FOSTERING MATHEMATICAL CREATIVITY IN THE MIDDLE GRADES:

PEDAGOGICAL AND MATHEMATICAL PRACTICES

by

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ABSTRACT

Increased automation and outsourcing have increased the need for creativity in many domestic jobs, so the purpose of this study is to explore middle school students' opportunity to be mathematically creative. The process standards of the Texas Essential Knowledge and Skills (TEKS) and National Council of Teachers of Mathematics (NCTM) and the Standards for Mathematical Practice of the Common Core State Standards (CCSS) can be inferred to indicate that mathematics content should be taught in a way that develops mathematical creativity. A qualitative case study was done to describe ways that three teachers fostered mathematical creativity in the middle grades. Classroom observations were triangulated with teacher and student interviews, researcher's log, and documents. Transcripts for whole class discussions of 40 hours of observation and transcripts for teacher and student interviews were open coded initially before categories were standardized and themes emerged. The three themes that emerged were that the teachers helped the students make mathematics personally meaningful, the teachers helped create an environment where students were comfortable expressing their personally meaningful understanding of mathematics and making mistakes, and they maintained expectations of mathematics practices. The teachers helped students make mathematics personally meaningful by allowing students to make some choices in how they do mathematics (use alternative methods, use alternative answer forms, solve problems with multiple correct answers, and flexibility with creating graphs and tables), to use their own words to describe mathematical concepts rather than emphasizing

memorization from a textbook, and to make connections (students' interests and experiences, school experiences and other content areas, and other real world experiences through the eyes of the teacher). A safe environment was created by allowing students adequate thinking time, making it clear that the students' voices were important (ask questions, share ideas and experiences, differentiate between off-task conversations and enthusiasm, insist students respect each other, and ensure all students participated in whole class discussion), promoting the idea that mistakes are okay (okay for students and teacher, provide a learning experience, and point out silver lining in incorrect or incomplete solutions), encouraging the use of resources, and emphasizing effort over perfection. Finally they maintained mathematics practices such as explaining reasoning, using appropriate terminology and notation, and using estimation to determine reasonableness of answers.

I. INTRODUCTION

Creativity is an important part of the modern world and a valued skill in the workplace according to authors such as Professor Emeritus Ken Robinson (2001/2011), economist Richard Florida (2002/2012), and Daniel Pink (2005) and agencies such as the U.S. Department of Labor (1991). The general public sees creativity and other 21st century skills as important according to a survey of 1000 adults: 70% ranked creativity as important, 87% for the ability to adapt to a changing world, 89% for critical thinking and decision making, and 88% for problem solving (Sacconaghi, 2006). Creativity is not only considered a necessary skill for the business world in general but also to mathematicians (e.g., Poincaré, 1913; Sriraman, 2004). However, do students perceive that they have opportunities to be creative in their mathematics class?

My interest in the topic of mathematical creativity began after a frustrating semester trying to teach college algebra. I had taught the course in the past, but this group just wanted to sit there and copy notes. Any time I tried to get them to interact with me or work in groups they were highly resistant. I felt like they wanted me to spoon-feed the material to them and suspected that this was what they were used to. I decided the opposite of spoon-feeding was creativity, so I began research on the topic.

"Creativity is the ability to produce work that is both novel (i.e, original, unexpected) and appropriate (i.e., useful, adaptive concerning task constraints)" (Sternberg & Lubart, 1999, p. 3). Poincaré (1913) provided a similar definition for mathematical creativity, that mathematical creativity requires creating something new and useful. He emphasized the role of choice to pursue useful new ideas. Building on this, mathematical creativity is “the ability to create mathematical objects, together with the discovery of their mutual relationships” (Ervynck, 1991, p. 46).

Research Explicitly Linked to Mathematical Creativity

Recent research that explicitly studies mathematical creativity is scattered and especially sparse with American populations. A combined search on EBSCO’s ERIC, Education Research Complete, and PscycINFO databases for math* (to include various forms: such as math, maths, mathematical, and mathematics) and creativity yields nearly 500 results, however many of the articles are about creativity in other subjects (especially science, technology, and engineering fields), activity or philosophical articles, or informal use of the word creativity – especially in the abstract or conclusion. This review focused on studies that focused on methods used to foster mathematical creativity and perceptions on the role of mathematical creativity in the classroom.

The limited results indicate that while teachers tend to support creativity in theory, they do not feel responsible for teaching creativity (Aljughaiman & Mowrer-Reynolds, 2005) and may find creative solutions to be disruptive (Beghetto, 2007). Teachers teach creativity infrequently, but teachers who do foster creativity more frequently are more productive (Schacter, Thum, & Zifkin, 2006).

Mathematical Pedagogical Methods Related to Creativity

Some teaching methods may give students the opportunity to be mathematically creative or develop skills needed to be creative, but the research does not focus on the creative aspect of the method. For example, the process standards developed by the National Council of Teachers of Mathematics (NCTM, 2000), which the first five Common Core Standards for Mathematical Practice (NGA Center & CCSSP, 2011) are linked to, provide indicators that require students to be mathematically creative or develop skills that aid mathematical creativity. The most obvious examples are, "Build new mathematical knowledge through problem solving," "Apply and adapt a variety of appropriate strategies to solve problems," "Make and investigate mathematical conjectures," "Select and use various types of reasoning and methods of proof," and "Understand how mathematical ideas interconnect and build on one another to produce a coherent whole" (NCTM, 2000, p. 402).

Thus, any standards-based program should give students the opportunity to be mathematically creative. For example, the Interactive Mathematics Project (Clarke, Breed, & Fraser, 2004) at the high school level and Problem-Centered Learning (Ridlon, 2009) at the middle school level are standards-based programs that showed statistically significant increases student achievement and positive attitudes towards mathematics than comparison groups in each study.

Statement of Problem

Creativity is an increasingly necessary skill. In the modern world of automation and instant access to knowledge via the worldwide web, memorizing a set of facts is no longer a marketable skill (Florida, 2002/2012; Pink, 2005; Robinson, 2001/2011; U.S. Department of Labor, 1991). However, there is a need to understand better how teachers can foster mathematical creativity in the classroom.

Purposes of the Study and Research Questions

One purpose of this study is to help fill gaps in the research on mathematical creativity in the classroom. The study adds to the body of research done in the United States; the mismatch in the curriculum may make international studies seem less relatable to teachers in the United States. Also this study is done in a K-12 setting, which is less common for research studies explicitly studying mathematical creativity. Finally the review of the literature indicated that little research has been done on students' opportunity to be mathematically creative; instead studies generally addressed a particular teaching method or perceptions.

Another purpose of this study is to provide insightful descriptions of how teachers foster mathematical creativity that teachers and administrators find relatable and useful. While this study focuses on the positive aspects of the classrooms with respect to mathematical creativity, the teachers are not portrayed as infallible, hence attainable.

The questions that guided this research are:

1. How do teachers make mathematics personally meaningful to foster students' mathematical creativity?
2. How do teachers create a safe environment to help students develop mathematical creativity?
3. How do teachers use mathematics practices to foster students' development of mathematical creativity?

Significance

School districts may be pressured by various stakeholders such as the parents and the community in general, businesses, and higher education institutions to foster creativity in students. A goal of this study is to raise awareness of the developmental nature of mathematical creativity. The findings in this study may help teachers and administrators, especially those with more traditional education experiences, develop a broader and more democratic understanding of mathematical creativity. The strategies described can be used to help train teachers to foster mathematical creativity.

Delimitations

This is a qualitative case study that involves three teachers from one school, thus this study provides descriptions for the opportunities that their students had to develop mathematical creativity. Also, the teachers in this study are relatable but not necessarily exemplary with respect to mathematical creativity. Hence, this study does not claim to provide a complete picture on *all* the ways teachers can foster mathematical creativity.

Researcher's Perspective

This section is intended to provide transparency for my perspective. With respect to validity and the researcher's integrity, "Investigators need to explain their biases, dispositions, and assumption" (Merriam, 2009). I describe briefly my experiences with mathematical creativity in the classroom as a student and a teacher and my overall perspective on the role of mathematical creativity in the classroom.

For the most part, my own K-12 experience was fairly traditional, especially for mathematics. The majority of the time it was: here is how you do it, now practice it 40 times. Explanations for why the algorithm worked may have been provided by my teachers but were not emphasized. Therefore, I was caught off guard when I enrolled in my first proof-based course and the instructor copied proofs on the board from his handwritten notes. He did explain why the proof proved the theorem, but when I tried to write proofs on my own I was completely confused on where to even begin.

As an instructor, I have had students who seemed to have worse experiences than me. I had developmental mathematics students who were very nervous about selecting *x*-

values to use to determine points to graph a linear equation in two variables. Other students used the same values for *x*, such as 1, 2, and 3, for every problem. At least I had enough positive experiences (parents who encourage creativity, gifted program in third grade, independent study of mathematics in the beginning of sixth grade, and competing in MATHCOUNTS in middle school) to make some little choices.

In this study, the premise is that mathematical creativity is a valuable skill that teachers can and should develop in students and that all students can be mathematically creative to some extent (Torrance, 1970). Also, mathematical creativity is developmental in nature (Kaufman & Beghetto, 2009); it is unreasonable to expect students to jump from mimicking how a teacher solves a problem to creating a completely novel work. Students need to practice making choices.

I do not believe that any one teaching method or combination of teaching methods is a panacea, nor do I believe that all teaching methods must require students to be mathematically creative. However, I do believe that is a certain minimum amount of tasks that should allow or even require mathematical creativity in order for students to learn and understand mathematics.

Summary

Creativity is an important part of the modern world and the study of mathematics. Allowing students opportunities to be mathematically creative will likely improve their interest and persistence in mathematics as well as their academic achievement. Research on students' opportunity to be mathematically creative is limited and not encouraging. This study will help narrow the gap in research on opportunity to be mathematically creative and the developmental nature of creativity and provide descriptions that teachers may find relatable.

In Chapter II, I will discuss in further detail research related to mathematical creativity in the classroom, including definitions and theories, research explicitly linked to mathematical creativity in the classroom, and pedagogical methods and curriculums that may foster some level of mathematical creativity. In Chapter III, I will describe the methodology used for this study, including my design, data collection, and data analysis. In Chapter IV, the findings are presented in themes and categories with vignettes and quotes from interviews to support the descriptions of the observations. Finally in Chapter V, the study is wrapped up with a discussion of the findings, implications, and suggested future research.

II. LITERATURE REVIEW

Defining creativity is a complicated task; there are many perspectives. Creativity can be described as historical or psychological (Boden, 1994), or it can be described in finer gradations as in the four-C model: mini-c (personal learning), little-c (everyday creativity), pro-c (professional creativity), and big-C (eminent creativity) (Kaufman & Beghetto, 2009). There are different views on whether the person or the result is creative. Some view creativity as a domain general quality, whereas others think creativity is domain specific (for example, a person could be a creative jazz musician but not a creative writer).

This review of the literature focuses on creativity as it manifests specifically in mathematics, especially in the K-12 classroom. How is mathematical creativity incorporated into mathematics courses?

The review of the literature begins with definitions and theories of creativity in general before discussing definitions and theories of mathematical creativity. Next, research involving mathematical creativity in classroom is discussed followed by, teaching methods that may be conducive to fostering mathematical creativity. This chapter will conclude with a discussion of gaps in the literature.

General Creativity Definitions and Theories

Definitions

There is no universal definition of creativity. Reviews of the literature on the definition of creativity (e.g., Kampylis & Valtanen, 2010; Plucker, Beghetto, & Dow, 2004) indicate that novelty and usefulness are common concepts of many definitions of creativity. The following three definitions do not restrict creativity to creative genius:

> Creativity is the ability to produce work that is both novel (i.e, original, unexpected) and appropriate (i.e., useful, adaptive concerning task constraints). (Sternberg & Lubart, 1999, p. 3)

> The creative work is a novel work that is accepted as tenable or useful or satisfying by a group in some point in time. (Stein, 1953, p. 311)

> Creativity is the interaction among aptitude, process, and environment by which an individual or group produces a perceptible product that is both novel and useful as defined within a social context. (Plucker et al., 2004, p. 90)

Thus, creativity is the process of producing a result that is novel and appropriate in a given context. The requirement of *appropriateness* is what differentiates creativity from imagination. A mathematics proposition may be deemed appropriate if it appears to be true and upon verification would contribute to the field. A mathematics proof would be deemed appropriate if it is correct and understandable to the intended audience, whereas a short story or a piece of art may be considered appropriate if it appeals to an audience.

Viewing creativity in context provides a more democratic view. An idea that is novel to second grade students may not be novel to adults, so this idea may be viewed as creative within the second grade classroom but not creative in the workplace. From a cultural perspective, a product may be viewed as creative in one culture, common is another culture, and inappropriate in a third culture. While this latter context example may not seem relevant to mathematics, it is relevant to mathematics education.

Levels of Creativity

Terms such historical creativity versus psychological creativity (Boden, 1994) or Big-C creativity versus little-c creativity are used to differentiate between the creativity of groundbreaking, major contributions to a specific domain and everyday creativity. This dichotomy has been further refined to a Four-C model (Kaufman & Beghetto, 2009) with levels: mini-c creativity, little-c creativity, Pro-c creativity, and Big-C creativity.

“Mini-c is defined as the *novel and personally meaningful interpretation of experiences, actions, and events*” (p. 3). Kaufman and Beghetto indicates that creativity at this level involves personal learning without the need of a tangible product, and they suggest this level of creativity is appropriate for many students, especially younger students. Little-c creativity involves some product that others may see as creative in a specific context, though it is not novel and does contribute with respect to a broader domain. The Pro-c level of creativity is for those who contribute to their domain but have not reached eminence status. Finally, the designation of Big-C creativity is reserved for those few whose works are recognized posthumously.

Table 2.1

Levels of Mathematical Creativity

Creativity Level	Mathematics Examples
mini-c	• Gains conceptual understanding • Defines term in own words • Makes connections between concepts
little-c	• Solves a non-routine problem • Discovers alternative method to solve a problem • Solves a self-selected real-world problem
Pro-c	• Creates a conjecture believed to be true that would be useful if proven • Proves a previously unsolved conjecture • Defines a new term
Big-C	• Creates a new field of mathematics • Solves an important theorem

Beghetto and Kaufman (2009) claim, “Given that mini-c creativity is inherent in any act of learning, math educators need to be prepared to recognize, cultivate, and nurture the multicreative potential of their students” (p. 43). When a student gains conceptual understanding, they are doing a mini-c process. “At the little-c level, just about any student—with the requisite support and encouragement from teachers—can find ways to express their little-c creativity in math projects, tasks, and activities" (p. 42-43). When students create something new for their level of education, such as solve a non-routine problem that has been solved by many other students before them, the students are exhibiting little-c creativity. When a student or a mathematician is recognized for contributing to the domain of mathematics, they have reached the Pro-c level. While many mathematicians create meaningful conjectures and contribute accepted proofs, only a few will be remembered over time. For example, the trailblazers who forge new areas of mathematics or solve important theorems will be remembered as eminent mathematicians who reached the Big-C level of creativity.

Mathematical Creativity Definitions and Theories

Definitions

Invention or mathematical creation is the soul of mathematics, Poincaré (1913) argued, rather than having a prodigious memory or excelling at calculations. He describes the role of choice in mathematical creativity:

> In fact, what is mathematical creation? It does not consist in making new combinations with mathematical entities already known. Any one could do that, but the combination so made would be infinite in number and most of them absolutely without interest. To create consists precisely in not making useless combinations and in making those which are useful and which are only a small minority. Invention is discernment, choice. (p. 386)

Like many definitions of general creativity, Poincaré specified both novelty and usefulness as requirements for mathematical creativity. A more explicit definition of mathematical creativity is "the ability to create mathematical objects, together with the discovery of their mutual relationships" (Ervynck, 1991, p. 46). A new mathematical object or relationship would be deemed appropriate if it illuminates rather than obscures understanding in the discipline of mathematics, is deep, and is responsive or fruitful

(MacLane as cited in Ervynck, 1991; Poincaré, 1913). This aligns with the work of mathematicians. Mathematicians create conjectures about how they believe mathematics is structured; they do not publish just any random conjecture, but a conjecture must have face validity and seem useful, fill in an existing hole or suggest a new path of inquiry. Mathematicians obviously also prove their own or other people's conjectures. Sometimes they provide an alternative proof for a previously proven theorem, when a new proof is either more elegant or sheds new light about the relationships defined in the theorem.

Both Poincaré and Ervynck expressed that most people are not capable of mathematical creativity. Lithner (2008) expressed a more democratic view of mathematical creativity and described mathematical creative reasoning as follows:

> *Creative mathematically founded reasoning* (CMR) fulfils all of the following criteria.
> 1. Novelty. A new (to the reasoner) reasoning sequence is created, or a forgotten one is re-created.
> 2. Plausibility. There are arguments supporting the strategy choice and/or strategy implementation motivating why the conclusions are true or plausible.
> 3. Mathematical foundation. The arguments are anchored in intrinsic mathematical properties of the components involved in the reasoning. (p. 266)

Note that a teacher using Lithner's definition need not require a student to contribute novel work to the mathematics field in order to consider the student's reasoning creative.

Mathematical Creativity Process

Based on the written reflections of many thinkers, including Poincaré, from a variety of fields, Wallas (1926) described stages of the thought process of creativity in general. Hadamard (1945) discussed these stages specifically in the context of mathematical creativity, also drawing heavily from Poincaré's self-reflections of his thought process. These stages of the thought process needed for mathematical creativity are preparation, incubation, illumination, and verification. Preparation is conscious work, followed by the incubation stage where the mathematician does no conscious work. The mathematician may cease to work on a problem when she is stuck or to work on something else of higher priority. Although the conscious work has ceased, the subconscious continues to work on the problem until a solution path surfaces, which is the illumination stage. Finally, the conscious mind takes over to verify the illumination is correct and fill in the details.

Recently, Sriraman (2004) interviewed five mathematicians; based on his analysis he concluded that these stages are still relevant. When discussing how they begin working on a new topic, the mathematicians indicated they would research what has been done first before jumping in with their own approaches. When asked whether they tend to work on a single problem or multiple problems, four of the mathematicians indicated that they switched back between at least a couple problems. Generally when they would get stuck on one problem they would switch to another one. One of the interview questions directly addressed incubation and illumination, "Have your best ideas been the result of prolonged deliberate effort or have they occurred when you were engaged in other unrelated tasks?" (p. 34). Sriraman provided excerpts from three of the mathematicians' interviews that support the need for both deliberate work and unconscious work. As one of the mathematicians explained it, "You spend a lot of time working on something and you are not getting anywhere with it…with the deliberate effort, then I think your mind continues to work and organize. And maybe when the pressure is off the idea comes…but the idea comes because of the hard work" (p. 29). Finally, Sriraman provided excerpts from four of the mathematicians on how they verify the truth of a proposition. These mathematicians not only mention the importance of a correct proof, but the need to understand how the proposition fits into the big picture.

Osborn (1953) broke the stages down into seven phases: orientation, preparation, analysis, hypothesis, incubation, synthesis, and verification, though he clarifies that not all creative endeavors will involve all phases. His model is for the general creative process, though it seems similar to the four-stage model discussed in the previous paragraph, and still seems applicable to mathematical creativity. The preparations stage is split into the orientation (understanding the problem), preparation (gathering data), analysis (breaking down the data), and hypothesis (considering various way solution paths) phases. The incubation and illumination stages are collapsed into the incubation phase, which makes sense since incubation does not mean much without illumination. Finally, the verification stage is split into the synthesis (filling in the details) and verification (judging the solution) phases. Like the other authors (Hadamard, 1945; Poincaré, 1913; Wallas, 1926), Osborn mentioned that some work usually comes before incubation; some sort of conscious preparation usually is needed for your subconscious to work from.

Mathematical Problem Solving Process

How is problem solving related to mathematical creativity? One way to approach this question is to compare the problem solving process to the creative process. Researchers have studied the problem solving process by conducting task-based interviews with experts, where the expert is asked to solve one or more non-routine problems and asked to verbalize their thinking. Sometimes the expert is asked to provide additional insight into the process after the problem solving session is completed. These typically conclude with some variation of the four stages of mathematical creativity, emphasizing a particular sub-process common among the participants. The problem-solving models tend to include some sort of cycle within the problem solving process.

One study noted that when experts were not making progress, they would break their immersion in the problem either by taking a breather by doing something with low cognitive demand, such as change their variables from *x*'s and *y*'s to *a*'s and *b*'s, or the experts would allow themselves to be completely distracted for a minute by the environment, such as the temperature or a background noise (Rosamond, 1994). These breaks from immersion could serve as the incubation stage in the creativity process. A study in which high school mathematics teachers served as expert problem solvers included a participant who embraced the idea of incubation and even taught her students to stare at a problem and let their mind go blank when they got stuck (Aldous, 2005). Sometimes students get so focused on solving a problem, they do not give themselves the chance to access the information stored in their brain that they need to solve the problem.

One common misconception is that Polya's (1957) problem solving steps – understanding the problem, devising a plan, carrying out the plan, and looking back – are linear, but studies of experts (e.g., Carlson & Bloom, 2005) indicate that they cycle through the planning, executing, and verifying steps. Carlson and Bloom also noted a conjecture cycle during the planning step, consisting of conjecture (the solution method), imagine (how the solution will unfold), and evaluate (if the method is plausible). This appears to be a creativity process within the problem solving process.

Another subprocess that can occur within the mathematical problem solving process is the use of representations. Both middle school students and expert problem solvers use representations to understand the problem, organize information about the problem, explore the problem, or monitor and evaluate progress in solving the problem (Stylianou, 2011). Students are less likely than experts to use a representation for multiple purposes or use multiple representations within a single problem.

When expert problem solvers are compared to novice or less experienced problems solvers, one of the biggest differences is the ability to self-monitor and self-evaluate (e.g., Schoenfeld, 1985). Whether due to a lack of content knowledge or lack of metacognitive awareness, novice problems solvers are more likely to take off with the first idea that crosses their minds and stick with a poor choice without considering alternatives or the appropriateness of their choice. And as mentioned earlier, Poincaré (1913) claimed that choice was the key the mathematical creativity.

The problem solving process was described using similar stages as the mathematical creative thinking process, with a creative process within the determining a solution method stage of the problem solving process. So then, the real difference seems to lie in what is considered a problem. In their creative problem solving framework that is not specific to mathematics, Isaksen, Dorval, and Treffinger (2011) "take the stance that problem solving is closing the gap between what is and what is desired" (p. 19). National Council of Teacher of Mathematics (NCTM, 2000) also uses a broad description of problem solving, "Problem solving means engaging in a task for which the solution method is not known in advance" (p. 52). Both of these definitions allow a variety of tasks to be considered problem solving in addition to solving word problems, such as learning a concept, solving a non-contextualized problem, and writing a proof.

