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POSITIVE SOLUTIONS AND CONTINUOUS BRANCHES FOR BOUNDARY-VALUE PROBLEMS OF DIFFERENTIAL INCLUSIONS

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Abstract. In this paper, we consider second order differential inclusions with periodic boundary conditions. We obtain the existence of positive solutions and of continuous branches of positive solutions.

1. Introduction

Consider the boundary-value problem

$$\begin{gathered} Lu \in \lambda F(t,u), \quad 0<t<1\,, \\ \alpha u(0)-\beta u'(0)=0, \quad \gamma u(1)+\delta u'(1)=0, \end{gathered} \tag{1.1}$$

where $Lu=-(ru')'+qu$, $r\in C^1[0,1]$, $q\in C[0,1]$ with $r>0$, $q\geq 0$ on $[0,1]$, $\alpha,\beta,\gamma,\delta \quad \geq 0$ with $\alpha\delta+\alpha\gamma+\beta\gamma \quad >0$, $F\colon [0,1]\times[0,+\infty)\to P([0,+\infty))$, and λ is a positive parameter.

When F is a continuous map, the existence of positive solutions of (1.1) was studied in [5]. In this paper, the results in [5, 12] will be used to prove the existence of positive solutions of (1.1).

First, we recall the following notion (see, e.g. [4, 8]). Let X,Y be two Banach spaces. Let $P(Y)$, $K(Y)$, $Kv(Y)$, $C(Y)$, $Cv(Y)$ denote the collections of all nonempty, nonempty compact, nonempty convex compact, nonempty closed, nonempty convex closed subsets of Y, respectively.

A multimap $F\colon X\to P(Y)$ is said to be upper semicontinuous (u.s.c.) [lower semicontinuous (l.s.c.)] if the set $F_+^{-1}(V)=\{x\in X: F(x)\subset V\}$ is open [respectively, closed] for every open [respectively, closed] subset $V\subset Y$. F is said to be compact if the set $F(X)$ is relatively compact in Y.

Let $A\subset K(Y)$ and the max-normal and min-normal be

$$\|A\|=\max\{\|x\|: x\in A\} \quad \text{and} \quad \|A\|_0=\min\{\|z\|: z\in A\}.$$

Let $C_+[0,1]$ ($L^1_+[0,1]$) denote the cone of all positive continuous (respectively, integrable) functions on $[0,1]$. We will consider the cone $C_+[0,1] \quad (L^1_+[0,1])$ as subspace of the space $C[0,1]$ (respectively, $L^1[0,1]$) with induced topology.

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The nonempty subset $M \subset L^1_+[0,1]$ is said to be decomposable provided for every $f, g \in M$ and each Lebesgue measurable subset $m \subset [0,1]$,

$$f\chi_m + g\chi_{[0,1]\setminus m} \in M,$$

where χ_m is the characteristic function of the set m.

2. Existence of positive solutions

Let $G(t,s)$ be the Green's function for (1.1). Then u is a solution of (1.1) if and only if

$$u(t) \in \lambda \int_0^1 G(t,s)F(s,u(s))ds.$$

Recall that

$$G(t,s) = \begin{cases} c^{-1}\phi(t)\psi(s) & \text{if } t \le s \\ c^{-1}\phi(s)\psi(t) & \text{if } s \le t, \end{cases}$$

where ϕ and ψ satisfy

$$\begin{gathered} L\phi = 0, \quad \phi(0) = \beta, \quad \phi'(0) = \alpha, \\ L\psi = 0, \quad \psi(1) = \delta, \quad \psi'(1) = -\gamma \end{gathered}$$

and $c = r(t)(\phi'(t)\psi(t) - \psi'(t)\phi(t)) > 0$. Note that $\phi' > 0$ on $(0,1]$ and $\psi' < 0$ on $[0,1)$. Let $G = \max\{G(t,s) : 0 \le t, s \le 1\}$. We shall make the following assumptions:

