Supplier Selection and Order Allocation Based on Integer Programming

Hayden Beauchamp, Energy Solutions, Richland, WA, USA

Clara Novoa, Ingram School of Engineering, Texas State University, San Marcos, TX, USA

Farhad Ameri, Department of Engineering Technology, Texas State University, San Marcos, TX, USA

ABSTRACT

The ability to assess and select new suppliers quickly and efficiently is a critical requirement for improving the agility of manufacturing supply chains. The Digital Manufacturing Market (DMM) is a web-based platform for intelligent supply chain configuration. This research enhances the DMM's performance by developing a column generation method for solving the supplier selection problem. The objective of the proposed method is to maximize the technological competencies of the selected suppliers while meeting their capacity constraints. The column generation method resolves the issue of limited scalability of a traditional linear programming formulation and can be integrated into the DMM. Additionally, using test generated problems, this research evaluates the effect on reducing the threshold distance traveled by semi-finished parts in the work orders. The results show that an economy of distance can be imposed with little effect on average match compatibility.

Keywords: *Column Generation, Digital Manufacturing Market, Linear Programming, Nonlinear Programming, Supplier Selection, Supply Chain Configuration*

INTRODUCTION

Manufacturing companies continuously strive to improve the responsiveness and flexibility of their supply chains by finding alternative means of sourcing. Managing the sourcing process has been a challenge in the last decade for many corporations (Saen, 2009). In particular, supplier discovery and evaluation is increasingly becoming complex and resource-intensive in global supply chains. There is a need for computational tools and techniques for efficient identification of prospective suppliers to enable rapid formation and reconfiguration of agile supply chains. This need is more pronounced when supply chain transactions are conducted on the web, where a huge number of stakeholders are involved in trading manufacturing services. Electronic marketplaces (e-market) for manufacturing services have recently become popular venues for sourcing particularly among small and medium sized companies. A web-based framework allows for interaction with a far greater number of potential suppliers and also

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enables automation of computational tasks like supplier discovery and evaluation.

Despite their advantages, existing manufacturing e-markets fail in building accurate connections between buyers and sellers due primarily to the syntactic (keyword-based) nature of the search. Ameri and Dutta (2008) proposed a semantic approach to supplier selection by developing a market framework, called Digital Manufacturing Market (DMM), based on the Semantic Web (SW) technology. DMM is an agent-based environment in which buyers (i.e., manufacturing companies) describe their needs by posting queries and sellers (i.e., suppliers) state their capabilities by creating advertisements. Both buyers' needs and sellers' capabilities are represented by software agents using a formal ontology called Manufacturing Service Description Language (MSDL) (Ameri, 2006). Semantic search engines quantify the similarities between service provisions (advertisements) and service requests (queries). The details of the semantic search methodology are in (Ameri & Dutta, 2008).

The semantic search engine returns a numeric similarity or *matching score* between 0 (completely dissimilar) and 1 (completely similar) for each DMM advertisement-query pair. Suppliers (i.e., sellers) are ranked according to the similarity assessment, and the supplier with the highest similarity score is selected to fulfill a service. This methodology of one-to-one matching is adequate if manufacturers' work orders require only a few services and the supply chain has few suppliers. However, as the size of the supply chain increases, the one-to-one matching technique becomes less efficient. Also, the matching mechanism in DMM does not reflect the trade-offs on operational criteria, such as time, cost, and capacity. The similarity score is purely based on technological criteria, such as the available processes and equipment, achievable geometries and tolerances, and required materials.

This paper studies a supply chain configuration problem in which manufacturing companies place *work orders* requiring several *services* that can be provided by multiple *suppliers*. This problem relates to a Multiple Sourcing Supplier Selection Problem (MSSSP). While a Single Sourcing Supplier Selection Problem finds the best supplier to satisfy a *work order*, MSSSP finds more than one *supplier* that will satisfy portions of the *work order*. Three mathematical optimization models are proposed. The contribution of the models is that they simultaneously consider operational and technological aspects such as suppliers' technological competency, capacity, and geographic location, and manufacturers' expected lead time for *work orders*. All models maximize the semantic similarity score between requested services and suppliers' advertisements.

The first model is a linear program named the *traditional formulation without distance constraints*. It considers only suppliers' capacity and provides a baseline. The second model is a nonlinear program that, including suppliers' capacity, also incorporates manufacturers' expected lead times for *work orders* and suppliers' geographic locations. This model is named the *traditional formulation with distance constraints*. The third model is a linear program that uses the column generation method to solve the first model more efficiently. It is called the *column generation formulation*. We show that the column generation method efficiently resolves the issue of limited scalability observed in the *traditional formulation without distance constraints*. The paper is divided into four sections. They are literature review, problem assumptions and mathematical optimization models, numerical results, and conclusions.

LITERATURE REVIEW

Electronic markets are defined as a network information system that enable buyers and sellers to exchange information, transact, and perform other related activities (Lancastre & Lages, 2006). Electronic markets require dynamic coordination of their business agents (Mahdavi, Mohebbi, Cho & Paydar, 2008). Agile supply chain configuration (ASCC) plays a key role in the efficiency of electronic markets and corresponds to solve a Supplier Selection Problem (SSP). Both ASSC and SSP consider multiple suppliers. However, ASCC can be more extensive than SSP because ASCC may include multiple buyers (i.e., manufacturers) while SSP usually involves one buyer. SSP is a multi-criteria, decision-making problem (Çebi & Bayraktar, 2003), as well as ASCC. A typical optimization model for the SSP may have multiple objectives (Aguezzoul & Ladet, 2007), one for each criterion, such as minimizing the purchasing price and manufacturing lead time, and maximizing the quality of the finished goods (Huo &Wei, 2008). Dickson (1966) proposed 23 criteria for evaluating suppliers, including capacity, delivery time, and quantity-based price discounts. Additional studies have identified up to 60 criteria to assess suppliers (Roa & Kiser, 1980).

SSP has been studied for over 50 years. Ware, Singh & Banwet (2012) mention about more than 150 refereed journal articles related to SSP. These articles appeared in rated journals from 1991 to 2011. Aguezzoul & Ladet (2007) have classified the modeling approaches developed by researchers and practitioners into 3 categories, which are: linear weighting models, mathematical programming models and statistical/probabilistic approaches. The authors mention that few articles have proposed mathematical programming techniques.