Cognitive Levels

Ervynck's (1991) developmental stages of mathematical creativity are preliminary technical stage, algorithmic activity, and creative activity. As a professor in Belgium his focus seemed to be on college students, however in his conclusion he also mentioned that there are ways to "encourage younger children to play their own part in knowledge generation, to make conjectures, to expect errors, to need to check, to convince to prove" (p. 53). The technical stage refers to mathematics being used as a tool without conceptual understanding, the algorithmic stage is completing explicit procedures, and the creative stage making a "non-algorithmic decision." He noted that creative thinking requires students to accept the possibility of error. Ervynck cautioned when teaching students, "If we do not encourage them to participate in the generation of mathematical ideas as well as their routine reproduction, we cannot begin to show them the full range of advanced mathematical thinking" (p. 53). Similar to Ervynck's developmental stages of mathematical creativity is Lithner's (2008) mathematical reasoning framework that consists of imitative reasoning, algorithmic reasoning, and creative reasoning.

Although Lithner took a more democratic stance with his definition of mathematical creativity, his focus was on postsecondary students. Four cognitive levels were identified for the thinking process used the complete mathematical tasks in middle school classes (Henningsen & Stein, 1997; Stein, Grover, & Henningsen, 1996; Stein & Lane, 1996). The research team used the cognitive levels to describe how the teachers set up the tasks and how the students implemented the tasks (Stein et al., 1996). They used a framework that suggests that the cognitive level of a mathematical task may change between the intent of the curriculum materials and how the teacher sets up the task, and then again depending on how students actually implement the task. The framework indicates that how the students actually implement the task is what impacts student learning, rather than the intent of the curriculum. Their research, including what caused a task that a teacher set up to require a high cognitive level to stay high or regress to a lower cognitive level upon implementation (Henningsen & Stein, 1997) and the relationship between task cognitive levels and learning gains (Stein & Lane, 1996), will be discussed in the Pedagogical Methods Related to Creativity section.

The highest level of cognitive demand is “doing mathematics,” which is characterized by “the use of complex, non-algorithmic thinking to solve a task in which there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked out example” (Stein & Lane, 1996, p.58). Although the research team did not explicitly use the term creativity, the description fits with the descriptions of mathematical creativity and with the mini-c or little-c level of creativity since students are expected to create an original solution to the problem. In the K-12 classroom, this is less likely to be novel to the domain of mathematics, but it is new to the student and their peers, who may simultaneously be creating the same solution. If they are creating something new to themselves, that is mini-c, but if their process is novel among their peers, they have reached little-c creativity, refer to Table 2.1.

The second highest level is the use of procedures with connections to concepts, meaning, and/or understanding. Since “students follow a suggested pathway through a problem, the pathway tends to be broad, general procedure that has close connections to underlying conceptual ideas,” the researchers consider this level also to entail high cognitive demand, and this fits with the mini-c level of creativity because students are building conceptual understanding. For example, a teacher can walk all the students through the same steps of a hands-on activity; the students are not creating the process, but they could still be creating knowledge structures or making connections between concepts. The students are not creating novel products that another person can evaluate, but this is considered a developmental stage in developing creativity in a domain (Beghetto & Kaufman, 2007).

The second lowest level of cognitive demand is the use of procedures without connections to concepts, meaning, and/or understanding, and the lowest level is memorization. The research team considers both of these levels to require only a low level of cognitive demand. The tasks at these levels do not seem to give students the opportunity to be mathematically creative.

Stein and associates contrasted “complex thinking and reasoning strategies that would be typical of doing mathematics” (Stein & Lane, 1996, p. 57) with mindless use of algorithms. Similarly, Cuoco, Goldenberg, and Mark (1996) advocate for focusing on teaching high school students how to think like a mathematician rather than having students work problems where they simply apply the property they were just taught. The team theorizes that since we cannot know what technology the students will face in the future, we cannot know with certainty which topics will be most applicable in students’ professional and everyday adult lives. However, teaching students to employ habits of mind gives them ways to approach topic in the future that may not have even be developed yet. Cuoco et al. described a curriculum organized around habits of mind:

> Such a curriculum lets students in on the process of creating, inventing, conjecturing, and experimenting; it lets them experience what goes on behind the study door before new results are polished and presented. It is a curriculum that encourages false starts, calculations, experiments, and special cases. (p. 376)

This description indicates an expectation of students to use mini-c and little-c creativity to learn mathematics, since students are expected to at least gain conceptual understanding and make connections if not actually create an appropriate product that is novel among their peers.

The team presented the habits of mind in four themes: ones that may be applicable in other domains, ones that are more specific to mathematics in general, ones that are specific to geometry, and ones that are specific to algebra. The general habits of mind are that students should be pattern sniffers, experimenters, describers, tinkerers, inventors, visualizers, conjecturers, and guessers. These same themes are prevalent in the other habits of mind, which generally seem to be more specific cases of the general habits of mind.

One of the mathematical habits of mind of note seems to describe the incubation stage of the mathematical creativity process. Cuoco et al. (1996) refers to this habit of mind as mathematicians use intellectual chants:

> A mathematician who is engrossed in a problem spends long periods of time alternating between scribbling on paper and looking off into space, kind of meditating. This second activity really involves rehashing logical connections and partial calculations, dozens (maybe hundreds) of times. (p. 388)

The team conceded that this may be difficult to include in the curriculum, but they suggest providing the students with a personal description of the process or an interview with a successful, reflective student.

Mathematical Creativity in Curriculum Standards

Curriculum standards provide teachers with expectations of what the students should learn, the intended curriculum. As discussed in the last section, the cognitive levels may change if the intended curriculum changes when implemented (Stein et al., 1996). However, the curriculum standards describe students' theoretical opportunities to be mathematically creative.

Texas is one of the few states not to adopt the Common Core State Standards (CCSS, 2011), so the most important standards, for accountability reasons, for Texas teachers are the Texas Essential Knowledge and Skills (TEKS, 2009). Teachers may be influenced by the National Council of Teacher of Mathematics (NCTM, 2000) curriculum standards, for example if they were emphasized in their teacher preparation program. Texas teachers who search for additional resources probably will be at least indirectly influenced by the CCSS. Since most states have adopted the CCSS, publishers are likely to align many products with those standards. In this section is an analysis of the TEKS, NCTM, and CCSS curriculum standards with respect to mathematical creativity. Thus for the TEKS and NCTM standards the focus will be on the process standards, and the Mathematical Practices will be the focus for CCSS. These types of standards explain how the student should be learning, thus are the most likely places to have standards involving the creative process.

The current TEKS were adopted September 1998, amended August 2006, and will remain in effect until Fall 2014 when teachers will be required to implement an updated version. The forthcoming TEKS were approved by the State Board of Education April 2012 and may receive only superficial edits when they are codified. Although the exact wording of the process standards changed between the current and the forthcoming TEKS, they are very similar in content. The ideas are applying mathematics to everyday situations, using a four-step problem-solving model, selecting tools to solve problems, communicating mathematics, using multiple representations, analyze and make connections, use mathematical reasoning to justify ideas. These are similar in nature to the NCTM process standards.

However, the TEKS (2009) simply supply a statement, such as “use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness” (p. 3), whereas NCTM (2000) provides a few pages of explanation for each standard. Although this standard allows a teacher to give non-routine problems, open-ended problems, or allow multiple solution methods to encourage mathematical creativity, this is not guaranteed. The TEKS process standards seem to leave plenty of room for a teacher to teach to the curriculum and foster mathematical creativity in her students, however another teacher could teach to the letter of the process standards in a controlling manner that discourages mathematical creativity.

In contrast, one part of the NCTM’s (2000) problem solving standard explicitly states the students should “build new mathematical knowledge through problem solving” (p. 256). This specifies that students should at least be using mini-c creativity. Within the description, teachers are encouraged to allow multiple solution methods and give students unfamiliar problems. Also, the importance of conjecture in mathematics is discussed, and it is suggested that teachers ask students to propose and investigate conjecture that extend a particular topic. NCTM’s description of the problem solving standard seems to require mini-c mathematical creativity from students and leaves room for little-c creativity.

“Make and investigate mathematical conjectures” (NCTM, 2000, p. 262) is also part of the reasoning and proof standard. Multiple solution methods are mentioned again in the description of the reasoning and proof standard and the representation standard.

Part of the communication standard is to "communicate their mathematical thinking coherently and clearly to peers, teachers, and other" (p. 268), and the description clarifies that students should compare the variety of strategies used by the different students. The concept of students building mathematical ideas resurfaces in the connections standard in the description for "understand how mathematical ideas interconnect and build on one another to produce a coherent whole" (p. 274). In the discussion of the representation standard, part of which states that students should create and use representations to organize, record, and communicate mathematical ideas" (p. 284), encourages teachers to allow students to use nonstandard, idiosyncratic representations initially before guiding the students towards conventional notation. These are all examples of how students are expected use at least mini-c creativity, if not little-c creativity. Although the word creativity may not be prevalent in the process standards, the introduction to the middle grades chapter indicates that this is the intent, "Middle-grades students should see mathematics as an exciting, useful, and creative field of study" (p. 211).

Thus if Texas teachers interpret the TEKS process standards the same way as NCTM describes how to implement their process standards, then the forthcoming version of the TEKS is a positive step towards encouraging mathematical creativity in the classroom. The introduction note addressing the process standards in the forthcoming version is more than three times as long as the current version. Also, the language is more direct. The current note states, "Throughout mathematics in Grades 6-8, students use these processes … to develop conceptual understanding and solve problems as they do mathematics" (TEKS, 2009, p. 1). The forthcoming version states, "The process standards describe ways in which students are expected to engage in the content …. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life" (TEKS, 2012, p, 1). The current note could be interpreted that the process standards should be addressed several times during the year, but the forthcoming version make it clear that process standards describe how the content standards should be addressed daily. Also to show their importance in the forthcoming versions, the process standards are listed first, whereas they are listed last in the current version. Lastly, in the forthcoming version, each content standard has a reminder of the process standards, such as, "The student applies the mathematical process standards to represent and use rational numbers in a variety of forms" (TEKS, 2012, p. 2).

The CCSS Standards for Mathematical Practice fall between the TEKS and the NCTM standards with regards to the amount of description provided; a paragraph is used to describe each standard. Unlike NCTM that provides grade band descriptions, the descriptions for the Mathematical Practice standards span the entire K-12 grade range. Mini-c to little-c creativity is apparent in some of the standards. Multiple solution methods, making conjectures, looking for patterns are included in these standards (CCSS, 2011).

Mathematical Creativity in the Classroom Research

This section discusses research done explicitly on mathematical creativity in the classroom. The studies presented in this section are organized by an important feature that relates to creativity. The studies include encouraging students to use multiple solution methods, working problems that have more than one correct answer, completing a project, emphasizing the abstractness of mathematics instead of the computation aspect, students self-reflecting, and teachers attributes and perceptions related to mathematical creativity in the classroom.

Encouraging Multiple Solution Methods

In the United Kingdom, 38 preservice elementary teachers completed a questionnaire, of which 10 were also interviewed, during the semester of their student teaching (Bolden, Harries, & Newton, 2010). On a 5-point scale with 1 being the least creative, the median score for the level of creativity for mathematics was a 2, with English, Art, and Science being considered to be more creative subjects. Similarly, on a 5-point scale, the median agreement score, with 1 being the lowest, with it being difficult to encourage creativity in mathematics was a 4. The preservice teachers discussed both creative teaching and creative learning. The researchers discussed two subcategories for creative teaching: teachers' imaginative use of resources and technology and teachers' application of mathematics to everyday examples. The preservice teachers indicated that the purpose for both types of creative teaching is to make mathematics more enjoyable for students: the resources make mathematics fun and the everyday examples gets the students interested. The preservice teachers indicated that practical activities and investigations as methods for creativity in learning for some topics, such as shape, measurement, fractions, and data gathering. The other method of creativity in learning was learning multiple methods, but the teachers also framed it as creative teaching. The teachers expressed that they were responsible for helping students with multiple methods until the students find the method that works for them. On the questionnaire, 31 preservice teachers provided statements that indicated that they "conceived mathematics as a subject as a set body of knowledge that offered little or no room for freedom of expression, imagination and independence" (p. 152). For example, one preservice teacher stated:

> Maths is pretty regimented, you either get a right answer or you get a wrong answer, so that cuts it very short but if you are given something in dance or PE you can create your own dance or rhythm whereas it's difficult to get that freedom of expression in maths…at the end there's always a right answer. (p. 152)

While in this study the teachers modeled multiple solution methods, in some studies the students were expected to use multiple solutions.

In Israel, students in 11 high school Geometry classes (seven high level and four regular level classes) where they were encouraged to use multiple solution methods to solve a problem were compared to three traditionally taught classes (two high level and one regular level class). There were significant differences in the high level but not the regular level, possibly due to the smaller size of this group due to many drops (Levav-Waynberg & Leikin, 2011). Students' problem solving performance was evaluated based on six criteria: correctness, connectedness, fluency, flexibility, originality, and creativity (which is composed of the previous three criteria). The treatment and control groups improved similarly for correctness. For connectedness, the high-level treatment students improved slightly on average from the pre-test to post-test (56 to 58), but the average score for the students in the control high-level classes markedly decreased from 53 to 39. These changes were significantly different at the 0.01 level. The researchers suggested, based on classroom observations, that this may be due to the emphasis on efficiency in the control classes, thus fewer concepts would appear in these students' proofs. Each group improved significantly from pre- to post- in fluency and flexibility, and the high-level students in the treatment group improved on average significantly more than the high level control students. There were no significant effects for changes in originality and the overall creativity score. The scores decreased for the high-level students and increased for the regular-level students. The researchers hypothesized that this was due to the changes in fluency. The greater increase in fluency and flexibility for the high-level students caused fewer responses to be original, whereas the smaller increase in frequency and flexibility for the regular-level students allowed there to be more infrequent responses. However, when looking exclusively at the rarest solutions, there was one high-level treatment student and one high-level control student who each created the rarest solutions during the pre-test. During the post-test, the nine rarest solutions came from the eight students in the high-level treatment class.

In Rwanda, Uworwabayeho (2009) noted based on his extensive observations, that most mathematics teachers lecture and write notes on the chalkboard while students copy notes, and then the students work practice problems for the last five to 10 minutes of class. He reported that technology is primarily used in teacher-centered ways, such as PowerPoint presentations or having students enter their answers into the computer, rather than as a learning tool. This study involved two teachers implementation of Geometer's Sketchpad (GSP). Uworwabayeho stated that they had planned to integrate GSP into the class, however due to the limited resources, the classes could use the computers only outside of the normal class period eight times during the school year with two students seated at each computer. One of the teachers noted, "If I was using chalk and board I could not have a particular attention to any one learner but within the GSP all learners being simultaneously working to get direct feedback" (p. 323). While the teachers could see the benefit of the students interacting with each other and the mathematics, they also noted the extra time the dynamic geometry software required that competed with the need to prepare students for school, district, and national examinations. Uworwabayeho mentioned seeing students use multiple approaches to complete the same task, thus indicated that students can use GSP to be creative and get mathematical exploration skills. He also noted that after the initial activity the class did together to get familiar with the program students were able to describe the procedure in their own words. The teachers in the study benefited through learning how to reflect on their teaching practice. One teacher stated, “One of the lessons learnt from this experiment is that the use of questioning in the teaching of mathematics made my learners more participative and drew my attention to learners’ difficulties” (p. 324).

Working Problems with Multiple Solutions

A study of Korean middle school students indicated that students who had open-ended problems incorporated into their mathematics classes did better than students in comparison classes with regards to fluency, flexibility, and originality of solutions (Kwon, Park, & Park, 2006). Both classes were pre- and post-tested with researcher created tests of open-ended problems.

In Israel, interviews were conducted and lessons observed related to creative teaching of mathematics for 11 elementary and middle school teachers (Lev-Zamir & Leikin, 2011). The interview and observation video transcripts were coded for instances of flexibility, originality, and elaboration. The teachers gave examples of both lessons where the teacher is being creative and lessons where the student is being creative as creative teaching. A lesson plan may allow students to be flexible if they are expected to generate multiple solutions to a problem. The students may be original by generating or discovering ideas or exercises that are new to the students. Finally, students can elaborate by generalizing ideas and raising the level of the discussion. When students are creative, this in turn requires the teacher to be creative, especially flexible.

Projects

In Turkey, Kandemir and Gür (2007) interviewed 43 preservice teachers who attended an 11-week creativity training course at the end of the semester. The first five weeks of training the researcher led the students through various activities. Then the next five weeks each group of students led at least two activities. The last week was used for practice and evaluation. The interviews revealed that 70% of the preservice teachers had a positive attitude towards creativity, whereas 12% (5 preservice teachers) indicated the belief that creativity could not be developed. The educational system was the most commonly cited barrier to teaching creativity in problem solving, indicated by 72% of the preservice teachers. The preservice teachers did not feel like they had experienced creative thinking in their own educational experiences, thus they assumed teaching students to think creatively would not be considered acceptable. There was also concern about whether most teachers are capable of teaching students to think creatively. Finally, the preservice teachers expressed concern over the time it would take to allow students to use creative problem solving, and that teachers must give priority to prepare students for the college entrance exam.

A survey of 218 students in the college of science and mathematics at a university asked students to identify the most creative activity they had done for 17 disciplines and then rate that activity from 1 to 5, where 5 is the most creative (Munakata & Vaidya, 2012). The average rating for mathematics was a 3.00, with only chemistry, economics, and history rated as less creative. The researcher reported on two activities involving photography, that were inspired by features in NCTM and Mathematical Association of America publications, that were used in an attempt to improve students' perceptions on creativity in mathematics and science. Munakata assigned her pre- and in-service teachers of the masters-level course contemporary teaching of mathematics to take one or more photographs, analyze the mathematics behind it, and "write at least four thought-provoking, multi-step mathematics problems" (p.4). These problems were required to be above an elementary school level and open-ended. She discouraged the students from taking photographs of items related to typical textbook problems, such as price tags. The students created posters with the photographs, descriptions of the photograph, and a possible solution for each problem. These were scored with a rubric both by their classmates and Munakata. Based on a short questionnaire the students filled out, they realized that mathematics was everywhere but had difficulties representing it. The students expressed that the assignment required creativity and critical thinking to complete.

In Israel, Shriki (2010) analyzed classroom discussions and written reflections to investigate how 17 preservice middle and high school teachers' perceptions about mathematical creativity developed during a geometry mathematics methods course. At the beginning of the semester, the students focused on the finished product as being creative, but later in the semester when reflecting on their own projects, they referred more to the process as being creative. Another change is the students went from seeing mathematics as a closed domain, where mathematicians have already invented everything and put that knowledge into book, to seeing it as an open domain to which the students are able to contribute. Quotes from the written reflections were chosen to be representative; at least half of the class shared the theme. After completing the project, students expressed various definitions and views of creativity. Sample descriptions included one where the student considered the work creative because it involved breaking down a figure and analyzing relationships, whereas another student considered the work to be creative because the process was new to the student and the student was the one asking and answering all the questions. Students indicated that creativity is fun, "Enjoy while you create and love your creations" (p. 171). In a similar vein curiosity was also seen as needed for creativity. Students also saw that being creative involved taking risks, possibly failing. Students seemed to gain a deeper understanding of mathematics through the process of trying to create their own geometric property, "You gave us an opportunity to see the beauty of mathematics and to develop our ability to do something new, different, and unusual. In that sense, it was really a creative doing" (p.170-171). Four students never overcame their reservations about the possibility of students being mathematically creative. Lack of appreciation of creativity at the personal level and unwillingness to take risks seemed to play a role for these students. For example, "Even if I will discover something, there is not guarantee that it would be something significant…It might be that I will not be able to prove my findings" (p.176).

Emphasizing Abstractness Instead of Computation

Ward et al. (2010) investigated students changing perceptions on the nature of mathematics, their attitude towards mathematics, the usefulness of mathematics, and creativity in mathematics when enrolled in a mathematics inquiry course. The 72 students in this course were first year students that are 18 to 20 years of age and qualified to take calculus. The purpose of the course was to enable students to see the conceptual and abstract side of mathematics, rather than simply seeing it as computational and algorithmic. The survey results indicate that the course was successful in improving students view of the creativity involved in mathematics, a broader understanding of the mathematics field, and helped students understand the role of proof in mathematics. However, inadvertently, the abstractness emphasized in the course also lowered students' sense of the usefulness of mathematics. The article concluded by describing future attempts to remedy this, such as one instructor added a two-week unit measuring and modeling income inequality in the United States. Informal surveys indicate that these additions have helped maintain students' perception of the usefulness of mathematics without sacrificing the other gains.

In Taiwan, a course using a historical approach to teaching calculus to 44 engineering majors at a technical college helped the students see mathematics as more than algorithms but a process that involves creativity (Liu & Niess, 2006). Many students at the beginning of the semester had narrow views on mathematics. For example when describing how mathematicians think, one student responded, "They are able to solve problems by using very simple, quick and precise approaches" (p. 383). Another student discussing mathematics stated, "Teachers reminded us that, unless asked to use a particular method, you would attain scores if you can write down the answers…. From childhood, you always get scores as long as answers are correct" (p. 385). The researchers indicated that these responses are typical and representative of the educational system in Taiwan. The course included exposing students to the history of mathematics and working historical, non-routine problems. For example, Liu had students determine the area of the circle without using the formula. As the students presented their solutions, their classmates asked questions or challenged the reasoning. Then they were given handouts with information on historical approaches to determining the area of a circle. The instructor also presented problems more as if he were a novice, so the students could see his thought process, including false starts, rather than presenting polished solutions. At the end of the semester more students characterized how mathematicians think as creative or imaginative, rather than using the quickest method.

Students' Self-Reflection

In a quasi-experimental study in Spain, teachers of 58 sixth-grade students in two classrooms employed a Thinking Actively in Academic Context method. This method involved integrating thinking, creative thinking, and self-regulation skills into mathematics content lesson (Sanz de Acedo Lizarraga, Sanz de Acedo Baquedano, & Oliver, 2010).

> They stimulated *transfer*: (a) by teaching the steps of the skill in the diverse curricular contents; (b) applying the skill in and outside of the educational environment; (c) inviting the students at the end of each didactic unit to answer several questions, such as: 'On what aspects did I work well during this unit?', 'Which aspects were more difficult?', 'What should I do to improve in the next unit?', 'How could I use what I learned in this unit in other situations?' (p. 137)

The two teachers implemented the method in mathematics, environmental knowledge, and language for a school year. They were compared to two teachers who lectured and whose assessments focused on memorization. The treatment and control groups were compared pre- and post- based on intelligence tests, a creativity test, and an achievement test. There were no significant differences between the treatment and control group initially, but there were significant differences in the post-tests. Additionally, the treatment group show significant gains, but the control group did not. The researchers used CREA (Corbalán Berná et al. as cited in Sanz de Acedo Lizarraga et al., 2010) to assess creativity. The test is commercially available in Spain and "measures cognitive creativity through the respondent's generation of questions about some graphic material" (Sanz de Acedo Lizarraga et al., 2010, p. 135).

