

APPLICATION OF COOPERATIVE GAME THEORY
IN SMART GRIDS

by

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DEDICATION

This thesis is dedicated to my sister,
for her endless support and motivation.

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LIST OF ABBREVIATIONS

Abbreviation	Description
AGC	Automatic Generation Control
AMI	Advance Metering Infrastructure
AMR	Automatic Meter Reading
BPL	Broadband over powerline
DApp	Decentralized Application
DER	Distributed Energy Resources
DG	Distributed Generation
EBI	Energi Bureau of Investigation
ECS	Energy Consumption Scheduling
EV	Electric Vehicle
G2V	Grid to vehicle
HEMS	Home Energy Management System
IoT	Internet of Things
IP	Internet Protocol
MDP	Markov Decision Process
PAR	Peak-to-Average-Ratio
PEV	Plug-in Electric Vehicles
PHEV	Plug-in-Hybrid-Electric Vehicle
PLC	Power line communications

PMU	Power Management Unit
QoS	Quality of service
RES	Renewable Energy Sources
SCADA	Supervisory Control and Data Acquisition
SG	Smart Grid
TCP	Transmission Control Protocol
TGC	Tradable Green Certificates
TOU	Time-of-Use
UDP	User Datagram Protocol
V2G	Vehicle to Grid
VPP	Virtual Power Plants

ABSTRACT

Nowadays, as the conventional grid cannot handle the technology, the smart grid is introduced to provide two-way communication between utility and customers to meet the energy requirements. In general, prosumers trade the locally produced energy using the NRG-X-Change mechanism and receive payment depending on their contribution individually. In this research, we study the different scenarios of trading energy in which all prosumers of the same grid are coalited. Firstly, we adopt one of the game-theoretic approaches called ‘Co-operative game theory’ to analyze all scenarios of prosumers’ coalition. Secondly, we consider three pricing functions such as concave, linear and convex, and analyze the behavior of each pricing function with hypothetical values on the prosumers’ coalition. Shapley value is calculated to show the benefits obtained by the prosumers’ coalition. In this research, we consider the average energy production and consumption data from Pecan Street Inc. for three seasons in a year such as Fall, Spring, and Winter. The results show that coalitions are profitable only when we consider the convex pricing function, whereas linear and concave are not profitable. By calculating the Shapley value, we conclude that price function ‘ g ’ is maximized only if production is twice greater than consumption.

1. INTRODUCTION AND LITERATURE REVIEW

1.1 Research Motivation

“The smart grid allows for a bi-directional flow of data between the utility and its customers” [1]. It is expected to deliver power efficiently and respond automatically to changes that occur in the grid. To effectively implement the smart grid system, the application of the electric power engineering technologies with network communications can be achieved through smart meters placed between the electricity provider and the customer.

The variable nature of renewable energy sources such as wind farms and solar panels adds complexity to the operation of the grid, so the smart grid provides data and automation to enable energy sources to put energy onto the grid and optimize its usage. Traditionally, the locally produced renewable energy was traded on a day-ahead basis by prosumers (a person who produces and consumes energy) and consumers (a person who consumes the energy) by participating in an auction. Buying and selling orders were submitted to a public order book and matched in a discrete or continuous fashion using equilibrium price. Bidding of energy ahead of time depends on predictions of future supply or demand, the inaccuracy of which translates to higher or lower costs for both buyers and sellers [2].

Currently, the trading of locally produced energy uses the NRGX-Change mechanism [2]. This mechanism doesn't depend on the energy market or matching of orders, produced energy is fed continuously to the grid and prosumers are paid according to the actual usage.

In this mechanism, each prosumer provides his produced energy to the grid individually and gets paid according to his contribution. Instead, there may be cases where the prosumer could form coalitions with other prosumers of the same grid, which may result in higher profit.

My thesis focus is to study multiple scenarios and help prosumers decide what is the best scenario to form a coalition, that is analyzed by using the game-theoretic approach. The research mainly uses one branch of game theory called Cooperative game theory, which is a high-level approach that focuses on predicting which coalitions may form, the payoff that each player involved in the coalition receives, and joint actions that the group takes.

1.2 Literature Review

With the increase in the demand for electricity, there is an increase in the average cost of power supply and Time-of-Use pricing is used to manage demand. Game theory can be applied to TOU strategies, by using cost models obtained from fluctuations between user demand and consumption, and the level of user satisfaction. By considering scenarios with single and multiple users and their responses to time-dependent prices, utility functions can be designed to obtain a Nash equilibrium using the game theory method. This method proved to be effective in optimizing TOU prices, by reducing utility company prices and improving user benefits, thus improving market efficiency. Power system stability is achieved by keeping the load level. To further improve the model, additional factors like renewable distributed generators and electricity usage at different time periods also can be considered [3].

Suppliers monitor for price changes almost hourly while consumers respond to changes as seen on their monthly bills. By developing computable equilibrium models, time-of-use prices can be estimated, which are useful in cases where consumer prices are regulated, but suppliers offer competitive pricing. Also, it finds use in the estimation of forwarding prices in unregulated markets and the evaluation and welfare analysis of regular versus TOU pricing before making pricing changes [4].

Game theory is used to create an interactive, incentive-based energy consumption scheduling game where the users act as players, and their daily schedules and loads are gaming strategies [5]. In this game, a utility company serving various customers is assumed to adopt different pricing tariffs for separating the energy usage in time and level.

Each player or user is required to apply their best response strategy to the current total load and tariffs in the power distribution system. They exchange interactive messages to maximize their benefits in this game setting, which leads to an optimal load profile at the equilibrium state. The user information remains anonymous, and they receive incentives for taking part in such games and services. This method has shown simulated results of a reduction in peak-to-average ratio of the total energy demand, energy cost and daily individual electricity charges [5].

As each individual focuses on their independent energy-saving mechanisms independent of other energy consumers, an agent-based cooperative model is created for smart grid consumers [6]. Agents take control of the smart meters, and they form stable coalitions with other agents in the game.

Each coalition behavior is dependent on the behavior of other coalitions in the system, and they may compete to get a higher payoff to the members of their group. In this scenario, there might be a need for restructuring of coalition formation as they are interdependent.

When dealing with cooperative games with coalition configuration, there are only some feasible coalitions according to the agents. A coalitions feasibility can be determined by its members' cohesion and is not expected to be same for all coalitions [7]. In order to generalize the games with coalition configuration, the cohesion index is introduced with the class of games along with an allocation rule characterized by using appropriate properties [7].

