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CONTROLLABILITY OF SEMILINEAR INTEGRODIFFERENTIAL EQUATIONS WITH NONLOCAL CONDITIONS

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Abstract. We establish sufficient conditions for the controllability of some semilinear integrodifferential systems with nonlocal condition in a Banach space. The results are obtained using the Schaefer fixed-point theorem and semigroup theory.

1. Introduction

The first step in the study of the problem of controllability is to determine if an objective can be reached by some suitable control function The problem of controllability happens when a system described by a state $x(t)$ is controlled by a given law such as a differential equation $x' = G(t, x(t), u(t))$. We discuss the possibility of driving a solution of a given system from an initial state to a final state by an adequate choice of the control function u.

Several authors have studied the problem of controllability of linear semilinear and nonlinear systems of ordinary differential equations in finite or infinite dimensional Banach spaces with bounded operators. For instance, Naito [6] studied the controllability of semilinear systems, Yamamoto and Park [8] discussed this problem for a parabolic equation with uniformly bounded nonlinear terms, Chukwu and Lenhart [3] studied the controllability of nonlinear systems in abstract spaces, Zhou [10] discussed the approximate controllability for a class of semilinear abstract equations, Naito [7] established the controllability for nonlinear Volterra integrodifferential systems. Finally, Balachandran and Sakhtivel [1, 2] studied the controllability of functional semilinear integrodifferential systems in Banach spaces.

In this paper, we study the controllability of some semilinear integrodifferential system subject to nonlocal condition in Banach space whose mild solution has been proved by Mazouzi and Tatar [5] by using Schaefer fixed-point theorem [4].

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2. Preliminaries

Consider the following functional semilinear integrodifferential system subject to a nonlocal condition:

$$\begin{gathered} x'(t) = Ax(t) + Bu(t) \\ + F\Big(t, x(\delta_1(t)), \int_0^t g\Big(t, s, x(\delta_2(s)), \int_0^s k(s, \tau, x(\delta_3(\tau)))d\tau\Big)ds\Big) \\ x(0) + h(t_1, \dots, t_p, x(.)) = x_0, \\ 0 < t_1 < t_2 \dots < t_p \le b, \quad t \in I = [0, b]. \end{gathered} \tag{2.1}$$

The expression $h(t_1, \dots, t_p, x(.))$ indicates that the function x is valued only on the set $\{t_1, t_2 \dots, t_p\}$. Actually, the nonlocal condition has a better effect on the solution and is more precise for physical measurements than the classical condition $x(0) = x_0$ alone. The control function u is given in the Banach space of admissible control functions $\mathbb{L}^2(I, U)$, U being a Banach space. A is the infinitesimal generator of a strongly continuous semigroup of bounded linear operators $T(t)$, $t \ge 0$ in X, B is a bounded linear operator from U into X. Furthermore, $F : I \times X \times X \to X$, $g : I \times I \times X \times X \to X$, $k : I \times I \times X \to X$, $h : I^p \times X \to X$, and $\delta_i \in C(I, I)$ are given functions such that $0 \le \delta_i(t) \le t$, $t \in I$ for $i = 1, 2, 3$.

We need the following fixed-point theorem due to Schaefer [4]:

Theorem 2.1. *Let E be a normed linear space. If $A : E \to E$ is a completely continuous operator (that is, it is continuous and the image of any bounded set is contained in a compact set), then either the subset $\{x \in E : x = \lambda Ax$ for some $\lambda \in (0, 1)\}$ is unbounded or A has a fixed point.*

Definition. The system (2.1) is said to be controllable on the interval I if for every initial state $x(0)$ and a final state x_1 there exists a control $u \in \mathbb{L}^2(I, U)$ such that the solution $x(t)$ of (2.1) satisfies $x(b) = x_1$.