- (H1) For every $x \in [0,+\infty)$ the multifunction $F(\cdot,x)\colon [0,1] \to Kv([0,+\infty))$ has a measurable selection, i.e., there exists a measurable function f such that $f(t) \in F(t,x)$ for a.e. $t \in [0,1]$;
- (H2) For a.e. $t \in [0,1]$ the multimap $F(t,\cdot)\colon [0,+\infty) \to Kv([0,+\infty))$ is u.s.c.;
- (H3) There exists a positive function $\omega \in L^1[0,1]$ such that

$$\|F(t,x)\| \le \omega(s)(1+x),$$

  for all $x \in [0,+\infty)$ and a.e. $t \in [0,1]$;
- (H4) The multioperator $F\colon [0,1] \times [0,+\infty) \to K([0,\infty))$ is almost lower semicontinuous; i.e., there exists a sequence of disjoint compact sets $\{I_m\}, I_m \subset [0,1]$ such that:
  - (i) $\text{meas}([0,1] \setminus \bigcup_m I_m) = 0$;
  - (ii) the restriction of F on each set $J_m = I_m \times [0,\infty)$ is l.s.c.;

We will use the method in [12] to prove the following results.

Theorem 2.1. *Let (H1)–(H3) hold. If* (1.1) *has no zero solution, then for each* $0 < \lambda < \frac{1}{G\int_0^1 \omega(s)ds}$, (1.1) *has a positive solution.*

Theorem 2.2. *Let (H3)-(H4) hold. If* (1.1) *has no zero solution, then for each* $0 < \lambda < \frac{1}{G\int_0^1 \omega(s)ds}$, (1.1) *has a positive solution.*

Proof of Theorem 2.1. From (H1)–(H3) it follows easily that the multioperator superposition

$$\begin{gathered} \wp_F\colon C_+[0,1] \to Cv(L^1_+[0,1]), \\ \wp_F(u) = \{f \in L^1_+[0,1] : f(s) \in F(s,u(s)) \text{ for a.e. } s \in [0,1]\}. \end{gathered}$$

is defined and closed (see, e.g. [4]). Consider a completely continuous operator

$$Q_\lambda \colon L^1_+[0,1] \to C_+[0,1], \quad Q_\lambda(f)(t) = \lambda \int_0^1 G(t,s)f(s)ds,$$

Let $\Gamma_\lambda = Q_\lambda \circ \wp_F$. From [4, Theorem 1.5.30] it follows that the multioperator Γ_λ is closed. We can easily prove that for every bounded subset $U \subset C_+[0,1]$, the set $\Gamma_\lambda(U)$ is relatively compact in $C_+[0,1]$. Hence applying [4, Theorem 1.2.48], we have that the Hammerstein's multioperator

$$\begin{gathered}\Gamma_\lambda \colon C_+[0,1] \to Kv(C_+[0,1]),\\ \Gamma_\lambda(u) = \lambda \int_0^1 G(t,s)F(s,u(s))ds.\end{gathered}$$

is upper semicontinuous. Let $T_+ = \{u \in C_+[0,1] : \|u\|_C \le \rho, \text{ where } \rho > 0\}$ For u in T_+ we have

$$\big\|\Gamma_\lambda(u)\big\|_C = \max\big\{\big\|\lambda \int_0^1 G(t,s)f(s)ds\big\|_C : f \in \wp_F(u)\big\},$$

where

$$\|\int_0^1 G(t,s)f(s)ds\big\|_C = \sup_{t\in[0,1]}\big\{\int_0^1 G(t,s)f(s)ds\big\}.$$

Since $f(s) \in F(s,u(s))$ for a.e. $s \in [0,1]$ and (H3), for a.e. $s \in [0,1]$ we have

$$f(s) \le \|F(s,u(s))\| \le \omega(s)(1+u(s)) \le \omega(s)(1+\|u\|_C) \le \omega(s)(1+\rho).$$

