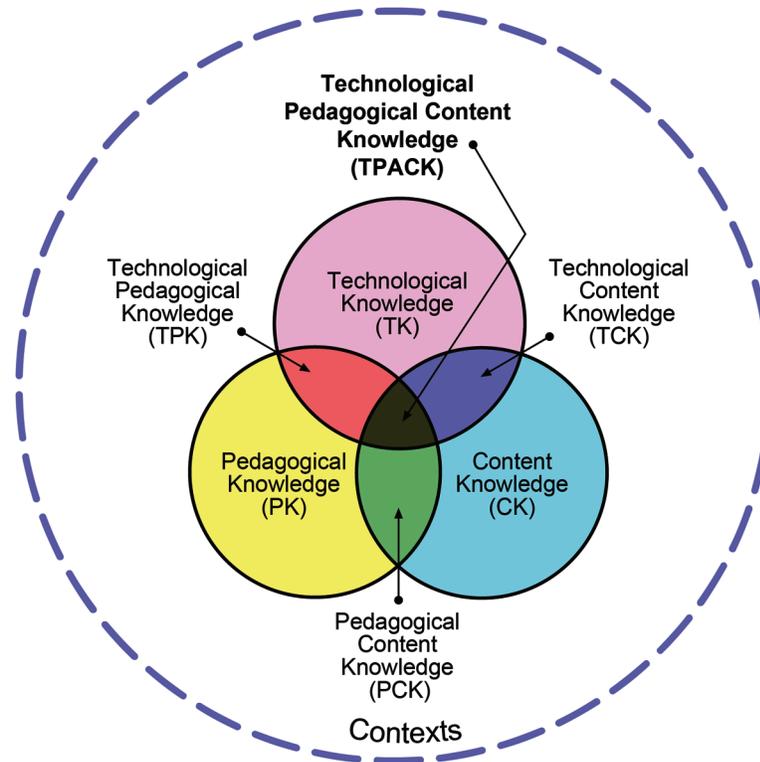
TPACK and its Assessment

In 2006, Mishra and Koehler proposed TPACK based on Shulman's concept of pedagogical content knowledge (1986) by including the knowledge of technology in the construct. The TPACK framework is comprised of seven subsets (see Figure 1): content knowledge (CK), pedagogical knowledge (PK), technological knowledge (TK), pedagogical content knowledge (PCK), technological pedagogical knowledge (TPK), technological content knowledge (TCK), and TPACK. TPACK addresses knowing how to teach a subject effectively with a given technology, "the basis of good teaching with technology" and "pedagogical techniques that use technologies in constructive ways to teach content" (Mishra & Koehler, 2006, p. 1029).

While the Venn diagram in Figure 1 represents the TPACK framework as "integrative" (a mixture of different types of knowledge), the wording leads to a more "transformative" perspective (Graham, 2011). Many researchers studying the development of TPACK look at each of the seven subsets separately, and thus they use the integrative view of TPACK; others use the transformative view of TPACK proposed by Angeli and Valanides (2005, 2009). Since the entire model is considered as TPACK and the middle subset is regarded as TPACK, then TPACK is the transformed knowledge, which is the target of teachers' knowledge development when teaching with digital technologies (Niess, 2015).

Since the inception of the TPACK construct in 2006, researchers have investigated methods of assessing TPACK and fostering its development (Lee & Hollebrands, 2008; Lee & Hollebrands, 2011). Koehler, Shin, and Mishra (2012) reviewed 66 empirical studies published between 2006 and 2010 and identified five types of measures used to assess TPACK: self-report measures, open-ended questionnaires, performance assessments, interviews, and observations.

Figure 1. The components of the TPACK framework
(Reproduced by permission of the publisher, © 2012 by tpack.org).



Self-report surveys (usually with a 5-point Likert scale) dominate the assessment of TPACK. Even though they are typically the most reliable and valid measures out of the five types of assessments (Koehler et al., 2012), research has shown that such instruments measure growth in confidence instead of growth in knowledge (Lawless & Pellegrino, 2007; Schrader & Lawless, 2004). Staus, Gillow-Wiles, and Niess (2014) compared inservice teachers' perceptions of their TPACK with their classroom practice. The results indicated that the observed TPACK was not always as high as the participants' perceptions about their TPACK. Since what is observed in practice affects student learning, conducting observations of teachers to assess their TPACK might be a more useful tool than measuring their perceptions. A group of researchers created rubrics for assessing lesson plans (Harris, Grandgenett, & Hofer, 2010) and classroom observations (Hofer, Grandgenett, Harris, & Swan, 2011) with respect to TPACK. However, those rubrics are neither content-specific nor technology-specific.

Sorto and Lesser (2009) developed an assessment of technological pedagogical statistical knowledge of middle school teachers. The instrument included six multiple-choice and open-ended items. The assessment did not rely on self-report and focused on content (statistics) and one type of technology (graphing calculator). Such measures are content-specific and technology-specific, and therefore, allow for measuring specific aspects of TPACK. However, such measures are costly to develop because of their specificity. Finding an appropriate sample of participants to validate these measures can also be challenging since a researcher has to find educators who are familiar with and use that specific technology in teaching.

TPACK and Mathematics Education

Empirical studies related to TPACK and mathematics education are scarce. Building on the TPACK framework (Mishra & Koehler, 2006), Niess et al. (2009) proposed a Mathematics Teacher TPACK Development Model. Teachers who develop their TPACK might progress through five levels: Recognizing, Accepting, Adapting, Exploring, and Advancing. This TPACK Development Model was derived from the researchers' observations of teachers learning to integrate spreadsheets into teaching (Niess et al., 2009) and on Rogers' (1995) model of the innovation-decision process that consisted of five stages: knowledge (corresponding to the Recognizing level of the TPACK Development Model), persuasion (Accepting), decision (Adapting), implementation (Exploring), and confirmation (Advancing). The descriptions of the five levels of the TPACK Development Model are as follows:

1. **Recognizing:** Teachers know how to use a given technological tool and can see how it fits in the mathematics curriculum, but do not integrate it into teaching.
2. **Accepting:** teachers form a positive or negative attitude toward teaching mathematics with a particular technology.
3. **Adapting:** Teachers decide to use or not to use technology in teaching mathematics.
4. **Exploring:** Teachers actively integrate technology as learning tools in teaching various mathematical topics.
5. **Advancing:** Teachers evaluate students' learning when using technology as a learning tool and support their decision to incorporate technology in teaching mathematics. (Niess, 2015; Niess et al., 2009)

In addition to the five TPACK development levels, the model is divided into four components, which were based on Grossman's (1989, 1990) four central components of PCK: conceptions of purpose for teaching subject matter, knowledge of students' understanding, curricular knowledge, and knowledge of instructional strategies. By amending the components with technology and mathematics as the content, the four components of TPACK (Niess, 2005; Niess et al., 2009) are as follows:

- **Access Component:** An overarching conception of teaching mathematics with technology.
- **Learning Component:** Knowledge of students' understanding, thinking, and learning mathematics with technology.
- **Curriculum Component:** Knowledge of where technology fits in the mathematics curriculum.
- **Teaching Component:** Knowledge of instructional strategies that integrate technology into teaching and learning of mathematics.

This model provided a useful framework for our study. It is important to remember that "a mathematics teacher may be at different levels for different themes" (Niess et al., 2009, p. 13). Also, teachers may be at different levels throughout their TPACK growth; they may progress and regress because TPACK growth tends to be nonlinear (Lyublinskaya & Tournaki, 2012; Özgün-Koca, Meagher, & Edwards, 2011).

The Dynamic Geometry Approach

Research suggests that alternatives to traditional instructional approaches can be successful in moving students toward meaningful justifications of ideas as students work cooperatively, make conjectures, present arguments and evidence, prove nonobvious statements, and formulate hypotheses to prove (Battista & Clements, 1995). Therefore, the role of teachers is to engage students in the “crucial elements of mathematical discovery and discourse” (Battista & Clements, 1995, p.50). These processes/elements are the characteristics of dynamic geometry environments advocated by the NCTM and CCSSO.

Dynamic Geometry (DG) is an investigative, exploratory study of geometry performed with the aid of interactive computer software. The major DG packages include the Geometers’ Sketchpad (GSP) (Jackiw, 2001), Cabri Geometry (Laborde & Bellemain, 2005), and GeoGebra (Hohenwarter, 2001). With the continuous real-time transformation often called “dragging” (Goldenberg & Cuoco, 1998) and other features such as dynamic measuring and animation, DG software can be used to help students engage in both constructive and deductive geometry (Schoenfeld, 1988) as they build, test, verify, and prove conjectures using easily constructible models. The instructional approach for using DG software, steeped in conjecturing-proving-based teaching, is referred to as the *DG approach* in this chapter. To guide the creation and selection of dynamic geometry professional development and dynamic geometry classroom implementation activities, the DG approach is operationally defined by the student expectations to:

- Construct dynamic geometric objects with dynamic geometry tools.
- Construct dynamic representations of problem situations with dynamic geometry tools.
- Perform actions (drag, measure, transform, and/or animate) on the constructed objects/situations.
- Observe variants and invariants related to the characteristics of objects under different actions.
- Investigate mathematical relationships and/or solutions in multiple ways.
- Formulate conjectures.
- Test conjectures.
- Receive immediate feedback from the software.
- Think mathematically and prove (or disprove) their conjectures.

The DG approach is further defined by teacher expectations to:

- Facilitate the student use of dynamic geometry software.
- Help students construct mathematical ideas through active explorations and investigations.
- Present prepared dynamic geometry environments for students to explore mathematical relationships when the constructions of the dynamic geometry environments are too complicated for students or when the constructions themselves are not the focus of the related explorations.
- Facilitate students’ argumentations by asking “why” questions – prompting students to furnish justifications for their statements and checking the validity of their justifications (Vincent, 2005).
- Extend students’ explorations by asking “what if” questions.

Professional Development Guiding Principles

Drawing on previous mathematics and TPACK professional development programs and related research studies (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Doering, Veletsianos, Scharber, & Miller,

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2009; Harris, 2008; Lampert, 2001; Stein & Lane, 1996; Visnovska, 2007), the project's professional development was driven by the following principles:

1. Bringing the areas of dynamic geometry technology, pedagogy, and geometric content knowledge together as one knowledge base;
2. Providing teachers with opportunities to engage in mathematically challenging tasks using DG tools;
3. Providing teachers with opportunities to enrich and improve their knowledge on students' learning and problem solving in DG environments;
4. Promoting autonomous and collaborative instructional decision-making while encouraging teachers to creatively use new instructional methods, tools, and resources; and
5. Supporting teachers to revise the core assumptions of their practice and helping them transform their classroom instruction.

Based on these guidelines, the project team developed and implemented a dynamic geometry -technology-centered professional development model for teachers. The model included approximately 76 hours of professional development over the course of two years and focused on building teachers' mathematical knowledge and knowledge of students' thinking, their capacity to use both categories of knowledge in practice, and their productive habits of mind (Doerr, Goldsmith, & Lewis, 2010) in DG environments. Under this model, important geometric concepts, processes, and relationships were presented or revisited through challenging problem situations, which were explored within the DG environment. Teachers learned DG skills while using them to tackle the problems. They eventually learned how dynamic geometry environments encouraged mathematical investigations by allowing users to manipulate their geometric constructions to answer "why" and "what if" questions, by allowing them to backtrack easily to try different approaches, and by giving them visual feedback that encouraged self-assessment. In addition to the tasks that required the teachers to construct, investigate, conjecture, and prove mathematical concepts using dynamic geometry, the professional development sessions also included discussion forums where teachers shared their insights on the DG approach, exchanged their recent experiences of what had and had not worked when using DG in their classes, communicated their understandings of the students' thinking on the related geometric investigations, and provided opportunities for teachers to offer each other feedback on their activities. Finally, these sessions allowed the professional development facilitators to provide comments to the participants on their DG tasks and on their forum discussions.

In terms of content, the professional development concentrated on important and commonly taught topics of high school geometry including those recommended by the CCSSI: triangle congruence and similarity, properties of special quadrilaterals, properties of circles, and geometric transformations, with related proofs.

To address the implementation fidelity of the dynamic geometry approach, a support team consisting of a project senior staff member, a doctoral student and a master teacher (an experienced teacher identified by school district as an expert in high school geometry) was formed. These three team members provided constant implementation support to the participant teachers. They regularly visited the participating schools to provide on-site assistance to the teachers; encouraged the teachers to phone and/or email the team members to ask questions, express concerns, and/or request help; provided case-by-case assistance to teachers that needed more help; and made great efforts to build stronger collaborative relationships between the university and the participating schools.

THE STUDY

Our study assessed and analyzed the development of TPACK of inservice teachers at the secondary level and focused on the transformed knowledge of technology, pedagogy, and content “where the inputs have been rearranged, merged, organized, assimilated, and integrated, such that they are no longer individually discernible” (Niess, 2015, p. 21). The study was a part of a larger research project, *Dynamic Geometry in Classroom Project* (DGP) funded by the National Science Foundation (NSF), which conducted repeated randomized control trials of the DG approach. The approach took advantage of the DG software (GSP for this project) to engage students in constructing mathematical ideas through experimentation, observation, data recording, conjecturing, conjecture testing, and proof. In DGP, 64 teachers were randomly assigned to treatment (DG) and control groups. The data collection lasted two years – the first year of DG implementation (or Year 1) and the second year of DG implementation (or Year 2). In this study, we focused on how teachers in the DG group transformed their knowledge for teaching geometry with DG. The role of professional development addressing the DG approach was a critical part of the study leading to the following research questions:

1. What is the impact of DG-oriented professional development on teachers’ observed TPACK?
2. What are the TPACK levels (e.g., Recognizing, Accepting, Adapting, Exploring, and Advancing) of teachers who incorporate the DG approach in teaching?

The TPACK framework (Mishra & Koehler, 2006) and the Mathematics Teacher TPACK Development Model (Niess et al., 2009) each served as lenses for this study. The TPACK framework provided a useful way to document and analyze technology integration in mathematics classrooms. The Mathematics Teacher TPACK Development Model provided comprehensive descriptors of each teacher’s knowledge related to technology integration in mathematics classrooms and assisted in identifying the TPACK levels of participants.

Participants

Participants of this study were 33 middle and high school mathematics teachers employed by public school districts in a large city who were in the treatment (DG) group of the DGP. The teachers participated in the professional development that helped them develop knowledge for effectively teaching geometry with the DG approach.

Transforming Teachers’ Knowledge in the Professional Development

All teachers participated in the professional development within DGP that started with a five-day summer institute. A team of university-based project leaders in mathematics and mathematics education collaborated with a group of school district master teachers to plan the professional development. The DGP Advisory Board consisting of mathematics educators with expertise in dynamic geometry technology reviewed and informed the final professional development plans. The focus of the professional development was to help the teachers transition from observing and exploring to theoretical thinking and proving, and transition to knowledge of dynamic geometry for teaching, through carefully designed

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activities, classroom climate that supports conjecturing and deductive justifications, and the guidance from the professional development facilitators (Arzarello et al., 1998; Hölzl, 2001; Öner, 2008).

Teachers began developing their TPACK by learning dynamic geometry skills. For example, they learned how to construct various geometric figures using GSP. Because of the nature of the dynamic geometry software and its natural connection to content (i.e., mathematics and geometry), teachers were also developing their TCK by learning dynamic geometry skills and using them to tackle mathematical problems. The professional development facilitators encouraged teachers to share ideas and help each other. They monitored the progress and asked questions to prompt teachers to furnish justifications for their statements and check the validity of their justifications (Vincent, 2005). This integration of content, technology and pedagogy (which is consistent with the TPACK framework) modeled what teachers were expected to do with students in their classrooms. Lastly, teachers began to design lessons and activities that incorporated GSP. The participating teachers presented their activities to their peers as a practice in teaching with dynamic geometry software and shared the activities among one another through a learning management system provided by the project.

Data Collection

The study utilized a mixed-methods design to gather data. Participants' TPACK development was captured and assessed through a Conjecturing-Proving Test, interviews, classroom observations, and task-based interviews. The instruments used in our study were specifically developed to assess teachers' knowledge for teaching geometry with dynamic geometry software. We focused on assessing TPACK through observations in practice as opposed to measuring teachers' perceptions solely with surveys. Since most TPACK-related studies focused on teachers' perceptions of their TPACK (Abbitt, 2011; Koehler, Shin, & Mishra, 2012; Schmidt et al., 2009;), our study intended to fill a gap in the TPACK assessment literature within the context of mathematics teaching and learning.

Conjecturing-Proving Test

A Conjecturing-Proving Test was developed and validated by the project team to measure teachers' knowledge that is closely aligned with the DG approach. This test consisted of 26 multiple-choice items and two free-response proofs. One of the multiple-choice items was adopted from the National Assessment of Educational Progress, but other multiple-choice items were developed or adapted specifically for the project. Many of the items were set in a teaching context, e.g. five items asked teachers to grade sample proofs using a provided rubric. The sample proofs were taken from an instrument developed by Healy and Hoyles (1998). The two free-response items were simple proofs taken from the Cognitive Development and Achievement in Secondary School Geometry Proof Test (Usiskin, 1982).

The instrument was administered to all participating teachers at the beginning and the end of the five-day summer institute. The purpose of administering the test was to see if the professional development focusing on the DG approach led to improved knowledge of geometry, especially conjecturing and proving in geometry. The goal was to compare pre- and post-test scores of teachers in the DG treatment group and the control group. The teachers in the control group also participated in the professional development, in order to address a confounding variable of teacher professional development. Since both of the groups received some professional development, the researchers eliminated the possibility that any differences between groups was attributable to the extra professional development provided by the

project. If differences appeared on the Conjecturing-Proving test between the DG and non-DG groups, the interactive dynamic geometry learning environment was the more likely source of variance since each group experienced comparable professional development. For the control group, the professional development facilitators used an approach that most teachers were currently using – direct instruction with the activity-based instructional methodology. The professional development for the control group was primarily lecturing, relying on the textbook, and providing participants with problem-solving exercises involving manipulatives but not using any technology tools.

Geometry Teaching Observation Protocol

An observation protocol was developed based on the DG approach and the Apple Classrooms of Tomorrow (ACOT) frameworks. The DG approach framework was chosen because one purpose of the observation was to capture the level of implementation fidelity of the DG approach, which included teachers' TPACK. The ACOT framework was chosen because it captured the teachers' stage of software use at the time of observation: entry, adoption, adaptation, appropriation, and invention (see Sorto, Jiang, White, & Strickland, 2015 for a complete Geometry Teaching Observation Protocol [GTOP] instrument). Items capturing teachers' observed knowledge used a 4-point Likert response scale from never occurred (0) to very descriptive (4). The scale reflected the degree to which the aspect was characteristic of the lesson observed, as opposed to the number of times the aspect occurred. Eight teachers were observed an average of four times during Year 1 by trained observers (two for each observation). Each observation was coordinated with each teacher ahead of time and took place in the teacher's classroom with his or her students. Teachers were chosen at random for observations based on two categories: years of experience (less than five and greater than or equal to five) and type of class they taught (Pre-AP Geometry or Regular Geometry).

The reliability and validity of GTOP have been established (Sorto et al., 2015). Estimates of inter-rater reliability were computed by a regression line of the observation of one observer on those of the other, results showed a correlation coefficient of 0.901. Estimates computed for each dimension or subscale and the instrument as a whole provided additional evidence of reliability. The dimension that included teachers' observed knowledge had a correlation coefficient of 0.815 with the overall instrument. Evidence of construct validity was established through expert review, a correlational analysis to measure the extent to which each dimension predicts the total score and two case studies.

Case Study Instruments

Four data collection tools – TPACK Development Interview Protocol, TPACK Observation Protocol, "Teaching with Dynamic Geometry" Task Interview Protocol, and Dynamic Geometry Implementation Questionnaire – were used with a group of four teachers in a case study. The case study was conducted to obtain an in-depth understanding of teachers' TPACK development. The participants were selected based on their high implementation fidelity of the DG approach during Year 1. The selection process utilized a purposeful sample to provide data that would give insights into teachers' practice and obtain a rich description of their DG integration. Table 1 shows the participants' characteristics.

The TPACK Development Interview Protocol (see Appendix 1 for sample questions) was created by one of the researchers to facilitate and guide semi-structured open-ended interviews. The focus of the interviews was on how teachers developed their TPACK and how they incorporated it into teaching

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Table 1. Case study teachers' characteristics

Teacher	Gender	DG Experience as a Learner	Total Teaching Experience (Years)	Teaching Geometry Experience (Years)	Teaching with DG Experience (Years)
A	M	No	2	2	1
B	M	No	4	4	1
C	F	Yes	2.5	2	1
D	F	Yes	19	9	7

geometry with dynamic geometry. The Mathematics Teacher TPACK Development Model (Niess et al., 2009) and the TPACK interview protocol developed by Niess, van Zee, and Gillow-Wiles (2010) informed the creation of several interview questions. The audio-recorded interviews were conducted with the four teachers at the beginning of Year 2.

The TPACK Observation Protocol was also developed to assist in classroom observations and identify strengths and areas for improvement of teachers' practices related to their TPACK and integrating dynamic geometry in teaching. The four teachers were observed two to four times during lessons that incorporated dynamic geometry. Each observation was video and/or audio recorded, and transcripts were produced for analysis. Unlike GTOP, this observation protocol focused solely on teachers' TPACK and was qualitative in nature. The protocol addressed questions such as:

- What is the role of technology in this lesson?
- What are the benefits of using technology in this lesson?
- Are there any apparent disadvantages of using technology in this lesson?
- How does teacher manage the classroom when teaching this lesson?

The "Teaching with Dynamic Geometry" task interview was developed as a result of a classroom observation of another teacher during Year 1. This task interview provided an avenue for assessing teachers' TPACK related to the same geometric topic. Therefore, the interview data allowed for comparison of different participants' TPACK using the same context. The interview consisted of two phases, exploration and conjecture, which came from two lesson components in which students in the observed classroom were participating. During the interview, teachers were asked to reflect on and analyze students' work from those two phases.

The DGP team developed the Dynamic Geometry Implementation Questionnaire. Selected data from the questionnaire responses were used to supplement data collected via interviews and observations. This process allowed for triangulation of the collected data.

DATA ANALYSIS AND FINDINGS

The researchers employed quantitative and qualitative analysis of data. The Conjecture-Proving Test and GTOP data were analyzed quantitatively while the rest of the data were analyzed using qualitative case study methods.

Conjecturing-Proving Test

For the Conjecturing-Proving Test, a statistic that represented the knowledge measured was calculated by adding the number of correct multiple-choice responses with points from free-response items. Despite the short interval between the administration of the pre- and post-tests, teachers averaged 20.49 points on the pre-test and 21.86 points on the post-test with an average gain of 1.37 points. A paired-sample t-test showed that this gain was statistically significant ($p = .003$). This result provided evidence that the professional development had a positive effect on teachers' conjecturing and proving capabilities as measured by the test, for both the DG and non-DG groups. The average gain was greater for the DG group (1.56 points) than the non-DG group (1.18 points), but this difference was not statistically significant ($p = .670$). However, in another component of the project, students of teachers in the DG group scored significantly higher than students of teachers in the non-DG group on a geometry achievement test (Jiang, White, & Rosenwasser, 2011) administered to students at the end of Year 1. This result further supported the efficacy of the DG approach and confirmed that professional development is necessary to continue transforming teachers' knowledge for teaching with dynamic geometry tools.

Geometry Teaching Observation Protocol

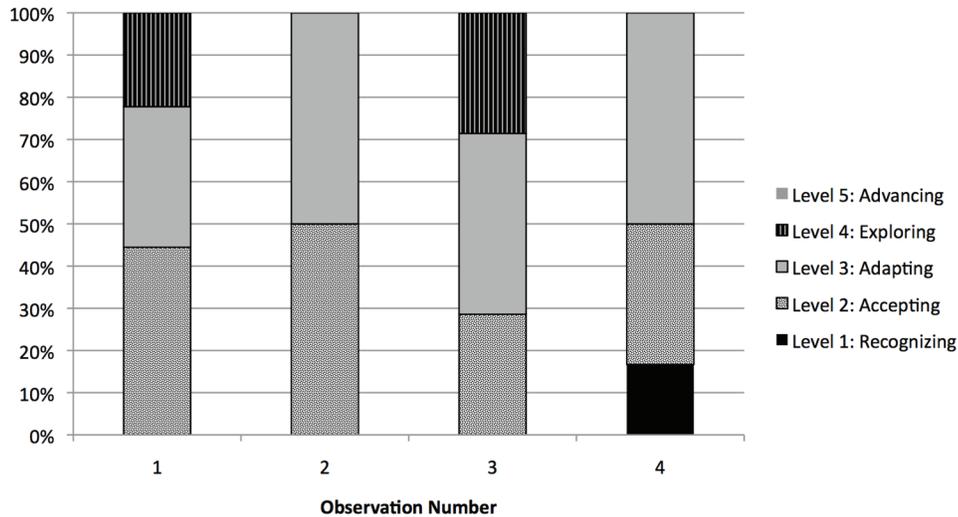
The Geometry Teaching Observation Protocol (GTOP) was used to document four different aspects of teachers' knowledge: knowledge of grade 10 geometry, knowledge of general instructional techniques, PCK, and TPACK. Teachers' knowledge of grade 10 geometry obtained the highest average score of 3.23 on a 4-point scale, followed by knowledge of general instructional techniques (2.82). The PCK and TPACK constructs obtained lower scores (2.64 and 2.80 respectively). From analysis of GTOP data, teachers' knowledge of how to integrate these domains to help students learn was less evident. In a correlational analysis, the teachers' knowledge domain was significantly ($p < 0.01$) correlated with all of the other aspects of the GTOP instrument. Evidence also identified that the teachers guided their students through explorations and production of conjectures. Finally, the data indicated that the teachers provided very little guidance to their students on proving (or disproving) conjectures. Over the four observational instances, the trend for teachers' knowledge remained almost at a constant level across all four observations. Figures 2 and 3 present the teachers' observed TPACK levels during the two years of DG implementation. Advancing (level 5) was not observed at all, and Recognizing (level 1) was only observed once during Year 1. The Accepting level was witnessed during 19 lessons, the Adapting level during 22 lessons, and the Exploring level during 11 lessons throughout the two years of DG implementation.

Case Study Analysis and Results

A rubric (see Appendix 2) based on the five TPACK development levels (Niess et al., 2009) and the four TPACK components (Niess, 2005) was developed to identify the teachers' TPACK levels. Lyublinskaya and Tournaki (2012) created a similar rubric for assessing TPACK of algebra teachers using TI-Nspire handheld technologies. Based on the collected data, TPACK levels were identified for each participant for each component (Access, Learning, Curriculum, and Teaching). As in the rubric by Lyublinskaya and Tournaki (2012), the overall TPACK level of each participant was determined by the lowest level observed across the four components. That is, for a teacher to achieve a particular level of TPACK, the collected data needed to provide evidence for at least that particular level across all four themes.

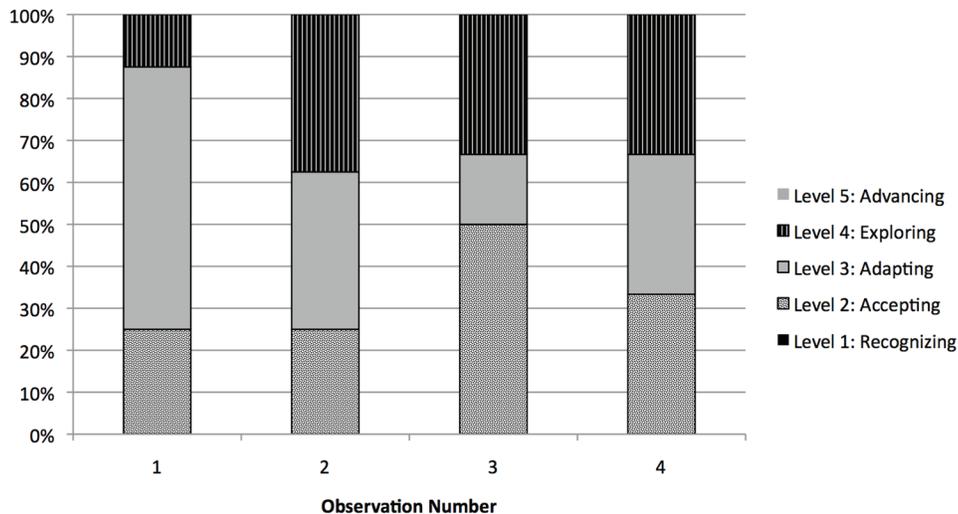
Dynamic Approach to Teaching Geometry

Figure 2. Observed TPACK levels during four observations during Year 1



To produce high-quality data that were credible and trustworthy, several data sources were used, and all of the data were triangulated to ensure the internal validity of the study (Merriam, 2009, p. 215). Checking what was learned through interviews with what was noticed during observations, comparing findings from interviews with those from observations, conducting multiple observations of the same participant were all parts of data triangulation. Member checking was employed to establish credibility (Lincoln & Guba, 1985) and involved “taking data, analyses, interpretations, and conclusions back to the participants so that they can judge the accuracy and credibility of the account” (Creswell, 2007, p. 208). Preliminary analyses were presented to the participant teachers, who were asked for their opinions and if they would add anything that was noteworthy. We identified a TPACK level for each teacher from the case study.

Figure 3. Observed TPACK levels during four observations during Year 2



Teacher A's TPACK

According to data collected through the interview and implementation questionnaires, Teacher A's TPACK was at the Adapting level for the Access TPACK component (Niess, 2005; Niess et al., 2009). Teacher A allowed his students to use dynamic geometry software for explorations and discovery of new geometric topics only in specific units. For example, he did not use the software for teaching and learning of 3-dimensional figures, but he used it for triangle congruence and geometric transformations. He experienced challenges with the dynamic geometry implementation because his school district adopted a new scope and sequence, so he integrated dynamic geometry software to enhance geometry learning only in some units. Also, he taught geometric concepts differently with the DG approach than without it because his students got to discover various geometric relationships instead of finding out about them through lecture. For example, during one of the observed lessons, his students investigated and made connections between different options for triangle congruence.

One of the observed lessons provided evidence of the Advancing level of TPACK for the Learning theme. In that lesson, it was apparent that Teacher A seriously thought about how to integrate dynamic geometry. Because the goal of the lesson was to come up with different conjectures related to the midsegments of a triangle, Teacher A knew that DG could help students with their thing and understanding of this topic and their learning would be enhanced through the use of dynamic geometry software. However, based on the remaining lessons and other data sources, Teacher A's TPACK was at the Exploring level for the Learning component. He guided his students in understanding by giving "hints as to where they should be looking."

The observed lessons and the accompanying student handouts provided evidence that Teacher A created his own DG-enhanced activities. This result implied that his TPACK was at the Exploring level for the Curriculum component.

Based on the observed lessons, it was clear that Teacher A used DG for higher-level thinking activities (e.g., his students formed and justified conjectures) placing his TPACK at the Exploring level for the Teaching component. He engaged students in explorations using dynamic geometry software where students took control of their learning, and he was in the role of a guide. He both probed students with questions and answered questions from them, thereby creating an environment where students could explore and discover geometry. He managed the DG activities so that his students were engaged and self-directed in their learning of geometric concepts.

Teacher A's TPACK levels were therefore identified at the Exploring level for three components (Learning, Curriculum, and Instruction) and at the Adapting level for the Access component. Therefore, Teacher A's overall TPACK was at the Adapting level, but with strong evidence of progressing to the Exploring level.

Teacher B's TPACK

Teacher B's TPACK was at the Adapting level for the Access component because he incorporated dynamic geometry only into specific units and lessons. He allowed his students to explore geometric concepts with dynamic geometry software infrequently because of limited access to a computer lab. He viewed the incorporation of dynamic geometry as more efficient, allowing him to demonstrate more examples. Also, he taught geometric concepts differently with dynamic geometry software than without it, and his students got to explore and discover the concepts instead of learning about them through lecture.

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Teacher B introduced geometric concepts to students and used DG as a learning tool only in some units. Initially, Teacher B provided evidence for the Recognizing level of TPACK for the Learning component. He stated in the interview,

If there was no project... Honestly, if there was no project [DGP], I would probably have a hard time getting to the lab and using dynamic geometry. I would still give class demonstrations, showing the different topics, but taking a class to a lab to explore... Honestly, if I were not in the project, it would probably not happen.

Immediately after that he added, “I am not sure how beneficial it was to any of the students the majority of the time. So I am not sure if it was worth the time spent versus [using a] traditional approach.” He also added, “We have been learning geometry without it [dynamic geometry] for a long time.” These comments suggested the Accepting level of TPACK because Teacher B expressed his concern for the lack of development of appropriate geometric thinking skills in his students when they used dynamic geometry for explorations. However, throughout the study, Teacher B allowed his students to explore selected geometric concepts with dynamic geometry, indicating his TPACK was at the Adapting level for the Learning component.

Data from the observed lessons suggested the Adapting level for the Curriculum component because Teacher B understood that there was a benefit of using dynamic geometry in teaching and learning geometry curriculum. He also stated in the interview, “I think it [dynamic geometry] is a benefit to a fair enough group of students. It [integration of dynamic geometry] is a worthy cause. So it is worth going to the lab to help students that will benefit from it [using dynamic geometry to learn geometry].”

Teacher B’s TPACK was between the Adapting level and the Exploring level for the Teaching component. He incorporated dynamic geometry into classroom instruction by engaging his students in high-level thinking activities. Also, during the professional development sessions, he shared his DG-based lessons and ideas with peers. This recognition indicated the Exploring level. Also, he engaged students in geometric explorations using dynamic geometry where students took control of their learning, and he was in the role of a guide. He provided his students with various instructional strategies to engage them in thinking about the geometric concepts under investigation. However, Teacher B tended to take more control of the activities and often did geometric constructions for his students instead of guiding them, indicating the Adapting level. Thus, Teacher B’s TPACK levels were rather consistent through all the components, and his overall TPACK was at the Adapting level.

Teacher C’s TPACK

Teacher C’s TPACK was at the Adapting level for the Access component. She used DG in specific units, e.g., to introduce points, lines, and angles. Also, finding time to integrate it was difficult because of the extensive curriculum. When she got to use dynamic geometry software in her instruction, she taught geometry differently and enabled her students to understand geometric concepts through investigations.

According to the interview and observation data, Teacher C was at the Adapting level for the Learning component. She introduced geometric concepts to students and used dynamic geometry as a learning tool only in some units. Teacher C’s students used dynamic geometry as a learning tool, but she did not assess their thinking by incorporating dynamic geometry software.

Teacher C set a goal for herself to have her students use computers every day as they came into class, at least for a few minutes, because she wanted to make the technology a more integral part of the learning. This information indicated the Exploring level of TPACK for the Curriculum component. However, according to data reported through the implementation questionnaires, she did not get to implement her goal. The observation data provided evidence for the Adapting and the Exploring levels. During a lesson on the $30^\circ - 60^\circ - 90^\circ$ triangle, she was using DG for an already-learned concept, suggesting the Adapting level. However, during a lesson on triangle congruence, she engaged her students in a higher-level thinking activity, indicating the Exploring level.

The observation data suggested the Exploring level of TPACK for the teaching component because Teacher C engaged students in explorations using DG approaches where students took control of their learning, and she was in the role of a guide. She provided extensive introductions and summaries to the lessons, during which students used DG software. She made sure to tell students at the end of the lesson what they were supposed to discover during their dynamic geometry explorations.

The findings showed that Teacher C's TPACK levels were at the Adapting and the Exploring levels. Therefore, her overall TPACK was at the Adapting level.

Teacher D's TPACK

The data showed evidence of Teacher D's TPACK at each of the first four levels for the Access component. Her TPACK was at the Adapting level because she used dynamic geometry only in specific units such as those that required accurate measuring. By recognizing challenges in teaching geometry with DG (such as the extensive curriculum), Teacher D discovered strategies to deal with those challenges (e.g., allowing students to use DG one day every week). This finding indicated the Exploring level of TPACK. Lastly, Teacher D's TPACK was at the Recognizing and Accepting levels for the Access component because she preferred to use dynamic geometry mainly for accurate measurements and efficiency instead of exploring more complex geometric topics.

Teacher D knew that the integration of dynamic geometry in learning was a worthwhile effort. Throughout all the years she had been using it in classroom instruction, she identified topics, for which dynamic geometry facilitated better learning and understanding of geometric concepts (e.g., parallel lines with a transversal). This finding suggested the Exploring level of TPACK for the Learning component. At the same time, her TPACK level was at the Recognizing level because she preferred to use DG as "a teaching tool rather than as a learning tool" (Niess et al., 2009, p. 21).

Teacher D's TPACK was at the Recognizing level for the Curriculum component because she focused on how things were displayed with DG (e.g., the use of different colors). Additionally, one of the observed lessons indicated the Accepting level because Teacher D tried one dynamic geometry activity with her first class, but then altered it to a less-challenging activity (for the rest of her classes), which in turn was a repetition of the lesson from the previous day. This result showed that Teacher D might have had difficulty identifying appropriate topics in the geometry curriculum for her students to explore with DG software.

Based on one of the observed lessons, Teacher D's TPACK was at the Adapting level for the Teaching component. She incorporated DG for enhancing or reinforcing already-learned topics as well as for activities outside the curriculum. However, she showed progress towards the Exploring level because she guided her students in dynamic geometry explorations and utilized various instructional strategies when using DG.

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Teacher D's TPACK levels ranged widely (from Recognizing to Exploring), making it difficult to identify an overall TPACK level for her. However, since much evidence was at the Recognizing level, Teacher D's overall TPACK was identified at the Recognizing level.

Summary of Teachers' TPACK Levels

Figure 4 summarizes the TPACK levels for the four teachers. Teacher A, Teacher B, and Teacher C's overall TPACK was at the Adapting level. Teacher A showed a potential for moving to the Exploring level if he increased the dynamic geometry integration in his classroom. Teacher D's overall TPACK level was at the Recognizing level; however, because she used dynamic geometry with her students on a weekly basis, she could move quickly to the Adapting level if she used DG approaches more as a learning tool instead of a teaching tool and for Exploring complex geometric concepts instead of using it merely as a substitution of traditional non-technology tools.

"Teaching with Dynamic Geometry" Task Interview

The audio recorded task interviews were transcribed and analyzed in two phases, exploration and conjecture. During the exploration phase, teachers were to analyze one student's work with GSP and identify a geometric transformation used by a student (see Figure 5) and for the conjecture phase, teachers analyzed the student's conjecture involving geometric reflections (see Figure 6).

After examining Figure 5, three teachers concluded that the depicted student's transformation was a translation. The fourth teacher, Teacher B, first stated that it was a reflection over the x -axis, but the labels did not correspond. He kept looking at the two figures seeing a reflection, but after a few moments he concluded that it was a translation because of how the vertices were labeled. However, he was right - the actual transformation was a reflection over the x -axis, but the corresponding labels of the pre-image and the transformed image implied a translation. GSP software automatically labels the vertices of transformed figures with corresponding letters so the most logical explanation by an experienced GSP-using teacher would be for the mismatched labels to have been changed by the student to the labels of the reflected figure illustrated in Figure 5.

Figure 4. Teachers' TPACK levels for each component

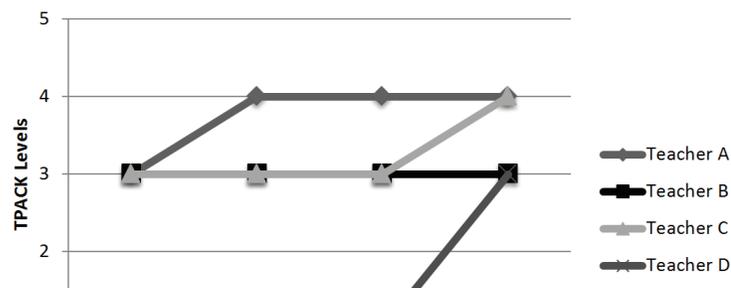
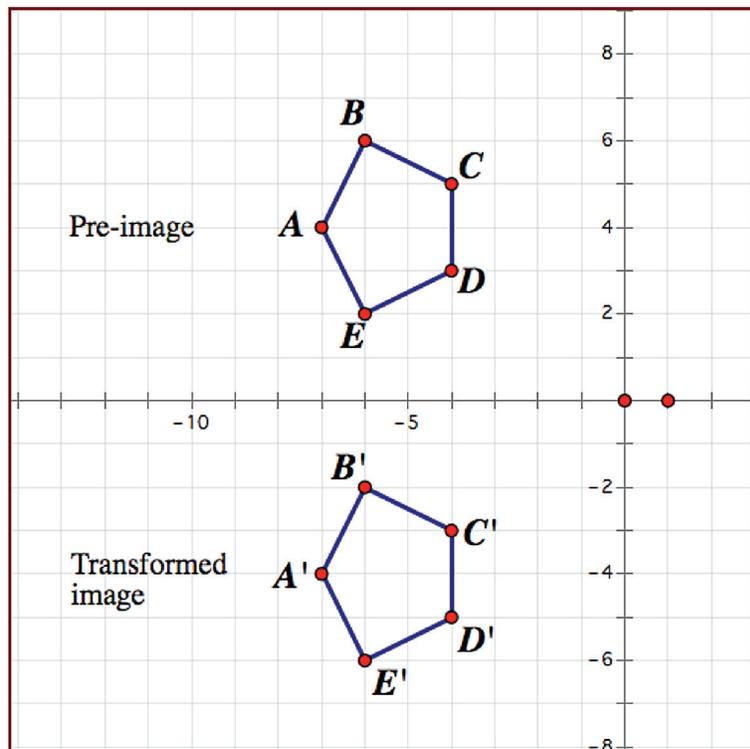


Figure 5. Transformation from the “Teaching with Dynamic Geometry” task



Next, teachers were asked: “If the student reflected the pre-image and then changed the labels of the reflected image to those shown in the figure, what would you do to help this student?” Three teachers replied that they would change the pre-image or use a different figure so that the reflected image did not resemble a translation. These teachers stated that they would utilize the dynamic features of the software to “correct” this situation implying that their TPACK was at least at the Exploring level. The fourth teacher, Teacher D, on the other hand, did not mention using GSP at all. She would try to explain to the student that the corresponding vertices had to be labeled appropriately, e.g., B' would have to be E' , C' would have to be D' . When asked to explain her decision, she mentioned folding the paper along the x -axis to make it more visible to the student which vertices were corresponding to which vertices. Teacher D’s response and her lack of reference to GSP was intriguing because during the interview she mentioned the topic of geometric transformations as one that lends itself to GSP use as a learning tool—“I like doing rotations and reflections [in GSP] because those are hard concepts.” The lack of GSP incorporation in her response to the student’s misconception was an unexpected finding since Teacher D had many years of experience in DG integration, and she mentioned that she could not teach without it. It was surprising that she did not incorporate it in this task. This result thus implied that her TPACK was at the Recognizing level, confirming the previously mentioned findings of her TPACK levels. This result also showed that she still had room for developing her TPACK into higher levels, and the experience of technology use in the classroom instruction did not necessarily translate into improved TPACK suggesting that a sustained professional development was necessary.

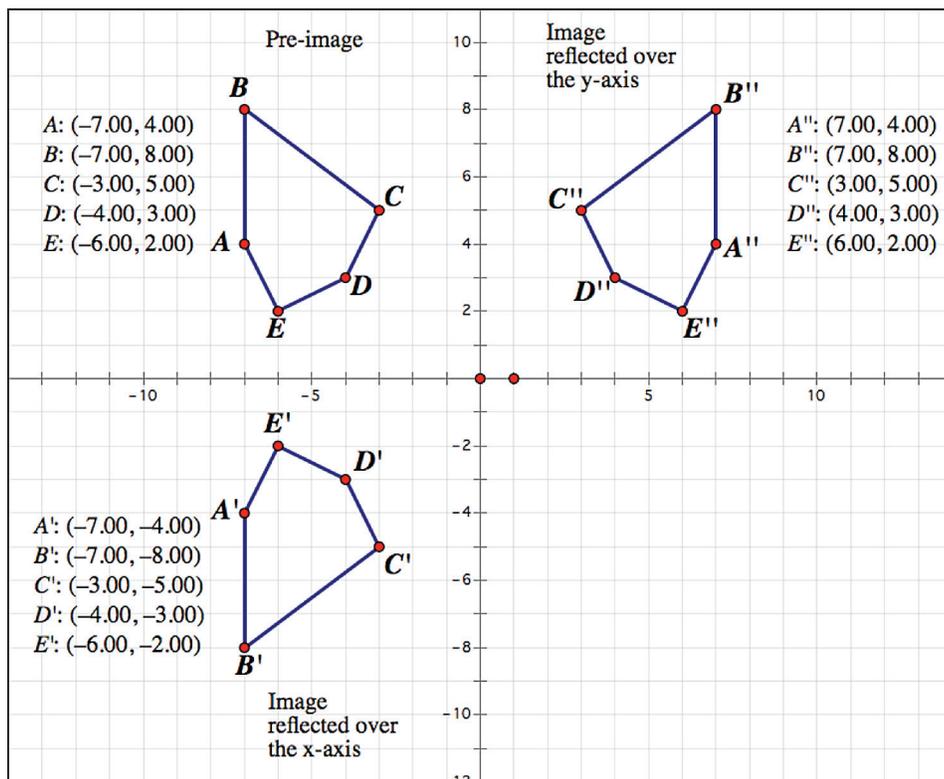
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During the conjecture phase, the teachers were to analyze a conjecture formed by a student with regards to reflections over the x -axis and the y -axis. The student, Student E, wrote the following conjecture: “When I move one of the corners of the pre-image, the corresponding corner of the image reflected over the x -axis also moves. The same thing happens with the image reflected over the y -axis. The coordinates of the reflected images change from positive to negative and vice versa.” Figure 6 shows Student E’s work in GSP, on which she based her conjecture.

All teachers provided evidence through their comments of understanding what Student E was trying to say and agreed that the first part of the conjecture was right; the only thing the teachers would change was the student’s last sentence: “The coordinates of the reflected images change from positive to negative and vice versa.” They thought that the statement was not precise. Teacher A said, “I think the student probably knows what’s happening but is having trouble putting it into words when saying that the coordinates change from positive to negative.” Teacher A would tell the student to look at the coordinates and ask, “Is everything changing? What is changing? And which axis?” This result showed that when teachers know their students, they understand the students’ thought processes and are competent translators of students’ ideas/expressions.

Teachers B and D had similar interpretations and would make sure that the student revised the last sentence and to be more specific about what exactly happened. Teacher C had a slightly different response and said she would have her students log the coordinates and look at more instances because:

Figure 6. Screenshot illustrating coordinates of the pre-image and the images reflected over the x -axis and the y -axis



If a student is looking just at her image, it is hard for her to see the big picture of what is happening to the x and y very specifically. So it is more general. Maybe if it was tabled with several people's numbers, then it is easier to see more specific. This is what is happening: across the x -axis, x stays the same, and y changes; across the y -axis, y stays the same and x changes. I think more data and other people's [data] would clarify that because they have much more confidence in other people's data than their own, too.

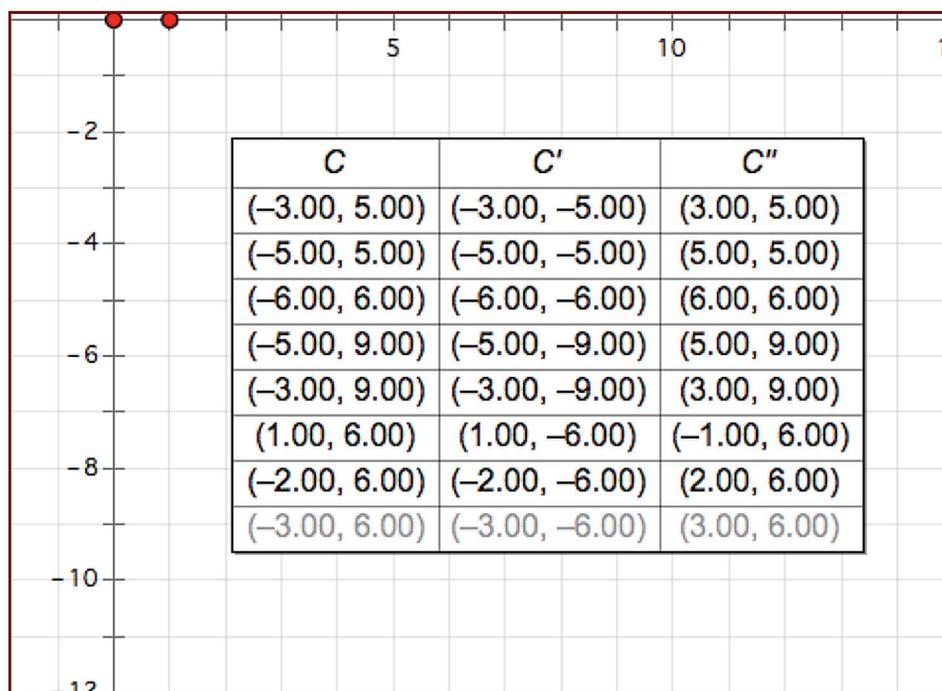
Even though Student E already had five points with coordinates, Teacher C thought it would be better to have even more examples; however, she did not mention using GSP in any way to achieve that. By using the dragging feature of GSP, students can move any of the vertices to obtain “more points” and quickly construct a data table (see Figure 7 for an example), which would address the teacher's desired outcome. Since none of the teachers mentioned the use of GSP to improve the precision of the student's conjecture, their TPACK was considered at the Recognizing level at this point.

Response to the Research Questions

Our study examined the impact of professional development focused on the DG approach and teachers' TPACK growth over a course of two years. The specific research questions guiding the study were:

1. What is the impact of DG-oriented professional development on teachers' observed TPACK?

Figure 7. Screenshot of a sample data table constructed in GSP for the coordinates of point C from the pre-image and its corresponding points after reflection over the x -axis and over the y -axis, coordinates of points C' and C'' respectively, after dragging point C



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2. What are the TPACK levels (e.g., Recognizing, Accepting, Adapting, Exploring, and Advancing) of teachers who incorporate the DG approach in teaching?

Addressing the first research question, the results of this study provided strong evidence about the effectiveness of the DG approach in teachers' development of TPACK and their classroom instruction for teaching geometry. Through multiple professional development sessions, the teachers were stretched beyond the typical 'high school math' material in a technology-intensive but comfortable and safe learning environment. Through the four-teacher case study of TPACK development, all participants displayed a strong knowledge of dynamic geometry during the interview and observations. They participated in all or almost all professional development sessions provided by the DGP and valued them because not only did they learn how to use DG approaches, they also learned and shared implementation strategies with their peers. That collaboration was valuable to them and assisted them in incorporating DG in their classroom instruction.

The GTOP and the case study data provided evidence for answering the second research question concerning the teachers' TPACK levels. Based on the GTOP data analysis, the highest level of TPACK (Advancing) was not observed. Conversely, the lowest level of TPACK (Recognizing) was only observed once. The three middle levels (Accepting, Adapting, and Exploring) were present with the Adapting level being most frequently observed, which was also observed in the case study. In the four-teacher case study of TPACK development, there were many variations in TPACK levels for most of the teachers, with overall TPACK levels identified as Adapting for three teachers and Recognizing for one teacher.

SOLUTIONS AND RECOMMENDATIONS

The main findings of the study revealed that:

1. The DGP Professional Development provided a contextual frame that facilitated the teachers' TPACK growth;
2. The DG teachers' mean gain score on a conjecturing-proving test was higher than that of the control teachers;
3. The DG teachers not only learned how to use dynamic geometry, but also learned and shared implementation strategies with peers;
4. The teachers' TPACK levels centered around the three middle levels (Accepting, Adapting, and Exploring); and
5. For most teachers, there were many variations in their TPACK levels among the four components of TPACK (Access, Learning, Curriculum, and Teaching).

Through this research, valuable lessons were learned about implementing professional development focused on TPACK development including addressing limitations and hurdles that arise within actual school settings. Further, the researchers identified areas that can benefit from further development and study in order to expand the foundation of TPACK research. Those areas include recommendations for professional development, school-based research, technology access and methodology.

The DGP professional development model used in this study was effective in helping secondary mathematics teachers' progress to more mature levels of the TPACK Development Model. Evidence from

this study indicated that teachers participating in the professional development experience transformed their TPACK toward the Exploring level (level 4). Our five-day summer institute proved to provide effective TK and a more transformed TPACK for participating inservice teachers based on the TPACK Development Model. Critically important to our professional development model was the sustained support received both from the master teachers and the university faculty members over the two years of the study. Our lesson management system also proved to be a key asset that allowed teachers to share lessons they had developed with peers. The process of sharing lessons as part of the summer institute and also throughout the school year using the lesson management system served not only to motivate teachers to create tools that could be used by their colleagues but also provided a motivational means of moving toward the Advancing level of TPACK where they evaluated the appropriateness of using technology for teaching and learning geometry.

Strong support from the school and district administrators was an important condition for research success. In this project, most schools, in fact, did provide the needed support as a result of the district leaders' agreement to participate in the NSF-funded project. In a few schools where the administrators were less inclined to provide support, teachers experienced various difficulties in accessing resources and time for students to participate in dynamic geometry investigations. Without access to technology and clear support from school leaders, those teachers were not able to fully develop their TPACK. Securing school and district administrators' support in integrating technology increases the likelihood of teachers using technology in the classroom.

In the continually evolving world of technology, it is important to take advantage of new technologies. Scheduling needs for school computer labs for test preparation and other subjects proved to be a barrier to implementing the DG approach and limited teachers' opportunities to grow their TPACK in practice. In most schools, teachers were able to conduct dynamic geometry explorations with their students in computer labs at most one or two times a week and had no further access to the direct interaction with dynamic geometry environments. This limitation can now be addressed by taking advantage of the dramatic role that tablet devices (e.g., iPads) can play in everyday instruction. A free tablet application that helps students develop deeper mathematical understanding using the dynamic features of GSP is Sketchpad Explorer. It allows students "to interact with, and investigate, any mathematical document created in GSP. With a simple, powerful, multi-touch interface, Sketchpad Explorer puts mathematics comprehension literally at fingertips" (Key Curriculum, 2015). Low-cost tablets and free apps effectively address the "inadequate time" issue that impacted the DG implementation fidelity and intensity during the study.

Multiple data collection instruments are essential for assessing TPACK development, and if instruments are not available, research must develop the necessary measurement tools. In this study, participants' TPACK growth was captured and assessed through a Conjecturing-Proving Test, interviews, classroom observations, and task-based interviews. In doing so, we were able to use various methods, instruments, and protocols to capture different dimensions of the participating teachers' TPACK transformation. One limitation of our study was the lack of observations of all teachers. Even though we conducted repeated classroom observations of a large sample of teachers in the DGP, it was not possible to observe all of them. The inclusion of classroom observations of all participating teachers, and not limiting observations to a smaller sample, would produce a more complete and detailed documentation of technology integration and TPACK growth. Also, increasing the sensitivity of instruments that measure changes in teachers' beliefs and disposition tied to mathematical practices would assist in documenting TPACK transformation.

FUTURE RESEARCH DIRECTIONS

While the teachers displayed a strong knowledge of dynamic geometry during the interview and observations in the case study of TPACK development, their TPACK levels remained at the three middle levels. None of them reached the highest level of TPACK (Advancing). The GTOP data analysis returned similar results. In addition, these teachers, although made significant progress in Exploring and conjecturing in a dynamic geometry environment, still struggled in proving conjectures. Most teachers' knowledge of geometry is weak and secondary mathematics teachers do not recognize "proof as a tool for learning mathematics" (Knuth, 2002, p. 379). These findings remind researchers that helping teachers develop their TPACK and strong proving abilities is certainly a challenging undertaking. Future research needs to investigate more fully what strategies should be developed and implemented to strengthen the professional development model presented in this chapter to produce high-quality teachers for geometry classrooms.

As a result of the DGP and its accompanying professional development, secondary teachers in an urban school district demonstrated TPACK growth. Explorations in other settings, particularly in rural schools where teacher peer group support is less localized and where technology support may not be as plentiful, are worthy of future research. Also, while our case studies provided examples of teacher growth tied to the TPACK Development Model, more detailed analysis is needed to match task and classroom enactments to the different levels of the model. Following on the findings of this study, additional research questions may be pursued by researchers interested in investigating teachers' TPACK transformation: What is the relationship between mathematics teachers' beliefs and TPACK growth when integrating dynamic geometry software in the classroom instruction? How do mathematics teachers in rural school districts develop TPACK for teaching in a DG environment? Lastly, this study focused exclusively on geometry. The assessment development by Sorto and Lesser (2008) cited earlier addressing statistics suggests that future investigations in this content area for middle as well as high school teachers using the observation protocols and case studies methods from this study are well positioned.

Debates about how best to prepare teachers to effectively use technological tools to enhance the learning of mathematics are likely to persist mainly because of the rapid change in educational technologies.

The increasing attention to mathematical practices (NCTM, 2014; NGA & CCSSO, 2010) requires that secondary teachers implement high-leverage tasks that address problem-solving, reasoning and proof, strategic use of technology tools, and mathematical modeling. With the use of technological tools to learn mathematics becoming an expectation for most secondary mathematics teachers, additional research should investigate the effects of other technology-oriented professional development models on teachers' TPACK development.

With respect to the DG approach to teaching geometry using a specific software (GSP) to develop TPACK, results of this study do not answer the question "Is the DG approach the best way to develop TPACK?" However, given that teachers who participated in the professional development program demonstrated TPACK growth to some extent, results of this study support the belief that TPACK can be developed, especially under the conditions provided by the project where school-based mathematics leaders and mathematics education researchers played a significant role.

CONCLUSION

This study investigated the impact of a DG-oriented professional development on the participating teachers' TPACK growth. As part of the project's professional development, the teachers had opportunities to become familiar with dynamic geometry tools and use dynamic geometry features to investigate problem situations for which the contextual frame facilitated the teachers' TPACK transformation, including ways in which they could use dynamic geometry technology with their students. They thrived on the challenge, the rigor, and the applicability of what they learned. They also appreciated the constant collaboration with colleagues as a continued growth process. In particular, they increased their levels of conjecturing about geometric relationships and re-conceptualized the way they presented geometry concepts to their students. This evidence can inform teachers' and school leaders' decisions about transforming curriculum and instructional practices in geometry as a core high school mathematics course.

A Conjecturing-Proving Test was given to the teachers to assess what they had learned from the summer institute. Both DG treatment and control group teachers made significant gains on the Conjecturing-Proving Test. Although the two groups of teachers did not differ significantly on their mean gain scores, the DG teachers' mean gain score was 32% higher than that of the control teachers.

Based on the GTOP and the case study data, most teachers displayed knowledge at the Adapting level of the TPACK model. This result also agreed with the findings from Lyublinskaya and Tournaki (2012) who discovered that the participants of their study displayed knowledge at these three levels of the TPACK model.

In the four-teacher case study of TPACK development, all cases implied that a sustained and collaborative professional development was necessary for teachers to develop and enhance their TPACK. Such professional development also should be connected to practice directly as all the cases in this study indicated that their TPACK growth occurred through attending the DGP professional development sessions and through their teaching practice.

In summary, the results of this study provided strong evidence about the effectiveness of the DG approach in teachers' development of TPACK and their classroom teaching. This evidence can inform teachers' and school leaders' decisions for transforming curriculum and instructional practices in geometry as a core high school mathematics course.

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KEY TERMS AND DEFINITIONS

Conjecturing-Proving: The mathematical practice or process of forming and stating a hypothesis, then providing justification that verifies or counters the hypothesis.

Dragging: Continuous and real-time transformations of geometric objects.

Dynamic Geometry: Active and exploratory geometry carried out with interactive computer software (such as the Geometer's Sketchpad) with the characteristic feature of "dragging," which allows for the continuous and real-time transformations of geometric objects.

Dynamic Geometry Approach: An approach to teaching and learning of mathematics that utilizes dynamic geometry that focuses on experimentation, conjecture making, conjecture testing, and proof.

Interview Protocol: A set of questions to facilitate and guide semi-structured open-ended interviews of teachers using technology for teaching mathematics.

Observation Protocol: Instrumentation and guidelines for systematically documenting teachers using technology as part of mathematics instruction.

Professional Development: Process of improving skills and developing knowledge to increase teaching effectiveness.

Technological Pedagogical Content Knowledge (TPACK): Knowledge of how to use technological tools effectively in teaching and learning of specific content matter.

APPENDIX 1

Sample Questions from the TPACK Development Interview Protocol

- When you hear the words “dynamic geometry” what comes to your mind?
- Why did you decide to teach geometry with DG?
- What is your current view and understanding about integrating DG as a learning tool in geometry?
- What specific geometry topics lend themselves to DG as a learning tool?
- What do you see as barriers to integrating DG in teaching and learning of geometry?
- How did your knowledge and skills with DG changed through your work in the DGP?
- How has your knowledge about students’ understanding, thinking, and learning about geometry topics with DG changed through your work in the DGP?
- How has your conception of incorporating DG in teaching specific topics in geometry changed through your work on the DGP? Give an example from a lesson you taught last year.
- When you start using DG with your students (at the beginning of a school year), what strategies do you use to guide them in learning about the software?
- How important is (to you) the integration of DG into teaching and learning of geometry?