For this article, we set the following assumptions:

- (H1) For each $t \in I$, $F(t, ., .) \in C(X \times X, X)$, and for each $(x, y) \in X \times X$, $F(., x, y)$ is strongly measurable
- (H2) There exist continuous functions p and $q : I \to [0, +\infty[$, and $\alpha \ge 1$ such that
$$\|F(t, x, y)\| \le p(t)\|x\|^\alpha + q(t)\|y\|,$$
for all $x, y \in X$ and $t \in I$.
- (H3) g and k are continuous functions such that
$$\begin{gathered} \|g(t, s, x, y)\| \le m_1(t, s)\|x\|^{\alpha-1}\varphi(\|x\|) + m_2(s)\|y\|, \quad \text{for all } x, y \in X, \\ \|k(t, s, x)\| \le m_3(t, s)\|x\|^{\alpha-1}\varphi(\|x\|), \quad \text{for all } t, s \in I, \end{gathered}$$
where $\varphi : [0, +\infty[\to]0, +\infty[$ it is a continuous nondecreasing function, $m_1 : I \times I \to [0, +\infty[$ is continuous and differentiable almost everywhere with respect to the first variable, $m_2 : I \to [0, +\infty[$ is continuous, $m_3 : I \times I \to [0, +\infty[$ is continuous
- (H4) $T(t)$, $t \ge 0$ is a compact semigroup and there exist some constants $M > 1$ and $\omega \in \mathbb{R}^+$ such that $\|T(t)\| \le Me^{\omega t}$, $t \ge 0$.
- (H5) $h \in C(I, X)$, and there exists a constant $H > 0$ such that $\|h(t_1, \dots t_p, x)\| \le H$, for $x \in B_r = \{x \in X : \|x(t)\| \le r\}$. Moreover, there exists $H_1 > 0$ such

that

$$\|h(t_1,\dots t_p,x_1(.))-h(t_1,\dots t_p,x_2(.))\|\le H_1\sup_{t\in I}\|x_1(t)-x_2(t)\|$$

(H6)

$$\int_0^b \widetilde{Q}(t)dt<\int_a^{+\infty}\frac{dz}{\varphi(z)+z^\alpha+z},$$

where $\widetilde{Q}(t)=\max\{\omega,\omega MM_1M_2,\omega Mp(t),\omega Mq(t),h(t)\}$ with

$$h(t)=\frac{1}{\alpha}m_1(t,t)+\frac{1}{\alpha}\int_0^t\Big|m_2(t)m_3(t,\tau)+\frac{\partial m_1(t,\tau)}{\partial t}\Big|d\tau,$$

and $a^\alpha=M^\alpha(\|x_0\|+H)^\alpha+N$, with

$$N=\Big(\|x_1\|+Me^{\omega b}(\|x_0\|+H)+M\int_0^b e^{\omega(b-\tau)}\|\phi(\tau,x)\|d\tau\Big).$$

(H7) The linear operator $W:\mathbb{L}^2(I,U)\to X$ defined by

$$Wu=\int_0^b T(b-s)Bu(s)ds$$

has an invertible operator W^{-1} which takes values in $\mathbb{L}^2(I,U)/\ker W$ and there exist positive constants M_1, $M_2>0$ such that $\|B\|\le M_1$ and $\|W^{-1}\|<M_2$.

3. Main result

Our main theorem is the following theorem:

Theorem 3.1. *Under hypotheses (H1)–(H7) the system* (2.1) *is controllable on I.*

Proof. Let us define the control function

$$u(t)=W^{-1}\Big(x_1-T(b)(x_0-h(t_1,\dots t_p,x(.)))-\int_0^b T(b-s)\phi(s,x)ds\Big)(t). \quad (3.1)$$

where

$$\phi(t,x)=F\Big(t,x(\delta_1(t)),\int_0^t g(t,s,x(\delta_2(s))),\int_0^s k(s,\tau,x(\delta_3(\tau)))d\tau ds\Big)$$

We shall show that with this control the solution $x(t)$ of system (2.1) satisfies $x(b)=x_1$. Indeed, we apply Schaefer theorem to show that the operator $\Phi:V\to V$, with $V=C(I,X)$, defined by

$$(\Phi x)(t)=T(t)(x_0-h(t_1,\dots t_p,x))+\int_0^t T(t-s)Bu(s)ds+\int_0^t T(t-s)\phi(s,x)ds$$

has a fixed point which is a solution of (2.1). We observe that $(\Phi x)(b)=x_1$ which means that u steers the integrodifferential system from x_0 to x_1 in time b.

