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CONTINUOUS DEPENDENCE FOR THE BRINKMAN EQUATIONS OF FLOW IN DOUBLE-DIFFUSIVE CONVECTION

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Abstract. This paper concerns the structural stability for convective motion in a fluid-saturated porous medium under the Brinkman scheme. Continuous dependence for the solutions on the gravity coefficients and the Soret coefficient are proved. First of all, an a priori bound in L^2 norm is derived whereby we show the solution depends continuously in L^2 norm on changes in the gravity coefficients and the Soret coefficient. This estimate also implies that the solutions decay exponentially.

1. Introduction

Within the context of fluid flow in porous media, or simply within theory of fluid flow, there has been substantial recent interest in deriving stability estimates where changes in coefficient are allowed, or even the model (the equations themselves) changes. This type of stability has earned the name structural stability, and is different from continuous dependence on the initial data. The concept of structural stability in which the study of continuous dependence (or stability) is on changes in the model itself rather than the initial data. Thus structural stability constitutes a class of stability problem every bit important. Structural stability is focus of attention now, and for the relevant results, the reader is referred to [1, 2, 3, 5, 6, 7, 8, 9, 10].

The Brinkman model is believed accurate when the flow velocity is too large for Darcy's law to be valid, and additionally the porosity is not too small. In this article, we are concerned with structural stability for the Brinkman equations modeling the double diffusive convection. The temperature field is non-constant and a solute is also diffused throughout the porous body.

The Brinkman equations governing the flow of fluid in double-diffusive convection are

$$\begin{gathered} u_{i,t} = \nu\Delta u_i - a u_i - p_{,i} + g_i T + h_i C \\ T_{,t} + u_i T_{,i} = \Delta T \\ C_{,t} + u_i C_{,i} = \Delta C + \rho \Delta T \quad \text{in } \Omega \times t > 0 \\ u_{i,i} = 0 \end{gathered} \tag{1.1}$$

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where u_i, T, C and p represent fluid velocity, temperature, salt concentration and pressure, respectively. The quantities $g_i(x)$ and $h_i(x)$ are gravity vector terms, and positive constants ρ, a are known the Soret coefficient and the Darcy coefficient, respectively. Ω is a bounded domain of of R^3, and with sufficiently smooth boundary $\partial\Omega$. Δ is the Laplace operator, $\|\cdot\|$ and $\langle u, v\rangle$ denote the norm and inner product on $L^2(\Omega)$.

Associated with (1.1), we impose the boundary data

$$u_i = 0; \quad T = 0; C = 0 \quad \text{on } \partial\Omega \tag{1.2}$$

and the initial data

$$u_i(x,0) = z_i(x); T(x,0) = T_0(x); C(x,0) = C_0(x); x \in \Omega \tag{1.3}$$

In (1.1) and in the equations throughout, a comma is used to denote partial differentiation: For example $u_{i,i}$ denotes $\frac{\partial u_i}{\partial x_i}$ and $u_{i,t}$ denotes $\frac{\partial u_i}{\partial t}$, and we employ the convention of summing over repeated indices from 1 to 3 .

2. A priori bounds

Multiplying $(1.1)_2$ by T and integrating over Ω, we have

$$\frac{1}{2}\frac{d}{dt}\|T\|^2 = -\|\nabla T\|^2 \tag{2.1}$$

Multiplying $(1.1)_3$ by C and integrating over Ω, then using arithmetic-geometric mean inequality, we obtain

$$\frac{1}{2}\frac{d}{dt}\|C\|^2 + \frac{1}{2}\|\nabla C\|^2 \le \frac{\rho^2}{2}\|\nabla T\|^2 \tag{2.2}$$

Multiplying $(1.1)_1$ by u_i and integrating over Ω, furthermore using Cauchy-Schwarz, arithmetic-geometric mean, then using Poincare's inequality, we find

$$\begin{aligned} \frac{1}{2}\frac{d}{dt}\|u\|^2 + \nu\|\nabla u\|^2 + \frac{a}{2}\|u\|^2 &\le \frac{1}{a}[g^2\|T\|^2 + h^2\|C\|^2] \\ &\le \frac{1}{a\lambda_1}[g^2\|\nabla T\|^2 + h^2\|\nabla C\|^2] \end{aligned} \tag{2.3}$$

where

$$g^2 = \max_\Omega g_i g_i; \quad h^2 = \max_\Omega h_i h_i.$$

and λ_1 is the first eigenvalue of the problem

$$\begin{gathered} \Delta\phi + \lambda\phi = 0 \quad \text{in } \Omega \\ \phi = 0 \quad \text{on } \partial\Omega \end{gathered}$$