This research identified that SSP has been modeled using linear programming (Kingsman, 1986; Sanayei, Farid-Mousavi, Abdi & Mohaghar, 2008), mixed integer programming (Bender, Brown, Isaac, & Shapiro, 1985; Chaudhry, Forst, & Zydiac, 1993), multi-objective programming (Aguezzoul & Ladet, 2007; Buffa & Jackson, 1983; Liao & Rittscher, 2007; Weber & Current, 1993), and nonlinear programming (Auguezzoul & Ladet, 2007; Benton, 1991; Ghodsypour & O'Brien, 2001; Liao & Rittscher, 2007; Razmi & Rafiei, 2010; Vencheh, 2011).

Some of the methods to solve SSP include goal programming (Aguezzoul & Ladet, 2007; Çebi & Bayraktar, 2003; Karpak, Kumcu, & Kasuganti, 1999), data envelopment analysis (Azadi & Saen, 2014; Liu, Ding, & Lall, 2000), fuzzy multi-objective integer programming (Huo & Wei, 2008), analytical hierarchy process (AHP) (Özgen, Önüt, Gülsun, Tuzkaya, U. & Tuzcaya, G., 2008; Nydick & Hill, 1992), analytic network process (Kirytopoulos, Leopoulos, Mavrotas, & Voulgaridou, 2010; Razmi & Rafiei, 2010), artificial neural network (Wu, Zhang, Zheng, & Xi, 2010), expert systems (Valluri & Croson, 2005), multi-attribute utility approach (Min, 1994), genetic algorithms (Ding, Benyoucef, & Xie, 2004; Liao & Rittscher, 2007) and tabu search (Ko, Kim, & Hwang, 2001). AHP can be combined with different methods, such as cluster analysis, neural networks, and data envelopment analysis. AHP was combined with linear programming to solve order allocation problems across multiple suppliers considering tangible and intangible factors (Ghodsypour & O'Brien, 1998). Hybrid models have varying degrees of success since they are affected by the inherent disadvantages of the combined methods (Sanayei et al., 2008).

Chamodrakas, Batis, and Martakos (2010) simplified the SSP in a business-to-business e-market environment by breaking it into two stages. In the initial screening stage, satisficing hard constraints was used as criteria to qualify vendors. In the second stage, a variant of the fuzzy preference programming method was applied. The computational complexity of the second stage problem reduced significantly due to the screening process in the first stage.

The aforementioned approaches deal directly with optimization of various supplier criteria but do not include other relevant criteria in agile supply chain configuration. In the SSP solved by Aguezzoul & Ladet (2007), distances and transportation costs between suppliers and the single-buyer were considered. Saen & Gershon (2010) also mention distance and supply variety as nondiscretionary criteria to consider in a SSP. In ASCC, inter-supplier distance is a high-level parameter that may be more crucial than other individual supplier criteria. The cost or risk associated with inter-supplier transportation may outweigh benefits attributed to an individual supplier.

While current supplier selection approaches include lead time, the models proposed in this work consider both suppliers' available capacity and manufacturers' (i.e., buyers) expected lead times. The lead times are represented as the maximum threshold distance a *work order* will be allowed to travel. Effective analytical tools for agile supply chain configuration should model multiple nuances that matter to manufacturing businesses. The models presented in this paper intend to fill this need.

PROBLEM ASSUMPTIONS AND MATHEMATICAL OPTIMIZATION MODELS

The assumptions to model the Supplier Selection Problem (SSP) and ultimately the Agile Supply Chain Configuration (ASCC) problem are below.

Assumption 1: *Sequence for services in work orders*

A manufacturer's *work order (WO)* consists of multiple manufacturing *services* (such as casting, machining, and coating) that must be completed in a single serial order. Any process involving *services* that can be performed in parallel or in any other flexible order (i.e., an assembly process) will be clearly delineated into separate *work orders*.

Assumption 2: *Number of suppliers per work order*

Services (S) in a *work order* can be performed by one or multiple *suppliers*.

Assumption 3: *Size of the supplier's pool*

The *supplier's* pool is composed of a large number of *suppliers*.

Assumption 4: *Distance traveled in a work order*

The distance in a *work order* is the sum of the distance traveled between *suppliers* by the semi-finished parts. Models do not include: (a) the distance traveled by the raw materials from the warehouse to the first *supplier* in a *work order*, and (b) the distance traveled by the finished part from the last *supplier* to the manufacturer posting the *work order.*

Assumption 5: *Transportation costs can be modeled through distance*

This is an assumption commonly used in supply chain papers such as the ones related to vehicle routing problems (Gendreau, Laporte, & Seguin, 1986; Novoa & Storer, 2009). In many practical cases, fuel costs are proportional to distances and are the main element of the variable transportation costs.

Assumption 6: *Suppliers are properly qualified to perform the services*

The DMM search agent qualifies all possible matches between *suppliers* and *services* above the minimum acceptance threshold. In order to preclude incompatible assignments the search agent reports only the acceptable matches.

Assumption 7: *Input data for the proposed models (i.e., parameters) is hypothetical*

Data for the models is based on reasonable assumptions made by the authors and some general knowledge of manufacturing industries. Once the DMM proposed by Ameri and Dutta (2008) collects information from soliciting manufacturers, the models can use more specific data.

In the remainder of this section, we provide the input data (i.e. input parameters), decision variables, and formulation for each one of the three proposed models.

Model 1: *Traditional formulation without distance constraints*

Table 1 presents the notation and description for all input parameters in Model 1. Table 2 exemplifies values for these parameters for a hypothetical supply chain with 5 potential *suppliers*, 3 *work orders*, 3 *services* in *work orders* 1 and 3 and two *services* in *work order* 2. The bolded area of Table 2 corresponds to the *matching score*, $score_{ij_ik}$. If the reader visualizes *work orders* as overlaid layers, *matching score*, $score_{ij_ik}$, is a three-dimensional matrix, capacity required by *service* j_i in *work order* i, c_{ij_i}, is a two-dimensional matrix, and *supplier's* capacity, C_k, is a one-dimensional array.

Model 1 decision variable is notated as x_{ij_ik}. This variable is binary and takes the value of 1 if *supplier k* ends assigned to *service* j_i in *work order i*. For the hypothetical example in Table 2, there are 40 decision variables since there are 5 *suppliers*, 3 *services* in *work orders* 1 and 3 and 2 *services* in *work order* 2 (5*3 + 5*2 + 5*3).