Teacher Attributes and Perceptions

Based on self-reported measures by elementary teachers, commitment to task, nature of knowledge, motivation for creative work, and learning goal orientation all predicted creative skill use (Hong, Harzell, & Greene, 2009). The study also indicated that intrinsic motivation for creative work and learning goal orientation strongest positive predictors for instruction practice fostering multiple perspectives in problem solving, with a belief that knowledge is simple or certain having a negative effect. A survey of 70 preservice middle school and high school teachers in the Pacific northwest revealed that high school and mathematics preservice teachers are more likely to see unique student responses as a distraction (Beghetto, 2007). Preservice teachers for other subjects were more likely to find value in these type of student responses.

In a study of 36 elementary teachers from a single school district in Idaho, 81% of the teachers surveyed believed that creativity can be developed in the classroom, but only 33% of the teachers believed that regular classroom teachers are responsible for developing creativity (Aljughaiman & Mowrer-Reynolds, 2005). This could be explained by the fact that 35% of teachers associated creativity with art projects, or they felt that developing creativity was the responsibility of the gifted and talented teachers. Most teachers, 88%, included original ideas in their definitions of creativity.

Mathematical Pedagogical Methods Related to Creativity

What teaching methods improve the chances of students experiencing mathematical creativity? The definition of creativity is anchored on the concept of novelty, so having students focus on memorization tasks and complete multiple problems using the process they have just been taught does not allow much room for creativity. If a teacher allows students to use alternative methods, some students may choose to use their own method, but this is not necessarily encouraged.

Using the open-ended approach (Becker & Shimada, 1997) not only allows, but also encourages different answers. This approach involves using well-defined problems that have multiple correct answers, such as determining the dimensions of a rectangle with an area of 24 square inches. The more open the problem is, such as determining the dimensions of a shape instead of a rectangle in the previous example, the more opportunities students have to be creative and learn more. For example, in a study in Poland of mathematically gifted students with minimal experience with rational functions, the students were given two examples, $y = (x+3)/(x+1)$ and $y = (x-1)/(x-2)$ and asked to describe properties of the graph based on the function (Duda, 2011). With the aid of a graphing calculator to graph many more examples, including those in the $y = (\mathrm{a}x+\mathrm{b})/(\mathrm{c}x+\mathrm{d})$ form, the students used a variety of techniques, such as solving for asymptotes algebraically or discovering transformations, to use the function to describe the asymptotes of the hyperbolas. This ended up being an ill-structured problem, because some students stayed closer to the original form while other students generalized more. Due to the nature of ill-structured problems, problem based learning allows students the opportunity to be novel in both their solution method and final result. However, open-ended questions must be done properly. Inexperienced or unprepared teachers may not know how to deal with unexpected methods and may worry more about their own performance than their students' performance (Inoue, & Buczynski, 2011).

When expert problem solvers are compared to novice or less experienced problems solvers, one of the biggest differences is the ability to self-monitor and self-evaluate (Schoenfeld, 1985). Whether due to a lack of content knowledge or lack of metacognitive awareness, novice problems solvers are more likely to take off with the first idea that crosses their minds and stick with a poor choice without considering alternatives or the appropriateness of their choice. One way to help build students metacognition skills is the use of portfolios. Learning portfolios, where students select work and provide reasoning for why the work is included, allows students to create a novel work, the portfolio, and judge the appropriateness of their choices of what they included. When researcher Lambdin and middle school teacher Walker (1994) described how to implement portfolios, they emphasized the guidance that students need and the benefits the students receive through the reflection process of evaluating their own work and the work of their peers. Hence a teacher who does not want to evaluate their students using a portfolio but would like their students to reap some of the benefits may be able to do so on a smaller scale by having students occasionally reflect on or self-evaluate their work.

A certain amount of basic knowledge is necessary to be creative with. Using contexts that students are familiar with, especially their interests, allows them more room for creativity. This way the students do not have unfamiliar contexts confounding the mathematics learning experience. Giving students the opportunity to make choices helps them take advantage of their natural inclinations and interests. A project is one way students can synthesize what they have learned to create a novel product related to their interests. For example, a teacher interviewed a 10 of 95 students after assigning a small project where the middle school students were required to write, solve, and present to the class five word problems related to their interests (Whaley, 2012). The interviews revealed that students indicated much deeper conceptual understanding was needed to create the problems than to simply solve a problem.

Looking at the same task from multiple perspectives may not come naturally to all students. Teachers can model this by working the same problem multiple ways. One way to do this is to infuse multiculturism, so students can see how different cultures use mathematics. An easy way for students to see multiple perspectives is by working in groups, if students are trained to listen to their classmates' suggestions. Some mathematicians value working with peers of varying backgrounds for different perspectives. During the interviews to validate the creativity process, Sriraman (2004) asked mathematicians about working with others and supervising research, which they found to be a mostly positive experience. One mathematician explained:

> It is a positive factor I think, because it continues to stimulate ideas …talking about things and it also reviews thing for you in the process, puts things in perspective, and keep the big picture. It is helpful really in your own research to supervise students. (p. 26)

So far the general definition of creativity has been discussed without addressing how creativity manifests itself in mathematics, in particular incorporating how mathematicians are creative into the K-12 classroom. One way mathematicians are creative is by creating conjectures. Allowing students to explore relationships and conjecture relationships is a way all students can experience this, such as asking elementary students to consider what happens when you add two even numbers (Schifter, 2009). Another way mathematicians are creative is in writing proofs for their own conjectures, other mathematicians' conjectures, or previously proven theorems (the new proof should shed some light on the theorem). Writing proofs is another way for students to synthesize what they know, use it in a different way, and create something new to them. Students not only can conjecture that the sum of two even numbers is an even number, but they can prove this to their classmates. Encouraging students to ask questions and problem pose can be more concrete as well. Students can learn to question what would happen if they modified a problem slightly and to pose questions about the world around them.

The second stage of creative thinking is incubation. This indicates that students may need time to mull problems that require more creative solutions or creating new knowledge. Not allowing students extended thinking time may reduce their ability to make connections and synthesize.

Programs that have included combinations of these aspects have shown some success in improving student achievement. The Interactive Mathematics Project (IMP) was used to serve 182 students in three high schools and compared to 74 similar students in their high schools and 52 students in a fourth high school in California (Clarke, Breed, & Fraser, 2004). The goals of the IMP program included focusing on problem solving, inquiry based learning, making connections within mathematics, and inclusion of alternative assessments (such as, portfolios, self-assessment, observations, presentations, and group projects). Clarke et al.'s study on the IMP high school curriculum investigated the programs impact on not only student achievement but also student perceptions. Two of the three participating high schools had IMP students as the intervention and Algebra 2 students as the comparison group. For these two schools the mean SAT score, presumably the mathematics section, was higher for the IMP group with *p*-values 0.0372 for one school and 0.1003 for the other school. For the perceptions, the IMP students from all three schools were compared the Algebra 2 students from the two schools. For many of the perception questions, the mean score for the IMP students was significantly higher at least at the 0.01 level, though often at the 0.001 level, than the mean score for the Algebra 2 students. On average, IMP students were more likely than Algebra 2 students to report a higher self-rating of mathematical ability and more positive feelings towards their mathematics class, "were more likely to hold a socially-oriented view of the origins and character of mathematical ideas rather than a Platonist belief in the existence of mathematical absolutes awaiting discovery" (p.12), to value talking to classmates as a learning method, to report higher engagement with drawing diagrams and working and talking with other students, and to identify the mathematics in everyday activities than Algebra 2 students. On the other hand, on average Algebra 2 students were more likely than IMP students to value drill and practice and to engage by working on their own, copying form the board, and working from the textbook.

Another study investigated the use of Problem-Centered Learning (PCL) in middle schools that were using a program described as the traditional explain and practice approach (Ridlon, 2009). The PCL method involved the teacher posing a problem, students working in small groups to solve the problem, and then the groups sharing and justifying their various solutions and solution methods. The students would discuss the various methods and reasoning and decide which methods are valid. The first year of the study, from four different teachers' classes 26 students were selected who scored below 40% on the national norm on the Iowa Test of Basic Skills (ITBS). They participated in a pull-out class with the researcher. The second year of the program, a teacher team-taught the program with the researcher. The students had been randomly assigned to the teacher and were representative of the demographics of the school. Gains between pre-test and post-test, which had questions similar to the ITBS, were statistically significant for the PCL students over the control groups. One of the themes from the qualitative data (student and parent surveys, parent interviews, observations from an outside evaluator, and student journals) was empowerment, "Their input and strategies were valued. Those in traditional class thought their sense-making had little value when compared with the teacher's superior procedures" (p. 218).

The Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) program emphasized thinking, reasoning, and problem solving in mathematics (Stein & Lane, 1996). They focused on tasks that allow multiple solution methods, encouraged the use of multiple representations, and required explanation from the student. They used their own QUASAR Cognitive Assessment Instrument (QCAI) that used open-ended tasks. The more tasks the students implemented at the doing mathematics level, see page 18, the more they gained on the QCAI during their three years on middle school. They found that each of the following factors was present in at least 70% of the tasks that were implemented at the doing mathematics level: task builds on students' prior knowledge, scaffolding, appropriate amount of time, high-level performance, and sustained pressure for explanation and meaning (Hennisen & Stein, 1997).

The Connected Mathematics Project emphasized classrooms as mathematical communities, reasoning about mathematics, conjecturing and problems solving, and connecting mathematics (Lappan & Ferrini-Mundy, 1993). A study in Texas looked at sixth grade students improvement on the Texas Learning Index based on the state standardized test, where at the time of the study a gain of one to three points was typical. The results found that 85 non-at risk students improved by 1.74 points on average (though the authors pointed out the ceiling effect of three students receiving a perfect score on the state standardized test) and the at risk students improved by 10.16 points on average. The pass rate on the state standardized test of the at risk group was 92% compared to the usual 60-75% pass rate. Observations of the three classrooms were made to determine the level of fidelity in the implementation of CMP curriculum which was determined to be reasonable.

Opportunity to Be Creative

Two studies used observations to explicitly investigate opportunity for creativity in the classroom. Based on Lithner's (2008) framework for creative reasoning described in the definitions section for mathematical creativity on page 12, Bergqvist and Lithner (2012) observed one lesson from 2 middle school teachers, 4 high school teachers, and 6 undergraduate mathematics instructors deemed typical in Sweden to see what level of reasoning the teachers presented. One to four teaching situations per class were analyzed for a total of 23 teaching situations. Overall, they found the reasoning to be primarily algorithmic with few examples where the mathematics foundations behind the algorithms are justified. In most cases no reflection was evident.

In order to measure elementary students opportunity to be creative, Schacter et al. (2006) created an observation rubric with 19 items under the five categories: creative thinking strategies, choice and discovery, intrinsic motivation, environment, and imagination and fantasy. Each item is a statement about the teacher, such as “The teacher develops activities where student have to create an original artifact and present this artifact as a potential new solution to a problem” (p. 56). This rubric was filled out while observing a lesson; eight lessons were observed throughout a school year for each of 48 teachers among grades 3 through 6. This was not done specifically for mathematics. On average, each of the 19 measures occurred once or twice during the eight observations and was evaluated to be low quality, which means that the creative aspect was not well aligned to the content or was not explained explicitly. Classrooms with higher proportions of minority, limited English proficiency, or low-performing students tend to receive less creative teaching. However teachers who used more creative methods were more “productive” based on academic achievement.

Both of these studies focused on what the teachers did and painted rather dismal pictures of the students’ opportunities to observe mathematical creative reasoning being modeled, to develop skills related to creativity, or to be creative. The students voices were only heard in response to the teacher, and the analysis focused on how the teacher responded rather than what the student learned from the situation.

Conclusion

The literature review uncovered many articles that offered theory or advice but were not research-based, but instead cited others who offered theories or advice on how to foster mathematical creativity in the classroom. The research articles explicitly linked to creativity revealed a few ways students can be mathematically creative, such as solving problems with using multiple solution methods (Bolden et al., 2010; Uworwabayeho, 2009; Waynberg & Leiken, 2011), solving problems with multiple solutions (Kwon et al, 2006; Lev-Zamir & Leiken, 2011), completing projects creating original problems (Munakat & Vaidya, 2012) or geometric property (Shriki, 2010), focusing on mathematical reasoning (Liu & Niess, 2006; Ward et al., 2010), and self-reflection and making connections (Sanz de Acedo et al., 2010).

Most of the research explicitly linked to mathematical creativity in the literature review was done with postsecondary students and internationally. Many of these studies focused on little-c creativity where students created something original compared to their peers. More research on mini-c mathematical creativity (Beghetto & Kaufmann, 2007) may help teachers see that learning mathematics can be creative instead of struggling to see mathematics as creative (Bolden et al., 2010), feeling like creativity is not their responsibility (Aljughaiman & Mower-Reynolds, 2005), or seeing creativity as a distraction (Beghetto, 2007).

In Chapter III I discuss how I used the definition of mini-c creativity to research the developmental nature of mathematical creativity in middle school classrooms in central Texas. Since the literature review did not uncover a model for fostering mathematical creativity in the classroom, this study used exploratory qualitative methods.

III. METHODOLOGY

The purpose of this study was to describe middle grades students' opportunities to be mathematically creative in the classroom. The questions that guided this research are:

1. How do teachers make mathematics personally meaningful to foster students' mathematical creativity?

2. How do teachers create a safe environment to help students develop mathematical creativity?

3. How do teachers use mathematics practices to foster students' development of mathematical creativity?

This chapter describes the research design, researcher's roles, participant selection, setting, participants, data collection, data analysis, building trustworthiness, and ethical issues for the study.

Research Design

Since qualitative research is often used when "there is a lack of theory or an existing theory fails to adequately explain a phenomenon" (Merriam, 2009, p. 15), a qualitative case study made sense for this study. My literature review of mathematics education research failed to uncover a complete picture on how teachers can foster the development of mathematical creativity in the classroom. The qualitative method allows for rich description of the teachers' practices. In particular, "A *case study* is an in-depth description and analysis of a bounded system" (Merriam, 2009, p. 40). All teachers that participated in this study teach at the same school, thus is naturally bounded. Multiple sources of data, including teacher interviews, student interviews, classroom observations, and documentation of displayed student work data, were collected to provide a rounded picture.

Researcher's Roles

My role as a researcher included being a human instrument for data collection and the analysis (Merriam, 2009). This allowed me to adapt and improve the focus of the study based on my initial observations. I was a participant observer, though my role was primarily as an observer. The class was aware of my observations, and I did help students sometimes during group work time. I was also a learner during this study. I have limited experience in middle school mathematics classrooms, thus what I learned extended beyond the scope of this study.

Participant Selection

Teachers

My initial goal was to see the tasks "average" teachers may do in their classroom that allow students to experience some level of mathematical creativity. Middle school was chosen because teachers specialize in a single subject but the curriculum is broader than high school.

I explained my study to attendees of a professional development on working with English language learners in the mathematics classroom and invited the teachers to participate in the study, indicating I would send them an email. I sent an email to seven attendees of the professional development, everyone except a bilingual instructor who teaches multiple subjects. As a result, I received two definite yeses, one definite no, and one response informing me that she had move to a vice principal position. The teacher who responded with a no actually joined the study a little later.

I included all three teachers who, at some point, indicated they were interested in participating. Since I was looking to describe examples of mathematical creativity in the classroom, I purposely chose to work with teachers who self-selected into the study. Intensive sampling provides "information-rich cases that manifest the phenomenon intensely, but not extremely" (Patton, 2002, p. 243). I was not interested in extreme cases, because I wanted the descriptions to be relatable for more teachers.

Students

In order to get the students' perspective, I asked the teachers to select the students to be interviewed as key informants, the presumption being that the teachers would know which students would be willing to cooperate and be able to bring back a signed parental consent form. The main criteria I asked the teachers to use to select students was based on participation. I wanted to talk to both students who fully participated in their groups and class discussions voluntarily and more reluctant students who generally did not contribute to class discussions unless called on. Maximum variation sampling was used to see what "important common patterns that cut across variation" (Patton, 2002, p. 243) or if these students had different perceptions on the opportunities they had in their mathematics classes.

Setting

All three teachers taught at the same middle school in central Texas. The school had about 1,000 students, nearly three-quarters of which were considered to be economically disadvantaged. About 75% of the students were Hispanic, about 15% were White, and then 10% were across a variety of races and ethnicities.

The school had consistently met expectations for the state standardized test, currently the State of Texas Assessments of Academic Readiness (STAAR). In 2013, 83% of the students in this middle school passed the mathematics portion of STAAR. In past years, the school has been “Recognized” for their students’ mathematics achievement on the state standardized test and has been named a Blue Ribbon School.

The school culture included using research-based programs. The adopted mathematics textbook was the Connected Mathematics Project (CMP) series (Lappan, Fey, Fitzgerald, Friel, & Philips, 2002). All teachers, all core content areas and electives, were required to use the graphic organizers from the Strategic Instruction Model (SIM). Also, the school participated in the Advancement via Individual Determination (AVID) program. AVID “is a college readiness system for elementary through higher education that is designed to increase schoolwide learning and performance” (AVID, 2014). All teachers at this school received AVID professional development for strategies to use with all their students. Also, there were specific AVID students who were placed into advanced classes, such as pre-algebra or algebra, that they may not have qualified for using traditional measures, but these students received additional support. Students were traditionally placed into advanced classes in this school based on their state standardized testing results and teacher recommendations; grades were used for borderline cases.

The school was on a block schedule, with four 90-minute periods. They had A and B days, so the periods are numbered one through eight. The students attended mathematics and language arts daily, though not necessarily at the same time on A and B days. They had each science and social studies either on A days or B days, and they alternate electives. There was also an additional period at the end of the day for

Table 3.1

Overview of Participants

Teacher	Teaching Experience	Education	Certification	Teaching Assignment	Students Interviewed
Ms. Hartzell	3 years elementary 10 years middle school	BA: Education with math specialization	Generalist 1-8	Grade 7 regular and pre-algebra	Mark, David, and Cristina
Ms. Patrick	8 years middle school	BA: Interdisciplinary Studies	Mathematics 4-8	Grade 7 regular and algebra	Juan
Ms. Calloway	10 years elementary 4 years middle school	BA: Education, MEd: Curriculum and Instruction	Early Education and Generalist pre-K-8	Grade 8 math test prep	Nathan

additional help in subjects based on students' needs, athletics, or AVID students had a special AVID class.

Participants

The three teachers whose classes I observed and the five students who were key informants are described in this subsection. The names given for the teachers and students are pseudonyms. Table 3.1 provides a summary of teachers' teaching assignments during the study and which students were in which teacher's class.

Teachers

For each teacher I describe her education and teaching experience and her current teaching assignment. During the initial interview I asked each teacher to describe the general impression of her own K-12 experience with mathematics. I included their responses to this question with their current teaching assignment because they seemed to describe how their K-12 experience as students influenced them as teachers.

Ms. Hartzell. Ms. Hartzell told me during the professional development that she would be willing to participate in the study and was the first teacher to respond to the invitation email. When I asked her for dates when she would like me to visit the classroom, especially ones that might give me the opportunity to observe students' mathematical creativity, she indicated that I was welcome to observe any day with or without prior notice.

She had taught seventh grade for 10 years at the middle school. Previously she taught three years at a private elementary school, fourth and third grade. She has an education degree with a math specialization, certified to teach grades 1 through 8. When I asked her about her experience with mathematics when she went through K-12, overall she considered her math teachers to be “dull” but her Algebra II teacher was who inspired her to be the teacher she is. This teacher was “absolutely crazy and she made it interesting.”

Her teaching assignment during the data collection phase of the study was two sections of grade 7 mathematics and one section of pre-algebra. The pre-algebra course covered all of the grade 7 and grade 8 TEKS. One of the two sections of grade 7 also had a co-teacher; this section included students who had special needs. I attended only one class period with the co-teacher. The co-teacher led most of the class period in a more traditional, direct teaching method. I did not include this observation in the data.

Ms. Hartzell’s classes had about 30 students. She arranged the desks in her classroom in groups of three or four. Students worked on a warm-up assignment in their groups, or individually if they chose to, for the first five to eight minutes of class. She set a timer, so they knew exactly how much time they were allowed. When time was up, she would give them three mental math problems orally. Then they would grade their own papers with red pens. During the grading period she would call on students to give answers and usually would ask students to explain how they solved the problems. Next they would go over a few homework questions. At the beginning and end of units, they would work on SIM graphic organizers for a few minutes. The remainder of the class was spent doing Connected Math investigations. Ms. Hartzell would spend a small amount of time introducing the investigation, by having students read introductory text and asking students questions to help them make connections to the real world or previous concepts. The students would work on part or all of the problems in the investigation and then they would have a whole class discussion on their work.

Sometimes they would switch between group work and whole class discussion a few times.

Ms. Patrick. Ms. Patrick initially indicated she would not be participating in the study. However, once she got used to the additional duties of having a student teacher in her classroom and mentoring a peer and Ms. Hartzell explained to her what I was doing in her classroom, Ms. Patrick volunteered to participate in the study.

All eight years of her teaching experience were teaching the same classes at the same middle school. She had a degree in interdisciplinary studies and was certified to teach mathematics for grades 4 through 8. Her own K-12 experience was all direct teach: taking notes and completing worksheets. She recalled very little hands-on experiences in mathematics classes, especially in middle school and high school.

Her teaching assignment was two sections of grade 7 mathematics and one section of algebra. One of the two sections of grade 7 also had a co-teacher; this section includes students who have accommodations. During my observations, the co-teacher led the class only while students corrected their warm-up assignment. Usually the co-teacher sat in a student desk during whole class discussions and helped students during individual or group work time.

Ms. Patrick had around 30 students per class period. She arranged the desks in pairs, all desks facing the same direction. Students were expected to work individually on their warm-up assignment for the first few minutes of class. The students graded their own papers with markers. During the grading period students were called on to provide answers and explain their solution methods. She used the SIM graphic organizers and

Connected Mathematics investigations in a similar manner as Ms. Hartzell, though she did any reading.

Ms. Calloway. Like Ms. Hartzell, Ms. Calloway was very open about me visiting her classroom at any time. She kept her door and blinds open at all times and welcomed parents and administrators to visit her classroom.