Multiplying $(1.1)_1$ by $u_{i,t}$ and integrating over Ω , furthermore using Cauchy-Schwarz and arithmetic-geometric mean inequality, then using Poincare's Poincare's inequality, we find

$$\begin{aligned} \frac{a}{2}\frac{d}{dt}\|u\|^2 + \frac{\nu}{2}\frac{d}{dt}\|\nabla u\|^2 &\le \frac{1}{2}[g^2\|T\|^2 + h^2\|C\|^2] \\ &\le \frac{1}{2\lambda_1}[g^2\|\nabla T\|^2 + h^2\|\nabla C\|^2] \end{aligned} \tag{2.4}$$

Multiplying (2.1) by Γ_{11}, (2.2) by Γ_{12} and (2.3) by Γ_{13}, then adding all results to (2.4) leads to

$$\frac{d}{dt}Q_1(t) + G(t) \le 0 \tag{2.5}$$

where $\Gamma_{1i}, (i = 1, 2, 3)$ are positive constants at our disposal,

$$Q_1(t) = \frac{\Gamma_{11}}{2}\|T\|^2 + \frac{\Gamma_{12}}{2}\|C\|^2 + \frac{(\Gamma_{13} + a)}{2}\|u\|^2 + \frac{\nu}{2}\|\nabla u\|^2 \tag{2.6}$$

and

$$\begin{aligned} G(t) = [&\frac{(2\Gamma_{11} - \rho^2\Gamma_{12})}{2} - \frac{1}{\lambda_1}g^2(\frac{1}{2} + \frac{\Gamma_{13})}{a}]\|\nabla T\|^2 + \frac{a\Gamma_{13}}{2}\|u\|^2 \\ &+ [\frac{\Gamma_{12}}{2} - \frac{1}{\lambda_1 1}h^2(\frac{1}{2} + \frac{\Gamma_{13})}{a}]\|\nabla C\|^2 + \Gamma_{13}\nu\|\nabla u\|^2 \end{aligned} \tag{2.7}$$

We can select $\Gamma_{1i}, (i = 1, 2, 3)$ to secure that all the all the coefficients of (2.7) are positive, such as

$$\Gamma_{13} = \frac{a}{2}; \quad \Gamma_{12} = \frac{4h^2}{\lambda_1}; \quad \Gamma_{11} > \frac{(2h^2\rho^2 + g^2)}{\lambda_1}.$$

Thus, with the help of Poincare's inequality, we can show that

$$\begin{aligned} G(t) \geq [&\lambda_1\frac{(2\Gamma_{11} - \rho^2\Gamma_{12})}{2} - g^2(\frac{1}{2} + \frac{\Gamma_{13})}{a}]\|T\|^2 + \Gamma_{13}\nu\|\nabla u\|^2 \\ &+ [\frac{2\lambda_1\Gamma_{12}}{2} - h^2(\frac{1}{2} + \frac{\Gamma_{13}}{a}]\|C\|^2 + \Gamma_{13}\frac{a}{2}\|u\|^2 \end{aligned} \tag{2.8}$$

We can easily show that

$$\kappa_1 Q_1(t) \leq G(t) \tag{2.9}$$

where κ_1 is a positive constant represented by

$$\begin{aligned} \kappa_1 = \min\Big\{&\frac{1}{\Gamma_{11}}[\lambda_1(2\Gamma_{11} - \rho^2\Gamma_{12}) - g^2(1 + \frac{2\Gamma_{13})}{a}], \\ &\frac{1}{\Gamma_{12}}[\Gamma_{12}\lambda_1 - h^2(1 + \frac{2\Gamma_{13})}{a}], \frac{a\Gamma_{13}}{\Gamma_{13} + a}, 2\Gamma_{13}\Big\} \end{aligned}$$