The linear programming formulation for Model 1 is given below. Model 1 is a generalized assignment (Cattrysse, Salomon, & Wassenhove, 1994; Fisher, Jaikumar, & Van Wassenhove, 1986; Rardin, 1998; Savelsbergh, 1997). Its single objective function *(TOF)* aggregates the individual scores of the selected *suppliers.*

$$Max$$

$$Tot_{Score} = \sum_{i=1}^{I}\sum_{j_i=1}^{J_i}\sum_{k=1}^{K} Score_{ij_ik}\, x_{ij_ik} \qquad (TOF)$$

s.t.

$$\sum_{k=1}^{K} x_{ij_ik} = 1 \qquad \begin{cases} i = 1,\ldots,I \\ j_i = 1,\ldots,J_i \end{cases} \qquad (T1)$$

$$\sum_{i=1}^{I}\sum_{j_i=1}^{J_i} c_{ij_i}\, x_{ij_ik} \le C_k \qquad k = 1,\ldots,K \qquad (T2)$$

$$x_{ij_ik} \triangleq \begin{cases} 1 & if\ k\ is\ selected\ for\ service\ j_i \\ 0 & otherwise \end{cases}$$

Constraint *T1* requires each *service*, in each *work order*, to be assigned to a single *supplier*. Constraint *T2* forbids each *supplier* from exceeding its capacity when fulfilling the

Table 1. Model 1 input parameters

Notation	Description
i	Index that represents a single *work order*. The maximum number of *work orders* in the system is notated as I and therefore *i=1,...,I.*
j_i	Index that represents a single *service* nested in *work order i*. The maximum number of *services* in a *work order* is notated as J_i and therefore $j_i=1,\ldots,J_i$.
k	Index that represents a single *supplier*. The maximum number of *suppliers* in the system is notated as K and therefore *k=1,...,K.*
$score_{ij_ik}$	*Matching score* if assigning *supplier k* to *service* j_i in *work order i*. *Matching scores* are real numbers between 0 and 1 obtained from the DMM as described in the introduction (second and third paragraphs) of this paper. The closer the score is to 1, the better the match. The score is a dimensionless quantity.
c_{ij_i}	Capacity required by *service* j_i in *work order i.* It is given in units of time, for example, hours necessary to perform *service* j_i in *work order i*.
C_k	Total available capacity of *supplier k*. The units for this input parameter are the same as for the capacity required by *service*, c_{ij_i}.

Table 2. Input parameter values for a hypothetical supply chain

	Supplier 1	Supplier 2	Supplier 3	Supplier 4	Supplier 5	Capacity Required By *Service*, c_{ij_i} (hours)
WO1						
S_{11}	**0.64**	**0.32**	**0.50**	**0.43**	**0.71**	3
S_{21}*	**0.23**	**0.63**	**0.95**	**0.58**	**0.01**	7
S_{31}	**0.15**	**0.56**	**0.40**	**0.42**	**0.54**	9
WO2						
S_{12}	**0.11**	**0.32**	**0.38**	**0.67**	**0.89**	11
S_{22}	**0.55**	**0.67**	**0.34**	**0.78**	**0.29**	10
WO3						
S_{13}	**0.39**	**0.58**	**0.22**	**0.33**	**0.24**	5
S_{23}	**0.41**	**0.08**	**0.92**	**0.62**	**0.78**	8
S_{33}	**0.29**	**0.45**	**0.34**	**0.07**	**0.11**	3
Supplier's Capacity, C_k (hours)	17	13	12	19	17	

*: S_{21}=*Service* two in *work order (WO)* one.

services assigned. The last line in the model indicates that the decision variables are binary. Using the parameter values in Table 2 and the model above, the model optimal solution was obtained with Excel Solver and with FICO Xpress Optimization Software. The resulting assignment is: for *work order* 1, *supplier* 5 is assigned to *service* 1, *supplier* 3 is assigned to *service* 2 and *supplier* 2 is assigned to *service* 3. For *work order* 2, *supplier* 5 is assigned to *service* 1 and *supplier* 4 is assigned to *service* 2. For *work order* 3, *supplier* 1 is assigned to *service* 1, *supplier* 4 is assigned to *service* 2 and *supplier* 2 is assigned to *service* 3. The total score for the resulting assignment is 5.35. Xpress took 0.032 seconds to find the optimal solution to Model 1.

Model 2: *Traditional formulation with distance constraints*

Model 2 keeps the notation for input parameters and decision variables used in Model 1. Three additional input parameters and their notation are introduced in Table 3. Table 4 provides the inter-supplier distances to input to Model 2 assuming the same hypothetical supply chain of 5 *suppliers* provided in Table 2.

Model 2 formulation is given below. The objective function and the constraints *D1* and *D2* in Model 2 are equivalent to *TOF*, *T1* and *T2* in Model 1.

$$Max$$

$$Tot_{Score} = \sum_{i=1}^{I}\sum_{j_i=1}^{J_i}\sum_{k=1}^{K} Score_{ij_ik}\, x_{ij_ik} \qquad (DOF)$$

s.t.

$$\sum_{k=1}^{K} x_{ij_ik} = 1 \qquad \begin{cases} i = 1, \ldots, I \\ j_i = 1, \ldots, J_i \end{cases} \qquad (D1)$$

Table 3. Additional input parameters for Model 2

Notation	Description
D_{kl}	Distance between *supplier k* and *supplier l.*
$D_{j_i k,(j_i+1) l}$	Distance traveled if *supplier k is* assigned to *service* j_i in *work order i* and *supplier l is* assigned to *service* (j_i+1) in *work order i*.
$Dmax_i$	Manufacturer's (i.e. buyer's) predefined threshold for the maximum distance traveled by the semi-finished parts in *work order i*.

$$\sum_{i=1}^{I}\sum_{j_i=1}^{J_i} c_{ij_i} x_{ij_ik} \le C_k \quad k = 1,\ldots,K \qquad (D2)$$

$$\sum_{j_i=1}^{J_i-1}\sum_{k=1}^{K}\sum_{l=1}^{K} D_{j_ik,j_{(i+1)}l}\, x_{ij_ik}\, x_{i(j_i+1)l} \le D_{max_i}$$

$$i = 1,\ldots,I \qquad (D3)$$

$$x_{ij_ik} \triangleq \begin{cases} 1 & if\, k\, is\, selected\, for\, service\, j_i \\ 0 & otherwise \end{cases}$$

Constraints *D3* require that the distance traveled in every *work order* be less than a particular threshold. Constraints *D3* make Model 2 an integer nonlinear program (NLP). NLP's are challenging to solve (Bonami, Kilinc, & Linderoth, 2012). The number of terms in the left side of constraints *D3* grows as the number of *services* and *suppliers* increases. Only a few terms in the left side of the constraint will be non-zero, but it cannot be known until the problem is solved. Constraints *D3* resemble the ones in the objective function of a Quadratic Assignment Problem (Rardin, 1998). Solving Model 2 to optimality turns difficult for large problems.

