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EXISTENCE OF NON-OSCILLATORY SOLUTIONS TO HIGHER-ORDER MIXED DIFFERENCE EQUATIONS

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Abstract. In this paper, we consider the higher order neutral nonlinear difference equation

$$\Delta^m(x(n)+p(n)x(\tau(n)))+f_1(n,x(\sigma_1(n)))-f_2(n,x(\sigma_2(n)))=0,$$
$$\Delta^m(x(n)+p(n)x(\tau(n)))+f_1(n,x(\sigma_1(n)))-f_2(n,x(\sigma_2(n)))=g(n),$$
$$\Delta^m(x(n)+p(n)x(\tau(n)))+\sum_{i=1}^{l}b_i(n)x(\sigma_i(n))=0.$$

We obtain sufficient conditions for the existence of non-oscillatory solutions.

1. Introduction

Consider the difference equations

$$\Delta^m(x(n)+p(n)x(\tau(n)))+f_1(n,x(\sigma_1(n)))-f_2(n,x(\sigma_2(n)))=0, \tag{1.1}$$
$$\Delta^m(x(n)+p(n)x(\tau(n)))+f_1(n,x(\sigma_1(n)))-f_2(n,x(\sigma_2(n)))=g(n), \tag{1.2}$$
$$\Delta^m(x(n)+p(n)x(\tau(n)))+\sum_{i=1}^{l}b_i(n)x(\sigma_i(n))=0, \tag{1.3}$$

for $n\ge n_0$, where $\tau(n),\sigma_i(n)$ are sequences of positive integers with $\tau(n)\le n$, $\lim_{n\to\infty}\tau(n)=\infty$, $\lim_{n\to\infty}\sigma_i(n)=\infty$, $i=1,2,\dots,l$. Also where $p(n),g(n),b_j(n)$, $j=1,2,\dots,l$ are sequences of real numbers, $f_i(n,x)$, $i=1,2$ are continuous and nondecreasing for x, $f_1(n,x)f_2(n,x)>0$. There exists $b\neq 0$ such that

$$\sum_{s=n}^{\infty}(s-n)^{(m-1)}|f_i(s,b)|<\infty,\quad i=1,2, \tag{1.4}$$
$$\sum_{s=n}^{\infty}(s-n)^{(m-1)}|g(s)|<\infty, \tag{1.5}$$
$$\sum_{s=n}^{\infty}(s-n)^{(m-1)}|b_j(s)|<\infty. \tag{1.6}$$

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Recently, there has been an increasing interest in the study of existence and oscillation of solutions to differential and difference equations. The papers [2, 5, 8, 9] discussed the existence of non-oscillatory solutions of differential equations. The papers [6, 7] discussed the oscillation of difference equations. But there are relatively few which guarantee the existence of non-oscillatory solutions of difference equations, see [3, 4].

This paper is motivated by the recent paper [10], where the authors gave sufficient conditions for the existence of non-oscillatory solutions of some first-order neutral delay differential equations. The purpose of this paper is to present some new criteria for the existence of non-oscillatory solution of (1.1)-(1.3).

A solution of (1.1) ((1.2) (1.3)) is said to be oscillatory if it has arbitrarily large zeros; otherwise it is said to be non-oscillatory.

2. Main Results

To obtain our main results, we need the following lemma.

Lemma 2.1 ([1]). *Let K be a closed bounded and convex subset of l^∞, the Banach space consisting of all bounded real sequences. Suppose Γ is a continuous map such that $\Gamma(K) \subset K$, and suppose further that $\Gamma(K)$ is uniformly Cauchy. Then Γ has a fixed point in K.*

In the sequel, without loss of generality, we assume that $f_i(n,x) > 0$, $i = 1,2$ and (1.4) holds for $b > 0$.

