ITALIAN ART THROUGH MATHEMATICS: ENGAGING STUDENTS THROUGH

ARTISTIC EXPLORATION

HONORS THESIS

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by

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This project shows a potential lesson plan to be used in a high school pre-calculus and evaluates the effectiveness of this strategy by synthesizing and analyzing similar projects. The students will select an interesting piece of art displayed in a designated museum or art gallery, and then recreate the image using conic equations. The students will then write a calculator program with the equations for the conic diagram. The program is expected to output an image, which will then be printed, and the class will create a (quite literal) art gallery with their conic images. The lesson is demonstrated with Botticelli's *Birth of Venus* (displayed in the Ufizzi in Florence, Italy) as the subject with the output image attached.

The lesson presented addresses the same mathematics content except it includes an artistic slant; this will not only allow for a real-world application, but also provide a more engaging mathematical experience to students who are not the typical "math

people." Furthermore, this project promotes a great amount of cooperation and allows students to almost instantly realize and correct their mistakes in their programs, and learn the effects of various transformations on conic equations. Lesson plans similar to this have been proven to engage students, provide fruitful educational results and foster real-world learning.

Introduction

Walking down the halls of a high school, the phrase "I hate school," resonates throughout the building. Furthermore, there is an extreme stigma surrounding mathematics classrooms; a large number of students seem to frequently hate mathematics classes. This proposes the question of "why do some students hate mathematics so much?" There is no seemingly way that this trend is solely based on the content which leads me to the conclusion that the issue must lie in the devices utilized in teaching mathematics, rather than solely the content.

Too often, people are categorized as "math people" or "art people." With that being said, students in American high schools often use this categorization as an explanation for their lack of motivation in a specific subject – most of the times students can do the mathematics but they are not motivated or interested because they have been labeled "not good at math" by society and this may lead to under performance. However, mathematics and art actually do have deep-seeded roots within each other, so therefore the utilization of art as a vessel to teach mathematics is actually very useful.

There have been countless studies on the pedagogical devices and their effectiveness, with a plethora of different results. One of the reasons for this discrepancy correlates to the research conducted by Dr. Howard Gardner (Gardner, 2006). Gardner's research, is focused on the ideas of multiple intelligences. Gardner asserts that "no two individuals—not even identical twins—have exactly the same intellectual profile because, even when genetic makeup is identical, individuals have different experiences," (Gardner, 2006, p. 23). Gardner's theory also suggests that individuals can possess any

combination of intelligences (or academic aptitudes and capacities) that are their strongest. His research focuses on the idea of having strengths in different areas, rather than having singular general ability. Gardner's theory states that there are 7 different intelligences which are as follows: visual-spatial, bodily-kinesthetic, musical, interpersonal, intrapersonal, linguistic, and logical-mathematics. The key to effective teaching, based off of Gardner's theory, is to engage various different intelligences.

A way to engage various different intelligences is to create projects that effectively relate to various student interests. When students spend time working on a project that is interesting, the learning is believed to be at a deeper level. Students tend to work harder on something they actually care about than on a summative or formative assessment. The struggle for mathematics is that the "real world" examples utilized in a mathematics classroom are not usually applicable and a red "X" denoting a wrong answer does not show the students what they did wrong. In order to combat this, I propose an example of a pre-calculus lesson plan that has the potential to engage various intelligences, be interesting to students, and also allow them to realize almost instantly when they made an algebraic error.

Art and Mathematics Education

As previously mentioned, Gardner's theory states that students have various different inept capabilities and aptitudes for different subjects. So, in an educational world where skills are predominantly assessed with exams and standardized tests, students who do not have strong testing abilities often perform worse than students who test well. In order to combat this system, a proposed solution is to provide a variety of assessments.

One content area that is often thought to have challenges with variety amongst assessments is mathematics. From my personal experience as a high school student and a pre-service teacher, mathematics teachers are conditioned to teach students for a unit test to demonstrate mastery; the idea of an end of unit project is often discounted as ineffective in checking for understanding. Though, allowing students to complete a creative end-of-unit project would actually allow students to demonstrate not only mastery of content, but create an engaging and memorable evaluation. However, this brings about the question, how can a mathematics project be engaging? Incorporating another content to connect to student schema is an ingenious way to do so.

It is of relative ease to incorporate art in mathematics lessons. In fact, there is a tremendous overlap in the two disciplines. For instance, in the article entitled *Artist? Mathematician? Developing both enhances learning!* Bush, Karp, and Nadler (2015) declare that

> "art naturally links to mathematics in a variety of ways, including identifying
>
> patterns, building sense making, grasping a context, solving problems, using prior
>
> knowledge to inform how to approach novel problems, using symbols and shapes

to communicate ideas, creating visual models and illustrations to support thinking,

using tools to think about concepts, and focus on hands-on learning experiences"

(Bush, Karp, & Nadler, 2015, p. 62).

Thus, the skills utilized in solving mathematical problems are also engaged in artistic

exploration; framing the introduction of a new mathematical process or idea with a

different perspective helps engage the students and learn the content without the hatred of

mathematics.

As aforementioned, engaging various different intelligences generates a higher

interest from students. Art, for instance, could fall under the category of musical, and

visual-spatial. Combined with the logical-mathematics intelligences, a lesson that

effectively synthesizes these attributes, would target at least 2 of the 7 intelligences (not

to mention the interpersonal, and intrapersonal, as well as an added layer of linguistic

intelligence). As stated in the News & Views article, utilizing "art as a vehicle for

learning about mathematics makes good sense," (Bush, Karp, & Nadler, 2015, p. 62).

Moreover, it is often thought that the reason many people are unable to grasp

mathematics is due to the lack of visualization. Often, people feel that there is no way to

visualize mathematics, and people who tend towards visual-spatial intelligences need

visuals. So, in order to contest this, teachers can use more aesthetically appealing models.

In the article *What Mathematics Education Can Learn from Art: The Assumptions,*

Values, and Vision of Mathematics Education, the author presents the point that

"aesthetic is contextual; it is the effect on an individual in the particular context that

makes an object or event moving (or not), and the effect is generated as the individual

takes in and makes something of the experience" (Dieteker, 2015, p. 2). This reaffirms

the importance of visuals in a lesson, because it provides context for which students need to learn. Context and aesthetics are two keys to helping students actually understand and process course material, rather than studying to pass an examination.

Speaking of context, it is important to note that students will learn and process more from a lesson given real world applications. Not all students want to learn about a hypothetical problem where a fictional character is trying to guess what the projected trajectory of a rocket from a quadratic formula. Instead, showing students actual applications of mathematics in the world around them will elicit a much higher interest and success rate. This assertion is strengthened by Dietiker's claim that "mathematics exists to describe the world around us, it is likely that nearly every visual or performing art offers potential metaphorical language for structural and aesthetic qualities of mathematical experiences" (Dieteker, 2015, p. 4). If students are made aware that an understanding of math opens a whole new perspective to understand art, they will be enthralled with a desire to learn more. Furthermore, it is not only to better understand the arts, but a firm grasp of mathematics also allows for a better understanding of the world.

In addition, studies have shown that incorporating these two specific contents lead to a higher success rate among students who have a disdain for the mathematical field of study. For example, it is worth mentioning that "art has been noted as an entry to mathematics and other STEM [science, technology, engineering, and mathematics] subjects for student who have not previously enjoyed or experienced success in the sciences" (Bush, Karp, & Nadler, 2015, p. 61). This has a direct relation to the integration of the two content areas and appealing to various student intelligences. Relating to various intelligences from Gardner's list leads to self-discovery, as students work on

something that actually appeals to their strengths. Students who know they are working on something not in their strong suit, is automatically set up for failure, whether they are able to complete the task or not. Students who feel comfortable with the assignment and what is being asked of them are more receptive to learning than students who feel there "is no hope" of their understanding.

Furthermore, now that the connection between mathematics education and art has been made apparent, it is important to put this information into context. There are many branches of mathematics that are easily integrated with art, however, the study of conics has strong connections. For that reason, the study of conics will be the content on which the students will focus.

Conics: A Background

Conics are at the fundamental building block of this pre-calculus project, however, a "conic" has yet to be defined. The study of conics all began with a cone. A cone is a three-dimensional shape that is formed from the projection of a point into a circle. In the textbook entitled *Conics and Cubics: A Concrete Introduction to Algebraic Curves* by Robert Bix, it is asserted "that all conics are sections of cones and therefore, projections of circles" (Bix, 2000, p. 69). Essentially, given a cone, it can be sliced a multitude of ways, and each way it is sliced produces a different conic 2-dimensional shape: circle, ellipse, parabola, and hyperbola. There is one interesting point to be made, however. Bix stated that "any conic can be transformed into the parabola $y = x^2$," (Bix, 2000). Which is important in this case because while constructing the conic diagrams, there is no specific conic for each case, considering they can be transformed to match the desired shape. Next, to discuss the various shapes created by conics.