Using the parameter values in Tables 2 and 4, values for D_{max_i} equal to 100, 101, and 102 for *work orders* 1-3, respectively, and the non-linear programming model presented above, the optimal solution (i.e., optimality gap 0.17%) to Model 2 was obtained with FICO Xpress Optimization Software. Programming the generic equation for the constraint D3 was easier to do in Xpress than in Excel Solver. The resulting assignment is: for *work order* 1, *supplier* 5 is assigned to *service* 1, *supplier* 2 is assigned to *service* 2 and *supplier 4* is assigned to *service* 3. The distance traveled in *work order* 1 is 92. For *work order* 2, *supplier* 5 is assigned to *service* 1 and *supplier* 4 is assigned to *service* 2. The distance traveled in *work order* 2 is 18. For *work order* 3, *supplier* 2 is assigned to

Table 4. Distances between suppliers' k and l, D_{kl} *for a hypothetical supply chain*

	Supplier 1	Supplier 2	Supplier 3	Supplier 4	Supplier 5
Supplier 1	0	34	45	87	22
Supplier 2	34	0	89	69	23
Supplier 3	45	89	0	13	35
Supplier 4	87	69	13	0	18
Supplier 5	22	23	35	18	0

service 1, *supplier* 3 is assigned to *services* 2 and 3. The distance traveled in *work order* 3 is 89. The total score for the resulting assignment is 5.27. As expected, this score is lower than the optimal score of 5.35 found in Model 1. However, the percentage difference is only 1.49% and the solution in Model 2 satisfy the distance constraint. Xpress took 0.10 seconds to find the optimal solution to Model 2.

This paper proposes an alternative methodology to avoid solving the nonlinear program in Model 2. The approach is to iteratively and dynamically introduce constraints that ultimately permit to find a solution that satisfies the distance thresholds. First, a model is solved without inclusion of any constraints of type *D3*. Thus the model solved is linear. Then, the program coded to solve the problem analyzes the solution and identifies the *work orders* that exceed D_{max_i}. In iteration 1, constraints $D3\hat{i}$ are added to preclude the selection of the same assignment of *suppliers* to *services* in the *work orders* that exceed D_{max_i}. In $D3\hat{i}$, x'_{ij_ik} denotes those decision variables with a current value of 1 in the *work order* i, T_i is the set grouping all the x'_{ij_ik}, variables, and $|Ti|$ represents the set cardinality (i.e., set size). At least one of the current assignments will be removed from each *work order* i that does not satisfy the distance threshold. The program with objective function *DOF* and constraints *D1*, *D2* and $D3\hat{i}$ is solved. If the new generated solution still has *work orders* violating the distance constraints, a new iteration is started and the step of adding new constraints of type $D3\hat{i}$ is repeated.

$$\sum_{j_i=1}^{J_i}\sum_{k=1}^{K} \boldsymbol{x}'_{ij_ik} \le \left|\boldsymbol{T}_i - 1\right| \qquad \left(D3\hat{i}\right)$$

$$\forall\, work\, order\, i\, that\, violates \qquad (D3)$$

The authors used the alternative methodology described above for solving the hypothetical supply chain problem exemplified in Tables 2 and 4 . The same assumed values for D_{max_i} were used (i.e., 100, 101, and 102). The methodology was coded in FICO Xpress Optimization Software. The resulting assignment, distances traveled, and optimal score exactly match the ones obtained with the nonlinear Model 2. Xpress took 0.045 seconds to find the optimal solution. The constraints of type $D3\hat{i}$ added at iteration 1 in the hypothetical supply chain example were: $x'_{115} + x'_{123} + x'_{132} \le 2$ for *work order* 1 and $x'_{314} + x'_{324} + x'_{332} \le 2$ for *work order* 3. A second iteration was necessary for *work order* 3. The new added constraint was $x'_{311} + x'_{323} + x'_{332} \le 2$.

Model 3: *Column generation*

For integer programming models with huge numbers of decision variables, it is sometimes inefficient to consider all columns of the linear programming relaxation model, and most of the columns will have an associated decision variable equal to zero in an optimal solution anyway (Savelsbergh, 2009). Column generation algorithms start with the linear programming relaxation of the integer programming model. The relaxation has an incomplete set of columns. This is called the Master model. Every time the Master model is solved, new information about its dual variables is gathered. Sub-problems are separately solved and they use information about the dual variables. The solutions to each sub-problem correspond to new valid columns in the Master model that prove to be beneficial to the problem. These new columns, if found, are added to the Master model. The new Master model is solved again. The steps of (1) solution of the current Master model and collection of dual variables values, (2) solution of sub-problems, and (3) addition of new columns to the Master model are repeated iteratively until all sub-problems cannot find any attractive columns to incorporate into the Master model. The reader can refer to Desaulniers, Desroisers, & Solomon (2010) to learn more about the column generation method.

In this research, the first author of the paper developed a column generation scheme in a transposed representation of the supplier selection problem depicted in Table 2 for a hypothetical supply chain. The author learned that in the supplier selection problem, there is a large number of *ways* in which *services* in the *work orders* may be satisfied by the *suppliers*. Oguz (2002) mentions multiple previous works in which column generation was useful because the problem had an extremely large number of possible columns, and explicit inclusion of all of them was impossible or consumed too much memory. Oguz (2002) also states that the efficiency of column generation is more noticeable in problems where the ratio between columns (n) and rows (m) exceeds 10.