Theorem 2.2. *Assume that $0 \le p(n) \le p < 1$, (1.4) holds, then (1.1) has a bounded non-oscillatory solution which is bounded away from zero.*

Proof. Choose $N > n_0$, such that

$$N_0 := \min\{\inf_{n\ge N}\{\tau(n)\}, \inf_{n\ge N}\{\sigma_1(n)\}, \inf_{n\ge N}\{\sigma_2(n)\}\} \ge n_0.$$

Let BC be the collection of bounded real sequence in Banach space l^∞ and $\|x(n)\| = \sup_{n\ge N}|x(n)|$. Define a set $\Omega \subset BC$ as follows:

$$\Omega = \{x(n) \in BC,\ 0 < M_1 \le x(n) \le M_2 < b, n \ge n_0\},$$

where $M_1 < (1-p)M_2$. Then Ω is a closed bounded and convex subset of BC. Set $c = \min\{M_2 - \alpha, \alpha - pM_2 - M_1\}$, where $pM_2 + M_1 < \alpha < M_2$. From (1.4), we get that there exists $N_1 > N$, such that for $n > N_1$,

$$\sum_{s=n}^{\infty} \frac{(s-n+1)^{(m-1)}}{(m-1)!} f_i(s,b) \le c, \quad i = 1,2.$$

Define two maps Γ_1 and Γ_2 on Ω as follows:

$$(\Gamma_1 x)(n) = \begin{cases} \alpha - p(n)x(\tau(n)), & n \ge N_1, \\ (\Gamma_1 x)(N_1), & N_0 \le n \le N_1 \end{cases}$$

$$(\Gamma_2 x)(n) = \begin{cases} \frac{(-1)^{m-1}}{(m-1)!}\sum_{s=n}^{\infty}(s-n+1)^{(m-1)} & \\ \times[f_1(s,x(\sigma_1(s))) - f_2(s,x(\sigma_1(s)))], & n \ge N_1 \\ (\Gamma_2 x)(N_1), & N_0 \le n \le N_1 \end{cases}$$

For any $x, y \in \Omega$, we have

$$(\Gamma_1 x)(n) + (\Gamma_2 y)(n) \le \alpha + c \le M_2,$$
$$(\Gamma_1 x)(n) + (\Gamma_2 y)(n) \ge \alpha - pM_2 - c \ge M_1.$$

That is $\Gamma_1 x + \Gamma_2 y \in \Omega$. Since $0 \le p(n) \le p < 1$, it is easy to check that Γ_1 is a contraction mapping.

Now we show that Γ_2 is continuous. For any $\varepsilon > 0$, we can choose $n_2 > N_1$, such that

$$\sum_{s=n_2}^{\infty} \frac{(s-n_0+1)^{(m-1)}}{(m-1)!} f_i(s,b) < \varepsilon, \quad i = 1,2.$$

Let $\{x_k(n)\}$ be a sequence in Ω, such that $\lim_{k\to\infty} \|x_k - x\| = 0$. Since Ω is a closed set, we get that $x \in \Omega$ and

$$\begin{aligned}
&|(\Gamma_2 x_k)(n) - (\Gamma_2 x)(n)| \\
&\le \Big| \sum_{s=n}^{n_2-1} \frac{(s-n+1)^{(m-1)}}{(m-1)!} (f_1(s, x_k(\sigma_1(s))) - f_1(s, x(\sigma_1(s))))\Big| \\
&\quad + \Big| \sum_{s=n}^{n_2-1} \frac{(s-n+1)^{(m-1)}}{(m-1)!} (f_2(s, x_k(\sigma_2(s))) - f_2(s, x(\sigma_2(s))))\Big| + 4\varepsilon.
\end{aligned}$$

Since f_i is continuous for x, we get that $\lim_{k\to\infty} \|\Gamma_2 x_k - \Gamma_2 x\| = 0$. We also know that Γ_2 is uniformly bounded and for for all $\varepsilon > 0$, there exists N_2 such that for $m_1 > m_2 \ge N_2$ and for all $x(n) \in \Omega$,

$$\begin{aligned}
&|\Gamma_2 x(m_1) - \Gamma_2 x(m_2)| \\
&\le \sum_{s=m_2}^{m_1-1} \frac{(s-n_0+1)^{(m-1)}}{(m-1)!} |f_1(s, x(\sigma_1(s))) - f_2(s, x(\sigma_2(s)))| \le \varepsilon.
\end{aligned}$$