Thus, from (2.5), we can derive that

$$\frac{d}{dt}Q_1(t) + \kappa_1 Q_1(t) \leq 0 \tag{2.10}$$

Furthermore, we obtain

$$Q_1(t) \leq Q_1(0)e^{-\kappa_1 t} \tag{2.11}$$

Therefore, recalling the definition of $Q_1(t)$ and combining (2.5), we can obtain

$$\begin{gathered} \|T\|^2 \leq M_1; \quad \|C\|^2 \leq M_1; \quad \|u\|^2 \leq M_1; \quad \|\nabla u\|^2 \leq M_1; \quad \int_0^t \|u\|^2 d\eta \leq M_1; \\ \int_0^t \|\nabla u\|^2 d\eta \leq M_1; \quad \int_0^t \|\nabla T\|^2 d\eta \leq M_1; \quad \int_0^t \|\nabla C\|^2 d\eta \leq M_1. \end{gathered} \tag{2.12}$$

where M_1 is a generic positive constant depending on the coefficients of (1.1) and the initial data terms in (1.3).

It also follows from inequality (2.11) that $\|\nabla u\|^2$, $\|T\|^2$ and $\|C\|^2$ decay exponentially as t tends to ∞.

3. Continuous dependence on the gravity coefficients

Let (u_i, T, C, P) and (u_i^*, T^*, C^*, P^*) be solutions to (1.1) with the same boundary and initial data (1.2), (1.3), but with different gravity coefficients (g_i, h_i) and (g_i^*, h_i^*), respectively.

Namely,

$$\begin{gathered} u_{i,t} = \nu\Delta u_i - au_i - p_{,i} + g_iT + h_iC \\ T_{,t} + u_iT_{,i} = \Delta T \\ C_{,t} + u_iC_{,i} = \Delta C + \rho\Delta T \quad \text{in } \Omega\times t>0 \\ u_{i,i} = 0 \end{gathered}$$

and

$$\begin{gathered} u^*_{i,t} = \nu\Delta u^*_i - au^*_i - p^*_{,i} + g^*_iT^* + h^*_iC^* \\ T^*_{,t} + u^*_iT^*_{,i} = \Delta T^* \\ C^*_{,t} + u^*_iC^*_{,i} = \Delta C^* + \rho\Delta T^* \quad \text{in} \quad \Omega\times t>0 \\ u^*_{i,i} = 0\,. \end{gathered}$$

Now set

$$\begin{gathered} w_i = u_i - u_i^*, \quad \Pi = p - p^*, S = T - T^*, \quad \Sigma = C - C^* \\ \gamma_i = g_i - g_i^*, \quad \mu_i = h_i - h_i^* \end{gathered} \tag{3.1}$$

Clearly, (w_i, Π, S, Σ) satisfies the equations

$$\begin{gathered} w_{i,t} = \nu\Delta w_i - aw_i - \Pi_{,i} + \gamma_iT + g_i^*S + \mu_iC + h_i^*\Sigma \\ S_{,t} + w_iT_{,i} + u_i^*S_{,i} = \Delta S \\ \Sigma_{,t} + w_iC_{,i} + u_i^*\Sigma_{,i} = \Delta\Sigma + \rho\Delta S \quad \text{in } \Omega\times\{t>0\} \\ w_{i,i} = 0 \end{gathered} \tag{3.2}$$

with the boundary-initial data

$$\begin{gathered} w_i = S = \Sigma = 0 \quad \text{on } \partial\Omega\times\{t>0\} \\ w_i(x,0) = S(x,0) = \Sigma(x,0) = 0 \quad \text{in } \Omega \end{gathered} \tag{3.3}$$

Multiplying $(3.2)_1$ by w_i and integrating over Ω, then using Cauchy-Schwarz's inequality, and the arithmetic-geometric mean inequality, we obtain

$$\nu\|\nabla w\|^2+\frac{a}{2}\|w\|^2+\frac{1}{2}\frac{d}{dt}\|w\|^2 \le \frac{2}{a}[(g^*)^2\|S\|^2+(h^*)^2\|\Sigma\|^2+\mu^2\|C\|^2+\gamma^2\|T\|^2], \tag{3.4}$$

where

$$\gamma^2 = \max_\Omega \gamma_i\gamma_i; \quad \mu^2 = \max_\Omega u_i; \quad (g^*)^2 = \max_\Omega g_i^*g_i^*; \quad (h^*)^2 = \max_\Omega h_i^*h_i^*.$$