Therefore,

$$\int_0^1 G(t,s)f(s)ds \le G(1+\rho)\int_0^1 \omega(s)ds,$$

and hence

$$\|\int_0^1 G(t,s)f(s)ds\big\|_C \le G(1+\rho)\int_0^1 \omega(s)ds.$$

Because the last inequality holds for all $f \in \wp_F(u)$,

$$\|\Gamma_\lambda(u)\|_C \le \lambda G(1+\rho)\int_0^1 \omega(s)ds.$$

Choose $\rho \ge \frac{\lambda G\int_0^1 \omega(s)ds}{1-\lambda G\int_0^1 \omega(s)ds}$ then $\|\Gamma_\lambda(u)\|_C \le \rho$, i.e., Γ_λ maps the set T_+ in to itself. The existence of positive solution of the problem (1.1) can be easily follow from the Bohnenblust-Karlin fixed point theorem $\square$

For the proof of Theorem 2.2 we need the following result proved in [6, 7].

Lemma 2.3. *Let X be a separable metric space; E be a Banach space. Then every l.s.c. multimap $\tilde{F}\colon X \to P(L^1([0,1],E))$ with closed decomposable values has a continuous selection.*

Proof of theorem 2.2. From conditions (H3)–(H4) it follows that

$$\wp_F \colon C_+[0,1] \to C(L^1_+[0,1])$$

is a l.s.c. multioperator with closed decomposable values (see, e.g. [4, 8]).

Consider again the Hammerstein's multioperator $\Gamma_\lambda = Q_\lambda \circ \wp_F$. By Lemma 2.3, the multioperator superposition $\wp_F$ has a continuous selection

$$\ell\colon C_+[0,1] \to L^1_+[0,1], \quad \ell(u) \in \wp_F(u).$$

Hence the operator

$$\gamma_\lambda\colon C_+[0,1] \to C_+[0,1], \quad \gamma_\lambda(u)(t) = \lambda \int_0^1 G(t,s)\ell(u)(s)ds,$$

is a completely continuous selection of the multioperator Γ_λ. As shown above, for each $0 < \lambda < \frac{1}{G\int_0^1 \omega(s)ds}$, we can choose $\rho > 0$ such that the multioperator Γ_λ maps the set T_+ in to itself. From the Schauder fixed theorem it follows that the operator γ_λ has a fixed point in T_+, i.e., (1.1) has a positive solution □

Now we use the result in [5] to prove the existence and multiplicity of positive solutions for (1.1), when F is lower semicontinuous. Assume that

(F1) $F\colon (0,1) \times [0,+\infty) \to Kv([0,+\infty))$ is l.s.c.;

(F2) For each $M > 0$, there exists a continuous function g_M on $(0,1)$ such that $\|F(t,x)\| \le g_M(t)$ for $t \in (0,1)$, $\quad x \in [0,M]$, and

$$\int_0^1 G(s,s)g_M(s)ds < \infty.$$

(F3) There exist an interval $I \subset (0,1)$ and a non-zero function $m \in L^1(I)$ with $m \ge 0$ such that for every $b > 0$, there exists $r_b > 0$ such that

$$\|F(t,x)\|_0 \ge bm(t)x \quad \text{for } t \in I,\ x \in (0,r_b);$$

(F4) There exist an interval $I_1 \subset (0,1)$ and a non-zero function $m_1 \in L^1(I_1)$ with $m_1 \ge 0$ such that for every $c > 0$, there exists $R_c > 0$ such that

$$\|F(t,x)\|_0 \ge c\,m_1(t)x \quad \text{for } t \in I_1,\ x \ge R_c;$$

Theorem 2.4. *Let (F1)–(F3) hold. Then there exists $\lambda_0 > 0$ such that* (1.1) *has a positive solution for $0 < \lambda < \lambda_0$. If, in addition, (F4) holds, then* (1.1) *has at least two positive solutions for $0 < \lambda < \lambda_0$*

For the proof of this we need the following result (see, e.g. [4, 11]).