APPENDIX 2

Table 1. TPACK development levels assessment rubric

TPACK Components	TPACK Development Levels				
	Recognizing (1)	Accepting (2)	Adapting (3)	Exploring (4)	Advancing (5)
Access	<ul style="list-style-type: none"> • Teacher permits students to use technology ‘only’ after mastering certain concepts. • Teacher resists consideration of changes in content taught although it becomes accessible to more students through technology. • Teacher notices that authentic problems are more likely to involve ‘unfriendly numbers’ and may be more easily solved if students used technology. 	<ul style="list-style-type: none"> • Teacher students use technology in limited ways during regular instructional periods. • Teacher worries about access and management issues with respect to incorporating technology in the classroom. • Technology permits a greater number of examples to be explored by students. 	<ul style="list-style-type: none"> • Teacher permits students to use technology in specifically designed units. • Teacher uses technology as a tool to enhance mathematics lessons to provide students a new way to approach mathematics. • Concepts are taught differently since technology provides access to connections formerly out of reach. 	<ul style="list-style-type: none"> • Teacher permits students to use technology for exploring specific mathematical topics. • Teacher recognizes challenges for teaching mathematics with technologies but explores strategies and ideas to minimize the impact of those challenges. • Through the use of technology, key topics are explored, applied and assessed incorporating multiple representations of the concepts and their connections. 	<ul style="list-style-type: none"> • Teacher permits students to use technology in every aspect of mathematics class. • Teacher recognizes challenges in teaching with technology and resolves the challenges through extended planning and preparation for maximizing the use of available resources and tools. • Students are taught and permitted to explore more complex mathematics topics or mathematical connections as part of their normal learning experience.

continued on following page

Table 1. Continued

TPACK Components	TPACK Development Levels				
	Recognizing (1)	Accepting (2)	Adapting (3)	Exploring (4)	Advancing (5)
Learning	<ul style="list-style-type: none"> Teacher views mathematics as being learned in specific ways and that technology often gets in the way of learning. Teacher is more apt to accept the technology as a teaching tool rather than a learning tool. 	<ul style="list-style-type: none"> Teacher has concerns about students' attention being diverted from learning of appropriate mathematics to focus on the technology in the activities. Teacher is concerned that students do not develop appropriate mathematical thinking skills when the technology is used as a verification tool for exploring the mathematics. 	<ul style="list-style-type: none"> Teacher begins to explore, experiment and practice integrating technologies as mathematics learning tools. Teacher begins developing appropriate mathematical thinking skills when technology is used as a tool for learning. 	<ul style="list-style-type: none"> Teacher uses technologies as tools to facilitate the learning of specific topics in the mathematics curriculum. Teacher plans, implements, and reflects on teaching and learning with concern for guiding students in understanding. 	<ul style="list-style-type: none"> Teacher plans, implements, and reflects on teaching and learning with concern and personal conviction for student thinking and understanding of the mathematics to be enhanced through the integration of the various technologies. Technology integration is integral (rather than in addition) to the development of the mathematics students are learning.
Curriculum	<ul style="list-style-type: none"> Teacher acknowledges that mathematical ideas displayed with the technologies can be useful for making sense of topics addressed in the curriculum. 	<ul style="list-style-type: none"> Teacher expresses desire but demonstrates difficulty in identifying topics in own curriculum for including technology as a tool for learning. 	<ul style="list-style-type: none"> Teacher understands some benefits of incorporating appropriate technologies as tools for teaching and learning the mathematics curriculum. 	<ul style="list-style-type: none"> Teacher investigates the use of topics in own curriculum for including technology as a tool for learning; seeks ideas and strategies for implementing technology in a more integral role in the development of the mathematics that students are learning. 	<ul style="list-style-type: none"> Teacher understands that sustained innovation in modifying own curriculum to efficiently and effectively incorporate technology as a teaching and learning tool is essential.
Teaching	<ul style="list-style-type: none"> Teacher is concerned that the need to teach about the technology will take away time from teaching mathematics. Teacher does not use technology to develop mathematical concepts. Teacher uses technology to reinforce concepts taught without technology. 	<ul style="list-style-type: none"> Teacher uses technology activities at the end of units, for "days off," or for activities peripheral to classroom instruction. Teacher merely mimics the simplest professional development mathematics curricular ideas for incorporating the technologies. Teacher tightly manages and orchestrates instruction using technology. 	<ul style="list-style-type: none"> Teacher uses technology to enhance or reinforce mathematics ideas that students have learned previously. Teacher mimics the simplest professional development activities with the technologies but attempts to adapt lessons for his/her mathematics classes. Instructional strategies with technologies are primarily deductive, teacher-directed to maintain control of the how the activity progresses. 	<ul style="list-style-type: none"> Teacher engages students in high-level thinking activities (such as project-based and problem-solving and decision-making activities) for learning mathematics using the technology as a learning tool. Teacher engages students in explorations of mathematics with technology where the teacher is in the role of a guide rather than the director of the exploration. Teacher explores various instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics. 	<ul style="list-style-type: none"> Teacher actively and consistently accepts technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students. Teacher adapts from a breadth of instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics. Teacher manages technology-enhanced activities in ways that maintain student engagement and self-direction in learning the mathematics.

(Adapted from the Mathematics Teacher TPACK Standards and Development Model (Niess et al., 2009)).

Teaching Geometry with Technology: A Case Study of One Teacher's Technological Pedagogical Content Knowledge

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Abstract: This qualitative case study investigated how high school teachers developed and used their knowledge in teaching geometry with technology. In particular, this study focused on teachers' technological pedagogical content knowledge (TPACK) and their integration of dynamic geometry in the classroom instruction. This paper reports findings from one of the four cases. The sources of data included: an initial interview, observations, documents, a closing interview, a survey, implementation questionnaires, professional development attendance records and the researcher's log. Data analysis utilized the TPACK Development Model to describe participant's dynamic geometry integration and to identify her TPACK development levels. The researcher was able to identify all TPACK development levels for the participant, which was an unexpected finding since the participant was an experienced teacher and long-term technology user.

Introduction

Use of technology in teaching and learning has been advocated by various professional organizations (ISTE, 2008; NCTM, 2000). The National Council of Teachers of Mathematics (NCTM) specifically advocates the use of technology in teaching mathematics in its Principles and Standards for School Mathematics (NCTM, 2000). The technology principle states, "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (NCTM, 2000, p. 373). Educational software such as *the Geometer's Sketchpad* (Sketchpad) can assist in developing students' understanding of mathematical concepts and increasing their reasoning skills (CBMS, 2001). Taking advantage of dynamic features such as dragging, one can see "the universality of theorems in a way that goes far beyond typical paper and pencil explorations" (CBMS, 2001, p. 132). Taking into consideration the increase in available technology in schools in the recent years and the requirements placed on teachers to use it in teaching, we still do not know how effective teachers are in integrating technology into teaching (Harris, Grandgenett, & Hofer, 2010). Finding out how teachers learn about technology and how they incorporate it into their teaching is necessary since the way teachers use technology can impact the quality of instruction and student learning (Roberts & Stephens, 1999).

Mishra and Koehler (2006) defined a construct of knowledge that teachers possess when teaching with technology, namely technological pedagogical content knowledge (TPACK, previously TPCK). The concept of TPACK and its assessment have been researched (Mishra & Koehler, 2006; Niess, 2005; Niess et al., 2009; Schmidt et al., 2009), although not in the area of geometry or dynamic geometry software. Niess et al. (2009) proposed a Mathematics Teacher TPACK Development Model built on the TPACK framework (Mishra & Koehler, 2006). The model consists of a five-stage process through which teachers might go while developing their knowledge when learning a new technology and integrating it into teaching. The five levels of the model derive from Everett Rogers' (1995) model of the innovation-decision process and are based on researchers' observations of teachers learning to integrate spreadsheets into teaching and learning mathematics (Niess et al., 2009). Level 1 is recognizing (knowledge), where teachers are able to use technology and "recognize the alignment of the technology with mathematics" (Niess et al., 2009, p. 9) but do not integrate it into teaching. Level 2 is accepting (persuasion) and involves forming a favorable or unfavorable attitude towards integrating technology in teaching mathematics. Level 3 is adapting (decision), where teachers decide to use or not to use technology in teaching mathematics. Level 4 is exploring (implementation) and involves active integration of technology and teaching mathematics. Lastly, level 5 is advancing (confirmation) and consists of teachers' evaluating "the results of the decision to integrate teaching and learning mathematics with an appropriate technology" (Niess et al., 2009, p. 9). Furthermore, the model is divided

into four themes: curriculum and assessment, learning, teaching, and access. The authors also stated, “a mathematics teacher may be at different levels for different themes” (Niess et al., 2009, p. 13). This model presented a useful framework for studying how teachers gain knowledge related to integration of technology in teaching mathematics and identifying the levels of TPACK they have.

Currently there are two contrasting paradigms for the assessment of teachers’ knowledge; one is mainly quantitative and the other one is qualitative. Surveys, which involve self-reported data, largely dominate the quantitative types of TPACK assessment (Archambault & Crippen, 2009; Schmidt et al., 2009). Such instruments are promising starting points in examining teachers’ and pre-service teachers’ TPACK; however, instead of knowledge, they measure teachers’ TPACK self-efficacy. The limitation of these surveys is that they rely on self-report or self-assessment of knowledge. Research shows that gains measured by such instruments reflect an increase in confidence instead of an increase in knowledge (Lawless & Pellegrino, 2007; Schrader & Lawless, 2004). Also, respondents might provide researchers with inaccurate data and their responses are hard to verify. Another way to assess teachers’ knowledge is mainly qualitative and “draws upon case descriptions of teachers’ classroom practices” (Groth, Spickler, Bergner, & Bardzell, 2009, p. 394). One advantage of this approach is that it allows researchers to explore “contextual factors that contribute to the knowledge that teachers exhibit in their classrooms” (Groth et al., 2009, p. 394).

The Study

This study sought to explain how secondary geometry teachers’ knowledge of geometry, pedagogy, and technology developed and was enacted in a classroom setting. In addition, this study described teachers’ TPACK development levels identified through the Mathematics Teacher TPACK Development Model (Niess et al., 2009). Three research questions guided this study:

1. How do high school teachers develop TPACK while teaching geometry using dynamic geometry software?
2. How do high school teachers enact their TPACK when teaching with dynamic geometry software?
3. How are the five TPACK development levels (i.e., recognizing, accepting, adapting, exploring, and advancing) characterized for high school teachers who incorporate dynamic geometry software in teaching?

The research questions indicated a qualitative method of inquiry. The goal of this study was to provide a rich and descriptive account of teachers’ use of dynamic geometry software in their teaching practices; therefore, the researcher employed a case study design (Yin, 2009). The researcher conducted this study within the context of the Dynamic Geometry Project (DGP), a NSF-funded project focusing on integration of the *Geometer’s Sketchpad* (Sketchpad), dynamic geometry software, in teaching high school geometry. Participants of DGP were geometry teachers from several school districts in a large city in southern U.S. There were approximately 70 teachers, half in the experimental group (using Sketchpad) and half in the comparison group (not using Sketchpad). The focus of this study was on teachers in the experimental group since those teachers were the ones using dynamic geometry software in teaching. All teachers participated in professional development workshops facilitated by the DGP staff. The professional development consisted of a 5-day summer workshop and six half-day Saturday workshops offered throughout the first year of Sketchpad implementation. During the second year of implementation, the year of this study, teachers participated in a 1-day summer workshop and three Saturday half-day workshops. Teachers received stipends for their participation. During the professional development participants became familiar with the software, learned about geometric concepts using the software, were demonstrated how to integrate it into teaching, created lesson plans, and collaborated with their peers. Participants were provided with curriculum materials that focused on integration of Sketchpad into learning and teaching of geometry as well as were provided access to the Sketchpad Lesson Link, a website (<http://www.keypress.com/x26771.xml>) with hundreds of Sketchpad activities aligned to textbooks, state standards, and the Common Core State Standards. Teachers were asked to use Sketchpad with their students in a lab setting (or in their classroom if laptops were available) twice a week so that students can experience dynamic geometry on regular basis. Teachers were also encouraged to use Sketchpad for presentation purposes whenever a computer lab or laptops were not available.

Sources of data included interviews, observations, documents, a self-report survey, implementation questionnaires, and informal conversations with participants. The first step in data collection was a semi-structured open-ended interview. The TPACK Development Model framework (Niess et al., 2009) informed the creation of several interview questions such as: Why did you decide to teach geometry with Sketchpad? In addition to your participation in the DGP, what kind of activities (e.g., professional development, conferences, self-directed study, Internet resources) have you engaged in that lead you to adopt teaching and learning geometry with Sketchpad? The interview protocol developed by Niess, van Zee, and Gillow-Wiles (2010), also based on the TPACK Development

Model, suggested additional questions such as: What is your current view and understanding about integrating Sketchpad as a learning tool in geometry? How has your knowledge about students' understanding, thinking, and learning about geometry topics with Sketchpad changed through your work in the DGP (or since you started using Sketchpad in the classroom instruction)? The observations were the second step in data collection process and their purpose was to see how teachers enacted their TPACK when teaching with Sketchpad and to identify their TPACK development levels. It was essential to observe how teachers enacted their TPACK during classroom instruction because that was how they used their knowledge in practice. Documents related to the observed lessons, such as student activity sheets that teachers either prepared themselves or printed from the Sketchpad Lesson Link, were also collected. These documents provided more information about the teacher's intended lesson and how her or his TPACK was reflected in the lesson plan. The final step in data collection was the administration of the TPACK Development Model Self-Report Survey (TPACK Survey) developed by Ivy (2011) and Riales (2011), which was based on the TPACK Development Model (Niess et al., 2009). The purpose for using the survey was to find out what the participants' perceptions were about their TPACK related to Sketchpad integration in teaching. The survey consisted of fifty-five statements organized into eleven groups of five statements. One statement in each group described one of the five TPACK development levels (i.e., recognizing, accepting, adapting, exploring, and advancing), and the eleven groups represented eleven theme/descriptor pairings from the TPACK Development Model. Participants were to choose one statement from each of the eleven groups. Their responses provided data related to their self-perceived TPACK development levels.

Since the participants of this study were part of a larger project, the DGP, they completed an implementation questionnaire every 4-6 weeks. The questions on the questionnaires were related to their integration of Sketchpad in teaching and learning. Selected data from the questionnaires were used in this study to aid in answering the research questions.

Data analysis took place simultaneously with data collection and was a reflective and ongoing process. The first step into data analysis was to transcribe the audiotaped interviews and observations. After transcribing, a codebook for content analysis (Patton, 2002), which was created by the researcher based on the TPACK Development Model, was used in a deductive analysis. This process assisted in identifying the correct levels of TPACK for each participant and facilitated in answering the research questions. Recording participants' responses to the TPACK Survey identified their self-perceived TPACK development levels. Because each response identified a TPACK development level for one theme-descriptor pairing, and the order of the statements (for each group of five statements) corresponded to the five TPACK development levels, from the lowest (recognizing) to the highest (advancing), no further analysis was needed of the survey alone. Later, the self-perceived levels of TPACK reported through the survey were compared with the TPACK development levels identified through other data sources. Student activity sheets contributed in describing the observed lessons. All collected data were triangulated to crosscheck for themes that emerged from all the different sources and to strengthen the research findings. As a result of the data analysis, TPACK development levels were identified across different themes for all participants and consequently aided in answering the three research questions.

Findings

The multiple case study included four participants. This paper provides findings from one of the cases, Susan (pseudonym). Susan was the most experienced teacher (20 years of experience) in this study and had the most experience in using Sketchpad in teaching (8 years). She started her teaching career as a middle school teacher and later transitioned to the high school level. At the time of this study, she was in her tenth year of teaching geometry and her fifth year of teaching at her current school. She was teaching geometry solely and was the only geometry teacher at her school. Susan was tremendously enthusiastic about incorporating technology into instruction. She had laptops, graphing calculators, clickers, an interactive white board, a document camera and a projector in her classroom.

Susan began developing her TPACK with regard to Sketchpad approximately eight years ago when she was working on her master's degree. She was part of a teacher quality grant with other in-service teachers during one summer. Her overall TPACK was at the recognizing level then as she learned about Sketchpad. In the following fall, she was elected to be the math teacher in a new program at her school. As part of that program, hundreds of freshmen received laptops, and Susan had to use some kind of educational computer program with her students. She chose to use Sketchpad (indicating an overall accepting level of TPACK), although she was teaching algebra at that time. She also had training in Cabri Junior, a dynamic geometry application for graphing calculators, but she preferred to use Sketchpad because she had laptops in her classroom.

As part of her participation in the DGP, Susan attended all of the professional development meetings offered by the DGP, for a total of approximately 73 hours. She also attended several conferences that offered Sketchpad sessions throughout her teaching career, indicating an overall adapting level of TPACK. She enjoyed learning more about Sketchpad; however, she stated:

I have been doing it long enough up to where I am usually ahead, so that is a disappointment. But I love going. That is why I love being part of this program because I am learning more to do that. But you got me thinking when I am using [Sketchpad] that I am able to point out to students more what they are supposed to see and not just see the magic. I think that I have improved a lot just by being in those [professional development sessions]. Any time you are with your coworkers and you can discuss ideas, it works better.

This statement indicates the exploring level of TPACK for the teaching theme, professional development descriptor, because Susan enjoyed learning more about Sketchpad and cooperating with colleagues on incorporating Sketchpad into teaching geometry. Susan's comment also revealed that she already knew a lot about Sketchpad; however, she still learned more through interacting with her colleagues using pedagogical dialogs.

Susan decided to use Sketchpad in teaching and learning because she was fascinated with it. Based on the initial interview, observed lessons and implementation questionnaire data, she incorporated Sketchpad into instruction as a learning tool for students' explorations and as a teaching tool for demonstrations. Exploring different instructional strategies with Sketchpad indicated the exploring level of TPACK for the teaching theme, environment descriptor. At the same time, her implementation questionnaire responses suggested that she used it as a teaching tool more so than a learning tool. Therefore, her TPACK for the learning theme, conception of student thinking descriptor, was at the recognizing level.

Susan liked Sketchpad because it is accurate in measurements as opposed to students' measurements on paper. This indicated the recognizing level of TPACK for the access theme, availability descriptor, because Susan saw Sketchpad as a useful tool for replacement of paper-and-pencil activities where student tended to make more mistakes on their measurement, and with Sketchpad their measurements were accurate. So when working on any activity that involved measuring, she preferred to use Sketchpad. This, in turn, suggested the adapting level of TPACK for the access theme, usage descriptor, because she incorporated Sketchpad only in specifically designed units. Also, Susan noticed that using Sketchpad was more efficient in teaching some geometric topics as she stated, "in a way it is quicker to get some points across—like angles—just something simple like to name an angle, at the beginning of the year—that the vertex has to be the middle letter." This indicated the accepting level of TPACK for the access theme, availability descriptor, because Susan saw Sketchpad as a valuable tool simply for its efficiency.

Susan decided to do Sketchpad activities with her students every Thursday this year; since "time is a big barrier," she set that one day (Thursday) aside for Sketchpad activities, and it was her "big goal." This indicated the exploring level of TPACK for the access theme, barrier descriptor, because Susan recognized that it took additional time to incorporate Sketchpad activities. By dedicating one day a week for Sketchpad activities, she overcame this challenge. In addition, implementation questionnaire data indicated that, in addition to engaging students in Sketchpad activities once a week, Susan used it for demonstration every day. This pointed to the exploring level of TPACK for the curriculum and assessment theme, curriculum descriptor, because Susan tried to integrate Sketchpad in a more integral role. However, at the same time she does not believe that students should use Sketchpad every day in class. Previously when she was teaching and had to use laptops every day, she "had a little issue with that." At that time, she was mainly teaching algebra and thought that students needed more "hands-on" activities as she stated:

I think you still need to do a little pencil and paper. A little bit of both is good, but you cannot just go completely immersed. I think [Sketchpad] is great to use. Again, I still feel that it does not need to be—I do not believe it can be used every single day, but I think that it is a great tool to use.

Susan's comment indicated that she recognized Sketchpad as a great tool for learning geometry, even though she did not use it every day. She incorporated it as a tool to facilitate learning of such geometric topics as angles and triangles. This indicated the exploring level of TPACK for the learning theme, mathematics learning descriptor, because she used Sketchpad "to facilitate the learning of specific topics in the mathematics curriculum" (Niess et al., 2009).

Susan's students used Sketchpad during all of the observed lessons except a follow-up lesson to the parallel-lines-with-a-transversal lab; Susan used Sketchpad then for demonstration. At the beginning of each lesson, Susan instructed her students to take a laptop from the cart located in the back of the classroom. Then she introduced the lesson and necessary vocabulary for that lesson, and distributed handouts with instructions for the activity. Students were working alone for the most part, talking to their nearby classmates about the activity from time to time. Susan acted as a guide walking around the classroom and answering any questions they had, which indicated the exploring level of TPACK for the teaching theme, instruction descriptor.

During the *parallel lines with a transversal lesson (lab)*, students were exploring angle relationships between two parallel lines and a transversal. This was an introductory lesson, and they had not explored this concept before. Susan introduced the lesson and pointed out a few things on the handout to make sure the students understood everything and that they followed the instructions closely. One of the things that she mentioned was the use of the construct menu:

When it says, “construct,” you need to be really careful and make sure you use the construct menu. You all like to just draw. It says, “Construct parallel lines.” It is not going to stay parallel [if you just draw]. You want to construct parallel lines. You have to use the construct menu. You have to use this. It is not going to work [if you just draw]. If you do not use this, I will know. I will come around, and I will check it, and you will end up redoing it so pay attention. If you do not use this, you have to redo it.

It was clear that Susan had experience with students simply drawing lines that appeared to be parallel, but when dragged the lines did not remain parallel. Susan probably witnessed this in several other lessons, and that is why she addressed it at the beginning of this activity. As students began to work on the activity, one student constructed the transversal so that it was perpendicular to the parallel lines, making all angles 90 degrees. Susan noticed that, and she said to the whole class, “If you did perpendicular, please undo. We do not want perpendicular lines. Parallel lines, but not perpendicular.” Later on she kept noticing that some students still had their transversal line perpendicular to the parallel lines. She addressed that issue with one of her students, Craig:

Susan: Do not construct a perpendicular [transversal].

Craig: I can’t make right angles?

Susan: You can later. But it’s not fun. That makes it boring. Math should be fun.

All students ended up having a correct construction, although Susan did not explain why they should not have a perpendicular transversal except that it was “boring.” After students had measured all angles, Susan directed them to move the angle measurements next to the corresponding angles because it makes it “more visual.” The rest of the lesson concentrated on identifying congruent, complementary, supplementary, alternate interior, same side interior, alternate exterior, same side exterior and corresponding angles. During the follow-up lesson, Susan was using Sketchpad and an interactive white board to review what the students did during the lab (see Figure 1). This confirmed that Susan recognized that geometric ideas were easily presented with Sketchpad and useful for making sense of topics in the curriculum and confirmed what she stated in the interview—the use of “different colors” makes it more visual and “it is quick to show.” The fact that she used Sketchpad for the visual effect, with different colors, indicated the recognizing level of TPACK for the curriculum and assessment theme, curriculum descriptor. The rest of the lesson consisted of reviewing answers to questions from the lab activity and working on problems that did not involve Sketchpad.

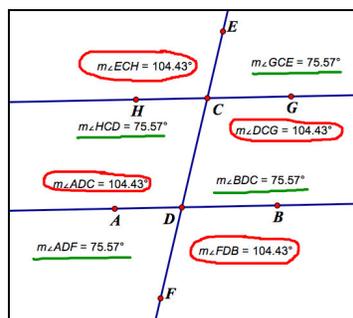


Figure 1: Screenshot of Susan’s review of the lab activity during the parallel lines and a transversal lesson.

Susan’s responses to the TPACK Survey showed that she perceives her TPACK to be at the exploring and advancing levels except for the assessment descriptor (see Figure 2). She indicated she did not allow her students to use Sketchpad on tests:

not because “I don’t like to allow...” [but because] 1) Designing a Sketchpad test would be difficult. 2) Students would have more ways to cheat while on the computer so I would have to watch them very closely.

Susan’s statement suggested that she assessed her students’ learning through asking procedural questions where students were more likely to cheat. This showed an opportunity for TPACK development and the incorporation of assessment questions that examine conceptual understanding rather than procedural one. Asking conceptual questions could eliminate cheating on tests and make the inclusion of Sketchpad even more integrated with learning.

Although Susan indicated high levels (exploring and advancing) of TPACK through the TPACK Survey, the non-survey data indicated that her TPACK development levels range from recognizing to exploring. The

following paragraphs describe Susan's TPACK development levels derived from the data sources and are split into four sections for the four themes of the TPACK development model, i.e., curriculum and assessment, learning, teaching, and access (Niess et al., 2009).

The initial interview and the second observed lesson—the parallel lines with a transversal follow-up lesson—indicated that Susan's TPACK development level was extremely low (recognizing) for the curriculum and assessment theme, curriculum descriptor, because she focused on how things were displayed with Sketchpad (e.g., the use of different colors). Additionally, the third observed lesson—properties of isosceles triangles lesson—indicated the accepting level because Susan tried one Sketchpad activity with her first period class, but then changed it to a less-challenging activity, which in turn was a repetition of the lesson from the previous day. This showed that Susan might have had a difficulty identifying appropriate topics in the geometry curriculum for her students to explore with Sketchpad. However, the initial interview and implementation questionnaire responses suggested the exploring level for the curriculum descriptor; Sketchpad integration had “more integral role for the development of the mathematics that students are learning” (Niess et al., 2009) because Susan tried to use it every day in addition to students' weekly explorations.

Susan knew that integration of Sketchpad in learning was a worthwhile effort. Throughout all the years she had been using it in the classroom instruction, she identified topics, for which Sketchpad facilitates better learning and understanding of geometric concepts (e.g., parallel lines with a transversal). This indicated the exploring level of TPACK for the mathematics learning descriptor, which agreed with her self-reported TPACK development level. However, her TPACK development level was low (recognizing) for the conception of student learning descriptor because she preferred to use Sketchpad as “a teaching tool rather than as a learning tool” (Niess et al., 2009). She reported the advancing level for this descriptor, indicating that Sketchpad integration in her classroom was integral “to development of the mathematics students are learning” (Niess et al., 2009). This was not evident in the observed lessons, and other data sources did not suggest it either.

Based on the observed lessons, Susan's TPACK was at the adapting level for the mathematics learning descriptor. She indicated the advancing level through the TPACK Survey; however, it was not evident that her students engaged in high-level thinking activities when using Sketchpad. Instead, she incorporated Sketchpad for enhancing or reinforcing already-learned topics as well as for activities outside the curriculum. Based on the observed lessons, Susan's TPACK was at the exploring level for the instruction descriptor because she guided her students in Sketchpad explorations. The initial interview data indicated the same level for the environment descriptor because Susan incorporated a variety of instructional strategies when using Sketchpad. According to data from the initial interview, Susan was at the exploring level for the teaching theme, professional development descriptor, because she associated and worked with other geometry teachers (through the DGP) who integrated Sketchpad in geometry instruction.

The initial interview and observation data indicated the adapting level for the usage descriptor because Susan used Sketchpad only in specific units such as those that require accurate measuring. By recognizing challenges in teaching geometry with Sketchpad (such as the extensive curriculum), Susan explored strategies to overcome them (e.g., setting one day a week to allow students to use Sketchpad), which indicated the exploring level for the barrier descriptor. Lastly, Susan's TPACK was low (at the recognizing and accepting levels) for the availability descriptor because she preferred to use Sketchpad mainly for its accurate measurements and efficiency instead of using it to explore more complex geometric topics.

Combined results from all non-survey data sources indicated that Susan's TPACK development levels were not aligned to her perceptions about her TPACK (see Figure 2). The TPACK development levels identified through interviews, observations, implementation questionnaire responses and student activity sheets aligned with her TPACK development levels reported through the TPACK Survey only for three descriptors (all at the exploring level): learning theme/mathematics learning, teaching theme/environment, and access theme/barrier. Her TPACK development levels were lower for the remaining themes/descriptors than the TPACK development levels identified through the TPACK Survey. Because Susan's TPACK development levels ranged widely (from recognizing to exploring), it was difficult to identify one TPACK development level for her. Among all four themes, the teaching theme showed most consistency, with one adapting and three exploring levels; therefore, Susan's TPACK was at the exploring level for this theme.

Summary

Susan learned about Sketchpad approximately seven years before this study and has been using it to teach algebra and geometry. She does not believe students should use it all the time, but she believes it is an excellent tool for visualizing some difficult-to-learn concepts (such as angles). She incorporated Sketchpad into her lectures on a

daily basis and her students used it for explorations about once a week. During the observed lessons, it was clear that Susan had experience in teaching with Sketchpad because she anticipated many areas where students had troubles (e.g., using the construct menu to construct parallel lines instead of drawing lines that appear to be parallel). However, she tended to use Sketchpad because it was easier and quicker to present geometric ideas with it. If her students had difficulties with Sketchpad activities (or if those activities were challenging and students struggled with the concepts), she decided to change them to something easier, i.e., something that she knew students would be able to do. Susan’s TPACK for the teaching theme was at the exploring level; she incorporated Sketchpad as a learning tool on a weekly basis, and enhanced her students’ learning through the use of Sketchpad. Based on the collected data, it was impossible to identify one TPACK development level for the remaining themes (i.e., curriculum and assessment, learning, and access).

Theme (descriptor)	TPACK Development Levels				
	Recognizing	Accepting	Adapting	Exploring	Advancing
Curriculum & Assessment (curriculum)					
Curriculum & Assessment (assessment)					
Learning (mathematics learning)					
Learning (conception of student thinking)					
Teaching (mathematics learning)					
Teaching (instruction)					
Teaching (environment)					
Teaching (professional development)					
Access (usage)					
Access (barrier)					
Access (availability)					

 Susan’s responses to the TPACK Survey

 Susan’s TPACK development levels reported by data sources other than the TPACK Survey

 Areas of agreement between the responses to the TPACK Survey and TPACK development levels reported by the other data sources

Figure 2: Summary of Susan’s TPACK development levels.

Conclusions

Susan participated in all or almost all professional development sessions provided by the DGP. She valued the DGP professional development sessions. The collaboration that took place during those sessions was especially valuable to her because she was the only geometry teacher at her school; meeting with other geometry teachers from local schools on a regular basis helped her learn how to better integrate Sketchpad into her geometry lessons. Even though she used Sketchpad for six years before the DGP, and she knew how to use it, she improved as a teacher as she mentioned in the initial interview—she was “able to point out to students more of what they were supposed to see.” However, as Susan’s TPACK development levels suggested, her TPACK was not fully developed. Even though she was an experienced teacher, her TPACK development levels were inconsistent, except for the teaching theme where her TPACK development level was at the exploring level. The findings imply that a sustained and collaborative professional development is necessary in order for teachers to develop and improve their TPACK. To facilitate a change in teacher practices, professional development should take place at the school or district levels (Penuel et al., 2007).

Susan used Sketchpad primarily for demonstrations even though her students had opportunities to do Sketchpad explorations every week. This suggested that she preferred to use it in a more teacher-directed instruction than in a student-centered instruction. This finding also agrees with findings from Hannafin et al. (2001); teaching style is difficult to change and it might take a long time for a teacher to adjust to a different approach to teaching and learning. Most of Susan’s TPACK development levels were below her self-perceived TPACK development levels. One explanation for this could be that Susan felt more comfortable and confident with Sketchpad since she was using it for a long time, and that possibly resulted in her higher perceptions about her TPACK. This confirms findings from other studies, which concluded that gains measured by self-report surveys reflect an increase in confidence instead of an increase in knowledge (Lawless & Pellegrino, 2007; Schrader & Lawless, 2004).

One of the goals of this study was to contribute to existing research on high school geometry teachers' TPACK when teaching with dynamic geometry software. Literature in this area of research is limited; therefore, the scholarly body of knowledge is increased by an addition of the case report offered through this study. Also, the findings along with current literature on TPACK development and professional development offer recommendations for teachers' professional development in the field of teaching with dynamic geometry software. The findings also offer suggestions for future research in the area of teachers' TPACK. Lastly, this study moves one step closer to developing items that can measure TPACK for teaching with dynamic geometry software. Further development in this area is needed and is left for future studies.

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Proposing a Model for TPACK Development: The Total PACKage Professional Development Model

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Abstract: The author of this paper proposes a professional development model for developing technological pedagogical content knowledge (TPACK). The model is based on a study of four mathematics teachers who used technology, specifically dynamic geometry software, in teaching high school level geometry. The model incorporates the ideas of virtual lesson study and TPACK-in-practice. The model is cyclical and consists of seven stages: technological knowledge (TK) development, technological content knowledge (TCK) development, technological pedagogical knowledge (TPK) development, developing a technology-enhanced lesson, teaching, observing and reflecting.

Introduction

This paper proposes a model for professional development of technological pedagogical content knowledge (TPACK). The Total PACKage Professional Development Model (TPACK PD Model) was development based on a study of four mathematics teachers who used dynamic geometry software in teaching a high school level geometry course. Therefore, the model was specifically designed for facilitating development of TPACK related to teaching geometry with dynamic geometry software. However, the model should be easily transferred to other subjects and other types of technology. The model also uses the concepts of TPACK-in-Practice (Figg & Jaipal, 2012) and a virtual lesson study (Yursa & Silverman, 2012). The following sections describe these concepts and the study in more detail.

The Study

The study sought to explain how secondary geometry teachers' knowledge of geometry, pedagogy, and technology developed and was enacted in a classroom setting. Participants of the study were four high school mathematics teachers who were part of the Dynamic Geometry Project (DGP), an NSF-sponsored project. The teachers participated in a weeklong professional development during summer and 11 follow-up sessions over the following two years. Three of the four participants indicated that the main sources of their TPACK development were the professional development workshops offered by the DGP. One of the participants taught with dynamic geometry before participating in the DGP so she was already familiar with the software; however, she had an opportunity to learn about the new features of the software as well as how to integrate it into classroom instruction better. Additionally, her TPACK development during the study indicated that, even with many years of incorporating technology into her classroom instruction, she had not developed her TPACK completely. The second main source of TPACK development was practice, i.e., planning activities that incorporated dynamic geometry and teaching with technology. This source of knowledge development suggested how TPACK development could be structured. In addition, based on TPACK development of the participants, they could still improve their knowledge for teaching with technology. Therefore, more professional learning opportunities for in-service teachers are needed. The following sections, TPACK-in-Practice and (Virtual) Lesson Study, briefly review two contemporary efforts for enhanced professional development related to teaching with technology.

TPACK-in-Practice

The TPACK-in-Practice Framework (Figg & Jaipal, 2012) offers a professional learning opportunity for teachers wanting to integrate technology in their instruction. This professional development model is closely related to content taught and does not focus merely on learning the technology. The model consists of four stages “(a) modeling a tech-enhanced activity type (learning WITH the tool), (b) integrating ‘pedagogical dialog’ in a modeled lesson, (c) developing TK (in context) through tool demonstrations, and (d) applying TPACK-in-Practice to design an authentic learning task” (Figg & Jaipal, 2012, p. 4685).

(Virtual) Lesson Study

Lesson study (LS) is a form of professional learning that is popular in Japan and other Asian countries; it is also gaining recognition in the United States and so far is mainly used for research purposes. The major benefits of LS are collaboration, teacher knowledge improvement and instructional improvement (Lewis, Perry, & Hurd, 2009). The features of LS include investigation, planning, research lesson and reflection (Lewis et al., 2009). Yursa and Silverman (2012) proposed a virtual model for LS, which creates an increased access to the LS community for teachers in rural and urban districts. Since the virtual LS is facilitated online, it allows for participation from remote locations and becomes more accessible.

The Total PACKage Professional Development Model (TPACK PD Model)

Based on the findings of the study and the professional development provided by the DGP, as well as drawing from the two models of professional development, TPACK-in-Practice and (Virtual) Lesson Study, a new type of professional development specifically designed for teachers interested in integrating dynamic geometry into their instruction is proposed. This new model is called the Total PACKage Professional Development Model or TPACK PD Model, and consists of seven stages: technological knowledge (TK) development, technological content knowledge (TCK) development, technological pedagogical knowledge (TPK) development, developing a technology-enhanced lesson, teaching, observing and reflecting. As Figure 1 illustrates, TPACK PD is a cyclical process; simply going through all stages once does not guarantee total TPACK development. Instead, TPACK PD participants continue learning and developing their knowledge by visiting all of the stages multiple times. This kind of professional development can facilitate teacher change because it uses additional sessions and collective participation (Penuel, Fishman, Yamaguchi, & Gallagher, 2007). By creating a professional learning community and devoting more time to professional development, teachers are more likely to integrate new knowledge into practice (Brown, 2004; Penuel et al., 2007).

The first four stages of the TPACK PD Model are adapted from the TPACK-in-Practice framework; however, they have been rearranged to better suit the purpose of developing TPACK for teaching with dynamic geometry software. In the TPACK-in-Practice model, TK development is the third stage; in the TPACK PD model, it is the first stage. The fourth stage from the TPACK-in-Practice model is also part of the LS model. The remaining three stages are adapted from the (Virtual) LS model. The following are descriptions of all stages of the TPACK PD Model:

- TK development through tool demonstration. Participants learn how to use the software in context. Since dynamic geometry software is very user friendly, participants can quickly learn the basics. Additionally, since the software is specifically designed for being used for teaching and learning of geometry, participants can quickly move to the next stage of knowledge development, TCK development. As mentioned earlier, TPACK development is cyclical; therefore, participants will be coming back to this stage to learn more about the software at later times.
- TCK Development through learning with technology. The next step is to introduce workshop participants to the tool (dynamic geometry or other technology) by learning with the tool. This is accomplished by a technology-enhanced model lesson where participants are learners and a workshop designer facilitates the lesson. “The experience provides participants with context for how the tool is useful in instruction” (Figg & Jaipal, 2012, p. 4685).
- TPK development through pedagogical dialog. Following the model lesson, participants engage in discussion about the lesson, which allows them to learn what decisions are involved in “designing and

implementing technology-enhanced activities” (Figg & Jaipal, 2012, p. 4685). This stage is especially valuable for novice teachers who are still developing their pedagogical content knowledge.

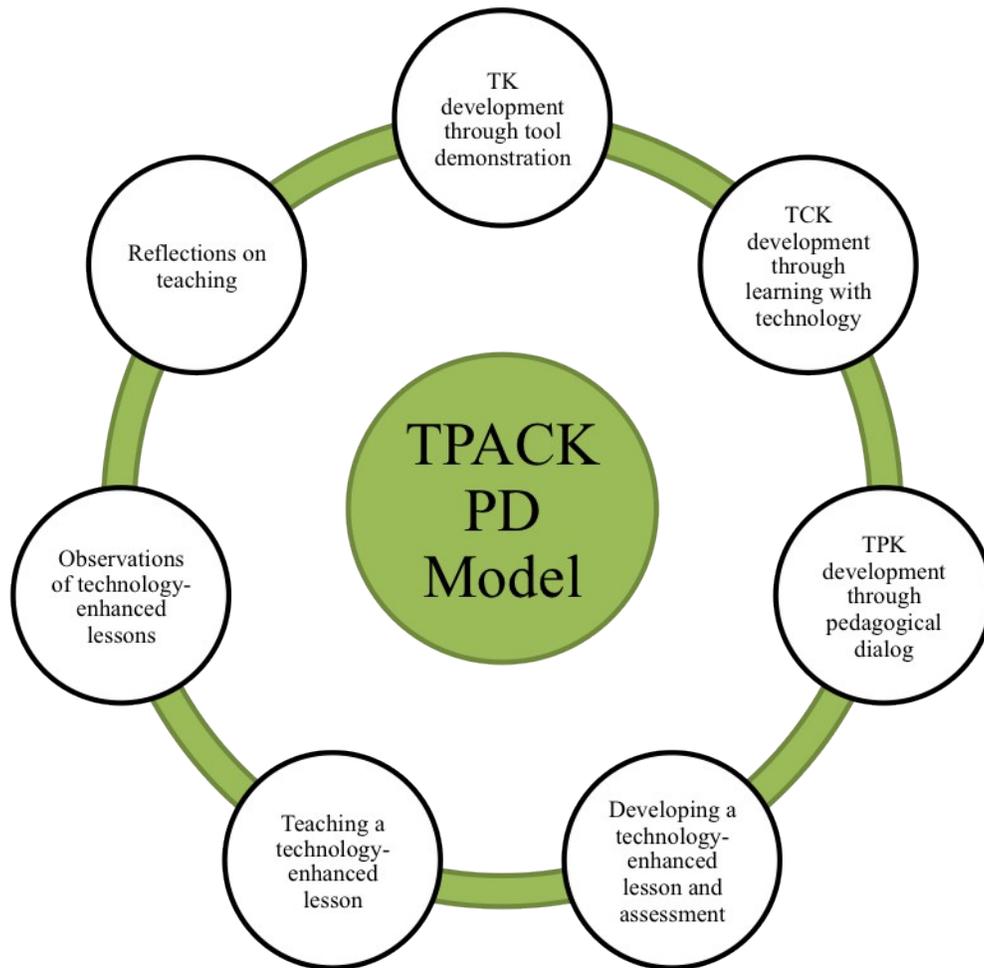


Figure 1: The TPACK PD Model.

- Developing a technology-enhanced lesson and assessment. Participants develop a lesson or an activity that incorporates technology. They will later teach and videotape this lesson. Participants also think of how technology can be used for assessment of topics taught in a given lesson.
- Teaching a technology-enhanced lesson. Each participant teaches the designed lesson with technology and videotapes it. Later, videotaped lessons will be shared with other teachers for watching and feedback.
- Observing technology-enhanced lessons. All participants watch their own, videotaped lessons as well as videotaped lessons of their colleagues. These lessons can be shared through an online management system, to which all participants have access. This virtual component of LS allows participants to watch the research lessons at their own time; it also allows them to observe a larger number of lessons.
- Reflecting on the observed lesson. Participants discuss the observed lesson, “draw out implications for lesson redesign, for teaching-learning more broadly, and for understanding of students and subject matter” (Lewis et al., 2009). Participants revise the lesson so that technology plays a more integral part in learning. In the virtual LS, this can be accomplished through asynchronous online discussion boards, synchronous chat rooms or video-conferencing.

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CHARACTERISTICS OF DIFFERENT LEARNING ENVIRONMENTS IN GEOMETRY CLASSROOMS

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This study used in-depth interviews of selected teachers to collect qualitative data to address the following research question: What characterize different learning environments in geometry classrooms? The main findings include: Since the experimental group teachers used dynamic geometry software to facilitate investigations, they were able to produce quality conjectures faster. However, as to proving, teachers varied considerably. Some could generate correct proofs, mostly for relatively simple geometric problems, some were able to work out parts of a proof but had difficulties to put the parts together, and the others were very weak in proofs.

To address the crucial need to improve geometry teaching and learning in our classrooms, we conducted a four-year research project – *Dynamic Geometry (DG) in Classrooms*. The basic hypothesis of the project was that use of DG software to engage students in constructing mathematical ideas through experimentation, observation, conjecturing, conjecture testing, and proving results in better geometry learning for most students. The project tested that hypothesis by assessing student learning in 64 classrooms randomly assigned to experimental and control groups. Teachers in both DG and control groups received relevant professional development. Data were analyzed mainly by appropriate Hierarchical Linear Modeling methods.

The study reported in this paper was a part of the four-year research project. The main purpose of this study was to answer the following research question: What are the characteristics of the learning environments in the DG and control groups with regard to conjecturing and proving? We will focus on teachers as learners in this paper.

The DG Approach and Related Literature

Research suggests that alternatives to traditional instructional approaches can be successful in moving students toward meaningful justification of ideas. “In these approaches, students worked cooperatively, making conjectures, resolving conflicts by presenting arguments and evidence, proving nonobvious statements, and formulating hypotheses to prove. Teachers attempted to involve students in the crucial elements of mathematical discovery and discourse” (Battista & Clements, 1995, p.50). These were exactly the elements experienced by the DG

group teachers in the professional development (PD) workshop.

Dynamic geometry is an active, exploratory study of geometry carried out with the aid of interactive computer software such as the Geometers' Sketchpad (GSP) (Jackiw, 2001). The instructional approach for using DG software to facilitate students' learning is referred to as the DG approach in this paper. Many researchers conducted studies on using the DG approach in geometry learning. Vincent (2005) found that the DG motivating context and the dynamic visualization fostered conjecturing and intense argumentation; and that the teacher's intervention was an important feature of the students' augmentations—prompting the students to furnish justifications for their statements and checking the validity of their justifications. Hollebrands (2007) identified different purposes for which students used dragging, the main feature of the DG software, and different purposes for which students used measures. These purposes appeared to be influenced by students' mathematical understandings that were reflected in how they reasoned about the physical representations, the types of abstractions they made, and the reactive or proactive strategies employed. Baccaglioni-Frank and Mariotti (2010) presented “a model describing some cognitive processes that can occur during the production of conjectures in dynamic geometry and that seem to be related to the use of specific dragging modalities” (p. 225) and used it to analyze students' explorations of open problems. Thus, when used as a cognitive tool, DG technology can facilitate students' exploration and investigation activities, promote their conjecturing, verifying, explaining, and logical reasoning abilities, and enhance their conceptual understanding of important geometric ideas. However, very few studies concentrated on whether there are different characteristics of the DG and non-DG learning environments, and if so, what they are and how we can conceptualize strategies for dealing with the differences.

Methodology

To gather evidence regarding the characteristics of the learning environments of the two treatment groups (DG and control), we used in-depth interviews of teachers to collect qualitative data. A stratified random sampling method was used to select 12 teachers (six from each of the two treatment groups; stratified based on mathematical abilities and implementation fidelity). Each interviewee was interviewed a minimum of three times during the school year. Protocols for semi-structured clinical interviews (Goldin, 1997) were created. In each protocol, the interviewee was given an activity involving the posing of conjectures from an exploration of a

geometric situation and testing of such conjectures to confirm generalization of the findings. Proving was then requested. When interviewing, the researcher watched the interviewee work throughout the activity and asked questions to help uncover his or her thought process and problem solving strategies. All interviews were video recorded and the videos were transcribed.

Data Analysis

The case study (Stake, 1995) method was used in the data analysis. Four teachers with a range of mathematical abilities were selected for the case studies, with pseudonyms Dan, Greg, Chris, and Nancy respectively. Data derived from the interview transcripts were analyzed inductively to formulate categories and themes to describe the phenomenon under study. This process allows for the data segments to be categorized by a system that is derived from the data themselves. A cross case analysis was also be conducted to address the research question.

Descriptive Analysis and Findings

Based on the related literature cited earlier, we identified the following components of Conjecturing and Argumentation: 1) Construction (Constructing the problem situation); 2) Investigation (Investigating the problem situation to generate conjecture); 3) Stating conjecture 4) Testing conjecture, and 5) Proving. What follows is a descriptive analysis (Koedinger, 1998) of the four teacher interviewees' performances within each of the five components. Due to the page limit of this paper, we will mainly discuss the problem given in the third interview: *A 4 by 4 picture hangs on a wall such that its bottom edge is 2 ft above your eye level. How far back from the picture should you stand, directly in front of the picture, in order to view the picture under the maximum angle?*

Dan and Greg's Performances

Construction (Constructing the problem situation): Dan was an interviewee selected from the DG group. He was very efficient in constructing problem situations. When given the maximum angle problem, he proficiently constructed the problem situation using GSP tools in the following steps (Figure 1): (1) constructing a vertical segment AB, using the DG measuring tool to make it 4 cm long, representing 4 feet long (even though the 4cm length was not necessary, it was a good idea); (2) using dilation to construct segment $AB' = 6$ ft (so $BB' = 2$ ft); (3) constructing a line through B' and perpendicular to segment AB, which is "eye level"; and (4) constructing a free point (labeled "You", simplified as Y) on line "eye level" and segments AY and BY. Then $\angle AYB$ would be the viewing angle. Greg is another interviewee selected

from the DG group. In the teacher content pretest administered at the opening PD session of the larger project, his score was among the lowest. At all PD sessions offered in project year 2, when picking up GSP skills, he usually needed extra help from the PD facilitators or peer participants. However, in all three interviews conducted in project year 3, similar to Dan, he showed high efficiency in using GSP software to construct the given problem situations including the situation of the maximum angle problem.

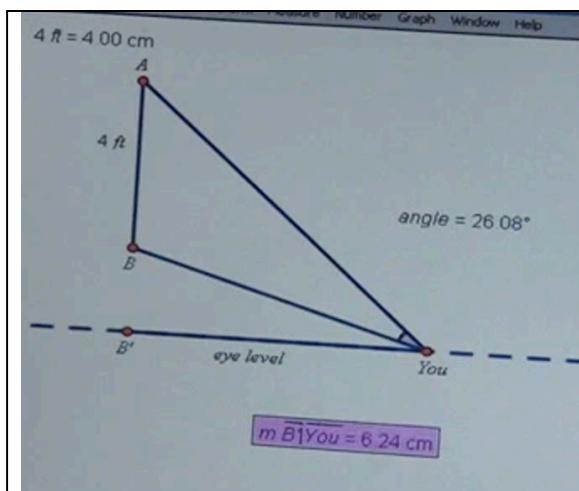


Figure 1. A problem situation

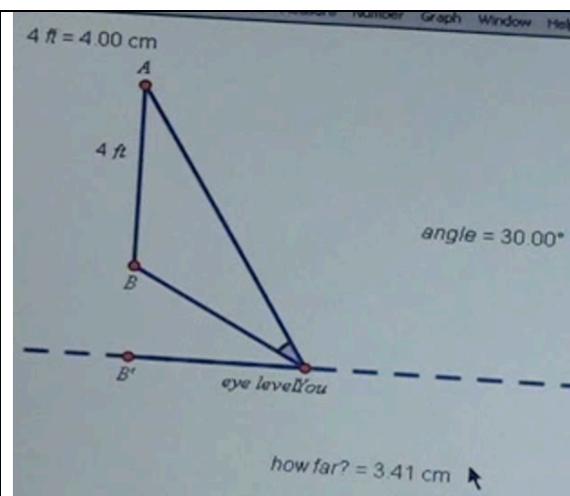


Figure 2. Investigating the problem situation

Investigation (Investigating the problem situation to generate conjecture): In Figure 1, since Y is a free point on line “eye level”, Dan dragged it back and forth on the line and found $\angle AYB$ became smaller when point Y moved to right, and became larger when point Y moved to left but after passing a particular location became smaller again. By dragging and comparing the dynamic measurements, Dan found that “particular” location shown in Figure 2 (Y: 3.41 cm [by measuring] representing 3.41 feet away from point B’), where $\angle AYB$ is a 30° angle, which is the maximum viewing angle. To illustrate this fact and formulate a conjecture, Dan continued to explore the situation shown in Figure 2. By guessing $\triangle AB'Y$ being a 30° - 60° - 90° triangle when $\angle AYB$ is a 30° angle and doing calculations using $\tan 30^\circ$, Dan came up with a conjecture. Greg experienced almost the same investigation process as Dan.

Stating conjecture: As the result of the investigation, Dan wrote his conjecture on his laptop screen: “You should be a distance from the painting equal to the total above eye-level height times $\tan 30^\circ$, which is $6 \cdot \tan 30^\circ = 6 \cdot \frac{\sqrt{3}}{3} = 2\sqrt{3}$ (feet).” Greg stated the same conjecture using a slightly different language.

Testing conjecture: Dan dragged the free point Y (i.e., point You) back and forth on line “eye level” and always observed the viewing angle AYB being maximized ($= 30^\circ$) when Y was a distance of $2\sqrt{3}$ (≈ 3.46) feet from point B'. This made him feel positive that his conjecture was correct. Greg did a similar drag test for the conjecture that he stated.

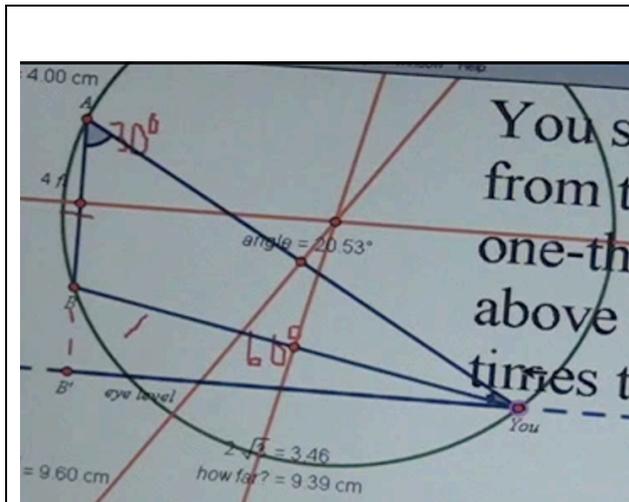


Figure 3. The circle going through A, B, and Y

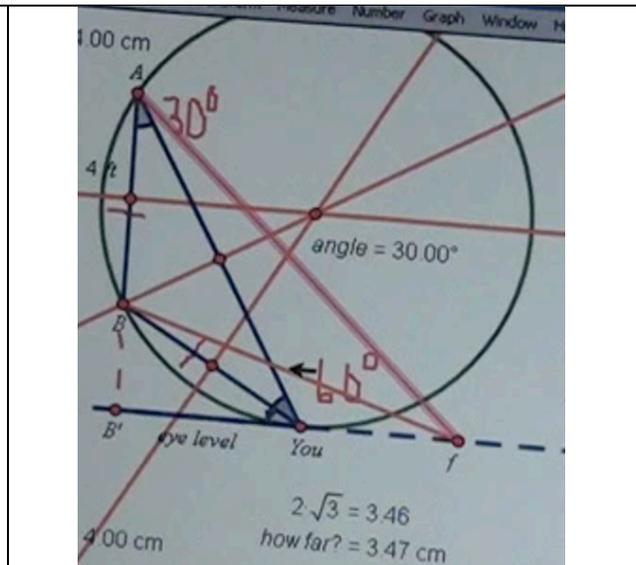


Figure 4. The situation constructed for proving

Proving: For simple proofs such as proving the conjecture that was made for the first interview (“If two altitudes are equal in length in a triangle, then the triangle is isosceles”), Dan was able to figure them out correctly in a relatively short time. For more complicated proofs, if time allowed was limited, he might need some help. Dan received such help during the process of developing a proof for the maximum angle problem. He had first tried several different ways to do so but was not able to proceed successfully. Due to the limited time of the interview, the researcher (interviewer) decided to intervene, “Let’s construct the circle going through points A, B, and Y (see Figure 3). It might help.” Dan constructed the circle by first constructing two perpendicular bisectors on segments BY and AY (Figure 3). Then he dragged point Y back and forth on line “eye level” again. By observing the constructed circle that kept changing size (Figure 3), he found that the viewing angle AYB would become maximized ($= 30^\circ$) when the circle was tangent to line “eye level” with point Y as the point of tangency (Figure 4). He wrote a different version of his conjecture on his laptop screen: “The maximum viewing angle occurs when you stand at the point of tangency created by line “eye level” and Circle ABY.” Based on another hint from the researcher, Dan constructed a free point (f [should be “F”]) on line “eye

level” (Figure 4), and constructed segments Af and Bf. After thinking for a while, he discovered the most important idea in finishing the proof: “It’s related to arcs and inscribed angle somehow. This outside angle [$\angle AfB$] is half of this arc [pointing to minor arc AB] minus this arc [pointing to the small arc that is “inside” $\triangle AfB$ and very close to point f]. This is subtracting something, so it is smaller than this angle [pointing to $\angle AYB$], which is half of this arc [pointing to minor arc AB again].” He measured the two angles using the GSP measuring tool, and dragged point f back and forth to observe the change of $\angle AfB$, its changing measure, and the fact that $m(\angle AfB)$ was always smaller than or equal to $m(\angle AYB)$. He stated, “I see. That’s brilliant. I will write the whole process of the proof.” As to Greg, the situation was different. Even for a simple proof such as the one required for the first interview problem, he needed hints or more significant help. For the maximum angle problem, he experienced considerable difficulties, which were related to his weak mathematical background. For example, he did not know how to construct the circle going through three non-linear points.

Chris and Nancy’s Performances

Construction (Constructing the problem situation): Chris and Nancy were from the control group. Chris used a ruler and a pencil to construct the situation of the maximum angle problem (Figure 5). Nancy did not perform any initial constructions. During her investigation of finding the maximum angle, she constructed a situation similar to Figure 6.



Figure 5. The problem situation by Chris

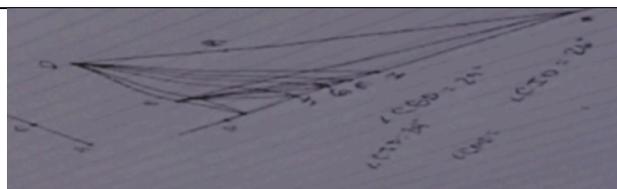


Figure 6. The investigation that Chris did

Investigation (Investigating the problem situation to generate conjecture): Since no free point could be made on line “eye level” as only static tools (a ruler and a pencil) were used, the viewing angle constructed in Figure 5 was only one example. Chris added four more examples (of the viewing angle), and measured all five examples using a protractor (see Figure 6). The measures of the five angles were approximately 28° , 29.5° , 29° , 27° , and 26° (from left to right). Then he estimated the range of the maximum angle, “Basically, around here to here [pointing to points G and J] is the optimal spot. I guess where is being maximized. Somewhere around 29 or 30 degrees.” The researcher prompted him by asking if he noticed any special triangles. Chris

responded, “ $\triangle CED$ [E being a point between G and J] might be an isosceles triangle.” After measuring the angles of that triangle, he said, “Yeah, it’s going to be isosceles, 120, 30 and 30.” It took Chris quite a long time in the investigation. Nancy’s investigation process was similar.

Stating conjecture: With a hint given by the researcher, both Nancy and Chris conjectured that the maximum viewing angle was at the point of tangency on line “eye level” with circle H (a circle formed by “your eye”, the bottom of the picture frame, and the top of the frame).

Testing conjecture: Neither Chris nor Nancy conducted the conjecture testing activity before working on their proofs.

Proving: Both Chris and Nancy were able to do simple proofs, but they had difficulties in doing more complicated proofs without prompting. For the maximum angle problem, without the researcher’s hint on constructing a circle mentioned above (i.e., Circle H), they would be unsure how to begin their proofs. The difference between them and Dan was that after receiving help, while Dan achieved conceptual understanding of the proof ideas, Chris and Nancy were still focused on the actual measurements in the figure to help with their proofs, which caused some confusion on developing a general formal proof.

Discussion

The above descriptive analysis reveals that when teachers are using DG software to explore geometric concepts and problems, the software’s dragging and dynamic measuring features provide great convenience and efficiency for teachers to construct and investigate problem situations. The infinitely many examples or counter examples generated and the dynamic visualization available in the investigation processes help teachers (e.g., Dan and Greg) clearly see what is invariant when other objects kept changing, so as to develop and test conjectures with sound understanding that is necessary for teachers to further create proof ideas. Influenced by the PD designed for the DG group, the teachers have developed a new learning style - conducting problem solving through a learning process characterized by constructing the problem situation – investigation – making conjecture – testing conjecture – proving. The larger project has provided evidence of the effectiveness of this learning process. The situation in the control group is different. Although some teachers recognize the importance of conjecturing and proving, the limitations of using static tools (motionless, time consuming, etc.) in constructing and investigating problem situations do not support the new learning style. For example, Chris and Nancy came up with their conjectures much more slowly than Dan and Greg, and so didn’t find

time to do the conjecture testing activity.

From the descriptive analysis, we also learned that after our PD sessions, the teachers' reasoning and proving abilities were improved at a slower pace in comparison to conjecturing. Some teachers could generate correct proofs, mostly for relatively simple geometric problems, some were able to work out parts of a proof but had difficulties to put the parts together, and the others were very weak in proofs. Therefore, it is by no means easy to really increase teachers' mathematical reasoning abilities. It would be a long-term task to develop effective strategies for achieving this goal. Furthermore, to take full advantage of the dynamic features of GSP to verify whether a conjecture is true before using it in the reasoning process is an important learning habit, which many teachers did not have. We should spend enough time and energy to help teachers develop this habit.

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TEACHING WITH DYNAMIC GEOMETRY SOFTWARE: A MULTIPLE CASE
STUDY OF TEACHERS' TECHNOLOGICAL PEDAGOGICAL CONTENT
KNOWLEDGE

DISSERTATION

Presented to the Graduate Council of
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for the Degree

Doctor of PHILOSOPHY

by

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San Marcos, Texas
May 2012

TEACHING WITH DYNAMIC GEOMETRY SOFTWARE: A MULTIPLE CASE
STUDY OF TEACHERS' TECHNOLOGICAL PEDAGOGICAL CONTENT
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DEDICATION

To my parents, Wiesława and Lech,
and to my husband, Jeremy.

ACKNOWLEDGEMENTS

First and foremost, I would like to thank God for giving me the ability and perseverance to complete this challenging journey. I have the strength for everything through Him who empowers me (Philippians 4:13).

There are many people who have supported me as I completed this study and no words are adequate to express my gratitude and appreciation for them. I would like to thank my dissertation chair, Dr. Zhonghong Jiang, for his suggestions and feedback. I also would like to express my appreciation to my committee members, Dr. Clarena Larrotta, Dr. M. Alejanda Sorto and Dr. Samuel Obara, for their insights and assistance with this project. I thank my committee for everything they have done.

I would like to thank the participants of this study. I appreciate very much their willingness to participate and the time they spent on providing data for this project. Without them, this project would not be possible, and I cannot thank them enough.

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ABSTRACT

TEACHING WITH DYNAMIC GEOMETRY SOFTWARE: A MULTIPLE CASE STUDY OF TEACHERS' TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE

by

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This qualitative case study investigated how four high school teachers developed and used their knowledge in teaching geometry with technology. In particular, this study focused on teachers' technological pedagogical content knowledge (TPACK) and their integration of dynamic geometry in the classroom instruction. The sources of data included: an initial interview, observations, documents, a closing interview, a survey, implementation questionnaires, professional development attendance records and the researcher's log. Data analysis utilized the TPACK Development Model to describe

participants' dynamic geometry integration and to identify their TPACK development levels.

All participants displayed good knowledge of geometry content, although they did not always know how to connect it with their pedagogical and technological knowledge. TPACK development levels were identified through the descriptions of participants' TPACK development and enactment. The levels varied within the themes and their descriptors for each participant; however, overall TPACK development levels were identified for three participants—two at the adapting level and one at the exploring level. The TPACK levels for the fourth participant were consistent only for the teaching theme descriptors and were at the exploring level.

Three unexpected findings surfaced. First, the participant with least teaching experience displayed the highest levels of TPACK. Second, the participant with most teaching experience with dynamic geometry showed the most inconsistency among the TPACK development levels, ranging from recognizing to exploring. Third, ongoing professional development and easy access to computers did not translate to frequent incorporation of dynamic geometry in teaching and learning. The participants claimed the curriculum and standardized testing to be the main barriers to increased technology use. Findings suggested that participants developed their TPACK through attending professional development workshops and implementing what they learned in the classroom instruction. Based on those findings, this study proposed a professional development model designed for teachers interested in integrating dynamic geometry in the classroom instruction.

CHAPTER I

INTRODUCTION

If we teach today as we taught yesterday, we rob our children of tomorrow.

– *John Dewey*

In today's world digital technologies are widely available and young people interact with them on a daily basis (Roblyer & Doering, 2010). Using technology in education not only motivates students by tapping into their comfort zone, but also enhances instructional methods, increases productivity, and allows students to gain required information age skills (Roblyer & Doering, 2010). Use of technology in teaching and learning has been advocated by various professional organizations (ISTE, 2008; NCTM, 2000). The International Society for Technology in Education (ISTE) prepared the National Educational Technology Standards and Performance Indicators for Teachers (NETS). Among them are the following four performance indicators:

Teachers design or adapt relevant learning experiences that incorporate digital tools and resources to promote student learning and creativity. Teachers develop technology-enriched learning environments that enable all students to pursue their individual curiosities and become active participants in setting their own educational goals, managing their own learning, and assessing their own progress. . . . Teachers demonstrate fluency in technology systems and the transfer of current knowledge to new technologies and situations. . . . Teachers model and

facilitate effective use of current and emerging digital tools to locate, analyze, evaluate, and use information resources to support research and learning. (ISTE, 2008, p. 1)

These performance indicators clearly show that teachers need to learn about new technologies and be prepared to provide their students with technology-based learning opportunities. The NETS were prepared for all teachers, no matter what grade or what subject they teach.

The National Council of Teachers of Mathematics (NCTM) specifically advocates the use of technology in teaching mathematics in its Principles and Standards for School Mathematics (NCTM, 2000). The technology principle states, “technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning” (NCTM, 2000, p. 373). In more detail, it declares that:

Students can learn more mathematics more deeply with the appropriate and responsible use of technology. They can make and test conjectures. . . . In the mathematics classrooms envisioned in Principles and Standards, every student has access to technology to facilitate his or her mathematics learning. . . . The teacher must make prudent decisions about when and how to use technology and should ensure that the technology is enhancing students' mathematical thinking. (NCTM, 2000, p. 373)

The NCTM standards advocate for wide access to technology for all students, but they also stress the appropriate use of technology. That indicates technology should be used only when it improves learning and not just for the sake of using it. Technology is not an end, but a means to enhanced learning.

Use of technology is also encouraged by many school districts. Although state standards for mathematics vary across the U.S., on average approximately 25% of content in state standards is on instructional technology (Porter, McMaken, Hwang, & Yang, 2011). The Common Core State Standards mention the use of technology in teaching and learning of mathematics, but their primary focus is on content to be learned and not on instructional strategies to be used (Common Core State Standards Initiative, 2010). The Common Core Standards for Mathematical Practice encourage “use of appropriate tools strategically” and among such tools dynamic geometry software (Common Core State Standards Initiative, 2010). In Texas, the Professional Development and Appraisal System (PDAS), an instrument for evaluating teacher performance, includes the quantity and quality of technology use during class time (TEA, 2005). Therefore, according to PDAS, mathematics teachers not only need to be prepared to integrate technology into teaching and learning of mathematics, but they have to do it effectively by providing good quality instruction in order to obtain a good evaluation.