From the discrete Krasnoselskii's fixed point theorem, there exists $x \in \Omega$, such that $x = \Gamma x$, i.e.

$$\begin{aligned}
x(n) &= \alpha - p(n)x(\tau(n)) \\
&\quad + (-1)^{m-1} \sum_{s=n}^{\infty} \frac{(s-n+1)^{(m-1)}}{(m-1)!} \Big(f_1(s, x(\sigma_1(s))) - f_2(s, x(\sigma_2(s)))\Big).
\end{aligned}$$

Note that $x(n)$ is a bounded non-oscillatory solution of (1.1) which is bounded away from zero. $\square$

Theorem 2.3. *Assume that $1 < p_1 \le p(n) \le p_2$, (1.4) holds, $\tau(n)$ is strictly increasing, then (1.1) has a bounded non-oscillatory solution which is bounded away from zero.*

Proof. We choose $N_1 > n_0$, such that

$$N_0 = \min\{\tau(N_1), \inf_{n\ge N_1}\{\sigma_1(n)\}, \inf_{n\ge N_1}\{\sigma_2(n)\}\} \ge n_0.$$

Let BC be the collection of bounded real sequences in the Banach space l^∞ and $\|x(n)\| = \sup_{n\ge N_1} |x(n)|$. Define a set $X \subset BC$ as follows:

$$\begin{aligned}
X = \big\{ &x(n) \in BC : \Delta x(n) \le 0, 0 < M_1 \le x(n) \le p_1 M_1 < b \text{ for } n \ge N_1 \\
&x_{(n)} = x_{(N_1)} \text{ for } N_0 \le n \le N_1 \big\}
\end{aligned}$$

Then X is a closed bounded and convex subset of BC.

Let $c = \min\{\alpha - M_1, p_1M_1 - \alpha\}$, where $M_1 < \alpha < p_1M_1$. We choose $N \geq N_1$, such that for $n \geq N$,

$$\sum_{s=n}^{\infty} \frac{(s-n+1)^{(m-1)}}{(m-1)!} f_i(s,b) \leq c.$$

For $x \in X$, define

$$\psi(n) = \begin{cases} \sum_{i=1}^{\infty} \frac{(-1)^{i-1}x(\tau^{-i}(n))}{H_i(\tau^{-i}(n))}, & n \geq N \\ \psi(N), & N_0 \leq n \leq N \end{cases}$$

where $\tau^0(n) = n$, $\tau^i(n) = \tau(\tau^{i-1}(n))$, $\tau^{-i}(n) = \tau^{-1}(\tau^{-(i-1)}(n))$, $H_0(n) = 1$, $H_i(n) = \prod_{j=0}^{i-1} p(\tau^j(n))$, $i = 1, 2, \dots$. From $M_1 \leq x(n) \leq p_1M_1$, we know $0 < \psi(n) \leq p_1M_1$, $n \geq N$.

Define a mapping Γ on X as follows

$$\Gamma x(n) = \begin{cases} \alpha + (-1)^{m-1} \sum_{s=n}^{\infty} \frac{(s-n+1)^{(m-1)}}{(m-1)!} \\ \times[f_1(s,\psi(\sigma_1(s))) - f_2(s,\psi(\sigma_2(s)))], & n \geq N \\ \Gamma x(N), & N_0 \leq n \leq N \end{cases}$$

Note that Γ satisfies the following three conditions:

(a) $\Gamma(X) \subseteq X$. In fact, for any $x \in X$, $\Gamma x(n) \geq \alpha - c \geq M_1$, $\Gamma x(n) \leq \alpha + c \leq p_1M_1$.