Lemma 2.5. *Let X be a metric space; Y be a Banach space. Then every l.s.c. multi-map $W\colon X \to Cv(Y)$ has a continuous selection.*

Proof of Theorem 2.4. Let $f\colon (0,1) \times [0,+\infty) \to [0,+\infty)$ be a continuous selection of F, i.e.,

$$f(t,x) \in F(t,x) \quad \text{for all } (t,x) \in (0,1) \times [0,+\infty).$$

It is easy to see that for all $(t,x) \in (0,1) \times [0,+\infty)$ the following inequality holds

$$\|F(t,x)\|_0 \le f(t,x) \le \|F(t,x)\|.$$

Consider now the problem

$$Lu = \lambda f(t,u), \quad 0 < t < 1, \tag{2.1}$$

with the conditions in (1.1). By (F1)–(F4) we have

(f1) The map $f\colon (0,1) \times [0,+\infty) \to [0,+\infty)$ is continuous;

(f2) For each $M > 0$, there exists a continuous function g_M on $(0,1)$ such that $f(t,x) \le g_M(t)$ for $t \in (0,1)$, $0 \le x \le M$ and

$$\int_0^1 G(s,s)g_M(s)ds < \infty.$$

(f3) There exist an interval $I \subset (0,1)$ and a non-zero function $m \in L^1(I)$ with $m \geq 0$ such that for every $b > 0$, there exists $r_b > 0$ such that
$$f(t,x) \geq bm(t)x, \quad \text{for } t \in I,\ x \in (0, r_b);$$

If $(F4)$ holds then we have

(f4) There exist an interval $I_1 \subset (0,1)$ and a non-zero function $m_1 \in L^1(I_1)$ with $m_1 \geq 0$ such that for every $c > 0$, there exists $R_c > 0$ such that
$$f(t,x) \geq c\, m_1(t)x, \quad \text{for } t \in I_1, \quad x \geq R_c;$$

From [5, Theorem 1.1] it follows that if (f1)–(f3) hold then there exists $\lambda_0 > 0$ such that (2.1) has a positive solution for $0 < \lambda < \lambda_0$. If, in addition, $(f4)$ holds then (2.1) has at least two positive solutions for $0 < \lambda < \lambda_0$. Hence we obtain our result □

3. Continuous branch of positive solutions

A sphere and a ball with center at the point 0 (the zero function) and radius r in the cone $C_+[0,1]$ will be denoted respectively by
$$S_+(0,r) = \{u \in C_+[0,1] : \|u\|_C = r\},$$
$$T_+(0,r) = \{u \in C_+[0,1] : \|u\|_C \leq r\}.$$

Recall the following notion (see, [1, 2, 10]).

Definition A set V of positive solutions of (1.1) is said to form a continuous branch connecting the spheres $S_+(0,r)$ and $S_+(0,R)$, with $0 \leq r < R \leq \infty$, if for every nonempty open bounded subset
$$\Delta \subset C_+[0,1] : T_+(0,r') \subset \Delta \subset T_+(0,R'),\ r < r' < R' < R$$
the set $V \cap \partial\Delta$ is nonempty, where $\partial\Delta$ is a boundary of Δ. If, in addition, $r = 0$ and $R = \infty$ then the set V is said to be a continuous branch with infinite length.

Let E be a Banach space; $\mathbf{K} \subset E$ be a cone.

Definition An operator $A\colon E \to E$ is said to be positive, if $A\mathbf{K} \subset \mathbf{K}$.

Lemma 3.1 ([1, 9])**.** *Let A be a positive completely continuous operator on the cone $\mathbf{K}$. Assume that on the border $\partial\Xi_{\mathbf{K}}$ of every bounded subset $\Xi_{\mathbf{K}} \ni 0$ of the cone $\mathbf{K}$ the following inequality holds*
$$\inf_{x \in \partial\Xi_{\mathbf{K}}} \|Ax\| > 0.$$
Then the positive solutions of the equation
$$Ax = \mu x, \quad x \in \mathbf{K} \setminus \{0\}$$
form a continuous branch with infinite length.