Circle

The first shape is the circle. Circles are defined a set of points equidistant from a center of a set of points. The length of the distance from the center to any point on the circle edge, is known as the "radius" and can be found by using the distance formula. The circle can be created by a plane parallel to the cone's base intersecting at a right angle. This action can be seen in **Figure 1** seen below; the circle is labeled with the letter "C."

The circle can be written in two separate forms, the first of which being standard (or center-radius form). Standard form is written as $(x-h)^2 + (y-k)^2 = r^2$ with the center being the point (h, k) and radius being represented by the letter r. This form is beneficial, because the radius and the center of the circle can easily be read from this form, which

makes graphing the equation easier. More importantly, it is easy to read these values from the graph, and create an equation. The second form of a circle is typically referred to as the general form. The general form is notated as follows: $ax^2 + by^2 + cx + dy + e = 0$. The letters a, b, c, and d each represent coefficients and e representing a constant. This is the "proper" form of a circle equation.
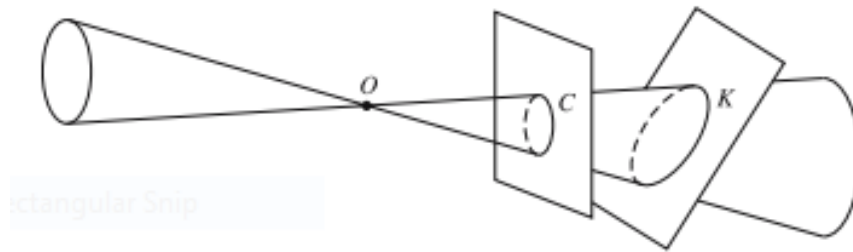
Figure 1

Ellipse

The next shape is the ellipse, or more commonly known as "oval." Ellipses are a member of the conic family, because when a cone is intersected by a plane at an oblique angle to the base, it creates an ellipse. This is seen above in **Figure 1**, with the ellipse being labeled "K."

The ellipse is similar to the circle in that it has a center and points surrounding the center. However, the ellipse is different from the circle in that all points are not equidistant from the center. There are two different orientations of ellipses, a vertical ellipse, and a horizontal ellipse. Vertical ellipses are ellipses that are taller than they are wide, and horizontal ellipses, which are wider than they are tall. The factor that classifies an ellipse as vertical or horizontal is its "major axis."

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All ellipses have two separate axes, one being labeled the major axis and the other being the minor. If an ellipse is vertical (taller than wide), the major axis will correspond to the y-values, and is defined by the distance from the center to either the top or the bottom of the ellipse. This can be seen in **Figure 2** with the y-axis representing the major axis. This in turn makes the minor axis correspond to the x-values, and is the distance from the center to either the left or the right of the figure. On the contrary, if the ellipse is horizontal (wider than tall), the major axis will correspond to the x-values. The major axis will be the distance from the center to the left or right edge of the ellipse, and the minor axis will correspond to the y-values and will be the distance from the center to the top or the bottom of the shape. This can be seen in **Figure 3** with the x-axis representing the major axis. It is worth noting that the calculator will not graph the pixels near the x-axis, but the actual ellipse is intended to be continuous.
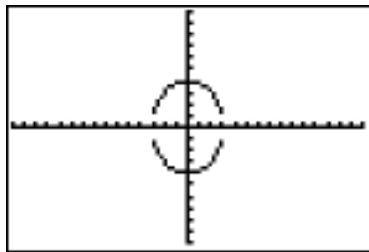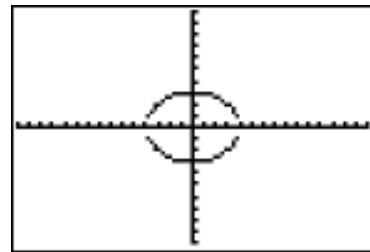
Figure 2   Figure 3

Similar to the circle, ellipses have two forms, general and standard form. The general form of an ellipse is $ax^2 + by^2 + cx + dy + e = 0$. It is to be noted, however that this equation is exactly the same as the general form of a circle. The difference between the two is indistinguishable until the equation is written in standard form. In standard form, an ellipse will have values for the major and minor axes that the circle equation is lacking.

Standard equations for ellipses are divided into two categories, the vertical ellipse and the horizontal ellipse. These equations diverge because, as aforementioned, the selection of the major axis depends which variable is "transformed." The standard form equation for a vertical ellipse is: $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$. The major axis is always labeled a and the minor is always labeled b, and similar to the circle, the vertex is (h, k). Since this is a vertical ellipse, the major axis corresponds to the y, which explains why a is with the y part of the equation. Furthermore, the standard form of a horizontal ellipse is: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$. The two equations are remarkable similar with the only difference being with which term the a corresponds. In this case, since it is horizontal, the major axis is with the x-value. The standard form of the circles are more often utilized because the axes and center can be easily picked out, which in turn leads to an ease of graphing.

Parabola

The subsequent conic is the parabola. The parabola is more commonly-known because it is a fundamental part of algebra. The parabola is the shape formed by graphing a quadratic function; it can also be formed from a cone, however. In the below diagram, **Figure 4**, it can be seen that when a plane perpendicularly intersects the cone's base, a parabola is formed in the cross-section. The general shape of a parabola is often described as a "horseshoe" or a "U" shape. However, parabolas can be transformed to become more flat or wide, depending on the equation. Additionally, parabolas have one "center" or "vertex," which is the singular point where the shape converges.

As is the case for most conics, the parabolic equation can be written in a standard or vertex formula and a general equation. It should be noted that there are two separate

types of parabolas, though. There is a vertical parabola, which opens either up or down; the horizontal parabola is of the same shape, but instead opens either left or right. The vertical parabola in standard form is $y = ax^2 + bx + c$, with a and b representing coefficients, and c representing a constant. When the standard form is rewritten using the "completing the square" technique, the standard form is created which is $y = a(x - h)^2 + k$. The horizontal parabola is very similar to the vertical. The horizontal parabola in standard form is $x = ay^2 + by + c$, with the standard form written as $x = a(y - k)^2 + h$. Akin to the previous 2 conics, the point (h, k) represents the center of the conic, which in this case is the vertex.
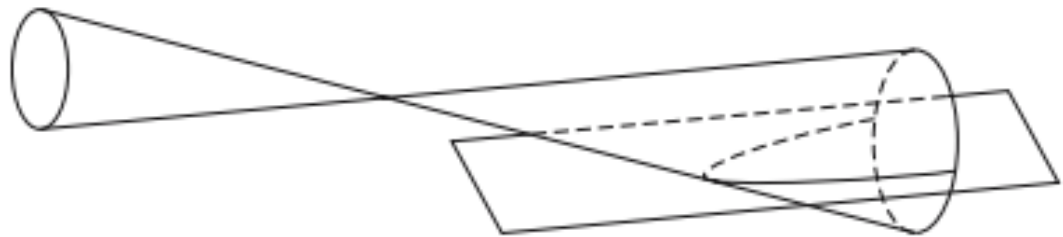
Figure 4

Hyperbola

Finally, the last conic to be discussed is the hyperbola. A hyperbola is created from not one, but two cones connected at the tip. The theory is that if a plane intersects the two cones perpendicularly to the base of the cones, then a hyperbola is formed. This concept can be visualized in the below diagram, **Figure 5**. In terms of shape and form, the hyperbola is very closely related to the parabola; the hyperbola is formed by two parabolas opening in different directions. In other words, it is as if the parabola also included its mirror image.

The graphing of a hyperbola is a bit more complicated than the previous forms. Hyperbolas are formed from two lines that form asymptotes that guide the parabolic shapes and intersect at a central point. When these two lines intersect, a rectangular area is formed that determines where the vertices of the graph will be placed, with vertices defining the point that each parabola begins (comparable to parabolic vertices). Alike to ellipses, there are two variations of the basic shape, a horizontal hyperbola and a vertical hyperbola. The horizontal hyperbola has parabolic structures on the left and right of the center whereas the vertical hyperbola has parabolic structures to the top and bottom of the center.