Cohesion index determines the cohesion of each coalition, and it helps us understand the relationships between different elements of a module. A model of games can be presented with a value inspired by the Owen value, where the total gain can be shared according to the cohesiveness degree of all the coalitions. This model can be used in software design and for the situations where uncertainty of payoffs is present.

Several customers of smart grids may have a surplus amount of energy for sale while the others are opting to buy the energy from the market to meet their demands. Customers in need of energy can buy from their neighbors at a lower price, instead of buying energy from the grid. Sellers not only make profits by selling their extra available energy, but there will be less load on the grid too. In order to minimize the energy bill while considering the transmission cost, buyers can play a game by deciding the amount of energy they will buy from the seller [8].

Simulated results of this game theory model have shown that the algorithm has minimized the individual energy bills of the buyers, which in turn yielded an increase of seller's profits after analyzing the algorithm's performance for several scenarios and comparing it with that of a centralized optimization model.

Markov Decision Process (MDP) is a discrete optimization method that helps us understand the problem of optimal energy distribution by dynamically changing the size of coalition. In a situation where there is one micro-grid and several customers, the micro-grid acts as a player and the customer acts as another player. The grid needs to decide the coalition size in order to optimally utilize the energy generated while the customer needs to decide strategies to optimize the trade-off cost of communication and energy distribution and effective power supply [9]. MDP helps us understand how to form a dynamical coalition and ensure efficient power distribution to the customer.

Energy Consumption Scheduling (ECS) devices are built-in devices used in smart meters, which connect to the power-grid and the local area network that handles two-way communication in smart-grids. These ECS devices are used for demand side management within a neighborhood of buildings that share a common energy source [10]. These devices interact automatically with the help of an algorithm that determines the optimal energy consumption schedule for each subscriber. The end goal is to decrease the total energy cost and the peak-to-average-ratio (PAR) in load demand in the system. In order to encourage the subscribers to use these devices with a new game-theoretic pricing model, incentives are provided.

1.3 Thesis Outline

In Chapter 1, we introduce the problem and discuss the examined scenario along with a literature review on game theory in smart grids. In Chapter 2, the theory of the smart grid is explained along with its infrastructure, role in renewable energy, and communication protocol. In this chapter, we also discuss the NRGCoin.

In Chapter 3, we briefly explain the concept of game theory along with its types such as Non-cooperative and Cooperative game theory and discuss solution concepts of each type.

In Chapter 4, we introduce the NRG-X-Change mechanism and study the system model. An example of Shapley value computation for 3 prosumers is presented. Next, to test the effect of the coalition, we consider hypothetical values for three different pricing functions such as concave, linear, and convex and study the Shapley value with and without a coalition. In the next part of this chapter, we analyze the 3 codes that can be used to compute the Shapley value by comparing the time complexities and execution times.

In Chapter 5, in the first section, we test the behavior of the ‘ g ’ function by considering four different scenarios. In the next section, we take into account the real-time data obtained from Pecan street and compare the effect of Shapley value. In Chapter 6, we conclude the research and discuss the potential future work.

2. SMART GRID TECHNOLOGIES AND NRG COIN

2.1 Smart Grid Technology

2.1.1 *Introduction*

In general, “The grid” refers to the electric grid that consists of a network of transmission lines, substations, transformers, etc. which transfer the electricity from the power plant to the home or business. Nowadays, the typical grid cannot handle the technology, the smart grid is introduced to manage the groundswell of digital and computerized equipment and can easily automate the requirements and rising complexity of electricity [1].

Smart grid is defined as an advanced power system that integrates electrical network and smart digital communication technology. It provides bi-directional communication between the utility and consumers and senses the transmission line with its smart sensors. Smart grid technology is used worldwide to fulfill the purpose of sustainable electric power with the involvement of active consumers.

Smart grid ensures customer satisfaction concerning electricity supply, reliability, quality, and fulfills energy demands of customers. Together with Home Energy Management Systems (HEMS) and, Advance Metering Infrastructure (AMI), it is used to handle the growing demand that connects consumers and utilities.

The consumption pattern of the consumers is informed to the utilities by HEMS and smart meters, which is enabled by AMI. Similarly, consumers are informed about the energy available from the grid with prices and incentives.

To overcome the energy deficiency, utilities provide various demand response programs that are based on the requirements of different demand periods [11].

Utilities provide various demand response programs based on the necessity of specific demand cycles to address energy deficiency. Smart infrastructure, smart management, and smart protection systems are three main categories of smart grid.

(i) The smart infrastructure supports the bi-directional flow of electricity and information. The bi-directional feature of the smart grid allows users to feed the excess electricity generated by houses with solar panels back into the grid.

In addition, the smart infrastructure is further categorized into three subsystems which are smart energy systems, smart communication, and smart information systems. The smart energy subsystem provides advanced electricity generation, delivery, and, consumption. The smart information subsystem is accountable for providing advanced metering, tracking and management of information .Whereas the transmission of information among various systems, devices and applications is provided by the smart communication subsystem [12].

(ii) Smart management system: The latest management and control services are provided by a smart management system. It uses the smart infrastructure of the smart grid to obtain various management objectives. The objectives include an increase in energy efficiency, balancing of supply and demand, reduction of operation cost and maximizing utility.

(iii) Smart protection system: “The smart protection system provides protection against security, privacy and failure” [12].

2.1.2 Advantages of Smart Grids

(i) Increased Reliability:

When a problem occurs in the conventional electrical system, it is resolved by the utility person by visiting the location. Whereas, in the smart grid, as the system is built with self-healing capacity, most problems that occurs in the system can be resolved by itself.

(ii) Reduced Electricity Consumption:

Through smart meters that are equipped in the smart grid system, the energy consumption can be adjusted.

(iii) Reduced Expenses to Energy Producers:

The consumption during peak hours can be decreased as smart grid allows direct communication of the prosumers with end- users.

(iv) Intelligent Infrastructure:

The smart grid system is built in such a way that the energy can be stored and distributed safely.

(v) Predictive:

The impact of weather on the system can be anticipated before its occurrence with the use of machine learning. Thus, suitable actions can be taken to modify the system before the next events arise.

2.1.3 Disadvantages of Smart Grids

(i) Increased Cost:

Compared to the conventional electricity meters, the cost of installation of the smart meters is very high.

(ii) Privacy Concern:

The smart meters, which are part of the smart grid infrastructure can be easily hacked.