In a supply chain configuration problem with 40 *work orders*, 50 *services*, and 250 *suppliers*, the transposed representation will have more than 2000 columns (40*50*ways* to satisfy a *service* in a *work order*) and 250 rows. The ratio between columns and rows, *n/m*, will easily exceed 10. Thus a transposed representation would facilitate to exploit the benefits of column generation. In Model 3, *suppliers* are represented in the rows and *work orders* and *services* are represented in the columns. In the proposed column generation scheme, a generated column is a *way* to satisfy a particular *service* in a given *work order*. Model 3 includes the Master and the sub-problems models. Model 3 keeps most of the notation used for input parameters in Model 1. Table 5 presents additional notation needed and its meaning.

Table 6 depicts the $z_{ij_i}^{(b_{ij_i})}$ variables and their possible values (bolded rows 3 and 4). This table also exemplifies some possible values for the $r_{ij_ik}^{(b_{ij_i})}$ binary coefficients (see columns in the constraints' coefficients matrix) for the hypothetical supply chain example with 5 *suppliers*, 3 *services* in *work orders* 1 and 3 and 2 *services* in *work order* 2. Even for this small hypothetical example, it is difficult to depict entirely the array of decision variables and the constraints coefficients matrix. To keep the table under the page width limitations, information is partly exemplified just for the first *work order* (WO1). To clarify the notation used in the table, first sub-column under column S_{21} under *work order* 1 (i.e. z_{12}^{1}) means the *way* number 1 of satisfying s*ervice* 2 in *work order* 1. In Table 6, the subscripts accompanying superscript *b* were dropped to simplify the notation. The Master model and the generic sub-problem model for Model 3 are presented below.

Model 3: *Master problem*

$$Max\ \ Grandscore_{cg} =$$
$$= \sum_{i=1}^{I} \sum_{j_i=1}^{J_i} \sum_{b_{ij_i}}^{B_{ij_i}} \sum_{k=1}^{k} score_{ij_ik}\, r_{ij_ik}^{(b_{ij_i})}\, z_{ij_i}^{(b_{ij_i})} \quad (CGOF)$$

s.t.

$$\sum_{b_{ij_i}=1}^{B_{ij_i}} z_{ij_i}^{(b_{ij_i})} = 1 \quad \begin{cases} i = 1,\ldots,I \\ j_i = 1,\ldots,J_i \end{cases} \quad (CG1)$$

Table 5. Additional input parameters and decision variables for Model 3

Notation	Description
$r_{ij_ik}^{(b_{ij_i})}$	Binary parameters that correspond to the columns of the constraint coefficients matrix in the Master model. They are also the decision variables in a sub-problem model. A single-value of this parameter is 1 if the *way b (b=1,...,B)* for satisfying *service* j_i in *work order i* includes *supplier k* and 0 otherwise.
$z_{ij_i}^{(b_{ij_i})}$	This new binary decision variable equals 1 if *way b (b=1,...,B)* of satisfying *service* j_i in *work order i* is selected and 0 otherwise.

Table 6. Representation of Model 3 for a hypothetical supply chain

	WO1									WO2		WO3			
		S_{11}			S_{21}**			S_{31}		S_{12}	S_{22}	S_{13}	S_{23}	S_{33}	
	z_{11}^1 =	… z_{11}^b… =	z_{11}^B =	z_{12}^1 =	… z_{12}^b… =	z_{12}^B =	z_{13}^1 =	… z_{13}^b… =	z_{13}^B =	. . . .					
*Su**	**0**	**1**	**0**	**1**	**0**	**0**	**0**	**0**	**1**	. . . .					C_k
1	0	0	0	r_{121}^1 =1	0	0	0	0	0	. . . .					17
2	0	r_{112}^b =1	0	0	0	0	0	0	0	. . . .					13
3	r_{113}^1 =1	0	0	0	0	r_{123}^B =1	0	0	0	. . . .					12
4	0	0	r_{114}^B =1	0	r_{124}^b= 1	0	r_{134}^1 =1	0	0	. . . .					19
5	0	0	0	0	0	0	0	r_{135}^b= 1	r_{135}^B =1	. . . .					17
c_{ij_ik}	3	3	3	7	7	7	9	9	9	. . . .					

*: *Su* = *Supplier* number.
**: S_{21}=*Service* two in *work order* one.

$$\sum_{i=1}^{I}\sum_{j_i=1}^{J_i} r_{ij_ik}^{(b_{ij_i})} c_{ij_i} z_{ij_i}^{(b_{ij_i})} \le C_k \qquad \mathrm{k}=1\ldots.K \quad (CG2)$$

The decision variables $z_{ij_i}^{(b_{ij_i})}$ take any value between zero and one. The removal of the integrality constraints for the variables $z_{ij_i}^{(b_{ij_i})}$ enable to solve the Master problem as a linear program and find the dual variables. The dual variables π_{ij_i} obtained from the convexity constraints (CG1) let to price the sub-problem solutions and to identify a stopping point for the iterative process. The dual variables δ_k are associated with the capacity constraints (CG2).

Model 3: *Sub-problem to solve for each service j_i and work order i.*

$$Max$$

$$Totval_{ij_i} = \sum_{k=1}^{K}\left[\left(score_k - \delta_k\right) r_k\right] - \pi_{ij_i} \qquad (SPOF)$$

s.t.

$$\sum_{k=1}^{K} r_k = 1 \qquad (\boldsymbol{CG}3)$$

$$r_k \triangleq \begin{cases} 1 \ \ if\, supplier\, k\, is\, selected \\ 0 \ \ otherwise \end{cases}$$

Constraint *CG3* is essentially the same as *T1*. It ensures that each *service* in each *work order* is assigned to a single *supplier*. The last line in the sub-problems for Model 3 corresponds to the binary sign constraints for the decision variables, r_k. Constraints *CG3* and the sign constraints ensure the generation of new valid columns for the Master model. Since the transposed decomposition scheme solves a simple sub-problem for each *service* j_i and *work order* i, the column generation model researched in this paper is capable of solving problems with a large number of *services* and *work orders*. A sub-problem solution generates a *way* $r_{ij_ik}^{(b_{ij_i})}$ in which a given *service* in a *work order* can be satisfied given the price in the sub-problem objective function *(SPOF)* is positive. This *way* is added as a column to the Master problem. The process of solving assignment sub-problems stops when there are no solutions that provide a positive value for *SPOF*. In this work, the resulting Master problem after no more columns can be added was solved as an integer program. However, this is not a general scenario; for certain problem sizes and input parameter values, the integer program may be very difficult to solve to optimality. Thus, the solution approach in this work is heuristic in general if compared to using the branch and price method (Savelsbergh, 2009).