As a doctoral research assistant, I had an opportunity to work on a research project investigating the use of the *Geometer's Sketchpad* (Jackiw, 2009), one of the major dynamic geometry software packages, in teaching and learning of geometry at the high school level. My work on the Dynamic Geometry Project (DGP) motivated me to investigate the area of teachers' knowledge as it relates to integration of dynamic geometry software in teaching and learning of mathematics. Why is this important? The Conference Board of the Mathematical Sciences (CBMS) explains that: “Technology has revolutionized many jobs and substantially increased the mathematical skills needed across the workforce. In contrast, its impact on instructional practices has been more

modest and varies greatly from classroom to classroom” (CBMS, 2001, p. 47). As part of my role as a doctoral research assistant, I have observed several teachers in their classrooms, and I have seen first-hand that integration of technology into teaching and learning of mathematics varies considerably.

Educational software such as *the Geometer’s Sketchpad* (Sketchpad) can assist in developing students’ understanding of mathematical concepts and increasing their reasoning skills (CBMS, 2001). Taking advantage of dynamic features such as dragging, one can see “the universality of theorems in a way that goes far beyond typical paper and pencil explorations” (CBMS, 2001, p. 132). For example, when exploring the exterior angles of a convex polygon, students can measure those angles for different polygons (e.g., a triangle, a quadrilateral, and a pentagon), add them, and notice that the sum equals 360 degrees. Then, they can select any vertex, drag it to change the form of the original polygon (but keeping it convex), and at the same time notice that the sum of the measures of the exterior angles does not change (see Figure 1).

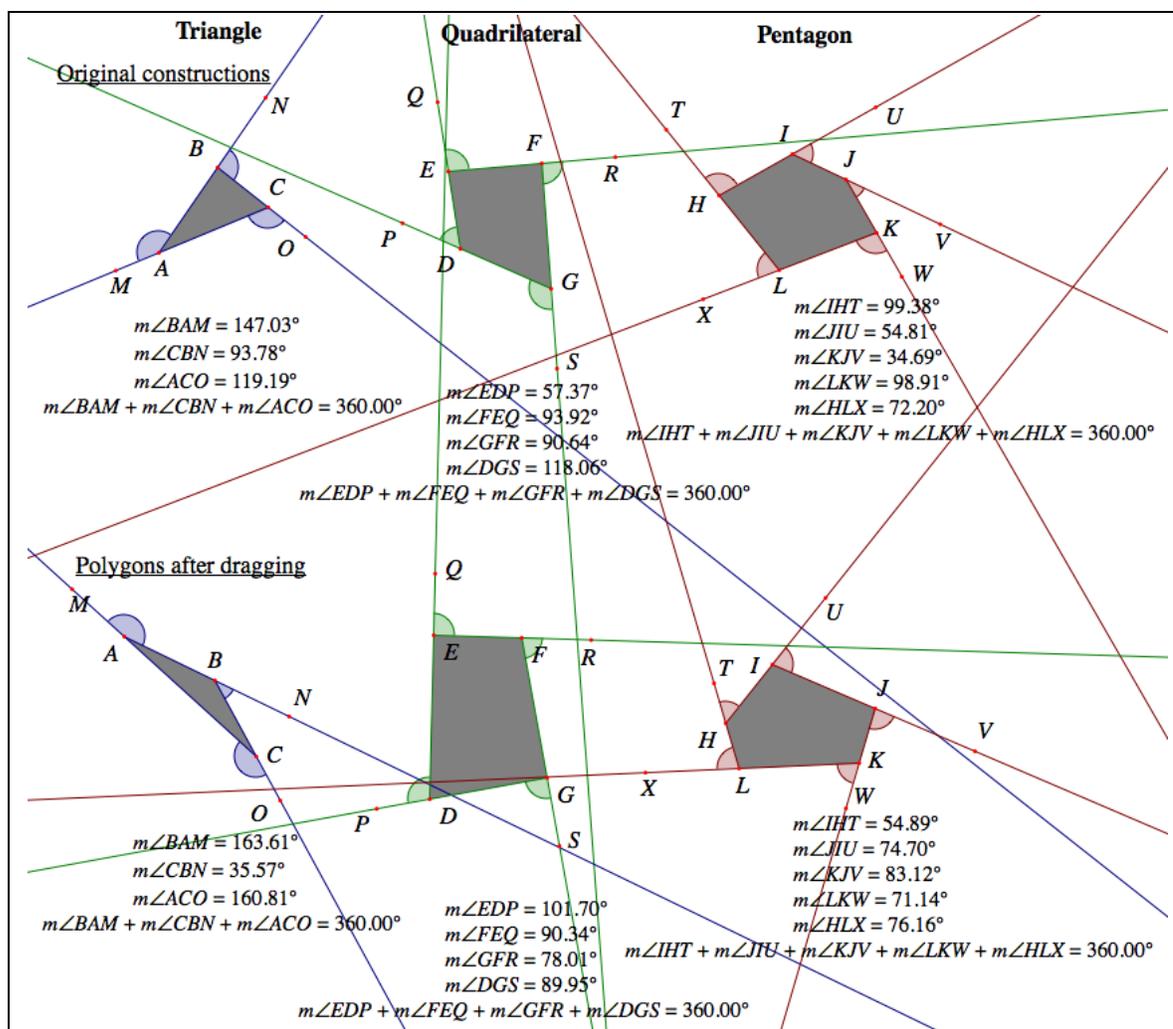


Figure 1. Screenshot of a Sketchpad file showing an activity for exploring the sum of exterior angle measures of convex polygons. The top part of the figure shows the original constructions of three polygons (triangle ABC, quadrilateral DEFG, and pentagon HIJKL), the exterior angle measures of each polygon, and the sums of exterior angle measures for each polygon. The bottom part shows these polygons after their vertices had been dragged.

This activity shows that the exterior angle measures are different from the original construction, but their sum remains the same for each polygon, and it remains constant

for all polygons explored here. However, one might argue that a similar exploration can be done with paper and pencil by constructing several different polygons, measuring their exterior angles, and calculating their sum. How is this exploration with Sketchpad different then? And is it better? The use of dynamic geometry tools certainly saves time because only one construction is needed, and the software does all the measurements automatically. The user only has to specify what to measure, drag vertices, and observe what changes and what remains the same. One might argue that the efficiency (the saving of time) when using Sketchpad is good enough because it allows more time for other activities, possibly ones that require more cognitive demand such as writing of a proof. However, there is more that the dynamic tools can offer. Students can continue their exploration of the sum of the exterior angle measures of convex polygons by taking full advantage of the dynamic features of Sketchpad. To do so, they can mark one of the vertices of a polygon as the center for dilation (by double-clicking on a vertex), change the Arrow tool to Dilate Arrow, select the entire construction, and drag it towards the marked center by clicking on any of its part. The final drawing should resemble a star - one point, rays with their vertices at that point, and all angles forming a circle. The number of rays will equal the number of sides of a polygon. Figure 2 represents one such example. Students can see that no matter what kind of convex polygon they have (triangle, quadrilateral, pentagon, etc.), as a result of this exploration, the exterior angles of a polygon will always create a circle, i.e., 360 degrees. This exploratory illustration is extremely powerful and is not possible with paper-and-pencil explorations.

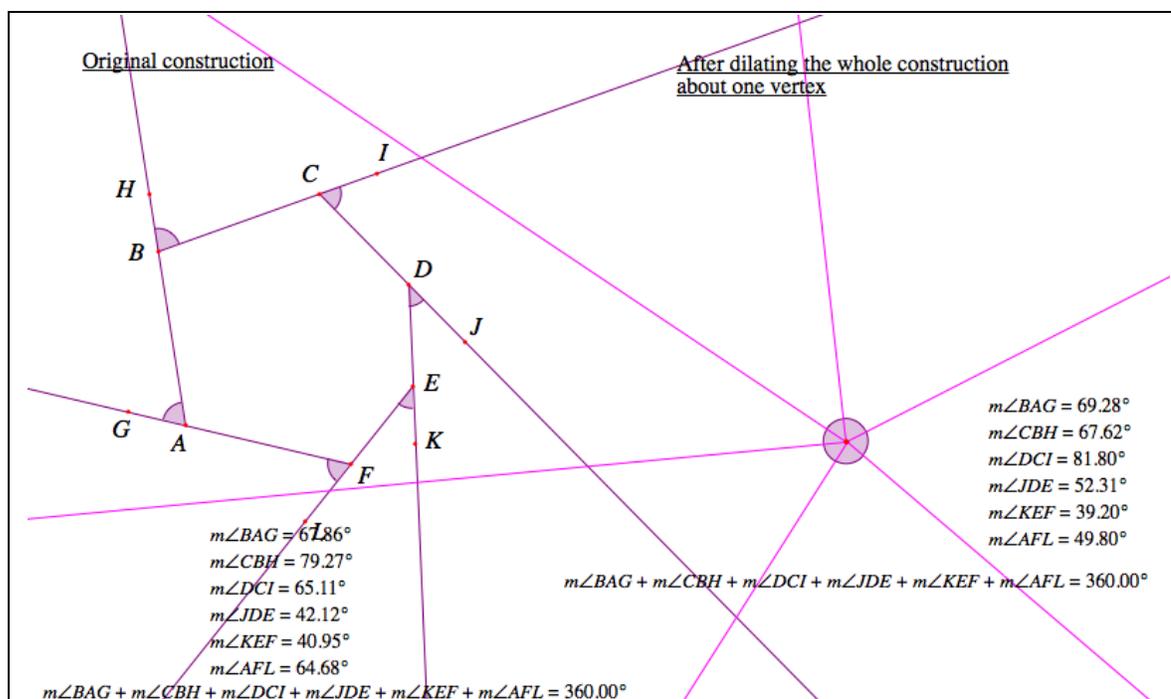


Figure 2. Screenshot of a Sketchpad file illustrating further exploration in the activity for the sum of exterior angle measures of convex polygons.

This kind of investigation also lays a better foundation for students' understanding of proof. Although, this is only one instance of Sketchpad use and how it can enhance student learning and understanding of mathematics, there are many more similar examples described in the literature, e.g., Sinclair and Yurita (2008) and Forsythe (2007, 2009).

Statement of the Problem

Taking into consideration the increase in available technology in schools in the recent years and the requirements placed on teachers to use it in teaching, we still do not know how effective teachers are in integrating technology into teaching (Harris, Grandgenett, & Hofer, 2010). Finding out how teachers learn about technology and how

they incorporate it into their teaching is necessary since the way teachers use technology can impact the quality of instruction and student learning (Roberts & Stephens, 1999).

It has always been assumed that teachers need to know the content they teach (Shulman, 1986). They also need to know excellent instructional strategies for teaching that content (Shulman, 1986). Now, with the emergence and wide availability of educational technologies, teachers also need to possess knowledge for effective integration of those technologies into teaching the content. Increasing the effectiveness of technology-supported content area teaching has been a national goal for several years (Riley, Holleman, & Roberts, 2000). According to the most recent national survey (Fast Response Survey System) conducted by the National Center for Education Statistics (NCES), 72 percent of high school teachers feel sufficiently trained in technology usage and 66 percent feel sufficiently trained to integrate technology into classroom instruction (Gray, Thomas, & Lewis, 2010). Additional information reported from this survey included the following: most teachers (93 percent) are interested in using technology in classroom instruction and almost three-quarters of teachers (74 percent) design lessons in which students use a variety of educational technologies (Gray, Thomas, & Lewis, 2010). However, the NCES does not provide statistics specifically for high school mathematics teachers.

Based on data reported by the participants in the DGP during the first year of implementation, teachers did not feel like they were effective in using Sketchpad in teaching geometry. Even though teachers felt sufficiently trained to integrate technology and many of them had created technology-based lessons, it did not necessarily mean they integrated technology into teaching and learning effectively. Based on preliminary

observations of the DGP participants, high school mathematics teachers do not have very good knowledge about integrating technology into teaching. Merely learning a new technology is not enough; teachers need to learn how a new technology is connected to the subject they teach and how it can be integrated into their instruction.

Mishra and Koehler (2006) defined a construct of knowledge that teachers possess when teaching with technology, namely technological pedagogical content knowledge (TPACK, previously TPCK). The concept of TPACK and its assessment have been researched (Mishra & Koehler, 2006; Niess, 2005; Niess et al., 2009; Schmidt et al., 2009), although not in the area of geometry or dynamic geometry software. Also, the conceptualization of teacher knowledge related to using technology in teaching mathematics is in the early stages. More research is needed to examine how teachers acquire TPACK and use it to teach geometry with technology effectively. Niess et al. (2009) proposed a TPACK development model for mathematics teachers that can be used in evaluating and describing their TPACK growth. This model consists of five stages (recognizing, accepting, adapting, exploring, advancing) and four themes (curriculum and assessment, learning, teaching, and access) as well as descriptors and examples describing actions in which teachers engage in “while adapting technology in their teaching in order to enhance student learning” (Niess et al., 2009, p. 12). Using this model can assist in identifying the level of teachers’ knowledge related to integrating technology into teaching and in providing insights into knowledge growth.

Purpose of the Study

This qualitative case study documented how high school geometry teachers develop and enact knowledge for teaching with dynamic geometry software.

Additionally, this study identified TPACK development levels based on interviews, classroom observations, student activity sheets and a survey. The goal of this study was to contribute to the existing body of knowledge about TPACK and teaching in a dynamic geometry environment.

Researcher's Background

I have four years of experience as a math instructor. Before becoming a high school teacher, I taught developmental mathematics at a community college. Then, I taught mathematics and economics at a large high school in Central Texas. I had an opportunity to work in a program where students' studies were self-paced. This program was designed for students who wanted to accelerate their education (for various reasons). I worked with a diverse population of students and taught algebra 1, geometry, algebra 2, math models, calculus, and TAKS (Texas Assessment of Knowledge and Skills) preparation course. After two years, I decided to become a full-time doctoral student. I began my studies in mathematics education at Texas State University-San Marcos where I had an opportunity to teach developmental mathematics again. After three semesters, I changed my position from teaching to research. As a research assistant, I had the opportunity to work with high school geometry teachers who participated in the DGP (conducted by a research team from Texas State and funded by the National Science Foundation). I established a relationship with those teachers through professional development workshops, classroom visits, interviews, and e-mail communications throughout the 2010-2011 school year. During that year, I became more familiar with the geometry curriculum in Texas high schools, although as a former teacher, I already had a good idea of what it was. Additionally, I learned about instructional strategies used by

teachers and the challenges they face when trying to integrate dynamic geometry into teaching and learning of geometry. I also observed that knowledge of geometry, pedagogy, and technology varied substantially among the DGP participants. Therefore, as a mathematics educator, I made my main goal to ensure that teachers have the necessary content knowledge, know appropriate instructional strategies for the subject they teach, and have necessary technological skills to incorporate new technologies effectively into teaching and learning of mathematics.

Researcher's Perspective

As a former high school teacher, I know firsthand that teacher knowledge plays a crucial role in teaching mathematics. I believe that content knowledge (CK), knowledge of mathematics, is the foundation of good quality teaching and that pedagogical content knowledge (PCK) develops as teachers gain more experience. Similarly, I believe that technological pedagogical content knowledge (TPACK) takes time to develop (Olive, 2002). In addition, I think that development of TPACK might take less effort for teachers who already have well-developed PCK than for teachers whose PCK is not well-developed. However, because technology changes all the time, TPACK might not ever be fully developed. Even if we restrict ourselves to one type of technology, such as Sketchpad, continuous learning is necessary because of new features and enhancements in this software.

Having good knowledge of mathematics is very important in today's world. Students who earn their degrees in mathematics are in high demand. Before going to college, students must graduate from high school and in order to do that (in Texas and many other states) they must pass a geometry course. Teachers' geometric knowledge for

teaching is essential for students' success and, in today's information-age world, knowledge for teaching in a technology-enhanced classroom is a must. However, teachers face many challenges when trying to integrate technology in their teaching. Some of those challenges include limited access to technology, lack of time for professional development on integrating technology into teaching, lack of support from campus administrators, and lack of knowledge for effective integration of technology into teaching. As difficult as sometimes these challenges might be, I believe teachers should strive to overcome them and improve their TPACK by continuously learning new content, new pedagogies, and developing their technological skills.

Definitions of Terms

The following terms are used throughout this dissertation, and their definitions are provided here:

Technological Pedagogical Content Knowledge (TPACK)

This is “knowledge of the technology-pedagogy-content interaction in the context of content-specific instructional strategies” (Cox & Graham, 2009) and “the total package required for integrating technology, pedagogy, and content knowledge in the design of instruction for thinking and learning mathematics with digital technologies” (Niess et al., 2009).

Technology

Technology consists of “equipment and resources such as calculators, computers, telecommunications, internet, cameras, multimedia, satellites and distance learning facilities, CDROMs, and scanners used for the purpose of instruction when available and appropriate” (TEA, 2005).

Technology Integration

The process of technology integration involves continuous learning about new technologies and resulting in improvement. Technology resources include computers and specialized software. Technology integration entails “the incorporation of technology resources and technology-based practices into the daily routines, work, and management of schools” (NCES, 2002, p. 75).

Dynamic Geometry (DG)

This term refers to active and exploratory geometry carried out with interactive computer software (such as Sketchpad). The characteristic feature of DG software is the continuous and real-time transformation called “dragging” that is not available elsewhere.

Delimitations

This study focused on one type of technology - dynamic geometry software, namely Sketchpad. Also, it involved only high school teachers who taught geometry and integrated Sketchpad in their instruction at the time of this study. In particular, participants in this study were teachers who also participated in the DGP.

Significance of the Study

One of the goals of this study was to contribute to existing research on high school geometry teachers’ TPACK when teaching with dynamic geometry software. Literature in this area of research is limited; therefore, the scholarly body of knowledge is increased by an addition of the case reports offered through this study. Also, the findings along with current literature on TPACK development and professional development offer recommendations for teachers’ professional development in the field of teaching with dynamic geometry software. The findings also offer suggestions for future research in the

area of teachers' TPACK. Lastly, this study moves one step closer to developing items that can measure TPACK for teaching with dynamic geometry software. Further development in this area is needed and is left for future studies.

Summary

This chapter identified the need for studying TPACK related to teaching geometry in a dynamic geometry environment. Use of technology has been advocated by many professional organizations and access to technology in schools has increased. Teachers tend to feel sufficiently trained in using technology and integrating it into classroom instruction. However, there is little research on technology integration from the viewpoint of teachers' TPACK. The goal of this study was to investigate how high school geometry teachers integrate dynamic geometry software into instruction and to determine their TPACK development levels.

This dissertation consists of six chapters. The next chapter, Chapter II, discusses literature related to teaching geometry with technology and TPACK. Then, Chapter III describes methodology used for this study; it provides context for the study, introduces the participants, describes the data sources and how they were analyzed. Next, Chapters IV and V present findings of this study. Chapter IV consists of case reports describing TPACK development, TPACK enactment and TPACK developmental levels for each participant. Chapter V provides cross-case analysis. Finally, Chapter VI offers discussion of findings, implications, and suggestions for future research.

CHAPTER II

LITERATURE REVIEW

Those who can, do. Those who understand, teach.

-Lee S. Shulman

This chapter reviews literature relevant to integrating technology into teaching geometry and teachers' technological pedagogical content knowledge (TPACK). The chapter is divided into six sections: pedagogical content knowledge, technological pedagogical content knowledge, assessing teachers' TPACK, use of technology in geometry, the gap in the literature and the theoretical framework. The gap in the literature section provides conclusions from the literature review and offers a design for this research project. The theoretical framework of this study is based on the Mathematics Teacher TPACK Development Model, which describes five stages of how TPACK can grow.

Pedagogical Content Knowledge

In 1986, Shulman introduced the concept of pedagogical content knowledge as a combination of content knowledge and pedagogical knowledge. He claimed teachers not only need to know the subject matter they teach, but they also need to know how to teach it. Pedagogical knowledge is generic, meaning that it can be applied to any subject. For example, a teacher with strong pedagogical knowledge has excellent classroom

management skills, knows how to organize student records, etc. Content knowledge is specific, e.g., a mathematics teacher has knowledge of mathematics, whereas a biology teacher has knowledge of biology. Therefore, pedagogical content knowledge is specific because it is related to content knowledge. Pedagogical content knowledge is about knowing how to teach the specific content (e.g., how to teach mathematics); it's about knowing student misconceptions, how to present the content in various ways, etc. (Shulman, 1986).

Technological Pedagogical Content Knowledge

Pierson (2001) investigated how teachers at various levels of technology use (Dwyer, Ringstaff, & Sandholtz, 1991) and teaching abilities (Berliner, 1994) used technology in their classrooms. As a result of her study, she introduced the concept of technological-pedagogical-content knowledge (Pierson, 2001), which defined “effective technology integration” (p. 427) and was built on Shulman’s concept of pedagogical content knowledge. Others who discussed the relationships between content, pedagogy, and technology include: Hughes (2004), McCrory (2004), Margerum-Leys and Marx (2002), Niess (2005), and Slough & Connell (2006). In 2006, Mishra and Koehler proposed a conceptual framework for technological pedagogical content knowledge (TPACK), which is comprised of seven domains (see Figure 3).

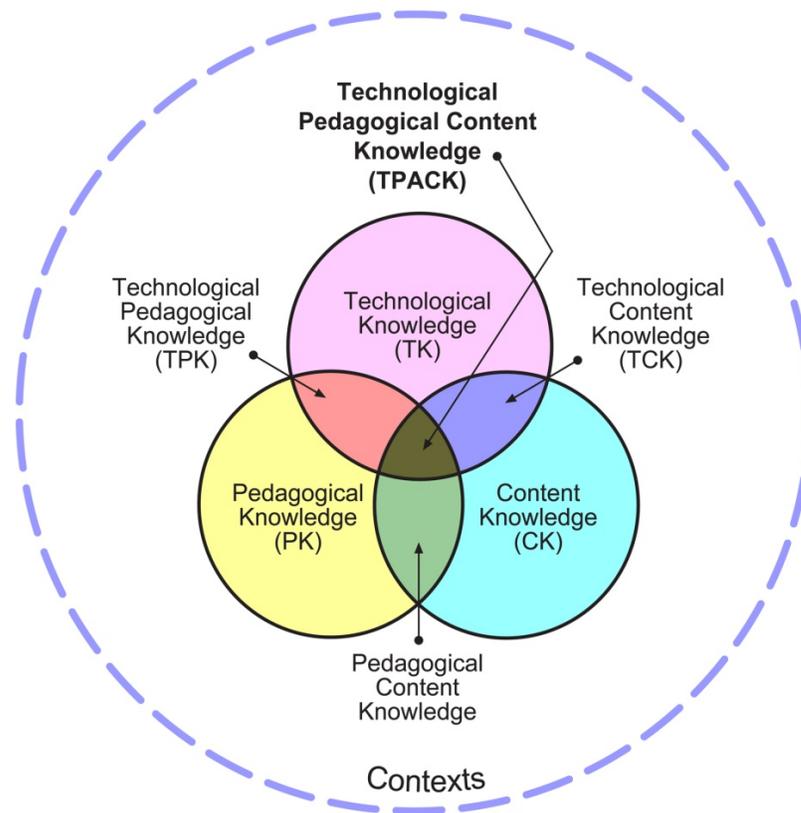


Figure 3. The components of the TPACK framework. Graphic from <http://tpack.org/>

Content knowledge (CK) is the knowledge about the subject matter to be learned or taught (e.g., knowledge about geometry). Pedagogical knowledge (PK) is a generic form of knowledge (it relates to all subjects) and involves things such as classroom management and student evaluation (Mishra & Koehler, 2006). Technological knowledge (TK) is about knowing standard technologies and skills required for operating particular technologies, e.g., knowing how to install software programs (Mishra & Koehler, 2006). Pedagogical content knowledge (PCK) is about knowing “what teaching approaches fit the content” and “how elements of the content can be arranged for better teaching” (Mishra & Koehler, 2006, p. 1027). Technological pedagogical knowledge (TPK) is the knowledge that can be used with any subject matter, e.g., “knowledge of tools for

maintaining class records, attendance, and grading” (Mishra & Koehler, 2006, p. 1028). Technological content knowledge (TCK) is about knowing how to use a technology for a given subject, e.g., MS Excel for statistics, or how to use subject-specific technologies, e.g., Sketchpad (Mishra & Koehler, 2006). Lastly, TPACK is about knowing how to teach a subject with a given technology, “the basis of good teaching with technology” and “pedagogical techniques that use technologies in constructive ways to teach content” (Mishra & Koehler, 2006, p. 1029). Although TCK and TPACK might seem like the same concept, the main difference between the two is that TPACK is about knowing how to *integrate* technology into teaching and learning, and TCK is about knowing how to *use* the technology for a given subject. Students who use technology in learning are likely to possess TCK, but they are not likely to possess TPACK. Teachers, who teach with technology, should have TCK, but they also should have TPACK in order to teach effectively.

Assessing Teachers’ TPACK

During the last century, certification tests for teachers have changed their focus. Towards the end of the 19th century, teacher certification tests focused on the subject matter with only five percent of the total points addressed by pedagogical practice. Towards the end of the 20th century, the focus of those tests reversed and “the emphasis was on how teachers manage their classrooms, organize activities, allocate time, . . . plan lessons, and judge general student understanding” (Shulman, 1986, p. 8). Reviewing how teachers’ knowledge has been assessed provides more information on how it has been conceptualized and why it is necessary to be concerned with teachers’ knowledge.

The concept of Shulman's pedagogical content knowledge (as previously described) was further developed and measured by Hill, Ball, and Schilling (2008). Hill et al. (2008) broke down subject matter knowledge (what Shulman called content knowledge) and PCK to more specific components. Subject matter knowledge consists of common content knowledge (CCK) (what Shulman meant by his subject matter knowledge (Hill et al., 2008)), specialized content knowledge (SCK), and knowledge at the mathematical horizon. PCK, on the other hand, consists of knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of curriculum. The instruments created by Hill et al. (2008) measure mathematical knowledge for teaching (MKT), which is a combination of subject matter knowledge and PCK. Therefore, measuring teachers' MKT focuses on content and also includes understanding of the content "in particular ways needed for teaching it," "what students are likely to make of the content," and "instruction that takes into account both students and mathematics" (Hill et al., 2007, p. 125). MKT items were multiple-choice and were developed from related literature and the researchers' classroom experiences (Hill et al., 2008) in mathematics education at the elementary and middle school levels.

Currently there are two contrasting paradigms for the assessment of teachers' knowledge; one is mainly quantitative (such as the measures developed by Hill et al.), and the other one is qualitative. The following sections provide an overview of assessment of teachers' TPACK and of their geometric knowledge using both paradigms.

Surveys, which involve self-reported data, largely dominate the quantitative types of TPACK assessment. Archambault and Crippen (2009) developed one such measure. Their study examined 596 teachers from around the U.S. who teach various K-12 online

courses. A similar survey was constructed by Schmidt et al. (2009) and was piloted on 124 pre-service teachers. Both surveys were designed to measure teachers' knowledge in all seven domains of TPACK and consisted of 5-point Likert scale items (Archambault & Crippen, 2009; Schmidt et al., 2009). Archambault and Crippen (2009) discovered that online teachers who participated in their study felt well about their knowledge in three domains: content, pedagogy, and pedagogical content; however, they felt less confident about their technological knowledge. The findings also revealed a large correlation between content and pedagogy, and a small relationship between technology and content as well as technology and pedagogy (Archambault & Crippen, 2009).

The main goal of Schmidt et al. (2009) was to develop and validate an instrument that would measure pre-service teachers' development of TPACK. After piloting the instrument and making revisions, the authors believe that their instrument is a reliable measure of TPACK. Both instruments (Archambault & Crippen, 2009; Schmidt et al., 2009) are promising starting points in examining teachers' and pre-service teachers' TPACK; however, instead of knowledge, they measured teachers' TPACK self-efficacy. The limitation of these surveys is that they rely on self-report or self-assessment of knowledge. Research shows that gains measured by such instruments reflect an increase in confidence instead of an increase in knowledge (Lawless & Pellegrino, 2007; Schrader & Lawless, 2004). Also, respondents might provide researchers with inaccurate data and their responses are hard to verify.

Sorto and Lesser (2009) developed a measure related to TPACK that did not rely on self-report; they used it for measuring technological pedagogical statistical knowledge (TPSK) of middle school teachers. The assessment included six multiple-choice and

open-ended items; the goal for open-ended items was to convert them later to multiple-choice items after eliciting responses from participants (Sorto & Lesser, 2009). The authors conjectured that years of teaching experience and statistics coursework could predict the level of TPSK, but no significant relationships were found in this study. The authors also found different characteristics of teachers' knowledge related to teaching statistics with technology. This was initial work on developing measures of TPSK. The authors planed on constructing "a longer instrument whose items would span technologies in addition to graphing calculators" (Sorto & Lesser, 2009, p. 7) and participants' responses to open-ended questions informed the researchers about answer choices for creating forced-choice questions.

Harris et al. (2010) created a rubric to aid in assessing pre-service teachers' lesson plans with respect to TPACK. The rubric was tested by fifteen experienced technology-using educators with various years of experience and teaching various subjects across all grades; the instrument found to be reliable and valid. The researchers believe that the rubric can be used with experienced teachers, although it was tested on pre-service teachers only. The researchers also were curious about its use as a teaching observation tool for evaluating the quality of technology integration.

Another way to assess teachers' knowledge is mainly qualitative and "draws upon case descriptions of teachers' classroom practices" (Groth, Spickler, Bergner, & Bardzell, 2009, p. 394). One advantage of this approach is that it allows researchers to explore "contextual factors that contribute to the knowledge that teachers exhibit in their classrooms" (Groth et al., 2009, p. 394). Groth et al. proposed an assessment framework based on the lesson study concept (Lewis, 2002) that is frequently utilized in Japan. They

collected data from “teachers’ written lesson plans, university faculty members’ reviews of lessons, transcripts and videos of implemented lessons, and recordings and transcripts of debriefing sessions about implemented lessons” (Groth et al., 2009, p. 392). Strengths of this assessment framework include the close connection of assessment and professional development (which can save time and resources) and learning opportunities for teachers during debriefing sessions. The main weakness of this framework was the inability to measure individual teachers’ knowledge as teachers can influence one another during professional development. One notable advice the researchers gave is “to avoid a purely deficit-oriented approach to describing teachers’ TPACK” (Groth et al., 2009, p. 407), meaning that researchers should not concentrate on teachers’ weaknesses, and their lack of knowledge; instead, they should also include teachers’ strengths when they are apparent.

Use of Technology in Geometry

Educational technologies such as dynamic geometry software can be integrated into teaching not only at the high school level, but also at the middle school level as well as elementary level (Olive, 2002). However, there is a need for more research as the literature in this area contains mainly “personal accounts of the powerful learning . . . with dynamic geometry technology” (Olive, 2002, p. 30) and not research studies. Also, teachers take a long time to adapt their teaching to take advantage of the technology (Olive, 2002). This implies that the development of TPACK takes time. The author mentioned that the full integration of the software into teaching is not immediate, and it requires time as well as willingness from the teachers’ side.

Coffland and Strickland (2004) examined what factors affected teachers' use of technology in high school geometry classrooms. The data was collected through a survey that was mailed to geometry teachers in southeast Idaho. Several significant relationships were found: (a) teachers teaching more geometry sections used less technology, (b) teachers with more awareness of computer capabilities had more positive attitude toward technology, and (c) teachers with technology training "in the integration of subject-specific software into their geometry classes" (p. 357) were more likely to use it (Coffland & Strickland, 2004). This study showed that professional development in the area of instructional technology integration could increase the use of technology when teaching geometry; however, the study did not address the type of training needed for technology integration. It also did not provide an answer to why teachers teaching more sections of geometry (and therefore more students) did not use technology as much as teachers teaching less geometry sections. These questions are left for future studies and can be researched from a TPACK perspective.

Roberts and Stephens (1999) investigated the effects of the frequency of computer use in high school geometry and student achievement. Three groups of students were taught by the same teacher: the first group did not use computers, the second group used Geometry Inventor (Brock, Cappo, Dromi, Rosin, & Shenkerman, 1994) once a week, and the third group used the software two times a week. Geometry Inventor is a software program that allows students to construct, measure, and use dynamic features to explore and discovery geometric concepts (Clements, 1995). All students were given the same chapter tests as well as end-of-the-semester exams. The only significant results were found for the introductory chapter and the transformational geometry chapter; students in

the non-technology group performed better than students in the other two groups. This study shows that although “using computer software did improve student interest and participation in geometry” (p. 23), some topics might be taught more effectively without using technology. The authors explained that during the activities for the introductory chapter students focused on becoming familiar with the software, which can give a possible explanation to lower scores in the technology groups. Also, test for the transformational geometry chapter was closely related to paper-and-pencil activities, on which the two technology groups did not spend as much time as the nontechnology group. The authors did not mention the role of the teacher in the instruction and said that when students were using computers they worked individually and did not interact with others. This finding implies that the computer had a role of an instructor, possibly because the teacher did not know how to utilize it in a more beneficial way, further implying the need for TPACK development.

Hannafin, Burruss, and Little (2001) conducted an exploratory phenomenological study whose goal was to identify the roles of participants while they used Sketchpad during their geometry lessons. The participants were one teacher and her twelve students in two 7th-grade classes. The study was conducted over three weeks; during the first week, the teacher received training on the dynamic geometry software and, during the following two weeks, she incorporated the software into her teaching. The researchers made classroom observations, distributed surveys and conducted interviews. The teacher also kept a journal where she recorded her reactions. The teacher’s role had to change from a lecturer to a facilitator, which was not easy for her because she did not feel comfortable letting her students take charge of their own learning; the teacher felt more

comfortable being in control of the classroom. It was difficult for her to change her role during the three weeks and she was not sure how to handle certain situations that arose when using Sketchpad. The authors suggested providing teachers with a sequence of scaffolding questions to help them transition to this type of teaching and so they know what and when to ask. This study shows we cannot simply give teachers a new technology, train them for a week, and expect a miracle. Also, it is difficult to change teachers' style of instruction (if it can be changed at all) and to ask them to incorporate certain technologies into their teaching might not always have a positive impact on teachers themselves or even on students as a result of teachers' resistance to change. Since the researchers advocate for a more student-centered approach to integrating Sketchpad into instruction, it might be beneficial to study teachers' knowledge as part of the technology integration into teaching and learning of geometry. The following study presents one such example.

Knapp, Barrett, and Kaufmann (2007) studied teachers' mathematical knowledge for teaching (MKT), defined by Hill, Rowan, and Ball (2005) as "the mathematical knowledge used to carry out the work of teaching mathematics" (p. 373) and its development by geometry teachers using dynamic geometry software. The purpose of the study was to find the ways in which teachers develop MKT as they prepared and implemented inquiry-based lessons using dynamic geometry with assistance from a collaborative coach. The authors presented the results from one of the four participants who taught 22 Sketchpad-based lessons in her middle school classes over the period of two years. The results indicated that the teacher "focused largely on 'teaching Sketchpad'" (Knapp et al., 2007, p. 1103) during the first year and "teaching geometry

using Sketchpad as a tool” during the second year (Knapp et al., 2007, p. 1103). This finding indicates possibly that the teacher has developed her knowledge related to teaching with Sketchpad as she gained more experience in using it in her teaching.

Gap in the Literature

The literature review of teachers’ knowledge and technology use revealed that significant improvements had been made in conceptualizing PCK and TPACK during the past two decades. Also, several attempts had been made in assessing TPACK through quantitative and qualitative approaches. However, more research is needed in the area of TPACK and teaching mathematics with technology. An extensive literature review showed that no studies had been conducted on teacher knowledge and integration of dynamic geometry into classroom instruction. Studies on the use of technology in teaching geometry indicated that teachers who incorporated technology into teaching geometry encounter many challenges. Therefore, it is beneficial to look at this issue through the TPACK lens. Also, because of the lack of research in this area, it is rather impossible to create a TPACK assessment similar to that of Hill et al. (2005). Since “psychometrically sound items are costly to develop” (Harris et al., 2009, p. 402) and their development takes time, exploring in depth specific cases should help further define and clarify the construct of TPACK.

Theoretical Framework

Niess et al. (2009) proposed a Mathematics Teacher TPACK Development Model built on the TPACK framework (Mishra & Koehler, 2006). The model consists of a five-stage process through which teachers might go while developing their knowledge when learning a new technology and integrating it into teaching. The five levels of the model

derive from Everett Rogers' (1995) model of the innovation-decision process and are based on researchers' observations of teachers learning to integrate spreadsheets into teaching and learning mathematics (Niess et al., 2009). Level 1 is recognizing (knowledge), where teachers are able to use technology and "recognize the alignment of the technology with mathematics" (Niess et al., 2009, p. 9) but do not integrate it into teaching. Level 2 is accepting (persuasion) and involves forming a favorable or unfavorable attitude towards integrating technology in teaching mathematics. Level 3 is adapting (decision), where teachers decide to use or not to use technology in teaching mathematics. Level 4 is exploring (implementation) and involves active integration of technology and teaching mathematics. Lastly, level 5 is advancing (confirmation) and consists of teachers' evaluating "the results of the decision to integrate teaching and learning mathematics with an appropriate technology" (Niess et al., 2009, p. 9). Furthermore, the model is divided into four themes: curriculum and assessment, learning, teaching, and access. The authors also stated, "a mathematics teacher may be at different levels for different themes" (Niess et al., 2009, p. 13). This model presented a useful framework for studying how teachers gain knowledge related to integration of technology in teaching mathematics and identifying the levels of TPACK they have.

Summary

This chapter reviewed literature related to TPACK and teaching geometry with technology. Through the examination, this chapter identified a gap in literature and a theoretical framework for studying teachers' TPACK. The literature review supported the need for studying teachers' TPACK, how it is acquired, and how it is used in classroom instruction. In particular, more research is desirable in the area of teaching geometry with

dynamic geometry software through qualitative research methods. The following chapter, Chapter III, describes the design and methodology implemented in this study. More specifically, it identifies the research questions that guided this study and discusses the research design, pilot study, researcher's roles, context of the study, participants, data collection and analysis, building trustworthiness, and ethical issues.

CHAPTER III

METHODOLOGY

The purpose of this study was (a) to document how high school geometry teachers develop and use their technological pedagogical content knowledge (TPACK) for teaching geometry with dynamic geometry software and (b) to describe teachers' TPACK development levels related to integration of dynamic geometry into classroom instruction. Three research questions guided this study:

1. How do high school teachers develop TPACK while teaching geometry using dynamic geometry software?
2. How do high school teachers enact their TPACK when teaching with dynamic geometry software?
3. How are the five TPACK development levels (i.e., recognizing, accepting, adapting, exploring, and advancing) characterized for high school teachers who incorporate dynamic geometry software in teaching?

The remainder of this chapter describes the methods used to conduct this study and consists of the following sections: research design, pilot study, researcher's roles, context of the study, participant selection, participants and setting, data collection, data analysis, building trustworthiness, and ethical issues.

Research Design

The research questions about teachers' knowledge when integrating dynamic geometry software into teaching indicated a qualitative method of inquiry. Denzin and Lincoln (2005) defined qualitative research as:

a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that make the world visible. These practices transform the world. They turn the world into a series of representations, including field notes, interviews, conversations, photographs, recordings, and memos to the self. At this level, qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative researchers study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them. (p. 3)

The goal of this study was to provide a rich and descriptive account of teachers' use of dynamic geometry software in their teaching practices. I used the case study methodology (Yin, 2009) to examine the TPACK development levels of high school geometry teachers and how they used their TPACK when integrating dynamic geometry into their teaching. Creswell (2007) defined case study research as:

a qualitative approach in which the investigator explores a bounded system (a *case*) or multiple bounded systems (cases) over time, through detailed, in-depth data collection involving *multiple sources of information* (e.g., observations, interviews, audiovisual material, and

documents and reports), and reports a case *description* and case-based themes. (p. 73)

In this study, bounded systems were teachers and their professional practices. Multiple sources of data included interviews, observations, documents, a self-report survey, implementation questionnaires, and informal conversations with participants. Yin (2009) suggested the use of multiple sources of evidence to ensure construct validity because they “essentially provide multiple measures of the same phenomenon” (p. 116). To ensure the reliability of the entire case study (Yin, 2009), I developed a case study database containing all the raw data, so that other researchers can inspect the evidence and “not be limited to the written case study reports” (p. 119).

Pilot Study

I conducted a qualitative pilot case study in spring 2011 with one participant. The study incorporated purposeful sampling and the participant selection criteria used were geographic proximity and a well-established relationship between the participant and me. Data sources consisted of two observations, two interviews, and student handouts. The pilot study helped in refining data collection plans, what data to collect and what procedures to follow. After conducting the initial interview for the pilot case study, I made significant changes to the interview protocol to better elicit participants’ responses related to their TPACK. Based on those revisions, during the closing interview I asked additional questions, which became part of the initial interview protocol for the main study. I also extended the observation protocol to focus more on TPACK enactment. The pilot study also provided an interesting Sketchpad teaching task (see Appendix A); I

included this task in the closing interview for the main study. Finally, the pilot study assisted in assessing time and resources needed to conduct the main study.

Researcher's Roles

I assumed several roles while conducting this research study. First, because of the qualitative nature of this study, I was a human instrument for data collection (Merriam, 2009); I interacted with participants through interviews and observations. During observations, I assumed a role of an “observer as participant” (Merriam, 2009, p. 124). Based on Merriam’s definition of “observer as participant,” the teachers and their students were aware of my activities; my participation in observed lessons was secondary to gathering information through taking notes and videotaping. Second, besides being an instrument for data collection, I was also an instrument for analyzing the collected data. Lastly, I was a learner; as I collected and analyzed the data, I gained deeper knowledge of the concept of TPACK and the integration of dynamic geometry into classroom instruction.

Context of the Study

I conducted this study within the context of the Dynamic Geometry Project (DGP). The DGP is a research project whose main objective was to compare two approaches to the teaching and learning of high school geometry. One approach utilized dynamic geometry (DG) in teaching and learning and the other one did not. The DGP is a four-year research project, which contains two consecutive years of implementation of the DG treatment. I refer to the two years as “the first year of the DG implementation” and “the second year of the DG implementation” respectively in this dissertation. I conducted this study during the second year of the DG implementation. Participants of

the DGP were geometry teachers from several school districts, all teaching high school level geometry. There were approximately sixty teachers, half of them in the DG group and half in the comparison group.

The focus of this study was on teachers in the DG group since they were the ones integrating dynamic geometry into classroom instruction. The teachers participated in professional development workshops facilitated by the DGP staff. During the first year of the DG implementation, the professional development consisted of a five-day summer workshop and six half-day Saturday workshops offered throughout the school year. During the second year of the DG implementation, the professional development consisted of one-day summer workshop and three half-day Saturday workshops offered throughout the school year. Teachers received stipends for their participation in those workshops. During the professional development sessions, participants became familiar with Sketchpad, learned about geometric concepts using Sketchpad, created lesson plans, collaborated with peers and shared some of their teaching strategies and activities with one another. Participants received curriculum materials that focused on integration of Sketchpad into learning and teaching of geometry; they also obtained access to the Sketchpad Lesson Link, a website (<http://www.keypress.com/x26771.xml>) with hundreds of Sketchpad activities aligned to textbooks, state standards, and the Common Core State Standards. The DGP staff asked teachers to use Sketchpad with their students in a lab setting (or in their classroom if laptops were available) twice a week so that students could experience dynamic geometry on a regular basis. The DGP staff also encouraged teachers to use Sketchpad for demonstration purposes whenever students did not have access to laptops or a computer lab.

Participant Selection

I used purposeful sampling to select four participants for this multiple case study.

Patton (2002) explained that:

The logic and power of purposeful sampling derive from the emphasis on in-depth understanding. This leads to selecting information-rich cases for study in depth.

Information-rich cases are those from which one can learn a great deal about issues of central importance to the purpose of the research, thus the term purposeful sampling. (p. 46)

To select information-rich cases, I wanted to ensure that each teacher's students have opportunities every week to explore geometric concepts using Sketchpad, and that those teachers displayed a high fidelity of implementing the DG approach in teaching.

Therefore, to identify potential participants, I looked at data collected by the DGP staff during the first year of the DG implementation to see which teachers fulfilled the requirements, and I compiled a list of approximately ten teachers. Initially, I sent an invitation letter (see Appendix B) via email to three teachers. One of them agreed to participate, one declined, and one did not respond. A couple of days later, I sent three more invitations. Two teachers agreed to participate and one declined. Once again I sent two more invitations, and both teachers agreed to participate. As a result, there were five participants at the beginning of this study. However, when trying to schedule the initial interview, one of the teachers never responded; therefore, four participants remained for the rest of the study.

Participants and Setting

This study took place in four high schools in a large city in Texas. The participants of this study were four geometry teachers (Brian, James, Susan and Laura, all pseudonyms) who participated in the DGP during both implementation years. Table 1 shows the four participants, their length of Sketchpad use in teaching prior to this study, number of years they had taught, and whether or not they used Sketchpad as college students. All participants were teaching at different schools, and all, except Susan, worked in the same school district. The order in which I listed the participants is the order in which I conducted the initial interviews. I use this order throughout this dissertation.

Table 1

Participants and Their Teaching Experiences

Participant (Pseudonym)	Sketchpad experience as a learner	Total teaching experience (years)	Teaching geometry experience (years)	Teaching with Sketchpad experience (years)
Brian	No	2	2	1
James	No	4	4	1
Susan	Yes	19	9	7
Laura	Yes	2.5	2	1

Table 1 shows that the four participants represented a wide spectrum of experiences. Three teachers (Brian, James and Laura) were novice teachers with less than five years of teaching experience, and Susan was an experienced teacher with more than five years of experience. Susan was also the only teacher with experience in using Sketchpad in teaching for more than one year (meaning she had used it before

participating in the DGP). Additionally, Susan and Laura used Sketchpad in learning geometry in college. The following paragraphs describe each participant in more detail.

Brian

He was the least experienced teacher in this study and had taught geometry for two years before the study. However, he was the most experienced geometry teacher at his school. Brian heard about Sketchpad for the first time while he was doing his student teaching, which was also geometry; however, he did not use Sketchpad at that time. During his first year of teaching, he used some sketches provided by his district for demonstrations in the classroom. He did not create any sketches nor had his students do any explorations with Sketchpad before his participation in the DGP.

James

He had taught mathematics (including geometry) for four years before this study. This was his fifth year teaching and his third year working at his current school; he taught for two years in another city in the same state before teaching at his current school. Before signing up for the DGP, James did not know anything about Sketchpad. One of the teachers at his school told him about the project and about Sketchpad. Once he took a look at Sketchpad, he wanted to integrate it into his instruction and thought that signing up for the DGP would be a convenient way to learn about this dynamic geometry software.

Susan

She was the most experienced teacher and had the most experience in using Sketchpad in teaching (8 years). She started her teaching career as a middle school teacher. Later she began teaching geometry at the high school level, and this was her fifth

years as the only geometry teacher at her current school. She also used Sketchpad as a graduate student while working on her master's degree, but that was after she used it in teaching.

Laura

She used Sketchpad as a college student; however, she did not use it for teaching until she signed up for the DGP and ended up in the DG group. Laura had computers in her classroom, so her students had easy access to technology (which was not always the case), but she had hoped to be in the comparison group of the DGP because she preferred to use manipulatives with her students instead of using technology.

Data Sources

The main sources of data included interviews and observations. The supporting sources of data included documents, a survey, implementation questionnaires, professional development attendance records, and the researcher's log. All data collection took place in the participants' classrooms during regular school hours. Table 2 summarizes all data sources and the following paragraphs describe them in detail.

Table 2

Summary of Data Sources

Data Sources	Details
Initial interview	30-40 minutes, semi-structured, open-ended
Observations	2-4 per teacher, 45 minutes each, videotaped, focus on teacher
Documents	Student activity sheets used during the observed lessons
Closing interview	20-30 minutes, open-ended questions, different for each participant, teaching-with-Sketchpad task
TPACK Survey	11 groups of 5 statements organized by themes and descriptors from the TPACK Development Model
Implementation questionnaires	Selected questions from the DGP implementation questionnaire related to TPACK
The DGP professional development attendance records	Number of hours/sessions of participation in the DGP professional development
Researcher's log	Ideas, initial thoughts after observations and interviews, and memos to self

Initial Interview

The initial interview was the first step in data collection and coincided with the beginning of a new school year. Each interview lasted approximately 30-40 minutes and took place during the teacher's conference period on a regular school day. Each participant received the interview questions as well as a consent form (see Appendix C) via email the day before the interview. At the beginning of the initial interview, each participant signed the consent form.

The data collected during the interviews provided information on how participants developed their TPACK and how they incorporated it into teaching geometry with Sketchpad. I developed an interview protocol (see Appendix D), which guided the initial

interview. The TPACK Development Model framework informed the creation of several interview questions such as: Why did you decide to teach geometry with Sketchpad? In addition to your participation in the DGP, what kind of activities (e.g., professional development, conferences, self-directed study, Internet resources) have you engaged in that lead you to adopt teaching and learning geometry with Sketchpad? The interview protocol developed by Niess, van Zee, and Gillow-Wiles (2010), also based on the TPACK Development Model, suggested additional questions such as: What is your current view and understanding about integrating Sketchpad as a learning tool in geometry? How has your knowledge about students' understanding, thinking, and learning about geometry topics with Sketchpad changed through your work in the DGP (or since you started using Sketchpad in the classroom instruction)? This was a semi-structured open-ended interview. Some characteristics of such interviews include: "interview guide includes a mix of more and less structured interview questions," "usually specific data required from all respondents," and "largest part of interview guided by list of questions or issues to be explored" (Merriam, 2009, p. 89).

Observations

The purpose of the observations was to see how teachers enacted their TPACK when teaching with Sketchpad and to identify their TPACK development levels. Stake (2006) asserted that observations are "the most meaningful data-gathering methods" (p. 4) because "it is important to describe what the case's activity is and what its effects seem to be" (p. 4). It was essential to observe how teachers enacted their TPACK during classroom instruction because that was how they used their knowledge in practice. Guba and Lincoln (1981) said:

In situations where motives, attitudes, beliefs, and values direct much, if not most of human activity, the most sophisticated instrumentation we possess is still the careful observer - the human being who can watch, see, listen, question, probe, and finally analyze and organize his direct experience. (p. 213)

Therefore, observations of participants in their natural environments (here, teachers in classrooms) provide rich information and assist in triangulation of the data from the interviews and other sources.

I developed an observation protocol (see Appendix E) to assist me in focusing on relevant information during observations. The focus of each observation was on the teacher. The main goal was to identify strengths and areas for improvement of teachers' practices related to their TPACK and integrating Sketchpad in teaching. I observed and videotaped multiple lessons of each teacher, which took place in classrooms and computer labs while teachers were teaching geometry with Sketchpad. Susan was the only participant that I did not videotape because her principal did not allow it. However, I was able to audiotape the observed lessons, and I took more detailed notes during those observations. I observed the same group of students (for each teacher) and took notes on a laptop or an iPad. The length of each observation for all teachers was approximately 45 minutes, the full length of one class period.

Documents

I collected documents related to the observed lessons, which consisted of student activity sheets that teachers either prepared themselves or printed from the Sketchpad

Lesson Link. These documents provided more information about the teacher's intended lesson and how her or his TPACK was reflected in the lesson plan.

Closing Interview

The closing interview took place at the end of the study. The main purpose of this interview was to clarify selected episodes that I noticed during observations. In addition, I wanted to understand better why teachers guided their classroom instruction the way they did when teaching with Sketchpad and how their actions related to their TPACK. The interview questions came from my preliminary analysis of the initial interview and observations. The questions were open-ended and gave an opportunity for each participant to explain how she or he would use her or his TPACK to improve her or his teaching with Sketchpad in the future. I also presented a teaching episode from the pilot study (see Appendix A) to the participants and asked them to reflect on it and explain how they would improve the given learning-teaching situation. Finally, participants were asked to fill out the TPACK Development Model Self-Report Survey.

TPACK Development Model Self-Report Survey

The TPACK Development Model Self-Report Survey (TPACK Survey, see Appendix F) was developed by Ivy (2011) and Riales (2011) and based on the TPACK Development Model (Niess et al., 2009). Margaret Niess reviewed the survey and provided feedback to Ivy and Riales. My purpose for using the survey was to find out what the participants' perceptions were about their TPACK related to Sketchpad integration in teaching. The survey consisted of fifty-five statements organized into eleven groups of five statements. One statement in each group described one of the five TPACK development levels (i.e., recognizing, accepting, adapting, exploring, and

advancing), and the eleven groups represented eleven theme/descriptor pairings from the TPACK Development Model. Participants were to choose one statement from each of the eleven groups. Their responses provided data related to their self-perceived TPACK development levels.

Implementation Questionnaires

Since the participants of this study were part of a larger project, the DGP, they completed an implementation questionnaire every 4-6 weeks. The questions on the questionnaires were related to their integration of Sketchpad in teaching and learning. Selected data from the questionnaires were used in this study, in particular the participants' responses to two questions: (a) How many times per week did the students work in a computer lab/classroom using Sketchpad software; (b) How many times per week was the geometry class taught in a classroom with one demonstration computer? Answers to these questions provided information on how often the participants got to use their TPACK in practice.

The DGP Professional Development Attendance Records

I compiled the participants' attendance data from the DGP professional development sessions, which took place over the two years of the DG implementation. Summing up all of hours of attendance for each participant aided in answering the first research question.

The Researcher's Log

After conducting interviews and observations, I recorded ideas, initial thoughts, and memos to self. I also recorded pertinent information from my informal conversations

with participants, e.g., e-mail correspondence and conversations before or after the observed lessons.

Data Analysis

Data analysis took place simultaneously with data collection and was a reflective and ongoing process. Since teachers were the primary units of analysis, I analyzed data collected from each participant separately and wrote a rich description of each case. First, I focused on answering the first two research questions, specifically how each participant developed and enacted her or his TPACK. Second, I identified TPACK development levels for each participant based on the collected data and the case descriptions. This portion of data analysis focused on answering the third research question. Then, I performed cross-case analysis (Yin, 2009) to identify common themes among the four cases. Since each case was unique, I also looked for any significant differences among the four cases. The cross-case analysis supplemented the research findings and provided additional information for answering the research questions. Table 3 summarizes data analysis for each data source, and the following paragraphs provide details of the analysis of interviews, observations, and supporting data sources.

Table 3

Summary of Data Analysis

Data Sources	Data Analysis
Initial interview	Transcribed, reviewed for accuracy, used codebook to identify and describe TPACK development levels
Observations	Reviewed observation notes and videos, clipped episodes that involved TPACK, transcribed conversations
Documents	Assisted in describing TPACK enactment
Closing interview	Cross-case analysis of the teaching-with-Sketchpad task
TPACK Survey	Identified self-perceived TPACK development levels
Implementation questionnaires	Provided the frequency of Sketchpad use and challenges to integrating Sketchpad in teaching
The DGP professional development attendance records	Identified the number of hours/sessions of participation in the DGP professional development
The researcher's log	Assisted in the overall analysis

Initial Interviews

The first step into data analysis was to transcribe the audiotaped interviews. After transcribing, I listened to the audio recording again and at the same time read through each transcript carefully to check for accuracy. Next, I read the transcript again in order to identify parts associated with the levels, themes and descriptors of the TPACK Development Model (Niess et al., 2009). I highlighted the relevant quotes and noted the TPACK development level (i.e., recognizing, accepting, adapting, exploring, advancing) and the theme (curriculum and assessment, learning, teaching, access) by using a codebook for content analysis (Patton, 2002) that I created based on the TPACK Development Model (see Appendix G). This was a deductive analysis “where the data are

analyzed according to an existing framework” (Patton, 2002, p. 453), where the existing framework was the TPACK Development Model. This process assisted in identifying the correct levels of TPACK for each participant and facilitated in answering the research questions. At times I noticed other intriguing themes emerging from the transcripts, so to keep myself focused on the research questions, I returned to the TPACK Development Model, the five levels, the four themes and their descriptors.

Observations

After coding the initial interview, I reviewed my notes from the observations and the videos several times. I clipped parts of the videos related to TPACK and transcribed the associated conversations. I coded them using the same codebook (see Appendix G) as for the initial interview.

Supporting Data Sources

Recording participants’ responses to the TPACK Survey identified their self-perceived TPACK development levels. Because each response identified a TPACK development level for one theme-descriptor pairing, and the order of the statements (for each group of five statements) corresponded to the five TPACK development levels, from the lowest (recognizing) to the highest (advancing), no further analysis was needed of the survey alone. Later, I compared the self-perceived levels of TPACK reported through the survey with the TPACK development levels identified through other data sources.

Next, I compiled the responses to the two Implementation Questionnaire questions related to the frequency of computer use for students and teachers. The data spanned from the end of August 2011 to the end of February 2012 for a total of twenty-

two (22) weeks. The data assisted in identifying the TPACK development levels for the access theme, usage descriptor. In addition, I looked at participants' open-ended responses and identified those that were related to TPACK. They provided information about the challenges in incorporating Sketchpad in the classroom instruction.

Student activity sheets contributed in describing the observed lessons. The DGP professional development attendance records assisted in finding the total number of hours and sessions that participants took part in. The researcher's log provided initial thoughts and analysis that contributed to the overall analysis.

Finally, I triangulated the data from all data sources to crosscheck themes that emerged from several sources and to strengthen the research findings. As a result of the data analysis, I was able to identify TPACK development levels across different themes for all participants and consequently answered the three research questions. Additionally, I performed cross-case analysis to compare and contrast TPACK development and enactment among the participants. The cross-case analysis was especially helpful in identifying challenges in Sketchpad integration and providing suggestions for TPACK professional development.

Identifying TPACK Development Levels

To answer the third research question, I identified TPACK development levels for each participant. As mentioned earlier, I utilized a codebook to perform this part of data analysis and to identify the correct TPACK development levels across the eleven descriptors from four themes of the TPACK Development Model (Niess et al., 2009). I began this process after performing the preliminary analysis for the first two research questions. I reviewed the case descriptions carefully and identified TPACK development

levels that were present. Next, I checked for which descriptors TPACK development levels were still not identified. I returned to the interview transcripts and the observation data to look for the specific instances where the missing TPACK development levels were present. This process aided in completing the case reports.

After identifying all of the TPACK development levels for each participant, I checked across the four cases to make sure that the same levels were assigned for the equivalent actions. Additionally, I double-checked that the corresponding actions had the same levels assigned across the four cases. Finally, I compiled a table that indicates which data sources contributed in identifying TPACK development levels for each theme-descriptor pairing (see Table 4 for details). The collected data, with the exception of the TPACK Survey, did not identify TPACK development levels for the curriculum and assessment theme, assessment descriptor.

Table 4

Sources of Data that Identified TPACK Development Levels by Themes and Descriptors

Theme (descriptor)	Interviews	Observations	Supporting data sources
Curriculum & Assessment (curriculum)	✓	✓	✓
Curriculum & Assessment (assessment)			
Learning (mathematics learning)	✓	✓	
Learning (conception of student thinking)	✓	✓	✓
Teaching (mathematics learning)	✓	✓	
Teaching (instruction)		✓	
Teaching (environment)	✓	✓	
Teaching (professional development)	✓		✓
Access (usage)	✓	✓	✓
Access (barrier)	✓	✓	✓
Access (availability)	✓	✓	

Note. Data sources that provided evidence for TPACK development levels for each theme-descriptor pairing are indicated by a check mark.

Building Trustworthiness

When conducting this research study, I adopted a stance of *neutrality* (Patton, 2002); I attempted “become aware of and deal with selective perception, personal biases, and theoretical predispositions” (Patton, 2002, p. 51). Because of my role as an instrument in data collection and analysis, I engaged in careful reflections on potential sources of bias and dealt with them. My main predisposition was to focus on participants’ flaws in instruction during observed lessons. To deal with it, I constantly reminded myself that it was not my goal for this study and kept referring to the TPACK Development Model as my lens for identifying accurate and reliable findings. I also returned to the research questions to ensure I was staying on track and not looking at irrelevant issues.

To produce high-quality data that are credible and trustworthy, I conducted a pilot study to refine the data collection instruments. In addition, I collected data from different sources, triangulated the data, and performed member checking. Triangulation of the data, also known as “the most well known strategy to shore up the internal validity of a study” (Merriam, 2009, p. 215), took place in many ways: through checking what I learned through interviews with what I noticed during observations, comparing findings from interviews and observations with those from the TPACK Survey, conducting follow-up interviews and multiple observations of the same participant. Member checking is “the most critical technique for establishing credibility” (Lincoln & Guba, 1985, p. 314). Member checking involves “taking data, analyses, interpretations, and conclusions back to the participants so that they can judge the accuracy and credibility of the account” (Creswell, 2007, p. 208). According to Stake (1995), participants should

“play a major role directing as well as acting in case study” research (p. 115). I took preliminary analyses to participants in this study and asked them for their opinions, and if they would add anything that is noteworthy that I omitted.

Ethical Issues

Before beginning the study, I obtained formal approval from the Institutional Review Board at Texas State to conduct this research project. I informed all the participants that their participation was voluntary and that they could withdraw at any time. I also asked them to sign a consent form, which explained the study in more detail. I assured them that all the data collected was going to be kept confidential and that I would protect their privacy. I masked their names in all of my reports and used pseudonyms instead. Through the participants and their school districts, I obtained parents’ permissions to videotape their students. One participant’s principal did not grant permission to videotape; therefore, I only recorded the audio during observations of that teacher.

Summary

This chapter introduced three research questions that guided this qualitative case study. A pilot study assisted in refining data collection plans and provided additional suggestions for data sources. The DGP was the context of this study and four of its teachers were participants in this study. The participants had varying experience with respect to teaching geometry and teaching with Sketchpad. Sources of data included: initial interview, observations, documents, closing interview, a survey, implementation questionnaires, professional development attendance records and the researcher’s log. A deductive content analysis based on the TPACK Development Model aided in data

analysis. The pilot study, multiple data sources, triangulation of the data, and member checking assisted in producing high-quality data and assuring credibility and internal validity. The data collected through this study aided in answering the research questions and in identifying the participants' self-perceived and enacted TPACK development levels. Findings are presented in the following two chapters, Chapter IV and Chapter V. Chapter IV consists of four case reports describing TPACK development, TPACK enactment and TPACK developmental levels for each participant. Chapter V provides cross-case analysis.

CHAPTER IV

CASE REPORTS

The main goal of this study was to provide a rich and descriptive account of teachers' technological pedagogical content knowledge (TPACK), i.e., the knowledge related to their use of dynamic geometry software in classroom instruction. This study employed a case study methodology, and multiple data sources assisted in answering the research questions. This chapter describes the findings of this study through four case reports. Each report is divided into three sections: TPACK development, TPACK enactment and TPACK developmental levels. The three section titles correspond to the three research questions:

1. How do high school teachers develop TPACK while teaching geometry using dynamic geometry software?
2. How do high school teachers enact their TPACK when teaching with dynamic geometry software?
3. How are the five TPACK development levels (i.e., recognizing, accepting, adapting, exploring, and advancing) characterized for high school teachers who incorporate dynamic geometry software in teaching?

Case 1: Brian

Brian was in his third year as a teacher at the time of this study. He has taught geometry since he started teaching. At the time of this study, he was in the DGP for the

second year, and was teaching three sections of geometry and three sections of algebra 1. During the previous year, he had six sections of geometry. Although he was a novice teacher, he had the most experience in teaching geometry at his school and was serving as a geometry leader. In the geometry syllabus, he mentioned the use of dynamic geometry in the classroom. Other than Sketchpad, he had used only one other dynamic geometry program; he mentioned using Cabri Junior on a graphing calculator only once or twice in the past.