(iii) Unreliable:

During conditions such as windstorm, heavy rain and lightning, the network providers don't provide guaranteed service.

(iv) The communication should be available continuously.

2.1.4 Applications of Smart Grids

(i) The load can be shared as the smart grid provides the ability to be integrated with renewable energy sources.

(ii) The system can be easily adapted to the sudden change that might occur in the lines and feeders.

(iii) In the consumer's side, AMI (Advanced Metering Infrastructure) is a designed framework that involves and incorporates several technologies to achieve its required results.

(iv) In the transmission and distribution side of the smart grid, energy fraud detection is one of the important parts of smart grid security.

(v) In the suppliers' side, V2G (Vehicle to Grid) operation is the system where electric plug-in vehicles interconnect with the smart power grid to reverse its power flow, channeling the stored energy from the car to the power grid.

2.2 Smart Grid Infrastructure

The smart grid infrastructure is presented in Figure 2.1, which consists of Distributed Generation (DG) sources, conventional sources that include fossil fuel-based power plants, renewable energy sources and, electric vehicles.

It also includes intelligent buildings, smart homes, and data centers, which handle the entire infrastructure for communication [13].

The communication system of Smart Grid (SG) includes intelligent nodes that are operated by using AMI (Advanced Metering Infrastructure) or SCADA (Supervisory Control and Data Acquisition).

In SG's communication system network specifications such as service quality (QoS), efficiency, coverage and stability, security and privacy should be provided [13].

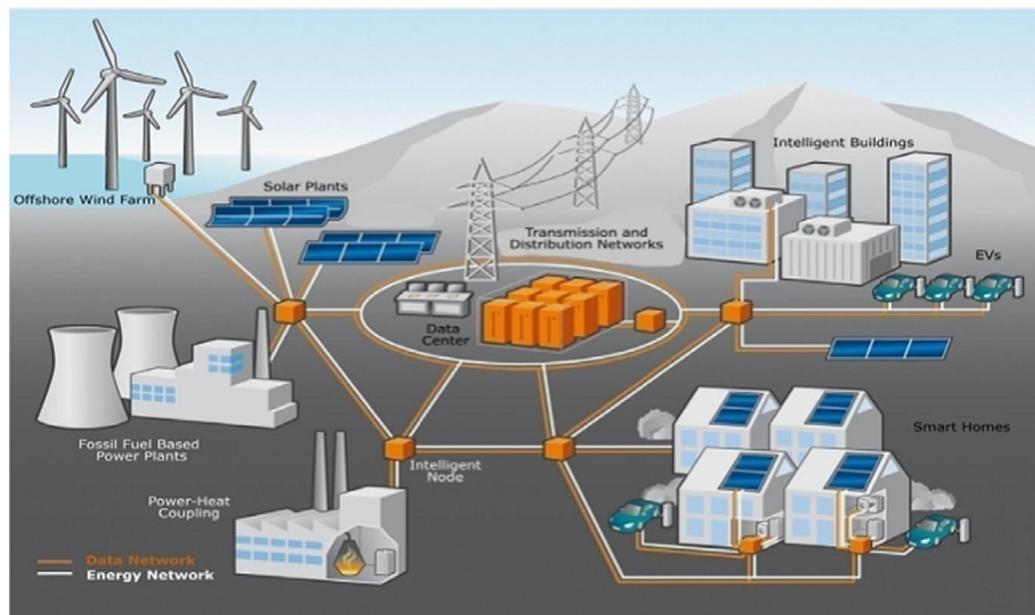


Figure 2.1: Smart grid infrastructure [14]

The flow of electricity generation and communication in SG is shown in Figure 2.2, which is divided into different sections namely power generation, transmission and distribution.

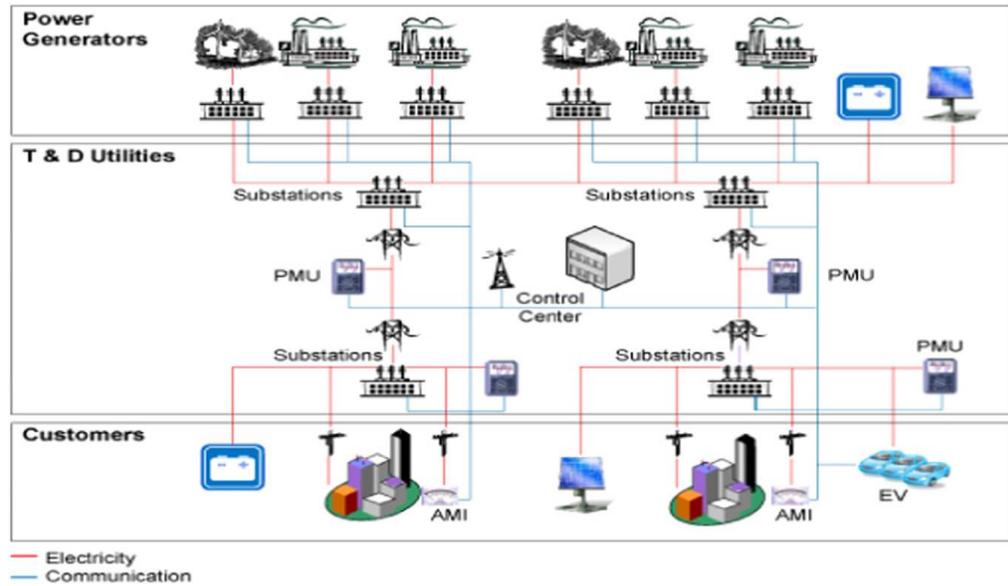


Figure 2.2: Flow of electricity and information in smart grid [13]

2.2.1 Power Generation Section

Distributed generation is one of the recent developments by the SG system that is related to power generation. The upcoming power grids are expected to be flexible, manageable, reliable and have innovative structure [13]. The Renewable Energy Sources (RES) meets these expectations by decreasing the energy generation and conversion cost, providing better storage options such as batteries and flywheels.

Complex analysis is required during installation of the DG as they consist of large deployments that are achieved from RES such as photovoltaic, wind turbines, fuel cells and microturbines.

Compared to the traditional large-scale power plant, the generation cost of the DG is higher. Smart power generation should be combined with demand forecasting and

Automatic Generation Control (AGC) to decrease the cost of generation while meeting energy requirements of consumers [13].

The development of DGs requires several Distributed Energy Resources (DER) that operate together when compared to the conventional power generators.