Multiplying $(3.2)_1$ by $w_{i,t}$ and integrating over Ω, then using Cauchy-Schwarz inequality, and the arithmetic-geometric mean inequality, we have

$$\frac{1}{2}\nu\frac{d}{dt}\|\nabla w\|^2 + \frac{a}{2}\frac{d}{dt}\|w\|^2 \le (g^*)^2\|S\|^2 + (h^*)^2\|\Sigma\|^2 + \mu^2\|C\|^2 + \gamma^2\|T\|^2 \tag{3.5}$$

Multiplying $(3.2)_2$ by S and integrating over Ω, then using Cauchy-Schwarz, the arithmetic-geometric mean inequality, the Sobolev inequality ,which holds for all $\varphi\in C_0^1(\varphi)$,

$$\|\varphi\|_4 \le \Lambda\|\varphi_{,i}\|$$

where $\|\cdot\|_4$ is the norm on $L^4(\Omega)$.

we derive

$$\frac{d}{dt}\|S\|^2 = -2\|\nabla S\|^2 + 2\langle T_{,i}, w_i S\rangle;$$

therefore

$$\begin{aligned}\frac{d}{dt}\|S\|^2 &\le -2\|\nabla S\|^2 + 2\|\nabla T\|\cdot\|w\|_4\|S\|_4 \\ &\le -2\|\nabla S\|^2 + 2\Lambda^{\frac{1}{2}}\|\nabla T\|\cdot\|\nabla w\|\cdot\|\nabla S\| \\ &\le -2\|\nabla S\|^2 + 2\alpha_{11}\|\nabla S\|^2 + \frac{\Lambda}{2\alpha_{11}}\|\nabla w\|^2\|\cdot\|\nabla T\|^2\end{aligned} \tag{3.6}$$

where α_{11} is a positive constant at our disposal.

Using similar method as in (3.6), from equation $(3.2)_3$, we find that

$$\frac{d}{dt}\|\Sigma\|^2 \le (-2+\alpha_{12}+\alpha_{13})\|\nabla\Sigma\|^2 + \frac{\Lambda}{\alpha_{13}}\|\nabla w\|^2\cdot\|\nabla C\|^2 + \frac{\rho^2}{\alpha_{12}}\|\nabla S\|^2 \tag{3.7}$$

where α_{1i}, $i=2,3$ are positive constants at our disposal.

Multiplying (3.6) by Γ_{21} and adding (3.7) leads to

$$\begin{aligned}&\frac{d}{dt}(\|\Sigma\|^2 + \Gamma_{21}\|S\|^2) \\ &\le \frac{\Lambda}{2}\|\nabla w\|^2(\frac{\Gamma_{21}}{\alpha_{11}}\|\nabla T\|^2 + \frac{2}{\alpha_{13}}\|\nabla C\|^2) \\ &\quad + 2[(\alpha_{11}-1) + \frac{\rho^2}{2\alpha_{12}\Gamma_{21}}]\Gamma_{21}\|\nabla S\|^2 + (-2+\alpha_{12}+\alpha_{13})\|\nabla\Sigma\|^2\end{aligned} \tag{3.8}$$

where Γ_{21} is a positive constant at our disposal.

In (3.8), we choose

$$\alpha_{11} = \frac{1}{4}; \quad \alpha_{12} = \alpha_{13} = \frac{1}{2}; \quad \Gamma_{21} = 4\rho^2.$$

it follows that

$$\frac{d}{dt}(\|\Sigma\|^2 + 4\rho^2\|S\|^2) + \|\nabla\Sigma\|^2 + 4\rho^2\|\nabla S\|^2 \le 2\Lambda\|\nabla w\|^2(4\rho^2\|\nabla T\|^2 + \|\nabla C\|^2) \tag{3.9}$$