TPACK Development

This section describes Brian's TPACK development journey with respect to Sketchpad integration in classroom instruction. All quotes in the following paragraphs come from the initial interview with Brian.

Brian used Sketchpad during his first year of teaching “just a small handful of times” by incorporating the sketches provided by his school district for demonstration. This indicated an overall recognizing level of TPACK. Just before signing up for the DGP, he attended a conference session where he learned more about Sketchpad. However, it was not until the DGP that he learned “what the program [Sketchpad] really had to offer,” indicating an overall accepting level of TPACK. After one year in the DGP, he felt confident when teaching with Sketchpad, indicating an overall adapting level of TPACK—“not a hundred percent, but confident enough.”

According to the DGP professional development attendance records, Brian attended all of the sessions offered by the DGP except one half-day session, for a total of approximately 69 hours. The fact that Brian focused on learning only one type of technology suggests the adapting level of TPACK for the teaching theme, professional

development descriptor. As a geometry leader at his school, Brian has been actively promoting the use of Sketchpad for learning geometry and sharing what he learned during the professional development sessions with his colleagues at work, indicating an overall exploring level of TPACK.

Brian decided to use Sketchpad in the classroom because:

Once I figured out what the program had to offer—that it was going to be useful in the classroom—and also the availability—here they basically gave me my own computer lab, so I have access to it all the time. They made it really easy for me.

This statement shows that Brian saw the potential of incorporating Sketchpad into classroom instruction. Also, he had easy access to a computer lab, so he could take his students there anytime. However, Brian realized that knowing how to use Sketchpad and how to integrate it in classroom instruction were not the same thing as he stated in the initial interview:

Being good with the program and being able to teach are two separate things, and I want to make sure I focus on that—that I can teach with it. I think I want to integrate it into the classroom more, not just the computer lab. It has a lot more application than I am using it for. I definitely use it more in the computer lab than I do in the classroom. I think it has just as much place in the classroom. That is what I need to look at doing more.

Brian expressed in this statement a desire to learn how to teach with Sketchpad and develop his TPACK in addition to knowing how to use Sketchpad. Also, he mentioned a goal for integrating Sketchpad as a teaching tool in the classroom, not just as a learning

tool in a computer lab. This showed that when he incorporated Sketchpad into classroom instruction, it was in a learner-centered environment (computer lab). However, he wanted to start using it in the classroom, too, as a demonstration tool.

Brian's conceptions about teaching geometry have not changed through incorporating Sketchpad into classroom instruction because he started using it only one year after he had started teaching, i.e., he was "still building a lot of my conceptions of my teaching of different topics." This indicated that he was developing his TPACK at the same time as he was developing his pedagogical content knowledge.

Brian mentioned, "teaching with Sketchpad is helpful to students and I want to continue to use it." At the same time, he saw that not all geometry topics were easily compatible with Sketchpad: "Some of the lessons or some of the units cater to it [Sketchpad] a little bit more." He thought that lessons on lines and angles accommodated the integration of Sketchpad better than lessons on three-dimensional figures.

Sketchpad integration was important to Brian; he was glad that he decided to use it in teaching because "it allows me to teach things that I think would be difficult to teach in the classroom or difficult to show" without Sketchpad, indicating an overall adapting level of TPACK. In addition, he mentioned in the initial interview that his students' parents enjoyed the fact that their children got to use technology in the classroom. Furthermore, school and district administrators liked to see parents happy and technology in math classrooms.

TPACK Enactment

This section describes how Brian enacted his TPACK—i.e., how he put his combined knowledge of Sketchpad, geometry, and pedagogy into practice. Although this

section focuses on TPACK enactment, it also provides additional information about Brian's TPACK development through his DG implementation. All quotes that appear in this section, with the exception of quotes and dialogs that appear in the triangular thinking lesson subsection, come from the initial interview.

According to the initial interview, during the first year of the DG implementation, Brian incorporated Sketchpad in teaching in the following two ways: introducing new topics in the classroom first and then going to a computer lab to explore them or introducing new topics in a computer lab first and then debriefing them in the classroom. Through this experimentation, he found that he preferred the latter:

Yes, generally I like to do it that way. They are introduced to it, they find it on their own, and then we discuss it. I think it is better when they find it first on their own. Although it is a little harder, I think it is better for them to see it for the first time by discovering it versus being told.

He also brought this up in one of his implementation questionnaire responses. He stated that if there was a concept that could be investigated with Sketchpad, he preferred to allow his students to discover it in the lab and then "refine and define the idea as a class." The observation on the triangle congruence lesson confirmed Brian's belief. Students were exploring concepts new to them in a computer lab, and after the lesson, Brian mentioned to me that he was going to review students' work and discuss it further with the class the following day.

Although Brian tried to take his students to a computer lab approximately twice a week during this school year, according to the implementation questionnaire data, he got to do it on average once every other week. The infrequent use of Sketchpad indicated the

adapting level of TPACK for the access theme, usage descriptor. Even though he had good intentions and access to a computer lab at any time, there were other things that he had no control over that prohibited him from fulfilling his goal. One of them was the implementation of a new curriculum in which the order of topics was different from what he was accustomed to. In one of his implementation questionnaire responses, Brian expressed his dislike for the new pacing calendar by saying “it has an odd order” and that it did not accommodate the DG implementation. The new scope and sequence required more time spent on planning lessons, which in turn made it challenging to plan ahead and to integrate Sketchpad in any lessons. Because of the challenges with the new scope and sequence, Brian integrated Sketchpad into teaching only in some units. This suggested the adapting level of TPACK for the access theme, barrier descriptor. Still, he had noticed that students were “learning better” from this new approach “because it is allowing them to discover, so it is benefiting their learning, it is getting them more engaged in it than if they are sitting at their desks.”

At the beginning of each observed lesson, Brian gave a handout to his students with instructions for each exploration. He created his own activity sheets for those Sketchpad-enhanced lessons, which indicated the exploring level of TPACK for the curriculum and assessment theme, curriculum descriptor. Originally those activities did not incorporate any technology, so Brian thought about how he could integrate Sketchpad into them and created technology-based lessons.

Brian described his current view about integrating Sketchpad as a learning tool in geometry as follows:

In the lab, they find it on their own, they come up with some idea, and we decide “Yes, that is actually a theorem” and we go on proving from there. So instead of writing theorems in the classroom and accepting them, we discover the relationships, come up with conjectures, and actually prove it. It allows them to understand geometry much better than just know geometry. I think that has been a big difference.

He saw the benefits of having his students explore geometric concepts and discover relationships instead of just telling them what those relationships were. What he stated in the initial interview was clearly visible during the observed lessons. Students were exploring, measuring, observing, finding relationships, and forming and justifying conjectures. Creating an environment where students were engaged and self-directed in learning geometry suggested the advancing level of TPACK for the teaching theme, environment descriptor.

As his students worked on Sketchpad activities during the observed lessons, Brian circulated throughout the room and answered students’ questions if they had any. He also asked guiding questions if he noticed that students were not following the instructions on the handout carefully. His actions during the observed lessons confirmed what he said during the initial interview, “I like to think that when they are in the lab, they are learning it more on their own, and I just make sure they stay on track.” The following lesson description provides several dialogs that Brian had with his students during class and confirms what he stated in the initial interview about “discovery” and guiding his students in their learning.

Triangular thinking lesson. During the triangular thinking lesson, students were examining the relationships formed by constructing midsegments of a triangle. Because the goal of the lesson was to come up with different conjectures related to the midsegments of a triangle, Brian integrated Sketchpad to help students with their thinking and understanding of this topic. This use of Sketchpad indicated the advancing level of TPACK for the learning theme, mathematics learning descriptor.

Based on the student activity sheets created by Brian, students were to construct an arbitrary triangle, its midsegments, and make conjectures about the angles, segments, and any shapes that they formed. While students were working on this activity, Brian noticed that some students were measuring many things, but they were not coming up with any conjectures (see Figure 4 for example), so he asked guiding questions, e.g., “Do you notice any relationships? What conjectures can you make?” These questions helped students focus on the objective of the lesson, i.e., forming conjectures.

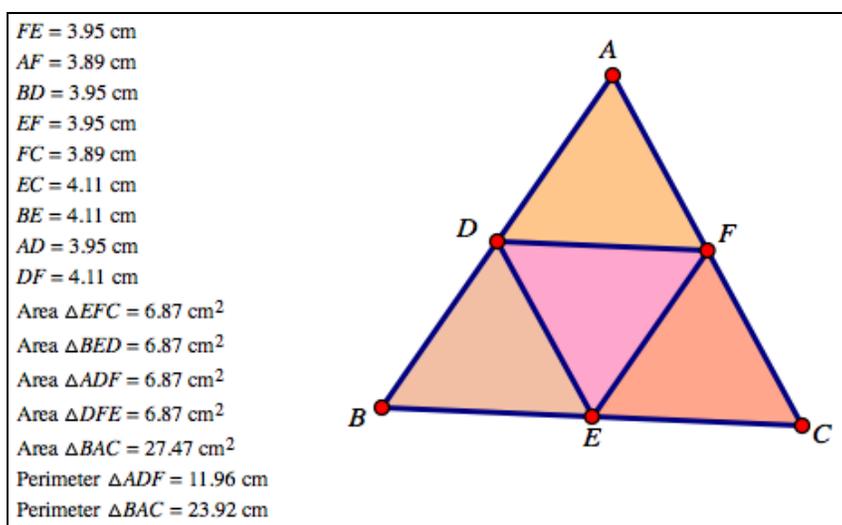


Figure 4. Screenshot of one student's measurements during the triangular thinking lesson.

Brian noticed that a student, Anna, calculated the areas of the original triangle and one of the smaller triangles formed after constructing the three midsegments. The following is the dialogue between him and Anna:

Brian: How do they compare?

Anna: We are going to do the rest.

Brian: Well, there are a lot of things that you can do. But how do those two compare? [He was pointing to the two area measures of triangles.]

Anna: It is smaller. [Referring to one of the four smaller triangles.]

Brian: Just smaller? Randomly smaller?

Anna: Because there are four of them.

Brian: There are four of them! You think if you had four of these [small triangles], what should it come up to?

Anna pointed to the area measurement of the original triangle and then multiplied the area of the small triangle by four. The product was equal to the area of the original triangle allowing Anna to form a conjecture with a justification.

This episode illustrates how Brian steered his students' learning and conjecture making by questioning. By prompting Anna, Brian guided her in making a conjecture and providing a justification for that conjecture. A similar conversation happened with another student, Beth, who measured areas of the original triangle and one of the smaller triangles. However, Beth already had a conjecture—"The area of one of the smaller

triangles is $1/4^{\text{th}}$ the area of the big triangle”—but she did not calculate the ratio of the two area measurements. Brian asked her how she knew that her conjecture was true.

Brian: You said that is one fourth, but you have not checked it. Go to
 Number, Calculate, click [the measure of the small triangle’s area]
 and divide by [the measure of the large triangle’s area]. So is that
 one-fourth? (see Figure 5)

Beth: Yes.

Brian: Is it true always? Try dragging it around.

Beth dragged vertex B and the ratio of the areas remained constant.

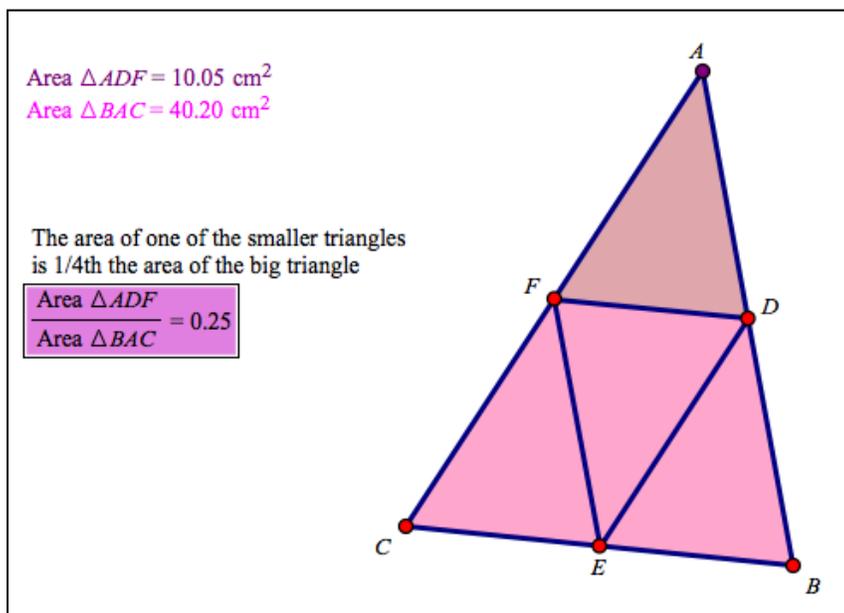


Figure 5. Screenshot of Beth’s conjecture with a measurement justification.

This episode shows that Brian incorporated some of the main features of the dynamic geometry, i.e., measuring and dragging. By calculating the ratio of the triangle area measurements and dragging a vertex of the large triangle, Beth could see that the ratio did

not change although the figure and the area measurements changed. This extra step assisted in justifying her initial conjecture. By questioning and guiding his students in explorations of geometric concepts, Brian displayed the exploring level of TPACK for the learning theme, conception of student thinking descriptor.

Brian continued to monitor his students' work and assisted them if they needed any help as he did with Anna and Beth in the aforementioned episodes. Brian made use of his TPACK in many more conversations with his students during this lesson. Dialogs not described here were of a similar nature to those conducted with Anna and Beth where his TPACK was unquestionably "visible." At the end of the lesson, all students came up with at least one conjecture, while many had three or four conjectures. One student's conjectures included the following:

- The area of one of the smaller triangles is $1/4^{\text{th}}$ the area of the big triangle.
- Both the large and small triangles have the same size angles.
- The midsegment is parallel to the opposite side of the original triangle.
- The midsegment of the smaller triangle is half of the opposite side of the original triangle.

Brian summarized all of the conjectures that students came up with during a whole-class discussion. He had a sketch of a triangle and its midsegments; on this sketch he also had buttons for five possible conjectures and corresponding measurements for justification (see Figure 6). He went through each one by asking students what conjectures they came up with and revealing those conjectures on his sketch one at a time. Table 5 summarizes four conjectures Brian discussed with his class; he did not reveal the one for the area

(which was included in the episode with Beth) because none of the students mentioned it in the limited time they had for the summary at the end of this lesson.

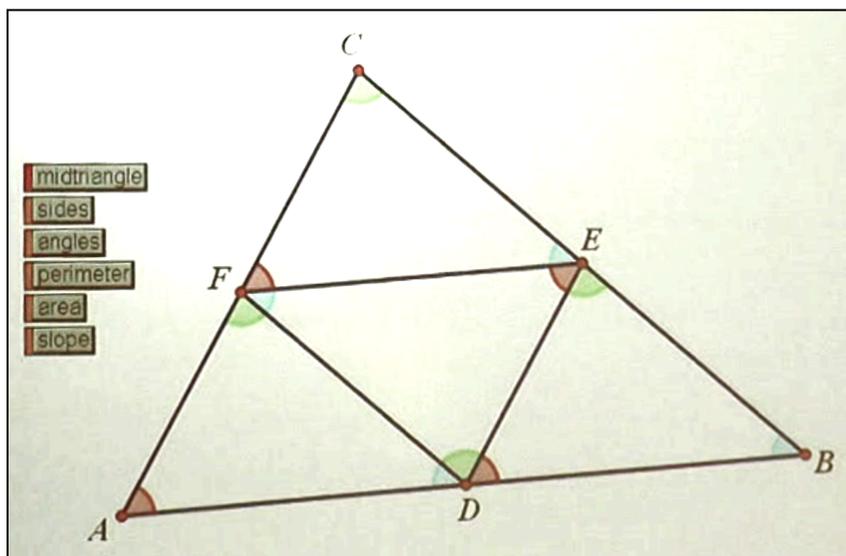


Figure 6. Screenshot of Brian's triangular thinking lesson summary sketch.

Table 5

Summary of Conjectures for the Triangular Thinking Lesson

Conjecture	Measurement justification
The measures of the sides of the smaller triangle are half that of the larger.	Ratios of the sides
The [corresponding] angles of all the triangles are congruent.	Measures of angles
The perimeter of the smaller triangle is half that of the larger triangle.	Ratio of the triangle perimeters
The midsegment is parallel to the opposite side.	Measures of slopes

TPACK Development Levels

Brian's responses to the TPACK Survey (see Figure 7) indicated that he considered his TPACK development levels to range from adapting to advancing, with most of them (six descriptors out of eleven) being "in the middle" at the exploring level. The following paragraphs describe Brian's TPACK development levels derived from the other data sources (e.g., interviews, observations) and are split into four sections for the four themes of the TPACK development model, i.e., curriculum and assessment, learning, teaching, and access (Niess et al., 2009).

Theme (descriptor)	TPACK Development Levels				
	Recognizing	Accepting	Adapting	Exploring	Advancing
Curriculum & Assessment (curriculum)				██████████	
Curriculum & Assessment (assessment)				██████████	
Learning (mathematics learning)					██████████
Learning (conception of student thinking)				██████████	
Teaching (mathematics learning)					██████████
Teaching (instruction)				██████████	
Teaching (environment)					██████████
Teaching (professional development)				██████████	
Access (usage)			██████████		
Access (barrier)			██████████		
Access (availability)				██████████	
 Responses to the TPACK Survey					

Figure 7. Brian's responses to the TPACK Survey.

Curriculum and assessment theme. The observed lessons and the accompanying student handouts provided evidence that Brian created his own Sketchpad-

enhanced activities. This implied that Brian was at the exploring level for the curriculum descriptor. The finding agreed with his ranking for this descriptor. On the TPACK Survey, he also reported to be at the exploring level for the assessment descriptor; however, no other data provided any information to where his TPACK actually is for this descriptor.

Learning theme. Based on the observed lessons, Brian was at the advancing level for the mathematics learning descriptor. In the triangular thinking lesson, it was apparent that Brian seriously thought about how to integrate Sketchpad. Because the goal of the lesson was to come up with different conjectures related to the midsegments of a triangle, Brian knew that Sketchpad could help students with their thinking and understanding of this topic and their learning would be enhanced through the use of Sketchpad.

Based on the observed lessons, initial interview, and implementation questionnaire responses, Brian was at the exploring level for the conception of student thinking descriptor. He guided his students in understanding by giving “hints as to where they should be looking” as he stated in one of the implementation questionnaires. His response to the TPACK Survey also indicated this level.

Teaching theme. Based on the observed lessons, it was clear that Brian used Sketchpad for higher-level thinking activities (e.g., his students form and justify conjectures) placing him at the exploring level for the mathematics learning descriptor. He ranked himself to be at the advancing level in this area; however, the infrequent use of Sketchpad that he reported through implementation questionnaires suggested his Sketchpad integration was not “active and consistent” (indicator of the advancing level).

The data collected through observations suggested that Brian was at the exploring level for the instruction descriptor. He engaged students in explorations using Sketchpad where students took control of their learning, and he was in the role of guide. He probed students with questions and answered questions from his students.

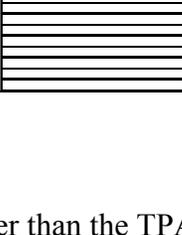
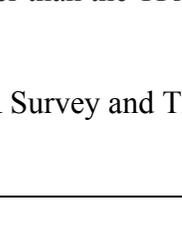
Brian created an environment where students could explore and discover geometry. He managed the Sketchpad activities so that his students were engaged and self-directed in their learning of geometric concepts. Therefore, he was at the advancing level for the environment descriptor. His response to the TPACK Survey also indicated this level.

According to data from the initial interview and the DGP professional development attendance records, Brian was at the adapting level for the professional development descriptor because he only attended professional development designed to focus on Sketchpad and, as a geometry leader, he shared ideas with other teachers at his school about incorporating Sketchpad in teaching.

Access theme. According to data collected through the initial interview and implementation questionnaires, Brian was at the adapting levels for the usage and barrier descriptors; these levels agreed with his responses to the TPACK Survey. Brian allowed his students to use Sketchpad for explorations and discovery of new geometric topics only in specific units (indicator of the usage descriptor). For example, he did not use Sketchpad for teaching and learning of 3-dimensional figures, but he used it for triangle congruence and geometric transformations. He experienced challenges with the DG implementation because of the new scope and sequence, so he integrated it to enhance geometry learning only in some units (indicator of the barrier descriptor).

Brian was at the adapting level for the availability descriptor. He taught geometric concepts differently because students got to discover them instead of finding out about them through lecture. For example, during one of the observed lessons, students investigated and made connections between different options for triangle congruence. Brian indicated his TPACK to be at the exploring level for this area descriptor; however, there was no other evidence to support his statement. His students did not explore concepts using multiple representations, which would indicate the exploring level of TPACK.

Summary of TPACK development levels. Brian's perceptions about his TPACK were close to the TPACK development levels extracted from the non-survey data (see Figure 8). The TPACK development levels identified through interviews, observations, implementation questionnaire responses and documents aligned with his TPACK development levels reported through the TPACK Survey for seven descriptors: curriculum and assessment theme/curriculum, learning theme/mathematics learning and conception of student thinking, teaching theme/instruction and environment, and access theme/usage and barrier. His TPACK development levels were one step lower than the levels reported on the survey for three descriptors, i.e., teaching theme/mathematics learning and professional development, and access theme/availability. Interviews, observations, implementation questionnaire responses and documents indicated four adapting levels, four exploring levels and two advancing levels; therefore overall, Brian's TPACK development level could be described as exploring.

Theme (descriptor)	TPACK Development Levels				
	Recognizing	Accepting	Adapting	Exploring	Advancing
Curriculum & Assessment (curriculum)					
Curriculum & Assessment (assessment)					
Learning (mathematics learning)					
Learning (conception of student thinking)					
Teaching (mathematics learning)					
Teaching (instruction)					
Teaching (environment)					
Teaching (professional development)					
Access (usage)					
Access (barrier)					
Access (availability)					

 Responses to the TPACK Survey

 TPACK development levels reported by data sources other than the TPACK Survey

 Areas of agreement between the responses to the TPACK Survey and TPACK development levels reported by the other data sources

Figure 8. Summary of Brian's TPACK development levels.

Summary

Brian recognized the need for professional development and learning how to teach with Sketchpad because knowing how to use Sketchpad was not enough to teach with it effectively. He preferred to use Sketchpad for introducing new geometric topics instead of reinforcing already-learned topics. He saw that students got to discover different concepts this way, were more engaged in a lesson, and “learned better.” He displayed good knowledge of Sketchpad and geometry content through creating his own Sketchpad-enhanced lessons. While students explored geometric concepts with Sketchpad, Brian circulated throughout the classroom and took on the role of a guide by prompting his students through questioning and facilitating their learning. He assisted them with forming their conjectures and integrating Sketchpad whenever possible to clarify their statements. Brian’s overall TPACK was at the exploring level; he created his own Sketchpad activities, used Sketchpad for higher-level thinking activities, and took on the role of a guide while students explored and discovered geometric concepts.

Case 2: James

James has been teaching geometry since the beginning of his teaching career, although he had never taught geometry solely. At the time of this study, he was teaching two sections of geometry and four sections of math models, a course that uses mathematics to solve real-life applied problems. This was his fifth year teaching and his third year working at his current school; he taught for two years in another city in the same state before his current school.

TPACK Development

This section describes James's TPACK development journey with respect to Sketchpad integration in classroom instruction. All quotes in the following paragraphs come from the initial interview.

Before signing up for the DGP, James did not know Sketchpad. One of the teachers at his school told him about the project and Sketchpad, indicating an overall recognizing level of TPACK. Once he took a look at Sketchpad, he wanted to integrate it into his instruction and thought that signing up for the DGP would be a convenient way to learn about this dynamic geometry software, indicating an overall accepting level of TPACK.

James was interested in learning about Sketchpad and incorporating it in his instruction. Once he attended the first DGP summer professional development workshop (five days), he began using Sketchpad in the classroom and became "comfortable integrating it to the class certainly within the first semester," indicating an overall adapting level of TPACK. According to the DGP professional development attendance records, he attended all of the workshops offered by the DGP, a total of approximately 73 hours. However, since he concentrated on learning only one type of technology, he was at the adapting level of TPACK for the teaching theme, professional development descriptor.

James considered himself to be proficient with Sketchpad. During the initial interview, he made several references to using animations in Sketchpad, incorporating movement and observing what changes and what stays the same:

Sometimes, when we are talking about areas of shapes, like of a trapezoid or a kite or rhombus, I use Sketchpad to create a sketch that you can figure out where those formulas are derived from. If you take a kite, take one-half the product of the diagonals—you take Sketchpad, and you can take the triangles formed by the diagonals. You can rotate them outwards to form a rectangle (see Figure 9), and then show students that you essentially have one-half of the rectangle. The diagonals become your base and height, and so that gives them at least a visual reference as to where that formula is being derived from as opposed to just simply—“here’s the formula for a kite.” So I like to use it for some of those animations.

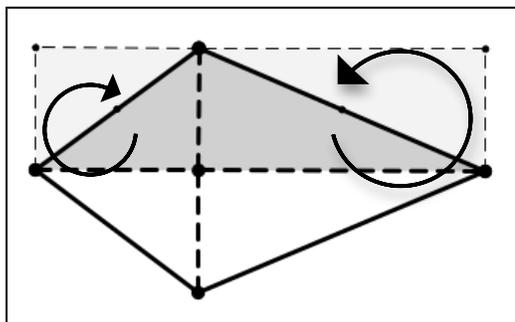


Figure 9. Illustration of rotation of two triangles in a kite to form a rectangle.

Based on this statement, the classroom observations and the work James did and shared during the DGP professional development sessions, he could be considered an expert when it comes to knowing how to use Sketchpad to present mathematical concepts.

TPACK Enactment

This section describes how James enacted his TPACK—i.e., how he put his combined knowledge of Sketchpad, geometry, and pedagogy into practice. Although this section focuses on TPACK enactment, it also provides additional information about James’s TPACK development through his DG implementation. All quotes that appear in this section, with the exception of quotes and dialogs that appear in the descriptions of the observed lessons, come from the initial interview.

James was comfortable with using Sketchpad in the computer labs while students did their explorations as well as in the classroom as a demonstration tool. He viewed learning with Sketchpad as more efficient because:

You can look at so many more examples in a short period of time than you can on paper. You can burn a whole class period on some really simple constructions. It takes a lot of time, so when you can construct the same thing in Sketchpad quickly, if you have that computer, you can actually get through stuff a lot quicker and have them explore it with a greater understanding.

His statement suggested the accepting level of TPACK for the access theme, availability descriptor, because James saw that the use of Sketchpad allowed covering a greater number of examples in a limited time. However, he also made several references to incorporating Sketchpad as a tool for discovery. He viewed the use Sketchpad during classroom instruction as “a natural extension or a better way to get students to understand what you teach them.” Without Sketchpad, “you are just lecturing about some sort of postulate or theorem and you cannot discover or explore any [postulate or theorem].”

This sort of Sketchpad use was visible during the observed lessons, and it indicated the adapting level of TPACK for the access theme, availability descriptor, because geometric concepts were taught differently with Sketchpad.

James described his typical classroom instruction as “traditional,” consisting mainly of lectures. On the other hand, when he took his students to a computer lab, they got to explore and discover different geometric concepts by incorporating Sketchpad. He believes that it is “important to have the two methods: the traditional approach, how we have been teaching it without Sketchpad, as well as using Sketchpad.” After one year of teaching with Sketchpad, he noticed that most of the students in “regular” geometry classes “get lost” when they did explorations using Sketchpad while students in Pre-AP (Advanced Placement) classes did well and “flourish with it.” He believes that, for those students who struggle with Sketchpad explorations, it is best to use a traditional approach as he stated:

When you lay it down with a traditional approach—“Here is the property. Here is how we use it to solve a problem. You go do it.”—You have modeled, you asked them, you showed them exactly what you want them to do, and they are able to mimic. So can they mimic you? Yes. Can they think for themselves and make the connection when they explore in Sketchpad? They struggle.

Based on this statement, James was more apt to introduce the key concepts to students in regular classes without Sketchpad, which indicated the accepting level of TPACK for the learning theme, mathematics learning descriptor. At the same time, he let students in

Pre-AP classes explore some geometric concepts with the Sketchpad, which indicated the adapting level in the same category.

Because of difficult access to a computer lab, James tended to take his students there only when he taught topics in “units that we are supposed to do in the lab,” meaning the units in the DGP-developed curriculum materials. The five units were (a) points, lines, and angles, (b) triangles and similarity, (c) transformations, (d) polygons, and (e) circles. James felt that the units on polygons and circles lend themselves to Sketchpad as a learning tool the most. Since properties related to circles were typically taught at the end of the school year, there was not much time to explore them, “but with Sketchpad, we can actually explore and learn.” The fact that he incorporated Sketchpad into teaching and learning only in specifically designed units suggested the adapting level of TPACK for the access theme, usage descriptor. Additionally, he would still limit some of the lab time even if he had access to a computer lab, and he would “pick and choose” the topics where his students struggled and where the use of Sketchpad could heighten their ability to make connections. This indicated the adapting level of TPACK for the access theme, barrier descriptor.

At the beginning of each observed lesson, James gave an activity sheet to his students with directions for each exploration. He obtained the activities from the Sketchpad Lesson Link website. Each handout had step-by-step instructions for students. As students worked on the activities, James walked around and answered their questions. Most of the questions revolved around how to do something or find something in Sketchpad. It was apparent that James had an excellent knowledge of the software while assisting his students as well as when demonstrating something on Sketchpad for the

whole class. However, students also had some questions related to the geometric concepts being explored; the following subsections describe a few examples.

Triangle inequalities lesson. During the lesson on triangle inequalities, students were to construct a triangle, measure the lengths of the three sides, calculate the sum of any two side lengths, and then drag a vertex of the triangle to try to make the sum they calculated equal to the length of the third side. Next, they had to answer the question—“Is it possible for the sum of two side lengths in a triangle to be equal to the third side length?” As students dragged one triangle vertex around, they discovered that the sum of two side lengths was equal to the third side length when the vertex they were dragging landed on that third side. However, some of the students seemed to have trouble answering the question. James noticed that and decided to do this part of the exploration on the demonstration computer so that the whole class could see it on the screen.

James: Ok, class, look up on the main board if you are struggling with number one [the first question]. You should have constructed a triangle. Find the length of all three sides and add two sides together. In this case, I have AB and BC. We are going to try to make that [the sum of AB and BC] the same length as the third side. We begin to move it over [James was dragging vertex B until it landed on segment AC - see Figure 10]. They are now equal. Do I have a triangle?

Class: No.

James: No. That's where you should be at with number one.

One student, Donna, was still confused.

Donna: But...

James: Do I have a triangle?

Donna: No.

James: So then, is it possible?

Donna: No.

James: Are you asking me or telling me?

Donna: I don't know... I'm telling you! I think. I'm making sure I'm right. So it's not possible.

James: Is it?

Donna: No! It's impossible. Then there would be no triangle.

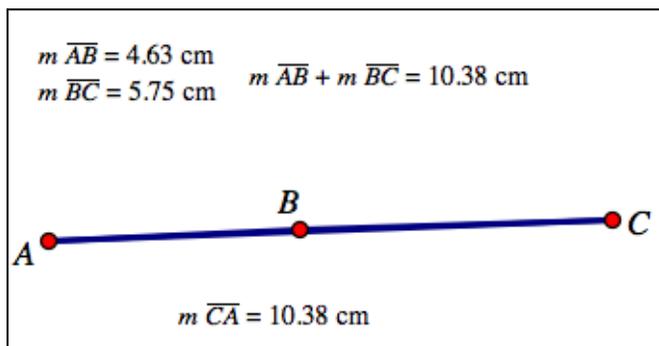


Figure 10. Screenshot from the triangle inequality lesson.

It was obvious that students saw that there was no triangle when the sum of two side lengths was equal to the third side, but they had trouble in answering the first question—“Is it possible for the sum of two side lengths in a triangle to be equal to the third side length?”—perhaps because it did not clearly ask if the triangle still existed.

Trigonometric ratios lesson. During this lesson, students were exploring the trigonometric ratios in a right triangle. According to the activity sheet from the Sketchpad Lesson Link website that James provided, they were to construct a right triangle, measure the ratios of various sides, and then use Sketchpad's calculator to find the sine, cosine, and tangent for a given angle. Although students had clear instructions for constructing a right triangle, many of them did not construct it correctly. Instead of constructing a line perpendicular to a given segment, they constructed two segments that appeared to be perpendicular, but did not pass a drag test. (A drag test is a form of assessment used to determine if a construction has been done properly. One drags parts of the construction to see if it holds true in different instances.) Because it was crucial to have a right triangle for this exploration, James decided to show this construction to the whole class after noticing that students were not following the instructions. Several minutes after showing the class how to construct a right triangle (as well as how to label its sides and find ratios of its sides), James was walking around and noticed that some students were still working with arbitrary triangles. So he showed them individually how to complete that construction. During the closing interview, he stated that if he were to teach this lesson again, he would probably provide his students with a pre-made right triangle so that they did not have to construct it themselves. This seemed like a good idea since the objective of this lesson was to explore trigonometric ratios and the construction of a right triangle was not the main objective.

Summary of the observed lessons. As lessons progressed and more students started asking the same questions, James tended to take more control of how the activities progressed. When students were not sure how to answer a question, or how to construct

or calculate something, he either showed the whole class how to do it, or did the required construction or calculation for students who struggled instead of guiding them in explorations. This indicated the adapting level of TPACK for the teaching theme, environment descriptor. However, the way James reflected on the trigonometric ratios lesson and the fact that he acknowledged that it could be improved showed that he was developing his TPACK. Also, because of his original intent to engage students in examining geometric concepts by using Sketchpad, his TPACK could be classified at the exploring level for the teaching theme, environment descriptor. In addition, James's instructional purposes for his students were clear; he wanted them to explore new geometric concepts on their own while he was walking around answering their questions. This indicated the exploring level of TPACK for the teaching theme, instruction descriptor.

In the initial interview, James mentioned creating his own curriculum materials that incorporate the use of Sketchpad, especially the ones that used animations. This was not evident in the observed lesson because James used existing activities from the Sketchpad Lesson Link website every time, indicating the adapting level of TPACK for the curriculum and assessment theme, curriculum descriptor.

TPACK Development Levels

James's responses to the TPACK Survey showed that his TPACK development levels spanned from accepting to advancing, although the majority of his responses were at the adapting level and the exploring level (see Figure 11 for details). The following paragraphs describe James's TPACK development levels derived from the other data sources (e.g., interviews, observations) and are split into four sections for the four themes

of the TPACK development model, i.e., curriculum and assessment, learning, teaching, and access (Niess et al., 2009).

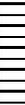
Theme (descriptor)	TPACK Development Levels				
	Recognizing	Accepting	Adapting	Exploring	Advancing
Curriculum & Assessment (curriculum)					
Curriculum & Assessment (assessment)					
Learning (mathematics learning)					
Learning (conception of student thinking)					
Teaching (mathematics learning)					
Teaching (instruction)					
Teaching (environment)					
Teaching (professional development)					
Access (usage)					
Access (barrier)					
Access (availability)					
 Responses to the TPACK Survey					

Figure 11. James's responses to the TPACK Survey.

Curriculum and assessment theme. James identified his TPACK at the exploring level for the curriculum descriptor. Although the initial interview provided some evidence for that level, data from the observed lessons suggested the adapting level because he understood that there was a benefit of using Sketchpad in teaching and learning geometry curriculum:

I think it [Sketchpad] is a benefit to a fair enough group of students. It [integration of Sketchpad] is a worthy cause. So it is worth going to the lab to help students that will benefit from it [using Sketchpad to learn geometry].

Learning theme. According to the interview and observation data, James was at the adapting level for the learning theme, mathematics learning descriptor. He introduced geometric concepts to students and used Sketchpad as a learning tool only in some units. James's response to the TPACK Survey also indicated this level.

James indicated the exploring level for the conception of student thinking descriptor; however, it was difficult to identify one level of TPACK from the non-survey data sources. In fact, James provided evidence for four TPACK development levels in this category. He stated in the initial interview:

If there was no project... Honestly if there was no project [DGP], I would probably have a hard time getting to the lab and using Sketchpad. I would still give class demonstrations, showing the different topics, but taking a class to a lab to explore... Honestly, if I were not in the project, it would probably not happen.

His statement indicated the recognizing level of TPACK. Immediately after that he added, “I am not sure how beneficial it was to any of the students majority of the time. So I am not sure if it was worth the time spent versus [using a] traditional approach.” He was referring to the first year of the DG implementation, and then he also added, “we have been learning geometry without it [Sketchpad] for a long time.” These comments suggested the accepting level of TPACK because James expressed his concern for the lack of development of appropriate geometric thinking skills in his students when they used Sketchpad for explorations. During the closing interview, James mentioned that he was planning to switch classrooms, permanently, with another teacher who had computers in the classroom but was not going to use them the following year. This showed a change in how James viewed the integration of Sketchpad as a learning tool because he said that by having computers in the classroom, he would be able to make it an integral tool for learning rather than as something additional as it was at the time of this study. If James is successful in this endeavor, then it will become easier for him to transition to the advancing level of TPACK for the conception of student thinking descriptor. Based on the observation and implementation questionnaire data, James was at the adapting level, because he allowed his students to explore selected geometric concepts with Sketchpad.

Teaching theme. James indicated the advancing level for the mathematics learning descriptor. However, his acceptance of Sketchpad as a tool for learning was not active and consistent. Based on the interview and observation data, he was at the exploring level. He incorporated Sketchpad into classroom instruction by engaging his

students in high-level thinking activities. Also, during the DGP professional development sessions, he shared his Sketchpad-based lessons and ideas with peers.

James's responses to the TPACK Survey indicated the exploring level for the instruction and environment descriptors. The data collected through observations confirmed the exploring level for those descriptors. James engaged students in explorations using Sketchpad where students took control of their learning, and he was in the role of a guide (the instruction descriptor). He provided his students with various instructional strategies to engage them in thinking about the geometric concepts under investigation (the environment descriptor). However, James tended to take more control of the activities and often did geometric constructions for his students instead of guiding them. This fact indicated the adapting level for the environment descriptor.

Data collected through the initial interview and the DGP professional development attendance records indicated that James was at the adapting level for the teaching theme, professional development descriptor because he attended professional development focusing only on Sketchpad (one type of technology).

Access theme. According to the data collected through the initial interview and implementation questionnaires, James was at the adapting level for the usage descriptor because he incorporated Sketchpad only into specific units and lessons. Additionally, the initial interview, the observed lessons and the implementation questionnaire responses indicated the adapting level for the barrier descriptor. James allowed his student to explore geometric concepts with Sketchpad infrequently because of limited access to a computer lab. His responses to the TPACK Survey indicated the adapting level for the usage and barrier descriptors as well. For the availability descriptor, James indicated the

accepting level of TPACK, and the initial interview data confirmed that. James viewed the incorporation of Sketchpad as more efficient and allowing him to demonstrate more examples. However, the initial interview and the observed lessons also suggested the adapting level for the availability descriptor because geometric concepts were taught differently with Sketchpad than without it, and students got to explore and discover them instead of learning about them through lecture.

Summary of TPACK development levels. Combined results from all data sources indicated that James's TPACK development levels were closely aligned to his perceptions about his TPACK (see Figure 12). The TPACK development levels identified through interviews, observations, implementation questionnaire responses and student handouts aligned with his TPACK development levels reported through the TPACK Survey for seven descriptors: learning theme/mathematics learning, teaching theme/instruction, environment, and professional development, and access theme/ usage, barrier, and availability. His TPACK development levels were one step lower than the levels reported on the survey for three descriptors, i.e., curriculum and assessment theme/curriculum, learning theme/conception of student thinking, and teaching theme/mathematics learning. James did not use the Sketchpad for any student assessments, but he mentioned that if he did then he would make sure that the tests consisted of different types of questions (conceptual as well as procedural), which indicated the adapting level. Interviews, observations, implementation questionnaire responses, and documents indicated one accepting level, eight adapting levels, and three exploring levels; therefore overall, James's TPACK was at the adapting level.

Theme (descriptor)	TPACK Development Levels				
	Recognizing	Accepting	Adapting	Exploring	Advancing
Curriculum & Assessment (curriculum)					
Curriculum & Assessment (assessment)					
Learning (mathematics learning)					
Learning (conception of student thinking)					
Teaching (mathematics learning)					
Teaching (instruction)					
Teaching (environment)					
Teaching (professional development)					
Access (usage)					
Access (barrier)					
Access (availability)					

 Responses to the TPACK Survey

 TPACK development levels reported by data sources other than the TPACK Survey

 Areas of agreement between the responses to the TPACK Survey and TPACK development levels reported by the other data sources

Figure 12. Summary of James's TPACK development levels.

Summary

James was an excellent example of how quickly one could learn to use Sketchpad and become highly proficient with it. He did not know about Sketchpad before the DGP, and after only one year of Sketchpad professional development and usage, he was able to create elaborate sketches. He also knew shortcuts and different ways of doing things in Sketchpad (e.g., measuring an angle). He used Sketchpad as a demonstration tool in the classroom as well as a learning tool in a computer lab. He believes in having two instructional approaches, one more “traditional” with lectures and one more learner-centered when students explore geometric concepts. His main reason for doing so was to assist students in making connections between what they “discover” with Sketchpad and what they need to do “on paper.” Because sometimes students had difficulties with Sketchpad explorations, James adjusted his lessons appropriately for “next time.” While students explored geometric concepts with Sketchpad, he took on the role of a facilitator and assisted students with any technical problems and with answering questions related to the content of a given lesson. James’s overall TPACK was at the adapting level; he incorporated Sketchpad as a teaching and a learning tool, and enabled his students to discover geometric concepts through the use of Sketchpad.

Case 3: Susan

Susan began teaching twenty years ago. She started her teaching career at the middle school level, and later switched to teaching geometry at the high school level. At the time of this study, she was in her tenth year of teaching geometry and her fifth year of teaching at her current school. She was teaching geometry solely and was the only geometry teacher at her school. Susan was tremendously enthusiastic about incorporating

technology into instruction. She had laptops, graphing calculators, clickers, an interactive white board, a document camera and a projector in her classroom.

TPACK Development

This section describes Susan's TPACK development journey with respect to Sketchpad integration in classroom instruction. All quotes in the following paragraphs come from the initial interview with Susan.

Susan began developing her TPACK with regard to Sketchpad approximately eight years ago when she was working on her master's degree. She was part of a teacher quality grant with other in-service teachers during one summer. Her overall TPACK was at the recognizing level then as she learned about Sketchpad. In the following fall, she was elected to be the math teacher in a new program at her school. As part of that program, hundreds of freshmen received laptops, and Susan had to use some kind of educational computer program with her students. She chose to use Sketchpad (indicating an overall accepting level of TPACK), although she was teaching algebra at that time. She also had training in Cabri Junior, a dynamic geometry application for graphing calculators, but she preferred to use Sketchpad because she had laptops in her classroom.

As part of her participation in the DGP, Susan attended all of the professional development meetings offered by the DGP, for a total of approximately 73 hours. She also attended several conferences with Sketchpad sessions throughout her teaching career, indicating an overall adapting level of TPACK. She enjoyed learning more about Sketchpad; however, she stated:

I have been doing it long enough up to where I am usually ahead, so that is a disappointment. But I love going. That is why I love being part of this

program because I am learning more to do that. But you got me thinking when I am using [Sketchpad] that I am able to point out to students more what they are supposed to see and not just see the magic. I think that I have improved a lot just by being in those [professional development sessions]. Any time you are with your coworkers and you can discuss ideas, it works better.

This statement indicates the exploring level of TPACK for the teaching theme, professional development descriptor, because Susan enjoyed learning more about Sketchpad and cooperating with colleagues on incorporating Sketchpad into teaching geometry. Susan's comment also revealed that she already knew a lot about Sketchpad; however, she still learned more through interacting with her colleagues using pedagogical dialogs.

TPACK Enactment

This section describes how Susan enacted her TPACK—i.e., how she put her combined knowledge of Sketchpad, geometry, and pedagogy into practice. Although this section focuses on TPACK enactment, it also provides additional information about Susan's TPACK development through her DG implementation. All quotes in this section, with the exception of quotes and dialogs that appear in observed lesson subsection, come from the initial interview.

Susan decided to use Sketchpad in teaching and learning because she was fascinated with it. Based on the initial interview, observed lessons and implementation questionnaire data, she incorporated Sketchpad into instruction as a learning tool for students' explorations and as a teaching tool for demonstrations. Exploring different

instructional strategies with Sketchpad indicated the exploring level of TPACK for the teaching theme, environment descriptor. At the same time, her implementation questionnaire responses suggested that she used it as a teaching tool more so than a learning tool. Therefore, her TPACK for the learning theme, conception of student thinking descriptor, was at the recognizing level.

Susan liked Sketchpad because it is accurate in measurements as opposed to students' measurements on paper. This indicated the recognizing level of TPACK for the access theme, availability descriptor, because Susan saw Sketchpad as a useful tool for replacement of paper-and-pencil activities where student tended to make more mistakes on their measurement, and with Sketchpad their measurements were accurate. So when working on any activity that involved measuring, she preferred to use Sketchpad. This, in turn, suggested the adapting level of TPACK for the access theme, usage descriptor, because she incorporated Sketchpad only in specifically designed units. Also, Susan noticed that using Sketchpad was more efficient in teaching some geometric topics—"in a way it is quicker to get some points across—like angles—just something simple like to name an angle, at the beginning of the year—that the vertex has to be the middle letter." This indicated the accepting level of TPACK for the access theme, availability descriptor, because Susan saw Sketchpad as a valuable tool simply for its efficiency.

Susan decided to do Sketchpad activities with her students every Thursday this year; since "time is a big barrier," she set that one day (Thursday) aside for Sketchpad activities, and it was her "big goal." This indicated the exploring level of TPACK for the access theme, barrier descriptor, because Susan recognized that it took additional time to incorporate Sketchpad activities. By dedicating one day a week for Sketchpad activities,

she overcame this challenge. In addition, implementation questionnaire data indicated that, in addition to engaging students in Sketchpad activities once a week, Susan used it for demonstration every day. This pointed to the exploring level of TPACK for the curriculum and assessment theme, curriculum descriptor, because Susan tried to integrate Sketchpad in a more integral role. However, at the same time she does not believe that students should use Sketchpad every day in class. Previously when she was teaching and had to use laptops every day, she “had a little issue with that.” At that time, she was mainly teaching algebra and thought that students needed more “hands-on” activities as she stated:

I think you still need to do a little pencil and paper. A little bit of both is good, but you cannot just go completely immersed. I think [Sketchpad] is great to use. Again, I still feel that it does not need to be—I do not believe it can be used every single day, but I think that it is a great tool to use.

Susan’s comment indicated that she recognized Sketchpad as a great tool for learning geometry, even though she did not use it every day. She incorporated it as a tool to facilitate learning of such geometric topics as angles and triangles. This indicated the exploring level of TPACK for the learning theme, mathematics learning descriptor, because she used Sketchpad “to facilitate the learning of specific topics in the mathematics curriculum” (Niess et al., 2009).

Susan’s students used Sketchpad during all of the observed lessons except a follow-up lesson to the parallel-lines-with-a-transversal lab; Susan used Sketchpad then for demonstration. At the beginning of each lesson, Susan instructed her students to take a laptop from the cart located in the back of the classroom. Then she introduced the

lesson and necessary vocabulary for that lesson, and distributed handouts with instructions for the activity. Students were working alone for the most part, talking to their nearby classmates about the activity from time to time. Susan acted as a guide walking around the classroom and answering any questions they had, which indicated the exploring level of TPACK for the teaching theme, instruction descriptor.

Parallel lines with a transversal lesson (lab). In this lesson, students were exploring angle relationships between two parallel lines and a transversal. This was an introductory lesson, and they had not explored this concept before. Susan introduced the lesson and pointed out a few things on the handout to make sure the students understood everything and that they followed the instructions closely. One of the things that she mentioned was the use of the construct menu:

Susan: When it says, “construct,” you need to be really careful and make sure you use the construct menu. You all like to just draw. It says, “construct parallel lines.” It is not going to stay parallel [if you just draw]. You want to construct parallel lines. You have to use the construct menu. You have to use this. It is not going to work [if you just draw]. If you do not use this, I will know. I will come around, and I will check it, and you will end up redoing it so pay attention. If you do not use this, you have to redo it.

It was clear that Susan had experience with students simply drawing lines that appeared to be parallel, but when dragged the lines did not remain parallel. Susan probably witnessed this in several other lessons, and that is why she addressed it at the beginning of this activity.

As students began to work on the activity, one student constructed the transversal so that it was perpendicular to the parallel lines, making all angles 90 degrees. Susan noticed that, and she said to the whole class, “If you did perpendicular, please undo. We do not want perpendicular lines. Parallel lines, but not perpendicular.” Later on she kept noticing that some students still had their transversal line perpendicular to the parallel lines. She addressed that issue with one of her students, Craig:

Susan: Do not construct a perpendicular [transversal].

Craig: I can't make right angles?

Susan: You can later. But it's not fun. That makes it boring. Math should be fun.

All students ended up having a correct construction, although Susan did not explain why they should not have a perpendicular transversal except that it was “boring.” After students had measured all angles, Susan directed them to move the angle measurements next to the corresponding angles because it makes it “more visual” (see Figure 13). The rest of the lesson concentrated on identifying congruent, complementary, supplementary, alternate interior, same side interior, alternate exterior, same side exterior and corresponding angles.

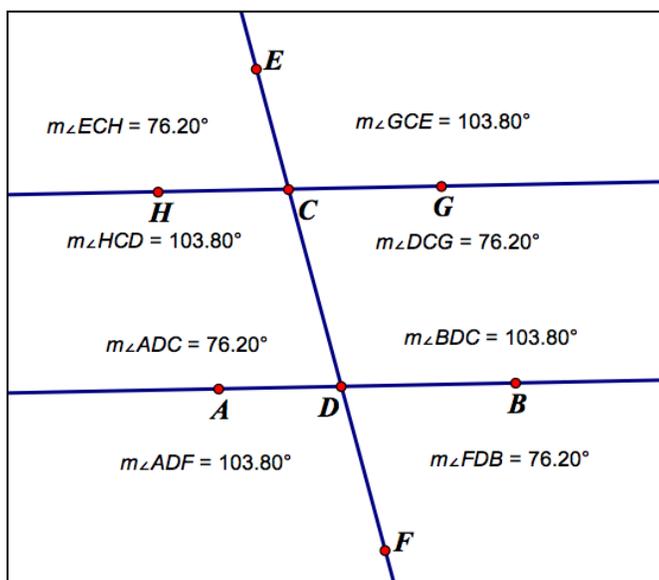


Figure 13. Screenshot of one student's work during the parallel lines with a transversal lesson.

Parallel lines with a transversal lesson (follow-up lesson). This lesson was a follow-up to the lab activity, which was four days before (there was a test day and a weekend in between the two). Susan was using Sketchpad and an interactive white board to review what the students did during the lab (see Figure 14). This confirmed that Susan recognized that geometric ideas were easily presented with Sketchpad and useful for making sense of topics in the curriculum and confirmed what she stated in the interview—the use of “different colors” makes it more visual and “it is quick to show.” The fact that she used Sketchpad for the visual effect, with different colors, indicated the recognizing level of TPACK for the curriculum and assessment theme, curriculum descriptor. The rest of the lesson consisted of reviewing answers to questions from the lab activity and working on problems that did not involve Sketchpad.

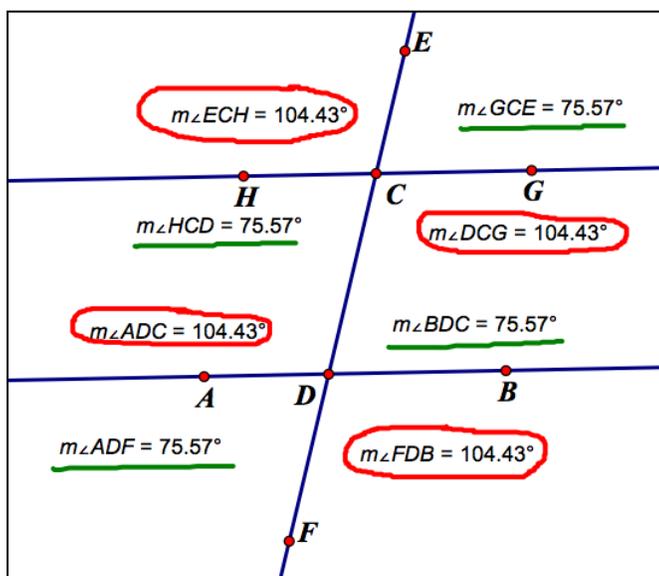


Figure 14. Screenshot of Susan's review of the lab activity during the parallel lines and a transversal lesson.

Properties of isosceles triangles lesson. During the break between classes and before this lesson, Susan told me that the Sketchpad activity that her students were going to work on was very similar to what they did the day before on paper. She tried to do a more challenging activity with her first period class that day, but it was too difficult for them. She stated that her student simply were not able to do it, so she decided to switch it with another activity. In the original and more challenging activity, students were to develop different ways of constructing isosceles triangles. In the "new" activity, students were to "discover" that the base angles of an isosceles triangle were congruent, which they already learned the previous day. Because the activity was reinforcing already known topic, Susan's TPACK indicated the adapting level for the teaching theme, mathematics learning descriptor. At the same time, the fact that Susan had a difficulty

identifying a topic for including Sketchpad as a learning tool indicated the accepting level for the curriculum and assessment theme, curriculum descriptor.

TPACK Development Levels

Susan's responses to the TPACK Survey showed that she perceives her TPACK to be at the exploring and advancing levels except for the assessment descriptor (see Figure 15). She indicated she did not allow her students to use Sketchpad on tests:

not because "I don't like to allow..." [but because] 1) Designing a Sketchpad test would be difficult. 2) Students would have more ways to cheat while on the computer so I would have to watch them very closely.

Susan's statement suggested that she assessed her students' learning through asking procedural questions where students were more likely to cheat. This showed an opportunity for TPACK development and the incorporation of assessment questions that examine conceptual understanding rather than procedural one. Asking conceptual questions could eliminate cheating on tests and make the inclusion of Sketchpad even more integrated with learning.

Although Susan indicated high levels (exploring and advancing) of TPACK through the TPACK Survey, the non-survey data indicated that her TPACK development levels range from recognizing to exploring. The following paragraphs describe Susan's TPACK development levels derived from the data sources and are split into four sections for the four themes of the TPACK development model, i.e., curriculum and assessment, learning, teaching, and access (Niess et al., 2009).

Susan's TPACK development level was extremely low (recognizing) for the curriculum and assessment theme, curriculum descriptor, because she focused on how things were displayed with Sketchpad (e.g., the use of different colors). Additionally, the third observed lesson—properties of isosceles triangles lesson—indicated the accepting level because Susan tried one Sketchpad activity with her first period class, but then changed it to a less-challenging activity, which in turn was a repetition of the lesson from the previous day. This showed that Susan might have had a difficulty identifying appropriate topics in the geometry curriculum for her students to explore with Sketchpad. However, the initial interview and implementation questionnaire responses suggested the exploring level for the curriculum descriptor; Sketchpad integration had “more integral role for the development of the mathematics that students are learning” (Niess et al., 2009) because Susan tried to use it every day in addition to students' weekly explorations.

Learning theme. Susan knew that integration of Sketchpad in learning was a worthwhile effort. Throughout all the years she had been using it in the classroom instruction, she identified topics, for which Sketchpad facilitates better learning and understanding of geometric concepts (e.g., parallel lines with a transversal). This indicated the exploring level of TPACK for the mathematics learning descriptor, which agreed with her self-reported TPACK development level. However, her TPACK development level was low (recognizing) for the conception of student learning descriptor because she preferred to use Sketchpad as “a teaching tool rather than as a learning tool” (Niess et al., 2009). She reported the advancing level for this descriptor, indicating that Sketchpad integration in her classroom was integral “to development of

the mathematics students are learning” (Niess et al., 2009). This was not evident in the observed lessons, and other data sources did not suggest it either.

Teaching theme. Based on the observed lessons, Susan’s TPACK was at the adapting level for the mathematics learning descriptor. She indicated the advancing level through the TPACK Survey; however, it was not evident that her students engaged in high-level thinking activities when using Sketchpad. Instead, she incorporated Sketchpad for enhancing or reinforcing already-learned topics as well as for activities outside the curriculum. Based on the observed lessons, Susan’s TPACK was at the exploring level for the instruction descriptor because she guided her students in Sketchpad explorations. The initial interview data indicated the same level for the environment descriptor because Susan incorporated a variety of instructional strategies when using Sketchpad. According to data from the initial interview, Susan was at the exploring level for the teaching theme, professional development descriptor, because she associated and worked with other geometry teachers (through the DGP) who integrated Sketchpad in geometry instruction.

Access theme. The initial interview and observation data indicated the adapting level for the usage descriptor because Susan used Sketchpad only in specific units such as those that require accurate measuring. By recognizing challenges in teaching geometry with Sketchpad (such as the extensive curriculum), Susan explored strategies to overcome them (e.g., setting one day a week to allow students to use Sketchpad), which indicated the exploring level for the barrier descriptor. Lastly, Susan’s TPACK was low (at the recognizing and accepting levels) for the availability descriptor because she preferred to use Sketchpad mainly for its accurate measurements and efficiency instead of using it to explore more complex geometric topics.

Summary of TPACK development levels. Combined results from all non-survey data sources indicated that Susan's TPACK development levels were not aligned to her perceptions about her TPACK (see Figure 16). The TPACK development levels identified through interviews, observations, implementation questionnaire responses and student activity sheets aligned with her TPACK development levels reported through the TPACK Survey only for three descriptors (all at the exploring level): learning theme/mathematics learning, teaching theme/environment, and access theme/barrier. Her TPACK development levels were lower for the remaining themes/descriptors than the TPACK development levels identified through the TPACK Survey. Because Susan's TPACK development levels ranged widely (from recognizing to exploring), it was difficult to identify one TPACK development level for her. Among all four themes, the teaching theme showed most consistency, with one adapting and three exploring levels; therefore, Susan's TPACK was at the exploring level for this theme.

Theme (descriptor)	TPACK Development Levels				
	Recognizing	Accepting	Adapting	Exploring	Advancing
Curriculum & Assessment (curriculum)					
Curriculum & Assessment (assessment)					
Learning (mathematics learning)					
Learning (conception of student thinking)					
Teaching (mathematics learning)					
Teaching (instruction)					
Teaching (environment)					
Teaching (professional development)					
Access (usage)					
Access (barrier)					
Access (availability)					

 Responses to the TPACK Survey

 TPACK development levels reported by data sources other than the TPACK Survey

 Areas of agreement between the responses to the TPACK Survey and TPACK development levels reported by the other data sources

Figure 16. Summary of Susan's TPACK development levels.

Summary

Susan learned about Sketchpad approximately seven years before this study and has been using it since. She had used it for teaching algebra and geometry. She does not believe students should use it all the time, but she believes it is an excellent tool for visualizing some difficult-to-learn concepts (such as angles). She incorporated Sketchpad into her lectures on a daily basis and her students used it for explorations about once a week. During the observed lessons, it was clear that Susan had experience in teaching with Sketchpad because she anticipated many areas where students had troubles (e.g., using the construct menu to construct parallel lines instead of drawing lines that appear to be parallel). However, she tended to use Sketchpad because it was easier and quicker to present geometric ideas with it. If her students had difficulties with Sketchpad activities (or if those activities were challenging and students struggled with the concepts), she decided to change them to something easier, i.e., something that she knew students would be able to do (e.g., the lesson on properties of isosceles triangles). Susan's TPACK for the teaching theme was at the exploring level; she incorporated Sketchpad as a learning tool on a weekly basis, and enhanced her students' learning through the use of Sketchpad. Based on the collected data, it was impossible to identify one TPACK development level for the remaining themes (i.e., curriculum and assessment, learning, and access).

Case 4: Laura

Laura was in her third year of teaching geometry at the time of this study. She taught geometry solely during the previous year, but during this study she was teaching three sections of geometry and three sections of math models, a course that uses mathematics to solve real-life problems. Laura went to college after raising five children.

As a Girl Scout and a Boy Scout leader for thirty-five years, Laura gained a significant amount of pedagogical knowledge prior to becoming a mathematics teacher. She had computers in her classroom, so her students had easy access to Sketchpad.

TPACK Development

This section describes Laura's TPACK development journey with respect to Sketchpad integration in classroom instruction. All quotes in the following paragraphs come from the initial interview.

Laura used Sketchpad as a college student in her teacher preparation program (indicating an overall recognizing level of TPACK) approximately four years before this study. She “felt very comfortable with Sketchpad” when she began her work in the DGP. Even though she used Sketchpad in college, she did not use it for teaching until the DGP because she preferred to do hands-on activities with her students. Laura used the term “hands-on” to describe activities, in which students “use their hands” or engage in tactile learning. Therefore, the term “hands-on” is used in this context in the remaining parts of this case report. The fact that she chose not to use Sketchpad in teaching indicated an overall accepting level of TPACK—in this case, an unfavorable attitude towards integrating technology in teaching mathematics (Niess et al., 2009). When she signed up for the DGP, she wanted to be in the comparison group, but because of the random assignment she ended up in the DG group. She attended all of the professional development sessions offered by the DGP, except for a one-day summer workshop, for a total of approximately 66 hours. This indicated the adapting level of TPACK for the teaching theme, professional development descriptor, because she focused on learning

only one type of technology. Through professional development and practice, her knowledge about teaching with Sketchpad changed for the better:

The main way that [my knowledge] has changed is just that experience of learning what students are going to make a mistake on, and what students are going to struggle with doing. Knowing that is extremely helpful to circumvent the flaw in your lesson because you understand how students are going to make a mistake.

Laura's comment suggested the adapting level of TPACK for the learning theme, conception of student thinking descriptor, because she "begins developing appropriate mathematical thinking skills when technology is used as a tool for learning" (Niess et al., 2009). Furthermore, by incorporating Sketchpad in teaching and learning, she noticed how little students understand about geometry.

What [Sketchpad] has shown me was how little [students] truly understand about geometry. When you tell them something, they are trying to memorize something to do, and they do not actually understand anything. I think that was the most eye-opening thing for me last year—trying a little harder to get more feedback from them, explaining to me what is going on, so that I can see if they know or they are just mimicking back.

Laura's statement indicates that when she incorporated Sketchpad, student learning and understanding of geometry was more "visible," provided her with more feedback about her students, and allowed her to adjust her instructional strategies accordingly. She also mentioned that she saw "much more deeper thinking" and "more understanding of what is going on with the Sketchpad." By seeing how her students benefit from using

Sketchpad, Laura started to change her attitude towards incorporating it into classroom instruction. Before the DGP, she did not use it in teaching, although she had used it as a college student and had access to it at her school. Through her participation in the DGP, her conception about incorporating Sketchpad changed so that she “probably will still use Sketchpad” if she has access to it after the DGP.

Laura based her above-mentioned reflections on the first year of the DG implementation. During the second year of the DG implementation, at the time of this study, Laura struggled immensely with her students’ attitude and lack of motivation towards learning geometry with Sketchpad. She mentioned in one of the implementation questionnaire responses, “This year has been very challenging to engage students in experimentation. Students have a very lazy attitude towards learning and wait for information to be given.” This also created a barrier to Sketchpad explorations; however, approximately two months after making that comment, Laura found a solution:

Students are terribly impacted if they believe they will be receiving credit for something. Having them turn their work in electronically certainly seems to motivate them, and I enjoy it much more. They are becoming much more receptive of the work part of the dynamic geometry tools.