The Virtual Power Plants (VPP) replace the traditional power plant by providing efficient and manageable structure. The DER that is present in the VPP can be linked to various nodes in the distribution network, thus changing the overall characteristics in terms of topology, impedance, and losses. The generation cost can be controlled and the loss of DER can be avoided by the energy management system that is improved by VPP [13].

2.2.2 *Transmission and Distribution Section*

The transmission and distribution systems are expected to meet the requirements of the smart grid structure with regards to technical and economic aspects. Throughout transmission or distribution layers, the smart power generation offers multiple DERs to incorporate the entire system.

The traditional transmission and distribution systems didn't include DGs as RES and DER would affect the reliability of the system, unpredictable climatic changes make RES based on DGs become dependent on conditions and causes fluctuations in power, voltage and frequency [15]. Thus, the transmission and distribution utilities require accommodation of power reliability and efficiency against high penetration of DER.

The adaptation of current transmission and distribution utilities to the SG involves some challenges. The first challenge is to convert the transmission grid to a digital

platform in order to manage the communication protocols. It provides increased controllability, flexibility and data management opportunities [13].

The transmission grid should be able to monitor the power flow on its own which is known as self-awareness, which consists of voltage, frequency and stability monitoring.

One of the important challenges of a conventional transmission grid is to ensure sustainability to control the growth of the smart transmission grid. The sustainability can be achieved by including smart control centers, smart networks and smart substations.

Power Management Units (PMU) are used for monitoring phasor synchronization, voltage stability, load sharing, power flows, restoring power systems and estimating algorithms to recover power lost [16].

The PMU can be a single device or operational segment within a compact unit such as meters or security equipment. To provide better monitoring, the PMUs are located at important substations in many power generation and transmission utilities.

2.2.3 Customer Section

This section covers various applications such as microgrid with RES and hydrogen, electric vehicles interacting grid as vehicle to grid (V2G) and grid to vehicle (G2V), energy storage systems etc., The electrical loads are divided into static and electronic loads, the load models are assumed as residential, official and industrial. The official load is considered important as it includes military or hospitals which require large reliable microgrids.

The microgrid is located closer to the load sites to decrease the transmission line losses. Smart home and microgrid integration are accomplished through the use of energy

management devices, sensors that control the air conditioning and smart meters that have the ability to control the energy efficiently remotely [13].

Due to the increase in fuel costs and environmental act, the usage of Electric Vehicles (EV), plug-in electric vehicles (PEVs) and plug in hybrid electric vehicles (PHEVs) are promoted.

Compared to PEVs and PHEVs, the individuals gained interest in electric vehicles. EV's batteries are not only meant to provide propulsion, they are also considered to be energy storage devices capable of supplying power during discharge mode [13].

The EVs are operated bidirectionally in V2G and G2V during discharge and charge modes. By using its own storage devices such as batteries, fuel cells or hydrogen tanks, EVs supply electrical energy to the grid in V2G mode. It requires smart charger systems to combine EVs to the grid, essential discharge and charge operations are available during parking periods. In G2V, EVs are powered by the electricity that is stored from an external power source. Important concern of G2V is the load on the existing distribution grids due to charging operations.

2.3 Smart Meters (SM)

The smart meter measures the consumption of energy by a customer. By capturing the voltage, phasor angle and frequency, the SMs are expected to identify energy consumption rates.

A smart meter consists of metering and communication infrastructure. Metering consists of time-of-use pricing control, data management system and AMR (Automatic Meter Reading) framework [13]. To obtain the data about customer and utility grid, bi-directional flow of data is to be allowed by communication infrastructure. To enable SM to communicate with remote centers and run control commands, the communication section of the SM consists of network connection and control infrastructure.

The data collected by SMs can be used by customers to predict and decrease their electricity bills by supervising energy consumption. The utilities, on the other hand, use the data obtained from SMs to know the real-time pricing which helps them to decrease the consumption of electricity and inspire consumers to reduce load during peak hours.

“The SM module contains the power supply, control, metering, communication, indicating, encoding and timing module” [13]. The logging module stores the information of energy consumption, date, power by consumer. The metering module measures voltage and current by detaching from utility grid, the billing module performs electricity billing by considering timestamps [13].

2.3.1 *Classification of Smart Meters*

The classification of SMs depends on their usage- The first type is concentrated on customer-based services, that is In-home, and the second type is the Utility AMI that are used by utility companies. The In-home meters are set up within the buildings of the customers, whereas the Utility AMI meters are set up outside the building of the customers.

The In-home meters are made up of clamp sensors, which are clamped around the main electricity supply cable. The clamp sensor (shown in Figure 2.3) measures the

alternating current that flows in the electric cable, and the smart meter that is connected to the clamp sensor converts this current to power, to calculate the amount of power consumed. This data is transmitted to the gateway at a frequency of 433MHz [17]. Through gateway the data is sent to the server and database of the service provider by means of the internet.

The data is stored by the service provider and converted into formats that can be further processed by the end user. These type of SMs require networks with high data rate communication and meter level storage.

The bi-directional communication between the utility meter and company is provided by Advance Metering Infrastructure (AMI). The main aim of AMI is to provide the information regarding the actual power consumption to the utility companies. The price which is available to the customers at the time they are using the energy helps customers decide about their energy usage.



Figure 2.3: Clamp meter [18]



Figure 2.4: Smart meter installed by utility provider [19]

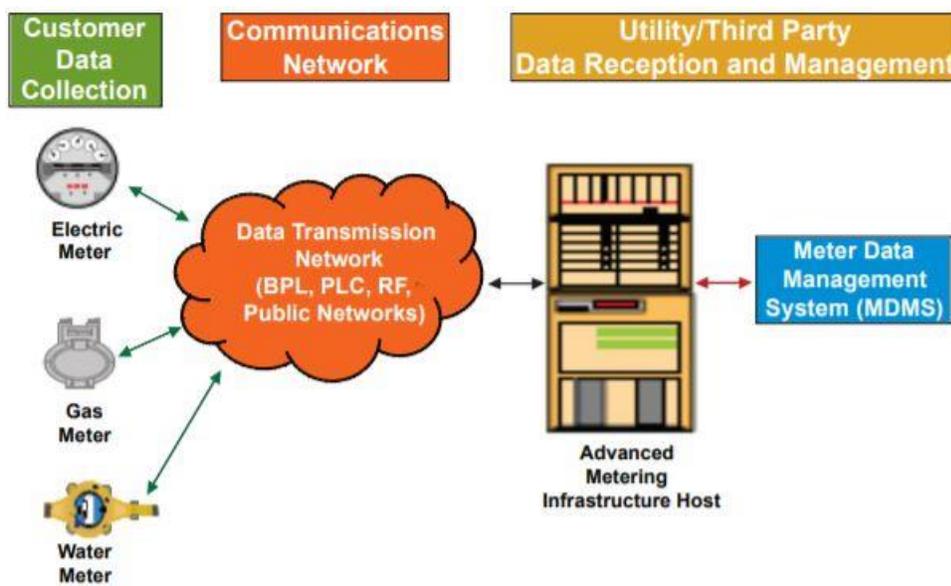


Figure 2.5: Elements of AMI [20]