Multiplying (3.4) by Γ_{22} and (3.5) by Γ_{23}, then adding all the results to (3.9), then using Poincare's inequality, we have

$$\begin{aligned}&\frac{d}{dt}\big[\frac{\Gamma_{23}}{2}\nu\|\nabla w\|^2 + \frac{(\Gamma_{22}+a\Gamma_{23})}{2}\|w\|^2 + (\|\Sigma\|^2 + 4\rho^2\|S\|^2)\big] \\ &+ \big[\lambda_1 - (\frac{2\Gamma_{22}}{a} + \Gamma_{23})(g^*)^2\big]\|\Sigma\|^2 \\ &+ \big[4\rho^2\lambda_1 - (\frac{2\Gamma_{22}}{a} + \Gamma_{23})(h^*)^2\big]\|S\|^2 + \Gamma_{22}\nu\|\nabla w\|^2 + \frac{a\Gamma_{22}}{2}\|w\|^2 \\ &\le w\|^2\big[4\rho^2\|\nabla T\|^2 + \|C_{,i}\|^2\big] + (\frac{\Gamma_{22}}{a} + \Gamma_{23})\big[\mu^2\|C\|^2 + \gamma^2\|T\|^2\big]\end{aligned} \tag{3.10}$$

where Γ_{2i}, $i=2,3$ are positive constants at our disposal.

We can choose Γ_{22} and Γ_{23} sufficient small to make sure that all the coefficients in (3.10) are positive, such as

$$\Gamma_{22} = \min\{\frac{a\lambda_1}{8(g^*)^2}, \frac{\rho^2 a\lambda_1}{2(h^*)^2}\}; \quad \Gamma_{23} = \min\{\frac{\lambda_1}{4(g^*)^2}, \frac{\rho^2\lambda_1}{(h^*)^2}\}.$$

Let

$$Q_2(t) = \{\frac{\Gamma_{23}}{2}\nu\|\nabla w\|^2 + \frac{(\Gamma_{22}+a\Gamma_{23})}{2}\|w\|^2 + \|\Sigma\|^2 + 4\rho^2\|S\|^2\}$$

and

$$\begin{aligned}\kappa_2 = \min\Big\{\frac{2\Gamma_{22}\nu}{\Gamma_{23}\nu}; \frac{a\Gamma_{22}}{(\Gamma_{22}+a\Gamma_{23})}; (\lambda_1 - (\frac{2\Gamma_{22}}{a}+\Gamma_{23})(g^*)^2);\\ \frac{1}{4\rho^2}[4\rho^2\lambda_1 - (\frac{2\Gamma_{22}}{a}+\Gamma_{23})(h^*)^2]\Big\}\end{aligned} \tag{3.11}$$

It follows from (3.10) that

$$\frac{d}{dt}Q_2(t)+\kappa_2 Q_2(t) \le (\frac{\Gamma_{22}}{a}+\Gamma_{23})[\mu^2\|C\|^2+\gamma^2\|T\|^2]+2\Lambda\|\nabla w\|^2[4\rho^2\|\nabla T\|^2+\|\nabla C\|^2] \tag{3.12}$$

It is easy to show that

$$2\Lambda\|\nabla w\|^2[4\rho^2\|\nabla T\|^2 + \|\nabla C\|^2] \le Q_2(t)f_2(t) \tag{3.13}$$

where

$$f_2(t) = \tau_2(\|\nabla T\|^2 + \|\nabla C\|^2), \quad \tau_2 = \frac{4\Lambda}{\Gamma_{23}\nu}(4\rho^2+1).$$

Thus, combining (3.12) and (3.13), we obtain

$$\frac{d}{dt}Q_2(t) + \kappa_2 Q_2(t) \le M_2(\mu^2+\gamma^2) + f_2(t)Q_2(t) \tag{3.14}$$

where $M_2 = 2(\frac{\Gamma_{22}}{a} + \Gamma_{23})M_1$.

Now, multiplying both sides of (3.14) by $e^{\kappa_2 t}$ and integrating from 0 and t, we get

$$Q_2(t) \le \frac{M_2}{\kappa_2}(\mu^2+\gamma^2) + \int_0^t f_2(\eta)Q_2(\eta)d\eta \tag{3.15}$$

Hence, applying Gronwall's lemma and using (2.12), we obtain

$$Q_2(t) \le \frac{M_2}{\kappa_2}e^{\int_0^t f_2(\eta)d\eta}\cdot(\mu^2+\gamma^2) \le \frac{M_2}{\kappa_2}e^{2\tau_2 M_1}\cdot(\mu^2+\gamma^2), \forall t>0 \tag{3.16}$$

Consequently, from inequality (3.16), we can see that $Q_2(t) \to 0$, as $g_i \to g_i^*$ and $h_i \to h_i^*$. Recalling the definition of $Q_2(t)$, so $(w_i, \Pi, S, \Sigma) \to 0$ and $(u_i, T, C) \to (u_i^*, T^*, C^*)$. So, continuous dependence on the gravity coefficients is proved.