They have always been receptive, but they thought it meant a free day.

This comment shows that Laura was developing her TPACK in the barrier descriptor area. First, she did not feel like students were learning much from using Sketchpad because “they have been very lazy” and “they think they are just there to play.” However, she started exploring strategies, such as having them turn in their work electronically, to “minimize the impact of those challenges” (Niess et al., 2009).

TPACK Enactment

This section describes how Laura enacted her TPACK—i.e., how she put her combined knowledge of Sketchpad, geometry, and pedagogy into practice. All quotes that appear in this section, with the exception of quotes and dialogs that appear in the subsections with descriptions of the observed lessons, come from the initial interview.

Laura preferred to do hands-on activities with her students, so she used Sketchpad only in a few units. She thought Sketchpad was perfect for exploration of some, but not all, topics. This indicated the adapting level of TPACK for the learning theme, mathematics learning descriptor. She described one lesson where the incorporation of Sketchpad did not work well:

If you try to do an exploration with parallel lines cut by a transversal, it seems like a terrific idea—“Let’s measure all the angles and see how they change.” Number one, with the current version of Sketchpad, you cannot anchor the measurement to the angle. You can do it if you know how, but my students are not going to do that, so by moving they lose the angle measurement, and then they cannot seem to get the angles measured correctly. So some things that require accurate measurement, especially of angles, for them to explore is almost setting them up for failure because they are not going to see the relationship if they are measuring incorrectly.

Laura’s description indicated that she did not like using Sketchpad for exploring parallel lines cut by a transversal; however, she liked using it for introducing her students to the “building blocks” of geometry because “students have absolutely no concept of what a point, a line, an angle are, especially angles.” This indicated the adapting level of TPACK

for the access theme, usage descriptor, because she used Sketchpad to teach geometric topics in specific units. At the same time, Laura incorporated Sketchpad to give her students “access to connections formerly out of reach” (Niess et al., 2009). This indicated the adapting level of TPACK for the access theme, availability descriptor. So Laura saw the benefit of letting her students use Sketchpad for explorations because they could “really understand the topic” that way. At the same time, it was difficult to find enough time for those explorations because of the extensive curriculum:

Sometimes I tell myself, “Just forget the curriculum if you need three days to explore, so [students] understand the topic.” But then it is scary because students are held responsible, so I do not want them to be disadvantaged in any way.

Laura’s statement indicated that it was challenging for her to incorporate Sketchpad into teaching and learning with the current curriculum. However, she strived to provide her students with a new way to approach geometry—“I see much more, much more deeper thinking about what is going on and more understanding of what is going on” when using Sketchpad. This suggested the adapting level of TPACK for the access theme, barrier descriptor. She understood benefits of integrating Sketchpad into classroom instruction, although she did it rather infrequently. Therefore, she was at the adapting level for the curriculum and assessment theme, curriculum descriptor.

During the observed lessons, Laura guided her students through geometric explorations with Sketchpad. This indicated the exploring level of TPACK for the teaching theme, instruction descriptor, because she did not direct their every move. She started off the lessons with students sitting in their desks while she introduced the lesson

topics. Then students moved to computers to do Sketchpad explorations. Before the end of the lessons, students came back to their desks for a lesson summary, a lecture, or a problem-solving session directed by Laura, with or without Sketchpad, exactly as she described it in the initial interview:

I have them explore on computers, and they make notes. But then I do not get much input from them until they come back to their desks at the end of the class, and we talk about it, and they start hearing other people's ideas.... It has to have that final push of either people putting something on the board or sharing something with the class or it never closes, never comes to a conclusion.

This statement suggested the adapting level of TPACK for the teaching theme, environment descriptor, because she tried to save time and “maintain control of how the activity progresses” (Niess et al., 2009).

At the end of each lesson, Laura reflected on her teaching with Sketchpad and asked herself “Was that good? Or was that bad [about this lesson]?” By planning, implementing, and reflecting on Sketchpad-enhanced lessons she exemplified the exploring level of TPACK for the learning theme, conception of student thinking descriptor. The following two subsections present parts from the observed lessons related to Laura's TPACK.

Triangle congruence lesson. During this lesson, students were investigating different conditions for triangle congruence. After listing all possible three-item combinations of sides and angles—i.e., SSS, AAA, SAS, SSA, AAS, ASA—students were to explore on Sketchpad which of them guarantee triangle congruence and which do

not. The Sketchpad file had one already-constructed triangle for each of the six conditions and students' goal was to construct a noncongruent triangle given the same condition (see Figure 17 for an example). Students could manipulate parts of the second figure, but they could not change lengths of the congruent segments or sizes of the congruent angles. The guiding question was: Can you make a different triangle?

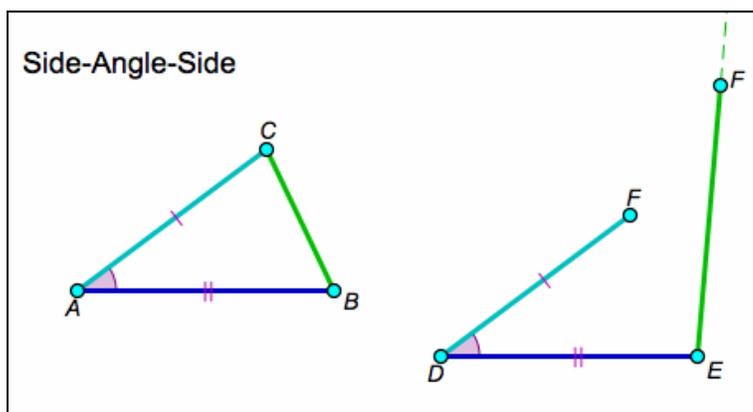


Figure 17. Sample screenshot from the triangle congruence lesson.

This lesson showed that students engaged in a higher-level thinking activity, and it implied the exploring level of TPACK for the learning theme, mathematics learning descriptor. At the end of the lesson, students posted their votes on the board as to which conditions proved congruence and which ones did not. Laura discussed each one with the whole class and made clarifications for three conditions where the students' votes were divided between “proves congruence” and “does not prove congruence”, i.e., AAA, SSA, and AAS (see Figure 18 for details). Using the condition AAA, Laura constructed another triangle in Sketchpad and dragged one of its vertices to change its size; students saw that the angles remained the same, but the side lengths did not, therefore showing

that AAA did not prove congruence. For SSA, Laura showed that a noncongruent triangle could be created. So SSA did not prove congruence. For AAS, Laura divided it into two cases—AAS(corresponding) and AAS(noncorresponding)—because students had these two cases to explore, which were listed in Sketchpad as A-A-(Corresponding)S and A-A-(Noncorresponding)S. Laura concluded that AAS(corresponding) proved congruence and AAS(noncorresponding) did not, and she mentioned that this was probably why the votes were divided for AAS. Laura used a “smiley face” to indicate that a condition proved congruence and an “X” to indicate that it did not.

Conclusion	Vote	Proves Congruence	Does NOT Prove Congruence
☺	SSS	✓✓✓✓✓✓✓✓	
✗	AAA	✓✓✓✓✓✓✓✓	✓
☺	SAS	✓✓✓✓✓✓✓✓	
✗	SSA	✓✓✓✓✓✓✓✓	✓✓✓✓✓✓✓✓
☺	AAS (circled)	✓✓✓✓✓✓✓✓	✓✓✓✓✓✓✓✓
☺	ASA	✓✓✓✓✓✓✓✓	
✗	AAS non-corresponding		

Figure 18. Students' votes on triangle congruence and Laura's conclusions.

The 30° – 60° right triangle lesson. In this lesson, according to the activity sheet provided by Laura, students were to construct the 30° – 60° right triangle by constructing an equilateral triangle and one of its medians. Then, they were to use half of the equilateral triangle for the rest of the exploration. Students constructed squares on the three sides of the 30° – 60° right triangle and calculated two ratios—the ratio of the area of the square on the hypotenuse to the area of the square on the shorter leg and the ratio

of the area of the square on the longer leg to the area of the square on the shorter leg. Students discovered that these ratios were constant and equal to four and three, respectively.

During this lesson, Laura mentioned to one of her students, Donna, that they discussed the $30^\circ - 60^\circ$ right triangle together with the $45^\circ - 45^\circ$ right triangle the day before. That meant that students already knew what the ratios of sides were, and it indicated the adapting level of TPACK for the teaching theme, mathematics learning descriptor, because students were using Sketchpad to reinforce already-learned concepts. The following conversation with Donna provides evidence of this.

Laura: Remembering from yesterday, what is the relationship between the smallest side [the shorter leg] and the biggest side [the hypotenuse]?

Donna mentioned the Pythagorean Theorem.

Laura: Okay, so what you are remembering is that, for every right triangle, areas of the two smaller ones [squares] add up to equal the area of the biggest one [square]. That is right, but then from yesterday, when we were looking at the side measurements of the special $30^\circ - 60^\circ - 90^\circ$ triangle, what did we say was the relationship between the smallest side [the shorter leg] and the biggest side [the hypotenuse]?

Dave: It was half.

Laura: Right, it was half. From here [the shorter leg] to here [the hypotenuse] was times two, correct?

Dave: Yes.

Laura: Why do you think the ratio of areas is four instead of two.

Dave: Times two.

Laura: Did you times it by two? Or what did you do to it?

Dave: Squared it.

Laura: So maybe the reason why the relationship of the areas is four is because the relationship between the sides is two.

It was not clear what the goal of the lesson was as far as the use of Sketchpad since students already knew what the ratios of sides were. For the rest of the lesson, students moved back to their desks and were to find the missing lengths in $30^\circ - 60^\circ - 90^\circ$ triangles, where one of the side lengths was given. Laura worked a few such problems on the board and reminded her students how to set up a proportion to solve for a missing value.

TPACK Development Levels

Laura's responses to the TPACK Survey spanned across all levels but most of them were at the adapting level (see Figure 19 for details). The remainder of this section describes Laura's TPACK development levels derived from the interviews, observations and implementation questionnaire responses. They are split into four subsections for the four themes of the TPACK development model, i.e., curriculum and assessment, learning, teaching, and access (Niess et al., 2009).

Theme (descriptor)	TPACK Development Levels				
	Recognizing	Accepting	Adapting	Exploring	Advancing
Curriculum & Assessment (curriculum)				██████████ ██████████ ██████████ ██████████	
Curriculum & Assessment (assessment)	██████████ ██████████ ██████████ ██████████				
Learning (mathematics learning)			██████████ ██████████ ██████████ ██████████ ██████████		
Learning (conception of student thinking)			██████████ ██████████ ██████████ ██████████	██████████ ██████████ ██████████ ██████████	
Teaching (mathematics learning)					██████████ ██████████ ██████████ ██████████
Teaching (instruction)			██████████ ██████████ ██████████ ██████████ ██████████		
Teaching (environment)			██████████ ██████████ ██████████ ██████████ ██████████		
Teaching (professional development)		██████████ ██████████ ██████████ ██████████			
Access (usage)			██████████ ██████████ ██████████ ██████████ ██████████		
Access (barrier)			██████████ ██████████ ██████████ ██████████ ██████████		
Access (availability)			██████████ ██████████ ██████████ ██████████ ██████████		
 Responses to the TPACK Survey					

Figure 19. Laura's responses to the TPACK Survey.

Curriculum and assessment theme. Based on the initial interview, Laura set a goal for herself to have her students use computers every day as they come into class, at

least for a few minutes, because she wanted to make the technology a more integral part of the learning. This would indicate the exploring level of TPACK for the curriculum and assessment theme, curriculum descriptor, which was also the level she indicated in the TPACK Survey. However, according to data reported through the implementation questionnaire, she did not get to implement her goal. Her TPACK for the curriculum descriptor was at the adapting level, based on the interview and implementation questionnaire data.

Learning theme. According to the interview and observation data, Laura was at the adapting level for the learning theme, mathematics learning descriptor. She introduced geometric concepts to students and used Sketchpad as a learning tool only in some units. Her response to the TPACK Survey also indicated this level.

For the conception of student thinking descriptor, Laura indicated the adapting and the exploring levels through the TPACK Survey. The initial interview and observation data also provided evidence for both of those levels. Laura's students used Sketchpad as a learning tool, but she did not assess their thinking by incorporating Sketchpad, which indicated the adapting level. At the same time, she reflected "on teaching and learning with concern for guiding students in understanding" (Niess et al., 2009) indicating the exploring level.

Teaching theme. Laura classified herself to be at the advancing level for the mathematics learning descriptor. The observation data, however, provided evidence for the adapting and the exploring levels. During the lesson on the $30^\circ - 60^\circ$ right triangle, she was using Sketchpad for an already-learned concept, suggesting the adapting level.

During the lesson on triangle congruence, she engaged her students in a higher-level thinking activity, indicating the exploring level.

Laura classified her TPACK to be at the adapting level for the instruction and the environment descriptors. However, the observation data suggested the exploring level for the instruction descriptor because she engaged students in explorations using Sketchpad where students took control of their learning, and she was in the role of a guide. The observation data confirmed the adapting level for the environment descriptor; Laura provided extensive introductions and summaries to the lessons, during which students used Sketchpad. She made sure to tell students at the end of the lesson what they were supposed to discover during their Sketchpad explorations.

Data collected through the initial interview and the DGP professional development attendance records indicated that Laura was at the adapting level for the teaching theme, professional development descriptor because she only attended professional development designed to focus on Sketchpad (one type of technology).

Access theme. Laura's responses to the TPACK Survey indicated the adapting level for all three descriptors for the access theme. Data collected through interviews, observations and implementation questionnaires also suggested the adapting level for all descriptors. She used Sketchpad in specific units (the usage descriptor), e.g., to introduce points, lines, and angles. Also, finding time to integrate it was difficult because of the extensive curriculum (the barrier descriptor). When she got to use Sketchpad in her instruction, Laura taught geometry differently then and enabled her students to understand geometric concepts through investigations (the availability descriptor).

Summary of TPACK development levels. The findings show that Laura's TPACK development levels were at the adapting and the exploring levels, with the majority of evidence at the adapting level, which was consistent with her perceptions about her TPACK (see Figure 20 for details). The TPACK development levels identified through interviews, observations, implementation questionnaire responses and documents aligned with her TPACK development levels reported through the TPACK Survey for seven descriptors: curriculum and assessment theme/curriculum, learning theme/mathematics learning and conception of student thinking, teaching theme/environment, and access theme/usage, barrier, and availability. Her TPACK development levels were one step higher than the levels reported on the survey for two descriptors, i.e., teaching theme/instruction and professional development. Her TPACK development level for the teaching theme, mathematics learning descriptor, however, was two steps lower than the one reported on the TPACK Survey. Interviews, observations, implementation questionnaire responses and documents indicated eight adapting levels and three exploring levels; therefore overall, Laura's TPACK development level could be described as adapting.

Theme (descriptor)	TPACK Development Levels				
	Recognizing	Accepting	Adapting	Exploring	Advancing
Curriculum & Assessment (curriculum)					
Curriculum & Assessment (assessment)					
Learning (mathematics learning)					
Learning (conception of student thinking)					
Teaching (mathematics learning)					
Teaching (instruction)					
Teaching (environment)					
Teaching (professional development)					
Access (usage)					
Access (barrier)					
Access (availability)					

 Responses to the TPACK Survey

 TPACK development levels reported by data sources other than the TPACK Survey

 Areas of agreement between the responses to the TPACK Survey and TPACK development levels reported by the other data sources

Figure 20. Summary of Laura's TPACK development levels.

Summary

Laura learned about Sketchpad in college and used it as a student for learning geometry. She did not want to use it with her students because she preferred to do hands-on activities before the DGP. Once she signed up for the project, she was asked to use Sketchpad on a regular basis because she was part of the DG group. She had no problem with incorporating it into her classroom instruction. During the observed lessons, Laura introduced the lesson topic and then sent her students to computers for Sketchpad explorations. Because she had computers in her classroom, incorporating Sketchpad in learning was seamless. At the end of each lesson, students went back to their desks, and Laura reviewed the lesson with them as a class. She believes that students need a summary of the completed exploration; this gives them a chance to compare their findings with their classmates' findings and to come to a common conclusion. Laura's overall TPACK was at the adapting level; she incorporated Sketchpad as a learning tool and enabled her students to better understand geometric concepts they investigated.

Summary

This chapter described how the participants of this study developed their TPACK and how they enacted it in the classroom instruction. Additionally, TPACK development levels were identified for all participants within the four themes and its descriptors (except for curriculum) from the TPACK Development Model (Niess et al., 2009). Four case reports provided answers to the three research questions. Each case report offered a rich description of a given participant's TPACK development and enactment related to teaching geometry with Sketchpad.

Susan had the most Sketchpad training and used it in the classroom instruction often. Laura learned geometry with Sketchpad in college but did not use it in teaching until the DGP. Brian was aware of Sketchpad before the DGP but did not have training until he signed up for the project; he started integrating it into teaching then. James did not know about Sketchpad until the DGP, and he became proficient in it within a year. All teachers acted as guides while their students explored geometric concepts with Sketchpad; they circulated throughout the room and answered students' questions. All teachers displayed sound knowledge of geometry content, although they did not always know how to connect it with their pedagogical and technological knowledge. The findings also revealed that easy access to computers does not always result in frequent Sketchpad use; the participants of this study claimed that the curriculum and standardized testing were responsible for that.

Through the descriptions of participants' TPACK development and enactment, TPACK development levels as defined by the TPACK Development Model (Niess et al., 2009) were identified. The levels varied within the themes and their descriptors for each participant; however, an overall TPACK development level was identified for Brian (exploring), James (adapting) and Laura (adapting). For Susan, an overall TPACK development level (exploring) was identified only for the teaching theme, as the levels for the remaining descriptors varied significantly.

This chapter provided findings for individual participants. The next chapter, Chapter V, continues reporting the findings for this study through cross-case analysis.

CHAPTER V

CROSS-CASE ANALYSIS

Chapter IV provided four case reports describing participants' TPACK development, TPACK enactment and their TPACK development levels. This chapter provides cross-case analysis and findings for the four participants as a group, although individual references are present at times. Therefore, this chapter augments the findings for individual participants and offers further answers to the research questions. In addition to providing findings for TPACK development and TPACK enactment, this chapter provides analysis of a teaching-with-Sketchpad task. This task was developed as a result of the pilot study; it provides an avenue for assessing TPACK related to the same geometric content for different participants. Therefore, it allows for comparison of different participants' TPACK using the same context.

TPACK Development

All teachers participated in all or almost all DGP professional development sessions. Even though they received stipends for attending these sessions, they expressed willingness to learn more about Sketchpad and to teach with it effectively. Susan and Laura had previous experience with Sketchpad as graduate/college students. In addition, Susan used it in the classroom instruction for six years before the DGP. Brian and James had limited or no knowledge of Sketchpad before the DGP, therefore, they developed most of their knowledge through the DGP professional development sessions and through

their own explorations and lesson planning. At the time of this study, out of all participants, James seemed to have the best knowledge of Sketchpad, interestingly enough he was the only teacher who did not know Sketchpad when signing up for the DGP; all other teachers had some experience with it before the DGP. Although James appeared to have the best knowledge of Sketchpad, Brian had the best TPACK related to teaching with Sketchpad. This was an unexpected finding since Brian was the least experienced teacher—in his third year of teaching at the time of this study. The reason why this finding was unexpected is the fact that the literature suggests that teachers with more developed pedagogical content knowledge (PCK) are more likely to develop their TPACK quicker. This was only the second year of the DG implementation for Brian, and he was still developing his PCK. However, based on the initial interview, he was developing his TPACK at the same time he was developing his PCK.

TPACK Enactment

All teachers enjoyed using Sketchpad, but they did not use it in a computer lab setting often (see Table 6 for frequency of Sketchpad use) even though all of them, except James, had easy access to computers—Brian had a devoted computer lab, Susan had laptops in her classroom, and Laura had desktop computers in her classroom. This was an unexpected finding—the fact that easy access to technology did not guarantee its (increased) integration into the classroom instruction.

Table 6

Frequency of Sketchpad Use

Teacher	Frequency of student Sketchpad use per week	Frequency of teacher Sketchpad use per week	Total Sketchpad use per week
Brian	0.5	0.5	1.0
James	0.6	0.9	1.5
Susan	1.2*	4.0*	5.2*
Laura	0.7	1.0	1.7

Note. Data for Susan's Sketchpad use cover only the first nine (9) weeks of school while for the rest of the teachers it spans across twenty-two (22) weeks.

Each participant had a different reason for limited Sketchpad integration. James had a difficult access to a computer lab; he had to plan his lab days weeks or months ahead. Brian struggled with incorporating Sketchpad into his lessons because of new scope and sequence of the curriculum. Laura preferred to do hand-on activities that did not involve technology. Susan believes, "you can't just go completely immersed."

Brian had easy access to a computer lab, so he was able to take his students there whenever he wanted to; the main obstacle with access was that the computer lab was located far away from his classroom and he had to switch in between the two very quickly on the days he took his geometry students to the lab because he had to teach his algebra students in the classroom. In the lab, Brian's students were able to have one computer per student. The computers were located on three walls of the room so that Brian could see his students' monitors as he walked around the room.

James's situation was similar to that of Brian's in a way that he had to take his students to a different room in order to use computers. The main challenge for James, however, was access to a computer lab. He had to share a computer lab with other teachers at his school and had to sign up for the lab time weeks in advance. During the

first two lessons, students were working on computers (one computer per student) in computer labs. As in Brian's case, the computers were located on three walls of the room so that James could see his students' monitors as he walked around the room. During the third lesson, James took his students to another teachers' classroom, which had a similar setup as the computer labs (computers on three walls of the room), but it also had individual student desks in the middle of the room making it a little more difficult to walk around to monitor student work on computers.

Susan and Laura had easy access to computers. Susan had a cart with laptops in her classroom at all times so whenever her students were working on Sketchpad activities, they took one laptop from the cart and sat at their regular student desk in the classroom. Laura, on the other hand, had computers along the three walls of her classroom. Whenever her students were working on Sketchpad activities, they moved from their student desks, which were located in the middle of the room, to the desks with computers on the edges of the room.

Brian and James preferred using Sketchpad instead of manipulatives. They acknowledged that Sketchpad and manipulatives could accomplish the same learning goals; however, there were some issues associated with the use of manipulatives, e.g., lack of availability of manipulatives. Also, Brian mentioned in the initial interview that the use of manipulatives "wastes just as much time as learning the program," so going to a computer lab was a much better option given that he had easy access.

Based on their experience from the first year of the DG implementation, Brian and James decided that the topics that did not work well with Sketchpad during the previous year would be covered in class first this year. James and Laura both noticed that,

when teaching with Sketchpad, they were able to see how little their students know about geometry because when students were exploring concepts on their own (in Sketchpad), it was easier to see what they knew and did not know. Students' learning and understanding were less visible during class lectures when students sat in their desks and took notes.

All teachers displayed good knowledge of mathematical content taught in the observed lessons. They provided activity sheets to their students for the concepts being explored with Sketchpad. Brian made his own handouts, while James, Susan and Laura used pre-made handouts from the Sketchpad Lesson Link website or other Sketchpad-related resources available to them.

The class instruction was similar among the four teachers, but it also varied in some ways. All teachers monitored their students during Sketchpad explorations and answered their questions. James provided the least instruction at the beginning of each lesson. During the lesson, if he saw that students were struggling with something, he jumped in and explained to the whole class. Brian and Susan introduced each lesson briefly and then let their students explore. James, Brian and Susan, did not summarize any lessons with the whole class (except one time for Brian). Laura, on the other hand, provided a thorough introduction to and summary for each lesson; in between, her students got to explore with Sketchpad for about half of the class time. Laura believes that without the whole-class summary students never arrive at a conclusion and "it never closes."

Teaching-with-Sketchpad Task

Exploration. Part of the closing interview involved reflecting on a teaching-learning episode that took place during the pilot study:

Ms. Johnson's geometry class is covering the unit on transformations. Yesterday they discussed translations and today they are discussing reflections. Students are using Sketchpad for an exploration. They are asked to create a pentagon in one of the quadrants and reflect it over the x-axis and then over the y-axis. The picture below (Figure 21) is a screenshot of one of Ms. Johnson's students, Ellen.

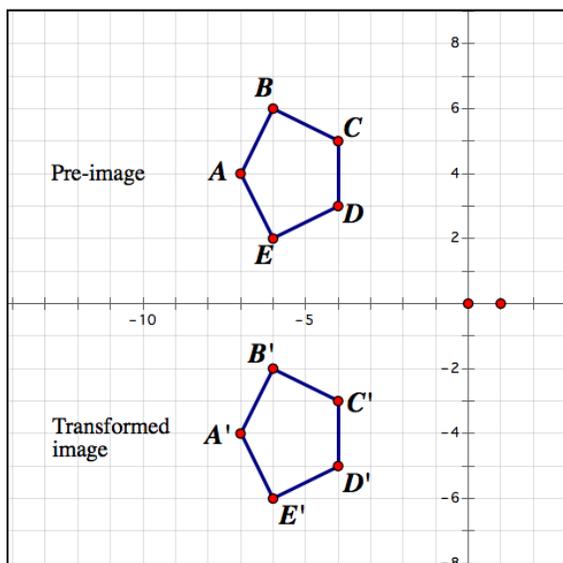


Figure 21. Transformation from the teaching-with-Sketchpad task.

What type of transformation do you think it is? Brian, Susan and Laura concluded that it was a translation. James's first response was that the presented transformation was a reflection over the x-axis, but the labels did not correspond. He kept

looking at the two figures seeing a reflection, but after a few moments he concluded that it was a translation because of how the vertices were labeled. This was in fact a reflection over the x-axis; however, just like James noticed, the labels of the vertices were not corresponding, making it look like a translation. The only possible explanation for this mismatch was that Ellen changed the labels of the reflected figure to those illustrated in Figure 21 because Sketchpad labels vertices of transformed figures correctly.

If the student reflected the pre-image and then changed the labels of the reflected image to those shown in the figure, what would you do to help this student?

Brian would change the shape of the pre-image so that it was not “symmetrical.” James would construct a more “abstract” shape, maybe a triangle that “does not look like it can be translated.” Therefore, Brian and James had similar ideas and would use the dynamic features of the software to “correct” this situation. Laura’s response was along the same lines; she would change the pre-image figure. Susan, on the other hand, did not mention using Sketchpad at all. She would try to explain to Ellen that the corresponding vertices had to be labeled appropriately, e.g., B’ would have to be E’, C’ would have to be D’.

When I asked her to explain more, she mentioned folding the paper along the x-axis to make it more visible to Ellen which vertices were corresponding to which vertices. When reminded that Ellen was using Sketchpad for this activity, Susan repeated that she would explain to Ellen that the corresponding vertices had to match. Susan’s response and the lack of Sketchpad use was intriguing because during the initial interview she mentioned the topic of geometric transformations as one that lend itself to Sketchpad use as a learning tool—“I like doing rotations and reflections [in Sketchpad] because those are hard concepts. It is harder than I thought for some students.” The lack of Sketchpad

incorporation in correcting Ellen's misconception was an unexpected finding since Susan had many years of experience in Sketchpad integration and she mentioned that she could not teach without it. It was surprising that she did not incorporate it in this task. At the same time, it showed that she still had room for developing her TPACK and the experience of technology use in the classroom instruction did not necessarily translate into increased TPACK suggesting that a sustained professional development is necessary.

Conjecture. The second part of the task involved reflecting over Ellen's conjecture about reflections over the x-axis and the y-axis:

Later in the exploration, students were asked to find coordinates of all the points (see Figure 22) and write a conjecture about the relationship between coordinates of the pre-image and coordinates of the image reflected over the x-axis as well as about the relationship between coordinates of the pre-image and coordinates of the image reflected over the y-axis. Ellen wrote the following conjecture, "When I move one of the corners of the pre-image, the corresponding corner of the image reflected over the x-axis also moves. The same thing happens with the image reflected over the y-axis. The coordinates of the reflected images change from positive to negative and vice versa."

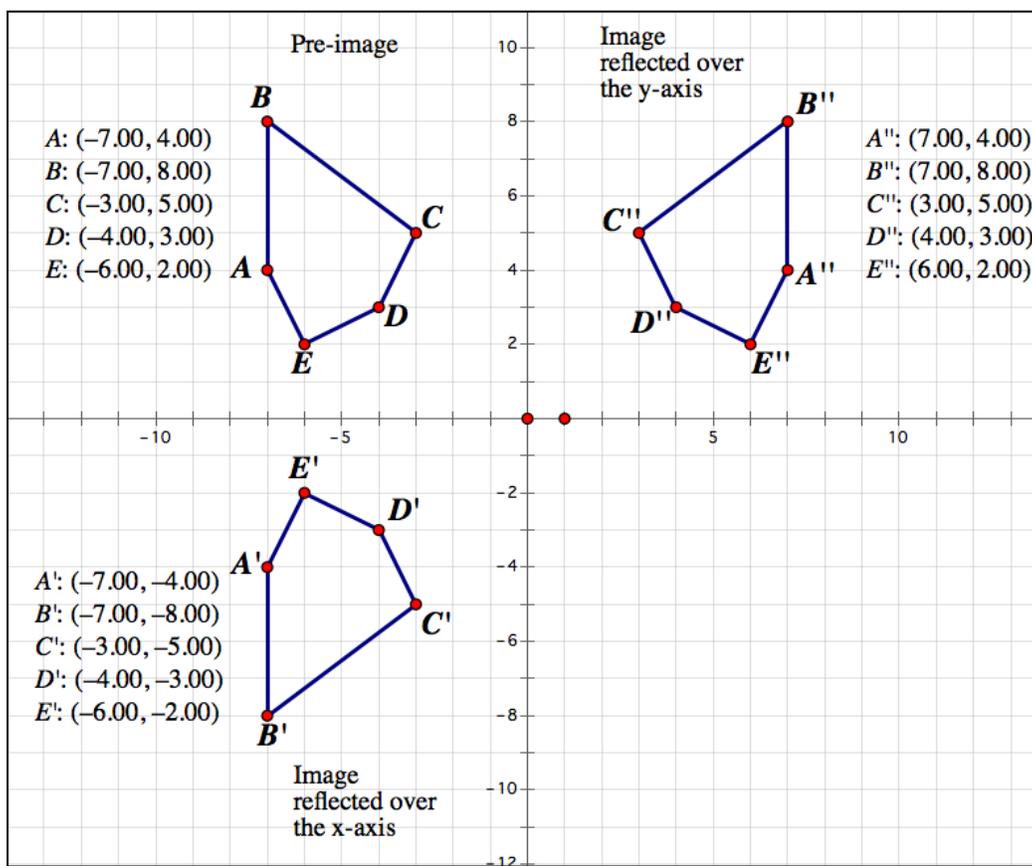


Figure 22. Sample screenshot illustrating coordinates of the pre-image and the reflected images.

What would you do to help Ellen improve her conjecture? How would you use sketchpad to help you accomplish this goal? All teachers understood what Ellen was trying to say and agreed that the first part of the conjecture was right; the only thing they would change was the last sentence—“The coordinates of the reflected images change from positive to negative and vice versa.” They thought that it was not clear enough. Brian said, “I think the student probably knows what’s happening but is having trouble putting it into words when saying that the coordinates change from positive to negative.” He would tell the student to look at the coordinates and ask, “Is everything changing?”

What is changing? And which axis?" This showed that when teachers know their students, they understand their train of thought and are good translators of students' ideas/expressions.

James's response was similar to Brian's in a sense that he could see what the student was trying to say, "I think I know where they're going with it," and he would continue to talk with the student so that she could better articulate the last sentence of the conjecture and be more specific. He would also spend a few minutes with the whole class to recap and make sure that they understood what happened when the image was reflected over the x-axis and the y-axis.

Susan's response was also similar to that of Brian's and James's and she would make sure that the student revised the last sentence and was more specific about what exactly happened. Laura's response was a little different and she said she would have her students log the coordinates and look at more instances because:

If a student is looking just at her image, it is hard for her to see the big picture of what is happening to the x and y very specifically. So it is more general. Maybe if it was tabled with several people's numbers, then it is easier to see more specific. This is what is happening: across the x-axis, x stays the same, and y changes; across the y-axis, y stays the same, and x changes. I think more data and other people's [data] would clarify that because they have much more confidence in other people's data than their own, too.

Even though Ellen already had five points with coordinates, Laura thought it would be better to have even more examples; however, she did not mention using Sketchpad in any

way to achieve that. By using Sketchpad, students can drag any of the vertices to obtain “more points” or they could construct a data table (see Figure 23 for example).

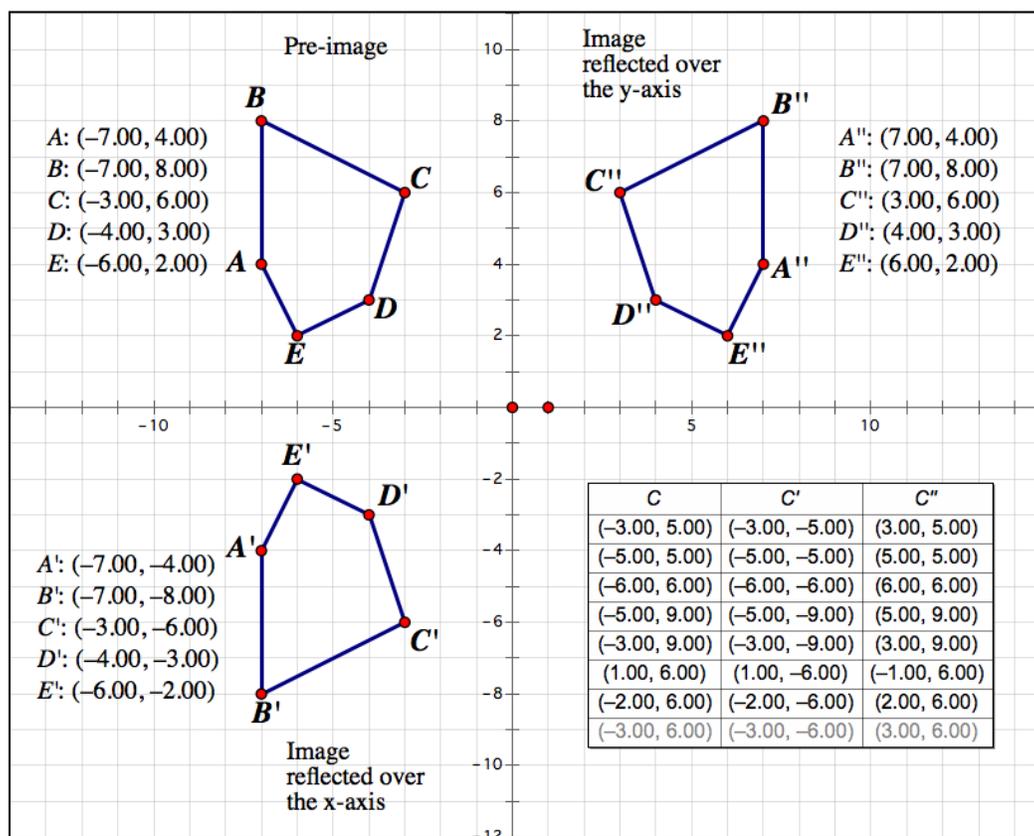


Figure 23. Screenshot of a sample data table constructed in Sketchpad for the coordinates of point C and its reflected images over the x-axis and the y-axis, C' and C'' respectively, after dragging point C.

TPACK Development Levels

Combined TPACK development levels were created after identifying individual TPACK development levels for all participants (see Figure 24 for a tabular representation and Figure 25 for a graphical representation).

Theme (descriptor)	TPACK Development Levels				
	Recognizing	Accepting	Adapting	Exploring	Advancing
Curriculum & Assessment (curriculum)	1		2	2	
Curriculum & Assessment (assessment)					
Learning (mathematics learning)			2	1	1
Learning (conception of student thinking)	1		2	2	
Teaching (mathematics learning)			1	4	
Teaching (instruction)				4	
Teaching (environment)			2	2	1
Teaching (professional development)			3	1	
Access (usage)			4		
Access (barrier)			3	1	
Access (availability)	1	2	3		
Level totals	3	2	22	17	2

Figure 24. Combined TPACK development levels for all participants from all non-survey data sources.

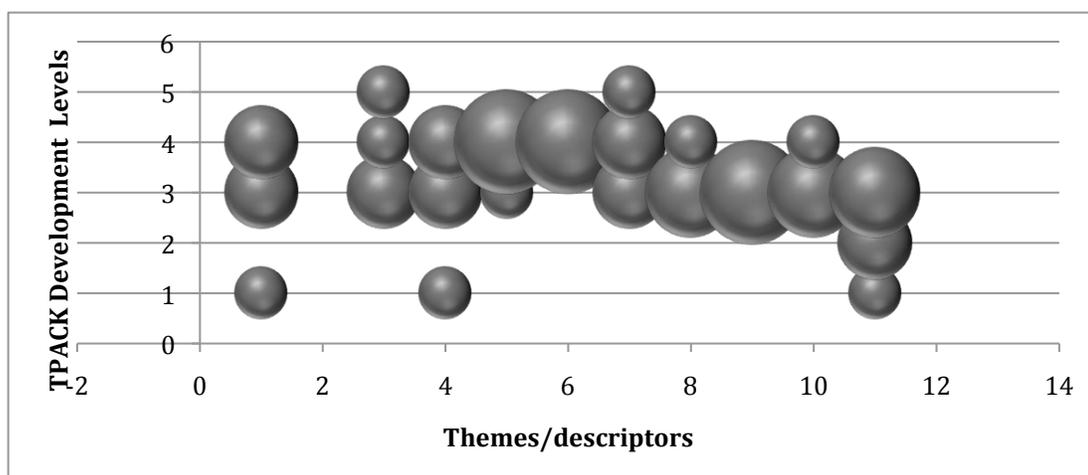


Figure 25. Combined TPACK development levels for all participants. Themes (descriptors) are: 1. Curriculum & Assessment (curriculum), 2. Curriculum & Assessment (assessment), 3. Learning (mathematics learning), 4. Learning (conception of student thinking), 5. Teaching (mathematics learning), 6. Teaching (instruction), 7. Teaching (environment), 8. Teaching (professional development), 9. Access (usage), 10. Access (barrier), 11. Access (availability).

It is clear that the combined TPACK concentrated around the adapting and exploring levels, which aligned with the TPACK development levels identified for each participant in Chapter IV—Brian (exploring), James (adapting), Susan (exploring for the teaching theme) and Laura (adapting). Although there is room for growth in knowledge, these figures are promising, considering this was only the second year of the DG implementation for most of the participants.

As mentioned earlier, none of the participants used Sketchpad for assessment, although they might have had beliefs about how they would incorporate Sketchpad in assessment. This finding offered an opportunity that can be explored in future studies.

Combined TPACK for the learning theme was identified at all levels except accepting. The TPACK development levels for the teaching theme were the most consistent and the highest—adapting (six indicators), exploring (eleven indicators) and advancing (one indicator). Combined TPACK for the access theme was also consistent for the usage and barrier descriptors— adapting (seven indicators) and exploring (one indicator).

Summary

This chapter provided cross-case analysis, common themes and differences among the four cases. It also identified several unexpected findings. Teachers can develop their TPACK at the same time they develop their PCK, so that even novice teacher can have high TPACK development levels. Easy access to technology does not guarantee technology integration in the classroom instruction. Also, prolonged incorporation of technology in teaching does not guarantee high TPACK development levels.

The next chapter, Chapter VI, presents discussion based on the findings of this study. Chapter VI also discusses tensions related to teaching with dynamic geometry software, implications for technology-enhanced professional development and recommendations for future research.

CHAPTER VI

DISCUSSION

This was a qualitative case study investigating high school teachers' technological pedagogical content knowledge (TPACK). Participants of this study were four high school geometry teachers who participated in the Dynamic Geometry Project (DGP) and were incorporating Sketchpad, a dynamic geometry software program, into their instruction. Three research questions guided this research study:

1. How do high school teachers develop TPACK while teaching geometry using dynamic geometry software?
2. How do high school teachers enact their TPACK when teaching with dynamic geometry software?
3. How are the five TPACK development levels (i.e., recognizing, accepting, adapting, exploring, and advancing) characterized for high school teachers who incorporate dynamic geometry software in teaching?

To answer these questions, I gathered data from multiple sources: initial interview, observations, documents, closing interview, a survey, implementation questionnaires, professional development attendance records and the researcher's log. I presented the findings in the individual case reports in Chapter IV and cross-case analysis in Chapter V. In this chapter, I address the following topics: discussion of findings, tensions related

to curriculum and standardized testing, implications for technology-enhanced professional development and recommendations for future research.

Discussion of Findings

TPACK Development

All participants displayed strong knowledge of Sketchpad during the interviews and observations. They participated in all or almost all professional development sessions provided by the DGP. Susan was the only participant who used Sketchpad in teaching before the DGP; therefore, she had additional training and experience in using Sketchpad. Laura also used Sketchpad as a college student, but she did not use it in teaching before the DGP because she preferred doing hands-on activities that did not involve technology with her students. All participants valued the DGP professional development sessions; not only did they learn how to use Sketchpad, they also learned and shared implementation strategies with their colleagues. That collaboration was valuable to them and assisted them in incorporating Sketchpad in their classroom instruction. For Susan, the collaboration was especially valuable because she was the only geometry teacher at her school; meeting with other geometry teachers from local schools on a regular basis helped her learn how to better integrate Sketchpad into her geometry lessons. Even though she used Sketchpad for six years before the DGP, and she knew how to use it, she improved as a teacher as she mentioned in the initial interview—she was “able to point out to students more of what they were supposed to see.” She mentioned several times that her students saw “magic” while using Sketchpad and did not understand where those things that they were exploring came from. Based on her comments, her TPACK had increased and she can teach more effectively with Sketchpad now. However, as Susan’s

TPACK development levels and the closing interview suggested, her TPACK was not fully-developed. Even though she was the most experienced teacher in this study, overall, her TPACK development levels were inconsistent, except for the teaching theme where her TPACK development level was at the exploring level. Moreover, Brian, James and Laura did not use Sketchpad in teaching before the DGP, and they developed their teaching strategies with Sketchpad through the modeled lessons from the DGP professional development sessions. That was clearly visible in the observed lessons when teachers were guiding students in their explorations and creating learner-centered environments. All the cases implied that a sustained and collaborative professional development was necessary in order for teachers to develop and improve their TPACK. Such professional development also should be connected to practice directly as all the cases in this study indicated that their TPACK development occurred through attending the DGP professional development sessions and through their teaching practice. This dissertation offers a model for TPACK professional development that is based on the findings of this study. It is designed to be ongoing and connected to practice. I discuss it in detail in the section titled “The Total PACKage Professional Development Model (TPACK PD Model) for Teaching Geometry with Dynamic Geometry Software.”

TPACK Enactment

One of the main themes that emerged from analyzing the data was the surprisingly low usage of Sketchpad by participants’ students (see Appendix H). On average, Brian’s and Laura’s students used Sketchpad once every other week while having easy access to computers; James’s students used Sketchpad with the same frequency while meeting many challenges with computer lab access. Based on the

interviews with participants and other conversations, curriculum and standardized testing played a key role in deciding whether they took their students to a computer lab or not. This “barrier” to technology integration is discussed in more detail in the forthcoming section titled “Curriculum and Standardized Testing.”

Susan used Sketchpad primarily for demonstrations even though her students had opportunities to do Sketchpad explorations every week. This suggested that she preferred to use it in a more teacher-directed instruction than in a student-centered instruction. This finding also agrees with findings from Hannafin et al. (2001); teaching style is difficult to change and it might take a long time for a teacher to adjust to a different approach to teaching and learning.

TPACK Development Levels

Since there are eleven descriptors across the four themes of the TPACK Development Model (Niess et al., 2009), participants had different TPACK development levels for the different descriptors. TPACK development level for the assessment descriptor was not identified for any of the participants. The overall TPACK development levels for the participants were: exploring for Brian, adapting for James and Laura, and exploring for Susan for the teaching theme only. Taken as a whole, the combined TPACK was at the adapting and exploring levels, which was highly promising. At the same time, the findings showed that the participants needed to further develop their TPACK through an ongoing professional development.

For Brian, James and Laura, the TPACK development levels aligned closely with their self-perceived TPACK development levels for most descriptors. For Susan, most of the TPACK development levels were below her self-perceived TPACK development

levels. One explanation for this could be that Susan felt more comfortable and confident with Sketchpad since she was using it for a long time, and that possibly resulted in her higher perceptions about her TPACK. This confirms findings from other studies, which concluded that gains measured by self-report surveys reflect an increase in confidence instead of an increase in knowledge (Lawless & Pellegrino, 2007; Schrader & Lawless, 2004). The rest of the participants, who were using Sketchpad in teaching only for the second year, felt like they still had a lot to learn and expressed lower TPACK development levels than Susan did in the TPACK survey.

Curriculum and Standardized Testing

Based on the interviews and implementation questionnaire responses, two tensions surfaced—one of them was related to the curriculum and the other one to standardized testing. Although all participants were open to using Sketchpad with their students and had no problems with identifying topics in the curriculum where Sketchpad could be used, they did not let their students use Sketchpad often. The main reasons for that were the extensive curriculum and the standardized testing accountability. In Texas, most students take a geometry course in 10th grade, but at the end of the school year they take a test that largely consists of algebra. In addition the geometry objectives tested in 10th grade consist of middle school geometry content; in 11th grade, the Exit Level assessment consists of high school level geometry content (see Table 7). The teachers in this study needed to prepare their students to do well on this test and had to follow the scope and sequence of curriculum that consisted of algebra in addition to geometry. That made it more challenging for integrating Sketchpad and having students explore and “discover” geometric concepts because these activities took more time. At the same time,

the teachers wanted their students to be successful and they wanted to ensure they covered all topics in the curriculum (including algebra) so that their students were ready to do well on the test.

Table 7

Texas Assessment of Knowledge and Skills (TAKS) Blueprint for Grade 10 and Exit Level

Mathematics

TAKS Objectives	Number of Items: Grade 10	Number of Items: Exit Level
1: Functional Relationships	5	5
2: Properties and Attributes of Functions	5	5
3: Linear Functions	5	5
4: Linear Equations and Inequalities	5	5
5: Quadratic and Other Nonlinear Functions	5	5
6: Geometric Relationships and Spatial Reasoning	5	7
7: 2-D and 3-D Representations	5	7
8: Measurement	7	7
9: Percents, Proportions, Probability, and Statistics	5	5
10: Mathematical Processes and Tools	9	9
Total number of items	56	60

On the other hand, the standardized testing in Texas is currently changing, which can mean fantastic news for geometry teachers and students. The new testing system includes end-of-course exams instead of grade-level exams and students will take a geometry exam at the end of the school year in which they take the geometry class. The spring 2012 is the first time for the new tests; at the high school level, only freshmen will take them in 2012. The new testing system brings changes into how geometry is taught and focuses fully on the geometry content (see Table 8).

Table 8

State of Texas Assessments of Academic Readiness (STAAR): Geometry Blueprint

Category	Number of Standards	Number of Items
1: Geometric Structure	9	10
2: Geometric Patterns and Representations	5	8
3: Dimensionality and the Geometry of Location	6	10
4: Congruence and the Geometry of Size	12	16
5: Similarity and the Geometry of Shape	4	8
Total	36	52

As the curriculum changes, teachers need to adapt to these changes and possibly adjust their teaching practices. The new test in Texas and related curriculum might create more favorable conditions for integrating dynamic geometry programs into instruction because of the full focus on the geometry content (see Table 9). Out of the four participants in this study, Brian was the only teacher who was teaching freshmen this year, and so he was the only one implementing new curriculum. Brian exhibited the highest TPACK development levels out of the four participants, which partially supports the hypothesis that the new testing system and curriculum focusing solely on geometry content facilitate more effective technology integration. At the same time, however, adjusting to the new curriculum requires time and, just as in Brian's case, adjusting to the new curriculum can take away time from planning and incorporating technology into instruction.

Table 9

High School Level Geometry Content in Texas Assessments

Assessment	Number of High School level Geometry items	Percentage of High School level Geometry Items
Grade 10 TAKS Exit Level	0	0
TAKS	21	35
STAAR	52	100

Because teachers have little or no influence on the curriculum or standardized testing, it is difficult for them to overcome this barrier to technology integration. With the current state of affairs, they can still integrate dynamic geometry into instruction; however, it might take a significant amount of time for them to readjust to the new curriculum and figure out how dynamic geometry can be integrated in it. Nonetheless, continued professional development can ease some of the readjustment challenges.

Implications for Technology-Enhanced Professional Development

Three participants in this study indicated that the main source of their Sketchpad knowledge development were the professional development workshops offered by the DGP. Since Susan had taught with Sketchpad before the DGP, she was already familiar with the software; however, she had an opportunity to learn about the new features of the software as well as how to integrate it into instruction better. Additionally, Susan's TPACK development levels indicated that, even with many years of incorporating Sketchpad into the classroom instruction, she had not developed her TPACK completely.

The second main source of TPACK development was practice, i.e., planning Sketchpad activities and teaching with Sketchpad. This source of knowledge

development suggested how TPACK development could be structured. In addition, since the combined TPACK development levels were around adapting and exploring, the participants could still improve their knowledge for teaching with Sketchpad; therefore more professional learning opportunities for in-service teachers are needed. The following sections, TPACK-in-Practice and (Virtual) Lesson Study, briefly review two contemporary efforts for enhanced professional development.

TPACK-in-Practice

The TPACK-in-Practice Framework (Figg & Jaipal, 2012) offers a professional learning opportunity for teachers wanting to integrate technology in their instruction. This professional development model is closely related to content taught and does not focus merely on learning the technology. The model consists of four stages “(a) modeling a tech-enhanced activity type (learning WITH the tool), (b) integrating ‘pedagogical dialog’ in a modeled lesson, (c) developing TK (in context) through tool demonstrations, and (d) applying TPACK-in-Practice to design an authentic learning task” (Figg & Jaipal, 2012, p. 4685).

(Virtual) Lesson Study

Lesson study (LS) is a form of professional learning that is popular in Japan and other Asian countries; it is also gaining recognition in the United States and so far is mainly used for research purposes. The major benefits of LS are collaboration, teacher knowledge improvement and instructional improvement (Lewis, Perry, & Hurd, 2009). The features of LS include investigation, planning, research lesson and reflection (Lewis et al., 2009). Yursa and Silverman (2012) proposed a virtual model for LS, which creates an increased access to the LS community for teachers in rural and urban districts. Since

the virtual LS is facilitated online, it allows for participation from remote locations and becomes more accessible.

The Total PACKage Professional Development Model (TPACK PD Model) for Teaching Geometry with Dynamic Geometry Software

Based on the findings of this study and the professional development provided by the DGP, and drawing from the two models of professional development, TPACK-in-Practice and (Virtual) Lesson Study, I propose a new type of professional development specifically designed for teachers interested in integrating dynamic geometry into their instruction. This new model is called the Total PACKage Professional Development Model or TPACK PD Model and consists of seven stages: technological knowledge (TK) development, technological content knowledge (TCK) development, technological pedagogical knowledge (TPK) development, developing a technology-enhanced lesson, teaching, observing and reflecting. As Figure 26 illustrates, TPACK PD is a cyclical process; simply going through all stages once does not guarantee total TPACK development. Instead, TPACK PD participants continue learning and developing their knowledge by visiting all of the stages multiple times. This kind of professional development can facilitate teacher change because it uses additional sessions and collective participation (Penuel, Fishman, Yamaguchi, & Gallagher, 2007). By creating a professional learning community and devoting more time to professional development, teachers are more likely to integrate new knowledge into practice (Brown, 2004; Penuel et al., 2007).

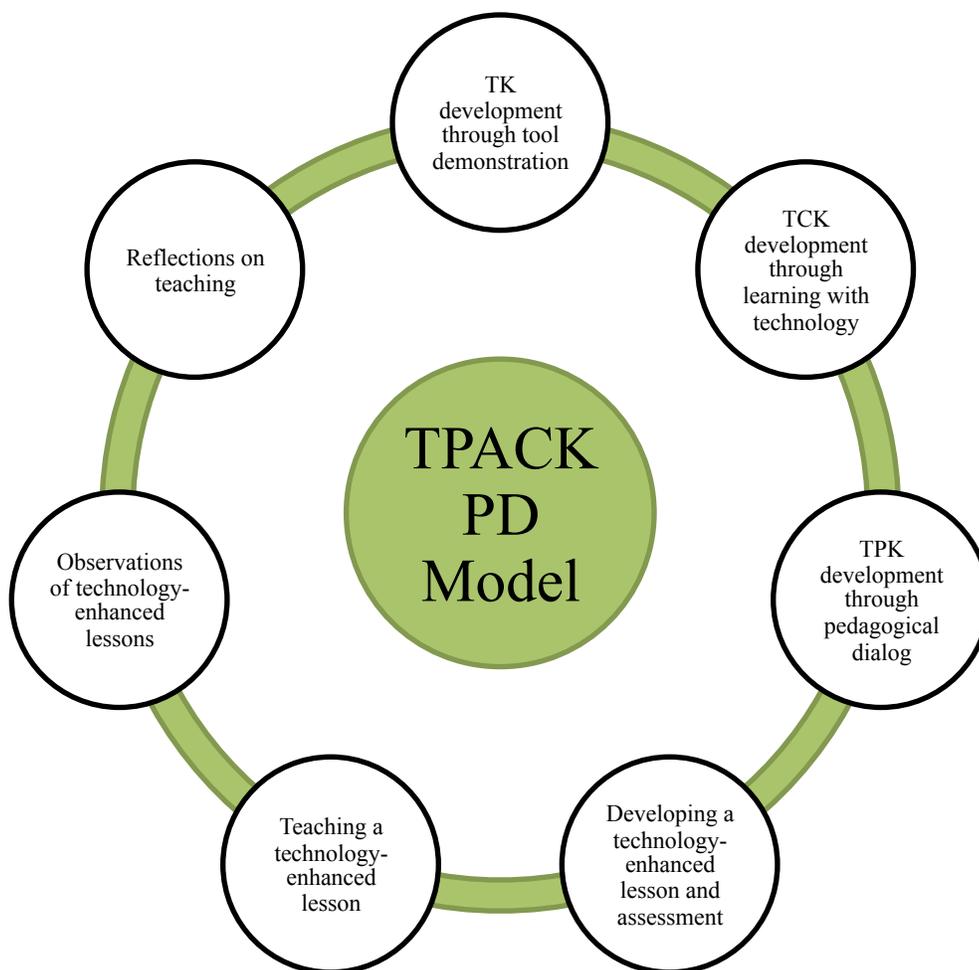


Figure 26. TPACK PD model.

The first four stages of the TPACK PD Model are adapted from the TPACK-in-Practice framework; however, they have been rearranged to better suit the purpose of developing TPACK for teaching with dynamic geometry software. In the TPACK-in-Practice model, TK development is the third stage; in the TPACK PD model, it is the first stage. The fourth stage from the TPACK-in-Practice model is also part of the LS model. The remaining three stages are adapted from the (Virtual) LS model. The following are descriptions of all the stages in the TPACK PD Model:

- TK development through tool demonstration. Participants learn how to use the software in context. Since dynamic geometry software is very user friendly, participants can quickly learn the basics. Additionally, since the software is specifically designed for being used for teaching and learning of geometry, participants can quickly move to the next stage of knowledge development, TCK development. As mentioned earlier, TPACK development is cyclical, therefore, participants will be coming back to this stage to learn more about the software at later times.
- TCK Development through learning with technology. The next step is to introduce workshop participants to the tool (Sketchpad) by learning with the tool. This is accomplished by a technology-enhanced model lesson where participants are learners and a workshop designer facilitates the lesson. “The experience provides participants with context for how the tool is useful in instruction” (Figg & Jaipal, 2012, p. 4685).
- TPK development through pedagogical dialog. Following the model lesson, participants engage in discussion about the lesson, which allows them to learn what decisions are involved in “designing and implementing technology-enhanced activities” (Figg & Jaipal, 2012, p. 4685). This stage is especially valuable for novice teachers who are still developing their pedagogical content knowledge.
- Developing a technology-enhanced lesson and assessment. Participants develop a lesson or an activity that incorporates Sketchpad. They will later teach and

videotape this lesson. Participants also think of how Sketchpad can be used for assessment of topics taught in a given lesson.

- Teaching a technology-enhanced lesson. Each participant teaches the designed lesson with Sketchpad and videotapes it. Later, videotaped lessons will be shared with other teachers for watching and feedback.
- Observing technology-enhanced lessons. All participants watch their own, videotaped lessons as well as videotaped lessons of their colleagues. These lessons can be shared through an online management system, to which all participants have access. This virtual component of LS allows participants to watch the research lessons at their own time; it also allows them to observe a larger number of lessons.
- Reflecting on the observed lesson. Participants discuss the observed lesson, “draw out implications for lesson redesign, for teaching-learning more broadly, and for understanding of students and subject matter” (Lewis et al., 2009). Participants revise the lesson so that the dynamic geometry program plays a more integral part in learning. In the virtual LS, this can be accomplished through asynchronous online discussion boards, synchronous chat rooms or video-conferencing.

Summary

Many experienced teachers did not have a chance to experience technologies such as Sketchpad in their teacher preparation. Also, many new teachers did not go through a traditional teacher preparation education; instead, they received their teacher certification through an alternative certification program. That usually means that they were not required to take a geometry course in college. And if they did not take such a course in

college then they were not likely to know about Sketchpad. Therefore, providing high-quality in-service professional development that focuses not only on TK development, but also on total TPACK development is imperative. As the findings of this study suggested, experienced teachers needed an ongoing and collaborative professional development as well. To facilitate a change in teacher practices, professional development should take place at the school or district levels (Penuel et al., 2007). Also at the school level, teachers can form groups teaching the same subject with technology so they can collaborate, share ideas, and support each other throughout the process of learning a new technology and integrating it into practice.

Recommendations for Future Research

This study fills in the gap in literature on TPACK and teaching geometry with dynamic geometry software by providing four case reports on how geometry teachers develop and use their TPACK. Researchers should look to see how teachers' TPACK affects student learning in a dynamic geometry environment. Evaluating students' learning, in addition to teacher's TPACK, will help in detecting to what extent teachers' TPACK impacts students' learning.

This study was a first step towards gaining more resources for future studies that wish to explore teachers' TPACK quantitatively, especially with respect to dynamic geometry. However, more studies in this area are needed in order to start developing items that measure TPACK related to teaching with dynamic geometry software.

As a result of this study, I proposed a new TPACK PD Model. Future studies can put it into practice and evaluate its effectiveness. Additionally, because participants of this study were part of a grant project, they received external motivation and support

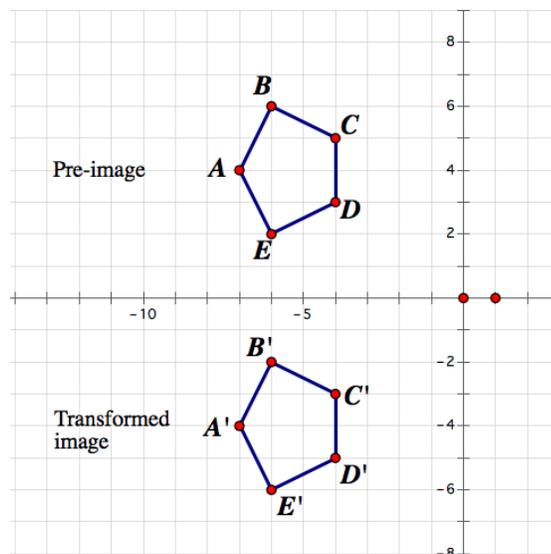
(e.g., professional development, monetary stipends) in integrating dynamic geometry in the classroom instruction. Since most teachers do not participate in such projects and do not receive that extra support, it would be worthy to investigate TPACK of such teachers by employing the TPACK Development Model (Niess et al., 2009).

APPENDIX A

TEACHING-WITH-SKETCHPAD TASK

Ms. Johnson's geometry class is covering the unit on transformations. Yesterday they discussed translations and today they are discussing reflections. Students are using Sketchpad for an exploration. They are asked to create a pentagon in one of the quadrants and reflect it over the x-axis and then over the y-axis. The picture below is a screenshot of one of Ms. Johnson's students, Ellen.

Do you think that Ellen might have any misconceptions when doing this exploration or that she might form some misconceptions based on what you see on her screen?



Later in the exploration, students were asked to find coordinates of all the points and write a conjecture about the relationship of coordinates of the pre-image and coordinates of the image reflected over the x-axis as well as about the relationship of coordinates of the pre-image and coordinates of the image reflected over the y-axis. One student wrote the following conjecture, "When I move one of the corners of the pre-image, the corresponding corner of the image reflected over the x-axis also moves. The same thing happens with the image reflected over the y-axis. The coordinates of the reflected images change from positive to negative and vice versa."

**What would you do to help this student improve her conjecture?
How would you use Sketchpad to help you accomplish this goal?**

APPENDIX B

INVITATION LETTER TO POTENTIAL PARTICIPANTS

I would like to invite you to participate in my dissertation study (which is part of the Dynamic Geometry Project). You are a perfect candidate because of the high level of implementation of dynamic geometry in your teaching in the past year. Below are some details about my study so you can decide if you'd like to participate.

Background Information:

The purpose of this study is to describe how high school teachers acquire knowledge of geometry, pedagogy, and technology, and how this knowledge affects their use of dynamic geometry software in teaching geometry.

Procedures:

If you agree to be in this study, you will be asked to:

- Participate in two audiotaped interviews (approximately 50 minutes in length each). The first interview will be conducted at the beginning of the school year and the second interview will be conducted after all observations.
 - Allow me to observe (and videotape if possible) three of your geometry classes while you teach with Sketchpad.
 - Provide me with lesson plans and/or handouts that accompany the observed lessons.
- All data will be collected August through December this year. We can schedule the interviews and observations at times most convenient for you. My schedule is flexible.

Voluntary Nature of the Study:

Your participation in this study is voluntary. This means that everyone will respect your decision of whether or not you want to be in the study and if you decide to join the study, you are free to withdraw at any time.

Risks and Benefits of Being in the Study:

There are no known risks and/or discomforts associated with this study. The expected benefit associated with your participation is that you will gain a deeper insight of your teaching practice with Sketchpad and of how to teach geometry with Sketchpad more effectively.

Please let me know if you have any questions. My phone number is 000-000-0000 if you prefer to call. I appreciate you taking the time to read this and considering participating in my study. I hope to hear from you soon.

APPENDIX C

CONSENT FORM

Teaching geometry with dynamic geometry software

You are invited to take part in a case study that will describe how teachers learn about dynamic geometry for teaching and how they use it in the classroom. You were chosen for the study because you use the Geometer's Sketchpad in teaching geometry and you showed a high level of implementation in the past year. The following information is provided for you to decide whether or not you wish to participate in this study.

Background Information

The purpose of this study is to describe how high school teachers acquire knowledge of geometry, pedagogy, and technology, and how this knowledge affects their use of dynamic geometry software in teaching geometry.

Procedures

If you agree to be in this study, you will be asked to:

- Participate in two audiotaped interviews (approximately 60 minutes in length each).
- Allow me to observe and videotape 3-5 of your geometry classes while you teach with Sketchpad.
- Provide me with lesson plans and/or handouts that accompany the observed lessons.

All data will be collected September through December this year.

Voluntary Nature of the Study

Your participation in this study is voluntary. This means that everyone will respect your decision of whether or not you want to be in the study and if you decide to join the study, you are free to withdraw at any time.

Risks and Benefits of Being in the Study

There are no known risks and/or discomforts associated with this study. The expected benefit associated with your participation is that you will gain a deeper insight of your teaching practice with Sketchpad and of how to teach geometry with Sketchpad more effectively.