All of her teaching experience had been in the same school district. She taught for 10 years at the elementary school level; this experience was varied, including science, social studies, math, bilingual, and at-risk specialist. Her degree was in elementary education and is certified in early childhood and as a generalist. She also had two masters' degrees and is familiar with education research. Her defining moment in her own K-12 education was having a seventh grade algebra teacher who point blank told her that girls are not supposed to do math. Despite having a mother who is an educator, she "slumped and slumped and slumped" and had to retake Algebra a couple of times, finally passing it as a freshman in high school.

Her teaching assignment was teaching grade 8 students who did not pass the grade 7 state standardized test. These students took her class in addition to their regular grade 8 mathematics class, meeting either on A day or B day. She had 5 periods of these classes. She also co-taught a grade 8 mathematics section with bilingual students; I did not observe this class.

The size of Ms. Calloway's classes varied from seven to 13 students. She arranged her desks in groups of four, though depending on the class size and behavior issues she may have only two students per group. Initially she did not give her students a warm-up assignment, then she decided to return to her practice from previous years of giving a warm-up assignment. Some days half the class would work on computers for part of the class period while other students get additional help on the daily objective. She did not use a particular textbook; she pulled from a variety of resources for activities and worksheets to supplement the students' regular grade 8 mathematics instruction.

Students

Each teacher was asked to recruit two students per level of class to be key informants. Because of behavior issues in her regular grade 7 class, Ms. Hartzell recruited students only from her pre-algebra class. Three of the four students returned a consent form. Ms. Patrick recruited from her grade 7 and algebra classes, but only one student returned the consent form. Since all of her classes are considered the same level, I asked Ms. Calloway to recruit two students. Despite her personally contacting the parents of the students she tried to recruit, only one student returned a consent form. Each student who brought back a consent form was interviewed and is described briefly below.

Mark. Mark was a student in Ms. Hartzell's pre-algebra class who tended not to volunteer contributions to class discussion or work with his classmates. He did well on his assignments and seemed confident in his ability to complete his work.

David. David was an outgoing student in Ms. Hartzell's pre-algebra class. He frequently volunteered answers and comments to class discussions and participated in group work. He did well on his assignments and seemed confident in his ability to complete his work.

Cristina. Cristina was a student in Ms. Hartzell's pre-algebra class who did not tend to ask questions in front of the whole class and worked with her classmates in a limited way. She seemed to lack confidence in her mathematics ability overall though did mention her strengths, computations such as multiplication and division.

Juan. Juan was a student in Ms. Patrick's grade 7 class that had a co-teacher in it. He participated fully in class discussions and works with his classmates. Juan seemed fairly confident and talkative is class but his answers were brief during the interview.

Nathan. Nathan was a student in Ms. Calloway's first period class. He volunteered in class discussions every chance he got, but did not always want to work with his classmates. He seemed fairly confident, but he did not do as well as his classmates thought he did (some students were surprised to learn that they did better than him on some parts of the state standardized test from the previous year).

Data Collection

Classroom observations were the primary data source for this study. Student interviews, teacher interviews, researcher's log, and documents were secondary data sources. Each data source is described in more detail below.

Classroom Observations

Observations were the main source of data for this study. Patton (2002) described six advantages of direct observation over interview: understanding the context, being discovery oriented, seeing things others do not, seeing things people are unwilling to discuss, making your own perceptions, and drawing on personal experience during analysis. Since I had limited experience in middle school mathematics classrooms, the observations were helpful to understand how the teachers ran their classrooms. Knowing this context helped me select appropriate interview questions. Once I started my observations, I took advantage of the discovery oriented nature of the process to shift my focus from specific tasks to the expectations and properties of the tasks that gave students the opportunity to be mathematically creative. Being able to have my own perceptions of the classroom tasks did help me see what others did not. For example, in the interviews all the participants struggled to provide an example of a type of problem that would have more than one correct answer; however I saw a few examples of these types of problems during the observations. Most of the participants were not practically chatty, which probably was not so much unwillingness as nervousness and lack of experience discussing their classroom experiences. Finally, remembering how is felt to be in each of the classrooms did help with the analysis and isolate each teacher's strengths.

Observations were scheduled ahead, but two teachers did express that I was welcome any time. Observations were audio recorded and whole class discussions were transcribed. Since the observations were not video recorded, field notes were essential to provide additional context to interpret the observation transcripts.

Table 3.2 shows the total number of observations and hours of observation for each teacher in the study. This is the number of observations included in the study. I observed an additional grade 7 class period and a tutorial class with Ms. Hartzell that were not included in the data analysis. Class periods are typically 1.5 hours, but one of Ms. Calloway's class periods I observed ended early due to an assembly.

Table 3.2

Classroom Observations per Teacher

Teacher	Number of Observations	Hours of Observations
Ms. Hartzell	13	19.5
Ms. Calloway	18	26.5
Ms. Patrick	9	13.5

Some of the observations overlapped in content. The classes tended to be at slightly different points in the lesson, but sometimes I observed mostly the same lesson in two or more class periods for the same teacher.

Interviews

Q methodology. Each interview began with a sorting task using the qualitative portion of Q methodology (Brown, 1996). A questionnaire that has participants rank various statement on a scale of 1 to 9 allows participants to rank all statement in a tight range, such as ranking everything a 4, 5, or 6. Q methodology forces participants to evaluate all statements and choose the extremes.

First the interviewee sorted 29 statements (see Appendix A) printed on small slips of paper into low, medium, and high piles depending on how well the statement described the student's experience in their mathematics class – the teachers were instructed to consider their expectations for their students. Then the interviewee sorted the 29 statements onto an array with 9 columns with varying lengths, similar to a normal curve as shown in Figure 3.1. The upper picture shows the statements placed onto the array as intended. They were instructed to pay attention to the columns but not necessarily the rows. For example, all the statements in the first column are less representative of their mathematics classroom than the statements in the second column, but they did not have to worry about the top statement being more or less representative than statement below it. I asked the interviewees to explain some statements that they ranked particularly high or low to determine their interpretation of the statements. I recorded the ordering for all the statements for each interviewee.

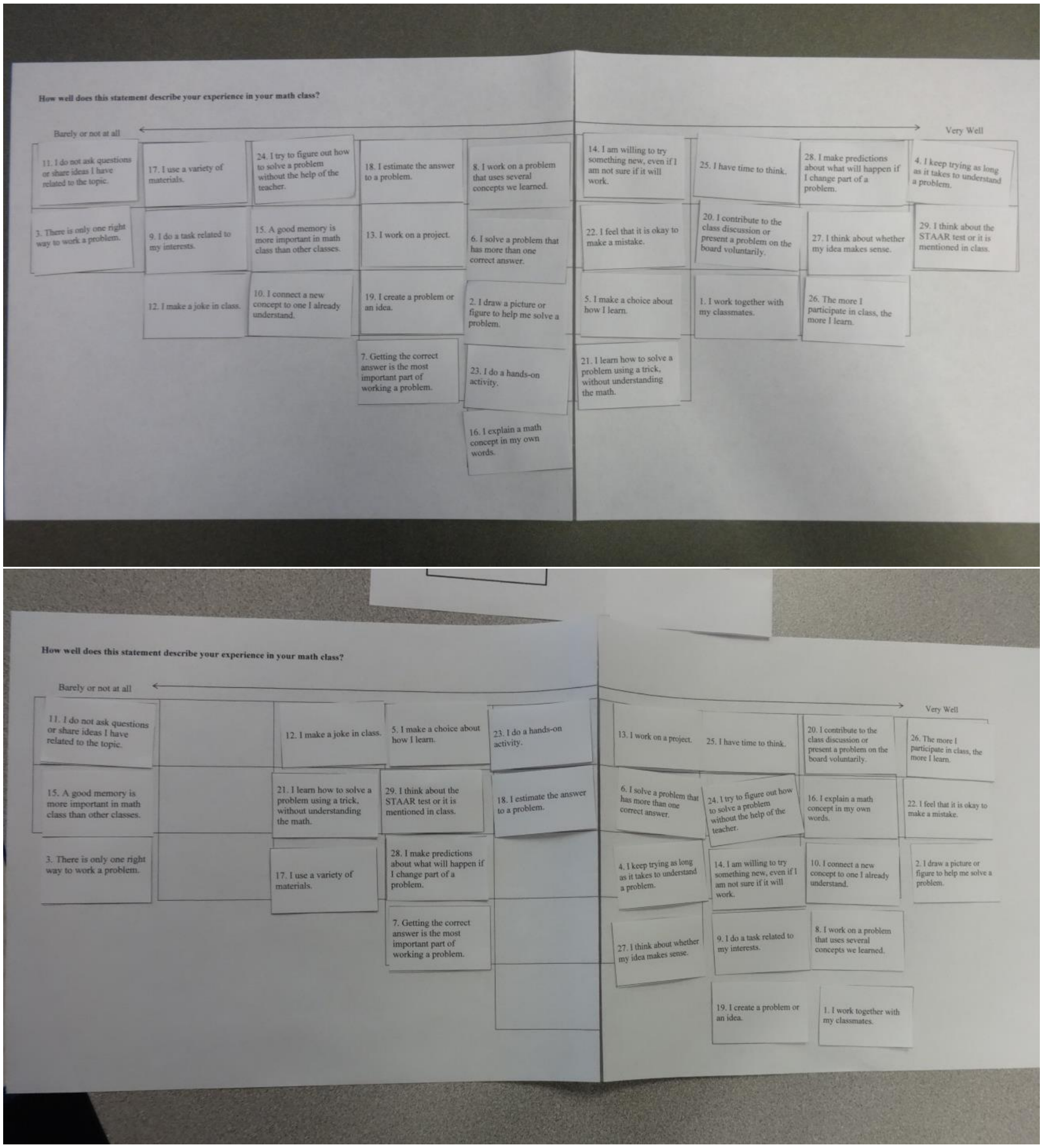


Figure 3.1. The upper picture shows the results for Juan, how the sorting task was intended to be completed. The lower picture shows Ms. Hartzell's unexpected method of sorting the statements.

Student interviews. The student interviews were used to triangulate and verify my interpretations of the classroom observations, thus were conducted after all observations had been completed. The student interviews were conducted one-on-one at the school library during their mathematics classes. The interviews lasted about 20 minutes and were semi-structured.

First the student did the sorting task described in the previous Q methodology subsection. Then we discussed a subset of a list of questions (see Appendix B). The students answered most of the same questions; the questions were selected based on the answers they gave and the discussion after the sorting task.

Teacher interviews. The first teacher interview was brief and informal. The main goals were to gain informed consent, to determine the teachers' education and teaching background, and to schedule observations. The second teacher interview was 30-45 minutes and semi-structured. I created a list of questions but did not ask all the questions to all the teachers and the order varied. This allowed me to get mostly the same type of data from all the teachers while leaving me with the flexibility to allow the teachers to focus on topics that are important to them (Merriam, 2009). The teacher interviews were conducted after all observations and the student interviews had been conducted and were similar to the student interviews described in the previous subsection.

The teachers completed the same sorting task as the students using the same statements but based on the teacher's expectations of her students. The teachers discussed the differences between what their expectations were and what actually happened in the classroom. I asked the teachers for their interpretations on some of the questions I asked the students, plus I had a few additional questions for the teachers (see Appendix C). The questions were based on my literature review and preliminary analysis of the data. The purpose of this interview was to triangulate what I saw in the classroom in the classroom and to verify my analysis of my observations.

Researcher's Log

Since the observations were not video recorded, a researcher's log was used to record impressions that I did not have time to record in my field notes. Also, notes from informal conversations I had with the teachers were recorded here.

Documents

The teachers provided copies of handouts used in classes. One teacher provided me with copies of a sample of a writing assignment. Also, I took photographs of student work displayed inside and outside of the classroom.

Data Analysis

The audio recordings for the interviews and whole class discussions from classroom observations were transcribed verbatim. Initially I had been planning on describing tasks that the teachers gave the students that allowed the students with opportunities to be mathematically creative; however, once I started making observations my focus shifted to more general aspects of the tasks and the classroom envrionment. Since my review of the literature had not uncovered a framework or model for how teachers can foster the development of mathematical creativity in the classroom, the transcripts were open coded.

> In open coding, event/action/interactions are compared with others for similarities and differences. They are also given conceptual labels. In this way, conceptually similar events/actions/interactions are grouped together to form categories and subcategories. (Corbin & Strauss, 1990, p. 12)

Data analysis was conducted using the constant comparative process (Glaser, 1965). Each piece of data collected and analyzed influenced how I analyzed new data.

As patterns emerged, codes were standardized and more focused on the topic to create categories. The categories were sorted into larger themes. At each step of labeling, labels were compared and evaluated to make sure they were clearly defined and differentiable. I also used the impressions noted in my researchers log for possible themes to aid in the organization of the emerging categories and themes.

The results of the sorting task were used to triangulate with the classroom observations. For the sorting task, statements sorted into the lowest three columns were considered low, the middle three were medium, and the highest three were considered high. The only exception was only the two highest columns were considered high for Ms. Hartzell due to her unconventional method of filling in the array (see Figure 1).

Credibility and Triangulation

Charmaz (2005) provides six questions to use as criteria for building credibility in study. The three main themes in these questions are the depth of the data gathered, the suitability of the data for the categories used to report the data, and providing the reader with enough evidence to support analysis. To address these questions I made multiple observations for each teacher, I created categories only if multiple observations were made that fit the category, and I included verbatim quotes from interviews and vignettes from observations to support my analysis. I used methodological triangulation which Stake (1995) defines as, "When we speak of methods in case study, we are again speaking principally of observation, interview, and documents" (p. 114). I triangulated my observation data with teacher interviews, student interviews, and documenting displayed student work.

Ethical Issues

Prior to beginning my study, I obtained approval from the Texas State University Institution Review Board to conduct this study. Each teacher gave informed consent prior to being observed (see Appendix D). The parents of the five students who were interviewed gave informed consent and the students gave informed assent prior to being interviewed. Pseudonyms are used for all teachers and students.

Summary

A case study was used to study middle school teachers' ways of fostering the development of mathematical creativity. The three teachers were purposefully sampled to increase the odds of seeing positive examples. Multiple observations were made for each teacher and this data was triangulated with teacher interviews, student interviews, and displayed student work. The findings will be presented in chapter IV and then discussed in chapter V.

IV. FINDINGS

In this chapter I will describe how the three teachers in this study made mathematics personally meaningful for the students, created a safe environment where all students had a voice and could make mistakes, and provided a solid mathematics base to foster mathematical creativity. Each of these themes is further refined into categories. Although the themes and categories are discussed individually, at any given moment in the classroom a few of these categories may be happening simultaneously or in rapid succession. A vignette that is chosen to illustrate a category may also illustrate other categories. The interlink of the categories is illustrated in the vignette on page 70.

Since around 200 students were observed, vignettes use generic labels, such as Student1. Unless noted otherwise, the student labels in the different vignettes are unrelated, that is Student1 in one vignette is unlikely to be the same child as Student1 in another vignette.

Personally Meaningful

"Mini-c is defined as the *novel and personally meaningful interpretation of experiences, actions, and events*" (Kaufman & Beghetto, 2009, p. 3). This section describes what opportunities the teachers provided the students with to experience a personally meaningful level of mathematical creativity. The first subsection discusses how teachers provided students with opportunities to make choices about how they do mathematics – this is related to the word novel in the mini-c definition. Poincaré (1913) pointed out that there an infinite number of ways to combine existing knowledge to create something new but most combinations would not yield something interesting, thus choice was key to mathematical creativity. The second subsection discusses students' opportunities to express their mathematical understanding in their own words – this is related to the personally meaningful interpretation part of the mini-c definition. The third section discusses how teachers help students connect the mathematics to the world around them – this is related to the experiences, actions and events in the mini-c definition.

Opportunity to Make Choices

All three teachers allowed their students to make some choices for themselves on how they completed their assignments. The most common choice observed was using alternative solution methods, but other choices included using alternative answer forms, answering questions that had multiple correct answers, and choices on how to display data.

Alternative methods. All three teachers encouraged students to work problems using the students' preferred methods. All three teachers and five students interviewed ranked the statement "There is only one right way to solve a problem" in the low end of the array; more specifically, all three teachers and two students put it in the "lowest" column. From the first interview, Ms. Hartzell made it clear that she supported students solving problems using whatever method works best for them:

> This is what I think of as creative, discovers an alternative method. The kids are always hesitant at first to say how they did something, because their last year teacher wanted it exactly that way. That's the beauty about math, there are 10 ways.

Ms. Patrick explained how her experience as a student influenced her view as a teacher on alternative solution methods:

> I don't care how they do it. I like to see all the different ways to do it, and I like for them to explain their different ways. And I really encourage that in class because you know one way clicks before the other way does. So I welcome all ways of solving one problem. And when I was in school is

> was one way or no way. So I promised myself if I ever became a teacher, I wouldn't be like that. And they're really excited to share their way, if it's different from what they've seen.

During the second interview, Ms. Calloway explained her perspective on the benefits of allowing students to use alternative solution methods:

> Oh, letting them know, there's not one way to solve a problem. My thing is, there's one result, but many paths to get there. And it's okay – you do it this way, you do it this – if we come up to the same result, I'm okay with that. And I let them know from the very beginning of class, from the first day. You all have different ways to solve problems, and actually let's all learn the different ways we can solve a problem. Because what you do may help someone later on if learn their strategy and vice versa.

These claims were supported by student interviews and classroom observations.

When asked how they would decide who did a problem the correct way if the person sitting next to them did a problem a different way than them, three students, Mark, Cristina, and Juan, said that if the student sitting next to them had the same answer as them, that they would assume both methods are correct. Cristina also stated that she does not necessarily use the method Ms. Hartzell shows them, but does the method she knows best. When David was asked what expectations Ms. Hartzell has for how they do math, David explained, "If there is a way you can solve it and there's a way you can prove it, then she's okay with it."

The acceptance or even promotion of the use of alternative methods was seen consistently throughout the observations. Students would volunteer an alternative method to the one just discussed in a whole class discussion. For example in Ms. Hartzell's regular grade 7 math class the following took place after a lengthy discussion about extending the sequence 1, 3, 7, 15, … using the pattern of the differences:

> Student: On this one, I didn't do that. I just like read those. I like multiplied each one by 2 and then added 1 to that.
> Teacher: See me after class. Explain what you mean again though.
> Student: Like if it's 1, I multiplied it by 2 and added 1. Then on 3, I multiplied it by 2 and added 1 and it was 7. And then 7 the same thing and it was 15. I did that with all the rest of them.
> Teacher: Ah, very nice. I like that.

The "See me after class" was so the student could get a small prize. She also responded positively to more routine alternative methods, such as solving simple percent problems with decimals versus setting up a proportion, with a thank you or good job. In the following example, students were filling in two missing numerators in three equivalent fractions. The first student had used a scale factor:

> Teacher: Were you able to get 35?
> Student1: I got three and two.
> Teacher: Three and two. How'd you know for sure?
> Student1: I …
> Teacher Let me start off by telling you, you are absolutely right. Justify it.
> Student1: Multiply the top and bottom by three.
> Teacher: Ah, scaled it up by a factor of 3 here. Is there anything else that can make this a true statement?
> Student2: I didn't do that.
> Teacher: What'd you do? I'm sorry, I can't hear you and you are trying to share a different method to get the same answer.
> Student2: When I did the three-ninths, I did bat and ball.
> Teacher: Bat and ball, what do you mean?
> …
> Teacher: I use the cross multiply and divide, the bat and ball as our eighth grade teachers call it. Absolutely. Thank you for sharing that other method.

Even though the second student didn't address her question, Ms. Hartzell allowed the student to share her alternative method.

Ms. Patrick's algebra class walked 100 feet at whatever pace they wanted and recorded the time to the nearest second. They used this data to calculate their walking rate, which they rounded to the nearest foot per second. In this episode alternate methods were both solicited by Ms. Patrick and volunteered by students:

Teacher: What would I do in order to solve that?
Student1: Times five.
Teacher: If I have 500 feet, and I am walking 5 feet per second.
Student1: Time times 5.
Student2: Not times.
Teacher: I'm trying to figure out how long it would take me to walk 500 feet. So what would I do?
Student 2: Divide 500 by five.
Teacher: Right, you're going to divide for A, I mean B1, I'm sorry. For B1 you would do 500 divided by your walking rate.
Student1: Or you could just do your time times five, because it's five times the difference.
Teacher: Five times by what?
Student1: Time times five. That's it. Because it's 5 times the distance; 500 feet instead of 100.
Teacher: Okay.
Student2: Yeah.
Teacher: So if it took you 20 seconds, you could have just done 20 times five. Is that what you're saying?
Student1: [inaudible]
Teacher: Okay. That makes sense. Or if it took you 22 seconds, you could do 22 times five. That works too.
Teacher: How far did you walk in 30 seconds? How did you get this one [Student3]?
Student3: It took me 26 seconds to walk 100 feet.
Teacher: So you're walking rate is what?
Student3: My walking rate is 4 feet per second.
Teacher: Okay.
Student3: Since I needed to add 4 seconds to 26 and I walk 4 feet every second, then I multiplied 4 by 4 is 16. So 16 feet plus 100 feet is 116 feet.
Teacher: Okay. What's another way? If you walked 4 feet per second for 30 seconds, what else could I do to solve that?
Student3: Multiply by 4.
Teacher: So if I did 30 times 4, I get 120 feet. What did you get?
Student: 116.
Teacher: Okay. I think, well, do you understand what I did? So if I have 10 minutes, what would I multiply my walking rate by then?
Student1: 600.
Teacher: Why 600?
Student1: Because there are 60 seconds in a minute. And 60 times 10 is 600.
Teacher: Okay 60 seconds in a minute for 10 minutes is 600. So if I did 600 times 4, we'll just stick with 4, 2,400 feet. And then in one hour I'm going to multiply by what?
Student4: 3,600.
Teacher: 3,600. How many?
Student1: Or you could multiply your 10-minute thingy by 6.

Teacher: Oh, yeah.

Although Ms. Patrick seemed to have specific methods in mind for how to solve the problems, once the students explain their processes she opened up to the alternative methods. Student3 used both his walking time and his rounded walking rate in his calculation, which caused his answer to differ from Ms. Patrick's that was based only on the rounded walking rate.

Ms. Hartzell also modeled the use of alternative methods for her students. When discussing a warm-up question where the students needed to determine the time 46 minutes before 2:10, she guided the class through a method they may have seen in sixth grade using 4 big mountains and 6 tiny mountains. When a student expressed uncertainty at the final result, she directed him to look at the clock and guided him through the solution that way.