We consider the parametrized problem with a parameter $\lambda\in(0,1)$ such that

$$\begin{gathered}x'(t)=Ax(t)+\lambda Bu(t)+\lambda\phi(t,x),\quad 0\le t\le b\\ x(0)+\lambda h(t_1,\dots t_p,x(.))=\lambda x_0,\end{gathered}\quad (3.2)$$

and we show that the solution to this equation is bounded. First, it is not hard to see that system (3.2) has a mild solution satisfying the integral equation

$$\begin{aligned} x(t) = \lambda T(t)(x_0 - h(t_1, \dots t_p, x(.))) + \lambda \int_0^t T(t-s)Bu(s)ds \\ + \lambda \int_0^t T(t-s)\phi(s,x)ds. \end{aligned} \tag{3.3}$$

It follows that

$$\begin{aligned} \|x(t)\| \le M.e^{\omega t}(\|x_0\| + H) + Me^{\omega t} \int_0^t e^{-\omega s}\Big[p(s)\|x(\delta_1(s))\|^\alpha \\ + q(s)\int_0^s m_1(s,\theta)\|x(\delta_2(s))\|^{\alpha-1}\varphi(x(\delta_2(\theta))) \\ + m_2(\theta)\int_0^\theta m_3(\theta,\tau)\|x(\delta_3(\theta))\|^{\alpha-1}\varphi(\|x(\delta_3(\theta))\|)d\tau d\theta\Big]ds \\ + MM_1M_2N.e^{\omega t}\int_0^t e^{-\omega s}ds. \end{aligned}$$

Denote the right hand side of the above inequality by $e^{\omega t}z(t)$, then

$$\|x(t)\| \le e^{\omega t}z(t), \quad 0 \le t \le b.$$

In particular, we have $z(0) = M(\|x_0\| + H)$. Differentiating $z(t)$ we obtain

$$\begin{aligned} z'(t) = Me^{-\omega t}\Big[p(t)\|x(\delta_1(t))\|^\alpha + q(t)\int_0^t (m_1(t,\theta)\|x(\delta_2(\theta))\|^{\alpha-1}\varphi(\|x(\delta_2(\theta))\|) \\ + m_2(\theta)\int_0^\theta m_3(\theta,\tau)\|x(\delta_3(\tau))\|^{\alpha-1}\varphi(\|x(\delta_3(\theta))\|)d\tau)d\theta + M_1M_2N\Big]. \end{aligned}$$

Since $0 \le \delta_i(t) \le t$, for $i = 1, 2, 3$ and $z(t)$ is nondecreasing, it follows that

$$\begin{aligned} &z'(t) \\ &\le Me^{-\omega t}\Big[p(t)e^{\alpha\omega t}z^\alpha(t) + q(t)\int_0^t (m_1(t,\theta)e^{(\alpha-1)\omega\theta}z^{\alpha-1}\varphi(e^{\omega\theta}z(\theta)) \\ &\quad + m_2(\theta)\int_0^\theta m_3(\theta,\tau)e^{(\alpha-1)\omega\tau}z^{\alpha-1}(\tau)\varphi(e^{\alpha\omega t}z(\tau))d\tau)d\theta + M_1M_2N\Big]. \end{aligned}$$

Setting $Q(t) = \max(p(t), q(t), M_1M_2)$ and

$$\begin{aligned} v^\alpha(t) = e^{\alpha\omega t}z^\alpha(t) + \int_0^t (m_1(t,\theta)e^{(\alpha-1)\omega\theta}z^{\alpha-1}\varphi(e^{\omega\theta}z(\theta)) \\ + m_2(\theta)\int_0^\theta m_3(\theta,\tau)e^{(\alpha-1)\omega\tau}z^{\alpha-1}(\tau)\varphi(e^{\alpha\omega t}z(\tau))d\tau)d\theta + N, \end{aligned}$$

we obtain

$$z'(t) \le Me^{-\omega t}Q(t)v^\alpha(t), \quad v^\alpha(0) = z^\alpha(0) + N, \quad v^\alpha(t) \ge e^{\alpha\omega t}z^\alpha(t),$$

so that $v(t) \ge \ e^{\omega t}z(t)$. Differentiating $v^\alpha(t)$ we obtain, after a few calculations,

$$v'(t) \le \omega v(t) + \omega M.Q(t)v^\alpha + h(t)\ \varphi(v(t)).$$

Therefore,

$$v'(t) \le \widetilde{Q}(t)(\varphi(v) + v^\alpha + v).$$

Integrating between 0 and t, we obtain

$$\int_a^{v(t)} \frac{dz}{\varphi(z)+z^\alpha+z} \le \int_0^b \widetilde{Q}(t)dt < \int_a^\infty \frac{dz}{\varphi(z)+z^\alpha+z}.$$