4. Continuous dependence on the Soret coefficient

Let (u_i, T, C, P) and (u_i^*, T^*, C^*, P^*) be solutions to (1.1), with the same boundary and initial data (1.2), (1.3), but with different Soret coefficient ρ and ρ^*, respectively:

$$\begin{gathered}u_{i,t} = \nu\Delta u_i - au_i - p_{,i} + g_iT + h_iC\\ T_{,t} + u_iT_{,i} = \Delta T\\ C_{,t} + u_iC_{,i} = \Delta C + \rho\Delta T \quad \text{in } \Omega\times t>0\\ u_{i,i} = 0\end{gathered}$$

and

$$\begin{gathered}
u^*_{i,t} = \nu\Delta u^*_i - au^*_i - p^*_{,i} + g_i T^* + h_i C^* \\
T^*_{,t} + u^*_i T^*_{,i} = \Delta T^* \\
C^*_{,t} + u^*_i C^*_{,i} = \Delta C^* + \rho^*\Delta T^* \quad \text{in } \Omega \times t > 0 \\
u^*_{i,i} = 0
\end{gathered}$$

Now set

$$w_i = u_i - u^*_i, \quad \Pi = p - p^*, \quad S = T - T^*, \quad \Sigma = C - C^* \tag{4.1}$$

Clearly, (w_i, Π, S, Σ) satisfies the equations

$$\begin{gathered}
w_{i,t} = \nu\Delta w_i - aw_i - \Pi_{,i} + +g_i S + +h_i\Sigma \\
S_{,t} + w_i T_{,i} + u^*_i S_{,i} = \Delta S \\
\Sigma_{,t} + w_i C_{,i} + u^*_i \Sigma_{,i} = \Delta\Sigma + (\rho - \rho^*)\Delta T + \rho^*\Delta S \quad \text{in } \Omega \times \{t > 0\} \\
w_{i,i} = 0
\end{gathered} \tag{4.2}$$

with the boundary-initial data

$$\begin{gathered}
w_i = S = \Sigma = 0 \quad \text{on } \partial\Omega \times \{t > 0\} \\
w_i(x,0) = S(x,0) = \Sigma(x,0) = 0 \quad \text{in } \Omega
\end{gathered} \tag{4.3}$$

Multiplying $(4.2)_1$ by w_i and integrating over Ω, furthermore using the Cauchy-Schwarz's inequality, and the arithmetic-geometric mean inequality, we have

$$\nu\|\nabla w\|^2 + \frac{a}{2}\|w\|^2 + \frac{1}{2}\frac{d}{dt}\|w\|^2 \le \frac{1}{a}[g^2\|S\|^2 + h^2\|\Sigma\|^2] \tag{4.4}$$

where

$$g^2 = \max_\Omega g_i g_i, h^2 = \max_\Omega h_i h_i.$$

Multiplying $(4.2)_1$ by $w_{i,t}$ and integrating over Ω, then using the Cauchy-Schwarz inequality, and the arithmetic-geometric mean inequality, we get

$$\frac{1}{2}\nu\frac{d}{dt}\|\nabla w\|^2 + \frac{a}{2}\frac{d}{dt}\|w\|^2 \le \frac{1}{2}[g^2\|S\|^2 + h^2\|\Sigma\|^2] \tag{4.5}$$

Multiplying $(4.2)_2$ by S and integrating over Ω, then using Cauchy-Schwarz, the arithmetic-geometric mean inequality, and using the Sobolev inequality, we can derive

$$\frac{d}{dt}\|S\|^2 d\eta \le -2\|\nabla S\|^2 + 2\alpha_{21}\|\nabla S\|^2 + \frac{\Lambda}{2\alpha_{21}}\|\nabla w\|^2\|\nabla T\|^2 \tag{4.6}$$

for α_{21} is a positive constant at our disposal.