The elements of AMI are represented in Figure 2.5.- The data regarding energy, gas and water usage is collected by advance meter devices at certain time periods and transferred to utility companies through communication networks. The AMI also collects data regarding pricing from the utility companies and passes it to the customers. AMI uses Broadband over Powerline (BPL), Power Line Communications (PLC), Radio Frequency and other networks to allow a bi-directional communication to transfer data

from smart meters to utility company. The metering information is received, stored and analyzed by a Meter data management system.

2.4 Prosumer in a Smart Grid

The demand for energy which is currently met by non-renewable energy sources is increasing rapidly. The excessive use of these limited non-renewable energy sources might affect the climate badly.

The traditional grid provides interaction between the distribution and transmission but not consumer. The smart grid provides the bi-directional flow of energy and information by including users. The consumers are informed about the availability of energy in the grid along with prices and incentives.

The renewable energy which is produced by consumers is considered as a new source of energy and can be shared with other consumers and the grid. When energy is produced and shared by the consumer, the consumer becomes a prosumer. The prosumers, like consumers, consume produced energy and they also feed the excessive energy to the grid or share with other consumers.

Examples of prosumers include houses which provide electricity and exchange power among themselves, buildings installed with EVs which provides storage service to utilities, and a microgrid which sells electricity to other microgrid or utility.

The price of electricity decreases when users produce electricity on their own rather than buying it from the utilities. Some countries allow users to earn money by selling the excessive produced energy back to the grid.

The two different energy states in which a prosumer operates are production and consumption state. When the production of energy is more than the consumption, then

the prosumer is said to be operating in production state. Whereas, when the production is less than the consumption, then the prosumer is operating in consumption state.

The state in which the prosumer operates depends on the production of energy (such as solar energy) and the energy consumed by residents. To minimize the electricity usage from the grid by the prosumer when operating in consumption state, battery storage can be installed at home. The level of the battery is indicated by $s(t)$ in Wh.

The battery is said to be in charging state when production is greater than consumption ($p(t) > c(t)$) and in discharging state when production is less than consumption ($p(t) < c(t)$). The energy states of a prosumer can be represented mathematically as,

$$\text{Production state: } p(t) + s(t) \geq c(t) \quad [17] \quad (2.1)$$

$$\text{Consumption state: } p(t) + s(t) \leq c(t) \quad [17] \quad (2.2)$$



Figure 2.6: Example of smart home with RES and EVs [21]

2.4.1 Microgrids

A Microgrid is an important element of a smart grid, that can operate individually by disconnecting itself from the conventional grid.

The conventional grid connects homes and other buildings to the central power system, enabling us to use electronics, heating and cooling systems and appliances. Because of this connection, all the users are affected when a part of the grid must be restored.

A microgrid is operated when connected to the grid, but it can detach and operate on its own using energy generated by distributed generators, RES such as solar panels, and batteries.

Microgrids can be helpful in times of power interruptions or other critical situations, they are useful in reducing power costs, and can be connected to a local resource which is limited and unstable for conventional grid.

The microgrid can function in two modes. The first is, grid connected mode which is connected to the grid, which uses electricity from the grid in case of high demand. Urban ecosystems use this mode, where grid electricity is used in case of crisis and backup requirements.

The other mode is the island or off-grid mode, in this mode the power plants operate individually by disconnecting from grid.

Island microgrids requires large DER installations, along with reliable demand response and control algorithms to decrease the probability of inadequate capacity to meet the demands.

The energy generated by solar panel is $p(t)$ in Wh, energy provided by wind turbines is defined as $W(t)$ in Wh and energy stored by a battery is $S(t)$. $n_i(t)$ is the negawatt of energy generated by each smart home and i is the set of M smart homes [17].

The total offered negawatt production is given as,

$$N(t) = \sum_{i \in M} n_i(t) [17] \quad (2.3)$$

The demand of a smart home is given as $c_i(t)$, hence total demand of microgrid is equal to,

$$C(t) = \sum_{i \in M} c_i(t) [17] \quad (2.4)$$

The total production of the microgrid is given as,

$$B(t) = p(t) + w(t) + s(t) + N(t) [17] \quad (2.5)$$

When, $B(t) \geq C(t)$ [17] the microgrid is said to operate in island mode.

2.5 Communication Protocols of Smart Grid

A communication between two entities has a set of rules called protocols. These sets of rules have a syntax through which messages can be exchanged between two systems. These protocols are implemented on networking platform. Various protocols are listed below.

a) Transmission Control Protocol (TCP):

TCP is employed in transmission of information within a network or across the internet. The protocol establishes a connection between a sender and a receiver. TCP divides the information into short pieces or short packets for sharing the media and for more efficient transmission. These short packets are reassembled at the destination. TCP is used in the transport layer.

b) Internet Protocol (IP):

IP is an addressing protocol, as it addresses to the systems and the devices. It is also known as routing protocol since it helps the packets in routing to nodes in a network till the destination.

c) User Datagram Protocol (UDP):

UDP is used as a substitution protocol to TCP. UDP does not guarantee the delivery of packets. The UDP is faster than TCP. UDP is mainly responsible for low latency linking and loss toleration [22].

Internet of Things (IoT) and Machine to Machine (M2M) applications are two of the challenging applications of smart grids. Generally, IPV4 sends large amounts of data at a time, but smart meters do not support the transmission of large amounts of data at once.

The problem can be resolved by altering few approaches in IEEE 802.15.4 standard, which is specially designed for low-rate and low-power wireless personal-area networks. IPV6 architecture can be used to meet the requirements of smart home.

IPV6 architecture allows the devices to access the information from the cloud and interact with the other users. IPV6 and IEEE 802.15.4 technologies' drawbacks are resolved by deployment of a layer in which the IPV6 and IEEE 802.15.4 have an adaptation between them. This adaptation is 6LoWPAN. The 6LoWPAN compresses the size of the header which results in reduction of transmission over-head. IPV6 header compression, reassembly, packet encapsulation and packet fragmentation are the main features of 6LoWPAN [17].