Compensation

There will be no compensation for participating in this study.

Confidentiality

Any information you provide will be confidential. Your name will not be associated with the research findings in any way. I would be happy to share my findings with you after the research is completed.

Contacts and Questions

My name is Ewelina McBroom and my faculty advisor is Dr. Zhonghong Jiang. You may ask any questions you have about this study by contacting me via phone at 000-000-0000, via email as XXXX@txstate.edu, or Dr. Jiang via email at XXXX@txstate.edu. Texas State University's approval number for this study is EXP2011R4108.

Please sign your consent with full knowledge of the nature and purpose of the procedures. A copy of this consent form will be given to you to keep.

Name of Participant

Signature of Participant

Date

APPENDIX D

INITIAL INTERVIEW PROTOCOL

The purpose of this interview is to develop an understanding of how you have developed your knowledge for teaching geometry with the Geometer's Sketchpad.

Background questions:

How long have you been teaching?

How long have you been teaching geometry?

How long have you been teaching with Sketchpad?

Have you ever used another dynamic geometry software program either in learning or teaching? If so, for how long?

Have you assumed additional roles/positions (e.g., department chair, math specialist, etc.) during your teaching career? If so, briefly describe your responsibilities.

Main interview questions:

1. When you hear the words “dynamic geometry” what comes to your mind?
2. How and when did you first hear about Sketchpad? Briefly describe this experience.
3. Why did you decide to teach geometry with Sketchpad?
4. What is your current view and understanding about integrating Sketchpad as a learning tool in geometry?
 - a. How do you define “integration” of Sketchpad in teaching and learning of geometry?
 - b. Describe a lesson you taught that represents your current view.
5. What specific geometry topics lend themselves to Sketchpad as a learning tool?
6. What do you see as barriers for integrating Sketchpad in teaching and learning of geometry?
7. How did your knowledge and skills with Sketchpad changed through your work in the DGP?
8. How has your knowledge about students’ understanding, thinking, and learning about geometry topics with Sketchpad changed through your work in the DGP?
9. How has your conception of incorporating Sketchpad in teaching specific topics in geometry changed through your work on the DGP project? Give an example from a lesson you taught last year.

10. When you start using Sketchpad with your students (at the beginning of a school year), what strategies do you use to guide them in learning about the software? (Learning about Sketchpad as students are learning about a specific geometry topic or focusing on learning about the technology and later as the geometry context?)
11. In addition to your participation in the DGP, what kind of activities (e.g., professional development, conferences, self-directed study, Internet resources) have you engaged in that lead you to adopt teaching and learning of geometry with Sketchpad? Briefly describe what influenced your choice.
12. How important is (to you) the integration of Sketchpad into teaching and learning of geometry?
13. Do you evaluate the results of your decision to integrate teaching and learning of geometry with Sketchpad? How do you do it?

APPENDIX E

OBSERVATION PROTOCOL

Date: _____ Time: _____ School: _____ Teacher: _____	
Time	Descriptive Notes
	<p>The physical setting <i>How is space allocated? What objects, resources, technologies are in the setting?</i></p> <p>The participants <i>How many students are present? Are there any other people present?</i></p> <p>Activities and interactions <i>What is going on? How do students and the teacher interact with the lesson and with one another?</i></p> <p>Conversation What is the content of conversations? Who speaks to whom? Who listens?</p> <p>What is the role of technology in this lesson?</p> <p>What are the benefits of using technology in this lesson?</p> <p>Are there any apparent disadvantages of using technology in this lesson?</p> <p>How does teacher manage the classroom when teaching this lesson?</p> <p>How does the teacher assess student learning in this lesson?</p>

APPENDIX F

TPACK DEVELOPMENT MODEL SELF-REPORT SURVEY

From *Secondary Mathematics Teachers' Perceptions of Their Integration of Instructional Technologies* (Doctoral dissertation), by J. T. Ivy, 2011, pp. 127-131, and from *An Examination of Secondary Mathematics Teachers' TPACK Development Through Participation in a Technology-Based Lesson Study* (Doctoral dissertation), by J. W. Riales, 2011, pp. 230-234. Copyright [2011] by J. T. Ivy and J. W. Riales. Reprinted with permission.

TPACK Development Model Self-Report Survey Specific to _____ (technology)	
Please place a check in the box to the left of each statement that describes your beliefs and/or integration of technology in your classroom. You may give additional information in the spaces provided to clarify your selections or if none of the statements describe your beliefs/integration.	
<input type="checkbox"/>	1. I can see how this technology might be useful with some of the topics in my curriculum, but I am not convinced its use will make much of a difference for my students' learning.
<input type="checkbox"/>	2. I believe this technology would make a difference in my students' learning and would like to use this technology with my students, but I'm not really sure how to integrate its use with the topics in my curriculum.
<input type="checkbox"/>	3. I believe this technology is beneficial to students' learning. I have allowed my students to use this technology for investigation of a few topics.
<input type="checkbox"/>	4. I believe this technology facilitates students' learning. I have allowed my students to use this technology for investigation of several topics. I have changed some of my lessons to integrate the technology and am searching for more ways to integrate the technology into the curriculum.
<input type="checkbox"/>	5. I am convinced that this technology is essential to promote learning for my students. My students use this technology on a regular basis. I extend the objectives in my curriculum by allowing my students the opportunities to develop deeper mathematical thinking through the technology use.
Use this space for any additional information related to the statements above.	

	6. I don't like to allow my students to use this technology on tests because I want to know what they know about mathematics, not what the technology can do.
	7. I allow my students to use this technology only on certain parts of tests or only on certain tests.
	8. If I allow my students to use this technology on tests, I make sure that the test questions measure what my students understand (concepts) along with what they know how to do (procedures).
	9. I allow my students to use this technology on tests. I make my tests to involve a variety of questions (some that require the technology, some that they could use the technology but it is not required, and some in which the technology use has no impact).
	10. I design my assessments so that the students must demonstrate the understanding of the mathematics through the technology use.
Use this space for any additional information related to the statements above.	
	11. I believe that if my students use this technology too often, they will not learn the math for themselves.
	12. I am afraid that if I try to introduce a new topic with this technology, that my students will be too distracted by the technology use to really learn the mathematics. I want them to learn how to do it on paper first, and then they can use the technology.
	13. I have allowed my students to explore a few topics using this technology even before the topics are discussed in class.
	14. My students explore several topics for themselves using this technology to help them develop a deeper understanding. Sometimes the students' thinking guides their explorations in directions other than what I had planned.
	15. I design my own technology lessons. When I plan my lessons, I really think about how to integrate the technology to help the students better understand the mathematics. After the lesson, I reflect on the lesson and how it could be changed to increase student understanding using this and/or other technologies.
Use this space for any additional information related to the statements above.	

	16. I might show my students how this technology relates to the topic, and I don't mind if my students use this technology outside of class, but I do not plan to allow class time for the students to use this technology.
	17. If my students use the technology to explore a new topic, they won't think about and develop the mathematical skills for themselves.
	18. I try to use this technology to promote my students' thinking, but have not had a lot of success.
	19. I often use pre-made technology activities to engage my students in their learning. I reflect on my students' thinking, communication and ideas during the technology use to make decisions about any changes that need to be made in the design of the lesson.
	20. I cannot imagine my classes without this technology! Using this technology is a vital piece of facilitating my students' learning and helps promote their thinking to more advanced levels.
Use this space for any additional information related to the statements above.	
	21. This technology might be useful, but before I could use this technology, I would have to teach my students about the technology and how it works. I have too many objectives to cover to do that.
	22. I use this technology occasionally, such as between units or at the end of the term. The technology use doesn't necessarily tie with the mathematical goals of the class.
	23. I use this technology to reinforce concepts that I have taught earlier or that my students should have learned in a previous class. I do not use it regularly when teaching new topics.
	24. I use this technology as a learning tool to engage my students in high-level thinking activities (such as projects or problem-solving).
	25. I use this technology to present mathematical concepts and processes in ways that are understandable to my students. I actively accept and promote use of this technology for learning mathematics. Other teachers come to me as a resource for ideas of how to help their students use the technology to promote understanding.
Use this space for any additional information related to the statements above.	

	26. My students and I use this technology for procedural purposes only.
	27. I have led my students through a few simple ideas of how to use this technology that I learned during professional development.
	28. I have led my students through uses of this technology that I learned during professional development, but I changed the activities to meet the needs of my students.
	29. When my students explore with this technology, I serve as a guide. I do not direct their every action with the technology.
	30. On a regular basis, I use a wide variety of instructional methods with this technology. I present tasks for my students to engage in both deductive and inductive strategies with the technology to investigate and think about mathematics to deepen their understanding.
Use this space for any additional information related to the statements above.	
	31. In my class, the focus is on the mathematics first. I can imagine that perhaps this technology might be used to reinforce those mathematical ideas only after the students have shown they can perform the skills on paper.
	32. I allow my students to use this technology to assist them with their skills. I direct my students step-by-step to use this technology.
	33. I use some exploration activities with this technology, but I usually guide my students through the steps to save class time.
	34. I have explored a variety of instructional methods with this technology, to allow my students to engage both inductively and deductively.
	35. I use this technology in a student-led environment, where the students explore with the technology both individually and in groups. When working in groups, all members of the group are actively involved.
Use this space for any additional information related to the statements above.	
	36. I would consider attending a workshop demonstrating the use of this technology, but only if it is local.
	37. I am interested and would be likely to attend workshops or professional developments to learn more about how to use this technology to further mathematics education.
	38. I am likely to attend professional developments related to technology use in mathematics education and to share those ideas with other teachers in my building, but I am likely to focus on learning one type of technology integration at a time.
	39. I have made contact with others who are using this technology and plan to meet and work with them throughout the year to integrate this and other technologies appropriately into our mathematics curriculum.
	40. I believe it is time to transform our mathematics curriculum to one that utilizes 21 st century technologies! I have found organizations and workshops that I can attend to learn more about how to integrate this and other technologies into my math curriculum. I plan to share what I learn with others in my district.
Use this space for any additional information related to the statements above.	

	41. My students can use this technology only after they have mastered the pencil-and-paper skills.
	42. I allow my students to use this technology on a regular basis, usually just for skill purposes and under tightly controlled circumstances.
	43. I have a few units in which I allow students to explore new topics with this technology.
	44. I encourage my students to use this technology during most class meetings. They often explore new topics using this technology.
	45. I allow my students to use this technology in every aspect of the class and encourage the technology use to challenge the boundaries of what they can learn and understand.
Use this space for any additional information related to the statements above.	
	46. Mathematics has not changed just because we have more technologies available. Students still need to know how to do everything they've always been taught. For example
	47. It takes too much time and hassle to allow the use of this technology every day. I will let my students use it from time to time
	48. Using this technology will present some management issues
	49. I know that using this technology presents some new management issues
	50. Using this technology presented some issues
Use this space for any additional information related to the statements above.	
	51. I see the use of this technology tool for simplifying some "messy math" problems (problems with "unfriendly" real-life numbers for example). I make this technology available on the rare occasion that we encounter those type problems (maybe for extra credit).
	52. Using this technology allows me to demonstrate more examples.
	53. I take a different approach to teaching using this technology. Through its use, my students not only explore and apply key concepts using multiple representations, but they are also able to examine more complex mathematics topics making mathematical connections than they would be able to without the technology use.
	54. Using this technology allows my students access to explore and apply key concepts using multiple representations (such as symbols, graphs, tables, and/or data lists) and making important connections among representations and concepts.
	55. My students regularly explore and apply key concepts of more complex mathematical topics than normally outlined for this class using multiple representations and connections.
Use this space for any additional information related to the statements above.	

APPENDIX G

CODEBOOK FOR TPACK DEVELOPMENT LEVELS AND THEMES

The codes were created for each TPACK development level in each of the eleven categories formed by themes and descriptors. The code descriptions are based on the TPACK Development Model (Niess et al., 2009).

Theme (descriptor)	Recognizing	Accepting	Adapting	Exploring	Advancing
Curriculum & Assessment (curriculum)	C1c	C2c	C3c	C4c	C5c
Curriculum & Assessment (assessment)	C1a	C2a	C3a	C4a	C5a
Learning (mathematics learning)	L1m	L2m	L3m	L4m	L5m
Learning (conception of student thinking)	L1c	L2c	L3c	L4c	L5c
Teaching (mathematics learning)	T1m	T2m	T3m	T4m	T5m
Teaching (instruction)	T1i	T2i	T3i	T4i	T5i
Teaching (environment)	T1e	T2e	T3e	T4e	T5e
Teaching (professional development)	T1p	T2p	T3p	T4p	T5p
Access (usage)	A1u	A2u	A3u	A4u	A5u
Access (barrier)	A1b	A2b	A3b	A4b	A5b
Access (availability)	A1a	A2a	A3a	A4a	A5a

Code	Code description
C1c	Acknowledges that mathematical ideas displayed with the technologies can be useful for making sense of topics addressed in the curriculum.
C2c	Expresses desire but demonstrates difficulty in identifying topics in own curriculum for including technology as a tool for learning.
C3c	Understands some benefits of incorporating appropriate technologies as tools for teaching and learning the mathematics curriculum.
C4c	Investigates the use of topics in own curriculum for including technology as a tool for learning; seeks ideas and strategies for implementing technology in a more integral role for the development of the mathematics that students are learning.
C5c	Understands that sustained innovation in modifying own curriculum to efficiently and effectively incorporate technology as a teaching and learning tool is essential.
C1a	Resists idea of technology use in assessment indicating that technology interferes with determining students' understanding of mathematics.
C2a	Acknowledges that it might be appropriate to allow technology use as part of assessment but has a limited view of its use (i.e., use of technology on a section of an exam).
C3a	Understands that if technology is allowed during assessments that different questions/items must be posed (i.e., conceptual vs. procedural understandings).
C4a	Actively investigates use of different types of technology-based assessment items and questions (e.g., technology active, inactive, neutral or passive).
C5a	Reflects on and adapts assessment practices that examine students' conceptual understandings of the subject matter in ways that demand full use of technology.
L1m	Views mathematics as being learned in specific ways and that technology often gets in the way of learning.
L2m	Has concerns about students' attention being diverted from learning of appropriate mathematics to a focus on the technology in the activities.
L3m	Begins to explore, experiment and practice integrating technologies as mathematics learning tools.
L4m	Uses technologies as tools to facilitate the learning of specific topics in the mathematics curriculum.
L5m	Plans, implements, and reflects on teaching and learning with concern and personal conviction for student thinking and understanding of the mathematics to be enhanced through integration of the various technologies.
L1c	More apt to accept the technology as a teaching tool rather than a learning tool.
L2c	Is concerned that students do not develop appropriate mathematical thinking skills when the technology is used as a verification tool for exploring the mathematics.
L3c	Begins developing appropriate mathematical thinking skills when technology is used as a tool for learning.
L4c	Plans, implements, and reflects on teaching and learning with concern for guiding students in understanding.
L5c	Technology-integration is integral (rather than in addition) to development of the mathematics students are learning.

Code	Code description
T1m	Concerned that the need to teach about the technology will take away time from teaching mathematics.
T2m	Uses technology activities at the end of units, for “days off,” or for activities peripheral to classroom instruction.
T3m	Uses technology to enhance or reinforce mathematics ideas that students have learned previously.
T4m	Engages students in high-level thinking activities (such as project-based and problem solving and decision making activities) for learning mathematics using the technology as a learning tool.
T5m	Active, consistent acceptance of technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students.
T1i	Does not use technology to develop mathematical concepts.
T2i	Merely mimics the simplest professional development mathematics curricular ideas for incorporating the technologies.
T3i	Mimics the simplest professional development activities with the technologies but attempts to adapt lessons for his/her mathematics classes.
T4i	Engages students in explorations of mathematics with technology where the teacher is in role of guide rather than director of the exploration.
T5i	Adapts from a breadth of instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics.
T1e	Uses technology to reinforce concepts taught without technology.
T2e	Tightly manages and orchestrates instruction using technology.
T3e	Instructional strategies with technologies are primarily deductive, teacher-directed in order to maintain control of the how the activity progresses.
T4e	Explores various instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics.
T5e	Manages technology-enhanced activities in ways that maintains student engagement and self-direction in learning the mathematics.
T1p	Considers attending local professional development to learn more about technologies.
T2p	Recognizes the need to participate in technology related PD.
T3p	Continues to learn and explore ideas for teaching and learning mathematics using only one type of technology (such as spreadsheets).
T4p	Seeks out and works with others who are engaged in incorporating technology in mathematics.
T5p	Seeks ongoing PD to continue to learn to incorporate emerging technologies. Continues to learn and explore ideas for teaching and learning mathematics with multiple technologies to enhance access to mathematics.

Code	Code description
A1u	Permits students to use technology 'only' after mastering certain concepts.
A2u	Students use technology in limited ways during regular instructional periods.
A3u	Permits students to use technology in specifically designed units.
A4u	Permits students to use technology for exploring specific mathematical topics.
A5u	Permit students to use technology in every aspect of mathematics class.
A1b	Resists consideration of changes in content taught although it becomes accessible to more students through technology.
A2b	Worries about access and management issues with respect to incorporating technology in the classroom.
A3b	Uses technology as a tool to enhance mathematics lessons in order to provide students a new way to approach mathematics.
A4b	Recognizes challenges for teaching mathematics with technologies, but explores strategies and ideas to minimize the impact of those challenges.
A5b	Recognizes challenges in teaching with technology and resolves the challenges through extended planning and preparation for maximizing the use of available resources and tools.
A1a	Notices that authentic problems are more likely to involve 'unfriendly numbers' and may be more easily solved if students had calculators.
A2a	Calculators permit greater number of examples to be explored by students.
A3a	Concepts are taught differently since technology provides access to connections formerly out of reach.
A4a	Through the use of technology, key topics are explored, applied, and assessed incorporating multiple representations of the concepts and their connections.
A5a	Students are taught and permitted to explore more complex mathematics topics or mathematical connections as part of their normal learning experience.

APPENDIX H

FREQUENCY OF SKETCHPAD USE

How many times per week did the students work in a computer lab/classroom using GSP software?

Week	1	2	3	4	5	6	7	8	9	10	11	12	13
Brian	1	3	0	0	1	0	0	0	0	0	0	1	1
James	0	1	1	1	1	0	0	1	0	0	1	0	1
Susan	1	1	1	1	1	1	1	2	2				
Laura	0	0	0	1	1	0	0	0	2	2	0	1	1

Week	14	15	16	17	18	19	20	21	22	Average
Brian	0	0	2	0	2	0	0	0	0	0.5
James	0	0	0	0	2	2	1	2	0	0.6
Susan										1.2
Laura	1	1	1	2	0	1	1	0	0	0.7

How many times per week was the geometry class taught in a classroom with one demonstration computer?

Week	1	2	3	4	5	6	7	8	9	10	11	12	13
Brian	2	1	0	0	1	0	0	1	1	0	0	1	1
James	0	1	1	1	1	1	0	1	1	1	0	1	0
Susan	4	4	4	4	4	4	4	4	4				
Laura	0	0	0	1	1	2	0	2	1	1	1	1	1

Week	14	15	16	17	18	19	20	21	22	Average
Brian	0	0	2	0	2	0	0	0	0	0.5
James	2	0	1	0	2	3	1	2	0	0.9
Susan										4.0
Laura	2	2	2	1	0	1	1	2	1	1.0

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VITA

Ewelina Suchacka McBroom was born in Opole, Poland on June 9, 1980, and is the daughter of Lech Suchacki and Wiesława Suchacka. After eleven years of education in Poland, she attended Ripley High School in Ripley, Tennessee for one year and graduated in 1999. Next, she attended Dyersburg State Community College where she received an Associate of Science degree in 2001. She continued her studies at the University of Memphis where she received a Bachelor in Business Administration degree in 2003 and a Master of Arts degree in 2005. In June 2006, she began her graduate studies at Tarleton State University-Central Texas and received a Master of Science degree in Mathematics in 2008. Between January 2006 and May 2008, she taught developmental mathematics courses at Central Texas College in Killeen, Texas, as well as high school mathematics and dual credit economics courses at Temple High School in Temple, Texas. In August 2008, she entered the Graduate College of Texas State University-San Marcos to pursue a doctoral degree in mathematics education. She also worked as a teaching assistant and a research assistant for the Dynamic Geometry in Classrooms Project while at Texas State.

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This dissertation was typed by Ewelina Suchacka McBroom.



Section 4

Dynamic Geometry Approach Learning Activities

The Dynamic Geometry in Classrooms project was funded to investigate the efficacy of the “Dynamic Geometry Approach” on students’ geometry learning. Numerous learning activities were employed as part of the project professional development of teachers as well as for use by teachers with their students to enact the dynamic geometry approach. This section is comprised of four papers that describe examples of learning activities derived from the project and created by project leaders that were published in peer-reviewed journals.

The first paper, “Multiple Proof Approaches and Mathematical Connections,” was published in the National Council of Teachers of Mathematics (NCTM) premier and oldest journal, *Mathematics Teacher* and appears here with permission. In this article for high school mathematics teachers the authors describe how to use dynamic geometry technology to explore the “Three Altitudes of a Triangle” problem and examine how pre-service teachers devised different proofs for the conjectures they made about the problem.

The second paper, “Where Are the Congruent Halves?” was published in an NCTM journal, *Mathematics Teaching in the Middle School*, and also appears here with permission. The article focuses on a learning activity that requires creating two polygons from one so as to apply knowledge of congruency and transformations to develop spatial reasoning, all while using technology and a dynamic geometry approach.

The third paper, “What’s Inside the Cube? Students’ Investigation with Models and Technology,” also was published in the NCTM *Mathematics Teacher* journal and appears here with permission. The Zometool and vZome software applications were used in conjunction with the Geometer’s Sketchpad to exemplify the dynamic geometry approach that allows learners to investigate tetrahedrons nested within cubes so as to develop spatial visualization skills.

The fourth paper, “The Role of Technology in a Geometry Investigation,” was published in the *Mathematics and Computer Education* journal and appears here with permission. This article describes how pre-service secondary school teachers used technology, in this case the Geometer’s Sketchpad and Maple software, to help them investigate relationships between triangles and their medians. The students conjectured that the ratio of the area of the median triangle to the area of the original triangle ABC remained 3:4 (75%) and the article examines how teachers were able to develop students’ reasoning and sense making through this activity.



Multiple Proof Approaches & Mathematical Connections

 [jstor.org/stable/10.5951/mathteacher.105.8.0586](http://www.jstor.org/stable/10.5951/mathteacher.105.8.0586)

Item Type

[Article](#)

[References](#)



Abstract

Using technology to explore the Three Altitudes of a Triangle problem, students devise many proofs for their conjectures.

Citation:

Jiang, Z. & O'Brien, G.E. (2012). Multiple Proof Approaches & Mathematical Connections. *The Mathematics Teacher*, 105(8), 586-593. doi:1. Retrieved from <http://www.jstor.org/stable/10.5951/mathteacher.105.8.0586> doi:1

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Where Are The Congruent Halves?

 [jstor.org/stable/10.5951/mathteachmidscho.17.8.0480](http://www.jstor.org/stable/10.5951/mathteachmidscho.17.8.0480)

Item Type

[Article](#)

[References](#)



Abstract

Try this activity that requires creating two polygons from one to apply knowledge of congruency and transformations and develop spatial reasoning.

Citation:

Obara, S. (2012). Where Are The Congruent Halves? *Mathematics Teaching in the Middle School*, 17(8), 480-485. doi:1. Retrieved from <http://www.jstor.org/stable/10.5951/mathteachmidscho.17.8.0480> doi:1

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What's Inside the Cube? Students' Investigation with Models and Technology

 [jstor.org/stable/10.5951/mathteacher.105.2.0102](http://www.jstor.org/stable/10.5951/mathteacher.105.2.0102)

Item Type

[Article](#)

[References](#)



Abstract

Applying Zometool, vZome software, and The Geometer's Sketchpad to tetrahedrons nested in cubes enhances students' spatial visualization skills.

Citation:

Obara, S. & Jiang, Z. (2011). What's Inside the Cube? Students' Investigation with Models and Technology. *The Mathematics Teacher*, 105(2), 102-110. doi:1. Retrieved from <http://www.jstor.org/stable/10.5951/mathteacher.105.2.0102> doi:1

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THE ROLE OF TECHNOLOGY IN A GEOMETRY INVESTIGATION.

Source: Mathematics & Computer Education . Fall2010, Vol. 44 Issue 3, p196-204. 9p.

Author(s): Obara, Samuel

Abstract:

This article describes how pre-service secondary school teacher used technology, in this case Geometer's Sketchpad and Maple software, to help them investigate relationships between triangles and their medians. The students conjectured that the ratio of the area of the median triangle to area of the original triangle ABC remained 3:4 (75%).

Citation: Obara, S. (2010). THE ROLE OF TECHNOLOGY IN A GEOMETRY INVESTIGATION. Mathematics & Computer Education, 44(3).

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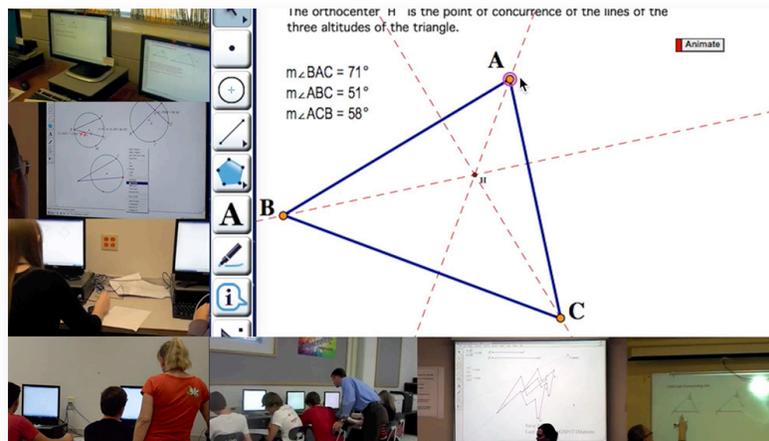
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Section 5

A Technological Pedagogical Content Knowledge (TPACK) Development Interview Protocol was established by project leaders to gather data tied to current research on how mathematics teachers progress in developing the knowledge from teaching mathematics effectively with technology. Similarly the TPACK Observation Protocol was established to allow the project to document the enactment of TPACK progressions observed in actual classrooms. Both protocols were validated by the project Advisory Board and external evaluator. Doctoral student Ewelina McBroom also contributed significantly to the development of these two instruments.

Lastly, a set of tasks were created and a protocol for interviews of students engaged with the tasks were developed based on input from the project Advisory Board. These tasks and the interview protocol were provided as the Teaching Dynamic Geometry Task Interview Protocol.



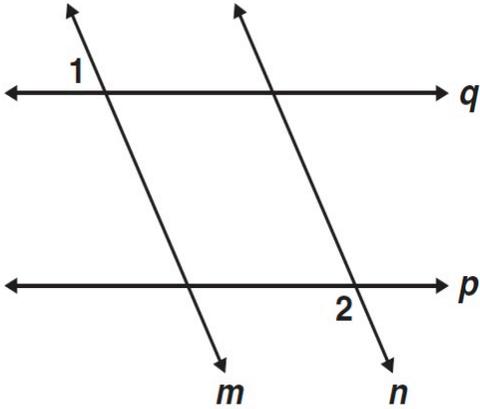
ENTERING GEOMETRY STUDENT TEST

A 20-item multiple-choice test titled the **Entering Geometry Student Test** was used as a pre-test in the Dynamic Geometry Project. The test used by the project was created by the University of Chicago (copyright 1982) School Mathematics Project (UCSMP) and included in Appendix A (pp. 148-152) of Dr. Zalman Usiskin's report titled VAN HIELE LEVELS AND ACHIEVEMENT IN SECONDARY SCHOOL GEOMETRY available at http://ucsmc.uchicago.edu/resources/van_hiele_levels.pdf. Dr. Usiskin and the University of Chicago kindly allowed the project to use the test with students.

More information about the original **Entering Geometry Student Test** and the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) Project under the direction of Dr. Zalman Usiskin can be found at <http://ucsmc.uchicago.edu/resources/van-hiele/>. This full web site provides access to the project report that can be freely downloaded with information about the **Entering Geometry Student Test**'s origin and rationales.

EXITING GEOMETRY TEST

- 1.) Given: $p \parallel q$;
 $m \parallel n$;
 $m\angle 1 = 75^\circ$

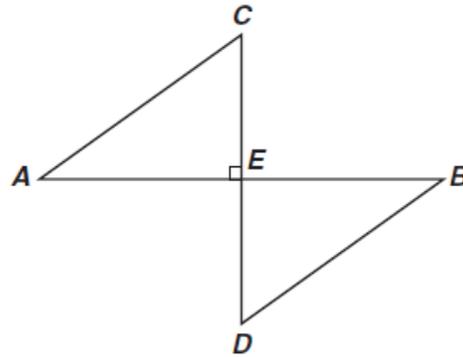


What is $m\angle 2$?

- A 15°
- B 75°
- C 90°
- D 105°

2.)

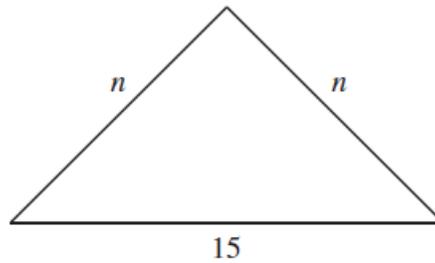
Given: E is the midpoint of \overline{CD} ; $\angle C \cong \angle D$



Which of the following statements *must* be true?

- A $\angle A \cong \angle D$
- B $\angle B \cong \angle C$
- C $\overline{CE} \cong \overline{BE}$
- D $\overline{AC} \cong \overline{BD}$

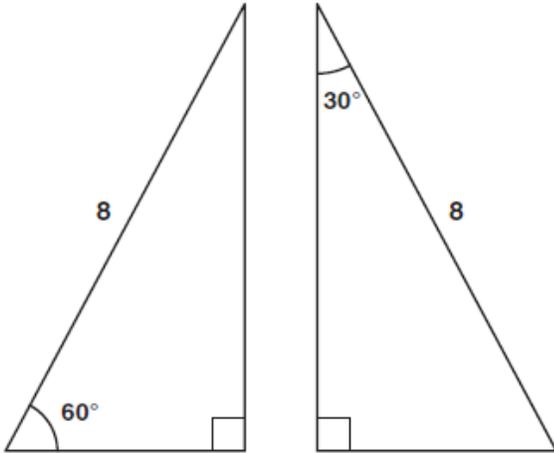
- 3.) In the figure below, n is a whole number. What is the *smallest* possible value for n ?



- A 1
- B 7
- C 8
- D 14

4.)

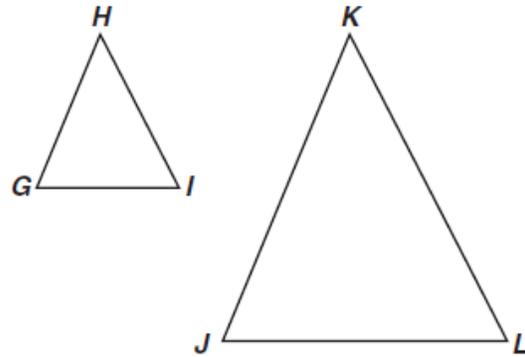
Which of the following *best* describes the triangles shown below?



- A both similar and congruent
- B similar but not congruent
- C congruent but not similar
- D neither similar nor congruent

5.)

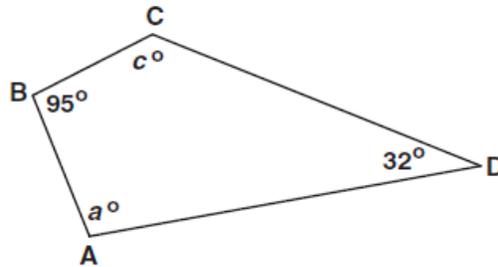
Which of the following statements must be true if $\triangle GHI \sim \triangle JKL$?



- A The two triangles must be scalene.
- B The two triangles must have exactly one acute angle.
- C At least one of the sides of the two triangles must be parallel.
- D The corresponding sides of the two triangles must be proportional.

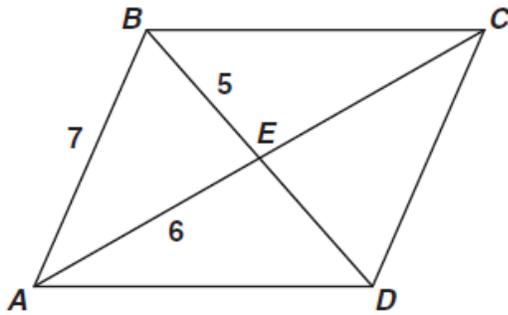
6.)

For the quadrilateral shown below, what is $m\angle a + m\angle c$?



- A 53°
- B 137°
- C 180°
- D 233°

- 7.) If $ABCD$ is a parallelogram, what is the length of segment BD ?

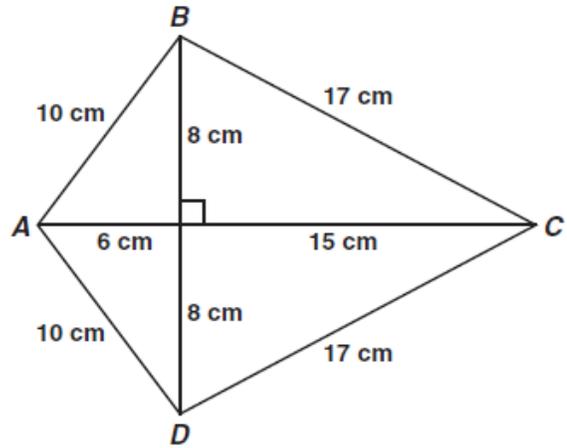


- A 10
- B 11
- C 12
- D 14

- 8.) The diameter of a circle is 12 meters. If point P is in the same plane as the circle, and is 6 meters from the center of the circle, which *best* describes the location of point P ?

- A Point P must be on the circle.
- B Point P must be inside the circle.
- C Point P may be either outside the circle or on the circle.
- D Point P may be either inside the circle or on the circle.

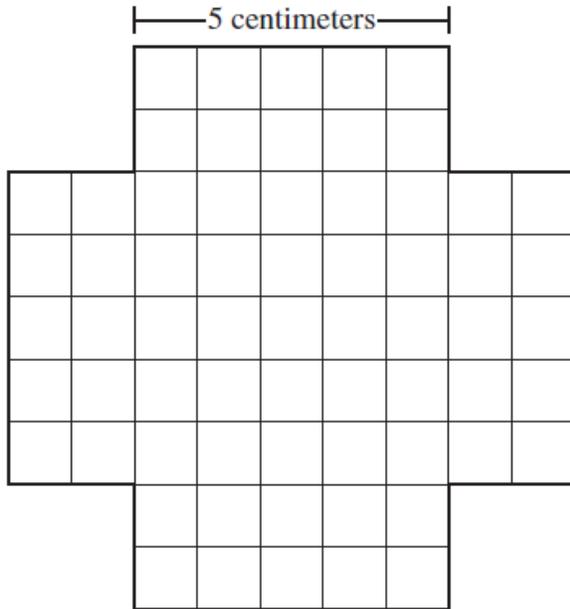
- 9.) Figure $ABCD$ is a kite.



What is the area of figure $ABCD$, in square centimeters?

- A 120
 - B 154
 - C 168
 - D 336
- 10.) The minute hand of a clock is 5 inches long. What is the area of the circle, in square inches, created as the hand sweeps an hour?
- A 10π
 - B 20π
 - C 25π
 - D 100π

- 11.) The four sides of this figure will be folded up and taped to make an open box.

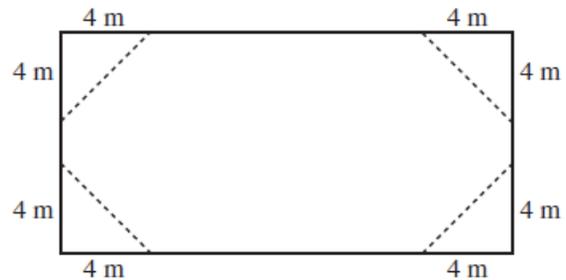


What will be the volume of the box?

- A 50 cm^3
- B 75 cm^3
- C 100 cm^3
- D 125 cm^3

- 12.)

The rectangle shown below has length 20 meters and width 10 meters.

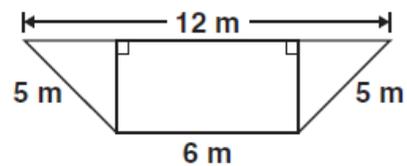


If four triangles are removed from the rectangle as shown, what will be the area of the remaining figure?

- A 136 m^2
- B 144 m^2
- C 168 m^2
- D 184 m^2

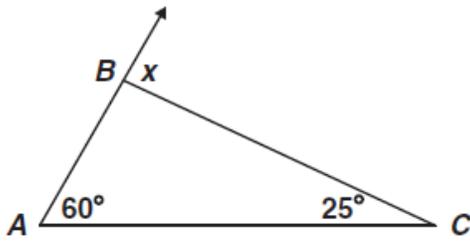
- 13.)

What is the area, in square meters (m), of the trapezoid shown below?



- A 28
- B 36
- C 48
- D 72

14.) What is $m\angle x$?



- A 35°
- B 60°
- C 85°
- D 95°

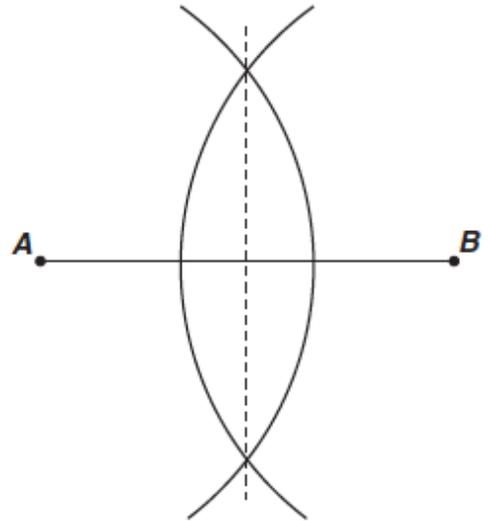
15.) What is the measure of an exterior angle of a regular hexagon?

- A 30°
- B 60°
- C 120°
- D 180°

16.) A right triangle's hypotenuse has length 5. If one leg has length 2, what is the length of the other leg?

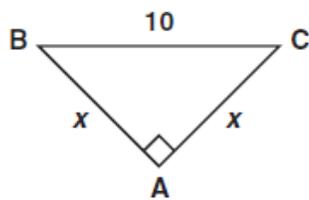
- A 3
- B $\sqrt{21}$
- C $\sqrt{29}$
- D 7

17.) What geometric construction is shown in the diagram below?



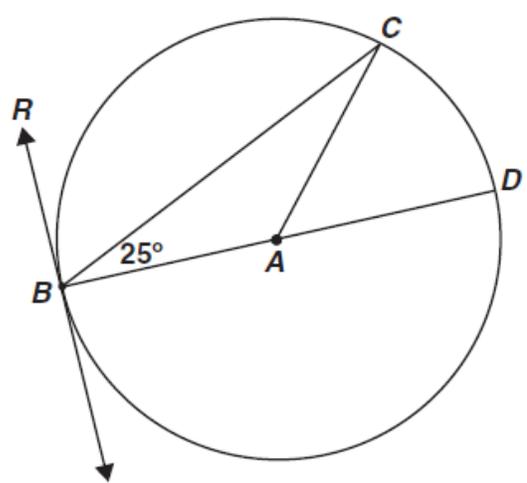
- A an angle bisector
- B a line parallel to a given line
- C an angle congruent to a given angle
- D a perpendicular bisector of a segment

18.) What is the value of x in the triangle below?



- A 5
- B $5\sqrt{2}$
- C $10\sqrt{3}$
- D 20

19.) \overline{RB} is tangent to a circle, whose center is A , at point B . \overline{BD} is a diameter.

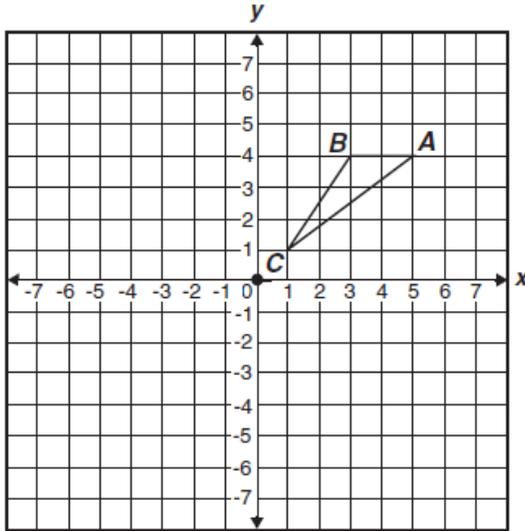


What is $m\angle CBR$?

- A 50°
- B 65°
- C 90°
- D 130°

20.)

If triangle ABC is rotated 180 degrees about the origin, what are the coordinates of A' ?

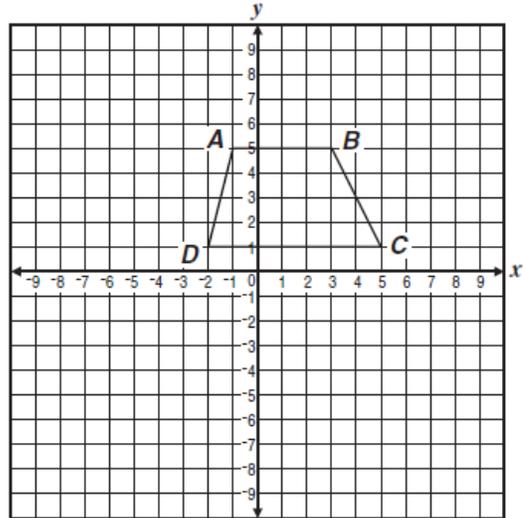


- A $(-5, -4)$
- B $(-5, 4)$
- C $(-4, 5)$
- D $(-4, -5)$

21.)

Trapezoid $ABCD$ below is to be translated to trapezoid $A'B'C'D'$ by the following motion rule.

$$(x, y) \rightarrow (x + 3, y - 4)$$



What will be the coordinates of vertex C' ?

- A $(1, -3)$
- B $(2, 1)$
- C $(6, 1)$
- D $(8, -3)$

22.)

Given: $TRAP$ is an isosceles trapezoid with diagonals \overline{RP} and \overline{TA} . Which of the following *must* be true?

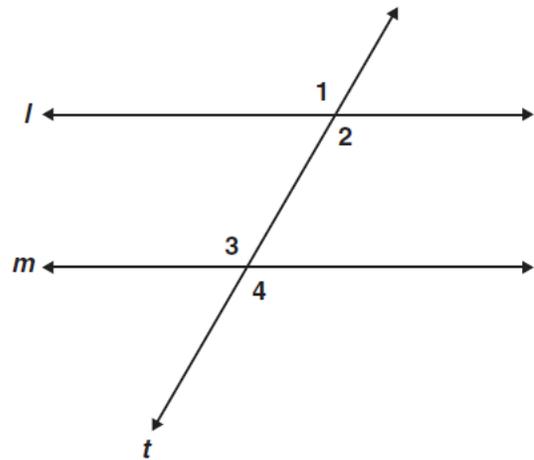
- A $\overline{RP} \perp \overline{TA}$
- B $\overline{RP} \parallel \overline{TA}$
- C $\overline{RP} \cong \overline{TA}$
- D \overline{RP} bisects \overline{TA}

23.) Which triangles must be similar?

- A two obtuse triangles
- B two scalene triangles with congruent bases
- C two right triangles
- D two isosceles triangles with congruent vertex angles

24.)

In the diagram below, $\angle 1 \cong \angle 4$.



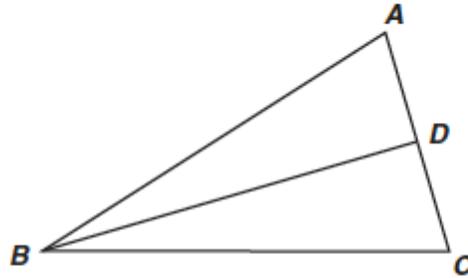
Which of the following conclusions does *not* have to be true?

- A $\angle 3$ and $\angle 4$ are supplementary angles.
- B Line l is parallel to line m .
- C $\angle 1 \cong \angle 3$
- D $\angle 2 \cong \angle 3$

25.) Use the proof to answer the question below.

Given: $\overline{AB} \cong \overline{BC}$; D is the midpoint of \overline{AC}

Prove: $\triangle ABD \cong \triangle CBD$



<u>Statement</u>	<u>Reason</u>
1. $\overline{AB} \cong \overline{BC}$; D is the midpoint of \overline{AC}	1. Given
2. $\overline{AD} \cong \overline{CD}$	2. Definition of Midpoint
3. $\overline{BD} \cong \overline{BD}$	3. Reflexive Property
4. $\triangle ABD \cong \triangle CBD$	4. ?

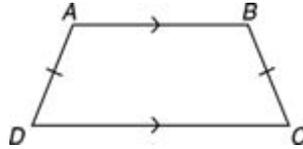
What reason can be used to prove that the triangles are congruent?

- A AAS
- B ASA
- C SAS
- D SSS

Dynamic Geometry in Classrooms: Professional Development Teacher Instrument

Conjecture and Proving Test for Teachers

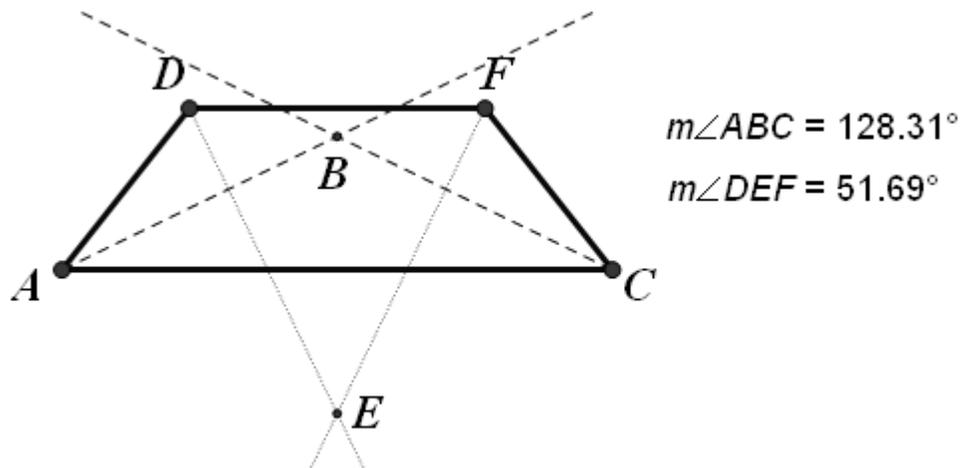
If the legs of a trapezoid are equal, it is called an “isosceles trapezoid”. (See example below).



1.) Ms. Westbrook asked Kara to determine whether the following conjecture is true or false:

“The angle bisectors of one pair of base angles of an isosceles trapezoid are perpendicular.”

Kara has found an isosceles trapezoid for which this statement is not true:



What *must* Kara do now?

- A) Kara should confirm that the conjecture is false for multiple isosceles trapezoids.
- B) Nothing... Kara has adequately shown the conjecture to be false.
- C) Kara should find out whether the conjecture is true for *any* isosceles trapezoids.
- D) Kara must prove that the conjecture is false for all isosceles trapezoids.

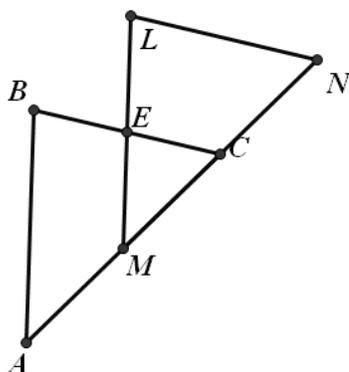
2.) Josh knows that trapezoids have two adjacent angles that are supplementary.

Can Josh conclude that any isosceles trapezoid has two adjacent angles that are supplementary?

- A) Yes.
- B) No.

You ask your students to describe some relationships in the figure below:

M is the midpoint of segment **AC** and triangle **MLN** is the image of triangle **ABC** under a translation by vector **AM**.



Choose the option that **best** describes each student’s response:

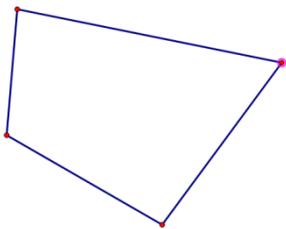
	unclear and needs to be restated	not true	trivial	a good observation of a relationship in the figure
3.) Maria says: “The two triangles are the same.”	A	B	C	D
4.) Gilbert says: “E is the midpoint of BC.”	A	B	C	D
5.) Crystal says: “MLN forms another triangle.”	A	B	C	D
6.) Thomas says: “It shows that it is twice as big.”	A	B	C	D
7.) Samuel says: “ ΔABC is similar to ΔMEC .”	A	B	C	D
8.) Bryan says: “The midpoint of ΔABC is similar to CE.”	A	B	C	D

A quadrilateral is “cyclic” if all of its vertices lie on a single circle. Your class is investigating the following conjecture in groups:

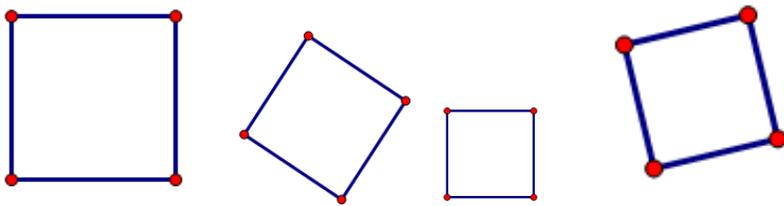
“All quadrilaterals are cyclic.”

9.) Your class plans to begin investigating this conjecture by examining some quadrilaterals and seeing if they are cyclic. Which collection of quadrilaterals would be the most helpful to examine:

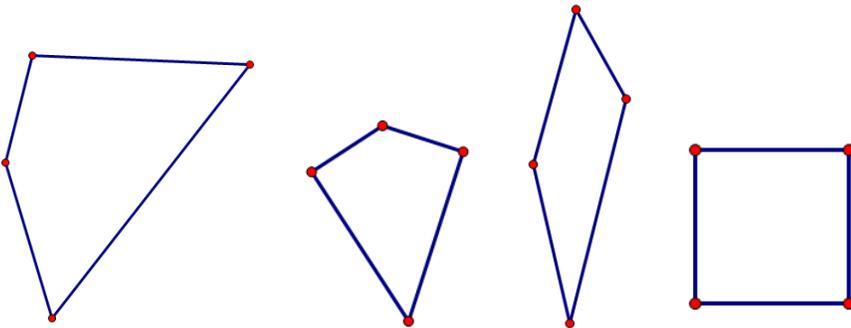
A)



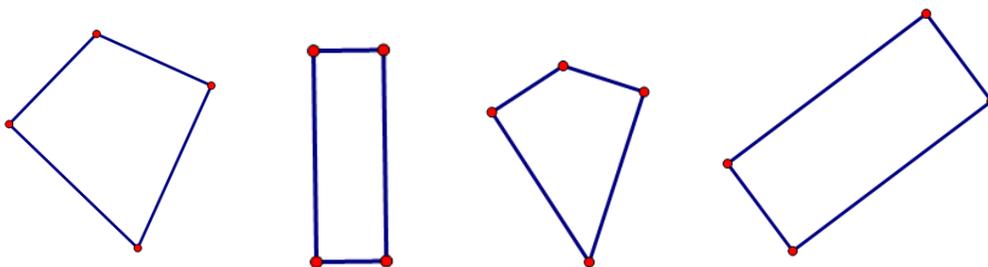
B)



C)



D)



E) Some of the above choices are equally good.

10.) Which of the following statements is TRUE?

- A) All quadrilaterals are cyclic.
- B) All parallelograms are cyclic, but not all quadrilaterals are cyclic.
- C) All rectangles are cyclic, but not all parallelograms are cyclic.
- D) All squares are cyclic, but not all rectangles are cyclic.

11.) A “kite” is a special kind of quadrilateral whose four sides form two pairs of congruent adjacent segments. In other words, a kite is a quadrilateral $ABCD$ with AB congruent to CB and AD congruent to CD . For a kite, which of the following statements is NOT always true?

- A) A pair of its opposite angles are congruent
- B) Its two diagonals are perpendicular to each other
- C) Its two diagonals bisect each other
- D) One of its diagonals cut the kite into two congruent triangles
- E) One of its diagonals bisects the other

12.) A parallelogram is a quadrilateral with both pairs of opposite sides parallel. In other words, a parallelogram is a quadrilateral $ABCD$ with $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$. For a parallelogram, which of the following statements is NOT always true?

- A) The opposite angles are congruent.
- B) The adjacent angles are supplementary.
- C) The opposite sides are congruent.
- D) The diagonals bisect each other.
- E) The diagonals are perpendicular to each other.

Students were given two properties of a figure:

Property S: It is a square.

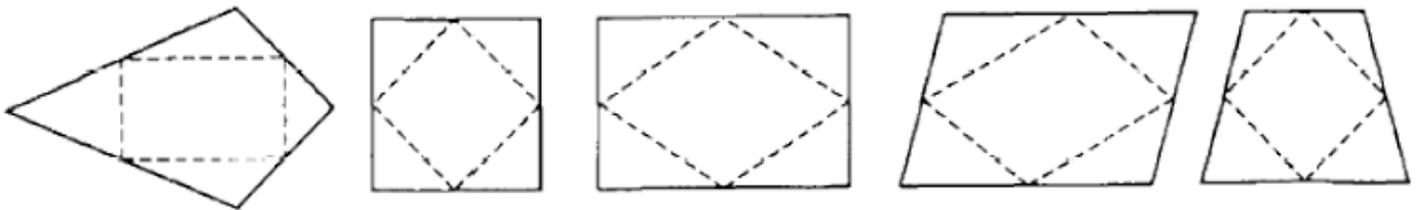
Property R: It is a rectangle.

Anna, Greg and Donna made an observation about these properties and gave their reasoning.

Identify whether their reasoning is correct or incorrect:

(Circle A or B to indicate CORRECT or INCORRECT for each observation.)

	Correct Reasoning	Incorrect Reasoning
13.) Anna said that S implies R because a square is a special case of a rectangle.	A	B
14.) Greg said that S implies R because both have four sides.	A	B
15.) Donna said that S implies R because not every rectangle is a square.	A	B



16.) In the five quadrilaterals shown above, the midpoints of the sides have been joined by broken line segments. Which best describes the five dotted figures formed?

- A.) All are parallelograms.
- B.) All are rectangles.
- C.) All are squares.
- D.) All are rhombuses.
- E.) No generalization can be made.

Suppose you are teaching “if-then” statements. In order to assess your students’ understanding of conditional logic you place three cards on a table and give them the following task:



2

A
B
C

If a card has an even number on one side then it is blank on the other side.

Which cards do you *need* to turn over to see if the above statement is true?

Identify whether their reasoning is correct or incorrect:

(Circle A or B to indicate CORRECT or INCORRECT for each observation.)

	Correct Reasoning	Incorrect Reasoning
17.) Maria says that A does not need to be turned over because the card has no even number on top.	A	B
18.) Luis says that B needs to be turned over because we need to see if there is an even number on the other side.	A	B
19.) Araceli says that B doesn’t need to be turned over because it doesn’t matter what’s on the other side of that card.	A	B
20.) Maggie says that all of the cards must be turned over to check every case.	A	B
21.) Aaron says that A needs to be turned over because it may be blank on the other side.	A	B

You have asked your class to prove the following statement:

When you add the interior angles of any triangle, your answer is always 180° .

Give student responses below the most appropriate score **using the following rubric:**

Rubric:

Description	Score
The proof has no mistakes and shows the statement is always true.	3 points
The proof has a mistake but the overall reasoning process is in the right direction.	2 points
The student only shows the statement is true for some triangles.	1 point
The student's response only attempts to show the statement is true for some triangles and has a mistake.	0 points

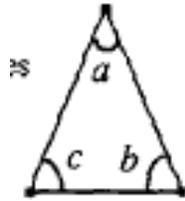
Circle the Score (Using Above Rubric)

<p>22.)</p> <p><i>Jill's Answer:</i></p> <p>I tore the angles up and put them together.</p>  <p>It came to a straight line which is 180°. I tried for an equilateral and an isosceles as well and the same thing happened.</p> <p><i>So Jill says it's true.</i></p>	3 points	2 points	1 point	0 points
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23.)

Barry's Answer:

I drew an isosceles triangle with c equal to \circ .



3 points	2 points	1 point	0 points
----------	----------	---------	----------

Statements

.....

Reasons

Base angles in isosceles triangle equal.

Base angles in isosceles triangle equal

So Barry says it's true.

24.)

Cynthia's Answer

I drew a line parallel to the base of the triangle.



3 points	2 points	1 point	0 points
----------	----------	---------	----------

Statements

.....

 \circ

Reasons

Alternate angles between two parallel lines are equal

Alternate angles between two parallel lines are equal

Angles on a straight line

\circ

So Cynthia says it's true.

25.)

Dylan's Answer

I measured the angles of all sorts of triangles accurately and made a table.

<i>a</i>	<i>b</i>	<i>c</i>	<i>total</i>
110	34	36	180
95	43	42	180
35	72	73	180
10	27	143	180

They all added up to 180° .

So Dylan says it's true.

3 points

2 points

1 point

0 points

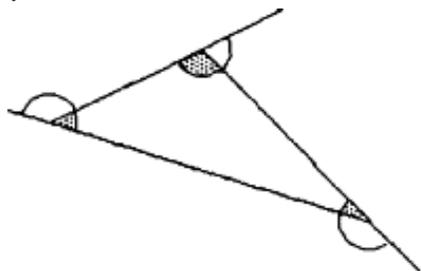
26.)

Ewan's Answer:

If you walk all the way around the edge of the triangle, you end up facing the way you began. You must have turned a total of 360° .

You can see that each exterior angle when added to the interior angle must give 180° because they make a straight line. This makes a total of 540° .

$540^\circ - 360^\circ = 180^\circ$.



So Ewan says it's true.

3 points

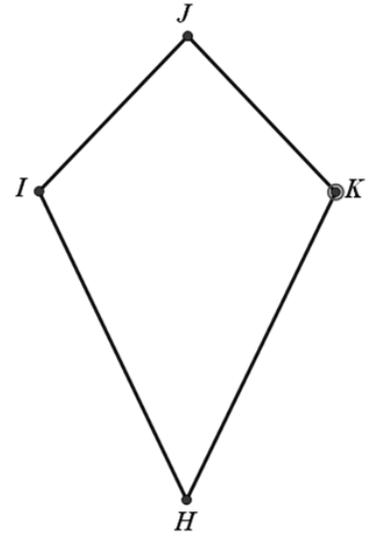
2 points

1 point

0 points

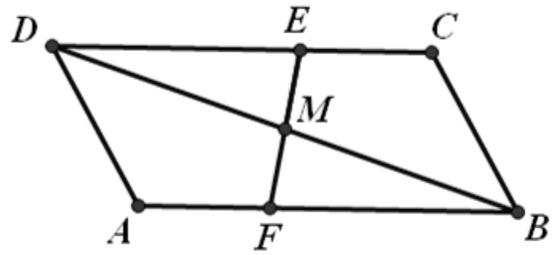
27.) Write this proof in the space provided.

Suppose that for quadrilateral $H I J K$, $H I = H K$ and $I J = J K$. Prove that $\angle I \cong \angle K$.



28.) Write this proof in the space provided.

Given $AB = DC$, $AD = BC$, M is the midpoint of \overline{DB} , and \overline{EF} contains M . Prove $FM = ME$.



Did you have enough time to complete this test?

Were there any questions that you didn't understand or found confusing? If so, which ones and why?

Any other comments about this test?

DGProjectID: _____

Conjecturing-Proving Test

Directions

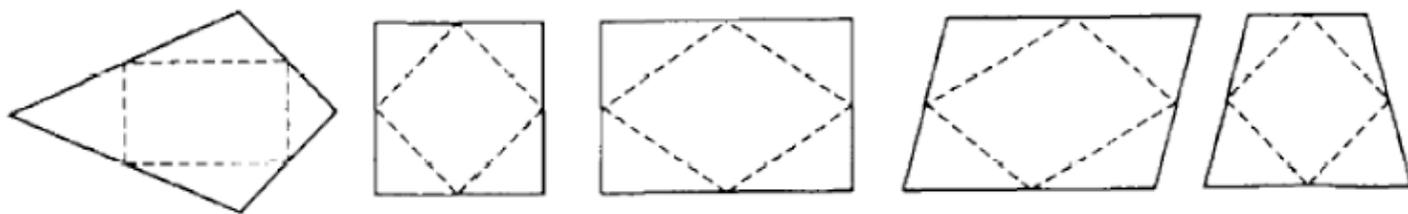
DO NOT BEGIN THE TEST UNTIL TOLD TO DO SO.

This test has 20 multiple-choice and 3 free-response questions. Answer the questions to the best of your knowledge.

Write your DGProjectID (assigned in the class) in the top right hand corner of this page. Write this number in the corresponding place on your Scantron.

When you are told to begin,

- Read each question carefully.
- Select the letter that goes with the BEST answer. Fill in that letter on your Scantron, and mark the answer on your test. There is only one correct answer to each question.
- If you need more scratch paper, raise your hand.
- If you want to change an answer, completely erase your first answer.
- If you need another pencil, raise your hand.



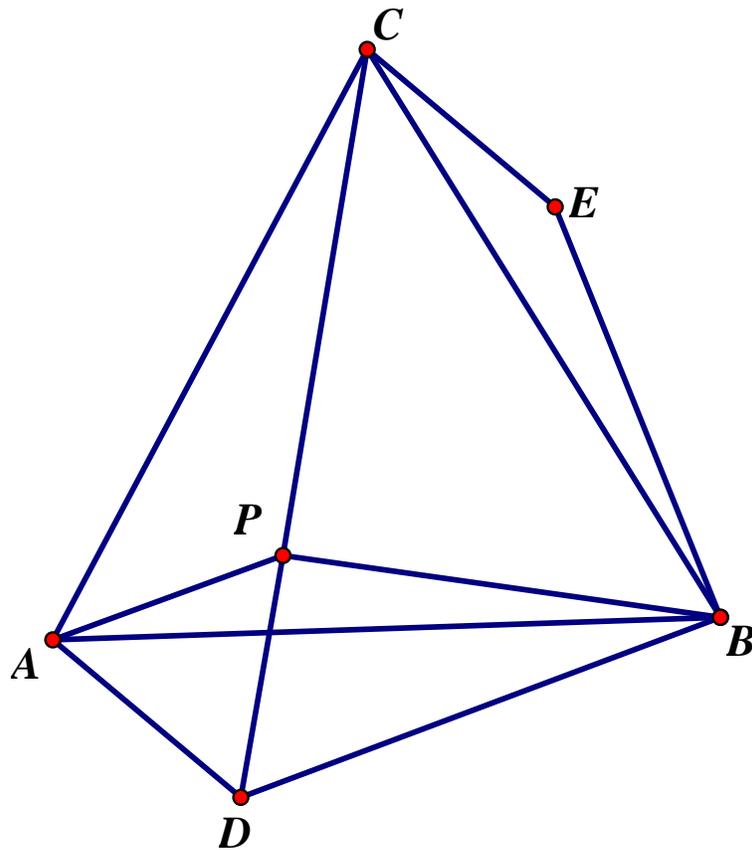
31) In the five quadrilaterals shown above, the midpoints of the sides have been joined by broken line segments. Which best describes the five dotted figures formed?