Let a be a positive constant. Consider now the problem (1.1) with the multimap
$$F\colon [0,1] \times [0,+\infty) \to K([a,+\infty))$$
satisfying the following assumptions:

(A1) F is almost lower semicontinuous;

(A2) For every nonempty bounded subset $\Omega \subset [0,+\infty)$ there exists a function $\vartheta_\Omega \in L^1_+[0,1]$ such that
$$\|F(t,x)\| \leq \vartheta_\Omega(t),$$
for all $x \in \Omega$ and a.e. $t \in [0,1]$;

(A3) There exists $q > 0$ such that the Green's function satisfies $G(t,s) \geq q$, for all $0 \leq t, s \leq 1$;

Theorem 3.2. *Let (A1)–(A3) hold. Then the positive solutions of* (1.1) *form a continuous branch with infinite length.*

Proof. Note that the condition (H3) is special case of the condition (A2). As is shown above, from (A1)–(A2) the multioperator Γ_λ has a completely continuous selection γ_λ on the cone $C_+[0,1]$. Let $\Xi \ni 0$ be an open bounded subset of $C_+[0,1]$. For all $u \in \Xi$, since $\ell(u)(s) \in F(s, u(s))$ for a.e. $s \in [0,1]$ we have

$$\gamma_\lambda(u)(t) = \lambda \int_0^1 G(t,s)\ell(u)(s)ds \geq \lambda a q > 0.$$

Hence

$$\inf_{u \in \partial\Xi} \|l(u)\|_C \geq aq > 0, \quad \text{where } l = \frac{\gamma_\lambda}{\lambda}.$$

On the cone $C_+[0,1]$ consider the equation

$$l(u) = \frac{1}{\lambda} u \tag{3.1}$$

By Lemma 3.1, the positive solutions of (3.1) form a continuous branch with infinite length. And hence we obtain our result □

4. Examples

Example 4.1. Let $D \subset [0,1]$ be a nonmeasurable set;

$$F\colon [0,1] \times [0,+\infty) \to Kv([0,+\infty))$$

be the multimap

$$F(t,x) = \begin{cases} [0, x+1] & \text{if } x = t \text{ and } t \in [0,1] \setminus D \\ [0, x+1] & \text{if } x = t+1 \text{ and } t \in D \\ x+1 & \text{otherwise.} \end{cases}$$

Consider the differential inclusion

$$\begin{gathered} -u''(t) \in \lambda F(t, u(t)), \quad \lambda > 0, \quad 0 < t < 1, \\ u(0) = u(1) = 0. \end{gathered} \tag{4.1}$$

It is easy to see that

$$G(t,s) = \begin{cases} t(1-s) & \text{if } 0 \leq t \leq s \leq 1 \\ s(1-t) & \text{if } 0 \leq s \leq t \leq 1 \end{cases}$$

is a Green's function for the operator $Lu = -u''$. Note that $\max\{G(t,s) : 0 \leq t, s \leq 1\} = 1$. Choose a function $\omega \equiv 1$ then the conditions (H1)-(H3) hold. Zero function is not a solution of (4.1). From Theorem 2.1 it follows that for each $0 < \lambda < 1$ the inclusion (4.1) has a positive solution

Example 4.2. Let $\varepsilon \in (0,1)$ and $F\colon (0,1) \times [0,+\infty) \to Kv([0,+\infty))$ be the multimap

$$F(t,x) = \begin{cases} t(x^2 + \frac{1}{1+x}) & \text{if } 0 < t \leq \varepsilon \text{ and } 0 \leq x \leq 1 \\ (t+1)(x^2 + \frac{1}{x+\varepsilon}) & \text{if } 0 < t \leq \varepsilon \text{ and } 2 \leq x \leq 3 \\ [t(x^2 + \frac{1}{1+x}), (t+1)(x^2 + \frac{1}{x+\varepsilon})] & \text{otherwise.} \end{cases}$$

It is clear that the multimap F is lower semicontinuous. Consider the inclusion

$$\begin{gathered}(-e^{\frac{-t^2}{2}}u')' + e^{\frac{-t^2}{2}}u \in \lambda F(t,u), \quad 0<\lambda, 0<t<1,\\ u(0)=u(1)=0.\end{gathered} \tag{4.2}$$