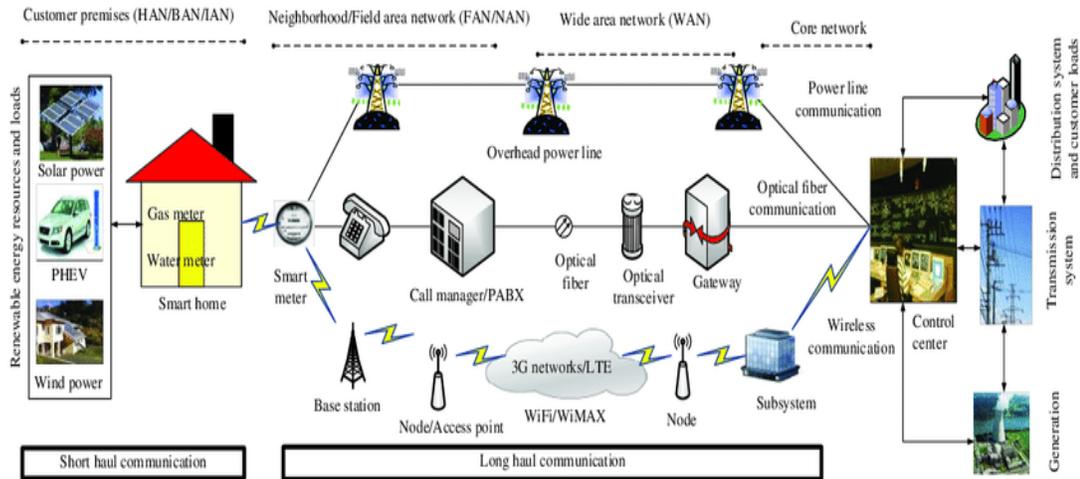


Figure 2.7: Communication protocol and standards of smart grid [22]

2.6 Role of Smart Grid in Renewable Energy

The main source of energy in most power generating systems is the fuel, whereas solar energy is the main source for renewable energy [23]. Cost and availability are the important problems that arises with renewable energy sources.

The smart grid provides the integration of renewable energy sources. Most smart grids are located far away from closely populated areas and exist near fuel source or dam site to take advantage of renewable energy sources.

The produced electric power is stepped up to a higher voltage when it is connected to the transmission network. The transmission network is built across long distances until it reaches the final customer. At substations, the power will be stepped down to a transmission level voltage, from which it enters the distribution wiring. The power is stepped down from distributed voltage to required voltage on arrival at the destination.

2.7 NRGCoin

NRGcoin is a virtual currency which is generated by injecting locally produced renewable energy into the smart grids. The mission of this currency is to be the world's leading crypto currency by promoting a self-funding and decentralized governance system. It is a master node coin which uses a modified version of dagger-hashimoto algorithm called Energi hash. It aims to be the most user-friendly crypto currency with a secure and trusted platform [24].

There is no maximum limit for this currency, but the supply is limited to a fixed amount of one million coins per month. Of the million coins generated each month, 10% of the coin is allocated to founder's rewards, 10% to PoS holders, 40% to NRG master nodes and the rest is allocated to treasury. The 50% of coin which is allocated to coin founders and team members will be used for development and marketing thus helping the currency to have a strong funding mechanism.

NRGcoin's market value is determined by trading the currency in an open exchange market. Higher demand increases its market value whereas a large number of sells decreases it.

2.7.1 Staking

NRGcoin was fully transitioned to proof of stake from proof of work on February 26th, 2019 helping participants to stake energy coins from Energi core wallet. Below is a short process on how to stake NRGcoins; you would need a minimum of 1 NRGcoin to start staking [24].

- a) Make sure the Energi core wallet is completely in sync with the blockchain network.

- b) Go to settings and click start staking [24]
- c) Unlock wallet by entering wallet passphrase
- d) Staking can be started once the arrow at the bottom right hand corner turns green

2.7.2 *Properties of NRGcoin*

- a) NRGcoins serve as the right to receive an equivalent quantity of energy irrespective of its market value thus making agents feel secure about increasing energy prices.
 - b) This currency can be traded for an equivalent amount of energy or can be converted to fiat currency at any point in time.
 - c) NRGcoins can also be used as a business for buying and selling currency for individual profits.
 - d) Distribute System Operators (DSO) use NRGcoins as debt instrument with high liquidity which helps them to quickly convert the currency to cash.
 - e) DSO's can use a large portion of their cash assets on investments by delivering energy instead of cash to compensate their debts.
 - f) This currency resembles Tradable Green Certificates (TGC) or Renewable Energy Certificates, thus being used as a measure of renewable energy and as an approach to support clean energy effort.
 - g) Being designed to have much finer granularity than TGC's, fractional amounts of energy can be exchanged quickly unlike with TGC's.
 - h) NRGcoins can serve as international currency to trade green energy across different countries.

i) NRGcoins are being decentralized without an issuer, it has less purchase obligations compared to TGC's.

2.7.3 Use Cases

(a) Masternode Use Case:

Masternodes are point of intersections on a decentralized network which provide features such as private and instant transactions to the blockchain network. Masternode holders earn rewards for second tier staking which include governance mechanisms for the owners of masternodes.

(b) Mining Use Case:

To secure the exchange network, miners are also rewarded for employing their compute power to proof of work. The fee given to the miners will serve as reimbursement for the compute power.

(c) Partner Use Case:

The Energi's smart contract platform will benefit DApp (Decentralized Application) and few other project partners. Energi provides value-adds such as technical expertise and decision making which will drive value to partnerships and projects based on their merit [25].

(d) End-Consumer Use Case:

The end consumers can use NRG as a store of value and as a utility for use in the smart grid. The more Energi DApp partners are on boarded into the network, the more usable the platform gets. The defense department will make sure that the end users have a secure platform by working with law enforcement and cyber security experts.

2.7.4 Energi Platform

a) Business Development:

Energi will establish an incubator program to support early stage blockchain startups on the Energi platform. High quality infrastructure projects which will help in building its platform will be highly encouraged by this program.