The authors tested the column generation method described using the input parameters for the hypothetical supply chain problem in Table 2. The methodology was coded in FICO Xpress Optimization Software. The resulting assignment and optimal score exactly match the one obtained with the linear programing Model 1. More extensive comparisons regarding computational time and solution quality for Model 1 vs. Model 3 are in the next section.

NUMERICAL RESULTS

Models experimentation was done in an Intel® CoreTM i7 2600 CPU @ 3.4 GHz, 16.00 GB of RAM, Microsoft Windows 7, 64-bit Operating System computer. Ten test problems with 5 replicates in each problem were generated. The methodology to compute the values for the model input parameters for these problems is explained in Table 7. The problems have 5-50 *work orders*, 10 *services*, and 500-5000 *suppliers*. Thus in each problem the ratio of *suppliers* to total *services* in *work orders* is kept as 10:1. A single test problem number and replicate is called an instance. The model runs are randomized and run serially with a program developed with the FICO Xpress Optimization Suite. The results of each run are transmitted automatically to an Excel database.

Results Comparison: *Model 1 Traditional Formulation without Distance Constraints vs. Model 3 Column Generation*

Tables 8 and 9 show the computational times and objective function values after running Models 1 and 3 on the test problems generated. Both models solved the problems to optimality. Model 3 was faster in general, especially if a problem has more than 15 *work orders*, 10 *services*, and 1500 *suppliers*.

With the computational resources used in this research, Model 1 is limited to efficiently solve problems with 5,000 *suppliers* and 2500,000 variables. Model 1 solves a problem with 100 *work orders* of 10 *services* each, and 10,000 *suppliers* in 17 minutes, 14 seconds. Model 1 solves a problem with the same number of *work orders* and *services* but 12,500 *suppliers* in about 26 minutes. Furthermore, the computer lacked sufficient memory to generate larger problems. On the other hand, the performance of Model 3 could improve further by solving the sub-models in parallel.

Results for Model 2: *Traditional Formulation with Distance Constraints*

For small problems, solving the NLP Model 2 with quadratic distance constraint took about the same time than using the method of adding constraints $\left(\boldsymbol{D}3\hat{\boldsymbol{i}}\right)$ dynamically. However, the NLP problem became quickly intractable. Nevertheless, the results obtained in the small problems solved confirm that the quadratic formulation with distance constraint functions as intended. Further research could involve looking for ways to linearize the distance constraint and/or to efficiently incorporate it into Model 3. However, the column generation model without distance constraints is still appealing to solve practical supplier selection problems. In some cases, by imposing constraints on the distance traveled by *work orders,*

Table 7. Methodology to compute the values for the input parameters in the models

Input Parameter	Description
$score_{ij_ik}$	*Matching score* values were obtained through the DMM as described in the Introduction section (second and third paragraphs).
c_{ij_i}	Average duration for each *service* in a *work order* is a number between 1 and 13 time units.
C_k	Individual *supplier* capacity varies between 25% and 50% in excess of the ratio between total capacity demanded in all *work orders* and number of *suppliers*.
D_{kl}	Distances between *suppliers k* and *l* were generated as random numbers between 1 and 100.
D_{max_i}	The first author computed the maximum distance traveled for different feasible assignments of *suppliers* to *services* in *work order i*, and the average maximum distance was selected as D_{max_i}. Since distance traveled parallels to time, D_{max_i} parallels to the expected manufacturer's lead time for a *work order*.

suppliers may end concentrated in the same geographical region. It can be undesirable if the manufacturing company (i.e. the buyer) is too vulnerable to cost risks in the particular area such as spikes in transportation costs due to a strike in the port of origin (Beil, 2010).

Effect of Reducing the Input Parameter D_{max_i}

This work also investigated the effect in the resulting average *matching score* if the input parameter, pre-defined threshold distance traveled in *work orders*, D_{max_i}, is reduced in the range of 5% to 75%. Fourteen new test problems were generated with the characteristics shown in Table 10 . When D_{max_i} was reduced by 75% no feasible solution was found. Figure 1 shows the results of this investigation. The poor results on the bottom half of Figure 1 come only from problems with the smallest number of *suppliers* (i.e., 25 *suppliers*). This supplier pool size is much smaller than it is expected in the practical web-based platform application. We conclude that for the problems generated and for pools larger than 175 *suppliers*, the average *matching score* stays over 0.99 even when the distance threshold value reduces up to 50%. This result agrees with the authors' intuition: "Problems with a larger pool of *suppliers* are less sensitive to threshold distance reductions".

Figure 2 shows the difference in resulting average *matching scores* by number of *suppliers* if comparing the Model 2 without reduction in threshold distance to the one with a 37.5% reduction. Figure 2 show that for problems with 100 *suppliers* the impact on average score is close to 0.1% and that for problems with 175 *suppliers* the impact is near 0.05%.

CONCLUSION

This research demonstrates how an Agile Supply Chain Configuration (ASSC) problem, a variant of the multiple sourcing supplier selection problem (MSSSP), can be effectively modeled as a generalized assignment problem when supplemented with a search agent in the digital manufacturing market (DMM). At the best of our knowledge this is the first work modeling a multiple sourcing supplier selection problem as a generalized assignment problem.