(b) Γ is continuous. Let $\{x_k(n)\}$ be a sequence in X, such that $\lim_{k\to\infty} \|x_k - x\| = 0$. Since X is a closed set, we know $x \in X$. For any $\varepsilon > 0$, we can choose $n_2 > N$, such that

$$\sum_{s=n_2}^{\infty} \frac{(s-n_0+1)^{(m-1)}}{(m-1)!} f_i(s,b) < \varepsilon, \quad i = 1, 2.$$

$$\begin{aligned} &|\Gamma x_k(n) - \Gamma x(n)| \\ &\leq \sum_{s=n}^{n_2-1} \frac{(s-n+1)^{(m-1)}}{(m-1)!} \sum_{i=1}^{2} |f_i(s,\psi_k(\sigma_i(s))) - f_i(s,\psi(\sigma_i(s)))| + 4\varepsilon. \end{aligned}$$

So $\lim_{k\to\infty} \|\Gamma x_k - \Gamma x\| = 0$.

(c) ΓX is uniformly Cauchy. For all $\varepsilon > 0$, there exists n_3 such that for $m_1 > m_2 \geq n_3$ and for all $x(n) \in X$,

$$\begin{aligned} &|\Gamma x(m_1) - \Gamma x(m_2)| \\ &\leq \sum_{s=m_2}^{m_1-1} \frac{(s-n_0+1)^{(m-1)}}{(m-1)!} |f_1(s,\psi(\sigma_1(s))) - f_2(s,\psi(\sigma_2(s)))| \leq \varepsilon. \end{aligned}$$

This shows that ΓX is uniformly Cauchy.

From Lemma 2.1, there exists $x \in X$, such that $x = \Gamma x$, i.e.

$$x(n) = \alpha + (-1)^{m-1} \sum_{s=n}^{\infty} \frac{(s-n+1)^{(m-1)}}{(m-1)!} [f_1(s,\psi(\sigma_1(s))) - f_2(s,\psi(\sigma_2(s)))],$$

for $n \geq N$. Since $\psi(n)+p(n)\psi(\tau(n))=x(n)$, we obtain

$$\begin{aligned}&\psi(n)+p(n)\psi(\tau(n))\\&=\alpha+(-1)^{m-1}\sum_{s=n}^{\infty}\frac{(s-n+1)^{(m-1)}}{(m-1)!}[f_1(s,\psi(\sigma_1(s)))-f_2(s,\psi(\sigma_2(s)))].\end{aligned}$$

So $\psi(n)$ satisfies (1.1) for $n \geq N$, and $\frac{p_1-1}{p_1p_2}x(\tau^{-1}(n)) \leq \psi(n) \leq x(n)$. □

Theorem 2.4. *Assume that $-1<p\leq p(n)\leq 0$, and (1.4) holds. Then (1.1) has a bounded non-oscillatory solution which is bounded away from zero.*

Proof. Let BC be the set of bounded real sequence in the Banach space l^∞ and $\|x(n)\|=\sup_{n\geq n_0}|x(n)|$. We choose M_1,M_2,α such that $0<M_1<\alpha<(1+p)M_2$. Define $\Omega=\{x\in BC, M_1\leq x(n)\leq M_2,\ n\geq n_0\}$. Let $c=\min\{\alpha-M_1, M_2-\alpha\}$, from (1.4) we get that there exists N such that for $n\geq N$,

$$\frac{1}{(m-1)!}\sum_{s=n}^{\infty}(s-n_0+1)^{(m-1)}f_i(s,b)\leq c,\quad i=1,2.$$