Hence there exists a constant $c>0$ such that $v(t)\le c$, for every $t\in I$. Consequently, $\|x(t)\|\le c$ for every $t\in I$.

In what follows we prove that the operator Φ is completely continuous. If $y(t)\in V : \|y(t)\|\le r$, for $r>0$, then

$$\begin{aligned}
&\|F\Big(t,y(t),\int_0^t g\Big(t,\theta,y(\theta),\int_0^\theta k(\theta,\tau,y(\tau))d\tau\Big)d\theta\Big)\|\\
&\le p(t)\|y(t)\|^\alpha + \ q(t)\int_0^t m_1(t,\theta)\|y(\theta)\|^{\alpha-1}\varphi(\|y(\theta)\|)\\
&\quad + \ m_2(\theta)\int_0^\theta m_3(\theta,\tau)\|y(\tau)\|^{\alpha-1}\varphi(\|y(\tau)\|)d\tau\, d\theta\\
&\le p(t)r^\alpha + q(t)r^{\alpha-1}\varphi(r)\int_0^t (m_1(t,\theta)+m_2(\theta)\int_0^\theta m_3(\theta,\tau)d\tau)\, d\theta.
\end{aligned}$$

We denote the last term of the latter inequality by $F_r(t)$. It is obvious that for each $r>0$, F_r is summable over I.

Consider a sequence $(x_n)_{n\ge1}\subset V$ converging to $\widehat{x}\in V$, then $(x_n)_{n\ge1}(t)$ and $\widehat{x}(t)$ must be contained in some closed ball $B(0,r)\subset X$, for all $t\in I$. It follows from hypotheses (H1) and (H2) that

$$\lim_{n\to\infty}\phi(t,x_n)=\phi(t,\widehat{x}) \quad \text{and} \quad \|\phi(t,x_n)-\phi(t,\widehat{x})\|\le 2F_r(t).$$

We conclude by the dominated convergence theorem that

$$\int_0^b \|\phi(s,x_n)-\phi(s,\widehat{x})\|ds \to 0, \quad \text{when } n\to\infty.$$

Define the sequence $\{u_n\}_{n\ge1}$ as follows

$$u_n(t)=W^{-1}\Big(x_1-T(b)(x_0-h(t_1,t_2,\dots,t_p,x_n))-\int_0^b T(b-s)\phi(s,x_n)ds\Big)(t).$$

Then

$$\begin{aligned}
&\|Bu_n(s)-Bu(s)\|\\
&\le \|BW^{-1}\|\Big[\|T(b)(h(t_1,t_2,\dots,t_p,x_n)-h(t_1,t_2,\dots,t_p,\widehat{x}))\|\\
&\quad +\|\int_0^b T(b-s)(\phi(s,x_n)-\phi(s,\widehat{x}))ds\|\Big]\\
&\le MM_1M_2e^{\omega b}\Big(H_1\sup_{t\in I}\|x_n-\widehat{x}\|+\int_0^b e^{-\omega s}\|\phi(s,x_n)-\phi(s,\widehat{x})\|ds\Big)\to 0,
\end{aligned}$$

as $n \to \infty$. We infer that

$$\begin{aligned}
\|\Phi x_n - \Phi\widehat{x}\| &\leq \sup_{t\in I}\|T(t)(h(t_1,t_2,\dots,t_p,x_n) - h(t_1,t_2,\dots,t_p,\widehat{x}))\| \\
&\quad + \sup_{t\in I}\|\int_0^t T(t-s)[(\phi(s,x_n)-\phi(s,\widehat{x})) + (Bu_n(s) - Bu(s))]ds\| \\
&\leq MH_1e^{\omega t}\sup_{t\in I}\|x_n(t) - \widehat{x}(t)\| \\
&\quad + Me^{\omega b}\Big[\int_0^b (\|\phi(s,x_n) - \phi(s,\widehat{x})\| + \|Bu_n(s) - Bu(s)\|)ds\Big] \to 0\,,
\end{aligned}$$

as $n \to \infty$. This shows that Φ is continuous.