In the test problems studied, the scalability problem in Model 1 Traditional formulation

Table 8. Computational times and objective function values for models 1 and 3 ran on test problems 1-5

						Model 1 Traditional formulation		Model 3 Column Generation	
Instance number		*WO*	*Services*	*Suppliers*	Total Variables	Time (sec)	Objective Value	Time (Sec)	Objective Value
1	1	5	10	500	25000	0.249	38.93323	0.655	38.93323
	2				25000	0.256	39.93190	0.499	39.93190
	3				25000	0.312	45.90898	0.624	45.90898
	4				25000	0.479	38.93323	0.515	38.93323
	5				25000	0.504	42.92369	0.686	42.92369
2	1	10	10	1000	100000	1.718	78.92174	2.105	78.92174
	2				100000	1.673	76.92379	1.809	76.92379
	3				100000	1.780	82.91943	1.902	82.91943
	4				100000	1.828	83.91909	2.043	83.91909
	5				100000	2.807	82.91943	1.934	82.91943
3	1	15	10	1500	225000	5.858	125.9108	4.944	125.9108
	2				225000	5.329	119.9137	4.303	119.9137
	3				225000	5.689	121.9132	3.945	121.9132
	4				225000	5.797	127.9081	4.570	127.9081
	5				225000	5.435	123.9121	4.383	123.9121
4	1	20	10	2000	400000	13.768	169.9145	8.876	169.9145
	2				400000	13.392	165.9163	8.469	165.9163
	3				400000	14.121	171.9138	8.781	171.9138
	4				400000	12.849	159.9200	8.579	159.9200
	5				400000	12.971	164.9178	8.486	164.9178
5	1	25	10	2500	625000	29.749	204.9168	15.272	204.9168
	2				625000	24.306	211.9142	15.553	211.9142
	3				625000	25.227	210.9146	15.350	210.9146
	4				625000	23.670	204.9168	14.929	204.9168
	5				625000	24.113	209.9148	15.349	209.9148

without distance constraints improves by solving Model 3 Column generation. The Generalized Assignment Problem (GAP) is NP-hard (Cattrysse, Salomon, & Wassenhove, 1994; Fisher, Jaikumar, & Van Wassenhove, 1986), and many approaches have been developed to solve it either approximately or exactly. In the GAP, the more limited in capacity the machines are, the more difficult is to solve the problem to optimality. However, *suppliers* do not seem to exactly resemble the machines in the GAP because a very limited suppliers' capacity is not always the case in the practical manufacturing marketplace and it wasn't the case studied in the generated test problems. This *suppliers'* characteristic, allows us to exploit a simpler customized decomposition scheme for the column generation method. The scheme gener-

Table 9. Computational times and objective function values for models 1 and 3 ran on test problems 6-10

						Model 1 Traditional Formulation		Model 3 Column Generation	
Instance Number		WO	Services	Suppliers	Total Variables	Time (sec)	Objective Value	Time (sec)	Objective Value
6	1	30	10	3000	900000	40.462	245.9219	24.617	245.9219
	2				900000	48.790	240.924	24.039	240.924
	3				900000	41.883	242.9225	24.211	242.9225
	4				900000	50.111	252.9204	24.819	252.9204
	5				900000	41.688	252.9204	25.037	252.9204
7	1	35	10	3500	1225000	65.648	276.9141	36.534	276.9141
	2				1225000	64.727	296.9077	38.906	296.9077
	3				1225000	77.340	288.9103	37.751	288.9103
	4				1225000	64.867	297.9066	38.766	297.9066
	5				1225000	67.637	286.9106	37.564	286.9106
8	1	40	10	4000	1600000	96.241	327.9195	54.678	327.9195
	2				1600000	104.588	341.9178	56.69	341.9178
	3				1600000	109.395	325.9207	54.304	325.9207
	4				1600000	98.8400	331.9191	54.787	331.9191
	5				1600000	101.914	335.9188	55.723	335.9188
9	1	45	10	4500	2025000	136.570	373.9103	77.142	373.9103
	2				2025000	135.382	360.9132	75.114	360.9132
	3				2025000	134.741	370.9108	76.612	370.9108
	4				2025000	141.341	375.9098	77.813	375.9098
	5				2025000	138.193	374.9099	77.734	374.9099
10	1	50	10	5000	2500000	180.727	418.9161	106.314	418.9161
	2				2500000	177.513	401.9190	101.244	401.9190
	3				2500000	184.104	412.9168	104.301	412.9168
	4				2500000	188.176	410.9173	103.787	410.9173
	5				2500000	193.575	421.9157	105.924	421.9157

ates *service* assignment sub-problems instead of *supplier* sub-problems.

A marginal intention of this study was to identify the degree to which reductions in distance thresholds could be imposed to the *work orders* without significantly influencing the resulting assignments. Considerations regarding the distance traveled by a *work order* are of critical importance in globalized supply chains. For the test problems studied, the results show that, due to the large size of the *suppliers* market, buyers (i.e., manufacturers) may impose an economy of distance with little effect on match compatibility.

Furthermore, computational results show that using a scoring agent from the DMM to find the parameter values for the objective function of the models, works well with very

Table 10. Characteristics of the problems used to test the effect of reducing D_{max_i}

Problem Size	Work Orders	Services	Suppliers	Decision Variables
1	5	5	25	625
2	5	10	25	1250
3	10	10	50	5000
4	10	15	50	7500
5	15	15	75	16875
6	15	20	75	22500
7	20	20	100	40000
8	20	25	100	50000
9	25	25	125	78125
10	25	30	125	93750
11	30	30	150	135000
12	30	35	150	157500
13	35	35	175	214375
14	35	40	175	245000

Figure 1. Impact on average score from reductions in original threshold distance, D_{max_i} *, between 5% -75%*