Most of the time I saw at most two methods presented during the whole class discussion. One of the rare opportunities I saw three solution methods discussed in a whole class discussion was during the last observation for this study. Ms. Calloway's eighth period class was shortened due to an assembly. She solicited alternate methods for converting nine-fifteenths to a percentage. One student simplified the fraction and then scaled up so the denominator was 100, so then the numerator is the percentage. The other two students divided 15 into 9, but then differed on how they converted the decimal to a percentage. One student moved the decimal two places, and the other student used place value to convert the decimal six-tenths to the fraction six-tenths and scaled up to make the denominator 100 so that the numerator is the percentage.

When the Ms. Hartzell's pre-algebra students had to determine whether they would get more pizza at a table of 10 people with 4 pizzas or a table of 8 with 3 pizzas, three different methods were shared during the whole class discussion. One group determined what fraction of a pizza each person gets depending on where they sit, and another group figured out how many people one pizza feeds depending on the table. The third group assumed that each pizza had 8 slices and calculated how many slices each person would get depending on where they sit. More students wanted to share their methods, but they ran out of time.

Alternative answer forms. When asked if every question has one correct answer, Mark, Cristina, and Nathan mentioned multiple forms of answers. Mark used the example of multiplication versus using an exponent, Cristina mentioned mixed numbers versus improper fractions, and Nathan compared equivalent fractions. While the teachers would consider both forms correct, in these cases they would consider a particular form to be the best answer in order to prepare students for the state standardized test. However, unless a problem specified a particular form, they would accept equivalent answers in simplified fraction or decimal form or equivalent answers using different units.

For example, in Ms. Patrick's algebra class a warm-up exercise had a bar graph with whole hours labeled on the y-axis and tick marks in quarter hour increments. The first question specified minutes, so only one answer was acceptable, however the other two questions did not specify units so multiple forms of the answer were considered:

> Teacher: How much more time for eighth graders than seventh graders?
> Student1: 45 minutes.
> Teacher: 45 minutes. What fraction of an hour is that?
> Student2: Three-fourths.
> Teacher: Three-fourths of an hour, so that would be fine too. Or, what's another way?
> Student3: Point 75 of an hour.

Teacher: Point 75, 75 hundreds of an hour. Twenty-four sixth graders would spend how much time total each day?
…
Student1: I multiplied 24 times 90 and divided the thing by 60.
Teacher: To get?
Student1: To get 1.5 hours times 24. Oops.
Teacher: Oh, you converted minutes to hours. [Student4]?
Student4: I did 90 minutes times 24 and got 2,160 minutes.
Teacher: Okay, that's fine. What's another way [Student5]?
Student5: I multiplied 1.5 times 24.
Teacher: That's exactly what I would have done.
Student1: Very good.
Teacher: One and a half, 1.5 times 24. Any of those ways will still give you 36 hours or, if you left it in minutes, how many [Student 4]?
Student4: 2,160.

During the second problem when Student1 says "oops," he seemed to realize he had done the problem the long way since 1.5 can be read directly off of the graph.

In Ms. Hartzell's pre-algebra class, they had worked through a similarity problem using a proportion, solved two slightly different ways, when a student had a question about the answer of 5 foot 3 inches. He said that during morning tutorials Ms. Hartzell had said the answer was 5.25. She guided him to see the equivalence of the two answers and remarked, "Thank you for calling me on that, I like that proof. Both answers are acceptable."

Alternative forms of equations were discussed. Ms. Hartzell asked her pre-algebra class for multiple forms of the equation $C = 21 \times n$. Most suggestions were various ways to write the product, but one student provided another equation using the fact family, $C \div 21 = n$. In Ms. Patrick's algebra class, I observed similar discussions. For example they discussed choosing their own variables for a context and also different forms of the equation based on fact families. In the algebra class when they were discussing slope and y-intercept, Ms. Patrick pointed out the differences in students' equations when the slope was one or the y-intercept was zero:

Teacher: I saw about half of ya'll leave it like this, and I saw the other half of ya'll put plus zero. Who's right?
Students: Both.
Teacher: Both of ya'll are. That being said, does plus zero change the value of your equation?
Students: No.
Teacher: Mathematicians are lazy, so they're going to leave that off. If it's not going to change the value of it, they're not going to put it on there and you're not going to see it. So what I'm telling you is if you don't see plus anything, you have to know that your y-intercept is zero. If there is not a plus anything, then it is at (0, 0). Does everybody understand that?
Student1: Are they ever going to give us, like $E = 2.4t$ and make up graph it on the chart?
Teacher: Yes. Like if I give you an equation and you have to graph it? Yes, it's really not that bad. [Student2], what did you get for the other equation?
Student2: Which one?
Teacher: The one I'm on, for Henri.
Student: For Henri I got $d = t + 45$.
Teacher: I also saw this. Which one is right?
Students: Both.
Teacher: Again they're both right. This is how you're going to see it. Here's another thing you have to know. If I multiply by 1, am I changing the value of it?
Students: No.
Teacher: So you don't need it. Which means if there isn't a coefficient, it's one. If there isn't one, it's one.

She declared both forms to be correct but helped students to interpret the simplified form.

Multiple correct answers. There were a couple types of problems where students had different answers from each other that were correct. One type of problem was open-ended questions that had multiple possible solutions and the student was only required to provide one, and another way students had different answers from each other was when the problems were based on student data.

Open-ended problems were fairly rare during my observations. Problem posing was one way the students were allowed to have completely different answer. When reviewing a homework assignment, four students in Ms. Hartzell's grade 7 class shared the percentage problem they wrote. A typical example is, "At Walmart a hat is 50% off.

The original price including tax is $32.78. What is the price after the discount is taken off?"

Ms. Calloway had her students write problems around a topic of their choosing. She gave them a choice of several graphic organizers to use to brainstorm subtopics and then problem ideas. I observed her modeling part of the process for her period 4 class. She chose the topic redecorating her home and guided them to help her pick subtopics of areas of her home: master bedroom, master bath, kitchen, guest bathroom, front yard, and living room. She guided them to be specific. For example, if a student suggested bedroom, she would ask, "Which one?" Then the students came up with three problem ideas for the master bedroom. They wrote a complete problem for the first idea, cost of paint. She asked them questions to help them see what they needed to write the problem. She started with asking them what kind of math they needed to do to calculate the cost of paint. The hint Ms. Calloway gave was asking if she could just go to the store and buy a little pint of paint. Then students realized she needed to know the dimensions of her walls. They discussed whether they were talking about area or volume and discussed lateral surface area versus total surface area in the context of the problem. They decided to do total surface area and subtract the floor. At this point she said that they could multiply the surface area by the price per gallon of paint, and they moved on to discuss other problem ideas. When a student expressed uncertainty on how to write an actual problem, Ms. Calloway returned to this scenario and started using actual numbers. A student volunteered a price for a gallon of paint. Initially a student gave 50 feet by 100 feet as the dimensions of the room, but Ms. Calloway had them think about the reasonableness of their answer, considering their classroom was 30 feet by 35 feet. So they revised the dimensions of a master bedroom to be 30 feet by 25 feet. She reminded them to refer back to the formula to figure out whether or not they needed the height of the walls. Once they wrote what they knew, Ms. Hooper asked them if they had all the information they needed to solve the problem. She gave them the last piece of information, that they needed the coverage for the paint. Here is the problem they came up with:

> Ms. [Calloway] is painting her master bedroom walls and ceiling. The walls are 16 feet high, 25 feet long, and 30 feet wide. One gallon of paint cost $23.90, it will cover 70 feet.

They discussed how they would use the surface area and the coverage to calculate how many cans of paint they needed, including rounding up the result if the answer was not a whole number. They ran out of time before writing the question, but she finished by asking, “Now that I have this number, now can I find out how much my paint cost?” and “Does everyone see what it takes to make a good problem?” I think the rush at the end caused the coverage of the paint to not be reasonable, but the students seemed to have a much better understanding of what was expected in their problem posing assignment after going through the complete process.

Another type of open-ended problem I observed the students doing is to create multiple representations of the same value. Ms. Hartzell’s students discussed a homework assignment where they created equivalent fractions by filling in blanks in partially filled fractions. Most of them were designed to have exactly one correct answer, such as the problem discussed on page 65, but the last couple of problems allowed for infinitely many correct answers:

$$\frac{3}{\square} = \frac{12}{\square} = \frac{9}{\square} \qquad\qquad \frac{\square}{3} = \frac{\square}{21} = \frac{\square}{6}$$

Students had written 3/4 = 12/16 = 9/12 and 2/3 = 12/21 = 4/6 on the board as the answers, when the following occurred in Ms. Hartzell's pre-algebra class:

> Teacher: Ladies and gentlemen, boys and girls, if you do not get these answers, you are wrong.
> Students: [talking]
> Teacher: Excuse me. At this point I need to turn around and see a quiet class with about 30 hands raised wanting to argue with me, but doing so in a polite fashion. [Student1], go ahead. What's your argument politely with me?

Voice: Enthusiasm, Share Ideas

Reasoning

> Student1: I don't know if I'm right, but on 37 I got 3/6, 12/24, and 9/18.
> Teacher: Six, 24, and 18 are your denominators. [Student1], I taught you you can say anything you want, but you have to prove it.

Terminology

> Student1: They're all a half.
> Teacher: Oh, okay, you win. I guess I'll let [Student1]'s answer in. Yes?
> Student 2: For.
> Teacher: I can't hear you, I'm sorry. 20 seconds. I want to hear what you have to say because you are clearly talking about this, but I want to wait. Go.

Voice: Respect

> Student2: For 37, I got 3/10, 12/40, and 9/30.
> Teacher: Nicely done. And [Student3]?
> Student3: On 38 I got 1/3.
> Teacher: On 38 you got 1/3, and that changed these other ones in the same way? Good. This morning I told you someone came up with it, and nobody has in here. Do you remember what the young lady said? One of my sweet little lazy peas, said Ms. [Hartzell] come on, is this a true statement? [Wrote 3/3 = 12/12 = 9/9 on the board.]

Mistake: Silver Lining

> Students: Yes.
> Teacher: It's too easy, but I love it. So really, I have to change my statement and say, if you got something that can be scaled up to be equivalent fractions then your answer is also correct. That's true. I was just joshing you.

Terminology

Choice: Multiple Correct Answers

After Ms. Hartzell made the first statement, which may be the only time I heard her say outright that the students are wrong, some students expressed excitement at getting the problems correct and others were saying "wait."

I observed Ms. Calloway guiding her period 1 class through multiple representations of the number 3,564, where a hexagon represents 1000, a pentagon represents 100, a quadrilateral represents 10, and a triangle represents one. The students did this problem during warm-up, and the only volunteered method was the most efficient method using place value. So, Ms. Calloway challenged them to think of a way to represent the number without using hexagons. A student realized they could use all triangles. She acknowledged this answer and asked for additional methods. When a student said quadrilaterals, they figured out the maximum number of quadrilaterals they could use to represent the number and figured out how to represent what was left over. Finally they figured out two ways to represent the number using the maximum number of pentagons. She closed this part of the lesson by stating:

> Look at all the different ways we came up with to answer the question. There is always more than one way to answer a question; you just have to know the strategy that works for you. Now how many people think we did a lot of math today? Yeah, we did, and believe it or not this is all problem solving. This is all higher order thinking. If you practice this enough you'll be able to whip out those tests and any assignment you get.

The nonverbal response to the question was that they had done a lot of mathematics.

Another type of open ended problem was a warm-up problem that had a line segment and asked students to draw a parallelogram. Ms. Hartzell's students gave examples that included a parallelogram that is not a rectangle, a rectangle that is not a square, and a square.

The open-ended problems tended to be homework problems discussed in class or warm-up problems. When students did classwork where they had different answers than their classmates, this was typically because the problems were based on student data. An activity I observed in Ms. Hartzell's prealgeba class had students choose items from a menu, then calculate the subtotal and then add on tax and tip. Another activity had students do jumping jacks and record their total every 10 seconds. This data was graphed and the students analyzed the table and graph for how their rate changed over time. Finally a third example observed in Ms. Patrick's classroom had students walk 100 feet. They used this to calculate their walking rate, which was used in various problems (see vignette on pages 65-67). The students were allowed to walk slower or faster based on their personal preference.

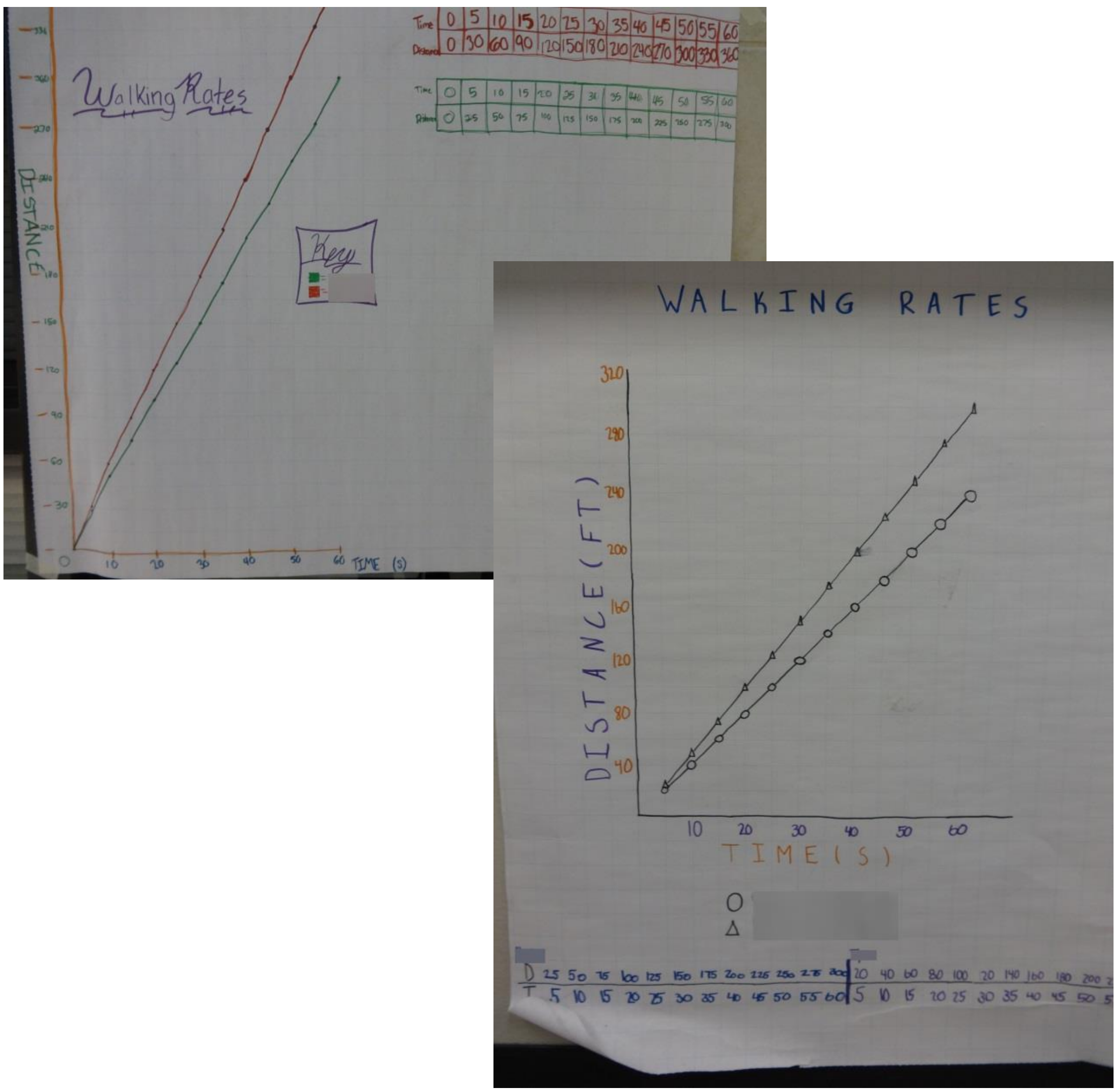


*Figure 4.1.*The graphs illustrate the small choices students make in order to create a graph that represents their personal walking rate.

Graphs and tables. One way teachers provided their students with the opportunity to make choices were to give students a fair amount of freedom when creating graphs. While providing guidance on what is appropriate, the teachers allowed students to choose their own titles, labels, and scales. Figure 4.1 provides two examples of graphs were based on the walking rate activity done in Ms. Patrick's class that was discussed on pages 65-67. Students made choices about the scale used on the *y*-axis and how to indicate which line corresponds to which set of data: color of line or shapes of data points.

Students were allowed to create table horizontally like shown in Figure 4.1 or vertically like in bottom picture in Figure 4.2. However, when a table involved an independent and a dependent variable, the teachers emphasized that the left column or top row should contain the independent variable. Infrequently students could choose the increment for the independent variable. I observed Ms. Patrick's continuation of an investigation where the algebra students the previous class period had created a table to determine the length a race should be so a younger brother who gets a head start closely wins a race with his faster older brother. A student allowed his table to be shown on the document camera and there was a brief discussion on his choice to "skip count by 2s."

Opportunities to Use Own Words

All three teachers were strong proponents of students putting concepts and definitions in their own words rather than memorizing the textbook. Ms. Patrick discussed the difficulty of understanding textbook definitions:

> Even me as a teacher reading the vocabulary in the back of the book, I'm like, "What is that?" You know 15 sentences, something I could say in one, you know. So, I absolutely need it in my own words. And the kids, they get so caught up in,

> "I don't know what that means." So if you know those definitions use big words, I think they'd understand it if they write it themselves.

Ms. Calloway addressed how putting something in your own words is easier to remember:

> If you can put it where it makes sense to you, it's so much more powerful. You ask most adults textbook definitions, they have no clue. They can give you some examples, but they couldn't explain it to you. So if you can put it in your own words, you've made it yours and no one can take that away.

Most of the students concurred; only Cristina stated that she preferred textbook definitions, because they sometimes provide an example. Mark said that "custom definitions" were easier to understand because textbook definitions "seem too to the point." Juan explained it simply, "Turns out I would make up the words; it let me remind it better." Also, Mark and David both said they used paraphrasing a problem in their own words as a problem solving strategy.

The Connected Mathematics curriculum included many problems where the students were expected to explain their work or patterns they see in investigations using their own words. Ms. Hartzell would tell her class, "You can use my sentence starter but you need to use your own thought." Usually students had a couple minutes to write their own sentences before the students would be invited to share their sentences. The following took place in the regular grade 7 class after the students figured out how many points would be left on a card for depending on the number of rides completed for several values:

> Teacher: So you want us to put into words to find any number. How do I figure it out? What if I want to go on 12 rides, what would you tell me to do? How would I figure out how many points I have left? Yes, [Student1].
> Student1: You'd do 100 minus six times the what you had, how many rides you want to ride, and then you get what it is equal to.
> Teacher: Subtract from 100 the number.
> Student1: Of rides that you had.

> Teacher: Of points. Subtract from 100 the number of - I'm sorry - I think you said number of rides times 6.
> Student2: Do we have to write exactly that or can we write our own sentence?
> Teacher: What did you write? Tell me what you wrote.
> Student2: I was just asking.
> Teacher: Oh, sure. Write your own.
> Student2: Okay.

In this case a student helped create the sentence with Ms. Hartzell on the spot, so another student clarified that they still could write their own sentences, which she cheerily confirmed. Additionally, Ms. Hartzell and Ms. Patrick did vocabulary on Fridays for the warm-up. Ms. Hartzell gave students time to create their own definitions and examples for a set of vocabulary words; they were allowed to talk within their group during this time. Then they went over the vocabulary words in a whole class discussion. The students were allowed to use dictionaries for reference but were expected to write a definition in their own words. For example, when a student read a dictionary definition, Ms. Hartzell's response was "I love it. So, determine, but that's dictionary speak that doesn't make a lot of sense so let's put in our own words."

When discussing what the difference between ratios and proportions, a student in Ms. Hartzell's pre-algebra class explained how the student typically sees to them being used, "Ratios give you everything, but proportions you usually have to find the missing part." A student in her grade 7 class described the difference as "Ratios is a fraction comparing two different numbers, and proportions are two different fractions that are being compared." Both of these responses got an "I like it" from Ms. Hartzell.

The students of Ms. Calloway's period 4 class determined whether or not they could form a triangle from three given lengths. Then they compared the examples and nonexamples of triangles to see if they noticed a pattern, trying to discover the triangle inequality theorem. One student described what she saw, "Because some of the lengths were bigger and the small lengths couldn't fit right with the big lengths." When asked to repeat due to a distraction, she rephrased, "The small lengths couldn't touch each other because the big lengths are so big." Ms. Calloway acknowledged, "You know what, that's pretty good. That is correct."

There were times when I observed students unable to communicate in their own words use hand gestures. Ms. Patrick encouraged a grade 7 student to use his hands to communicate what he knows when defining rectangle:

> Student: Two pairs of equal sides.
> Teacher: Okay, more specifically, which ones are equal? Which ones are congruent?
> Student1: The sides are congruent. I don't know.
> Teacher: Show me with your hands. Which ones would be congruent?
> Student1: This. [student holds hands vertical]
> Teacher: And?
> Student1: That. [student holds hands horizontal]
> Teacher: Okay. What can we say about this? Those sides are?
> Students: Congruent. Parallel.
> Teacher: Parallel, they're congruent, they're? Starts with an O.
> Student2: Obtuse.
> Student3: Opposite sides.

When Ms. Hartzell asked students to give real world examples of various types of triangles, the discussion began when she acknowledged a student pointing to college pendants hung on the wall.

Opportunities for Connections

The teachers presented mathematics concepts as being related to each other and the world. During our first interviews Ms. Calloway and Ms. Patrick included helping students relate the content to the real world when describing their typical class period. Ms. Hartzell told her pre-algebra class, "The reason I teach seventh grade math is that I know you are going to use it the rest of your life," and Ms. Patrick made a similar statement to me during our second interview. Ms. Calloway explained it during the second interview:

> And if you can make the life connections, you'll see where math is important. Because half of it is that you don't feel that math is important. And it's all around you; you just have to look for it. Or better yet, you just have to recognize it.

I observed these connections being made to students' interests and life experiences outside of school, to their school world, and to the real world that students have limited to no experience.

Students' interests and experiences. In our second interview, Ms. Patrick explained,

> So, I like to see what their interests are and, you know, you kind of learn like throughout the year. Like this kid's really into BMX biking and you know this person is into fashion and this person is into this. So, whenever I know that and it fits into the situation, I kind of bring that in. And if I don't know, I just ask the question and kids are almost always willing to share what it is that they want to do and I try to spin that, just to show them that they are going to use seventh grade math for the rest of their life.