Energi's platform provides advice and support required for building sound business models thus encouraging the projects to collaborate with other projects. Access to the metrics and assessments will be provided to the partners on the platform which will help them in evaluating projects to meet industry standards.

b) User Protections:

Like other emerging technologies, blockchain system has no exemption to unauthorized access to the funds in the network. Hacking and high security measures required to maintain crypto currency are two of the main reasons for delay in its mass adoption. The Energi Bureau of Investigations (EBI) helps in protecting its user base from bad actors on the network like hackers and scammers. To protect its users, Energi community works with relevant law enforcement agencies and other institutions wherever possible.

c) Engineering Support:

Energi platform takes help of DApp teams for technical guidance, assistance with contract auditing and for ensuring proper security measures are in place for the growth of its platform. Similar to Ethereum, Energi has solid contracts with its partners and it aims to establish an excellent developer-friendly platform for DApp's. Educational programs will be offered to the developers to improve their knowledge in blockchain technology.

d) Marketing Systems:

Energi's marketing system takes full advantage of social media to effectively communicate its vision to not just the crypto currency enthusiasts, but to the people who are new to crypto currency market as well. Energi has also created a YouTube video series to educate new users about the cryptocurrency and blockchain technology as a whole.

To share Energi's long term vision and to meet early stage projects, Energi's team members travel to global blockchain conferences.

The founder of NRGCoin, Tommy World Power, initially performed airdrop campaigns which are not only a way of distributing NRGcoin's, but also encouraging engagement with Energi social media platforms thus helping it to gain a significant web presence [15].

3. GAME THEORY AND ITS TYPES

3.1 Introduction to Game Theory

A game theory is a mathematical method of making decisions with multiple decision makers, called players or agents. A strategic situation is analyzed to determine a particular action. The situation involves a group of players each having possible choices. The result of an individual player depends on the decisions made by other players that participate in the game.

An essential assumption of game theory is that the players involved in the game are rational. Rationality means that the individuals who are involved in the game are aware of the strategies and choices available to them, which helps them to decide the best strategy that can be implemented to get the desired outcome.

Thus, the game theory can be determined as a study of strategic situation involving rational players and this theory can be used in various fields such as economics, networking, wireless networks, wireless communications and other real time scenarios.

In economics, the competition between companies can be modelled through game theory, we use game theory in networking to solve the problem of routing and resource allocation in a complex situation, and the communication devices of a wireless network are controlled by network operators by applying the concepts of game theory.

Based on cooperation the game theory is classified into two branches, which are: Non-Cooperative game theory and Cooperative game theory. The focus of Non-Cooperative game theory is the distribution of payoff among individual players.

Whereas, Cooperative game theory focus on how the payoff are divided among a group of players compared to individual players.

3.2 Non-Cooperative Game Theory

The concept of non-cooperative game theory is applied to predict individual player actions and payoff, as opposed to the actions involving a group of players. It deals with the strategic decisions of individual players who try to increase their payoffs, without considering the effects of their decisions on another players' outcome. The word non-cooperative doesn't always signify that the players don't cooperate with each other, it means that the decisions taken by the players should be self-enforcing with no communication between the players involved in the game [26].

3.2.1 Basics of Non-Cooperative Game Theory

The Non-Cooperative game theory is divided into static and dynamic games. In static games, the actions of the players are unaffected by time and information. The static games can be described as a process where all players make decisions simultaneously or at different time slots. To the contrary, in dynamic games all the players who are involved in the game have information about other players' choices: - here time plays a central role in making decisions.

A static game has three components which are: the set of players N , action sets $(A_i)_{i \in N}$ and utility functions $(u_i)_{i \in N}$. To maximize their utility function $u_i(a_i, a_{-i})$, each player i chooses an action $a_i \in A_i$ that depends on actions taken by all other players who are involved in the game, which is denoted by a_{-i} . Whereas in dynamic games, the players define additional components such as information sets, times and sets involving information about past actions [26].

3.2.2 *Solution Concept of Non-cooperative Game Theory*

Nash equilibrium is the solution concept of non-cooperative game theory. It involves two or more players, where each player is expected to know the strategies of other players involved in the game and neither of the players is going to gain any profits by changing their own strategies, while other players strategies remain constant.

The Nash equilibrium has both advantages and drawbacks. The advantage is that it describes the stable state of non-cooperative games in which no player can maximize their utility by changing their action a_i , while the actions of other players remains unchanged a_{-i} [27]. The drawback is that, in games where each player has a finite action set, Nash equilibrium is expected to exist in mixed strategies [28]. A non-cooperative game theory can have numerous Nash equilibria, thus selecting the appropriate Nash equilibrium is challenging.

3.3 *Cooperative Game Theory*

In non-cooperative games, the players are not able to form coalitions and communicate directly with each other. To the contrary, in cooperative games, players communicate with each other and receive utilities. The concept of cooperative game theory provides answer to the question, “What happens when players communicate with each other and decide to cooperate?” [26].

Cooperative game theory consists of two parts: Nash bargaining and coalitional game, Nash bargaining involves situations in which a group of players must agree on certain conditions under which they can cooperate. Whereas, the formation of cooperative groups or coalitions are discussed in coalitional game.

By forming coalitions, players can receive more benefits than they would obtain individually. The players involved in the coalition do not always have the same interest and don't contribute the same value.

The players involved in a coalition expect to receive benefits by forming coalitions, depending on the value they contribute. These players are important for the existence of the coalition and are called the core of the coalition.

3.3.1 *Elements of Cooperative Game Theory*

The elements of a cooperative game theory are a set of players and a characteristic function. Suppose $n \geq 2$ denotes the number of players in a game, numbered from 1 to n , and N denotes the set of players $N = \{1, 2, 3, \dots, n\}$. The coalition (S) is defined as a subset of N , $S \subseteq N$, and set of all coalitions is denoted by 2^N .

The set N is called the grand coalition and the empty set \emptyset is called an empty coalition. In case of 2 players i.e., when $n=2$, four coalitions $\{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$ can be formed. In general, for n players, 2^n set of coalitions can be formed with 2^n elements in it.

Definition: A n -person coalitional game is represented by (N, v) , here $N = \{1, 2, \dots, n\}$ is the set of players and v is the characteristic function. The characteristic function must satisfy two conditions, [29]

- (i) $v(\emptyset) = 0$
- (ii) if S and T are two separate coalitions ($S \cap T = \emptyset$), then $v(S) + v(T) \leq v(S \cup T)$ [29]

$v(S)$ is considered as value or worth of a coalition ($S \subseteq N$). The condition (i) states that the value of an empty coalition is zero and (ii) states that the value of two

separate coalitions should be greater when they join compared to the value obtained when they don't form a coalition.