Using similar method as in (4.6), from equation $(4.2)_3$, we can find that

$$\begin{aligned}
\frac{d}{dt}\|\Sigma\|^2 \le (-2 + \alpha_{22} + \alpha_{23} + \alpha_{24})\|\nabla\Sigma\|^2 + \frac{\Lambda}{\alpha_{23}}\|\nabla w\|^2\|\nabla C\|^2 \\
+ \frac{\rho^{*2}}{\alpha_{22}}\|\nabla S\|^2 + \frac{(\rho - \rho^*)^2}{\alpha_{24}}\|\nabla T\|^2
\end{aligned} \tag{4.7}$$

for α_{2i}, $(i = 2, 3, 4)$ are positive constants at our disposal.

Multiplying (4.6) by Γ_{31} and adding (4.7), leads to

$$\begin{aligned}
&\frac{d}{dt}(\|\Sigma\|^2+\Gamma_{31}\|S\|^2)\\
&\le(-2+\alpha_{22}+\alpha_{23}+\alpha_{24})\|\nabla\Sigma\|^2+2\big[(\alpha_{21}-1)+\frac{\rho^{*2}}{2\alpha_{22}\Gamma_{31}}\big]\Gamma_{31}\|\nabla S\|^2\\
&\quad+\frac{(\rho-\rho^*)^2}{\alpha_{24}}\|\nabla T\|^2+\frac{\Lambda}{2}\|\nabla w\|^2\big[\frac{\Gamma_{31}}{\alpha_{21}}\|\nabla T\|^2+\frac{2}{\alpha_{23}}\|\nabla C\|^2\big]
\end{aligned} \tag{4.8}$$

where Γ_{31} is a positive constant at our disposal.

In (4.8), we choose $\alpha_{21}=1/2$, $\alpha_{22}=1$, $\alpha_{23}=\alpha_{24}=1/4$, It follows that

$$\begin{aligned}
&\frac{d}{dt}(\|\Sigma\|^2+\Gamma_{31}\|S\|^2)+\|\nabla\Sigma\|^2+(\Gamma_{31}-\rho^{*2})\|\nabla S\|^2\\
&\le\Lambda\|\nabla w\|^2(\Gamma_{31}\|\nabla T\|^2+4\|\nabla C\|^2)+4(\rho-\rho^*)^2\|\nabla T\|^2
\end{aligned} \tag{4.9}$$

Multiplying (4.4) by Γ_{32} and (4.5) by Γ_{33}, furthermore adding all the results to (4.9), we get

$$\begin{aligned}
&\frac{d}{dt}\big[\frac{\Gamma_{33}}{2}\nu\|\nabla w\|^2+\frac{(\Gamma_{32}+a\Gamma_{33})}{2}\|w\|^2+(\|\Sigma\|^2+\Gamma_4\|S\|^2)\big]\\
&+\Gamma_{32}\nu\|\nabla w\|^2+\frac{a\Gamma_{32}}{2}\|w\|^2+\|\nabla\Sigma\|^2+(\Gamma_{31}-\rho^{*2})\|\nabla S\|^2\\
&\le(\frac{2\Gamma_{32}}{a}+\Gamma_{33})[g^2\|\Sigma\|^2+h^2\|S\|^2]+2\Lambda\|\nabla w\|^2[\Gamma_{31}\|\nabla T\|^2\\
&\quad+4\|\nabla C\|^2]+4(\rho-\rho^*)^2\|\nabla T\|^2
\end{aligned} \tag{4.10}$$

where Γ_{3i}, $(i=2,3)$ are positive constants at our disposal.