An example she provided is that a student shared they are interested in becoming a tattoo artist, so she had a whole class discussion about opening a tattoo shop to discuss revenue, expenses, and profit. In one of her grade 7 mathematics classes I observed her trying to get students interested in advertising. She tried to get them to talk about television, radio, internet, billboard, and magazine advertisements with little response. However, once she mentioned advertisements on games you play on your phone, students appeared to become interested in the topic. A little later in this discussion Ms. Patrick had students think about why they like sports teams they support. She started off by asking the students how many of them like the Cowboys:

> Teacher: Is anybody willing to share why you like the Cowboys? [Student1].
> Student1: Because I was born and raised in Dallas.

Teacher: Okay, that's a really good reason. Born and raised in Dallas, until recently I guess since you are here now. So born and partly raised. So what part about being in Dallas makes you like the Cowboys?
Student1: My dad always watches them, so I grew up watching them.
Teacher: I'm so glad that you said that. His dad watches the Cowboys, so he watches the Cowboys. Think about your favorite team. Is it probably because someone you love really loves that team, so you kind of adopted that from them.
Students: No. Yes.
Teacher: For me growing up, my dad always watch the Chicago Cubs. I've lived in Texas my whole life.
Student2: The Chicago what?
Teacher: Cubs. Baseball. I grew up watching the Cubs, because of that I just kind of fell in love with the Cubs too. Things happen, we tend to like stuff because other people like it. Would you agree with that to a certain extent? Some of us like to be different. But for the most part we tend to like stuff because other people like it, so they get us excited about it so we want to be part of it and it's this whole big deal. No matter what it is. Like clothes. Like there's in styles. One person's in love with it, then 3 people are in love with it, and before you know if everyone's in love with it. So if everyone else loves it, then maybe I'll love it too.

This discussion was used to prepare students to compare four different representations of the results of fictional taste test to see which one best convinces them that one product is more popular than the other.

Similarly, Ms. Calloway tapped into her students' interests to define the word reasonable; she asked the students if it were reasonable to pay $300 for a new video game or $400 for a pair of designer shoes. See Appendix E for two writing samples by Ms. Hartzell's students where they describe how their level of hunger changes over the course of a day and used that to create a graph.

The students' experiences were sometimes volunteered, such as Ms. Hartzell's grade 7 student sharing that he saw discounts displayed using fractions in Louisiana as opposed to percentages that are typically used in textbook problems. Other times they were solicited, such as when Ms. Hartzell asked pre-algebra students to share real world examples of various types of triangles, so they were able to share objects they were familiar with such as a yield sign or a sandwich cut into triangles. Ms. Patrick asked her grade 7 students to give an example of similar figures in their lives, expecting photos, especially on a phone. The students also gave other examples that under certain circumstances could be similar, such as fonts, clothes, cars, and mirrors.

When appropriate, mathematics vocabulary words were compared to how students had encountered these words in their life. For example, Ms. Hartzell had students think about what independent and dependent meant and connected that to independent and dependent variables. Ms. Patrick made a point of differentiating how similar is used in everyday language as opposed to the mathematical term using the example that two people looking similar does not meet the mathematical definition of what it means to be similar.

School. Three ways the teachers helped students connect mathematics to their school experience was through common experiences, connecting mathematical concepts, and connecting to other subjects. Ms. Hartzell and Ms. Patrick frequently referred to mnemonic devices and strategies their sixth grade teachers had taught them, the students who had been at the school the previous year. They often used these expressions in conjunction with the mathematical terms. Such as "top dog in the house" was a reminder to divide the numerator by the denominator when converting from fraction to decimal notation. When introducing the concept of similarity, Ms. Patrick referred to a drawing on the wall that is representative of a task the sixth grade students do. In sixth grade, the students took a picture and drew an enlarged version using a grid and coordinate points.

Sometimes the common experience was from the current class. For example, when Ms. Patrick reminded her students that she had pulled four students to the front of the class to give a visual of what a word means, which helped them remember the word adjacent. Ms. Calloway created the problem to convert the fraction 9/15 to a percentage, see page 67, based on the number of students who had completed a homework assignment.

On a daily basis when Ms. Hartzell transitioned the class from grading warm-ups to going over homework questions, she circulated around the class and announced to the class the percentage of students who were ready in each group. The following scenario shows how she used this common experience to help students understand a homework problem on converting between fractions, decimals, and percentages:

> Teacher: I love that number 3 and number 4 are put up there, because as far as I'm concerned those are the two hardest ones on the page…. [Student1], were you able to figure out how to convert number 3?
> Student1: Uh.
> Teacher: It's okay to say no.
> Student1: No. For the percent I got [inaudible].
> Teacher: And that's a good try. When I came to this group – I'm sorry to put you on the spot right now, but I was kind of glad you weren't ready when I ran around and did my percents like I always do. Do you remember what I said when I was at this group?
> Students: Sixty-six and one-third.
> Student2: Two-thirds.
> Teacher: What does that mean?
> Student3: Repeating decimal.
> Teacher: It's not a whole; the whole group wasn't ready. This group has how many people in it?
> Students: 3.
> Teacher: Three, so when I said 66 and 2/3, that's what fraction?
> Student2: 33 and 1/3 doubled.
> Teacher: What'd you just say? He said, 33 and 1/3 doubled. Which is, oh my, 66 and 2/3. I'm just impressed. Why did you come up with that? I want to hear your reasoning behind it?
> Student2: I don't know. I just noticed that 66 is double 33 and 2/3 is double 1/3.
> Teacher: That's exactly right. So when I was looking at that group, how many people are in that group?
> Students: 3.
> Teacher: Three, and so how many people were ready when I was at that group?
> Students: 2.

> Teacher: Two-thirds of them were ready, and what [Student2] was talking about was 1/3 as a percent. Interesting.

She made percentages a part of her daily routine which helped them visualize percentages they were less familiar with, 33 1/3% and 66 2/3%.

When Ms. Hartzell was discussing scientific notation, she asked them if they had seen it somewhere before, expecting the science connection, and a student mentioned the order of operations using the mnemonic PEMDAS. Another example of a student making a connection between concepts is when students figured out how to determine the measure of an angle in triangles or quadrilaterals given other angle measurements, a student brought up the Pythagorean Theorem. Ms. Hartzell acknowledged that they both had to do with measures of triangles, but noted the differences in that the Pythagorean Theorem only worked with right triangles and that it was for side lengths not angle measurements. The next time the class met, they were talking about similar figures. She gave them the lengths of the legs of a right triangle and had them use the Pythagorean Theorem to find the hypotenuse before finding side lengths of similar figures.

Another way the teachers helped the students connect mathematics concepts was through the use of Strategic Instruction Model (SIM) graphic organizers. The teachers used these to help the students organize how the concepts were related.

Writing is another school wide initiative, though the teachers already included some writing before the push from the administration. The grade 7 students write at least one sentence every day, if only the sentence containing the answer to the word problem in their warm-ups. However frequently the curriculum tasks the students with describing relationships or how they solved a problem in their own words. Another activity is one-pagers that the teachers credited as an Advancement via Individual Determination

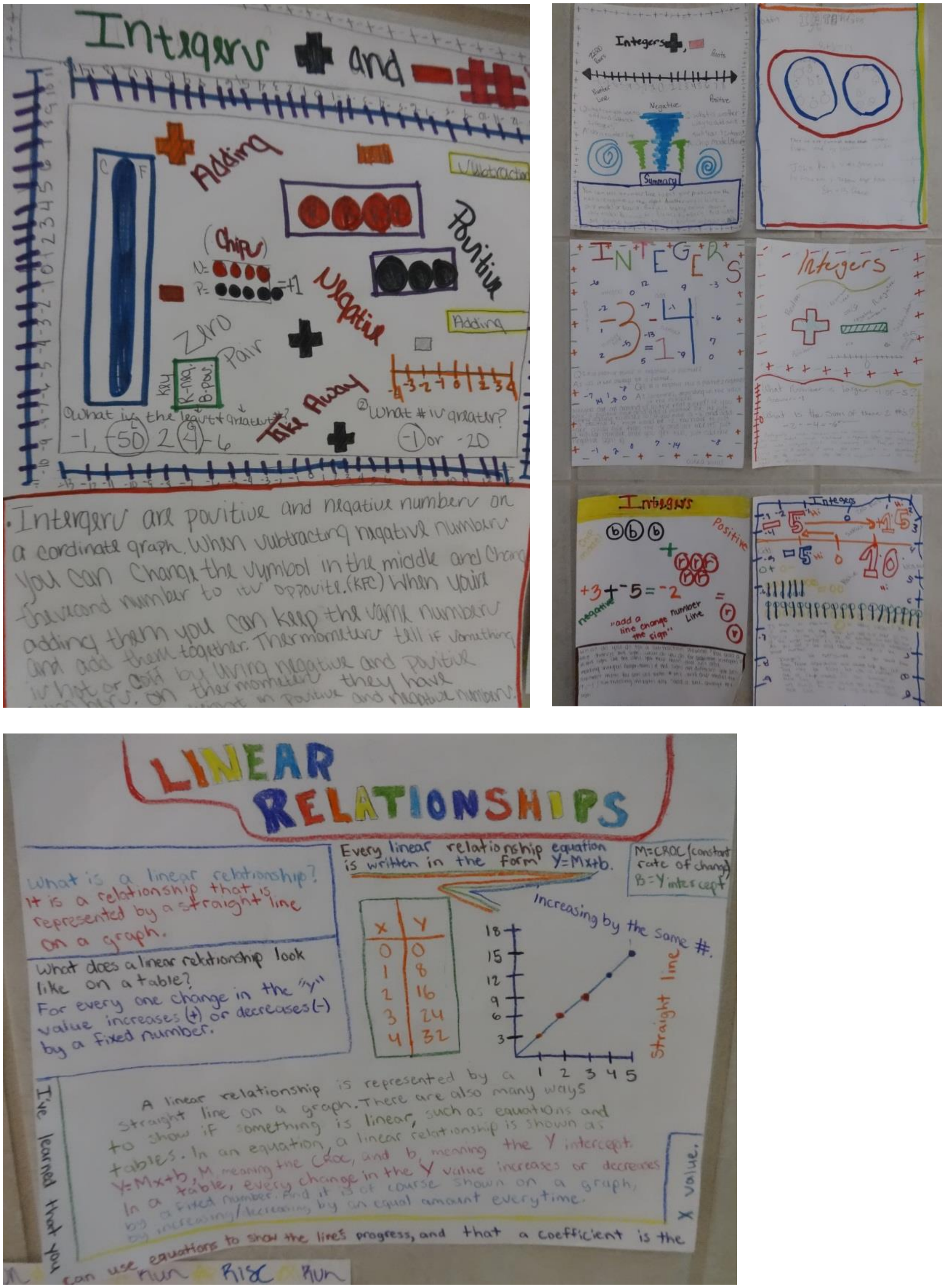


Figure 4.2. The grade 7 students' completed the AVID activity one-pagers to connect mathematics and writing.

(AVID) activity. The students would write and illustrate a mathematical concept. In Figure 4.2, the top left photo shows a "one-pager" on adding and subtracting integers done by a student in Ms. Patrick's class. This is a particularly busy example; often students would include only one or two representations of the concept. The top right photo shows a group of six of the same activity from Ms. Hartzell's students. The bottom picture shows a "one-pager" on linear relationships by a student in Ms. Patrick's class. The writing samples in Appendix E that provided examples of how students have the opportunity to connect mathematics to their everyday experiences is another example of when students are expected to write about mathematics.

When creating graphs, students are allowed to write their own titles. Ms. Hartzell emphasized that the titles should be informative and reminded them that their language arts teacher "always says we need to have a hook." Ms. Calloway used the students' knowledge of writing to explain what a subtopic is during the how to write word problems lesson discussed on pages 71-72. She said her paper would have a title, an introduction, and then the first big paragraph would be about her first subtopic.

The teachers pointed out connections to other subjects, such as noting that they also study graphing independent and dependent variables in science class. When creating a double line graph, Ms. Hartzell pointed out that they needed to include a key or legend like they might see in social studies.

Other real world experiences. The Connected Mathematics curriculum included typical real world examples that some students may have limited to no experience with. One strategy the teachers used to connect the students was to help students visualize the situations by telling the students about the concepts from the teachers' perspective. For example, when talking about discounts, Ms. Hartzell described, “When I walk into the grocery store and I see 50% off a book I was going to buy anyway over in that corner of [a local grocery store], you know where the books are. I get really excited. Why?” The next class period she indirectly referred to this visualization, “What do you do with the discount? That means I’m happy because the price got lower.”

When Ms. Calloway modeled how create word problems, see pages 71-72, she helped the students see the real world through her eyes. Another example of her seeing mathematics in the real world through her eyes happened impromptu when somebody realized that had money in their pocket.

> Teacher: I know, like one time I found $100 in my pocket, and I was like, oh wow – shoes!
> Student: [Expensive brand].
> Teacher: No, not [expensive brand]. Yeah, I’d love a pair of those. But I won’t do that they like build me some sidewalks. Ruin my heels. All my heels need caps.
> Students: [laughter]
> Teacher: No really, 107 pairs of shoes all getting them recapped. You know how much? That’s a math problem. So Ms. [Calloway] has 107 pairs of shoes. It costs $12 to get each pair recapped. Oh wait, let’s make it a decimal. [Writes $12.00.] How much does it cost her to get all of her shoes done?

Sometimes it is unclear which real world contexts students will relate to. Ms. Calloway had her students work in pairs on 16 problems, mostly contextualized, that were posted throughout the room and represented all the ways Ms. Calloway has seen Pythagorean Theorem tested on the state standardized test. Students were reminded of their objective, to solve problems using the Pythagorean Theorem, before they began. Ms. Calloway told the students that it was a competition, so they were not allowed to get help from her or me. Overall the students struggled, but there were some light bulb moments in her period 5 class. There are multiple possible factors contributing to these light bulb moments. Persistence as they continued to work on the problems as they struggled probably contributed. Also, it is possible that even though they moved on to another problem, on some level they knew they had worked the problem incorrectly but were ready to work on something new, thus incubation played a role in the light bulb moment. One pair of girls worked a few problems by computing the sum or difference of the squares of the side lengths, but they did not take the square root of result. Finally, when they encountered a problem involving a ladder against a house and came up with the answer 476, one of the girls asked her partner, “Don’t we have to find the square root of our answers?” Although not taking the square root gave them unreasonable answers for the other situations, for some reason the student was able to see it in this situation. Another pair of girls had worked three problems involving isosceles triangles with a perpendicular bisector before they realized that they should be using only half of the base in the Pythagorean Theorem. The girl in the pair who seemed less confident realized while working the earlier problems that they were not doing the problems correctly, but since she could not explain how to do it correctly, she went along with her partner’s way of working the problems. The first problem they worked of this form had a busy picture of a bridge, the second problem represented a roof with a support beam, and the third problem showed a tent. Finally on the tent problem the partner opened up to the idea that they may not be doing these problems correctly and they figured out that only half of the base was in the right triangle.

Summary of Making Mathematics Personally Meaningful

The teachers gave students choices (use alternative methods, use alternative answer forms, solve problems with multiple correct answers, and flexibility with creating graphs and tables) in how they do mathematics, allowed their students to put mathematical concepts in their own words instead of memorizing the textbook, and helped the students to make connections to both their school world (mathematics classroom and connections to other subjects) and the real world (students interests and seeing less familiar experiences through their teacher's eyes). These strategies gave students the opportunity to make mathematics personally meaningful and develop creativity at least at the mini-c level.

Safe Environment and Effort

This section describes how the teachers created a safe environment where students are able to make and express their personally meaningful interpretations of the mathematics content. The first subsection discusses how the teachers provide student with time to think. The incubation stage of the mathematical creativity process implies that at least sometimes, creativity takes time; flashes of inspiration generally come after a lot of conscious and subconscious thought (Hadamard, 1945). The second subsection discusses the actions teachers took in order to make students feel like they have a voice in the classroom, so they are comfortable sharing their personal interpretations of the mathematics. The third subsection discusses how the teachers react to mistakes and help students accept that mistakes are part of learning. Many believe risking making a mistake is an important part of the creative process (e.g., Ervynck, 1991). The fourth subsection describes how teachers supported students' use of resources. Finally, the fifth section discusses how the teachers focused on effort. This is in line with a mastery goal structure, which is believed to be an "environment that supports the development and expression of student creativity" (Kaufman and Beghetto, 2009).

Think Time

The teachers and students were almost unanimous in ranking the statement "I have time to think" highly. One student ranked it medium, but Nathan admitted that he automatically raised his hand without giving himself time to think. While sorting the statements into the grid, Ms. Hartzell lamented, "I wish I could give them all the time in the world."

The mathematics classes met daily for 90 minutes. Ms. Hartzell and Ms. Patrick took advantage of this time by generally allowing the students time to think independently or in groups before having class discussions. The discussions prior to the students working in groups tended to help students remember what they had been talking about recently and to help students relate to the topic of the day. For the more challenging questions, students generally had time to think independently and consult with their groups before classroom discussions. During classroom discussions, if the student did not complete the problem or did it incorrectly they usually were given the time to work through it then on the spot. These opportunities will be discussed further in the mistakes subsection.

Ms. Calloway's support class met for 90 minutes alternating days, which the students took in addition to their grade 8 mathematics class. Since the students also had a regular mathematics class, she was not responsible for covering the entire curriculum, so she used the time to allow students to think. She described the importance of giving students time to process during our second interview:

> Don't turn off someone else's light bulb. Don't take that away from them. And it's funny because it does take practice, because the kids are used to just yelling things out and not giving other people that moment. … If you don't give them time, then they may never get it. And people have different processing times.

I observed several instances where she reminded students to give their classmates time to think when they tried to cut each other off.

Although I observed more time in classroom discussions prior to group work in Ms. Calloway's class, the students had lots of thinking time. For example when estimating the square root of 130, they spent 3 minutes discussing what perfect squares the number falls between, what the square root of those numbers are, and what the square root sign is called. Then they spent 13 minutes reviewing sets of numbers to help them give the most precise name for the type of number the square root of 130 would be. During this time a student guesstimated 11.5 to be the square root of 130 because it's in the middle of 11 and 12. Then they spent 4 minutes deciding how they could check to see if 11.5 is the square root of 130 and 8 minutes checking it by multiplying 11.5 times itself. Then each group spent about 5 minutes checking different values. Then there was a 9 minute discussion, including checking some conflicting results, to get a better estimate for the square root of 130. Hence the class spent over 40 minutes to estimate the square root of 130.

The following vignette happened in Ms. Calloway's period 3 class after the students had been given some time to work independently or with their group to determine if various trios of numbers could represent side lengths of a triangle:

> Teacher: Number 3, did that make a triangle?
> Students: Yes.
> Student1: Yes. No.
> Student2: Yes.
> Student1: Triangle 3 did not make a triangle.
> Teacher: Did it make a triangle.
> Student2: Yes.
> Student1: No it didn't.
> Teacher: [Student3], did it make one?
> Student3: Yes.

> Student1: She put no.
> Student3: I was telling you yes, but you said no let's put no.
> Teacher: Okay [Student1], I want you to try to build this.
> Student1: Alright. So 5 and 3; so 5 is orange. Let me see this orange. 5 and 3, 3, yellow. Hello. [15 seconds] Miss, it does not make a triangle. Oh hold up now, hold up now. I might be getting somewhere. It does, right?
> Teacher: Okay. Yes it does.
> Student1: Oh, okay. I see what's up.

They used strips of paper to try to form the triangles; colors were used to easily identify various lengths. Although he already had time to work on it and the majority of the students correctly determined that the lengths could form a triangle, Ms. Calloway allowed Student1 to take about 40 seconds during the whole class discussion to figure this out for himself.

Voice

The statement "I do not ask questions or share ideas I have related to the topic" was ranked low by all the teachers and all but one student. Juan ranked this high because he tries to work independently without the help of the teacher sometimes. Ms. Calloway credited the administration for creating a school culture where all of the teachers have a voice, so it trickles down. The teachers gave students a variety of ways to share their voice. The teachers welcomed questions and comments even if they were slightly off topic and differentiated between loudness due to being off task versus enthusiasm. They ensured everyone had a voice by insisting students respect one another, calling on students who would not otherwise participate, and using non-verbal checks of understanding.

Ask questions. The teachers allowed students to ask questions of their classmates first and then the teacher during group work. During class discussions, students were allowed to ask questions if they raised their hand and waited to be called on. Ms. Hartzell had students write the numbers of homework problems that they struggled with on the board and they discussed at least some of them after the students completed and corrected their warm-up. One day she exclaimed, “Oh my gosh, those are exactly the ones I wanted to go over.”

Students also asked questions about the context surrounding the mathematics. These were answered briefly when the teacher knew the answer or the student was encouraged to research the answer and report back.

Share ideas. As discussed in the choice subsection, students would volunteer alternative methods, answer forms, or answers. When a student’s answer did not conform to the expected response, the teacher would comment on its validity or connection and move on or sometimes the teacher would go with it. For example when Ms. Hartzell asked her class about how to create a scale on the x-axis, “If I want to evenly space this, what could I do?” she was probably expecting students to say use a ruler. Rulers had already been passed out for them to use if they wanted to. She had the zero and the highest value, 120, indicated, and a student suggested she eyeball the middle to mark 60. She continued by asking what they should put between 0 and 60 using the student’s method. At the end, after she had marked her scale with increments of 10, she remarked, “This is the most evenly spaced scale I have ever done thanks to [the student].”

Although the teachers tried to keep the students positive, they were allowed to share their frustrations. Figure 4.3 shows four examples of Ms. Hartzell’s students’ comparisons of mathematics to food, which includes both positive and negative comparisons. All of these were hanging outside of her class, and she confided in me that she chose the better ones to hang outside instead of inside her classroom.