Let $Lu = (-e^{t^2/2}u')' + e^{-t^2/2}u$. Then

$$G(t,s) = \begin{cases} \frac{e^{t^2/2}}{\int_0^1 e^{-\tau^2/2}d\tau} \int_s^1 e^{-\tau^2/2}d\tau \int_0^t e^{-\tau^2/2}d\tau, & \text{if } 0\le t\le s\\ \frac{e^{t^2/2}}{\int_0^1 e^{-\tau^2/2}d\tau} \int_0^s e^{-\tau^2/2}d\tau \int_t^1 e^{-\tau^2/2}d\tau, & \text{if } s\le t\le 1 \end{cases}$$

is a Green's function for the operator L (see, e.g. [3]).

For each $M>0$, let $g_M(t) = (M^2+\frac{1}{\varepsilon})(t+1)$. We have

$$\|F(t,x)\| \le (t+1)(x^2+\frac{1}{x+\varepsilon}) \le g_M(t),$$

for $0<t<1$, $0\le x\le M$ and

$$\int_0^1 G(s,s)g_M(s)ds < +\infty.$$

Hence the condition (F2) holds. Let $I=(0,\varepsilon), m(t)=t$. Then for every $b>0$

$$\|F(t,x)\|_0 = t(x^2+\frac{1}{1+x}) \ge b\,m(t)x \quad \text{for } t\in I,\ x\in(0,r_b),$$

where $r_b = \min\{\frac{-b+(b^2+4b)^{1/2}}{2b}, 1\}$. The condition (F3) holds. For every $c>0$

$$\|F(t,x)\|_0 \ge t(x^2+\frac{1}{1+x}) \ge c\,m(t)x, \quad \text{for } t\in I,\ x\ge c.$$

The condition (F4) holds. By Theorem 2.4, there exists $\lambda_0>0$ such that (4.2) has at least two positive solutions for $0<\lambda<\lambda_0$

Example 4.3. Let $F\colon [0,1]\times[0,+\infty)\to K([1,+\infty))$ be the multimap

$$F(t,x) = \begin{cases} (t^2+2)(x^2+\frac{1}{x+1}) & \text{if } 0\le t\le 1,\ 0\le x\le 1\\ (t+2)(x^2+\frac{1}{x+1}) & \text{if } 0\le t\le 1,\ 2\le x\le 3\\ [(t^2+2)(x^2+\frac{1}{1+x}),\ (t+2)(x^2+\frac{1}{x+1})] & \text{otherwise.} \end{cases}$$

Consider the problem

$$\begin{gathered}-(1+e^t)u'' - e^tu' \in \lambda F(t,u), \quad 0<t<1, \quad 0<\lambda,\\ u(0)-2u'(0)=0, \quad u'(1)=0.\end{gathered} \tag{4.3}$$

It is clear that F is lower semicontinuous. Hence the condition (A1) holds.

$$G(t,s) = \begin{cases} x-\ln(1+e^x)+1+\ln 2 & \text{if } 0\le t\le s\\ s-\ln(1+e^s)+1+\ln 2 & \text{if } 0\le s\le t \end{cases}$$

is a Green's function for operator $Lu = -(1+e^t)u'' - e^tu'$ (see, [3]) and

$$G(t,s)\ge 1, \quad \text{for all } t,s\in[0,1].$$

The condition (A3) holds.

For every bounded subset $\Omega\subset[0,+\infty)$, let $\vartheta_\Omega(t) = (t+2)(1+\|\Omega\|^2)$. We have

$$\|F(t,x)\| \le (t+2)(x^2+\frac{1}{1+x}) \le \vartheta_\Omega,$$

for all $x \in \Omega$ and all $t \in [0, 1]$. Therefore the condition (A2) holds. From Theorem 3.2 it follows easily that the set of positive solutions of (4.3) forms a continuous branch with infinite length

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