- A) All are parallelograms.
- B) All are rectangles.
- C) All are squares.
- D) All are rhombuses.
- E) No generalization can be made.

32) A parallelogram is a quadrilateral with both pairs of opposite sides parallel. For a parallelogram, which of the following statements is **NOT** always true?

- A) Opposite angles are congruent.
- B) The diagonals are perpendicular to each other.
- C) Opposite sides are congruent.
- D) The diagonals bisect each other.

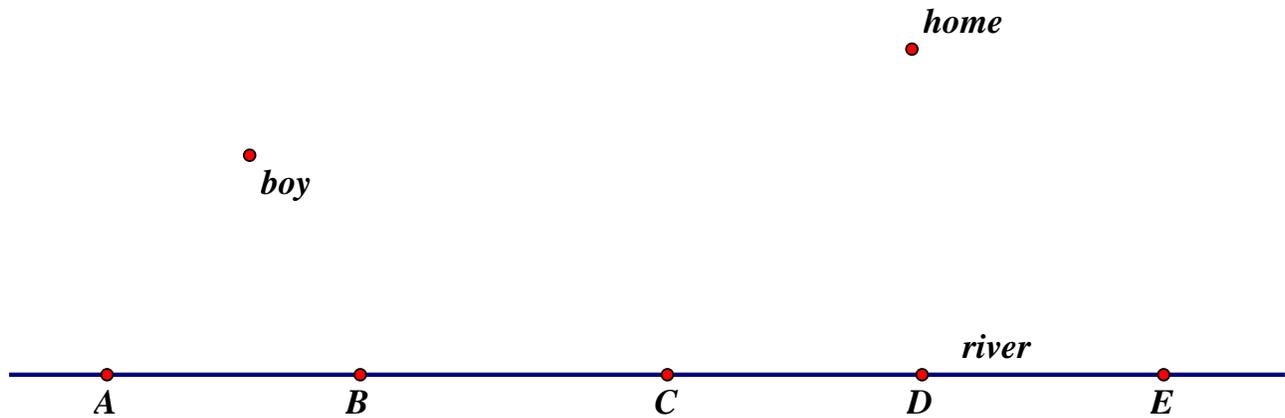
33) While solving a problem, Justin constructed the following diagram:



Which of the following do you think could be **TRUE**?

- A) is similar to
- B) is similar to
- C) is similar to
- D) is similar to

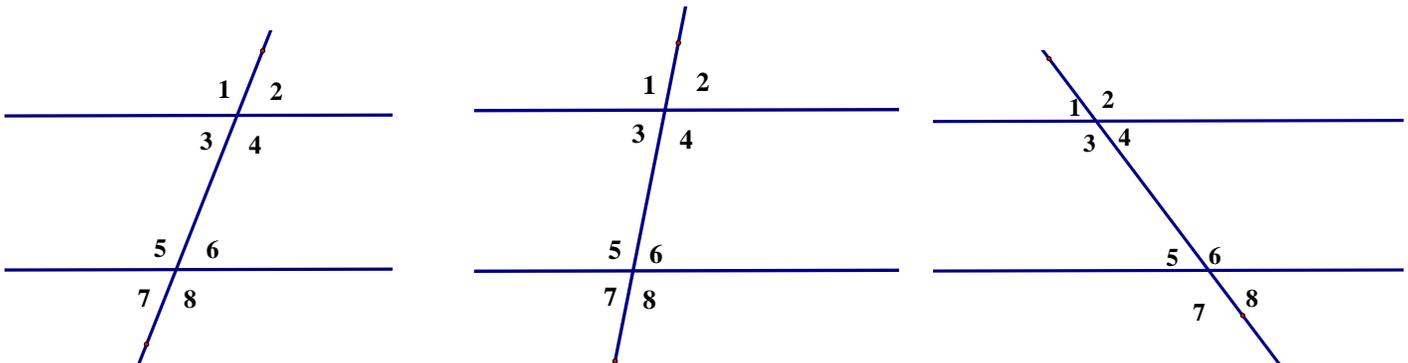
34) In the following figure, the boy must go to the river (to get water) and then go home.



In order to take the shortest path to the river and then home, the boy should stop at the river:

- A) somewhere between A and B.
- B) somewhere between B and C.
- C) somewhere between C and D.
- D) somewhere between D and E.

35) Elena is exploring what happens when two parallel lines are cut by a third line. She drew several examples:



Which of the following is **NOT** necessarily true?

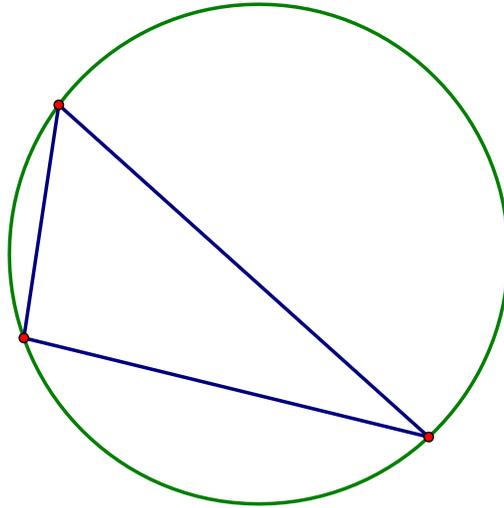
- A) $\angle 6$ is always congruent to $\angle 2$.
- B) $\angle 6$ is always congruent to $\angle 3$.
- C) $\angle 8$ is always congruent to $\angle 1$.
- D) $\angle 6$ is never congruent to $\angle 4$.

Note: A triangle is **inscribed** in a circle if all of its vertices lie on a single circle. (See example below).

36) Ms. Westbrook asked Kara to conclude whether the following statement is true or false:

“If a triangle can be inscribed in a circle then it must be equilateral.”

Kara has found a triangle for which this statement is **false**:



What **MUST** Kara do now to conclude whether the statement is true or false?

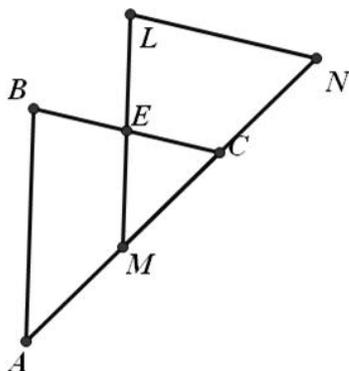
- A) Kara has to see if the statement is false for other triangles.
- B) Nothing... with this example Kara has shown the statement to be false.
- C) Kara needs to see if the statement is true for equilateral triangles.
- D) Kara must prove that the statement is false for all triangles.

37) Josh knows that any triangle can be inscribed in a circle. He then concludes that any isosceles triangle can be inscribed in a circle. Which of the following comments do you agree with?

- A) Josh is right; he can conclude this for isosceles triangles because he knows it for all triangles.
- B) Josh is wrong; he needs to know if it is true specifically for isosceles triangles.
- C) Josh is right because I have seen a figure in which an isosceles triangle is inscribed in a circle.
- D) Josh is wrong because isosceles triangles may not be triangles.

Use the following given statements to answer questions 38 - 43.

- M is the midpoint of segment \overline{AC} .
- Triangle MLN is a translation of triangle ABC.



Choose the option that **best** describes each of your friends' responses:

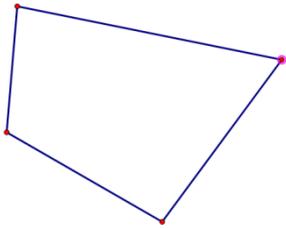
	unclear and needs to be restated	not true	trivial (It is given in the problem.)	a good conjecture
38) Maria says: "The two triangles are the same."	A	B	C	D
39) Gilbert says: "E is the midpoint of \overline{BC} ."	A	B	C	D
40) Crystal says: "MLN forms another triangle."	A	B	C	D
41) Thomas says: "It shows that it is twice as big."	A	B	C	D
42) Samuel says: " $\triangle ABC$ is similar to $\triangle MEC$."	A	B	C	D
43) Bryan says: "They must have the same length with same size/shape."	A	B	C	D

44) A quadrilateral is “cyclic” if all of its vertices lie on a single circle. Your class is investigating the following conjecture in groups:

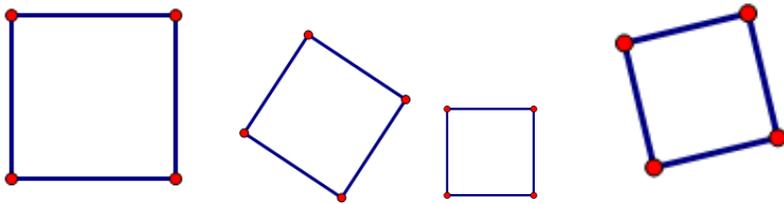
“All quadrilaterals are cyclic.”

Your class plans to begin investigating this conjecture by examining some quadrilaterals and seeing if they are cyclic. Which collection of quadrilaterals would be the **most** helpful to examine:

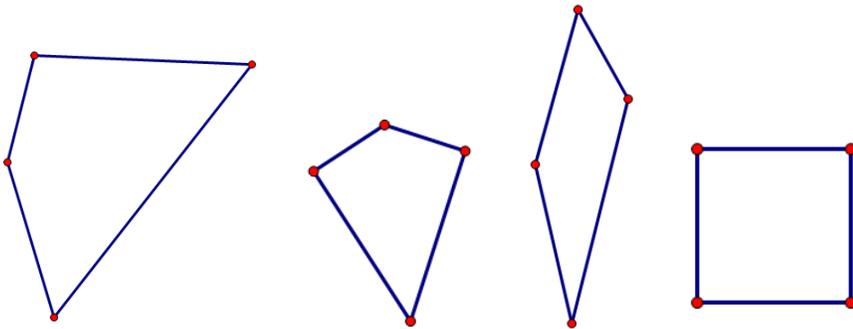
A)



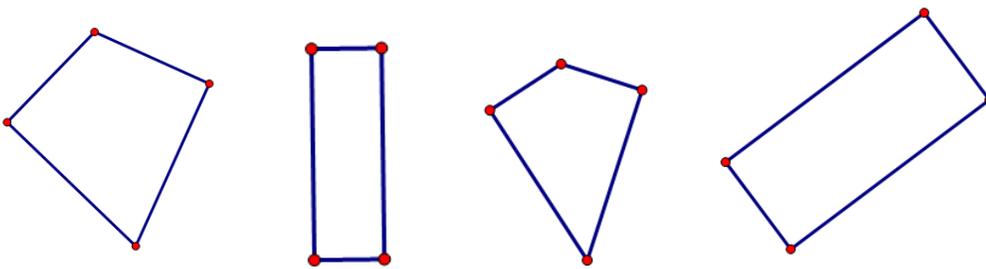
B)



C)

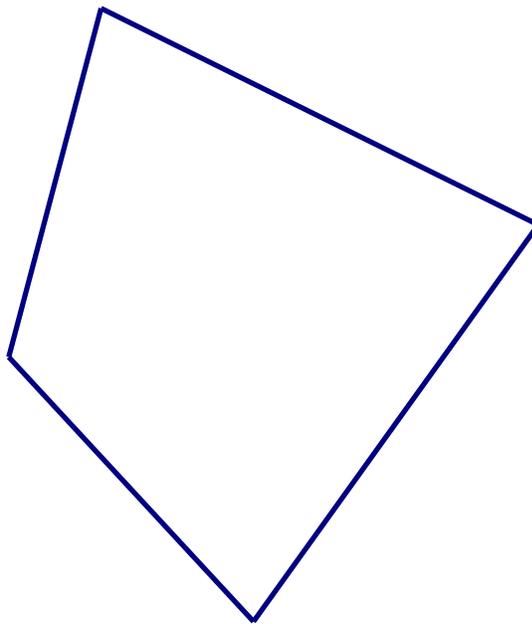
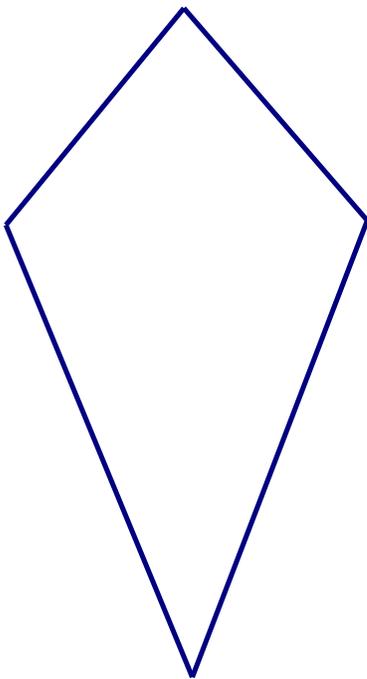
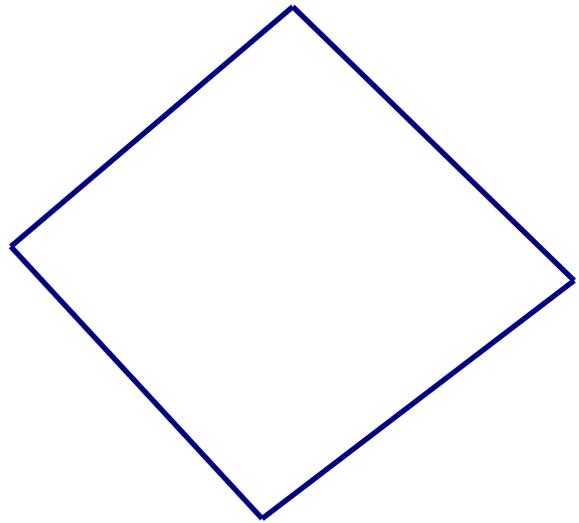
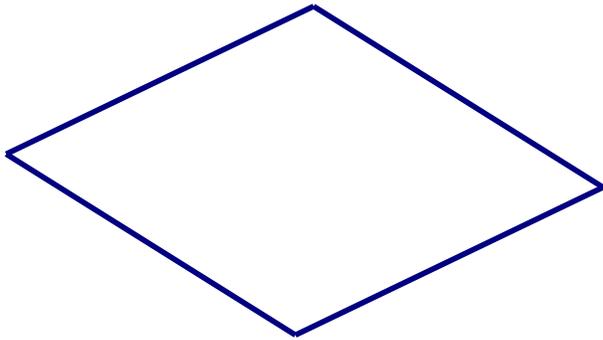


D)



E) Some of the above choices are equally good.

45) The following quadrilaterals are kites:



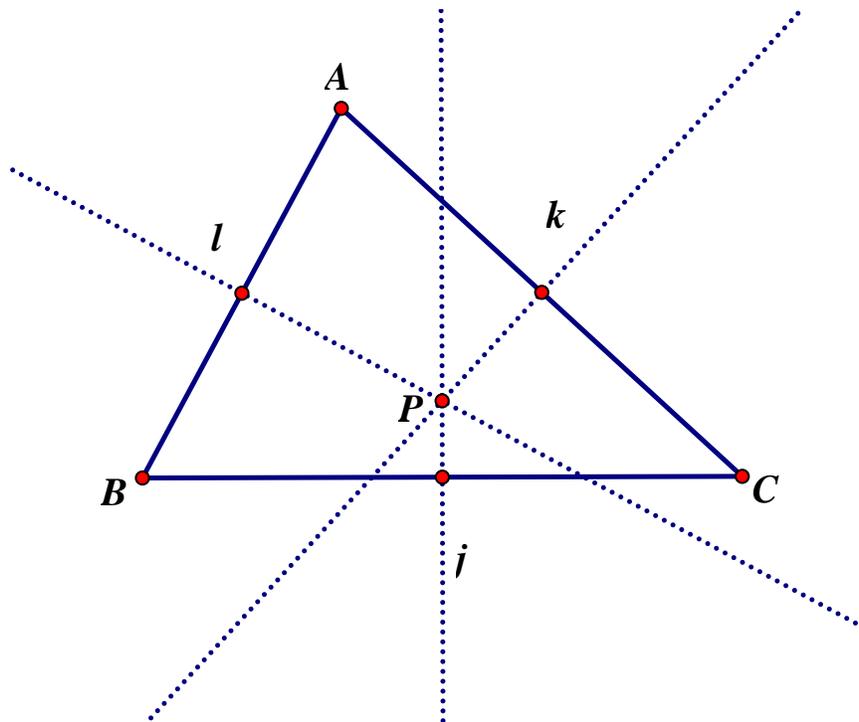
For a kite, which of the following statements is **NOT** always true?

- A) A pair of its opposite angles are congruent.
- B) Its two diagonals are perpendicular to each other.
- C) Its two diagonals bisect each other.
- D) One of its diagonals cuts the kite into two congruent triangles.

46) All squares are rectangles. Why? (Choose the BEST answer.)

- A) All Squares are rectangles because a rectangle can have sides of different lengths.
- B) All Squares are rectangles because a square is a special type of rectangle.
- C) All Squares are rectangles because both have four sides.
- D) All Squares are rectangles because not every rectangle is a square.

47) Lines j , k , and l are three perpendicular bisectors of $\triangle ABC$. These lines always intersect at a single point P . Use the diagram to determine which of the following is **NOT** always true?



- A) If $\triangle ABC$ is acute, then P lies inside the triangle.
- B) P is the same distance from the three vertices (A , B , and C) of the triangle.
- C) P is the same distance from the three sides (\overline{AB} , \overline{BC} , and \overline{CA}) of the triangle.
- D) We can construct a circle through A , B , and C with P at the center.

Note: The remaining questions are about **proofs**. To **prove** something in mathematics means to provide justification.

Your teacher asked you to prove the following statement:

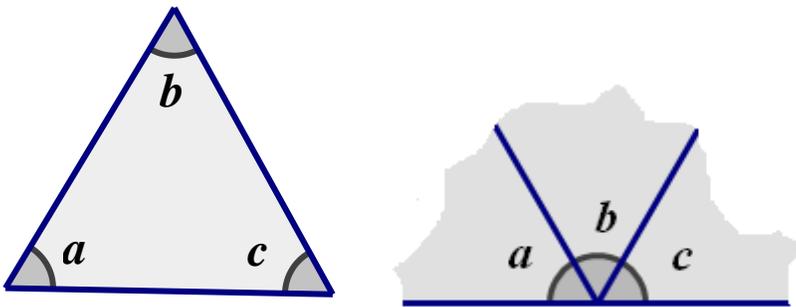
When you add the measures of the interior angles of any triangle, your answer is always 180° .

Choose the answer that BEST describes each of your classmates' answers.

48)

Jill's Answer:

I constructed an equilateral triangle. Then I tore the angles up and put them together.



It came to a straight line which is 180° . I tried for an isosceles as well and the same thing happened.

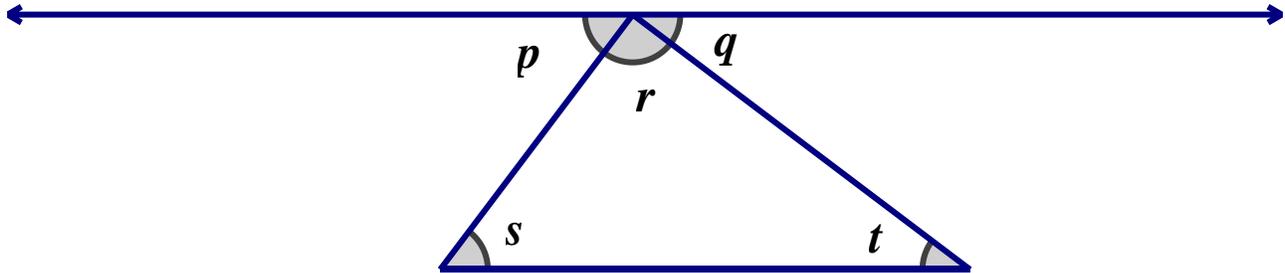
So Jill says it's true.

- A) She proved the statement is true for all triangles.
- B) She showed the statement is true for equilateral and isosceles triangles, but she needs to try more triangles.
- C) She can't prove the statement just by tearing up triangles.
- D) She didn't prove the statement; she needs to measure each angle.

49)

Cynthia's Answer:

I constructed a line parallel to the base of the triangle.



Statements	Reasons
$p = s$	Alternate angles between two parallel lines are equal
$q = t$	Alternate interior angles between two parallel lines are equal
$p + q + r = 180^\circ$	Angles on a straight line
$\therefore s + t + r = 180^\circ$	

So Cynthia says it's true.

- A) She proved the statement is true for all triangles.
- B) She showed the statement is true for one triangle, but she needs to try more triangles.
- C) She can't prove the statement by constructing a parallel line outside the triangle.
- D) She didn't prove the statement; she needs to measure each angle.

50)

Dylan's Answer:

I measured the angles of many kinds of triangles accurately and made a table.

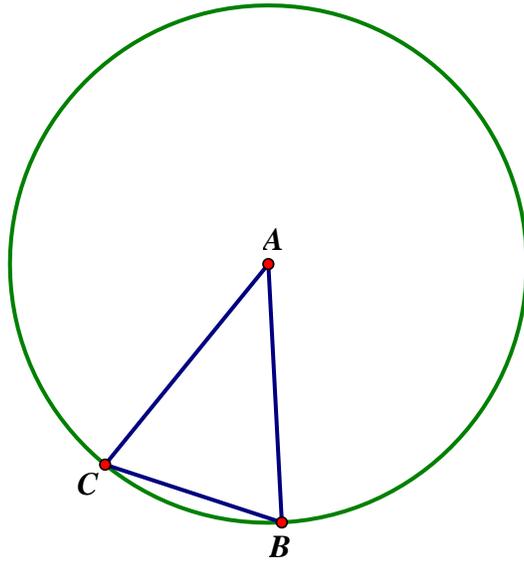
<i>a</i>	<i>b</i>	<i>c</i>	<i>total</i>
110	34	36	180
95	43	42	180
35	72	73	180
10	27	143	180

They all added up to 180° .

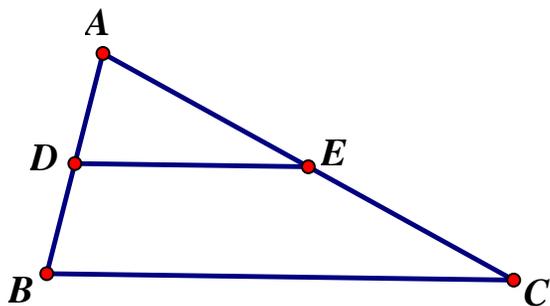
So Dylan says it's true.

- A) He proved the statement is true for all triangles.
- B) He showed the statement is true for some triangles, but he needs to try more triangles.
- C) He can't prove the statement by measuring angles.
- D) He did not measure the angles accurately enough; he should have measured to at least the nearest tenth.

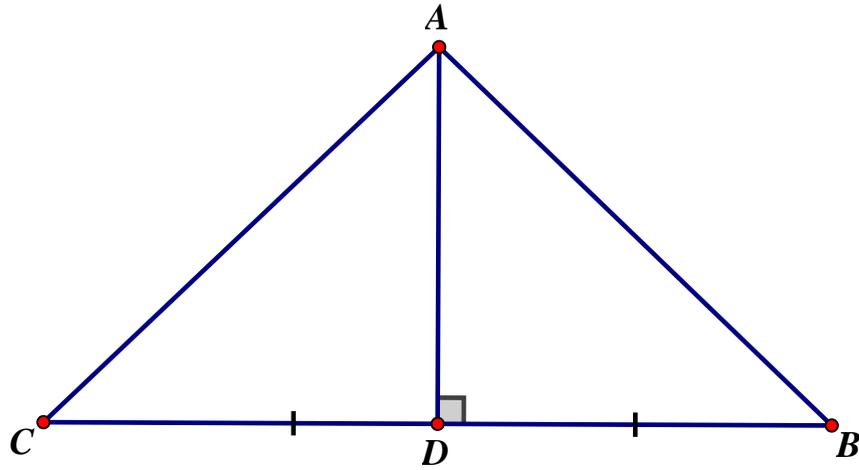
51) B and C lie on a circle with center A . Prove that $\triangle ABC$ is isosceles. (In other words, explain why $\triangle ABC$ is isosceles.)



52) In $\triangle ABC$, D is the midpoint of \overline{AB} and E is the midpoint of \overline{AC} (See the figure below for an example of this). What do you think must be true about \overline{DE} and \overline{BC} ? (Make as many good conjectures as possible.)



53) In $\triangle ABC$, $\overline{CD} \cong \overline{BD}$ and $\angle ADB$ is right. Show that $\triangle ACD \cong \triangle ABD$.



My Beliefs About Geometry

[Teacher Form]

Circle the appropriate category

Gender: Male Female

Directions: Each of the following statements represents a belief about geometry.
Circle the number that shows the extent to which you agree or disagree with each statement.

	5 Strongly Agree	4 Agree	3 Don't Know	2 Disagree	1 Strongly Disagree
1. Geometry involves logical thinking.	5	4	3	2	1
2. Logical arguments are a major part of geometry.	5	4	3	2	1
3. A geometric construction is a drawing used to show a geometric idea.	5	4	3	2	1
4. I can use a computer application to create a geometric construction.	5	4	3	2	1
5. Geometric constructions allow me to explore geometric relationships.	5	4	3	2	1
6. Making conjectures (i.e., educated guesses) is a good habit of geometric learning.	5	4	3	2	1
7. Measurement can help me make conjectures about geometric relationships.	5	4	3	2	1
8. Investigating examples is important when making a conjecture.	5	4	3	2	1
9. If I find a counterexample then the conjecture is false.	5	4	3	2	1
10. Technological tools are particularly useful in forming conjectures.	5	4	3	2	1
11. Technological tools can help me test conjectures.	5	4	3	2	1
12. Proofs are used to justify geometric ideas.	5	4	3	2	1
13. Proofs involve explorations of geometric situations.	5	4	3	2	1
14. Knowing definitions is important in geometric proof.	5	4	3	2	1
15. Understanding what a theorem says helps to prove it.	5	4	3	2	1
16. Whenever I study the proof of a theorem I try to understand each step and reason for it.	5	4	3	2	1
17. In geometry I can use true statements to justify each statement I make.	5	4	3	2	1
18. In geometry I can use facts, rules, definitions, and/or properties to reach a logical conclusion.	5	4	3	2	1

DG Implementation Questionnaire

Name: _____ School: _____

Part 1

Select the most appropriate choice that applies to each of the following questions:

1) How many times per week did the students work in a computer lab/classroom using GSP software?

	None	One time	Two times	More than two times
Week 1				
Week 2				
Week 3				
Week 4				
Week 5				

2) How many times per week was the geometry class taught in a classroom using one demonstration computer?

	None	One time	Two times	Three times	More than 3 times
Week 1					
Week 2					
Week 3					
Week 4					
Week 5					

3) How many activities from the GSP Curriculum materials did you use per week, and which activities? (List activities used in the appropriate cell.)

	None	One	Two	More than two
Week 1				
Week 2				
Week 3				
Week 4				
Week 5				

4) How often did you do the following (during this month)?

	Almost never	Once	Two or three times	Once a week	Two or more times a week
Engage students in using the dragging and dynamic measurement functions of the DG software to do explorations					
Engage students in making conjectures					
Encourage students to test and refine their conjectures					
Encourage students to attend to underlying relationships rather than to the particulars of a drawing					
Provide suggestions/demonstrations to open up more and/or new sorts of investigations					
Expect students to explain what they observe and help them develop a link between investigations and deductive reasoning					
Expect students to describe their reasoning when they express conjectures or answer questions					

5) How often did you use the following in your teaching (during this month)?

	Almost never	Rarely	Every few sessions	Most sessions	Nearly all sessions
Class discussion					
Individual work					
Small group work					
Teacher demonstration					
Student interaction with you					
Student demonstration					

6) What percent of your students did the following on a regular basis?

	None	1-25%	26-50%	51-75%	76-100%
Construct					
Drag					
Measure					
Animate					
Transform					
Observe what characteristics change and what remain the same					
Investigate mathematical relationships in multiple ways					
Form conjectures					
Test conjectures					
Receive immediate feedback					
Prove (or disprove) their conjectures					
Cooperate with peers by offering and requesting help					
Share findings/thoughts with the class					
Converse with you (teacher)					
Revise ideas based on evidence					
Revise approaches to finding solutions					
Apply knowledge to new settings and scenarios					

Part 2

For the following questions, please provide the information requested. If you have not experienced these situations, indicate so.

1) Please describe how you engaged students in doing experimentation (constructing, dragging, measuring, animating, transforming, observing, etc.) with DG tools during this month.

2) Please describe how you engaged students in forming and testing conjectures with DG tools during this month.

3) Please describe an instance in which you engaged students in coming up with their own proof of a conjecture.

4) Please describe how the use of DG tools has helped you improve your understanding of students (via formative assessment of students' learning). Provide explicit examples.

5) Please describe how the use of DG tools has influenced the way you plan and implement instruction. Provide explicit examples.

6) Please describe how students have responded to the use of DG tools?

7) Provide examples of students' work that clearly show the benefits of using DG tools.

8) Describe any challenges you have experienced and any concerns you might have regarding the use of DG tools.

Geometry Teaching Observation Protocol (GTOP)

DG Group

Dynamic Geometry Project

Texas State University

I. BACKGROUND INFORMATION

Name of teacher _____ Announced Observation? _____
(yes, no, or explain)

Location of class _____
(district, school, classroom, computer lab)

Number of students observed _____ Type of Class _____
(Regular, Pre-AP (TAKS), or Pre-AP (EOC))

Observer _____ Date of observation _____

Start time _____ End time _____

Control or treatment classroom? _____

Observation number _____ Second Observer: _____
(First, second, third, fourth)

II. DESCRIPTION OF TEACHING CONTEXT

In the space below please give a brief description of the lesson observed, the classroom setting in which the lesson took place (regular classroom, computer lab, setting arrangements, etc.) Capture, if you can, the defining characteristics of this situation that you believe provide the most important context for understanding what you will describe in greater detail in later sections. Use diagrams if they seem appropriate.

III. DESCRIPTION OF EVENTS

Record here events which may help in documenting the ratings.

Time	Description of Events

IV. DESCRIPTION OF *IMPLEMENTED* DYNAMIC GEOMETRY LESSON

	Never Occurred			Very Descriptive	
	0	1	2	3	4
1. The lesson has appropriate objectives for the concept(s) being explored. (use the lesson plan to help make this assessment).	0	1	2	3	4
2. The lesson includes tasks that involve the use of the dynamic moving and dynamic measurement functions of the software to do observations and explorations.	0	1	2	3	4
3. The activities develop the notion of “figure” rather than “drawing” – attending to underlying relationships rather than particulars of a specific drawing.	0	1	2	3	4
4. The activities in the lesson are designed to move students along the following trajectory (or part of it): from initial conjecture, to investigation, to more thoughtful conjecture, to verification and proof.	0	1	2	3	4
5. The activities are designed so that students observe interesting mathematical phenomena and are motivated and challenged to understand why these phenomena occur and to explain them logically.	0	1	2	3	4

V. DESCRIPTION OF *IMPLEMENTED* DYNAMIC GEOMETRY LESSON

The lesson leads the class to:

6. construct dynamic geometric objects with limited explicit guidance by the teacher or some materials (e. g., handout w/ specific steps),	0	1	2	3	4
7. “drag” for the purpose of determining whether observed quantities and/or relationships remain constant, are changing, or are otherwise impacted by the action,	0	1	2	3	4
8. measure for the purpose of further exploring relationships and/or conjecturing, and/or disproving,	0	1	2	3	4
9. observe the various quantities/qualities of their sketches, particularly relationships that might lead to conjectures or proofs,	0	1	2	3	4
10. investigate mathematical relationships in multiple ways including using transformations and/or animations,	0	1	2	3	4
11. form conjectures based on their interactions with the software,	0	1	2	3	4
12. test conjectures using the software or other means (i.e., deductive reasoning),	0	1	2	3	4

13. take advantage of immediate feedback (as offered by the software or teacher).	0	1	2	3	4
14. reason inductively and/or deductively throughout,	0	1	2	3	4
15. prove (or disprove) their conjectures.	0	1	2	3	4

VI. ASSESSMENT OF *QUALITY* OF TEACHING

LEVEL OF COGNITIVE DEMAND	Never Occurred			Very Descriptive	
16. Students engage in recollection of facts, formulae, or definitions. (<i>Memorization</i>)	0	1	2	3	4
17. Students engage in performing algorithmic type of problems and have no connection to the underlying concept or meaning. (<i>Procedures without connections</i>)	0	1	2	3	4
18. Students engage on the use of procedures with the purpose of developing deeper levels of understanding concepts or ideas. (<i>Procedures with connections</i>)	0	1	2	3	4
19. Students engage in complex and nonalgorithmic thinking, and explore/investigate the nature of the concepts and relationships. (<i>Doing Mathematics</i>)	0	1	2	3	4
TEACHERS' OBSERVED KNOWLEDGE					
20. The teacher has a solid grasp of the geometry content at the level he or she is teaching. (<i>Grade level geometry knowledge</i>)	0	1	2	3	4
21. The teacher has knowledge of the use of instructional techniques specific to teaching geometry along with a deep understanding of the subject to appropriately integrate instruction with the concepts. (<i>Mathematical pedagogical knowledge/knowledge for teaching</i>)	0	1	2	3	4
22. The teacher leads students to the appropriate geometric dynamic actions (construct, drag, measure, transform, and animate) according to the goals of the lesson. (<i>Dynamic geometrical knowledge for teaching</i>)	0	1	2	3	4

VII. ASSESSMENT OF ENGAGEMENT AND DISCOURSE

23. Students are encouraged to share questions, hints, ideas, and/or progress with other students.	0	1	2	3	4
24. Teacher circulates, observes (to monitor progress), ask questions, and provides necessary help as students work.	0	1	2	3	4
25. Teacher initiates class discussion when necessary.	0	1	2	3	4

VIII. TEACHERS' STAGE OF USE OF THE SOFTWARE

26. Does the teacher use technology to teach geometry? _____
(yes, no, or explain)

27. If the teacher used technology, specify the type:

- Graphing calculator
- Dynamic Geometry Software, specify: _____
- Other: _____

28. If the teacher used technology, identify the stage of use of software (PICK ONE)

- Entry stage (Teacher is in the stage of learning new technology.)
- Adoption stage (Teacher uses new technology to support traditional instruction.)
- Adaptation stage (Teacher integrates new technology into traditional classroom practice.)
- Appropriation stage (Teacher focuses on cooperative, project-based, and interdisciplinary work - incorporating the technology as needed and as one of many tools.)
- Invention (Teacher discovers new uses for technology tools.)

Circle your level of confidence in your answer choice for Question 28:

1 = not at all confident 2 = somewhat confident 3 = confident 4 = very confident

Additional comments you may wish to make about this lesson.

TPACK OBSERVATION PROTOCOL

Date: _____ Time: _____ School: _____ Teacher: _____	
Time	Descriptive Notes
	<p>The physical setting <i>How is space allocated? What objects, resources, technologies are in the setting?</i></p> <p>The participants <i>How many students are present? Are there any other people present?</i></p> <p>Activities and interactions <i>What is going on? How do students and the teacher interact with the lesson and with one another?</i></p> <p>Conversation What is the content of conversations? Who speaks to whom? Who listens?</p> <p>What is the role of technology in this lesson?</p> <p>What are the benefits of using technology in this lesson?</p> <p>Are there any apparent disadvantages of using technology in this lesson?</p> <p>How does teacher manage the classroom when teaching this lesson?</p> <p>How does the teacher assess student learning in this lesson?</p>

TPACK DEVELOPMENT INITIAL INTERVIEW PROTOCOL

The purpose of this interview is to develop an understanding of how you have developed your knowledge for teaching geometry with the Geometer's Sketchpad.

Background questions:

How long have you been teaching?

How long have you been teaching geometry?

How long have you been teaching with Sketchpad?

Have you ever used another dynamic geometry software program either in learning or teaching? If so, for how long?

Have you assumed additional roles/positions (e.g., department chair, math specialist, etc.) during your teaching career? If so, briefly describe your responsibilities.

Main interview questions:

1. When you hear the words "dynamic geometry" what comes to your mind?
2. How and when did you first hear about Sketchpad? Briefly describe this experience.
3. Why did you decide to teach geometry with Sketchpad?
4. What is your current view and understanding about integrating Sketchpad as a learning tool in geometry?
 - a. How do you define "integration" of Sketchpad in teaching and learning of geometry?
 - b. Describe a lesson you taught that represents your current view.
5. What specific geometry topics lend themselves to Sketchpad as a learning tool?
6. What do you see as barriers for integrating Sketchpad in teaching and learning of geometry?
7. How did your knowledge and skills with Sketchpad changed through your work in the DGP?
8. How has your knowledge about students' understanding, thinking, and learning about geometry topics with Sketchpad changed through your work in the DGP?
9. How has your conception of incorporating Sketchpad in teaching specific topics in geometry changed through your work on the DGP project? Give an example from a lesson you taught last year.
10. When you start using Sketchpad with your students (at the beginning of a school year), what strategies do you use to guide them in learning about the software?(Learning about Sketchpad as students are learning about a specific geometry topic or focusing on learning about the technology and later as the geometry context?)
11. In addition to your participation in the DGP, what kind of activities (e.g., professional development, conferences, self-directed study, Internet resources) have you engaged in that lead you to adopt teaching and learning of geometry with Sketchpad? Briefly describe what influenced your choice.
12. How important is (to you) the integration of Sketchpad into teaching and learning of geometry?
13. Do you evaluate the results of your decision to integrate teaching and learning of geometry with Sketchpad? How do you do it?

**Sample of Eight Different Activity Sheets and
Protocols Used for Interviews**

Properties of Kites

Introduction

In this activity, you will investigate kites. A **kite** is a quadrilateral with exactly two distinct pairs of congruent consecutive sides. The angles between two congruent sides are called **vertex angles** and the other two angles are called **nonvertex angles**.

Investigation

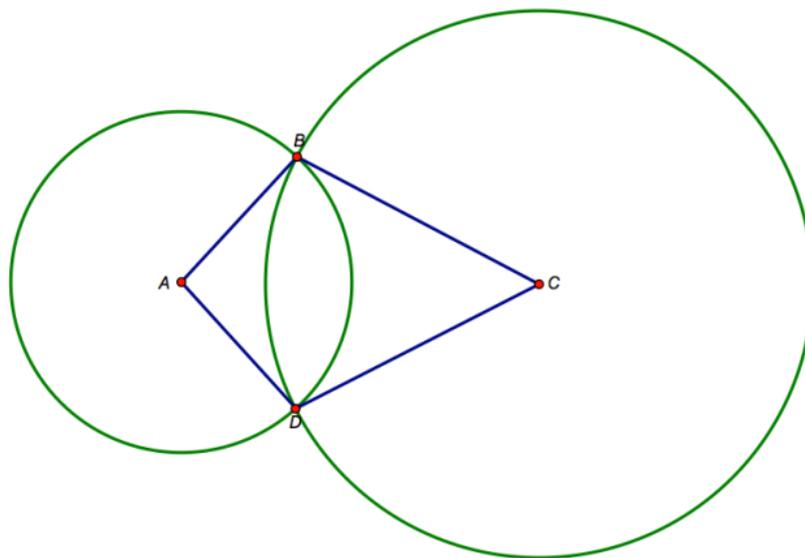
1. Using a straight edge and compass, construct *a kite based on its definition described above*.

How to Construct a Kite

- Draw two arbitrary segments \overline{AB} and \overline{BC} with B as the shared point.
- Construct \overline{AD} and \overline{DC} so that $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$.

How to construct the congruent segments.

1. Draw points A and B, use the compass to construct a circle with center A through B;
2. draw point C and use a compass to construct another circle with center C and which intersects the first circle at point B;
3. the two circles intersect at two points, with one of them point B. Label the second point of intersection D;
4. construct segments;



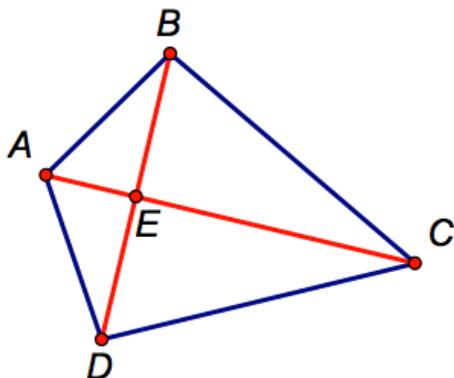
2. Which two angles of the kite ABCD are vertex angles? Which two angles are non-vertex angles?

3. What do you think may be true about each pair of opposite angles? What could you do to check for things that might be true?

4. How can you check to see if this is true for other kites?

5. Formulate a conjecture – we will call it your Kite Angles Conjecture.

6. Construct the two diagonals \overline{AC} and \overline{BD} , with E as their intersection point.



7. What do you think is true about the measure any of the angles between the two diagonals (say $\angle AEB$)?
What can we do to look for things that may be true?
What is the relationship between the two diagonals?
Do you think that this property still holds for other kites?
How can we check?
-
-

8. Formulate your conjecture - the Kite Diagonals Conjecture.



9. Measure the lengths of the segments on the diagonals (AE , EC , BE , and ED). What do you notice about the segments? Does either diagonal bisect the other? Do you think that this property still holds for other kites? How can we check?

10. Formulate your conjecture - the Kite Diagonal Bisector Conjecture.

11. Prove your Kite Angles Conjecture.

PROPERTIES OF KITES

Introduction

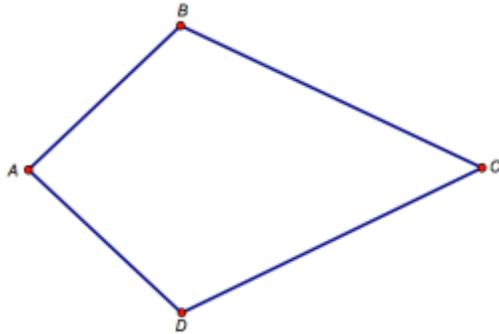
In this activity, you will use Geometer's Sketchpad (GSP) to investigate kites. A **kite** is a quadrilateral with exactly two distinct pairs of congruent consecutive sides. The angles between two congruent sides are called **vertex angles** and the other two angles are called **nonvertex angles**.

Investigation

1. In GSP, construct *a kite based on its definition described above*.

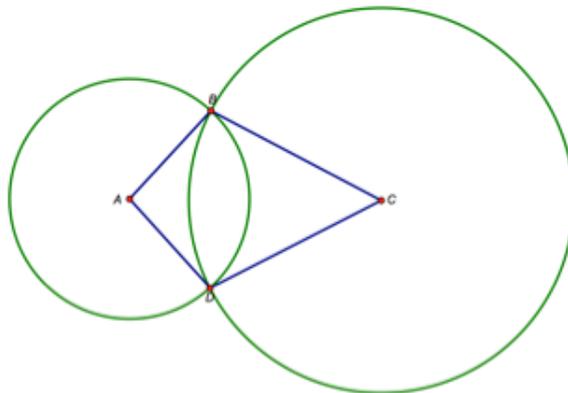
How to Construct a Kite

- Draw two arbitrary segments \overline{AB} and \overline{BC} with B as the shared point.
- Construct \overline{AD} and \overline{DC} so that $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$.



How to construct the congruent segments.

1. Select point A and then select point B, use menu Construct/Circle by Center + Point to construct a circle;
2. Select point C and then select point B, use menu Construct/Circle by Center + Point to construct another circle;
3. The two circles intersect at two points, with one of them point B;
4. Click on another intersection point of the two circles and label that point D;
5. Construct segments;
6. Hide the two circles.



4. How can you use GSP to see if this is true for other kites?

5. Formulate a conjecture – we will call it your Kite Angles Conjecture.

9. Measure the lengths of the segments on the diagonals (AE , EC , BE , and ED). What do you notice about the segments? Does either diagonal bisect the other? Do you think that this property still holds for other kites? How can we check?

10. Formulate your conjecture - the Kite Diagonal Bisector Conjecture.

11. Prove your Kite Angles Conjecture.

Properties of Kites

Introduction

In this activity, you will investigate kites. A kite is a quadrilateral with exactly two distinct pairs of congruent consecutive sides. The angles between two congruent sides are called vertex angles and the other two angles are called nonvertex angles.

Investigation

1. Using a straight edge and compass, construct *a kite based on its definition described above.*

Give student a chance (a least a few minutes) to proceed on their own.

Encourage them to think about the definition of a kite.

If the student does not progress, ask the student to describe each of the key vocabulary words in the definition: quadrilateral, congruent sides, consecutive sides, distinct pairs. If the student does not know the meaning of a word, guide them to the meaning.

If you are uncertain if the student understands the definition of a kite.

Have them draw two examples on their paper. Have them tell you why what they drew is a kite.

Then have the student construct a kite. Let the student attempt this for a few minutes without aid. It is likely that student will “draw” a kite instead of constructing one.

Let the student do this and see if they label congruent sides according to the definition.

Then ask the student how they know that the labeled sides are in fact congruent.

Ask the student if he/she can use the compass and straight edge to construct the picture to ensure that the sides are congruent. If they have difficulty in this construction, show the instructions on the Construct a Kite Page.

If the student has difficulty constructing the congruent segments triangle, show the student the page with construct congruent segments instructions.

If after 10 minutes with all hints, student cannot construct the kite, construct the kite for them.

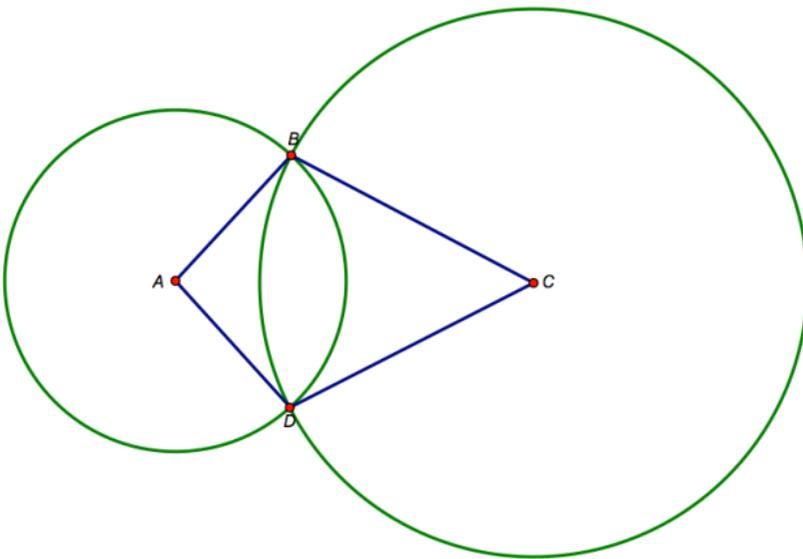
How to Construct a Kite

Draw two arbitrary segments \overline{AB} and \overline{BC} with B as the shared point.

Construct \overline{AD} and \overline{DC} so that $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$

How to construct the congruent segments.

2. Draw points A and B, use the compass to construct a circle with center A through B;
3. Draw point C and use a compass to construct another circle with center C which intersects the first circle at point B;
4. The two circles intersect at two points, with one of them point B.
5. Label the second point of intersection D; construct segments;



2. Which two angles of the kite ABCD are vertex angles?
Which two angles are non-vertex angles?

If the student does not understand the question. Refer them to the definition of vertex and non-vertex angles.

If the student incorrectly identifies the angles. Refer them to the definition.

If after 3 minutes the student cannot correctly identify the angles, identify the vertex and non-vertex angles for them. Then ask them to draw a kite on their paper and identify the angles on this new kite.

3. What do you think may be true about each pair of opposite angles?
What could you do to check for things that might be true?

For the first question we are looking for students to say that the non-vertex angles are congruent. The vertex angles are not necessarily congruent. They may say this in informal language.

That is OK.

For the next question, we are looking for them to say “measure the angles”. They may say, “The angles look the same”. Ask them how they could check to see if this is true.

If after a few minutes students are stumped, ask the student “what are the measures of the angles”.

Let them come up with the idea to measure the angles using a protractor. If they do not, then you can suggest this to them. Give them time to remember how to do this.

If they don't, then show them how to measure the angle.

4. How can you check to see if this is true for other kites?

We are looking for “draw some more kites” or “prove it”.

Often students suggest constructing another kite. Tell the student construct a second kite.

Ask the student how he/she chose what kind of kite to construct.

We are interested if students think about constructing a very different looking kite.

See if the student measures the angles on the new kite.

If not ask the student how he/she can check if their conjecture holds for this new kite as well.

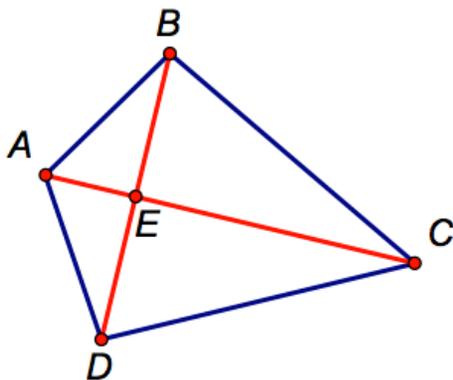
Ask the student if he/she thinks the conjecture holds for all kites.

5. Formulate a conjecture – we will call it your Kite Angles Conjecture.

If the student is not familiar with the word conjecture, explain what it means in this context.

The students will likely repeat what they said in 3). Have them write this down. Then ask them to formulate the conjecture in as mathematical a way as possible, like their teacher or textbook might do. They may state this in terms of the labels of the figure or using the vocabulary associated with the kite. (For kite ABCD, $\angle B \cong \angle D$ vs. for any kite, the non-vertex angles are congruent.)

6. Construct the two diagonals \overline{AC} and \overline{BD} , with E as their intersection point.



7. What do you think is true about the measure any of the angles between the diagonals (say $\angle AEB$)?

What can we do to look for things that may be true?

What is the relationship between the two diagonals?

Do you think that this property still holds for other kites? How can we check?

For the first question we are looking for students to say that these are right angles.

They may say this in informal language. That is OK.

For the second question, we are looking for the student to say “measure the angles” or “check that the angles are right angles”.

For the third question, we are looking for the student to say “the diagonals are orthogonal”.

If after a few minutes, students are stumped, ask the student “what are the measures of the angles”

(e.g. $\angle AED$ and $\angle BEC$) or “what kind of angles are they”.

Let the student come up with the idea to measure the angles on their own.

If they do not, then you can suggest this to them.

The student should check if the angles are right in both kites they have constructed.

8. Formulate your conjecture - the Kite Diagonals Conjecture.

- The student will likely repeat what he/she said in 7). Have them write this down. Then ask him/her to formulate the conjecture in as mathematical a way as possible, like their teacher or textbook might do.

9. Measure the lengths of the segments on the diagonals (\overline{AE} , \overline{EC} , \overline{BE} , and \overline{ED}). What do you notice about the segments? Does either diagonal bisect the other? Do you think that this property still holds for other kites? How can we check?

- For the first two questions we are looking for the student to say that one diagonal is bisected, the other is not.
- The student should check that this is true for each kite he/she has constructed.
- Ask the student to describe what is the difference between the two diagonals? Why might one be bisected, but not the other?

10. Formulate your conjecture - the Kite Diagonal Bisector Conjecture.

The students will likely repeat what he/she said in 9) using very informal language. Have he or she write this down. Then ask him/her to formulate the conjecture in as mathematical a way as possible, like their teacher or textbook might do. Ask the student to use words to distinguish between the role of diagonal \overline{AC} and \overline{BD} . We are looking for "The diagonal connecting the two vertex angles of a kite bisects the other diagonal." Or \overline{AC} , the diagonal connecting the vertex angles, bisects \overline{BD} , the diagonal connecting the non-vertex angles. The student could also refer to the congruent sides.

11. Prove your Kite Angles Conjecture.

Ask the student what it means to “prove” a conjecture?

Ask the student why someone would “prove” a conjecture?

Here we want to hear the student’s conception of proof.

Do not force the student to write a proof in a particular manner. The student can do a two column proof or a paragraph proof or simply a list of statements that help justify the result. He or she may use notation referring to the particular labeled kite (e.g. \overline{AC}) or descriptive words like the pair of congruent sides. If the student says things of the form “this side has to be the same as that side”. Have him/her write this down and ask him/her how we could know exactly which side or angle they are talking about. We are most interested in what comes to the student’s mind as they try to “prove” something. So the most important part is that they think out loud.

It is quite possible that the student will not know how to proceed. Let the student struggle a little while. Then ask the student, “What do we know from the definition of a kite?” Tell the student write this down. The student should talk about congruent sides.

If the student needs further help, ask him/her what we want to prove and what methods do we know for showing congruence. Let the students list some ways and write them down. We are looking for some mention of congruent triangles.

Then ask the student if he/she could “see” or make some triangles which might be useful.

Here is one version of the proof :

On the kite $ABCD$ with $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$, construct diagonal \overline{AC} to form two triangles: $\triangle ABC$ & $\triangle ADC$
The two triangles share the side \overline{AC} . The other two pairs of sides are, respectively, congruent by the construction (or definition) of the kite. Hence $\triangle ABC \cong \triangle ADC$ by Side-Side-Side.
Therefore, the corresponding angles $\angle B$ and $\angle D$ are congruent.

If the student is unable to make progress on the proof, this is OK.

We are looking to see how far the student can go if given some hints.

3. What relationship do you observe between \overline{DE} and \overline{BC} ?

4. Measure the lengths of \overline{DE} and \overline{BC} . What do you observe?

5. Measure $\angle ABC$ and $\angle ADE$. What do you observe? What does your finding imply?

6. Based on you findings in 4 and 5, state a conjecture about the relationship between \overline{DE} (a midsegment connecting two sides of a triangle) and \overline{BC} (the third side)?

Mary did this activity as well. Her conjecture was that
“A midsegment connecting two sides of a triangle is parallel to the third side and is half as long.” Can you use a protractor and compass to test the conjecture. Do you think her conjecture is true?

10. Prove or disprove Mary’s conjecture. You might write your explanation as an argument in paragraph form or as a two-column proof.

Three midsegments investigation

1. Construct arbitrary $\triangle ABC$ and its three midsegments using a straight-edge. Construct an interior $\triangle DEF$ by connecting the midpoints D, E, and F respectively.

2. Calculate the perimeter and area using a ruler and a calculator.

Perimeter of $\triangle DEF$ = _____ Area of $\triangle DEF$ = _____
Perimeter of $\triangle ABC$ = _____ Area of $\triangle ABC$ = _____

3. Find the ratios of perimeters and areas.

Ratio of Perimeters =

Ratio of Areas =

4. What is the relationship between $\triangle DEF$ and $\triangle ABC$? Can you prove it? If so, write a proof.

5. What is the relationship between $\triangle DEF$ and $\triangle ADE$? Can you prove it? If so, write a proof.

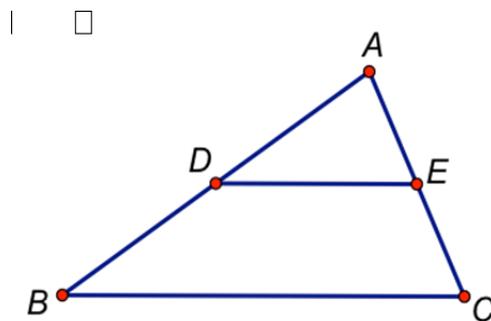
Midsegments of Triangles

Introduction

In this activity, you will use Geometer's Sketchpad (GSP) to investigate the midsegment, a segment that connects the midpoints of two sides of a triangle. First, students will construct and investigate one midsegment and the relationship of the new small triangle to the original triangle. Then, all three midsegments will be constructed and this figure will be explored.

One midsegment investigation

1. In GSP, construct arbitrary $\triangle ABC$
2. Construct the midpoints of \overline{AB} and \overline{AC} , and label them D and E respectively. Construct \overline{DE} . \overline{DE} is a midsegment of $\triangle ABC$

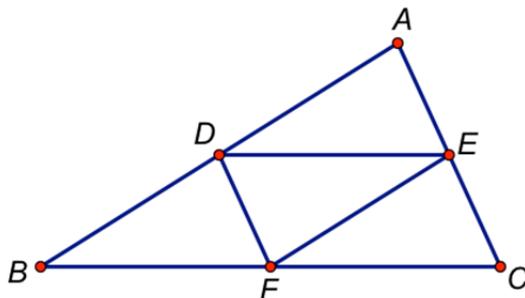


Mary did this activity as well. Her conjecture was that “A midsegment connecting two sides of a triangle is parallel to the third side and is half as long.” Can you use GSP to test the conjecture. Do you think her conjecture is true?

10. Prove or disprove Mary’s conjecture. You might write your explanation as an argument in paragraph form or as a two-column proof.

Three midsegments investigation

1. In GSP, construct arbitrary $\triangle ABC$ and its three midsegments.



2. Calculate the perimeter and area using the tools in the Measurement menu.

Perimeter of $\triangle DEF$ = _____ Area of $\triangle DEF$ = _____
Perimeter of $\triangle ABC$ = _____ Area of $\triangle ABC$ = _____

3. Find the ratios of perimeters and areas. What happens to these ratios as a vertex of $\triangle ABC$ is dragged?

Ratio of Perimeters =

Ratio of Areas =

4. What is the relationship between $\triangle DEF$ and $\triangle ABC$? Can you prove it? If so, write a proof.

5. What is the relationship between $\triangle DEF$ and $\triangle ADE$? Can you prove it? If so, write a proof.

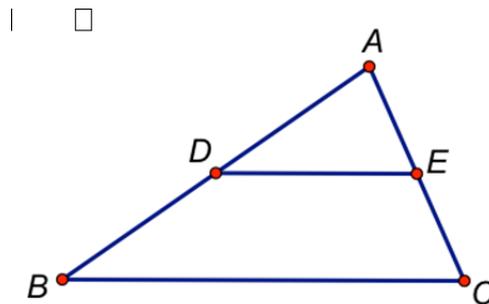
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One midsegment investigation

1. In GSP, construct arbitrary $\triangle ABC$
 - Give teacher time to construct triangle.
 - Show them how to do it if they have difficulty.
2. Construct the midpoints of \overline{AB} and \overline{AC} , and label them D and E respectively. Construct \overline{DE} . \overline{DE} is a midsegment of $\triangle ABC$



- Give teacher time to construct midpoints.
 - Pay attention to see if teacher constructs the midpoints, versus draws the midpoint. If they draw the midpoint, ask them how they know the point is exactly in the middle.
3. What relationship do you observe between \overline{DE} and \overline{BC} ?
 - We are looking for "A midsegment connecting two sides of a triangle is parallel to the third side and is half as long." But teacher may not jump to this until questions 4 and 5 below.

4. Measure the lengths of \overline{DE} and \overline{BC} . What do you observe?

- We are looking for “ \overline{DE} is half as long as \overline{BC} .”

5. Measure $\angle ABC$ and $\angle ADE$. What do you observe? What does your finding imply?

- We are looking for the measures of the angles are equal, hence the midsegment and \overline{BC} are parallel.
- If the teacher stops at “angles are equal”, ask them what “kind” of angles these are. Guiding them towards “corresponding angles”. Ask them what the relationship between the angles says about the relationship between the segments \overline{DE} and \overline{BC}

6. Based on you findings in 4 and 5, state a conjecture about the relationship between \overline{DE} (a midsegment connecting two sides of a triangle) and \overline{BC} (the third side)?

- Allow the teacher to state the conjecture in informal language if he/she want to. Have the teacher type this in GSP. Then have the teacher state the conjecture in formal mathematical language “as it would in the textbook.” Have them this as well if it is different.
- Ask the teacher if they think this conjecture is true for all midsegments of any triangle. Ask them how they could use GSP to check if this is true. In this case, we can test other triangles by dragging the vertices. Or check other midsegments, by constructing the other midsegments and measuring. See what the teacher does. If they only talk about one, ask about the other. If they talk about creating another triangle, ask them how they can use GSP to alter the one they have.
- Give the teacher 10 minutes total to measure, observe and conjecture. If the teacher cannot come up with a conjecture proceed to 7.

Mary did this activity as well. Her conjecture was that “A midsegment connecting two sides of a triangle is parallel to the third side and is half as long.” Can you use GSP to test the conjecture.

Do you think her conjecture is true?

10. Prove or disprove Mary’s conjecture. You might write your explanation as an argument in paragraph form or as a two-column proof.

If the teacher has difficulty producing such as a proof, consider the following questions. Give the hints one a time, letting the teacher consider it to see if this is sufficient to let the teacher proceed.

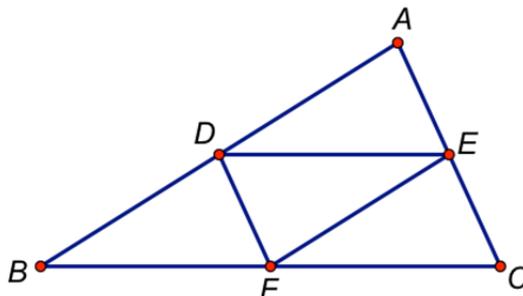
1. What is the relationship between $\triangle ADE$ and $\triangle ABC$? (If you need help, see question 2)
2. Do these two triangles have the same shape? So what is the relationship between them?
3. Can you prove that $\triangle ADE \sim \triangle ABC$? If so, please do so. (If you need help, see questions 4-6)
4. Observe $\triangle ADE$ and $\triangle ABC$. Based on the given conditions, what do we already know? (What is the relationship between angle DAE and angle BAC? What is the ratio $\frac{AD}{AB}$? What is the ratio $\frac{AE}{AC}$?)
5. Can the AA Similarity postulate be used to do the proof? If yes, why? If not, why not?
6. Can the SAS Similarity postulate be used to do the proof? If yes, why? If not, why not?
7. Based on you answers to questions 1-6, explain/prove that $\triangle ADE \sim \triangle ABC$.
8. What are the properties of similar triangles
9. What can you say about the ratio $\frac{DE}{BC}$?
10. What can you say about angle ADE and angle ABC?
11. What relationship between \overline{DE} and \overline{BC} does Question 10 imply?

Based on your answers to questions 7-11, write a proof explaining that \overline{DE} is parallel to \overline{BC} , and is half as long.

Three midsegments investigation

1. In GSP, construct arbitrary $\triangle ABC$ and its three midsegments.

- Again check if the teacher actually constructs the figure.



2. Calculate the perimeter and area using the tools in the Measurement menu.

- Give the teacher time to figure out how to do this in GSP. Only if they get stuck, tell them how.

Perimeter of $\triangle DEF$ = _____ Area of $\triangle DEF$ = _____

Perimeter of $\triangle ABC$ = _____ Area of $\triangle ABC$ = _____

3. Find the ratios of perimeters and areas. What happens to these ratios as a vertex of $\triangle ABC$ is dragged?

- They may come to this conclusion either by calculating in their head, using GSP to calculate the ratio, or through logic.

Ratio of Perimeters = $\frac{1}{2}$ Ratio of Areas = $\frac{1}{4}$

4. What is the relationship between $\triangle DEF$ and $\triangle ABC$? Can you prove it? If so, write a proof.

- We would like the teacher to use the previous result to see that the sides of $\triangle ABC$ are twice as long as those of $\triangle DEF$. However, the teacher may want to prove it all over again for the second and third side. Let them do this. After they are done, ask them if we could have used the result of the first part.

5. What is the relationship between $\triangle DEF$ and $\triangle ADE$? Can you prove it? If so, write a proof.
- We would like the teacher to use the previous result to see that $\triangle DEF$ and $\triangle ADE$ are congruent.
 - If the teacher does not make progress ask them what they know about the sides of the triangles.
 - If still no progress, ask which segments are congruent. Have them use congruence marks on their diagram to illustrate this.
 - If they say the two triangles are congruent. Ask them to justify their answer. Have them type their justification even in informal language in GSP. Then ask them to state it as they might see in a textbook and type this in GSP.