Additionally, like the ellipse there are two axes that affect the hyperbolic graph. The two axes form the dimensions of the rectangle guidelines from the asymptotic lines. Basically, the distance between the central point and the desired vertex (to either the left or the right), forms the first axis, represented by "a." The other axis is formed by the distance from center point to the top of the asymptotic guide-rectangle.

Next, the equations of the hyperbola are remarkably alike to that of an ellipse. However, the difference is rather than an addition sign between the two fractions, there is a subtraction sign. Interestingly enough, there is no general form of a hyperbola, but there are standard form equations for the two different types of hyperbolas. First, the horizontal hyperbola (opens to the left and right) is written as follows: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$. Note, that the a is associated with the x and the y is subtracted. The standard form for the vertical parabola is the opposite so that $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$. Conversely, the a is associated with the y in this case, and the x is subtracted.
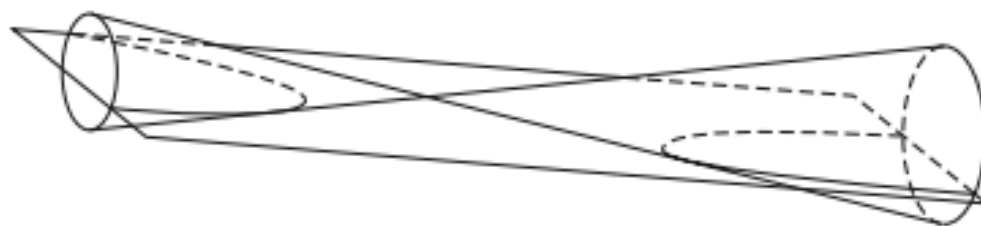
Figure 5

Now that a formal introduction or background to conics has been introduced, it is important to bring together the various pieces. In order to efficiently synthesize mathematics education, art, and conics, it is necessary to create a lesson plan combining all three. Effectively utilizing these various ideas could lead to a higher success rate amongst secondary students.

Sample Lesson Plan

Lesson Component
Objectives Students will be able to identify the transformations of a given conic equation. Students will be able to solve conic equations for y using a graphing calculator Students will be able to graph conic equations on graph paper.
TEKS/ELPS: PC.2.g.The student is expected to graph functions, including exponential, logarithmic, sine, cosine, rational, polynomial, and power functions and their transformations, including af(x), f(x)+d, f(x-c), f(bx) for specific values of a, b, c, and d, in mathematical and real world problems. PC.3.g. The student is expected to make connections between the locus definition of conic sections and their equations in rectangular coordinates.
ENGAGEMENT: Prior to students entering class, the teacher will print out various different paintings from the Ufizzi Gallery in Florence, Italy (examples would be Botticelli's *Primavera*, Titian's *Venus of Urbino*, *da Vinci's Adoration of the Magi,* etc.). Upon entrance, students will be exposed to the art, and the teacher will give a brief overview of the art. After the introduction, students will be allowed to use the classroom computers to research works of art in the Uffizi Gallery using the following links: http://www.uffizi.org/artworks/ https://www.virtualuffizi.com/the-artworks.html At the end of the class period, students will be expected to turn in their top three favorite works of art from their exploration. The teacher will then go through the students' selected pieces and make sure there are no duplicates of works. The teacher will assign the works by order of choice ranking in

an attempt to please all students, with special consideration to student potential for success with their selected conic diagrams.

EXPLORATION:

Students will re-create a selected image from Florence's Ufizzi Gallery using conic diagrams on graph paper. Students will create their image solely with conics.

As students create their image, they will write down the vertex or center, a point on the conic itself. The students are encouraged to write the equation in vertex form at this point to make for easier computations when it comes time for calculator programming.

After completion of the drawing, students will then begin programming their drawing. Students will enter all equations into their program in "y=" format. Students will periodically check for errors by looking at the conic image created. This will allow for instant error recognition and student troubleshooting to fix their errors in real time. This is imperative to learning and understanding the material.

EXPLANATION:

The teacher will spend class days explaining various skills essential for this project. The skills that will be specifically highlighted are conic equations, transformations, solving for y and calculator programming.

The first lecture will be over conics. The teacher introduces circles, parabolas, ellipses, and hyperbolas. Circles will be presented with the vertex formula and standard form. The students will then notice the assets and deficits of the equations. The teacher will then repeat the process for the other three conic equations.

The second lecture will be regarding the transformations of various equations. The students will be introduced to vertical transformations (translations up and down, stretches, and compressions) and horizontal transformations (translations up and down, stretches, and compressions). The teacher will introduce the parent functions and then show the transformed functions so that they can recognize how to transform the conic to match their conic diagram.

The third pertinent lecture is solving for y, specifically in these conic equations. The teacher will work examples of solving for y, and explain that in the calculator program equations must be entered in "y=" format.

Students will be asked to solve for y from various different stages, such as given only an image, given a vertex and a point, and given vertex and transformations.

Finally, the teacher will explain how to write programs on the calculator. The teacher will highlight that programs must be entered with very certain formatting. Additionally, the teacher will explain how to set up the window of the conic image, as well as positive and negative effects of scaling the image. Without forgetting how to restrict the domain to get a selected section of the conic diagram.

ELABORATION:

Students will be well adept in function transformations. Students will be able to apply the basic conic transformations, to most other functions. This is quintessential in understanding mathematics.

Furthermore, students will learn how to correct their mistakes and learn specifically, how important detail is in mathematics. Forgetting one equals sign changes the whole image.

Additionally, students will feel comfortable with solving and manipulating conic equations. Students will understand thoroughly the derivation of standard form equations from vertex formulas, as well as understanding the advantages of each.

EVALUATION:

At the end of the unit, students will turn in a hard copy of a completed program that draws the image of the art piece. The teacher will print out the completed images, and the students will create a gallery walk showing all of the classes' artwork.

Resources:

Class set of TI-84 (or newer) programmable calculators, a school computer lab, TI Software, TI USB cord, class set of rulers, graphing paper, and pencils.

Differentiation:

Students will be provided written instructions as well as verbal instructions.

A dyslexia/dysgraphia-friendly font will be used.

The nature of the product is more visual than conceptual, so students with difficult reading will be able to create visual representations, instead of readings.

Unit Lesson Plan: Conics	
Monday	Intro Activity Students Research Project Introduced
Tuesday	Lecture over conics and their properties
Wednesday	Transformations of conic equations Solving for y Programming into calculator
Thursday	In-class art sketches (using conics) Identifying conic shapes Identifying transformations of conic equations
Friday	In-class work day Students will finish solving their equations and programming their calculators

Monday	Completed program is due Teacher will print out class pictures Students will hang their work for a gallery walk

Sample Lesson Plan Evaluation

Due to the nature of this project, the most accurate form of results would be the completion of the project assigned to the secondary students. So, in accordance with the above lesson plan, it was essential to pick which work of art on which to focus. The painting selected is Botticelli's *The Birth of Venus* which is displayed in the Ufizzi Gallery in Florence, Italy. The image is shown below in **Figure 6**. It was an awe-inspiring work of art that lends itself fully to the creation of a conic equation. The curves of Venus's body are very similar to the mathematical curves of conic equations. Not to mention the variety of equations to be utilized in the creation of this image. Every conic, with the addition of a few lines were included in the conic diagram.