For $x\in\Omega$, define:

$$\varphi(n)=\begin{cases}\sum_{i=0}^{k_n-1}(-1)^i p_n^{(i)}x(\tau_n^{(i)})+(-1)^{k_n}p_n^{(k_n)}\frac{x_N}{1+p_N}, & n\geq N\\ \frac{x_N}{1+p_N}, & n_0\leq n\leq N\end{cases}$$

where we take k_n such that $n_0\leq\tau_n^{(k_n)}\leq N$, $\tau_n^{(0)}=n$, $\tau_n^{(1)}=\tau_n$, $\tau_n^{(2)}=\tau_{\tau_n},\dots,\tau_n^{(k)}=\tau_{\tau_n^{(k-1)}}$, $p_n^{(0)}=1$, $p_n^{(1)}=p_n,\dots,p_n^{(s)}=p_np_{\tau_n}\dots p_{\tau_n^{(s-1)}}$. It is easy to prove that $x(n)=\varphi(n)+p(n)\varphi(\tau(n))$, $n\geq N$ and $M_1\leq x(n)\leq\varphi(n)\leq\frac{M_2}{1+p}$. Define a mapping Γ on Ω as follows:

$$\Gamma x(n)=\begin{cases}\alpha+\sum_{s=n}^{\infty}\frac{(-1)^{m-1}(s-n+1)^{(m-1)}}{(m-1)!}\\ \times[f_1(s,x(\sigma_1(s)))-f_2(s,x(\sigma_2(s)))], & n\geq N\\ \Gamma x(N), & N_0\leq n\leq N\end{cases}$$

For any $x\in\Omega$, $M_1\leq\alpha-c\leq\Gamma x(n)\leq\alpha+c\leq M_2$, we get $\Gamma\Omega\subseteq\Omega$. Similar to the proof of Theorem 2.2, we can obtain Γ is continuous and uniformly Cauchy. So there exists $x\in\Omega$ such that $x=\Gamma x$. The proof is complete. □

Theorem 2.5. *Assume that $p_1\leq p(n)\leq p_2<-1$, and (1.4) holds. Then (1.1) has a bounded non-oscillatory solution which is bounded away from zero.*

Proof. We choose positive constants M_1,M_2,α such that $-p_1M_1<\alpha<(-p_2-1)M_2$. Let $\Omega=\{x\in BC, M_1\leq x(n)\leq M_2,\ n\geq n_0\}$, $c=\min\{\frac{(\alpha+M_1p_1)p_2}{p_1},(-p_2-1)M_2-\alpha\}$. Choosing N sufficiently large such that for $n\geq N$,

$$\frac{1}{(m-1)!}\sum_{s=n}^{\infty}(s-n+1)^{(m-1)}f_i(s,b)\leq c,\quad i=1,2.$$

Define two maps Γ_1, Γ_2 on Ω as follows:

$$\Gamma_1 x(n) = \begin{cases} -\frac{\alpha}{p(\tau^{-1}(n))} - \frac{x(\tau^{-1}(n))}{p(\tau^{-1}(n))}, & n \geq N \\ \Gamma_1 x(N), & n_0 \leq n \leq N \end{cases}$$

$$\Gamma_2 x(n) = \begin{cases} \sum_{s=\tau^{-1}(n)}^{\infty} \frac{(-1)^{m-1}(s-\tau^{-1}(n)+1)^{(m-1)}}{(m-1)!p(\tau^{-1}(n))} \\ \times [f_1(s, x(\sigma_1(s))) - f_2(s, x(\sigma_2(s)))], & n \geq N \\ \Gamma_2 x(N), & N_0 \leq n \leq N \end{cases}$$

For each $x, y \in \Omega$,

$$\Gamma_1 x(n) + \Gamma_2 y(n) \geq \frac{-\alpha}{p_1} + \frac{c}{p_2} \geq M_1, \quad \Gamma_1 x(n) + \Gamma_2 y(n) \leq \frac{-\alpha}{p_2} - \frac{M_2}{p_2} - \frac{c}{p_2} \leq M_2.$$

So that $\Gamma_1 x(n) + \Gamma_2 y(n) \in \Omega$. Since $p_1 \leq p(n) \leq p_2 \leq -1$, we get Γ_1 is a contraction mapping. We also can prove that Γ_2 is uniformly bounded and continuous. Further we know Γ_2 is uniformly Cauchy. So by discrete Krasnoselskii's fixed point theorem, there exists $x \in \Omega$ such that $\Gamma_1 x + \Gamma_2 x = x$. i.e.