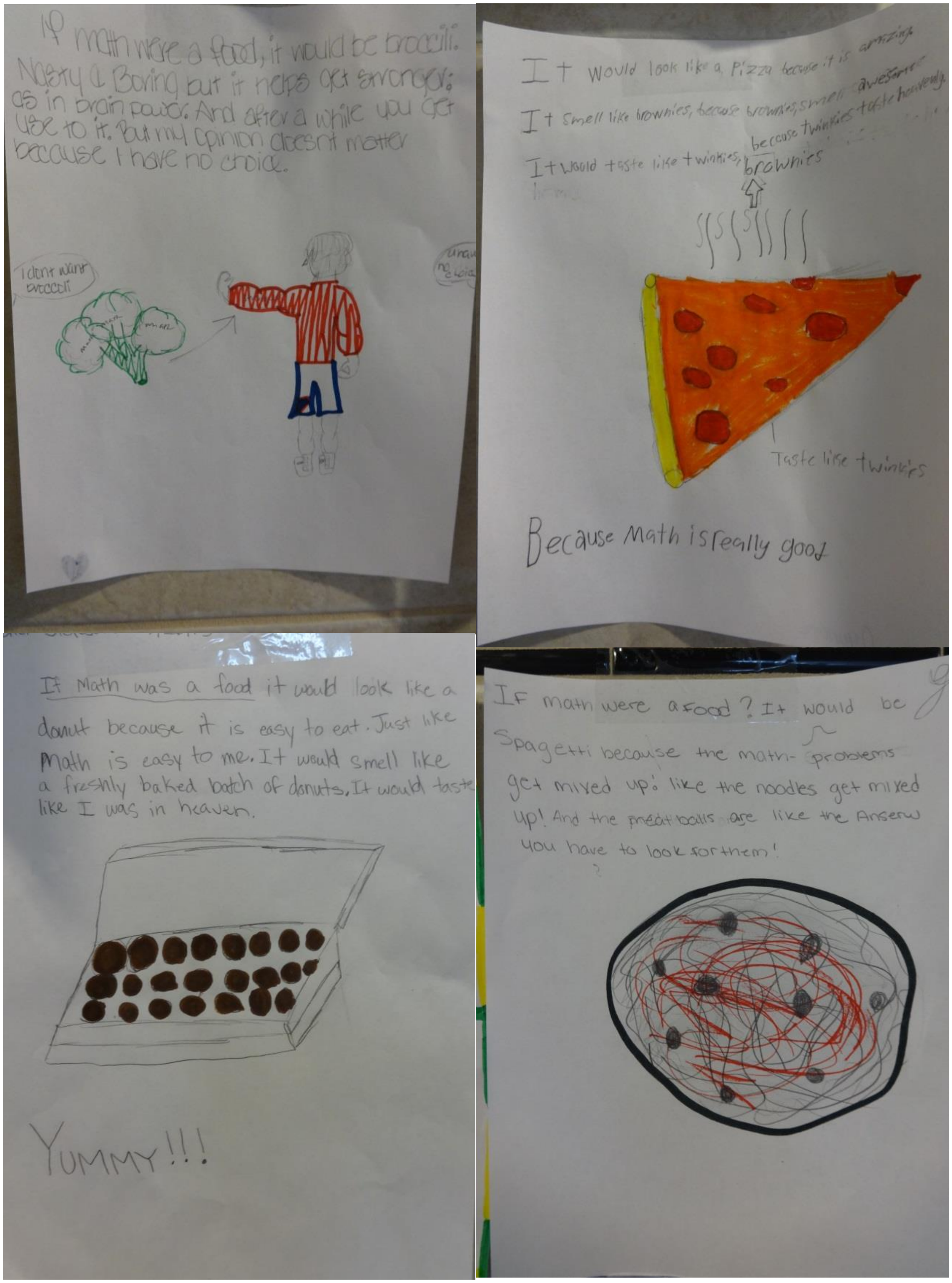


Figure 4.3. Ms. Hartzell's class did these the first day of class and include both positive and negative views of mathematics.

The first one reads:

> If math were a food, it would be broccoli. Nasty & Boring but it helps get stronger: as in brain power. And after a while you get use to it. But my opinion doesn't matter because I have no choice.

The pizza example says " math is really good" and the chocolate donut example says "math is easy." The spaghetti example reads:

> If math were a food? It would be Spaghetti because the math problems get mixed up like the noodles get mixed up! And the meatball are like the Anserw you have to look for them!

Enthusiasm. In all three classrooms, the teachers expected students to raise their hands and wait to be called on during whole class discussions. However, the teachers were careful to differentiate between when students were having unrelated conversations or were on topic but talking all at once. For example, Ms. Hartzell told her pre-algebra class, "I love the fact that we all have voices in this class and we are all very opinionated, but I do need to hear you one at a time." When a student shouted out an answer, Ms. Calloway would state, "I'm sorry, I have no hand raised." During a particularly loud discussion in her algebra class, Ms. Patrick queried, "Why are you yelling? I guess I appreciate your enthusiasm."

Respect. The teachers made it clear that the students were expected not only to respect the teacher but also their classmates. When students tried to talk over other students, the teachers would point out what is happening. Ms. Hartzell had various phrases she used to cue students that they were not respecting their classmates such as, "I find it difficult to hear her," "Sorry, I can't focus on you,", "Hold on [student], I'd like to hear your voice," and "I'm sorry [student], I think they need a moment." Ms. Patrick would point out that a student "is being rude." Ms. Calloway would ask the student who interrupted give other student a chance to speak or to think.

Additionally, Ms. Hartzell had an incentive plan to encourage the students to stay on task and be respectful:

> I give them 5 minutes of free time at the end of class, which is enough time for us to pack up, clean up the room, and get ready to leave to the next class. If I ask for a voice level 0 and I don't get it, I say that's 10 seconds; I'm taking 10 seconds off of their free time. But if they get to 120 seconds, that's 2 minutes. You wasted 2 minutes of my time; I'm taking your 5 minutes of free time, because you do not deserve it.

She took seconds during whole class discussion regardless if she is the one trying to talk or a student who had been called on is trying to talk but cannot due to students talking.

Ms. Hartzell made it clear that laughing at another student would not be tolerated. Generally students just cut each other off, but I observed a student say "Duh" to another student once. Ms. Hartzell responded quickly with, "You, last warning. That will never be okay in this room." Ms. Calloway did not allow students to call each other stupid or tell each other to shut up. Similarly, one time I observed a student tell another one to shut up and Ms. Calloway warned the student was close to getting written up.

On a more positive note, occasionally students would show support for each other. There were a few occasions when a few students would spontaneously applaud or offer positive feedback to a student's contribution to the whole class discussion.

Full participation. The teachers had a variety of ways to make sure that all students participated and stayed engaged. Ms. Hartzell and Ms. Patrick would randomly select a stick with the students names on it to make sure all students would have a chance to speak. To encourage students to stay on task, sometimes Ms. Hartzell would deduct seconds if students were not on task. Ms. Calloway's classes are smaller, so she could keep track more easily on who had not participated yet.

The teachers would offer encouragement to students who were timid. For example Ms. Hartzell told a grade 7 student, “Yes sir. Please don’t doubt yourself. That’s beautiful.” She told a reluctant pre-algebra student, “I wish you said that more loudly so they could hear. Please don’t act like you lost your voice.” When a student who is normally quiet participated in class, Ms. Calloway stated, “I’m glad you’re here; I’m glad all of you are here.” When student change their mind about contributing to the class discussion, the teachers often try to get the student to share, such as when Ms. Calloway commented, “See [a student] has her hand raised, even though she quickly lost confidence because she put it down and is now biting on her thumb. But she did raise her hand, and that’s why I’m getting to calling on her.”

Ms. Calloway will ask students if they agree, disagree, or unsure of another student’s response. During our second interview she explained her reasoning:

> With your thumbs up, agree. Disagree thumbs down. If you’re not sure, thumbs to the side. That keeps people from feeling bad about an incorrect answer. It shows me those who are quiet who are confused. And then those who get it correct, it validates at the same time. Yay, I got a thumbs up.

She used this strategy during many of the lessons I observed.

Mistakes

The teachers worked to create a safe environment where every student had a voice, even when they made mistakes. In the respect subsection I described how teachers expected other students interact with their classmates and how the teachers reacted. In this section I focused more on how the teachers interacted directly with a student who said something incorrect or incomplete.

Mistakes are okay. All three teachers ranked the statement “I understand that it is okay to make a mistake” highly but only two students did. Ms. Patrick noted a difference between her expectations and what actually happens, “They don’t like to be wrong, and I think it’s okay to be wrong.” David, who is outspoken in class and ranked the statement highly based on his experience, explained his mixed feelings:

> Like, sometimes like, in class it just feels weird. Like if you’re not any what, or any way, like … you don’t feel comfortable in the class. It’s like, you don’t want to make a mistake, so you don’t want to participate. But most of the time, like, I know that it’s okay to make a mistake, because …. I mean, even if somebody does say something or whatever, it doesn’t really matter. You just need to keep going, like, self-confidence. But sometimes it’s harder if the classmates like, uh, like say something or laugh. Ms. [Hartzell]’s real, like, on that. She doesn’t tolerate any of that.

When students would hesitate to answer instead stating that they think they are wrong, the teachers would let them know it is okay to make a mistake. For example, in response to a student stating “I don’t think it’s right though,” Ms. Hartzell said, “It’s okay to be wrong, I guarantee you.” When a student told Ms. Calloway, “I don’t want to get it wrong,” she explained, “I don’t care. I want to know what going on in your head.” Ms. Hartzell gave preemptive assurance that it okay to make a mistake when calling on a student to read, “Some of those words are tricky. Don’t be afraid to stumble over them.”

Learning experience. When a student gave an incorrect answer, regardless if the student volunteered or was called on, usually the teacher stayed with the student to find out what the student did and help the student correct their misconception. Ms. Calloway described her approach:

> In my class, I will say, okay let’s stop and think about your response. There is no, no, that’s not the correct answer. There’s no, that’s crazy. …. There is, no one call out the answer. Let’s go back and look at what you feel with this. That’s what you can expect in my class, because we already do have holes. And it’s not, okay well you got the wrong answer, let’s skip to someone else who may have the correct answer. No. It’s go back. Let’s give you some think time. Let’s go through the process. What did you do?

When asked why he thinks he learns more when he participates more, Juan explained, “When I go up there, I know if it’s going to be wrong or right and that will help me if I get it wrong and show me what I did wrong.”

All three teachers readily admitted to making mistakes and were okay with students pointing out mistakes. Ms. Patrick said she liked to make her mistakes a learning experience, so regardless if the students noticed first or she noticed first, she had the student explain to her why her work is wrong. For example, when students were solving a proportion problem based on a shadows and similar triangles, many of them used a scale factor to solve the proportion. Ms. Patrick wanted to show them they would get the same answer by cross-multiplying and dividing. Once she realized her mistake, she asked the class, “What did I mess up on?” A student noted, “You flipped it.” Ms. Patrick’s lesson for her students was to remember to use a key when setting up proportions and not to rush.

Silver lining. Ms. Patrick described how she responded when a student gave an incorrect answer during whole class discussions:

> I try to put a positive spin on it. I’m like, you know what, you’re really close, I said, but – then I try to ask the question in a different way so maybe that make them think in a different way, so they’re like, oh, okay yeah, then it would be 2. So I try to reword, or I try to find some silver lining in the answer that they did give me.

This was evident in the observations. For example, when Ms. Hartzell’s class was having a whole class discussion about vocabulary words, Ms. Hartzell reacts positively when a student provides an example instead of a definition:

> Teacher: What is discount in your own words honey?
> Student1: Take half off.
> Teacher: Beautiful example. Take half off. Somebody in a different way, say the same thing she did but don’t use a fraction? [Student2], how can we say take half off in a different way?

> Student2: I don't know.
> Teacher: Somebody else, what's half as a percent?
> Student3: 50%.
> Teacher: Take 50% off. There's one more way we can say it. We don't see in the stores very often, but just for giggles give it to me. What is it [Student4]?
> Student4: Take point five off.
> Teacher: Take five tenths off. We rarely see that one, but heck, I got a fraction, decimal, and percent out of you. Alright [Student2], back to the words, the definition. What is your definition of a discount?

This was not uncommon in the regular warm-ups either; if a student was called on and did not have the answer to the question she asked, sometimes they would give the answer to another problem they had successfully completed. Sometimes students had not completed a problem but described their process thus far. For example, a student set up a proportion and cross multiplied, to which Ms. Hartzell responded, "You my dear just set me up with a beautiful equation."

The teachers and students recognized that not all mistakes are the same. When asked if they ever heard their teacher say something that might be incorrect, David said she might make "a little mistake" and Cristina said Ms. Hartzell my say "the wrong thing, but then she means the right answer." David also said that if he and a classmate arrived at different answers, it is probable that one of them made a little mistake. The teachers would point out when students were "very close." For example, when a student shared a table he created with the class, Ms. Patrick discussed and agreed with his choice of increment and then asked the class to discuss the context the data towards the end of the table represented. In order to help the class correctly interpret the data she pointed out that the time was in the right column and stated "just for future reference, the independent variable is going to be on the left-hand side of the table every time."

Ms. Hartzell sometimes could find the silver lining in a completely incorrect process, such as when a student gave a detailed description of subtracting two fractions that should have been multiplied. By the end he somehow realized he was wrong:

> Student: I did it wrong.
> Teacher: You're okay. The fact that you were subtracting and finding common denominators gave us a beautiful review of fractions. So I'll never be mad for you throwing an idea out there.

Resources

As part of the safe environment where students were free to ask questions, make comments, and learn from their mistakes, teachers encouraged students to use resources available to them. An example where the wall clock was used as a resource was discussed on page 67. All of the mathematics classrooms had dictionaries, which were used when reviewing vocabulary. Students were also encouraged to use textbooks or dictionaries when they came across unfamiliar terminology during any lesson:

> Teacher: What's a conjecture?
> Student1: Uh, yeah.
> Teacher: Where can we find that information?
> Student2: In a dictionary.
> Teacher: Yeah, this may be a math class, but Ms. [Patrick] has dictionaries.

Rulers were available when needed to draw graphs or geometric shapes. The teachers would point out in a positive manner when a student would make use of tables in their school planner, a place value chart, or their STAAR formula chart, often during warm-ups. A corner of a piece paper was used in each classroom at some point to check to see if an angle was right. All the mathematics classrooms had a vocabulary wall, and the seventh grade students were allowed to consult the word wall while completing their unit tests. Calculators were used only occasionally, probably since they are not allowed to be used on the state standardized test. Students with accommodations were encouraged to use whatever additional resources they were allowed.

Effort

When asked what expectations her teacher had for how she does mathematics, the first thing Cristina mentioned was "We always have to like try our best." Ms. Hartzell gave a similar response, "As long as they are doing it, I'm a happy lady." Ms. Patrick explained it a little further:

> As long as they are trying. They're on task and they're trying, that's really all I can ask for, because at least they're giving me something to work with: something for me to praise, something for me to help them on, something for me to correct. Just as long as they are producing something, that's my expectation. As long as they are trying, you know, I can work with that. It's the kid that sit there and do absolutely nothing that I, that drives me batty, you know. Like at least write it down, you know; give me something.

Thus the teachers acknowledged when students put forth effort by listening to questions and comments and finding the silver lining in mistakes, as discussed in earlier subsections. They also gave students credit for trying; when they could not come up with anything else, effort was the last resort silver lining for an incorrect answer or an incomplete problem. Both Mark and Cristina noted that if students make a mistake, at least they tried. Typical responses from Ms. Hartzell are "It's a good try. What method did you use to get it?" or "I like that you're trying. You know that's all I care about."

When they try, struggle, and persist is when Ms. Hartzell sees students having light bulb moments:

> They happen at the strangest times, which are glorious when they do happen. It's the persistence thing. When a child is struggling so hard and doesn't get it, doesn't get it, but doesn't give up on me and I don't give up on them, and then they finally understand it.

Nathan also recognized that light bulb moments do not happen unless he works "really, really hard." Ms. Calloway declared in the first interview, "Failure is not an option in my class," and in the second interview she explained, "The first thing that they typically say is, 'I can't get this and I can't do this.' So, that's not an option in my class. I can't and I won't." Mark embraced this philosophy stating, "I don't give up. I can always complete them in a certain amount of time. I haven't been able to find one that I haven't been able to do."

Summary of Safe Environment and Effort

The teachers gave students time to think, allowed them to ask questions and make comments, found the silver lining in mistakes, encouraged students to use available resources, insisted students respect each other, and encouraged students to at least try and hopefully persist. These strategies helped create an environment where students were safe to take risks and be mathematically creative.

Mathematics Practices

While the teachers were helping make the mathematics personally meaningful for the students and provide them with a safe environment to express themselves and make mistakes, the teachers were focused on the mathematics. Mathematics content knowledge is the appropriateness in the definition of creativity in general, and was part of Lithner's (2008) model for Creative Mathematically Founded Reasoning (see page 12). The first subsection describes the role the teachers gave explanations, proof, and debate to engage students in mathematical discussions about their reasoning. The second section describes how the teachers and students used mathematical terminology and notation in the classroom. Finally, the third subsection discusses the teachers' and students' use of estimation and reasonableness.

Reasoning

David, Cristina, and Nathan all said that when the teacher asks the class a question her goal is the check for understanding. David explained, "To make, to let everybody understand it and not just get the right answer because someone else told them. For everybody else to understand the question more, understand the concept."

Explanations and proof. The most common way this was apparent in the classroom, other than the requirement to show their work, was that after students provided an answer during whole class discussions they usually would be asked to explain their process. Often the teacher would indicate whether the answer was correct or not prior to asking for an explanation but occasionally they would not.

This worked both ways; the teacher did not provide formulas for them to memorize without explanation. They frequently used inquiry to help students build on previous knowledge. While they did not usually prove the new concepts to be true, they did explore the concepts enough that the students believed them to be true. An example where Ms. Hartzell avoided forcing an idea on student was when her pre-algebra students were comparing representations for relationships with a constant rate of change. She asked the class "The number next to the letter, is it always going to be the constant rate of change?" The students seemed unconvinced this would always be true, so she concluded the lesson by saying, "A question to ponder for tomorrow."

Most of the time students simply explained what they did without justifying it explicitly based on definitions, etc. However, occasionally the explanations were a little more proof-like. For example Ms. Patrick discussed with her algebra on the Pythagorean Theorem and whether or not a certain leg is A and a certain leg is B:

> You don't need to, but I guess that would be a way to remember if you want to be consistent with every right triangle you use, then you can always use the height, the altitude, that can correspond with the a, the base can correspond with the b, but ultimately it doesn't matter, because addition is commutative. We can add in any order.

After this explanation, a student asked if they would get the same answer if they switched A and B, so Ms. Patrick illustrated with a specific example. Later that same period a student explained how he determined the measure of an angle in an equilateral triangle, "I knew there were 180 degrees in a triangle, and I knew there was three equal angles, so I divided by three." Instead of just saying he divided 180 by three, he explained why by referring to the properties of triangles.

Debate. Another way that students were able to explain their reasoning was through debates; these were fairly quick and informal. A method where students represented their different views while staying seated happened a few times, whereas students getting up and sorting into groups happened exactly once per teacher during my observations.

One way this was done was when the teacher asked the class how many students did something one way as opposed to another and then have a volunteer from each group explain their reasoning, though often one group could not explain their reasoning. For example in Ms. Hartzell's pre-algebra class:

> Question. I saw as I was walking around some of you connected your lines and some of you did not. Raise your hand if you connected. Raise your hand if you did not. Raise your hand if you can defend why you chose what you chose. [Student1], you chose not to. Why?
> Student1: I just forgot to. I would have connected it if I had remembered, because it is a constant rate
> Teacher: Nice explanation. Yes?

Student2: I agree
Teacher: Oh, you agree with her. Nice. I do too.

In this example, Student1 had displayed her work on the document camera actually supported the reasoning for the opposite of what she actually did.

Other times, the teacher would have the students get up and go to a specific part of the classroom and then have a volunteer from each group explain their reasoning. While completing a SIM graphic organizer with her students, Ms. Calloway's students could not agree if addition is always used when solving Pythagorean Theorem problems or sometimes used. The students sorted themselves into an always group and a sometimes group and each side explained their reasoning. They were allowed to switch sides if they were swayed by the other side's argument. In the end they decided to put it in the sometimes column.

Four corners is an AVID strategy that I saw used in Ms. Patrick's classroom. The students were given four statements to choose from that represented information about the same data: the ratio of the two numbers over ten thousand, the difference of the two numbers, the percentage one of the numbers is of the whole, and the ratio reduced to single digit numbers. They were asked which did the best job of convincing them that one product was better than another product. Eighteen students picked the ratio of large numbers, 6 picked the percentage, and 6 picked the ratio of small numbers. The teacher and students agreed that the difference did not give as much information as the other three options. No consensus was reached. Ms. Patrick stated that she did not believe there was a correct answer, but she considered the best answer to be the percentage.

Terminology and Notation

Although the students were encouraged to explain their process and vocabulary in their own words, they were also expected to use mathematically correct terminology. When a student used informal language instead of mathematics terminology, the teacher would paraphrase the student's response substituting appropriate terminology or ask the student for the term. While going over the vocabulary associated with the warm-up task, Ms. Calloway praised a student for using appropriate terminology:

> Teacher: Please tell me what does an improper fraction look like.
> Student1: Big number over a small number.
> Teacher: [Student2], thank you for raising your hand.
> Student2: It's like five-fourths.
> Teacher: Okay, that's an example. Very good.
> Student1: [inaudible]
> Teacher: [Student2] has his hand up.
> Student2: When the numerator is bigger than the denominator.
> Teacher: Yes, I love that terminology. It's where the numerator is greater than the denominator.

Student1 kept blurting out without raising his hand. Had he given his answer after being called on, Ms. Calloway likely would have given him credit for having the right idea and asked him for the terminology for top and bottom. She substituted greater for bigger when resvoicing Student2's answer.

The teachers all insisted students use precise language when stating numbers. Students usually knew to express the number 0.5 as "5 tenths" as opposed to "point 5." When students did express a decimal number using "point," the teachers usually had the student restate the number using place value. Ms. Hartzell credited the professional development Accessing English Language Learners in Mathematics Class with Dr. Alejandro Sorto of Texas State University with helping her and other teachers who attended to reflect on how often imprecise language or mnemonic devices, such as top dog in the house, were used in her classroom.

Correct notation was discussed in a few situations. During the warm-ups Ms. Hartzell and Ms. Patrick used with their classes, the students were expected to write an expression, usually with two or three operations, that could be used to solve a word problem. Sometimes it was discussed whether or not parentheses are needed. If they were used but not needed that would be discussed but accepted. Similarly, multiple representations of equations were discussed earlier in the Choice subsection, see pages 69-70. Although Ms. Patrick accepted both $1x$ and x from her students, they discussed that the 1 is not necessary and would not be used by mathematicians.

Reasonableness

One of the ways the teachers expected the students to check their work is by making sure their answer is reasonable. David explained how he uses estimation to check his answers:

> Most of the time you can tell if it's reasonable or not, because if you have numbers that are really small numbers and you get this giant number, you probably know that it's not going to be the correct answer. So I just redo it.

Since so much of their work is set in some sort of real world context, determining the reasonableness of their answer is often a combination of number sense and real world sense. For example, Ms. Calloway's class had to determine the hypotenuse of a right triangle that had legs 40 and 50 based on a football player running across and down the field and then the shortest distance back, when the students were determining the square root of 2500, Ms. Calloway asked them if 50 yards or 500 yards were reasonable answers.

Summary of Mathematics Practices

The teachers made sure the students had a solid mathematics foundation by having the students explain their reasoning (individually or in debate), use appropriate terminology and notation, and determine the reasonableness of their answers using estimation. The students knew that they when they were mathematically creative they had the justify their reasoning.