For every positive real number r we set $B_{r,V} = \{x \in V : \|x(t)\| \leq r\}$. To show that $\Phi(B_{r,V})$ is precompact in V we only have to check the precompactness of $\Phi(B_{r,V})(t)$ in V, for each $t \in I$, according to Arzela -Ascoli theorem. Let t be fixed in $]0,b]$ and $n \in \mathbb{N}^* : \frac{1}{n} < t$. For every $x \in B_{r,V}$ we have

$$\begin{aligned}
(\Phi x)(t) &= T(t)(x_0 - h(t_1,\dots t_p,x)) + T(\frac{1}{n})\int_0^{t-\frac{1}{n}} T(t-s-\frac{1}{n}) \\
&\quad\times (Bu(s) + \phi(s,x))ds + \int_{t-\frac{1}{n}}^t T(t-s)(Bu(s)+\phi(s,x))ds.
\end{aligned} \tag{3.4}$$

We set

$$(T_nx)(t) = \int_{t-\frac{1}{n}}^t T(t-s)(Bu(s)+\phi(s,x))ds.$$

For every $\epsilon > 0$, there exists $n_0 \in \mathbb{N}^*$ such that for every $n \geq n_0$, and $x \in B_{r,V}$, we have

$$\|(T_nx)(t)\| \leq \int_{t-\frac{1}{n}}^t \|T(t-s)\|(M_1M_2\tilde{N} + F_r(s))ds < \epsilon\,,$$

where

$$\tilde{N} = \Big(\|x_1\| + Me^{\omega b}(\|x_0\| + H) + M\int_0^b e^{\omega(b-\tau)}F_r(\tau)d\tau\Big).$$

Next, we define

$$\begin{aligned}
&(S_n(x))(t) \\
&= T(t)(x_0 - h(t_1,\dots t_p,x)) + T(\frac{1}{n})\int_0^{t-\frac{1}{n}} T(t-s-\frac{1}{n})(Bu(s)+\phi(s,x))ds\,.
\end{aligned}$$

Following the steps of the proof of the main theorem in [5] we can show that $\Phi(B_{r,V})(t)$ is compact and consequently the operator Φ is completely continuous. Therefore, Φ has a fixed point in $V = C(I,X)$ which is the expected mild solution we are seeking and accordingly the system is controllable on I. $\square$

4. Example

Consider the problem

$$\begin{aligned} z_t(t,y) &= z_{yy}(t,y) + u(t,y) + \frac{z^2(t,y)\sin(z(t,y))}{(1+t)(1+t^2)} \\ &\quad + \int_0^t \Big[\frac{z(s,y)}{(1+t)(1+t^2)^2(1+s)^2} \\ &\quad + \frac{1}{(1+t)(1+t^2)}\int_0^s \frac{z(\tau,y)}{(1+s)(1+\tau)}\exp z(\tau,y)d\tau\Big]ds \\ &\quad z(t,0) = z(t,1) = 0, \quad t \in I = [0,1] \\ z(0,y) - \sum_{i=1}^p t_i z(t_i,y) &= z_0(y), \quad 0 < y < 1,\ 0 < t_1 < t_2 < \dots < t_p \leq 1. \end{aligned} \tag{4.1}$$

Let X denote the Banach space $\mathbb{L}^2(I)$, $z(t,y) = x(t)(y)$ and $u \in \mathbb{L}^2(I,X)$ be the control function. Let

$$h(t_1,t_2,\dots,t_p,x(.)) = \sum_{i=1}^p t_i x(t_i).$$

We can easily check that there exists $H > 0$ such that $|h(t_1,t_2,\dots,t_p,x(.))| < H$; for instance, we may take $H = pt_p r$, if $\|x(t)\| \leq r$. On the other hand, we have

$$\|h(t_1,t_2,\dots,t_p,x_1(.)) - h(t_1,t_2,\dots,t_p,x_2(.))\| < pt_p\|x_1(t) - x_2(t)\|.$$