Figure 6

After noticing the various equations, the next step in the lesson is to create the image. The image should be as close to the original image as possible with the incorporation of mathematics. It is also important to create a scale and graph to match the calculator window. In order to have an image without skewing, a window must be selected very carefully. Following the window selection, the image can be drawn. Below, as seen in **Figure 7,** is the image created on graph paper using conic equations of *The Birth of Venus*.
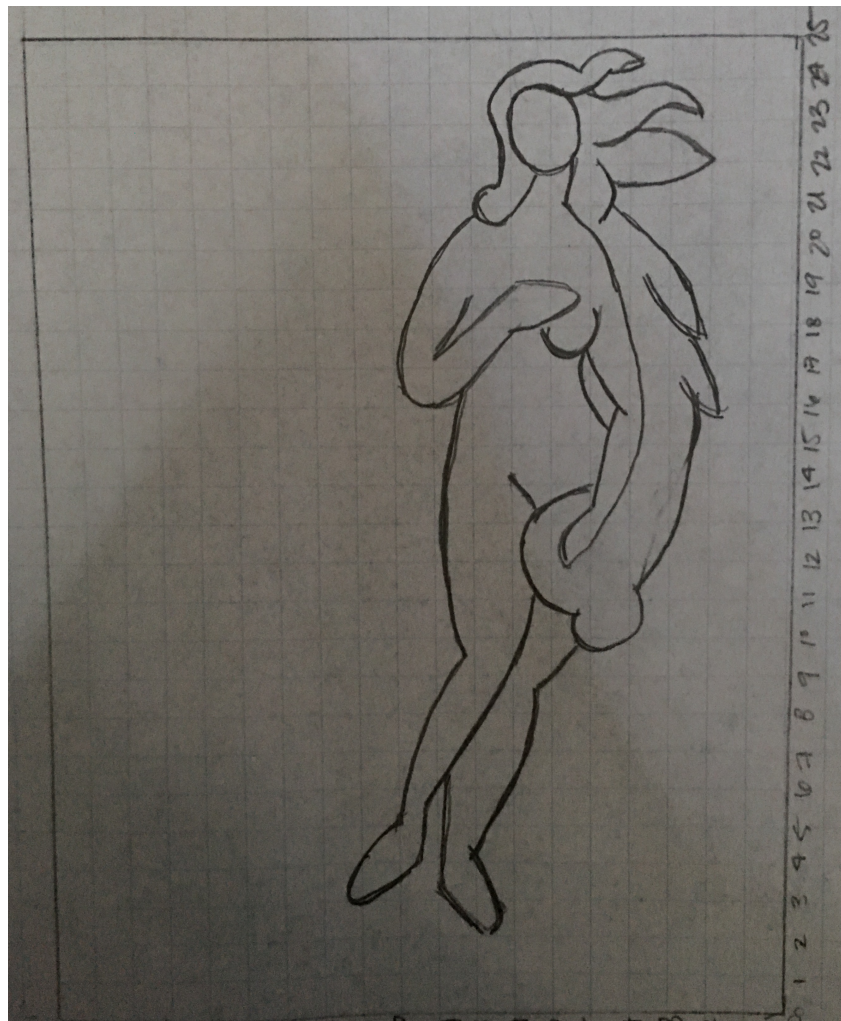

Figure 7

The following is the process through which I each conic equation was obtained. I picked centers, vertices, and radii from the conic image shown in **Figure 7**. From there shifts (a and b values) were calculated. Once the equation was found, the equation had to be solved for y in order to enter the equation into the calculator.

1. Ellipse – Hair

Center: (23, 6)

Vertices: (23, 7.75) & (24.5, 6)

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

$$\frac{(y-6)^2}{1.5^2} + \frac{(x-23)^2}{1.75^2} = 1$$

$$\frac{(y-6)^2}{1.5^2} = 1 - \frac{(x-23)^2}{1.75^2}$$

$$(y-6)^2 = 1.5^2(1 - \frac{(x-23)^2}{1.75^2})$$

$$y - 6 = \pm\sqrt{1.5^2(1 - \frac{(x-23)^2}{1.75^2})}$$

$$y = 6 \pm\sqrt{1.5^2(1 - \frac{(x-23)^2}{1.75^2})}$$

2. Ellipse – Head

Center: (22.75, 6.5)

Vertices: (22.75, 7.1) & (24, 6.5)

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-22.75)^2}{1.25^2} + \frac{(y-6.5)^2}{.6^2} = 1$$

$$\frac{(y-6.5)^2}{.6^2} = 1 - \frac{(x-22.75)^2}{1.25^2}$$

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$$(y - 6.5)^2 = .6^2(1 - \frac{(x - 22.75)^2}{1.25^2})$$

$$y - 6.5 = \pm\sqrt{.6^2(1 - \frac{(x - 22.75)^2}{1.25^2})}$$

$$y = 6.5 \pm \sqrt{.6^2(1 - \frac{(x - 22.75)^2}{1.25^2})}$$

3. Parabola – Hair

Center: (24.75, 4.5)

Vertex: (24.25, 4)

$$x = a(y - k)^2 + h$$

$$24.25 = a(4 - 4.5)^2 + 24.75$$

$$-.5 = a(.5)^2$$

$$a = -2$$

$$x = -2(y - 4.5)^2 + 24.75$$

$$x - 24.75 = -2(y - 4.5)^2$$

$$\frac{x - 24.75}{-2} = (y - 4.5)^2$$

$$\pm\sqrt{\frac{x - 24.75}{-2}} = y - 4.5$$

$$4.5 \pm \sqrt{\frac{x - 24.75}{-2}} = y$$

4. Parabola – Hair

Center: (23.6, 5.5)

Vertex: (24, 5)

$$x = a(y - k)^2 + h$$

$$24 = a(5 - 5.5)^2 + 23.6$$

$$.4 = a(.5)^2$$

$$a = 1.6$$

$$x = 1.6(y - 5.5)^2 + 23.6$$

$$x - 23.6 = 1.6(y - 5.5)^2$$

$$\frac{x - 23.6}{1.6} = (y - 5.5)^2$$

$$\pm\sqrt{\frac{x - 23.6}{1.6}} = y - 5.5$$

$$5.5 \pm \sqrt{\frac{x - 23.6}{1.6}} = y$$

6. Parabola – Hair

Center: (22, 7.1)

Vertex: (23, 7.5)

$$y = a(x - h)^2 + k$$

$$7.5 = a(23 - 22)^2 + 7.1$$

$$.4 = a(1)^2$$

$$.4 = a$$

$$y = .4(x - 22)^2 + 7.1$$

7. Circle - Hair

Center: (21, 7)

Radius: .5

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 21)^2 + (y - 7)^2 = .5^2$$

$$(y - 7)^2 = .5^2 - (x - 21)^2$$

$$y - 7 = \pm\sqrt{.5^2 - (x - 21)^2}$$

$$y = 7 \pm \sqrt{.5^2 - (x - 21)^2}$$

8. Parabola – Neck

Center: (22.5, 6.75)

Vertex: (21, 7)

$$y = a(x - h)^2 + k$$

$$7 = a(21 - 22.5)^2 + 6.75$$

$$.25 = a(-1.5)^2$$

$$.111 = a$$

$$y = .111(x - 22.5)^2 + 6.75$$

9. Parabola – Neck

Center (22.5, 5.75)

Vertex (21, 6)

$$y = a(x - h)^2 + k$$

$$6 = a(21 - 22.5)^2 + 5.75$$

$$.25 = a(-1.5)^2$$

$$.111 = a$$

$$y = .111(x - 22.5)^2 + 5.75$$

10. Parabola – Hair

Center: (21, 4.8)

Vertex: (21.5, 5)

$$y = a(x - h)^2 + k$$

$$5 = a(21.5 - 21)^2 + 4.8$$

$$.2 = a(.5)^2$$

$$.8 = a$$

$$y = .8(x - 21)^2 + 4.8$$

11. Parabola – Hair

Center: (24.5, 4)

Vertex: (24, 5)

$$y = a(x - h)^2 + k$$

$$5 = a(24 - 24.5)^2 + 4$$

$$1 = a(.5)^2$$

$$4 = a$$

$$y = 4(x - 24.5)^2 + 4$$

12. Ellipse – Hair

Center: (24, 4.5)

Vertices: (23.5, 4.5) & (24, 5.25)

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

$$\frac{(y-4.5)^2}{.5^2} + \frac{(x-24)^2}{.75^2} = 1$$

$$\frac{(y-4.5)^2}{.5^2} = 1 - \frac{(x-24)^2}{.75^2}$$

$$(y-4.5)^2 = .5^2\left(1 - \frac{(x-24)^2}{.75^2}\right)$$

$$y - 4.5 = \pm\sqrt{.5^2\left(1 - \frac{(x-24)^2}{.75^2}\right)}$$

$$y = 4.5 \pm \sqrt{.5^2\left(1 - \frac{(x-24)^2}{.75^2}\right)}$$

13. Parabola – Hair

Center: (23.75, 3.5)

Vertex: (23, 2.75)

$$x = a(y-k)^2 + h$$

$$23 = a(2.75 - 3.5)^2 + 23.75$$

$$-.75 = a(-.75)^2$$

$$a = -1.333$$

$$x = -1.333(y - 3.5)^2 + 23.75$$

$$x - 23.75 = -1.333(y - 3.5)^2$$

$$\frac{x - 23.75}{-1.333} = (y - 3.5)^2$$

$$\pm\sqrt{\frac{x - 23.75}{-1.333}} = y - 3.5$$

$$3.5 \pm \sqrt{\frac{x - 23.75}{-1.333}} = y$$

14. Parabola – Hair

Center: (22.5, 4.5)