Summary

This chapter described how teachers foster students' mathematical creativity by allowing the students to make mathematics personally meaningful, providing a safe environment where all students are expected to participate and effort is acknowledged, and requiring students to have a solid mathematics base. The Personally Meaningful section described the opportunities students had for mini-c creativity. The Safe Environment and Effort section described more general expectations the teachers had that supported students expressing their personal meaning. Finally the Mathematics Practices section described how the teachers to discuss the mathematics content.

V. DISCUSSION

In the modern world of technology, creativity is increasing becoming a necessary skill for the work place (Department of Labor, 1991; Pink, 2005). The general public sees creativity as important (Sacconaghi, 2006). The literature review revealed creativity is part of mathematics curriculum standards (CCSS, 2011; NCTM, 2000; TEKS, 2009). However, teachers may consider creative students to be disruptive (Beghetto, 2007) and may not feel like it is their responsibility to foster creativity in their students, even if they support the idea that creativity should be fostered in students (Aljughaiman & Mowrer-Reynolds, 2005). The review of the literature in Chapter II revealed some ways that teachers tried to foster mathematical creativity and other pedagogical methods that may be appropriate to foster mathematical creativity. Many of the research studies explicitly about mathematical creativity in the classroom involved college classes, including ones with preservice teachers, and were performed internationally with different curriculum standards. Also, these did not seem to provide a complete picture on how teachers develop creativity at more basic level.

The purpose of this study was to describe middle grades students' opportunities to be mathematically creative in the classroom. The questions that guided this research are:

1. How do teachers make mathematics personally meaningful to foster students' mathematical creativity?

2. How do teachers create a safe environment to help students develop mathematical creativity?

3. How do teachers use mathematics practices to foster students' development of mathematical creativity?

Discussion of Findings

Personally Meaningful

The teachers in this study provided their students with opportunities for personally meaningful creativity on a daily basis. The students were allowed to make choices on how they learn mathematics, especially the method they used to solve problems. They were encouraged explain mathematical concepts and relationships in their own words and relate the concepts to their own experiences. This theme is directly linked to the definition of mini-c creativity. The opportunities they have to make their own choices allow the students to experience mathematics in a novel way. The opportunities they have to use their own words allow the students to express these personally meaningful interpretations. The opportunities they have for connections allow the student to use their own experiences, actions, and event to create meaning in mathematics.

Opportunity to make choices. Considering that Poincaré (1913) equated creativity with choice, it is not surprising that the research on mathematical creativity is focused here. Some of the studies were clearly at a much higher level than what was observed for this study: multiple methods of high school geometry proofs (Levav-Waynberg & Leiken, 2011), non-routine calculus problems (Liu & Niess, 2006), and college classes with projects (Kandemir & Gür, 2007; Munakata & Vaidya, 2012; Shriki, 2010).

A couple studies came closer to level described in this study. Bolden et al. (2010) claimed the examples the teachers in his study came up with for mathematical creativity tended to be creativity on the part of the teacher, however using a developmental definition of creativity could mean that the creativity on the part of the teachers is modeling creativity for the students and helping the students make the mathematics personally meaningful for the students. Levav-Zamir and Leiken's (2011) use of mathematical creativity did specify that the student creates ideas, exercises, or facts that are new to the student. An example they provided is where students create number sentences using numbers from the date; this type of problem would fit in with the higher-level examples from this study.

The teachers in this study encouraged students to use whatever method they prefer on a daily basis. Sometimes the teachers asked students to share multiple methods during whole class discussions and other times students volunteered an alternative method to the one discussed. Often the alternate methods were routine, such as solving proportions using scale factor versus cross-multiplying and dividing. However, occasionally students volunteered less-routine methods; while all methods received positive feedback from the teacher, the teachers did mention when they thought a method was a little more special.

They were also flexible about the form of the answer in certain cases. They expected fractions to be in simplest terms and improper fractions to be converted to mixed numbers (because it is expected on the state standardized test), however they would allow students to choose between decimal form and fraction form or allow answers in different units.

The teachers occasionally gave students opportunities to work problems where they may get different answers than their classmates. Having students pose problems was one way they accomplished this. Another way was to give the student a problem that had more than one correct answer. Finally, the students sometimes worked problems based on their own data. In this case, the problems they worked had one best answer, but the answers varied from student to student based on the initial data.

Finally, teachers used graphs and tables as an opportunity for students to make choices. The students could make choices about at least some of the parts of the graph, such as the title, labels, or scales.

Opportunity to use own words. Allowing students to describe mathematical concepts in their own words was barely mentioned in the mathematical creativity research (Uworwabayeho, 2009). This is not surprising since personally meaningful creativity is more about the process of creating new knowledge and making sense of mathematics for oneself, whereas most research is about little-c creativity where the focus is on creating a product for someone else, teacher and classmates. In a study not explicitly linked to creativity, a math teacher had students write in a math journal at least once a week (Baxter, Woodward, & Olsen, 2005). She was pleasantly surprised to learn that her special education students who did not participate in class discussions did express higher levels of mathematical understanding in the journals. This indicates that giving students a variety of ways to express themselves is necessary to service different learning styles. While I focused on whole class discussions for my analysis, the students were frequently expected to write about their understanding and had the chance to discuss their understanding in small groups. Hand gestures were also used to communicate understanding.

Opportunity for connections. Like the opportunity to use own words, the opportunity for connections is more apparent at the personally meaningful level of creativity thus is not as apparent in the research as choice. Although creativity is about connecting current ideas to create a new one, the other levels of creativity focus more on the product than the process, so the connections are not in the forefront. One curriculum that was explicitly linked to creativity was also explicitly linked to making connections (Sanz de Acedo Lizarraga, Sanz de Acedo Baquedano, & Olviver 2010).

The teachers in this study tried to make connections on a daily basis. The teachers tried to make connections to students' interests and experiences either by planning a discussion they think will pique their students' interest or allowing students to volunteer their interests and experiences. The teachers took advantage of common experiences the students had at school by referring to tasks done in earlier grades at that school or experiences that previously happened in that class. They linked the mathematics concepts being studies to previous mathematics concepts. They compared mathematics terminology to similar words as they are commonly used. Finally, the teachers would point out when the content overlapped with another subject.

Safe Environment and Effort

The teachers created safe environments for students to express their personally meaningful creativity in whole class discussions. The teachers provided students with think time, so the students had time to process and were not put on the spot. The teachers allowed the students to have a voice in the class where they were free to expression their personally meaningful creativity. The students were assured that mistakes are okay and encouraged to make use of various resources. The teachers focused on the students' effort rather than some measure of their ability.

Think time. The need for think time in order to be mathematically creative is part of the mathematical creativity process, the incubation step (Hadamard ,1945;

Sriraman, 2004). The mathematics classes met for 90 minutes daily. Students often had time to work in independently or groups before participating on whole class discussions. Even during whole class discussions the teachers often took time to help the student through the process to solve the problem or give the student a little extra time to work on the problem.

Voice. The mathematical creativity research tended to focus on product rather than process, but one study discussed how students made questions and comments on each other's work (Liu & Niess, 2006). The teachers in this study gave the students a voice in the classroom by welcoming questions and comments, but students did not usually address each other during whole class discussion. All the teachers had the expectation students would raise their hand and wait to be called on during whole class discussion, which prevents students from interrupting each other, however occasionally the teachers did allow small portions of a whole class discussion to happen without hand raising. However if students were all talking at once but on task, the teachers would thank the students for their enthusiasm but remind the them to raise their hands if they wanted to speak.

I only observed two instances of students being rude to their classmates other than interrupting them, but the "shut up" and "duh" comments were quickly admonished by the teachers and the students were warned they were on the verge of being written up. However, I saw several instances where a few classmates applauded a student's discussion of their solution method or gave a positive comment like "good job."

When a few students dominate a discussion, the other students can feel like their voice does not matter and stop trying to participate. These teachers made it clear that every student was expected to participate in class discussions; if only a few students volunteered answers, other students would be called on to participate.

Mistakes. Risking being wrong is part of the learning process and the mathematical creativity process (Ervynck, 1991), and a study noted that students the who did not do well on a creative project were afraid of not being able to do the project correctly (Shriki, 2010). The teachers have a variety of ways to encourage students to risk making a mistake, such as outright telling students that it is okay to make a mistake. They turned mistakes into learning experiences, including when students caught the teacher making a mistake. Finally, the teachers looked for the silver lining in students' incorrect or incomplete answers, such as acknowledging when the solution or solution method is partially correct.

Resources. As part of the safe environment, teachers encouraged students to use resources available to them. The teachers would make a positive comment when a student would take the initiative to use a resource, such as a formula chart.

Effort. The teachers told the students that trying is what is important and offered encouragement to students who were hesitant to participate in whole class discussions. They seemed to lean more towards mastery goal orientation than performance goal orientation, which has been linked to creativity (Hong, Harzell, & Greene, 2009). The teachers never referred to one student or one class as being more able than another, but instead focused on how hard they were trying.

Mathematics Practices

In order to be mathematically creative, students need to understand mathematics, not just be able to do mathematics. A couple studies described how students were helped to see mathematics as creative by focusing on the abstract mathematics rather than computations (Liu & Niess, 2006; Ward et al. 2010)**.**

Reasoning. On a daily basis the teachers in this study expected students to explain their reasoning. Most often students explained the process they used to solve a problem, though occasionally they referenced a definition or property. The lack of proof is consistent with Stylianides's (2009) analysis of the opportunity to prove in the Connected Mathematics Program curriculum. She noted few opportunities for proof and that 90% of the time the teacher's manual did not provide guidance on how to teach proofs. One of the teachers told me she relied on the teacher's guide her first year teaching. Students compared and evaluated their classmates reasoning through debates. These were done either by a show of hands or moving into 2 or 4 sections of the classroom depending on the student's reasoning. Then a student from each group would explain their reasoning. Either a decision would be reached or there would be a discussion on why more than one answer is correct.

Terminology and notation. Although students were encouraged to explain ideas in their own words, they were also expected to use correct mathematical terminology daily. Using correct terminology helps students communicate their reasoning thus making their claim appropriate, which is necessary for creativity. Students were expected to use correct notation. Their novice ways of writing expressions were accepted, but they also discussed the fact that parentheses were not necessary or a coefficient of one does not need to be written.

Reasonableness. One way students can create personal meaning is by understanding when their solutions are reasonable. This was rarely mentioned in class though it was mentioned as an expectation for checking answer in the student and teacher interviews. Since much of the content was covered using context, reasonableness often could be determined using the context with number sense being less important.

Tensions and Challenges

I experienced some challenges with this study. I began the observations with the intent of describing specific tasks that allowed students to be mathematically creative. As I did my observations it became clear that at least in this case, certain characteristics of the assignments and the expectations of the teachers on how students do mathematics would better describe how the teachers fostered mathematical creativity. Had I realized this from the beginning I probably would have tried to get parental consent to video record at least some classes or at least taken slightly different field notes. However, this may have been a blessing in disguise, since the transcripts of the audio alone provided lots of data.

I genuinely liked the teachers who volunteered to participate in this study and felt they were trying their best to provide their students with positive, productive, and meaningful learning experiences in mathematics. However, I was distracted by some of the mathematical statements they made that were incorrect or misleading. It was often unclear if these statements were the teachers' attempt to scaffold the material or due to conceptual misunderstanding of the teachers. Both while making observations and when analyzing transcripts I redirected myself to focus on the positive aspects that I wanted to describe.

Implications and Recommendations

Implications

By focusing on personally meaningful creativity, mathematical creativity becomes attainable for all mainstreamed students. The descriptions provided in this study may help provide a broader, developmental conception of mathematical creativity in the classroom. Since personally meaningful creativity is more about the process than the product, teachers who want to foster creativity to students with little to no experience may start by changing their expectations on how students do mathematics in their classroom rather than focusing on specific tasks.

School districts could use this study to aid in developing professional development for mathematical creativity. By helping teachers have a broader understanding of creativity, they may be more willing to take on the responsibility of fostering mathematical creativity in their classrooms.

Recommendations for Future Research

This study describes the how three teachers fostered their students' mathematical creativity, especially at the personally meaningful level. This is likely an incomplete picture, so a natural extension would be do more observations to provide descriptions of missing components.

The expectations of the teachers in this study for how students did mathematics were consistent with the definition of mini-c creativity; however the students did not do projects. I think it would be interesting to compare students in a direct teach setting that occasionally do projects where creativity is an expectation to students are expected to use mini-c creativity on a daily basis but do not do projects. This could be a way to investigate the developmental nature of mathematical creativity.

Finally, willingness to make mistakes is part of the creativity process. In this study the two students who strongly supported the statement that it is okay to make mistakes both expressed discomfort in the interview with being wrong, but their actions were very different. One student volunteered answers on a daily basis and the other student never voluntarily participated in class discussion because she was afraid of making. Further research is needed to study how students can transition from understanding that mistakes are part of learning and creativity in theory to internalizing this belief.

APPENDIX SECTION

APPENDIX A

Sorting Task Statements

1. I work together with my classmates.
2. I draw a picture or figure to help me solve a problem.
3. There is only one right way to work a problem.
4. I keep trying as long as it takes to understand a problem.
5. I make a choice about how I learn.
6. I solve a problem that has more than one correct answer.
7. Getting the correct answer is the most important part of working a problem.
8. I work on a problem that uses several concepts we learned.
9. I do a task related to my interests.
10. I connect a new concept to one I already understand.
11. I do not ask questions or share ideas I have related to the topic.
12. I make a joke in class.
13. I work on a project.
14. I am willing to try something new, even if I am not sure if it will work.
15. A good memory is more important in math class than other classes.
16. I explain a math concept in my own words.
17. I use a variety of materials.
18. I estimate the answer to a problem.
19. I create a problem or an idea.
20. I contribute to the class discussion or present a problem on the board voluntarily.
21. I learn how to solve a problem using a trick, without understanding the math.
22. I feel that it is okay to make a mistake.
23. I do a hands-on activity.
24. I try to figure out how to solve a problem without the help of the teacher.
25. I have time to think.
26. The more I participate in class, the more I learn.
27. I think about whether my idea makes sense.
28. I make predictions about what will happen if I change part of a problem.
29. I think about the STAAR test or it is mentioned in class.

APPENDIX B

Student Interview Questions

- Did you attend Goodnight last year?
- What differences or similarities have you noticed between your math class last school year and this school year?
- What are norms in your math class? What are the behavior expectations? What are the expectations on how you do math?
- Think of a time when you were struggling to complete a math problem. What did you do?
- How long do you work on a math problem before giving up?
- Does every math problem have exactly one correct answer? If not, can you think of an example?
- How do you know if your answer to a math problem is correct?
- Have you ever given an incorrect answer out loud in class? What happened?
- If the person sitting next to you gets a different answer than you to a problem, how do you decide whose answer is correct?
- If the person sitting next to you does a problem a different way than you, how do you decide which way is the correct way?
- Can you think of a time when you used something from your life to understand a math concept?
- Can you think of a time when you had a light bulb moment? What caused you to suddenly understand the concept or problem?
- What do you think your teacher's goal is when she asks the class a question?
- Which help you remember math ideas better: a textbook definition or defining it in your own words?
- Have you ever heard your teacher say something that you thought might be incorrect? What did you do? textbook
- Have you ever seen an error in your textbook? What did you do?
- Is there anything else that's important to you about your math class that you would like to talk about?

APPENDIX C

Teacher Interview Questions

How did you decide on the set-up of your classroom, especially how the student desks are arranged?
- How do you choose group size?
- How do you decide how to group students?

What are the norms or expectations in your classroom?
- How are students expected to behave as individuals?
- How are students expected to interact with their classmates?
- How are students expected to interact with you?
- How are students expected to do math?
- Ms. Hartzell: Please explain the taking seconds and free time system.

What are the advantages of students working in groups?
What is your reaction to students who opt out of working with their peers?

My understanding is that writing and the SIM graphic organizers are school-wide expectations. How do these support students' learning of mathematics?
Are there other school-wide expectations that directly influence your classroom?

For each teacher I observed a debate. Did you receive professional development on this?

One aspect of your classroom I am not familiar with is assessment. Can you tell me a little about what goes into calculating a student's grade for your class?
- Participation, daily grades, quizzes, and tests
- Partial credit

Connected Math
- Do the sixth grade student use Connected Math?
- Did you receive professional development on teaching Connected Math?
- Do you think Connected Math has influence teaching style?
- Ms. Hooper: Do you think your teaching style would have developed differently if you had been only a mainstream mathematics instructor?

Additional comments?

APPENDIX D

IRB approval number: CON2013W7530

CONSENT FORM
Opportunity to Be Mathematically Creative

You are being asked to allow your child to participate in a research study conducted by Michelle Schrauth (ms35449@txstate.edu, 210-262-6721) as part of her doctoral dissertation at Texas State University. Your child's participation in this study is voluntary. You or your child may decide to withdraw from the study at any time without losing standing with the child's teacher, the child's school, or Texas State University. If your child participates in the study, your child may choose to not answer one or more questions at any time for any reason. Please contact me if you later wish to withdraw your child's participation or to receive summary results after the study in completed.

The purpose of the study is to determine middle school students' opportunity to be mathematically creative. Your child can provide valuable information about what happens in math class.

If you allow your child to be in the study, we will ask your child to do the following things:

- Allow the researcher to view their math notebooks or other work.
- Participate in a 30-minute interview. Your child will be asked to complete a math problem and explain their thinking. They will also be asked to discuss their math class. The interview will be audio-recorded.

Confidentiality and Privacy Protections:

- All recording will be coded with no personally identifying information on them.
- The recordings will be kept securely in a file cabinet in Dr. Sorto's locked office at Texas State University.
- For possible future research, the researcher will retain the recordings for 10 years before destroying the recordings.
- The data from this study may be made available to other researchers in the future for research purposes not detailed within this consent form. In these cases, the data will contain no identifying information for your child.

Risks, Benefits, and Compensation:
This study explores what happens in the mathematics class on a daily basis, and data will remain confidential. Thus, the risk in being in the study is minimal: no more than attending class. Your child may benefit from this study by being able to share his or her view on what happens in the math class. Your child will receive no compensation for participating in this study.

Contacts:
Please contact me, see first paragraph, with any questions. Additionally, pertinent questions about the research, research participants' rights, and/or research-related injuries to participants should be directed to the IRB chair, Dr. Jon Lasser (512-245-3413 – lasser@txstate.edu), or to Ms. Becky Northcut, Compliance Specialist (512-245-2102).

Consent to Participate in Study:

Your signature below indicates that you have read the information provided in the consent form and have decided to allow your child to participate in the study. If you later decide that you wish to withdraw your permission for your child to participate in the study, simply tell me. You may end your child's participation at any time.

____
Printed Name of Child

____ __________________
Signature of Parent or Legal Guardian Date

____ __________________
Signature of Researcher Date

ASSENT FORM
Opportunity to Be Mathematically Creative

I agree to be in a study about opportunities students have to be mathematically creative. This study was explained to my parent or guardian and they said that I could be in it. The only people who will know about what I say and do in the study will be the people in charge of the study.

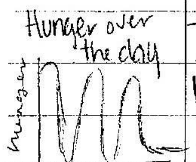
In the study I will be asked questions about what happens in my math class. I will allow the researcher to look at my math notebook. In an interview, I will also be asked to solve a problem, explain my thinking, and discuss creativity in my math class. If the researcher writes a paper based about this study, she will use a fake name for me. The recording will be stored in Dr. Sorto's office at Texas State University. They will be stored in a locked cabinet for 10 years. Then the recordings will be destroyed.

Writing my name on this page means that the page was read (by me/to me) and that I agree to be in the study. I know what will happen to me. If I decide to quit the study, all I have to do is tell the person in charge. I will not get in trouble with my teacher or parent if I do not want to participate in the study or quit being in the study.

Child's Signature	Date
Signature of Researcher	Date

APPENDIX E

These two writing samples describe the students' level of hunger throughout the day, which was used to create a graph.



Hunger Throughout the Day [redacted]

It was a usual school day morning. I woke up at approximately 6am, and boy, was I hungry! I went down to the kitchen, dressed and ready to go. I grabbed a bowl, milk, and cereal. I served myself, got a spoon, began to dig in. After I finished eating, I felt full and content. I threw my jacket on, grabbed my backpack, and left to the bus stop.

Around 12:00 pm, I started getting really hungry. I could not wait for lunch! We lined up and left. I sat at a booth with my friends, the six of us divided evenly along both sides. I pulled out my lunch from my lunch box, a sub sandwich, apple juice, and skittles. I split the skittles with my friends, and ate. I felt full and very happy afterward.

Around six p.m, I was home, and hungry again. My dad made dinner, and I chowed down until I felt like I was going to burst with food!

I had gone to bed, but woke up at midnight. Surprisingly, I wasn't hungry! I went back to sleep, ready to start the next day, hungry yet again.

Reflection on Graphs

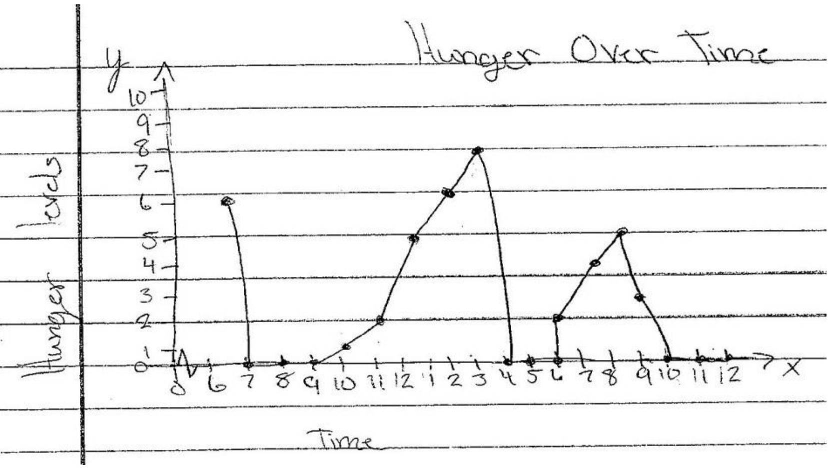
When I first wake up which is at 6 A.M., I get up wash my face and make eggs for me and my little brother. After that we get ready for school.

The next time I get hungry is around 12 pm which is lunch time, but I don't really like the food at school so I wait till I get home. Some days I'll eat the school lunches but other times I won't. After a while if I haven't eaten I lose my appetight.

Another time I get hungry is when I get home from school Sometimes there's nothing to eat at my house so I have to wait till my parents get home. Which is around six or seven. Either they pick up food or they just make food.

After eating I usually do homework then go to sleep. Well thats my cycle of eating food.

Over →



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