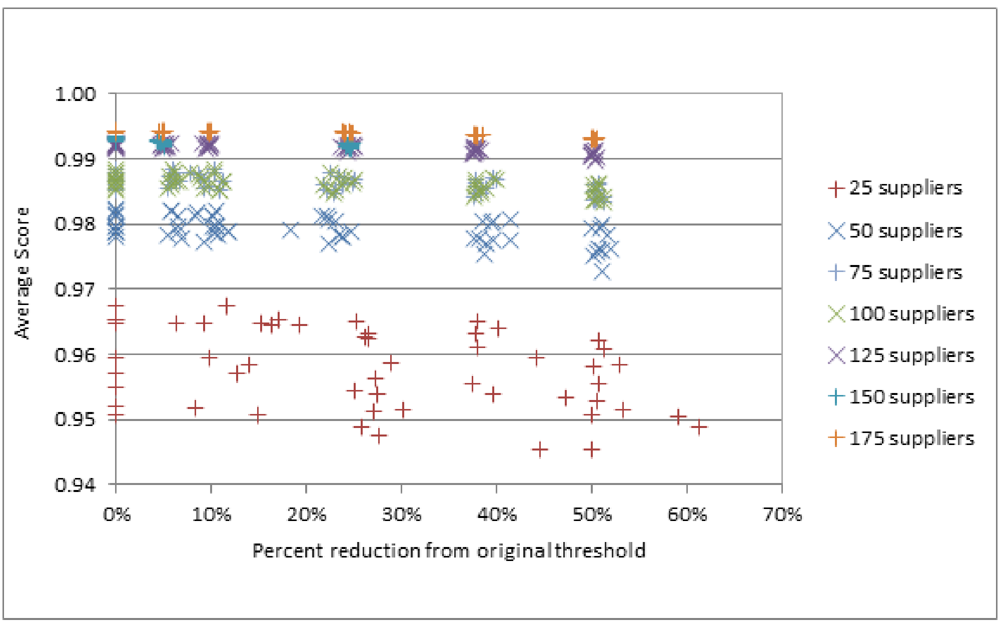
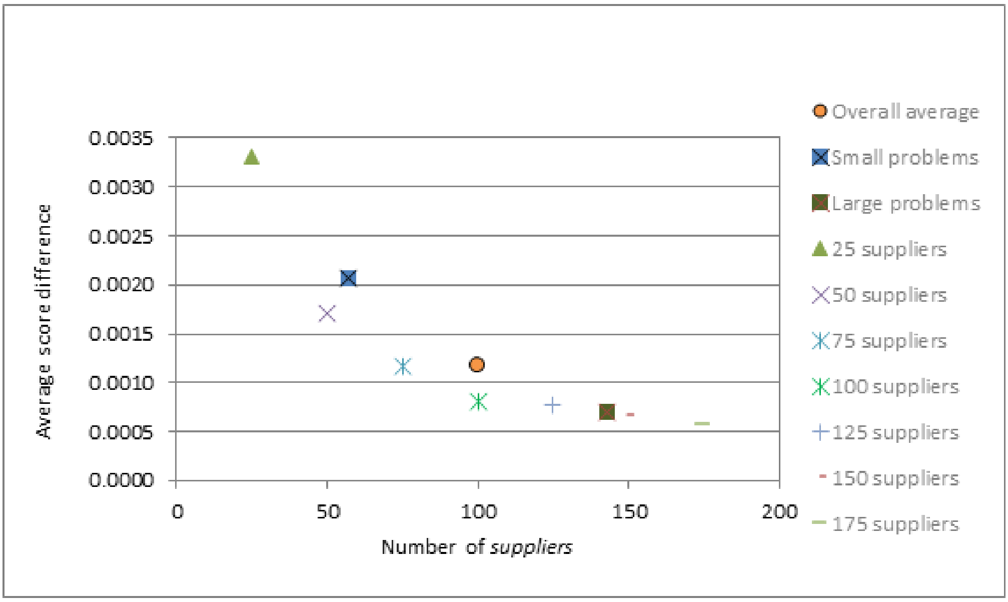


Figure 2. Model 2 average score difference for no threshold distance reduction vs. 37.5% threshold reduction



large problem sizes. The scoring agent avoids to run an optimization based on vague max/min objectives. Also, it avoids the staggered inclusion of constraints, like capacity and distance, and the use of complementary models as in Chamodrakas, et al. (2010).

As a further research the authors propose to find efficient ways: (1) to deal with the non-linear distance constraint *D3* presented in Model 2 and (2) to incorporate the distance constraint in Model 3. The authors are exploring the inclusion of the distance constraint in Model 3 by solving the sub-problems by *work order* and generating combinations of *suppliers* that satisfy the requested *services* within the *work order* under the pre-defined threshold distance. A comparison of the complexity and efficiency of including the distance constraint implicitly vs. explicitly in the sub-problems will be studied. Finally, this study used test generated problems; the creation of a database hosting test problems that multiple researchers on the Supplier Selection Problem can access would benefit the research community.

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Hayden Beauchamp is an Operations Research Analyst at Energy Solutions where he supports the Hanford environmental cleanup mission to immobilize 56 million gallons of legacy waste from nuclear weapons production during WWII and the Cold War, currently stored in 177 underground tanks in the Hanford Tank Farms. He received his B.S. in Industrial Engineering and M.S. in Industrial Technology from Texas State University. As part of the Mission Analysis Engineering team his current work includes simulation-based optimization of waste retrieval and disposition processes in order to develop heuristics for scheduling 50 years of day-to-day operations and to produce design inputs for state-of-the-art mobilization, treatment and immobilization technologies. This involves a heuristic approach to parametric optimization through a combination of waste chemistry/rheology process modeling and stochastic simulation.

Clara Novoa is an associate professor at the Ingram School of Engineering, Industrial Engineering Program, at Texas State. She is one of the two lead faculty in the Systems Optimization Learning and Reliability (SOLAR) research laboratory. She received her doctoral degree in Industrial and Systems Engineering from Lehigh University in 2005. Dr. Novoa got her B.S. in Industrial Engineering from Universidad de Los Andes, Bogota, Colombia and her M.E. in Industrial Engineering from the University of Puerto Rico at Mayaguez. Her research interest include Supply Chain Optimization and Parallel and Distributed Computing to accelerate exact and heuristic optimization methods. Dr. Novoa is a lead faculty in S-STEM and IUSE grants funded by the National Science Foundation (NSF) to improve undergraduate STEM education and increase the recruitment and retention of female in Engineering and Computer Science. Dr. Novoa's publications are in journals such as European Journal of Operational Research, Quality Engineering, Electric Power System Research and International Journal of Parallel, Emergent and Distributed Systems. Clara Novoa is the corresponding author for this paper.

Farhad Ameri is an assistant professor in the Department of Engineering Technology at Texas State University and the lead faculty in the Engineering Informatics Research Group (INFONEER). He received his doctoral degree in Manufacturing Engineering from the University of Michigan Ann Arbor in 2006 and served as a research fellow in the Clemson Engineering Design Applications and Research (CEDAR) Group at Clemson University. He received his B.S and M.S in Industrial Engineering form Iran University of Science and Technology and Sharif University of Technology respectively. Dr. Ameri's research interests include Engineering Informatics, Knowledge-based Engineering, Distributed Design and Manufacturing, and Model-Based Systems Engineering. His research is funded by the National Science Foundation (NSF) and the National Institute of Standards and Technology (NIST). Dr. Ameri's work has appeared in renowned journals such as ASME Journal of Computing and Information Science in Engineering, ASME Journal of Mechanical Design, International Journal of Advanced Engineering Informatics, Research in Engineering Design, and Computer-Aided Design and Applications.