$$\begin{aligned} x(n) = &-\frac{\alpha}{p(\tau^{-1}(n))} - \frac{x(\tau^{-1}(n))}{p(\tau^{-1}(n))} + \frac{(-1)^{m-1}}{(m-1)!p(\tau^{-1}(n))} \\ &\times \sum_{s=\tau^{-1}(n)}^{\infty} (s - \tau^{-1}(n) + 1)^{(m-1)} [f_1(s, x(\sigma_1(s))) - f_2(s, x(\sigma_2(s)))]. \end{aligned}$$

The proof is complete. □

Theorem 2.6. *Assume that $p(n)$ satisfies the conditions in one of Theorems 2.2–2.5, and (1.4), (1.5) hold. Then (1.2) has a bounded non-oscillatory solution which is bounded away from zero.*

Proof. Set $g_+(n) = \max\{g(n), 0\}$, $g_-(n) = \max\{-g(n), 0\}$. Then $g(n) = g_+(n) - g_-(n)$. Also (1.2) can be written as

$$\Delta^m(x(n) + p(n)x(\tau(n))) + [f_1(n, x(\sigma_1(n))) + g_-(n)] - [f_2(n, x(\sigma_2(n))) + g_+(n)] = 0.$$

Let $F_1(n, x(\sigma_1(n))) = f_1(n, x(\sigma_1(n))) + g_-(n)$, $F_2(n, x(\sigma_2(n))) = f_2(n, x(\sigma_2(n))) + g_+(n)$. Similar to the proof of Theorems 2.2–2.5, we obtain the conclusion. □

Theorem 2.7. *Assume that $p(n)$ satisfies the conditions in one of the Theorems 2.2–2.5, and (1.6) holds. Then (1.3) has a bounded non-oscillatory solution which is bounded away from zero.*

Proof. We prove only the case $0 \leq p(n) \leq p < 1$. Let BC be the set of bounded real sequence in the Banach space l^∞ and $\|x(n)\| = \sup_{n \geq n_0} |x(n)|$. We choose M_1, M_2, α such that $pM_2 + M_1 < \alpha < M_2$. Define $\Omega = \{x \in BC, M_1 \leq x(n) \leq M_2\}$, $c = \min\{\frac{\alpha - pM_2 - M_1}{lM_2}, \frac{M_2 - \alpha}{lM_2}\}$. N is sufficiently large such that for $n \geq N$

$$\frac{1}{(m-1)!} \sum_{s=n}^{\infty} (s - n + 1)^{(m-1)} |b_i(s)| \leq c, \quad i = 1, 2, \dots, l.$$

Define two maps Γ_1, Γ_2 on Ω as follows

$$\Gamma_1 x(n) = \begin{cases} \alpha - p(n)x(\tau(n)), & n \geq N \\ \Gamma x_1(N), & n_0 \leq n \leq N, \end{cases}$$

$$\Gamma_2 x(n) = \begin{cases} (-1)^{m-1} \sum_{s=n}^{\infty} \frac{(s-n+1)^{(m-1)}}{(m-1)!} \sum_{i=1}^{l} b_i(s)x(\sigma_i(s)), & n \geq N \\ \Gamma_2 x(N), & n_0 \leq n \leq N \end{cases}$$

For each $x, y \in \Omega$, $\Gamma_1 x(n) + \Gamma_2 y(n) \geq \alpha - pM_2 - lM_2 c \geq M_1$, $\Gamma_1 x(n) + \Gamma_2 y(n) \leq \alpha + lM_2 c \leq M_2$, that is $\Gamma_1 x(n) + \Gamma_2 y(n) \in \Omega$. Γ_1 is a contraction mapping and Γ_2 is continuous and uniformly Cauchy. So there exists $x \in \Omega$ such that $\Gamma_1 x + \Gamma_2 x = x$. The proof is complete. $\square$

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