We can choose Γ_{31} to make sure that $\Gamma_{31}>\rho^{*2}$, therefore, by the Poincare's inequality from (4.10), we have

$$\begin{aligned}
&\frac{d}{dt}\big[\frac{\Gamma_{33}}{2}\nu\|\nabla w\|^2+\frac{(\Gamma_{32}+a\Gamma_{33})}{2}\|w\|^2+(\|\Sigma\|^2+\Gamma_{31}\|S\|^2)\big]\\
&+\big[\lambda_1(\Gamma_{31}-\rho^{*2})-(2\frac{\Gamma_{32}}{a}+\Gamma_{33})h^2\big]\|S\|^2+\frac{a\Gamma_{32}}{2}\|w\|^2\\
&+\Gamma_{32}\nu\|\nabla w\|^2+\big[\lambda_1-(\frac{2\Gamma_{32}}{a}+\Gamma_{33})g^2\big]\|\Sigma\|^2\\
&\le\Lambda\|\nabla w\|^2[\Gamma_{31}\|\nabla T\|^2+4\|\nabla C\|^2]+4(\rho-\rho^*)^2\|\nabla T\|^2
\end{aligned} \tag{4.11}$$

We can choose Γ_{32} and Γ_{33} sufficient small to make sure that all the coefficients in (4.11) are positive, such as we may choose

$$\Gamma_{32}=\min\{\frac{a\lambda_1}{8g^2},\frac{a\lambda_1(\Gamma_{31}-\rho^{*2})}{8h^2}\};\quad \Gamma_{33}=\min\{\frac{\lambda_1}{4g^2},\frac{\lambda_1(\Gamma_{31}-\rho^{*2})}{4h^2}\}.$$

Let

$$Q_3(t)=\{\frac{\Gamma_{33}}{2}\nu\|\nabla w\|^2+\frac{(\Gamma_{32}+a\Gamma_{33})}{2}\|w\|^2+\|\Sigma\|^2+\Gamma_{31}\|S\|^2\}$$

and

$$\begin{aligned}
\kappa_3=\min\{&\frac{2\Gamma_{32}}{\Gamma_{33}};\frac{a\Gamma_{32}}{(\Gamma_{32}+a\Gamma_{33})};(\lambda_1-(\frac{2\Gamma_{32}}{a}+\Gamma_{33})g^2);\\
&\frac{1}{\Gamma_{31}}[\lambda_1(\Gamma_{31}-\rho^{*2})-(\frac{2\Gamma_{32}}{a}+\Gamma_{33})h^2]\}
\end{aligned} \tag{4.12}$$

Then, it follows from (4.11) that

$$\frac{d}{dt}Q_3(t)+\kappa_3 Q_3(t)\le \Lambda\|w\|^2[\Gamma_{31}\|\nabla T\|^2+4\|C_{,i}\|^2]+4(\rho-\rho^*)^2\|\nabla T\|^2 \tag{4.13}$$

Also, it can be easily shown that

$$\Lambda\|\nabla w\|^2[\Gamma_{31}\|\nabla T\|^2+4\|\nabla C\|^2]\le f_3(t)Q_3(t) \tag{4.14}$$

where

$$f_3(t)=\tau_3(\|\nabla T\|^2+\|\nabla C\|^2), \tau_3=\frac{2\Lambda}{\Gamma_{33}\nu}(\Gamma_{31}+4).$$

Thus, combing (4.13) and (4.14), we obtain

$$\frac{d}{dt}Q_3(t)+\kappa_3 Q_3(t)\le 4(\rho-\rho^*)^2\|\nabla T\|^2+f_3(t)Q_3(t) \tag{4.15}$$

Now, multiplying both sides of (4.15) by $e^{\kappa_3 t}$ and integrating over $[0,t]$, we obtain

$$Q_3(t)\le \frac{M_3}{\kappa_3}(\rho-\rho^*)^2+\int_0^t f_3(\eta)Q_3(\eta)d\eta \tag{4.16}$$

where $M_3=4M_1$.

Hence, applying Gronwall's lemma and using (2.12), we obtain

$$Q_3(t)\le \frac{M_3}{\kappa_3}e^{\int_0^t f_3(\eta)d\eta}\cdot(\rho-\rho^*)^2\le \frac{M_3}{\kappa_3}e^{2\tau_3 M_1}\cdot(\rho-\rho^*)^2,\quad \forall t>0 \tag{4.17}$$

As a result, from inequality (4.17), we can see that $Q_3(t)\to 0$, as $\rho\to\rho^*$. Recalling the definition of $Q_3(t)$, so $(w_i,\Pi,S,\Sigma)\to 0$ and $(u_i,T,C)\to(u_i^*,T^*,C^*)$. Consequently, continuous dependence on the Soret coefficient is proved.

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