Minimum Length Investigation

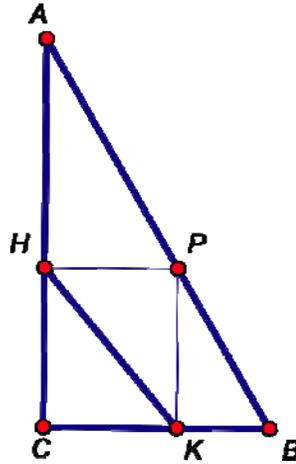
Introduction

In this activity, you will use Geometer's Sketchpad (GSP) to investigate the following problem:

You are given a right triangle ABC , \overline{AB} being the hypotenuse. Take a point P on \overline{AB} . Construct a line parallel to \overline{AC} through P . Name H the point of intersection with \overline{AC} . Similarly, construct a line parallel to \overline{BC} through P . Name K the point of intersection with \overline{BC} . For which position of P does the segment \overline{HK} have minimum length?

1. In GSP, construct a right triangle ABC with \overline{AB} being the hypotenuse.
2. Complete the construction described in the problem description.

3. Observe the illustration of the problem. Where should point P be on segment \overline{AB} so that segment \overline{HK} has minimum length? What can you do in GSP to help you determine this?



4. Based on your exploration, state a conjecture.

Mary did this activity as well. Mary's conjecture was that "the segment \overline{HK} has minimum length, if the segment \overline{CP} is perpendicular to \overline{AB} ?"

5. Prove or disprove Mary's conjecture.

Minimum Length Investigation

Introduction

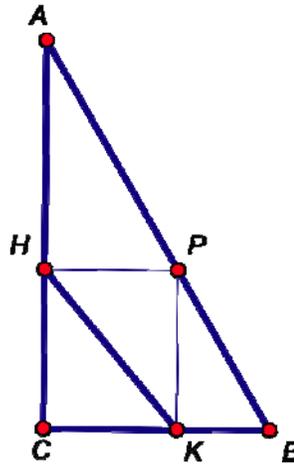
In this activity, you will investigate the following problem:

You are given a right triangle ABC , \overline{AB} being the hypotenuse. Take a point P on \overline{AB} . Construct a line parallel to \overline{AC} through P . Name H the point of intersection with \overline{AC} . Similarly, construct a line parallel to \overline{BC} through P . Name K the point of intersection with \overline{BC} . For which position of P does the segment \overline{HK} have minimum length?

1. Using a straight edge and compass, construct a right triangle ABC with \overline{AB} being the hypotenuse.
 - Give teachers chance to start construction.
 - Note if they make drawing or construction. If they make a drawing ask him how they could construct it.
 - If they do not know how to construct a right triangle, it is OK. Move on to part 3.

2. Complete the construction described in the problem description.

3. Observe the illustration of the problem. Where should point P be on segment \overline{AB} so that segment \overline{HK} has minimum length? What can you do to help you determine this?



- There are many approaches a teacher could take.
 - Empirical: measure HK for figure they have, and then repeat for another position of P. Encourage teacher to try multiple points and ask if the results would be similar for a different shaped right triangle (i.e. where $BC \gg AC$).
 - Coordinate: they could try to give coordinates to each of the points, e.g. $C = (0,0)$, $A = (0, a)$, $B = (b, 0)$. They are likely to get stuck here. Ask what they know about P? Lead them to thinking about the equation of a line: $y = -b/a x + a$. See if they can figure out the relationship between the coordinates of P, H and K. This method leads to use of the distance formula. They then must try to minimize a quadratic (or square root) equation. The details maybe too involved for the teacher. See if the teacher can at least describe the process.
 - Geometrical: they can look for geometrical relationships in the figure, e.g. looking for similar triangles etc. Let them struggle a little here. With this approach they may focus on the single example given in the figure. Ask then if these relationships would still appear to hold if P was much closer to B or to A. This approach is most likely to lead to the conjecture we are looking for, but may require a few leading questions.
 - Ask what is the shape of HPKC. *rectangle*
 - What properties does rectangle have?
 - What about the diagonals of a rectangle? *Congruent*
 - Can we interpret the diagonal CP in a special way? *It's length is the distance from the point C to the line segment AB.*
- If a teacher tries approach 1 or 2 and quickly gives up without making progress, suggest approach 3.

4. Based on your exploration, state a conjecture.
- Have the teacher state their conjecture. In this activity, the desired conjecture is difficult to see, so encourage the teacher to make any conjecture they think is reasonable.
 - The min length is always less than BC and AC.
 - The ratio between AP and PB is _____.
 - The min length is determined when the quadratic equation _____ is minimum.
 - ***The min length occurs when the CP is perpendicular to AB.*** 😊
 - Write down the teacher's conjecture.
 - If the conjecture is in informal language, write it down this way and then ask the teacher to state the conjecture as it might be stated in a textbook.
 - Once the teacher has made a conjecture, show Mary's conjecture.

Mary did this activity as well. Mary’s conjecture was that “the segment \overline{HK} has minimum length, if the segment \overline{CP} is perpendicular to \overline{AB} ?”

5. Prove or disprove Mary’s conjecture.

[If the teacher needs help, we may ask questions such as:

- *Why does \overline{CP} have minimum length when \overline{CP} is perpendicular to \overline{AB} ?*
- *What is the relationship between \overline{HK} and \overline{CP} ? Why is this the case?*
- *Write your explanation or proof based on your answers to these two questions.]*

[The following is a sample proof:

We know that the shortest distance between a point and a line (or segment) is a perpendicular line segment constructed from the point to the line (or segment). In this problem, when $\angle CPB = 90^\circ$, \overline{CP} exactly represents the shortest distance between point C and segment \overline{AB} . Based on the construction of quadrilateral PHCK, it is a rectangle since it is a parallelogram with right angles. \overline{HK} and \overline{CP} congruent because the two diagonals of any rectangle are congruent. Therefore, if the location of P makes $\angle CPB$ a right angle, then \overline{HK} has minimum length.]

** The teacher might consider a set of circles centered in C and going through P. The circle whose radius is minimum is the circle tangent to segment \overline{AB} ; therefore \overline{HK} has minimum length when P is the point of contact (point of tangency) between \overline{AB} and the circle centered in C. This would be excellent!*

Let A = (0, a), B = (b, 0) and C = (0,0). Then \overline{AB} lies on the line with slope $-\frac{a}{b}$ and y-intercept a. Then coordinates of P are $(x, -\frac{a}{b}x + a)$ for $0 < x < b$. The coordinates of H are $(0, -\frac{a}{b}x + a)$ and of K are $(x, 0)$. Hence the length of \overline{HK} is the given by the

distance formula $\sqrt{x^2 + (-\frac{a}{b}x + a)^2} = \sqrt{(1 + \frac{a^2}{b^2})(x - \frac{a^2 b}{a^2 + b^2})^2 + \frac{a^2 b^2}{a^2 + b^2}}$. This distance is minimized when $x = \frac{a^2 b}{a^2 + b^2}$ and the

minimum distance is $\sqrt{\frac{a^2 b^2}{a^2 + b^2}}$ (note this can be determined using calculus or by completing the square). The slope of the line

through points P and C is then $\frac{-\frac{a}{b} \frac{a^2 b}{a^2 + b^2} + a}{\frac{a^2 b}{a^2 + b^2}} = \frac{-a^3 + a^3 + a b^2}{a^2 b} = \frac{b}{a}$. Hence the line through points P and C is perpendicular to the line

through A and B.

Note using a similar triangle argument or using the distance formula again, you can show that the $\frac{\overline{AP}}{\overline{PB}} = \frac{a^2}{b^2}$.

Parallelogram Investigation

Take any parallelogram and construct squares externally on each side.

The centers of the four squares are the vertices of a quadrilateral.

Make a conjecture on the shape of the quadrilateral.

Initial conjecture:

1. Using a protractor, ruler, and compass, construct a parallelogram.
 - a. Construct two segments with a common vertex.
 - b. Select one segment and the vertex not on the segment. Construct a parallel line.
 - c. Repeat with the other segment and vertex.
 - d. Point of intersection is the fourth vertex of the parallelogram.
Construct the remaining two segments.

Create this construction on the given graph paper.

2. Construct squares externally on each side of the parallelogram.
 - a. Mark one of the vertices as a center of rotation. Select one of the segments containing that vertex and rotate it 90 degrees.
 - b. Repeat until all squares are constructed.
 - c. Construct diagonals of all the squares to find the centers of the four squares.

3. Connect the centers with segments.
 - a. What do you observe? Make a conjecture on the shape of the new quadrilateral.
 - b. Make any measurements to confirm your conjecture. Does your initial conjecture hold?

Final conjecture:

Mary did this activity as well. Her conjecture was that

*If squares are constructed externally on each side of a parallelogram,
then the quadrilateral created by the centers of the four squares is a square.*

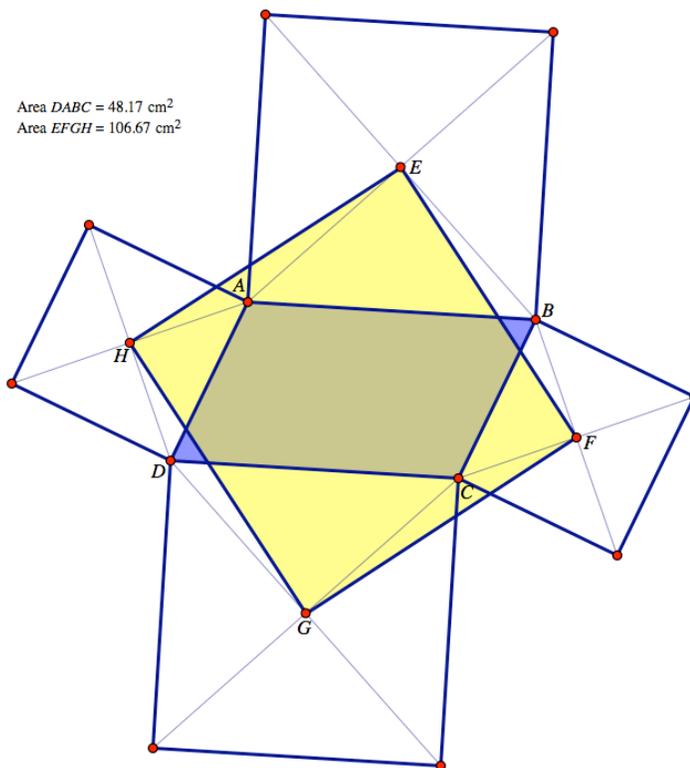
Prove or disprove Mary's conjecture.

Based on your answers to these questions, explain/prove that quadrilateral $HEFG$ is a square. *(There is a reference picture of labeled points on the next page)*

What is the relationship between the area of the “new” quadrilateral and the area of the original parallelogram?

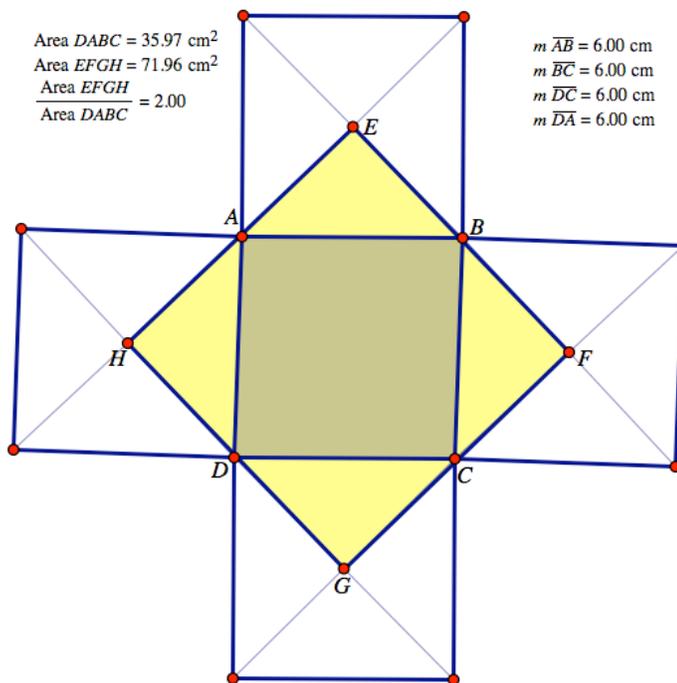
Make any necessary measurements and state your conjecture:

(Measure the areas and their ratio. The ratio will vary.)



Mary did this activity as well. She made two conjectures:

1. *The ratio of the area of the square (EFGH) to the area of the parallelogram (ABCD) is **greater than or equal to 2**.*
2. *The ratio is 2 when the parallelogram is a square.*



Mary did this activity as well. She made two conjectures:

1. *The ratio of the area of the square (EFGH) to the area of the parallelogram (ABCD) is **greater than or equal to 2**.*
2. *The ratio is 2 when the parallelogram is a square.*

Prove or disprove Mary's conjectures.

Parallelogram Investigation

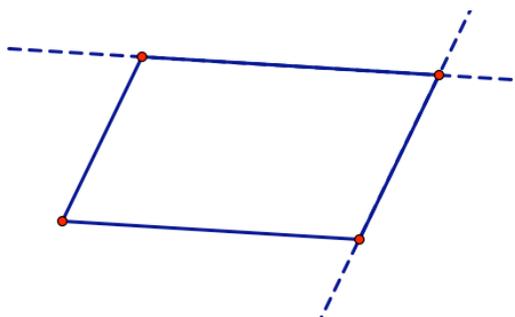
Take any parallelogram and construct squares externally on each side.

The centers of the four squares are the vertices of a quadrilateral.

Make a conjecture on the shape of the quadrilateral.

Initial conjecture:

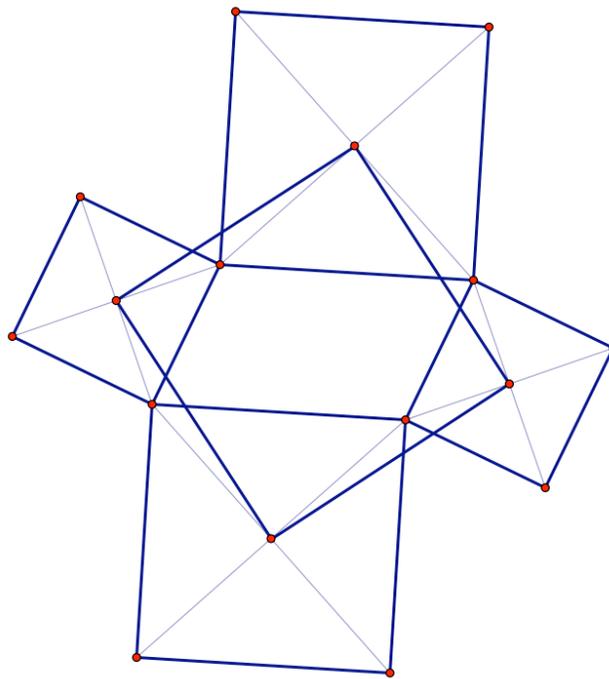
1. In GSP, construct a parallelogram.
 - a. Construct two segments with a common vertex.
 - b. Select one segment and the vertex not on the segment.
Construct a parallel line.
 - c. Repeat with the other segment and vertex.
 - d. Point of intersection is the fourth vertex of the parallelogram.
Construct the remaining two segments.



- e. Hide parallel lines.
2. Construct squares externally on each side of the parallelogram.
 - a. Mark one of the vertices as a center of rotation. Select one of the segments containing that vertex and rotate it 90 degrees.
 - b. Repeat until all squares are constructed.
 - c. Construct diagonals of all the squares to find the centers of the four squares.

3. Connect the centers with segments. Drag vertices of the parallelogram.
 - a. What do you observe? Make a conjecture on the shape of the new quadrilateral.
 - b. Make any measurements to confirm your conjecture. Does your initial conjecture hold?

Final conjecture:



Mary did this activity as well. Her conjecture was that

*If squares are constructed externally on each side of a parallelogram,
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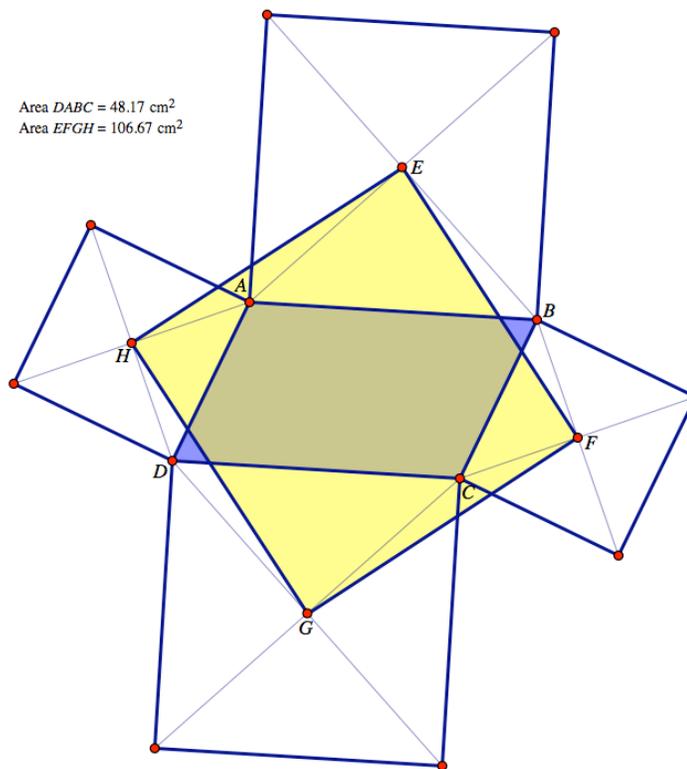
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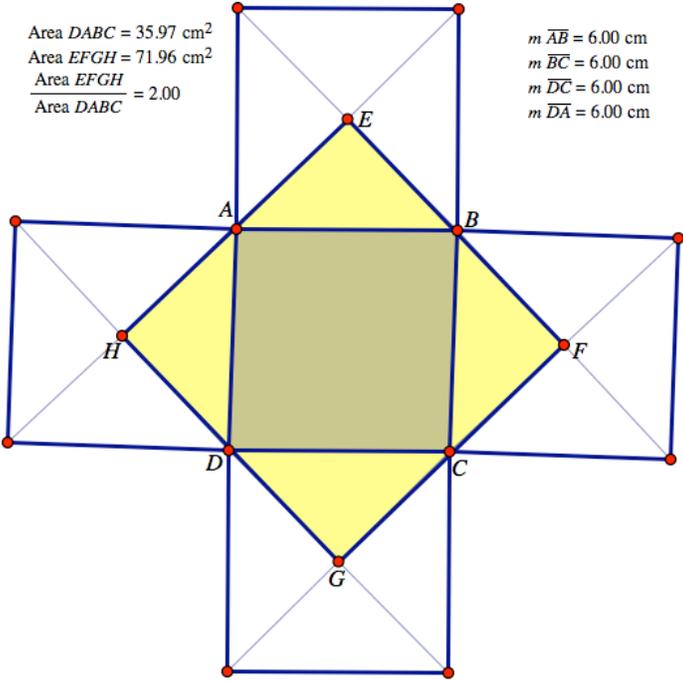
Make any necessary measurements and state your conjecture:

(Measure the areas and their ratio. The ratio will vary.)



Mary did this activity as well. She made two conjectures:

- 1. The ratio of the area of the square (EFGH) to the area of the parallelogram (ABCD) is **greater than or equal to 2**.
- 2. The ratio is 2 when the parallelogram is a square.



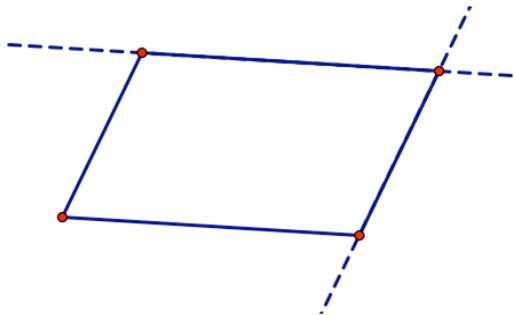
Prove or disprove Mary's conjectures.

Parallelogram Investigation

Take any parallelogram and construct squares externally on each side. The centers of the four squares are the vertices of a quadrilateral. Make a conjecture on the shape of the quadrilateral.

Initial conjecture:

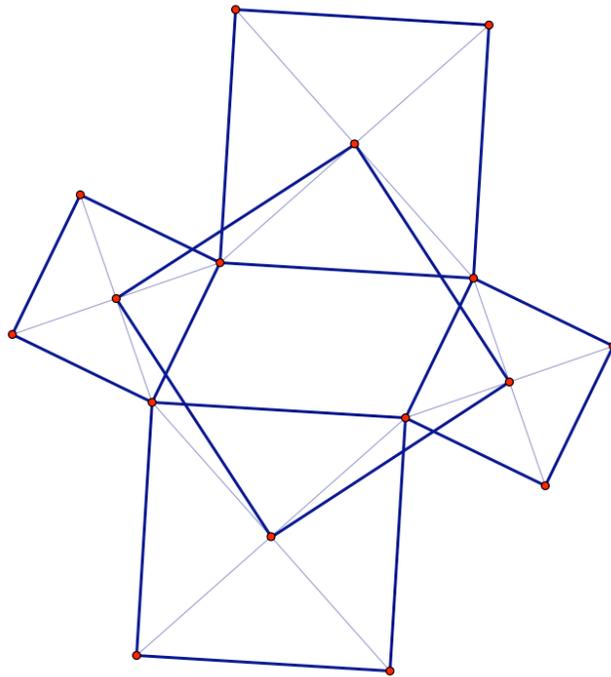
1. In GSP, construct a parallelogram.
 - a. Construct two segments with a common vertex.
 - b. Select one segment and the vertex not on the segment. Construct a parallel line.
 - c. Repeat with the other segment and vertex.
 - d. Point of intersection is the fourth vertex of the parallelogram. Construct the remaining two segments.



- e. Hide parallel lines.
2. Construct squares externally on each side of the parallelogram.
 - a. Mark one of the vertices as a center of rotation. Select one of the segments containing that vertex and rotate it 90 degrees.
 - b. Repeat until all squares are constructed.
 - c. Construct diagonals of all the squares to find the centers of the four squares.

3. Connect the centers with segments. Drag vertices of the parallelogram.
 - a. What do you observe? Make a conjecture on the shape of the new quadrilateral.
 - b. Make any measurements to confirm your conjecture. Does your initial conjecture hold?

Final conjecture:



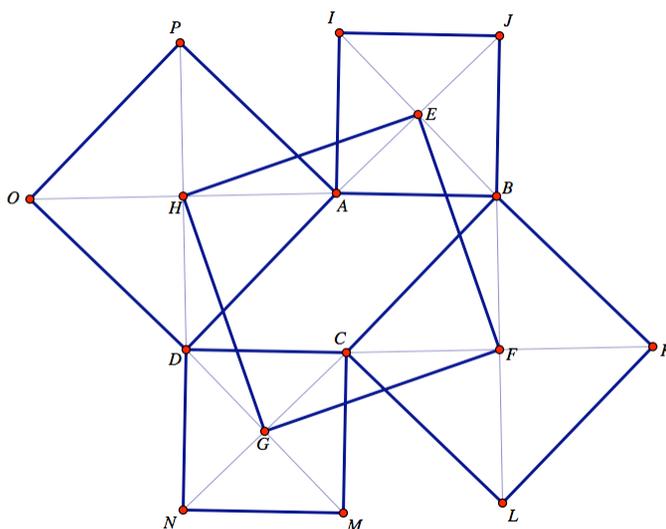
Mary did this activity as well. Her conjecture was that

If squares are constructed externally on each side of a parallelogram, the quadrilateral created by the centers of the four squares is a square.

Prove or disprove Mary's conjecture.

If the teacher has a difficulty producing a proof, ask the following questions:

- What is the relationship among $\triangle AHE$, $\triangle BFE$, $\triangle CFG$, and $\triangle DHG$?
- Can you prove that $\triangle AHE \cong \triangle BFE \cong \triangle CFG \cong \triangle DHG$?



If the student still has difficulty with the proof, ask the following questions:

- What is the relationship between $\overline{AE}, \overline{EB}, \overline{CG}, \overline{GD}$? $\overline{BF}, \overline{FC}, \overline{DH}, \overline{HA}$?
- What is the relationship between $\angle HDA$ and $\angle HAP$? $\angle IAE$ and $\angle ABE$?
Etc.
- What is the relationship between $\angle PAI$ and $\angle LCM$? $\angle PAI$ and $\angle CBA$?
Etc.
- Can you prove that $\triangle AHE \cong \triangle BFE \cong \triangle CFG \cong \triangle DHG$?
- What is the relationship between $\angle HEF$ and $\angle AEB$?

Based on your answers to these questions, explain/prove that quadrilateral $HEFG$ is a square.

A Sample Proof:

$\overline{AE} \cong \overline{EB} \cong \overline{CG} \cong \overline{GD}$ because they are $\frac{1}{2}$ of the square diagonal. Similarly,
 $\overline{BF} \cong \overline{FC} \cong \overline{DH} \cong \overline{HA}$.

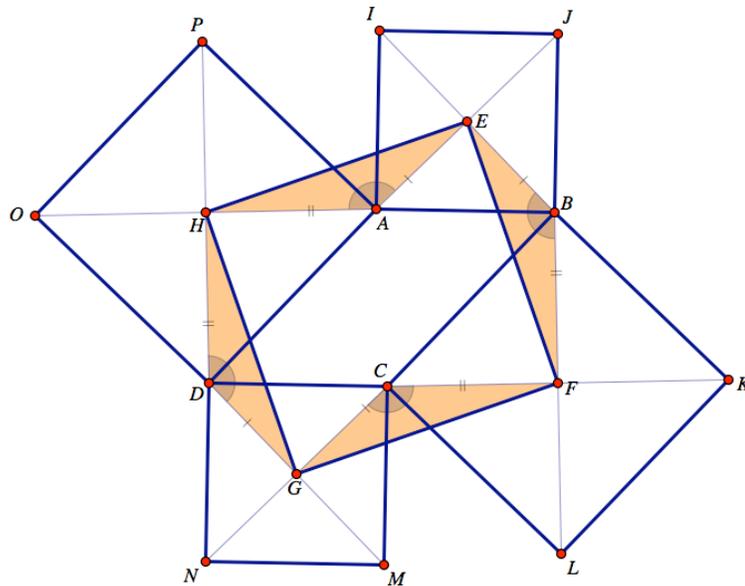
$\angle HDA \cong \angle HAP \cong \angle IAE \cong \angle ABE \cong \angle FBC \cong \angle FCL \cong \angle MCG \cong \angle CDG$ and
 $\angle PAI \cong \angle CBA \cong \angle LCM \cong \angle ADC$. By the Angle Addition Postulate,
 $\angle HAE \cong \angle FBE \cong \angle FCG \cong \angle HDG$.

Then $\triangle AHE \cong \triangle BFE \cong \triangle CFG \cong \triangle DHG$ by SAS.

By CPCTC, $\overline{HE} \cong \overline{EF} \cong \overline{FG} \cong \overline{GH}$ so the quadrilateral $HEFG$ is a rhombus.

$\angle HEF \cong \angle AEB$ by angle addition/substitution and $m\angle AEB = 90^\circ$.

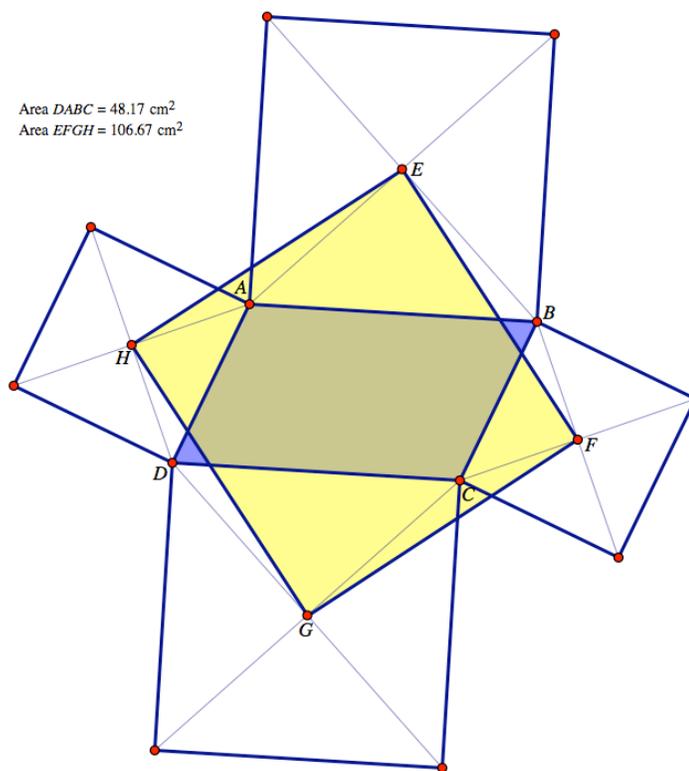
Therefore, quadrilateral $HEFG$ is a square.



What is the relationship between the area of the “new” quadrilateral and the area of the original parallelogram?

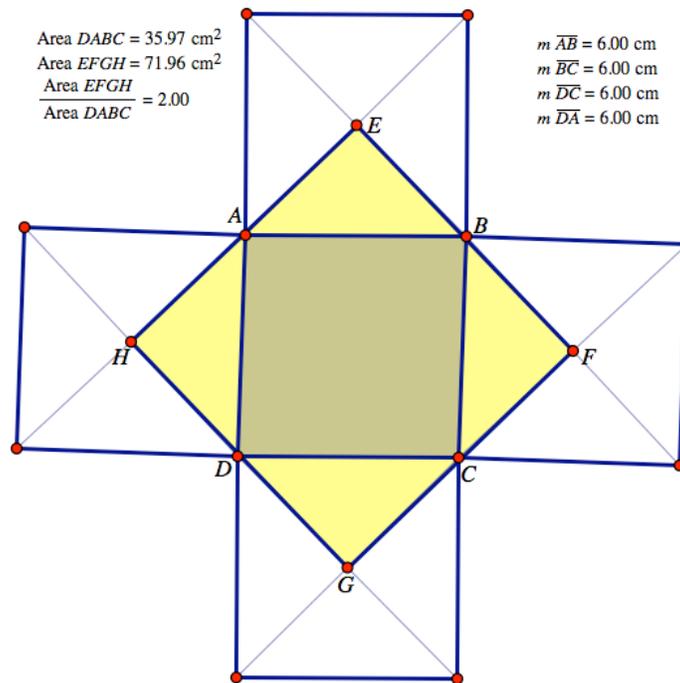
Make any necessary measurements and state your conjecture:

(Measure the areas and their ratio. The ratio will vary.)



Mary did this activity as well. She made two conjectures:

1. The ratio of the area of the square (EFGH) to the area of the parallelogram (ABCD) is **greater than or equal to 2**.
2. The ratio is 2 when the parallelogram is a square.



Prove or disprove Mary's conjectures.

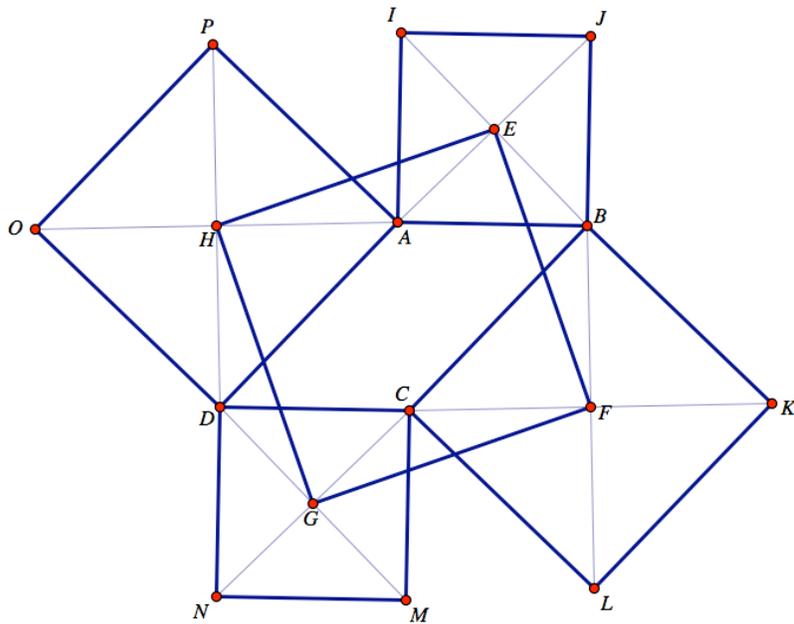
A Sample Proof:

This proof uses the following mathematical ideas:

1) The area of a parallelogram $A = ab \sin \gamma$, where a and b stand for the lengths of two adjacent sides of the parallelogram and γ stands for the angle contained between sides of lengths a and b .

2) The law of cosines: $c^2 = a^2 + b^2 - 2ab \cos \gamma$, where γ denotes the angle contained between sides of lengths a and b and opposite the side of length c (of a triangle).

3) $a + b \geq 2\sqrt{ab}$ [or $a^2 + b^2 \geq 2ab$] where $a \geq 0$ and $b \geq 0$ (" $=$ " holds when $a = b$).



Proof: (See the figure above)

Area (ABCD) = AD*DC*Sin(\angle ADC) [by Idea 1]

Area (EFGH) = HG² = HD² + DG² - 2*HD*DG*Cos(\angle HDG) [by Idea 2]

$$= \left(\frac{AD}{\sqrt{2}}\right)^2 + \left(\frac{DC}{\sqrt{2}}\right)^2 - 2*\left(\frac{AD}{\sqrt{2}}\right)*\left(\frac{DC}{\sqrt{2}}\right)*\text{Cos}(\angle\text{HDG})$$

$$= \frac{AD^2}{2} + \frac{DC^2}{2} - AD*DC*\text{Cos}(\angle\text{HDG})$$

$$= \frac{1}{2}(AD^2 + DC^2) - AD*DC*\text{Cos}(90^\circ + \angle\text{ADC})$$

$$= \frac{1}{2}(AD^2 + DC^2) + AD*DC*\text{Sin}(\angle\text{ADC})$$

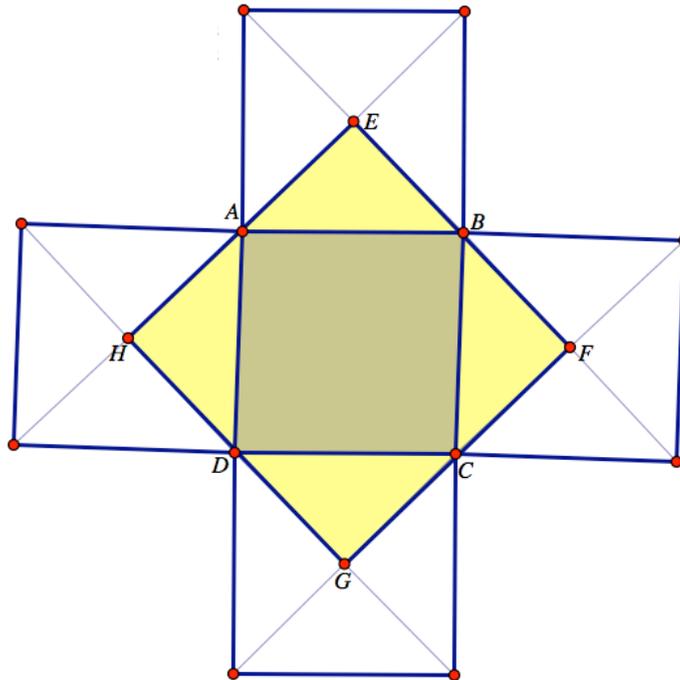
By Idea 3, $AD^2 + DC^2 \geq 2*AD*DC$ ["=" holds when $AD = DC$], and so Area (EFGH) $\geq AD*DC + AD*DC*\text{Sin}(\angle\text{ADC}) \geq 2*AD*DC*\text{Sin}(\angle\text{ADC})$ ["=" holds when $\text{Sin}(\angle\text{ADC}) = 1$, i.e., $\angle\text{ADC} = 90^\circ$]

Therefore $\frac{\text{Area (EFGH)}}{\text{Area (ABCD)}} \geq 2$ [The ratio = 2 when $AD = DC$ and $\angle\text{ADC} = 90^\circ$, i.e., ABCD is a square.]

If the subject has difficulty producing a proof such as this one, ask the following questions (for the second part of the conjecture only):

- What is the relationship between $\triangle AEB, \triangle BFC, \triangle CGD, \triangle DHA$?
- What is the relationship between these triangles and the square $ABCD$?

Based on your answers to these questions, explain/prove that the ratio of the area of the square $HEFG$ to the area of the square $ABCD$ is 2.



A Sample Proof:

All squares except the square $HEFG$ are congruent and $\triangle AEB \cong \triangle BFC \cong \triangle CGD \cong \triangle DHA$. The area of each triangle is $\frac{1}{4}$ of the area of the square $ABCD$ so the sum of the areas of the four triangles equals the area of the square $ABCD$. The area of the square $HEFG$ is equal to the sum of the area of the four triangles and the area of the square $ABCD$. Therefore, the ratio of the area of the square $HEFG$ to the area of the square $ABCD$ is 2.

Definition of Exterior Angles

If we continue a side of a triangle past a vertex as in the diagram below, we form an **exterior angle** of the triangle, like $\angle BCD$ in the figure below. The interior angles of the triangle which are not adjacent to the exterior angle are called **remote interior angles**. (In the diagram, $\angle BAC$ and $\angle ABC$ are the remote interior angles.)

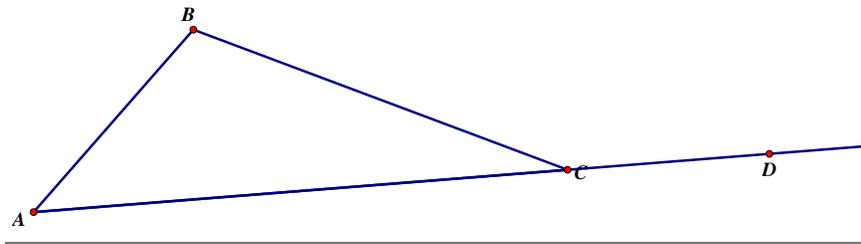


Diagram 1. Exterior Angle

4. How could you alter Diagram 1 to show another exterior angle? What would be the corresponding remote interior angles?

Now let's investigate the relationship between an exterior angle and the remote interior angles.

5. Make a conjecture about the relationship between the measure of exterior angle $\angle BCD$ and the measures of remote interior angles $\angle BAC$ and $\angle ABC$. You may use a protractor to measure the angles. Write your conjecture in mathematical terms.

6. Do you think your conjecture is true for all triangles or just the one shown in Diagram 1? How could you check?

Alana did this activity as well. Alana's conjecture was that "An exterior angle of a triangle is the sum of the remote interior angles."

7. Do you think Alana's conjecture is true? How could you check ?

8. Prove or disprove Alana's conjecture. You might write your explanation as an argument in paragraph form or as a two-column proof.

The Exterior Angles of a Triangle

This activity assumes the students are familiar with

- Constructing triangles, rays and points in GSP
 - the fact that the sum of the interior angles of a triangle is equal to 180 degrees
1. Introduce yourself to the student.
 2. Record the key identifier information of the student: Name, gender, school, teacher name, period, student id number and date.
 3. The student should be set up in front of a computer which has GSP, and internet access.
 4. Before student starts, begin recording connect session.
 5. Give student page one of student activity sheet (copy appears in gray below). Make sure student puts name, school, teacher and student id on sheet.

Introduction

In this activity, you will use Geometer's Sketchpad (GSP) to investigate some more properties of triangles. We begin with an arbitrary triangle.

1. In GSP, construct $\triangle ABC$.
 - give student 1-2 minutes to attempt to construct triangle.
 - if student does not construct triangle correctly (for example line segments are not connected in GSP), point out that something might be wrong, give student chance to correct error.
 - if student has no idea how to make sketch, show the student how to construct two sides and label the points, let the student finish the sketch.
2. Identify the *interior angles* of $\triangle ABC$.
 - have the student write answer on sheet.
 - if the student does not know what identify means, tell them to show you the interior angles. Then ask the student to name the interior angles.
 - if student does not remember what an interior angle is, ask them what "interior" means. If they do not know, tell them it means the inside.
 - if the student still can not identify the interior angles, name one of the angles for them.

3. What do you know about the interior angles?

- we are looking for “sum of the interior angles is 180 degrees”
- if the student gives you other information about the angles, prod for more.
- if the student gives you information which is specific to the given triangle (e.g. all the angles are acute, $\angle ABC$ is obtuse), ask if this will be true for all triangles.
- if the student states something that is incorrect, ask them a guiding question to help them correct themselves.
- if all else fails, ask them if they know what the sum of the measures of the interior angles is.
- once the student has stated that the sum of the interior angles is 180 degrees, give the student page 2 of the activity.

Definition of Exterior Angles

If we continue a side of a triangle past a vertex as in the diagram below, we form an **exterior angle** of the triangle, like $\angle BCD$ in the figure below. The interior angles of the triangle which are not adjacent to the exterior angle are called **remote interior angles**. (In the diagram, $\angle BAC$ and $\angle ABC$ are the remote interior angles.)

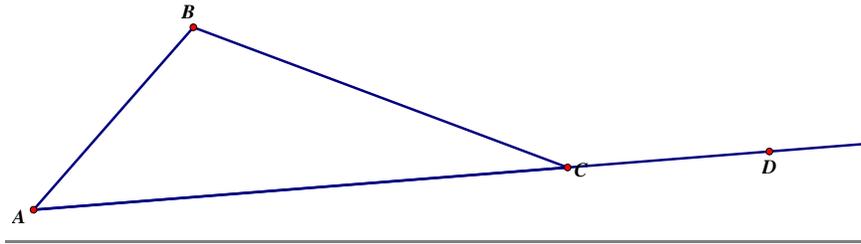


Diagram 1. Exterior Angle

4. How could you alter Diagram 1 to show another exterior angle? What would be the corresponding remote interior angles?
- we are looking for “extend one of the other sides, locate and label a point outside the triangle and then name the angles”. It is OK if the students can do this on the diagram without articulating the general procedure.
 - use this question to make sure the student understands the definition.
 - once the student demonstrates understanding of the definition, give them page 3 of the activity.

Now let's investigate the relationship between an exterior angle and the remote interior angles in GSP. Change your GSP construction to match Diagram 1.

5. Construct \overrightarrow{AC} to extend side AC .

* Give the student, 1-2 minutes to do this on their own. Then show the student how to do it.

* The student should use a ray going containing the segment AC . Make sure the extension, the ray or line segment exterior to the triangle is collinear with AC . A student may just put a segment which terminates at vertex C but is not collinear.

* Make sure the triangle is labeled the same as diagram A or the names of the angles will not match later on,

6. Construct point D on \overrightarrow{AC} , outside of the triangle.

* Give the student, 1 minute to do this on their own. Then show the student how to do it.

7. Make a conjecture about the relationship between the measure of exterior angle $\angle BCD$ and the measures of remote interior angles $\angle BAC$ and $\angle ABC$.

* We are looking for the conjecture that the exterior angle is equal to the sum of the remote interior angles. We want the students to measure the angles and compute the sum to check the conjecture and drag the points of the triangle to verify that this true for "all" triangles.

* Give the student 2 minutes to try to understand what might be a reasonable conjecture.

- * If the student does not make any progress towards a conjecture: ask the student what facts/ relationships they know about the angles, and what they could do in sketchpad to help them see relationships between the angles.
- * If after 2-3 minutes, the students do not measure the angles in GSP. Ask them to. If they do not know how to measure the angles, show them how to do one of the angles.
- * Ask them what they could compute using the measures that would help determine or test a conjecture.
- * If the students come up with another conjecture other than the one sought encourage them to think of another which relates the exterior angles and the remote interior angles.

8. Use GSP to test if your conjecture appears to be true.

- * Give the students 2 minutes to think about how they could use GSP to test the conjecture.
- * If they make no progress, or don't understand the question, ask them if it is true for just this triangle or all triangles. How could they use GSP to test it for other triangles?
- * If they suggest constructing a new triangle, ask if "they could change this triangle into another triangle." You can show them a sketch of a triangle that is different and ask if they can use GSP to change the current figure to look like the sketch.
- * If the student arrives at the desired conjecture, ask the student to write the conjecture in mathematical terms (number 9). If the student does not arrive at the desired conjecture after a total of 10 minutes of thinking about it with hints, give the student the final page of the activity.

9. Write your conjecture in mathematical terms.

* Allow the students to write the conjecture in any way that is natural for them. Ask the students to explain what any nontrivial mathematical term means (e.g. congruence). Ask the students if the statement as written applies to just the triangle in diagram one, or to a general triangle.

Another student, Alana did this activity as well. Alana's conjecture was that "An exterior angle of a triangle is the sum of the remote interior angles."

10. Use GSP to test the conjecture. Do you think Alana's conjecture is true?

* If the student arrived at Alana's conjecture before skip to number 11.

* If the student did not arrive at the conjecture, have the student use GSP to check the conjecture. At this point the student should have all the angles measured and only needs to check the sum. If the student does not know how to use GSP to calculate the sum, show them. Ask the student if this will be true for all triangles. The students should drag the points. If they suggest constructing a new triangle, ask if "they could change this triangle into another triangle." You can show them a sketch of a triangle that is different and ask if they can use GSP to change the current figure to look like the sketch.

11. Prove or disprove Alana's conjecture. You might write your explanation as an argument in paragraph form or as a two-column proof.

- Wait for 2-3 minutes to see if the students progress. If the student do not understand what "proof" means. Explain that you want them to use what they know about the angles of a triangle to show why Alana's conjecture works for all triangles.
- If the students continue to struggle, give them the following hints:

1. What facts do we know about the angles? What fact do we know about the interior angles?

2. What is the sum of $\angle ABC$, $\angle BAC$, and $\angle BCA$? Why?

2. What is the sum of $\angle BCA$ and $\angle BCD$? Why?

3. You may find it helpful to assign variables to each angle and write the relationships in hints (1) and (2) as algebraic equations.

-
- Have the student write out their proof on their activity sheet. Ask them to state the reason/justification for each step. It is OK if the final product does not look like a formal proof but contains the key steps.

This concludes the student activity.

6. Collect all the papers with student work. Staple them together.
7. Make sure key identifier information is correct: Name, gender, school, teacher name, period, student id number and date.
8. Stop recording connect session.
9. Thank the student for their cooperation.

Triangle Investigations

Introduction

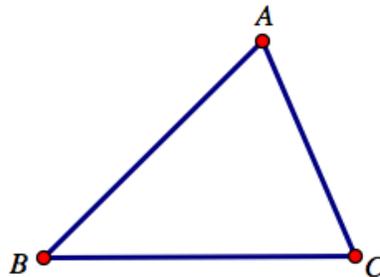
In this activity, you will use Geometer's Sketchpad (GSP) to make conjectures and prove properties of triangles.

Altitudes investigation

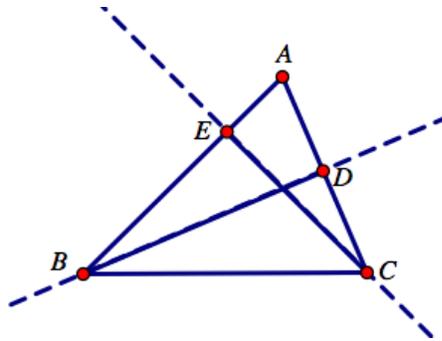
In a triangle, if two altitudes are equal in length, what conjecture can you make?

Initial conjecture:

1. In GSP, construct an arbitrary $\triangle ABC$



2. Construct the altitudes from vertex B to \overline{AC} and from vertex C to \overline{AB} .
 - a. Construct a line containing B and perpendicular to \overline{AC} .
 - b. Construct a line containing C and perpendicular to \overline{AB} .
 - c. Label the intersection points with \overline{AC} and \overline{AB} - D and E respectively.
 - d. Construct \overline{BD} and \overline{CE} .



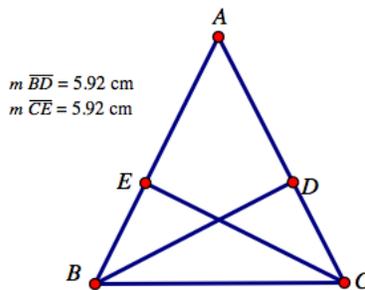
a. Hide perpendicular lines.

3. Measure the two altitudes. Drag vertices of the triangle so that the two altitudes have the same measure.

a. What do you observe?

b. Make any other measurements to confirm your initial conjecture. Does your initial conjecture hold?

c. Drag vertex A to a different location, keeping the altitudes congruent. Does your conjecture still hold?



Final conjecture:

Mary did this activity as well. Her conjecture was that

If two altitudes in a triangle are equal in length, then it is an isosceles triangle.

Prove or disprove Mary's conjecture.

You might write your explanation as an argument in paragraph form or as a two-column proof.

Triangle Investigations

Introduction

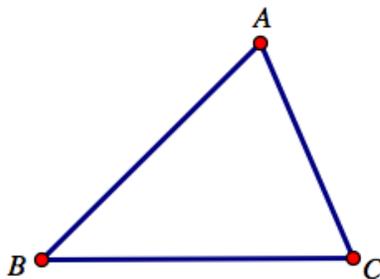
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Altitudes investigation

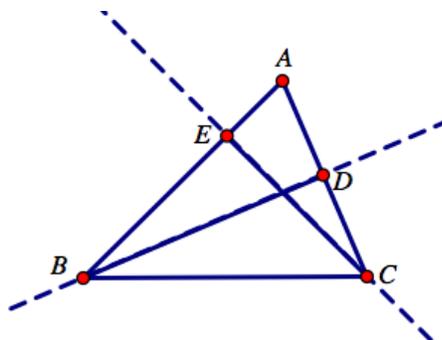
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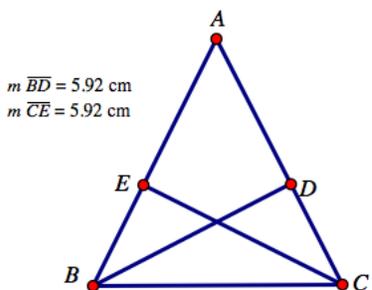
2. Construct the altitudes from vertex B to \overline{AC} and from vertex C to \overline{AB} .
 - a. Construct a line containing B and perpendicular to \overline{AC} .
 - b. Construct a line containing C and perpendicular to \overline{AB} .
 - c. Label the intersection points with \overline{AC} and \overline{AB} - D and E respectively.
 - d. Construct \overline{BD} and \overline{CE} .



a. Hide perpendicular lines.

3. Measure the two altitudes. Drag vertices of the triangle so that the two altitudes have the same measure.

- a. What do you observe?
- b. Make any other measurements to confirm your initial conjecture. Does your initial conjecture hold?
- c. Drag vertex A to a different location, keeping the altitudes congruent. Does your conjecture still hold?



Final conjecture:

Mary did this activity as well. Her conjecture was that

*If two altitudes in a triangle are equal in length,
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Prove or disprove Mary's conjecture.

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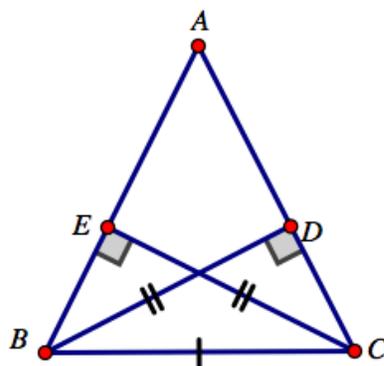
If the teacher has difficulty producing such as a proof, ask the following questions:
(You might not need to use all of them.)

1. What is the relationship between $\triangle BEC$ and $\triangle CDB$?
2. Do these two triangles have the same shape and size? So what is the relationship between them?
3. Can you prove that $\triangle BEC \cong \triangle CDB$?

Based on your answers to questions 1-3, explain/prove that $\triangle ABC$ is isosceles.

Sample Proof:

Since \overline{BD} and \overline{CE} are two altitudes of $\triangle ABC$, $\angle BEC$ and $\angle CDB$ are right angles. Since $\triangle BEC$ and $\triangle CDB$ are right triangles, and \overline{BD} and \overline{CE} are congruent as well as \overline{BC} is congruent to itself by the reflexive property, $\triangle BEC$ and $\triangle CDB$ are congruent by the Hypotenuse-Leg Theorem. By CPCTC, $\angle EBC$ and $\angle DCB$ are congruent. Therefore, $\triangle ABC$ is an isosceles triangle by the Base Angle Theorem Converse.



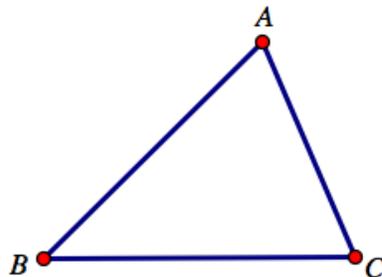
(You can also prove $\triangle BAD$ and $\triangle CAE$ are congruent and use the definition of isosceles triangle to do the proof.)

Angle bisectors investigation

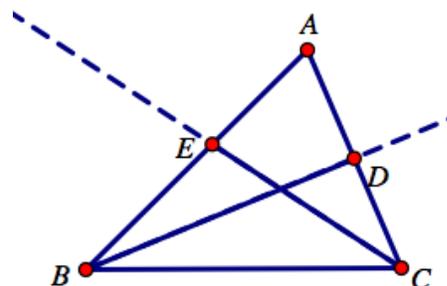
In a triangle, if two angle bisectors are equal in length, what conjecture can you make?

Initial conjecture:

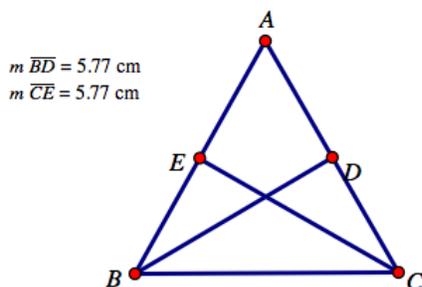
1. In GSP, construct an arbitrary $\triangle ABC$



1. Construct the angle bisectors (each constructed from a vertex to the opposite side) of $\angle ABC$ and $\angle ACB$. Label them \overline{BD} and \overline{CE} .
 - a. Select points A, B, C in that order and from the Construct menu select Angle Bisector. Similarly, select points A, C, B in that order and from the Construct menu select Angle Bisector.
 - b. Hide the rays.
 - c. Label the intersection points of the angle bisectors with \overline{AC} and \overline{AB} - D and E respectively.
 - d. Construct \overline{BD} and \overline{CE} .



2. Measure \overline{BD} and \overline{CE} . Drag vertices of the triangle so that the two segments have the same measure.
- What do you observe?
 - Make any other measurements to confirm your initial conjecture. Does your initial conjecture hold?
 - Drag vertex A to a different location, keeping \overline{BD} and \overline{CE} congruent. Does your conjecture still hold?



Final conjecture:

Mary did this activity as well. Her conjecture was that

*If two (internal) angle bisectors in a triangle are equal in length,
then it is an isosceles triangle.*

Prove or disprove Mary's conjecture.

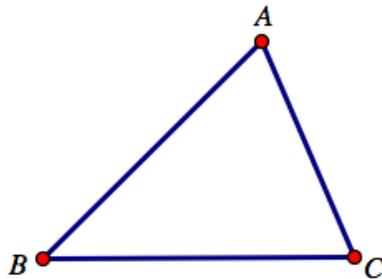
You might write your explanation as an argument in paragraph form or as a two-column proof.

Angle bisectors investigation

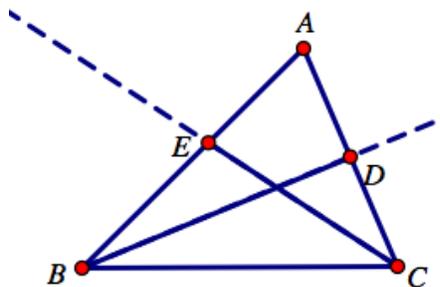
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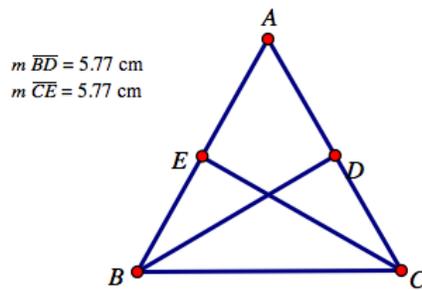
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1. Construct the angle bisectors (each constructed from a vertex to the opposite side) of $\angle ABC$ and $\angle ACB$. Label them \overline{BD} and \overline{CE} .
 - a. Select points A, B, C in that order and from the Construct menu select Angle Bisector. Similarly, select points A, C, B in that order and from the Construct menu select Angle Bisector.
 - b. Hide the rays.
 - c. Label the intersection points of the angle bisectors with \overline{AC} and \overline{AB} - D and E respectively.
 - d. Construct \overline{BD} and \overline{CE} .



2. Measure \overline{BD} and \overline{CE} . Drag vertices of the triangle so that the two segments have the same measure.
- What do you observe?
 - Make any other measurements to confirm your initial conjecture. Does your initial conjecture hold?
 - Drag vertex A to a different location, keeping \overline{BD} and \overline{CE} congruent. Does your conjecture still hold?



Final conjecture:

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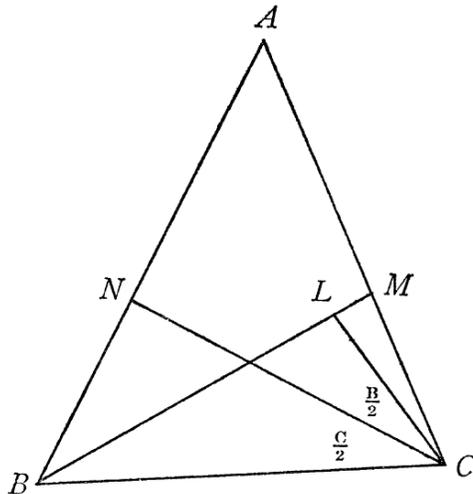
*If two (internal) angle bisectors in a triangle are equal in length,
then it is an isosceles triangle.*

Prove or disprove Mary's conjecture.

You might write your explanation as an argument in paragraph form or as a two-column proof.

The proof is a challenging one. You may discuss the proof insights with the teacher in a way that you think is suitable with his/her reasoning and proof ability.

The following is a very neat sample proof for your reference:



THEOREM. *Any triangle having two equal internal angle bisectors (each measured from a vertex to the opposite side) is isosceles.*

Proof. Let ABC be the triangle with equal angle bisectors BM and CN , as in the figure. If the angles B and C are not equal, one must be less, say $B < C$. Take L on BM so that $\angle LCN = \frac{1}{2}B$. Since this is equal to $\angle LBN$, the four points L, N, B, C are concyclic (lie on a circle). Since

$$B < \frac{1}{2}(B + C) < \frac{1}{2}(A + B + C),$$

$\angle CBN < \angle LCB < 90^\circ$. Since smaller chords of a circle subtend smaller acute angles, and $BL < CN$,

$$\angle LCB < \angle CBN.$$

We thus have a contradiction.

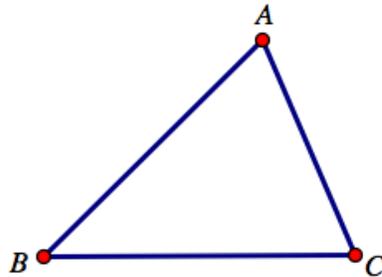
G. GILBERT, B.I.C.C. Research Organization, London AND
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Medians Investigation

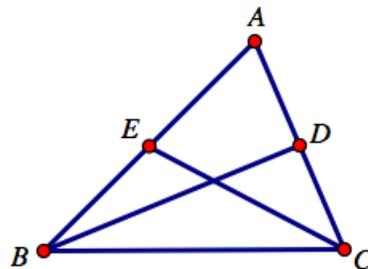
In a triangle, if two medians are equal in length, what conjecture can you make?

Initial conjecture:

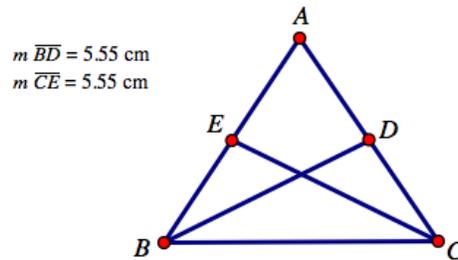
1. In GSP, construct an arbitrary $\triangle ABC$



1. Construct the medians on \overline{AC} and \overline{AB} .
 - a. Construct midpoints on \overline{AC} and \overline{AB} , and label them D and E , respectively.
 - b. Construct medians \overline{BD} and \overline{CE} .



2. Measure the two medians. Drag vertices of the triangle so that the two medians have the same measure.
 - a. What do you observe?
 - b. Make any other measurements to confirm your initial conjecture. Does your initial conjecture hold?
 - c. Drag vertex A to a different location, keeping the altitudes congruent. Does your conjecture still hold?



Final conjecture:

Mary did this activity as well. Her conjecture was that

*If two medians in a triangle are equal in length,
then it is an isosceles triangle.*

Prove or disprove Mary's conjecture.

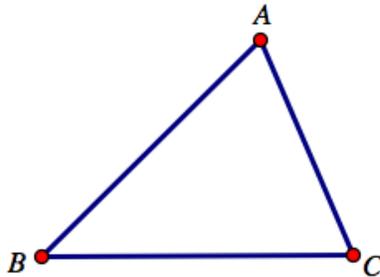
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Medians Investigation

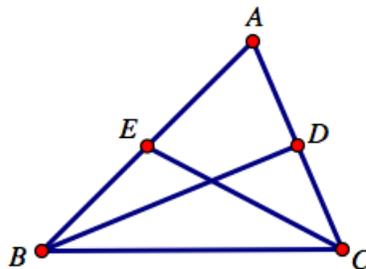
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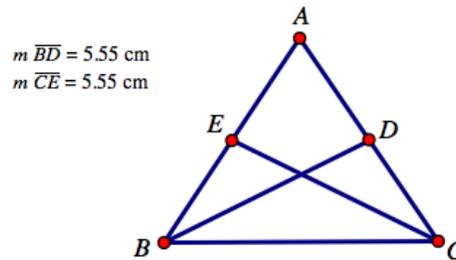
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You might write your explanation as an argument in paragraph form or as a two-column proof.

If the teacher has difficulty producing such as a proof, ask the following questions:
(You might not need to use all of them.)

1. What do you know about medians of a triangle?

The centroid divides each median in the ratio 2:1.

2. What can you say about angles formed by the medians?

They are vertical angles.

3. What is the relationship between $\triangle BEF$ and $\triangle CDF$?

They are congruent. (Prove it.)

4. What is the relationship between $\triangle ABD$ and $\triangle ACE$?

They are congruent. (Prove it.)

Based on your answers to questions 1-4, explain/prove that $\triangle ABC$ is isosceles.

Proof:

Since \overline{BD} and \overline{CE} are two medians of $\triangle ABC$, $BF = \frac{2}{3}BD$ and $FD = \frac{1}{3}BD$ as well as $CF = \frac{2}{3}CE$ and $FE = \frac{1}{3}CE$. Since \overline{BD} and \overline{CE} are equal in length, then by substitution, $BF = CF$ and $FE = FD$. $\angle BFE$ and $\angle CFD$ are vertical angles and, therefore, are congruent. Thus, $\triangle BFE$ and $\triangle CFD$ are congruent by SAS Postulate. By CPCTC, $\angle EBF \cong \angle DCF$, i.e., $\angle ABD \cong \angle ACE$. Since $\overline{BD} \cong \overline{CE}$ (given), and $\angle BAD \cong \angle CAE$ by the reflexive property, $\triangle BAD$ and $\triangle CAE$ are congruent by AAS Postulate. By CPCTC, $\overline{AB} \cong \overline{AC}$. Therefore, $\triangle ABC$ is an isosceles triangle by definition.