Moreover, since

$$\begin{aligned} &F\Big(t, x(t), \int_0^t g(t,s,x(s), \int_0^s k(s,\tau,x(\tau))d\tau)ds\Big) \\ &= \frac{x^2(t)\sin(x(t))}{(1+t)(1+t^2)} \\ &\quad + \int_0^t \Big[\frac{x(s)}{(1+t)(1+t^2)^2(1+s)^2} + \frac{1}{(1+t)(1+t^2)}\int_0^s \frac{x(\tau)}{(1+s)(1+\tau)}\exp x(\tau)d\tau\Big]ds, \end{aligned}$$

we have

$$\|F(t,x,j)\| = \|\frac{1}{(1+t)(1+t^2)}(x^2\sin x + j)\| \leq \frac{1}{(1+t^2)}\|x\|^2 + \frac{1}{(1+t)}\|j\|,$$

where

$$j = \int_0^t g\Big(t,s,x(s), \int_0^s k(s,\tau,x(\tau))d\tau\Big)ds.$$

Next, if $h = \int_0^s k(s,\tau,x(\tau))d\tau$, then

$$\begin{aligned} \|g(t,s,x,h)\| &= \|\frac{x}{(1+t)(1+t^2)^2(1+s)^2} + \frac{h}{(1+t)(1+t^2)}\| \\ &\leq \frac{1}{(1+t^2)(1+s)}\|x\| + \frac{1}{(1+t^2)(1+t)}\|h\|. \end{aligned}$$

Finally, we have

$$\|k(s,\tau,x)\| = \|\frac{xe^x}{(1+s)(1+\tau)}\| \leq \frac{1}{(1+s)(1+\tau)}\|x\|\exp(\|x\|).$$

Define the operator $A : D(A) \subset X \to X$ by $Av = v''$ with domain

$$D(A) = \{v \in X : v, v' \text{ absolutely continuous}, \ v'' \in X, \ v(0) = v(1) = 0\}.$$

Note that $D(A)$ is dense in X and A is a closed operator. We conclude by the Hille-Yosida theorem that A is an infinitesimal generator of an analytic semigroup $T(t)$, $t \geq 0$ which is also compact and satisfies hypothesis (**H4**). Furthermore,

$$Av = \sum_{n=1}^{\infty} n^2 (v, v_n) v_n, \quad v \in D(A)$$

$$T(t)v = \sum_{n=1}^{\infty} \exp(-n^2 t)(v, v_n) v_n, \quad v \in X,$$

where $\lambda_n = n^2$, $n = 1, 2, \dots$ are the eigenvalues of A, and $\{v_n(s) = \sqrt{2} \sin ns\}_{n \geq 1}$ is the orthogonal set of eigenfunctions of A.

Let $Bu : I \to X$ be defined by $Bu(t)(y) = u(t, y)$, $y \in (0, 1)$. Define the linear operator W by

$$Wu = \int_0^1 T(1-s)u(s)ds = \sum_{n=1}^{\infty} \int_0^1 \exp[-n^2(1-s)](u(s, y), v_n) v_n ds,$$

assuming that it has a bounded inverse operator W^{-1} in $L^2(I, X)/\ker W$ satisfying hypothesis (H7).

With this choice of A, B, F, and h, we observe that (2.1) is an abstract formulation of (4.1), and accordingly, system (4.1) is controllable on I whose control function is

$$\begin{aligned} u(t) = W^{-1}\Big(& x_1 - T(1)(x_0 - \sum_{i=1}^{p} t_i x(t_i)) \\ & - \int_0^1 T(1-s) \frac{1}{(1+s)(1+s^2)} \Big[z^2(s, y) \sin(z(s, y)) \\ & + \int_0^s \Big(\frac{z(\tau, y)}{(1+s^2)(1+\tau)^2} + \int_0^\tau \frac{z(v, y)}{(1+\tau)(1+v)} e^{z(v,y)} dv \Big) d\tau \Big] ds \Big)(t). \end{aligned}$$

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