Vertex: (21.75, 5)

$$x = a(y - k)^2 + h$$

$$21 = a(5 - 4.5)^2 + 22.5$$

$$-1.5 = a(.5)^2$$

$$a = 2$$

$$x = 2(y - 4.5)^2 + 22.5$$

$$x - 22.5 = 2(y - 4.5)^2$$

$$\frac{x - 22.5}{2} = (y - 4.5)^2$$

$$\pm \sqrt{\frac{x - 22.5}{2}} = y - 4.5$$

$$4.5 \pm \sqrt{\frac{x - 22.5}{2}} = y$$

15. Parabola – Hair

Center: (23.25, 3.5)

Vertex: (23, 4)

$$x = a(y - k)^2 + h$$

$$23 = a(4 - 3.5)^2 + 23.25$$

$$-.25 = a(.5)^2$$

$$a = -1$$

$$x = -(y - 3.5)^2 + 23.25$$

$$x - 23.25 = -(y - 3.5)^2$$

$$-x + 23.5 = (y - 3.5)^2$$

$$\pm\sqrt{-x + 23.5} = y - 3.5$$

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$$3.5 \pm \sqrt{-x + 23.5} = y$$

16. Parabola – Hair

Center: (22.75, 3.75)

Vertex: (22, 2.5)

$$x = a(y - k)^2 + h$$

$$22 = a(2.5 - 3.75)^2 + 22.75$$

$$-.75 = a(-1.25)^2$$

$$a = -.48$$

$$x = -.48(y - 3.75)^2 + 22.75$$

$$x - 22.75 = -.48(y - 3.75)^2$$

$$\frac{x - 22.75}{-.48} = (y - 3.75)^2$$

$$\pm\sqrt{\frac{x - 22.75}{-.48}} = y - 3.75$$

$$3.75 \pm \sqrt{\frac{x - 22.75}{-.48}} = y$$

17. Parabola – Hair

Center: (21, 3.5)

Vertex: (21.75, 5)

$$x = a(y - k)^2 + h$$

$$21.75 = a(5 - 3.5)^2 + 21$$

$$.75 = a(1.5)^2$$

$$a = .333$$

$$x = .333(y - 3.5)^2 + 21$$

$$x - 21 = .333(y - 3.5)^2$$

$$\frac{x - 21}{.333} = (y - 3.5)^2$$

$$\pm \sqrt{\frac{x - 21}{.333}} = y - 3.5$$

$$3.5 \pm \sqrt{\frac{x - 21}{.333}} = y$$

18. Ellipse – Shoulder & Arm

Center: (16.5, 7)

Vertices: (16.5, 10) & (21, 7)

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

$$\frac{(x - 16.5)^2}{4.5^2} + \frac{(y - 7)^2}{3^2} = 1$$

$$\frac{(y - 7)^2}{3^2} = 1 - \frac{(x - 16.5)^2}{4.5^2}$$

$$(y - 7)^2 = 3^2(1 - \frac{(x - 16.5)^2}{4.5^2})$$

$$y - 7 = \pm \sqrt{3^2(1 - \frac{(x - 16.5)^2}{4.5^2})}$$

$$y = 7 \pm \sqrt{3^2(1 - \frac{(x - 16.5)^2}{4.5^2})}$$

19. Parabola – Arm

Center: (15.8, 9.5)

Vertex: (18, 7)

$$x = a(y - k)^2 + h$$

$$18 = a(7 - 9.5)^2 + 15.8$$

$$2.2 = a(-2.5)^2$$

$$a = .352$$

$$x = .352(y - 9.5)^2 + 15.8$$

$$x - 15.8 = .352(y - 9.5)^2$$

$$\frac{x - 15.8}{.352} = (y - 9.5)^2$$

$$\pm\sqrt{\frac{x - 15.8}{.352}} = y - 9.5$$

$$9.5 \pm \sqrt{\frac{x - 15.8}{.352}} = y$$

20. Circle – Elbow

Center: (17, 9)

Radius: 1

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 17)^2 + (y - 9)^2 = 1^2$$

$$(y - 9)^2 = 1^2 - (x - 17)^2$$

$$y - 9 = \pm\sqrt{1^2 - (x - 17)^2}$$

$$y = 9 \pm \sqrt{1^2 - (x - 17)^2}$$

21. Parabola – Upper Arm

Center: (19, 8)

Vertex: (17.25, 9)

$$y = a(x - h)^2 + k$$

$$9 = a(17.25 - 19)^2 + 8$$

$$1 = a(-1.75)^2$$

$$.327 = a$$

$$y = .327(x - 19)^2 + 8$$

22. Line – Arm

Vertices: (19, 7) & (21, 5)

$$y = mx + b$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5 - 7}{21 - 19}$$

$$m = \frac{-2}{2}$$

$$m = -1$$

$$y - y_1 = m\,(x - x_1)$$

$$y - 5 = -1(x - 21)$$

$$y - 5 = -x + 21$$

$$y = -x + 26$$

23. Parabola – Hand

Center: (18.75, 5.5)

Vertex: (19, 6.5)

$$y = a(x - h)^2 + k$$

$$6.5 = a(19 - 18.75)^2 + 5.5$$

$$1 = a(.25)^2$$

$$16 = a$$

$$y = 16(x - 18.75)^2 + 5.5$$

24. Circle – Breast

Center: (18.5, 7)

Radius: .75

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 18.5)^2 + (y - 7)^2 = .75^2$$

$$(y - 7)^2 = .75^2 - (x - 18.5)^2$$

$$y - 7 = \pm\sqrt{.75^2 - (x - 18.5)^2}$$

$$y = 7 \pm \sqrt{.75^2 - (x - 18.5)^2}$$

25. Circle – Breast

Center: (18, 5.5)

Radius: .6

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 18)^2 + (y - 5.5)^2 = .6^2$$

$$(y - 5.5)^2 = .6^2 - (x - 18)^2$$

$$y - 5.5 = \pm\sqrt{.6^2 - (x - 18)^2}$$

$$y = 5.5 \pm \sqrt{.6^2 - (x - 18)^2}$$

26. Ellipse – Body

Center: (15, 7)

Vertices: (21.5, 7) & (15, 3.75)

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

$$\frac{(x - 15)^2}{6.5^2} + \frac{(y - 7)^2}{3.25^2} = 1$$

$$\frac{(y - 7)^2}{3.25^2} = 1 - \frac{(x - 15)^2}{6.5^2}$$

$$(y - 7)^2 = 3.25^2(1 - \frac{(x - 15)^2}{6.5^2})$$

$$y - 7 = \pm\sqrt{3.25^2(1 - \frac{(x - 15)^2}{6.5^2})}$$

$$y = 7 \pm \sqrt{3.25^2(1 - \frac{(x - 15)^2}{6.5^2})}$$

27. Hyperbola – Body Curve

Center: (16.5, 5.75)

a = .25

b = .5

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$

$$\frac{(y - 5.75)^2}{.25^2} - \frac{(x - 16.5)^2}{.5^2} = 1$$

$$\frac{(y - 5.75)^2}{.25^2} = 1 + \frac{(x - 16.5)^2}{.5^2}$$

$$(y - 5.75)^2 = .25^2(1 + \frac{(x - 16.5)^2}{.5^2})$$

$$y - 5.75 = \pm\sqrt{.25^2(1 + \frac{(x - 16.5)^2}{.5^2})}$$

$$y = 5.75 \pm \sqrt{.25^2(1 + \frac{(x - 16.5)^2}{.5^2})}$$

28. Ellipse – Upper Arm

Center: (17.5, 3)

Vertices: (17.5, 5) & (15, 3)

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

$$\frac{(x - 17.5)^2}{1.5^2} + \frac{(y - 3)^2}{2^2} = 1$$

$$\frac{(y - 3)^2}{2^2} = 1 - \frac{(x - 17.5)^2}{1.5^2}$$

$$(y - 3)^2 = 2^2(1 - \frac{(x - 17.5)^2}{1.5^2})$$

$$y - 3 = \pm\sqrt{2^2(1 - \frac{(x - 17.5)^2}{1.5^2})}$$

$$y = 3 \pm \sqrt{2^2(1 - \frac{(x - 17.5)^2}{1.5^2})}$$

29. Ellipse – Forearm

Center: (14, 3)

Vertices: (16.25, 3) & (14, 5)

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

$$\frac{(x - 14)^2}{2.25^2} + \frac{(y - 3)^2}{2^2} = 1$$

$$\frac{(y - 3)^2}{2^2} = 1 - \frac{(x - 14)^2}{2.25^2}$$

$$(y-3)^2 = 2^2\left(1 - \frac{(x-14)^2}{2.25^2}\right)$$

$$y - 3 = \pm\sqrt{2^2\left(1 - \frac{(x-14)^2}{2.25^2}\right)}$$

$$y = 3 \pm \sqrt{2^2\left(1 - \frac{(x-14)^2}{2.25^2}\right)}$$

30. Parabola – Hand

Center: (11.75, 5.75)

Vertex: (14, 7)

$$x = a(y-k)^2 + h$$

$$14 = a(7 - 5.75)^2 + 11.75$$

$$2.25 = a(1.25)^2$$

$$a = 1.44$$

$$x = 1.44(y - 5.75)^2 + 11.75$$

$$x - 11.75 = 1.44(y - 5.75)^2$$

$$\frac{x - 11.75}{1.44} = (y - 5.75)^2$$

$$\pm\sqrt{\frac{x - 11.75}{1.44}} = y - 5.75$$

$$5.75 \pm \sqrt{\frac{x - 11.75}{1.44}} = y$$

31. Ellipse – Body

Center: (12, 2)

Vertices: (24, 2) & (12, 9)

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-12)^2}{11^2} + \frac{(y-2)^2}{7^2} = 1$$

$$\frac{(y-2)^2}{7^2} = 1 - \frac{(x-12)^2}{11^2}$$

$$(y-2)^2 = 7^2(1 - \frac{(x-12)^2}{11^2})$$

$$y - 2 = \pm\sqrt{7^2(1 - \frac{(x-12)^2}{11^2})}$$

$$y = 2 \pm\sqrt{7^2(1 - \frac{(x-12)^2}{11^2})}$$

32. Ellipse – Thigh

Center: (11.5, 10)

Vertices: (16, 10) & (11.5, 13.5)

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-11.5)^2}{4.5^2} + \frac{(y-10)^2}{3.5^2} = 1$$

$$\frac{(y-10)^2}{3.5^2} = 1 - \frac{(x-11.5)^2}{4.5^2}$$

$$(y-10)^2 = 3.5^2(1 - \frac{(x-11.5)^2}{4.5^2})$$

$$y - 10 = \pm\sqrt{3.5^2(1 - \frac{(x-11.5)^2}{4.5^2})}$$

$$y = 10 \pm\sqrt{3.5^2(1 - \frac{(x-11.5)^2}{4.5^2})}$$

33. Parabola – Calve

Center: (5, 10)

Vertex: (10, 7)

$$y = a(x-h)^2 + k$$

$$7 = a(10-5)^2 + 10$$

$$-3 = a(5)^2$$

$$-.12 = a$$

$$y = -.12(x - 5)^2 + 10$$

34. Line – Calve

Vertices: (0, 12.5) & (1, 11.9)

$$y = mx + b$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{11.9 - 12.5}{1 - 0}$$

$$m = \frac{-.6}{1}$$

$$m = -.6$$

$$y - y_1 = m\,(x - x_1)$$

$$y - 12.5 = -.6(x - 0)$$

$$y - 12.5 = -.6x$$

$$y = -.6x + 12.5$$

35. Parabola – Thigh

Center: (13, 4)

Vertex: (9, 6)

$$y = a(x - h)^2 + k$$

$$6 = a(9 - 13)^2 + 4$$

$$2 = a(-4)^2$$

$$.125 = a$$

$$y = .125(x - 13)^2 + 4$$

36. Parabola – Calve

Center: (11, 6)

Vertex: (2, 10)

$$y = a(x - h)^2 + k$$

$$10 = a(2 - 11)^2 + 6$$

$$4 = a(-9)^2$$

$$.049 = a$$

$$y = .049(x - 11)^2 + 6$$

37. Line - Calve

Vertices: (0, 9.75) & (10, 7.75)

$$y = mx + b$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{7.75 - 9.75}{10 - 0}$$

$$m = \frac{-2}{10}$$

$$m = -.2$$

$$y - y_1 = m\ (x - x_1)$$

$$y - 9.75 = -.2(x - 0)$$

$$y - 9.75 = -.2x$$

$$y = -.2x + 9.75$$

38. Line – Foot

Vertices: (3, 12) & (5, 10)

$$y = mx + b$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{10 - 12}{5 - 3}$$

$$m = \frac{-2}{2}$$

$$m = -1$$

$$y - y_1 = m\ (x - x_1)$$

$$y - 10 = -1(x - 5)$$

$$y - 10 = -x + 5$$

$$y = -x + 15$$

39. Circle – Foot

Center: (3.25, 11.1)

Radius: .25

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 3.25)^2 + (y - 11.1)^2 = .25^2$$
$$(y - 11.1)^2 = .25^2 - (x - 3.25)^2$$
$$y - 11.1 = \pm\sqrt{.25^2 - (x - 3.25)^2}$$
$$y = 11.1 \pm \sqrt{.25^2 - (x - 3.25)^2}$$

40. Parabola – Foot

Center: (5, 9.5)

Vertex: (2.8, 11)

$$y = a(x - h)^2 + k$$
$$11 = a(2.8 - 5)^2 + 9.5$$
$$1.5 = a(-2.2)^2$$
$$.31 = a$$
$$y = .31(x - 5)^2 + 9.5$$

41. Parabola – Foot

Center: (3, 9)

Vertex: (2, 7.5)

$$y = a(x - h)^2 + k$$
$$7.5 = a(2 - 3)^2 + 9$$
$$-1.5 = a(-1)^2$$
$$-1.5 = a$$
$$y = -1.5(x - 3)^2 + 9$$

42. Parabola – Foot

Center: (2.3, 7.1)

Vertex: (5, 10)

$$y = a(x - h)^2 + k$$

$$10 = a(5 - 2.3)^2 + 7.1$$

$$2.9 = a(2.7)^2$$

$$.398 = a$$

$$y = .398(x - 2.3)^2 + 7.1$$

43. Parabola – Hair

Center: (15, 1.5)

Vertex: (21, 5)

$$y = a(x - h)^2 + k$$

$$5 = a(21 - 15)^2 + 1.5$$

$$3.5 = a(6)^2$$

$$.097 = a$$

$$y = .097(x - 15)^2 + 1.5$$

44. Parabola – Hair

Center: (19, 3.5)

Vertex: (17.5, 2)

$$y = a(x - h)^2 + k$$

$$2 = a(17.5 - 19)^2 + 3.5$$

$$-1.5 = a(-1.5)^2$$

$$-.667 = a$$

$$y = -.667(x - 19)^2 + 3.5$$

45. Parabola – Hair

Center: (15, 1.5)

Vertex: (19, 5)

$$y = a(x - h)^2 + k$$

$$5 = a(19 - 15)^2 + 1.5$$

$$3.5 = a(4)^2$$

$$.219 = a$$

$$y = .219(x - 15)^2 + 1.5$$

46. Parabola – Hair

Center: (17, 3)

Vertex: (15, 1)

$$y = a(x - h)^2 + k$$
$$1 = a(15 - 17)^2 + 3$$
$$-2 = a(2)^2$$
$$-.5 = a$$
$$y = -.5(x - 17)^2 + 3$$

47. Parabola – Hair

Center: (16, 2.5)

Vertex: (12, 5)

$$y = a(x - h)^2 + k$$
$$5 = a(12 - 16)^2 + 2.5$$
$$2.5 = a(4)^2$$
$$.156 = a$$
$$y = .156(x - 16)^2 + 2.5$$

48. Line – Hair

Vertices: (9, 1) & (14, 3)

$$y = mx + b$$
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{3 - 1}{14 - 9}$$
$$m = \frac{2}{5}$$
$$m = .4$$
$$y - y_1 = m\,(x - x_1)$$
$$y - 1 = .4(x - 9)$$
$$y - 1 = .6x - 3.6$$

$$y = .6x - 2.6$$

49. Parabola – Hair

Center: (16, 2.5)

Vertex: (11, 4)

$$y = a(x - h)^2 + k$$
$$4 = a(11 - 16)^2 + 2.5$$
$$1.5 = a(5)^2$$
$$.06 = a$$
$$y = .06(x - 16)^2 + 2.5$$

50. Circle – Hair

Center: (10.1, 4.8)

Radius: .7

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 10.1)^2 + (y - 4.8)^2 = .7^2$$
$$(y - 4.8)^2 = .7^2 - (x - 10.1)^2$$
$$y - 4.8 = \pm\sqrt{.7^2 - (x - 10.1)^2}$$
$$y = 4.8 \pm \sqrt{.7^2 - (x - 10.1)^2}$$

51. Circle – Hair

Center: (12, 5)

Radius: 1.5

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 12)^2 + (y - 5)^2 = 1.5^2$$
$$(y - 5)^2 = 1.5^2 - (x - 12)^2$$
$$y - 5 = \pm\sqrt{1.5^2 - (x - 12)^2}$$
$$y = 5 \pm \sqrt{1.5^2 - (x - 12)^2}$$

Below is a transcript of my finalized calculator program. The equations entered into the calculator were found in the previous section. Only equations solved for y can be entered into the calculator and can be graphed correctly, so only the final step from each of the previous conics will be incorporated in this section.

The first eight lines of the program (seen below) construct the calculator screen for optimal image viewing. The first line clears previous drawings, so that the program starts with a clean screen. The second line of code takes the axes off the image, so that the x- and y- axis will not be shown in the image. The third line allows for equations to be graphed that are not functions, with function defining a graph that has no repeating y-values. Next, the PlotsOff feature insures that the graphed lines will be continuous. Finally, lines 5 through 8 set the window of the image. The window is important because it tells the calculator the proportions between the x- and y-axes, as well as the section of the coordinate plane will be graphed.

Likewise, another interesting part of the programming of the calculator is the implementation of domain restrictions. Essentially, most of these graphs have domains that extend for large, and even infinite, intervals, so in order to get the desired section of the graph, it is necessary to tell the calculator precisely which section to use. In this section, those restrictions can be seen as the $x \geq and \ x \leq$ sections of the program lines. This is telling the calculator a specific interval to graph. This cuts down on unnecessary frivolous drawings throughout the program.

Furthermore, it is worth noting that the actual entry of the program onto the calculator will not be as easily understood. For instance, line 9 of the program (seen below) depicts the upper half of an ellipse. Written out, it is easily understood and

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deciphered. However, in order to have the calculator accurately read the equation, it must

be entered quite differently. **Figure 8** below depicts a screenshot of the calculator

depicting the same line of program. It is worth noting that the equation is the same, just

with extra precautionary measures to ensure the order of operations is followed correctly.

```
PROGRAM:BOV
:DrawF (√(2.25(1
-(((X-23)^2)/(3.
063))))+6)/(X≥22
.9 and X≤24.61)
:DrawF (-(2.25(1
-(((X-23)^2)/(3.
063))))+6)/(X≥24
```

Moreover, there are instances that when rewritten in y= form, the equation has the

"plus-or-minus" (±) sign. In this case, when entered into the calculator, the equation will

be broken into two separate equations, one with a plus in place of that sign, and the

second with a minus in place of that sign. This is because some conics when graphed do

not satisfy the definition of a function and repeat y-values. In order to incorporate the

duplicate y-values, the graph is broken into an upper and lower half. In addition, there are

instances that only the top or half of the shape is used in the image. In these cases, there

will only be one equation representing that conic.

PROGRAM:BOV

1. :ClrDraw
2. :AxesOff
3. :FnOff
4. :PlotsOff
5. :01->Xmin
6. :25->Xmax
7. :00->Ymin
8. :15->Ymax
9. :DrawF $(6 + \sqrt{1.5^2(1 - \frac{(x-23)^2}{1.75^2})})/(x \geq 23 \ and \ x \leq 24.5)$

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10. :DrawF $(6 - \sqrt{1.5^2 \left(1 - \frac{(x-23)^2}{1.75^2}\right)})/(x \geq 24.25 \ and \ x \leq 24.75)$

11. :DrawF $6.5 + \sqrt{.6^2(1 - \frac{(x-22.75)^2}{1.25^2})}$

12. :DrawF $6.5 - \sqrt{.6^2(1 - \frac{(x-22.75)^2}{1.25^2})}$

13. :DrawF $(4.5 + \sqrt{\frac{x-24.75}{-2}})/(x \geq 24.5 \ and \ x \leq 25)$

14. :DrawF $(4.5 - \sqrt{\frac{x-24.75}{-2}})/(x \geq 24.5 \ and \ x \leq 25)$

15. :DrawF $(5.5 + \sqrt{\frac{x-23.6}{1.6}})/(x \geq 23.6 \ and \ x \leq 24.25)$

16. :DrawF $(5.5 - \sqrt{\frac{x-23.6}{1.6}})/(x \geq 23.6 \ and \ x \leq 24)$

17. :DrawF $(.4(x-22)^2 + 7.1)/(x \geq 21.25 \ and \ x \leq 23)$

18. :DrawF $(7 + \sqrt{.5^2 - (x-21)^2})/(x \geq 20.5 \ and \ x \leq 21.5)$

19. :DrawF $(7 - \sqrt{.5^2 - (x-21)^2})/(x \geq 20.5 \ and \ x \leq 21.5)$

20. :DrawF $(.111(x-22.5)^2 + 6.75)/(x \geq 20.5 \ and \ x \leq 21.1)$

21. :DrawF $(.111(x-22.5)^2 + 5.75)/(x \geq 20.75 \ and \ x \leq 22)$

22. :DrawF $(.8(x-21)^2 + 4.8)/(x \geq 20.25 \ and \ x \leq 21.5)$

23. :DrawF $(4(x-24.5)^2 + 4)/(x \geq 24 \ and \ x \leq 24.5)$

24. :DrawF $(4.5 + \sqrt{.5^2(1 - \frac{(x-24)^2}{.75^2})})/(x \geq 23.25 \ and \ x \leq 23.6)$

25. :DrawF $(4.5 - \sqrt{.5^2 \left(1 - \frac{(x-24)^2}{.75^2}\right)})/(x \geq 23.25 \ and \ x \leq 24)$

26. :DrawF $(3.5 + \sqrt{\frac{x-23.75}{-1.333}})/(x \geq 24 \ and \ x \leq 24.5)$

27. :DrawF $(3.5 - \sqrt{\frac{x-23.75}{-1.333}})/(x \geq 23 \ and \ x \leq 23.75)$

28. :DrawF $(4.5 + \sqrt{\frac{x-22.5}{2}})/(x \geq 22 \ and \ x \leq 22.7)$

29. :DrawF $(4.5 - \sqrt{\frac{x-22.5}{2}})/(x \geq 22 \ and \ x \leq 23)$

30. :DrawF $(3.5 + \sqrt{-x + 23.5})/(x \geq 23 \ and \ x \leq 24)$

31. :DrawF $(3.5 - \sqrt{-x + 23.5})/(x \geq 23 \ and \ x \leq 24)$

32. :DrawF $(3.75 + \sqrt{\frac{x-22.75}{-.48}})/(x \geq 22.5 \ and \ x \leq 22.75)$

33. :DrawF $(3.75 - \sqrt{\frac{x-22.75}{-.48}})/(x \geq 22 \ and \ x \leq 23.5)$

34. :DrawF $(3.5 + \sqrt{\frac{x-21}{.333}})/(x \geq 21 \ and \ x \leq 22.5)$

35. :DrawF $(3.5 - \sqrt{\frac{x-21}{.333}})/(x \geq 21 \ and \ x \leq 23.5)$

36. :DrawF $(7 + \sqrt{3^2(1 - \frac{(x-16.5)^2}{4.5^2})})/(x \geq 17 \ and \ x \leq 21)$

37. :DrawF $(9.5 - \sqrt{\frac{x-15.8}{.352}})/(x \geq 15.8 \ and \ x \leq 18)$

38. :DrawF $(9 + \sqrt{1^2 - (x - 17)^2})/(x \geq 0 \ and \ x \leq 16)$
39. :DrawF $(.327(x - 19)^2 + 8)/(x \geq 17.25 \ and \ x \leq 18.9)$
40. :DrawF $(-x + 26)/(x \geq 17.25 \ and \ x \leq 19.25)$
41. :DrawF $(16(x - 18.75)^2 + 5.5)/(x \geq 18 \ and \ x \leq 19)$
42. :DrawF $(7 + \sqrt{.75^2 - (x - 18.5)^2})/(x \geq 18.2 \ and \ x \leq 19)$
43. :DrawF $(5.5 + \sqrt{.6^2 - (x - 18)^2})/(x \geq 16 \ and \ x \leq 20)$
44. :DrawF $(5.5 - \sqrt{.6^2 - (x - 18)^2})/(x \geq 17 \ and \ x \leq 20)$

45. :DrawF $(7 - \sqrt{3.25^2\left(1 - \frac{(x-15)^2}{6.5^2}\right)})/(x \geq 11.2 \ and \ x \leq 21)$

46. :DrawF $(5.75 - \sqrt{.25^2\left(1 + \frac{(x-16.5)^2}{.5^2}\right)})/(x \geq 15.75 \ and \ x \leq 17)$

47. :DrawF $(3 + \sqrt{2^2\left(1 - \frac{(x-17.5)^2}{1.5^2}\right)})/(x \geq 16 \ and \ x \leq 17.5)$

48. :DrawF $(3 + \sqrt{2^2\left(1 - \frac{(x-14)^2}{2.25^2}\right)})/(x \geq 13 \ and \ x \leq 16)$

49. :DrawF $(5.75 + \sqrt{\frac{x-11.75}{1.44}})/(x \geq 11 \ and \ x \leq 12.5)$

50. :DrawF $(2 + \sqrt{7^2\left(1 - \frac{(x-12)^2}{11^2}\right)})/(x \geq 8.5 \ and \ x \leq 16.5)$

51. :DrawF $(10 - \sqrt{3.5^2\left(1 - \frac{(x-11.5)^2}{4.5^2}\right)})/(x \geq 8 \ and \ x \leq 11)$

52. :DrawF $(10 - \sqrt{3.5^2\left(1 - \frac{(x-11.5)^2}{4.5^2}\right)})/(x \geq 13 \ and \ x \leq 14.2)$

53. :DrawF $(-.12(x - 5)^2 + 10)/(x \geq 5 \ and \ x \leq 9)$
54. :DrawF $(-.6x + 12.5)/(x \geq 5 \ and \ x \leq 8)$
55. :DrawF $(.125(x - 13)^2 + 4)/(x \geq 8.5 \ and \ x \leq 9.8)$
56. :DrawF $(.049(x - 11)^2 + 6)/(x \geq 4.25 \ and \ x \leq 8.5)$
57. :DrawF $(-.2x + 9.75)/(x \geq 3 \ and \ x \leq 6.5)$
58. :DrawF $(-x + 15)/(x \geq 3.5 \ and \ x \leq 5)$
59. :DrawF $(11.1 + \sqrt{.25^2 - (x - 3.25)^2})/(x \geq 2 \ and \ x \leq 3.5)$
60. :DrawF $(11.1 - \sqrt{.25^2 - (x - 3.25)^2})/(x \geq 2 \ and \ x \leq 3)$
61. :DrawF $(.31(x - 5)^2 + 9.5)/(x \geq 3 \ and \ x \leq 5)$
62. :DrawF $(-1.5(x - 3)^2 + 9)/(x \geq 2 \ and \ x \leq 3)$
63. :DrawF $(.398(x - 2.3)^2 + 7.1)/(x \geq 2 \ and \ x \leq 4)$
64. :DrawF $(.097(x - 15)^2 + 1.5)/(x \geq 17.25 \ and \ x \leq 21)$

65. :DrawF $(-.667(x - 19)^2 + 3.5)/(x \geq 17.25 \text{ and } x \leq 19)$
66. :DrawF $(.219(x - 15)^2 + 1.5)/(x \geq 15.5 \text{ and } x \leq 18)$
67. :DrawF $(-.5(x - 17)^2 + 3)/(x \geq 15.5 \text{ and } x \leq 16.8)$
68. :DrawF $(.156(x - 16)^2 + 2.5)/(x \geq 14 \text{ and } x \leq 16)$
69. :DrawF $(.6x - 2.6)/(x \geq 14 \text{ and } x \leq 15)$
70. :DrawF $(.06(x - 16)^2 + 2.5)/(x \geq 11 \text{ and } x \leq 14.5)$
71. :DrawF $(4.8 + \sqrt{.7^2 - (x - 10.1)^2})/(x \geq 9 \text{ and } x \leq 11)$
72. :DrawF $(4.8 - \sqrt{.7^2 - (x - 10.1)^2})/(x \geq 9 \text{ and } x \leq 10.5)$
73. :DrawF $(5 + \sqrt{1.5^2 - (x - 12)^2})/(x \geq 10 \text{ and } x \leq 15)$
74. :DrawF $(5 - \sqrt{1.5^2 - (x - 12)^2})/(x \geq 10.25 \text{ and } x \leq 12)$

After the completion of the program, if everything is entered correctly the image will be drawn on the calculator. However, throughout the process the student can execute the program to make sure images are graphed correctly. It is recommended to execute the program after every addition of a new equation. Unfortunately, the process of drawing the image on the calculator is rather sluggish, so it makes the process even longer. However, this precautionary measure expedites the process and insures all conics are entered correctly. This is the most daunting part of the process, but it does teach students the importance of detail in mathematics. The completed program of *The Birth of Venus* can be seen below in **Figure 8**.
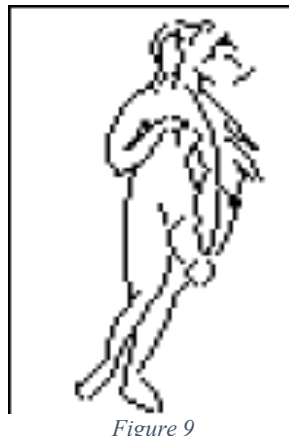

Figure 9

Throughout the process of this project, there were many pros but also cons. In order to discuss the effectiveness of this being implemented in a secondary classroom, a log was kept of thoughts throughout the process. The subsequent section will serve as a conclusive discussion of the principles motivating the integration of art and mathematics.

Conclusion

Throughout the duration of this project, it was a learning experience with various pros and cons. For example, some of the most important points to touch upon are the calculator graphics, the intricacy of the specified images, instantly discovering errors, self-correction, and the involvement of multi-media.

On one hand, there are some cons. As aforementioned it is worth noting the graphics on the calculator screen. When drawing the image on graph paper prior to the calculator programming, the students are encouraged to make an intricate diagram. However, upon programming the calculator, the easily identifiable features on the graph paper get muddled on the calculator screen. Even though the equations are solved correctly, they may not be depicted as perfectly as intended. Furthermore, on conics that do not follow the typical definition of a function (circles, sideways parabolas, ellipses, for example) tend to have sections that do not graph correctly. This is due to the nature of the calculator screen, and although the student may complete every step correctly, the calculator may not generate the correct image. Finally, another point worth mentioning in regards to calculator graphics is the window of the program. The calculator screen is wider than it is tall, so the window of the program must compensate for this discretion. The scale of width to height is 1.5:1 so the students must be made aware of this in the pre-drawing phase of this project.

Next, another downfall of this project was the intricacy of the conic image. The art that is intended to be portrayed is incredibly detailed, and it is possible to reduce the picture down to utilize conics. However, similar to the graphics issue, it is incredibly challenging to capture the essence portrayed in the initial image. In order to create a

cohesive image, the drawing must be simplified a great deal. This is apparent in the above **Figure 6** and **Figure 7. Figure 6** depicts an image of the original *The Birth of Venus*, displayed in Florence. However, when the image was transcribed onto graph paper, in order to capture Venus's whole body as well as meet the proportional requirements of the calculator window, the details in the background and facial features for example are sacrificed. This is an issue because it does not accurately depict the artistic renderings, yet it does instill the mathematics ideal which is the heart of the project.

On the other hand, there are also some tremendous perks to this assignment. For one, the students have the opportunity to instantly see errors. Typically, in mathematics classes, students attempt work and turn in worksheets without knowing if their work was executed correctly. Furthermore, even if they discover their answer is wrong, they may not even realize what issues they have made. However, with this project, students will graph their equations and see the equations drawn out before them. This is important because it lets students not only to see their issues, but allows for the trial and error method to fix their mistakes and create a better final project. This ability to visualize and correct mistakes is unique to this assignment and imperative for student learning.

Finally, another interesting aspect of this assignment is the integration of multimedia. Students integrate mathematics and art by recreating artistic images with mathematical concepts. This is important because as aforementioned it engages various intelligences. Furthermore, the integration of technology is very important. It seems that often times in a high school classroom, students are either taught to find answers solely on the calculator, or are discouraged from using the calculator altogether. However, with

the implementation of this project, students will use their calculators in a constructive manner, that shows student a different perspective of the calculator capabilities. It if thought that the integration of technology in the classroom is useful in conveying content; this lesson is an efficient way to do this.

Conclusively, this project is very tedious and time-consuming with an extreme attention to detail. However, it engages students with various different strategies and interests. It teaches students that mathematics is not simply solving for x and graphing, but instead can be applied virtually anywhere. Mathematics is thought of as computations and quantities, however the ultimate goal is that students realize that mathematics can be seen throughout the world around them and using art as a singular example of this could open their eyes to the endless possibilities.

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