If the set is formed by all the players of the coalition, it is called a grand coalition. The grand coalition may not always be profitable for all the players, thus it is preferable for some players to form small coalition sets. For instance, if 'S' is a coalition set with payoff 'v' and if the members of 'S' join another coalition S' and get payoff v' the players who choose to form coalition S' are called defectors and the payoff received by them in coalition S' is greater than the payoff received by them in coalition 'S' i.e., $v' > v$ [30].

In contrast to grand coalition, empty coalition is a set with no players in it. A proper subset is a coalition which consists of players less than the total number of players. If the coalition is made from one player, then it is called as singleton coalition [31].

Types of players in Cooperative Game Theory:

Depending on the contribution of the players in the cooperative game, they are divided into three types which are Regular, Dummy and Veto players. Regular players are those who expect payoffs by contributing to the coalition, these players are not important for forming the coalition. Dummy players don't contribute any value to the coalition but are important to a coalition, due to which they get some payoff at the end of a game.

A player i^* is said to be a veto player, if he is involved in all winning coalitions. i^* is a veto player if $v(N \setminus i^*) = 0$ [32].

3.3.2 *Nature of the Characteristic Functions*

The characteristic functions of a cooperative game are expected to be super-additive, sub-additive or monotonic. A game is said to be super-additive if the value obtained by the players in coalition is greater than the sum of all values that the players would receive individually.

A game $G=(N,v)$ is super-additive, if S and T are separate coalitions ($S \cap T = \varnothing$) and $v(S \cup T) \geq v(S) + v(T)$ for all $S, T \subseteq N$ where $v(S \cup T)$ is the payoff of the coalition and $v(S), v(T)$ are the values of individual players [32].

When the worth of a coalition is less than the sum of the worth's obtained by individual players, the game is said to be sub-additive. A game is sub-additive when ($S \cap T = \varnothing$) and $v(S \cap T) \leq v(S) + v(T)$. Monotonicity means that larger coalitions gain more value, when a coalition T is greater than coalition S it produces higher values, $S \subseteq T \rightarrow v(S) \leq v(T)$ [33].

3.3.3 *Definition of the Core*

One of the solution types of cooperative game theory is 'The Core'. It involves about the stability of the coalition instead of fair distribution of payoffs [34]. Sometimes the players may form a grand coalition, or some players may wish to form smaller coalitions. As, sometimes players in a smaller coalition gain more profits [33].

The payoff vector ' x ' is said to be in the core of the game (N, v) if,

$$S \subseteq N, \sum_{i \in S} x_i \geq v(S) \tag{3.1}$$

The sum of payoffs obtained by players in a sub-coalition S should be at least equal to the payoff obtained by the players by not forming a coalition. In some games, the core defines the empty set.

A game with an empty core is described as a situation with strong instability, as expected payoffs in a grand coalition are vulnerable to coalitional blocking [34].

3.3.4 *The Shapley Value*

According to Lloyd Shapley, the benefits are divided among players according to their marginal contributions or the payoff received by each individual player is equivalent to the value they add to the coalition. For example, consider a committee in which all the members must be present to pass a suggestion. Here, $v(N) = 1, v(S) = 0$ and $N \neq S$ Where, N is the total number of players, S is the coalition with missing member.

$v(N) - v(N \setminus \{i\}) = 1$ denotes that for each member i the marginal contribution is 1 and all the members are important to add value. In this scenario, the marginal contributions cannot be allocated to all the members. In cases where the benefits can be divided equally among the members so that each member gets $1/N$ of the value, the Shapley axioms can be used to allocate the value [30].

(a) *Shapley Axioms*

According to Shapley, the surplus can be distributed equally among the players in a fair way by considering the value added by each player. To achieve this, the following axioms are proposed, [35]

(i) Efficiency: The payoffs should add up to $v(N)$, which means that the surplus of the grand coalition should be allocated among the players.

$$\sum_{i \in N} \varphi_i(N) = v(N) \tag{3.2}$$

(ii) Symmetry: If two players i and j contribute the same value to every coalition, then i and j are said to be interchangeable w.r.t 'v'.

$$v(S \cup \{i\}) = v(S \cup \{j\}) \tag{3.3}$$

(iii) Additivity: The solution to the sum of two TU (Transfer Utility) games must be the sum of the value obtained by each game.

$$(v_1 + v_2)(S) = v_1(S) + v_2(S) \quad (3.4)$$

(iv) Null Players: A player i is said to be a null player if he doesn't add any value to the coalition.

$$v(S \cup \{i\}) = v(S) \quad (3.5)$$

(b) Mathematical Definition of Shapley value

The Shapley value is the solution concept of cooperative game theory. In a coalitional game (N, v) , the Shapley value divides the total payoff among the players of the game.

According to the Shapley value, the amount that player ' i ' receives in a coalition game (N, v) is defined by,

$$\varphi_i(v) = \frac{1}{N!} \sum_{S \subseteq N} |S|! (n - |S| - 1)! (v(S \cup \{i\}) - v(S)) \quad (3.6)$$

Where, n is the total number of players and the sum extends over all subsets S of N not containing player i . The Shapley value describes the marginal contributions of ' i ' ($v(S \cup \{i\}) - v(S)$) when added to a coalition that doesn't have ' i ' in it, which is multiplied by different ways of coming up with marginal contributions $|S|! (n - |S| - 1)!$ and dividing by all possible ways in which the coalition can be formed ($N!$) [34].

4. PROPOSED COOPERATIVE GAME THEORY FOR PROSUMER COLLABORATION

4.1 NRG-X-Change Mechanism

Traditionally, the locally produced renewable energy was traded on a day-ahead basis by the prosumer (a person who produces and consumes energy) and the consumer (a person who consumes the energy), by engaging in a double auction to exchange their energy [1]. In a double auction market, buying orders were submitted to an agent by the buyers who would offer high prices to pay, and sell orders were submitted to set lower prices for selling [2]. But the buyers and sellers would bid and order inefficiently, as the bidding was dependent on the assumption of future supply and demand.

For example, prosumers who were unaware of the market would bid their energy at a higher price, which would bring unmatched orders for their energy. Due to a lack of buyers at the time, the energy would be produced and injected into the grid and prosumers would make zero profit.

Currently, the NRG-X-Change mechanism is used for trading locally produced energy, which doesn't depend on the energy market or matching of orders but rather, produced energy is fed continuously to the grid and prosumers are paid according to the actual usage [1]. In this mechanism, each prosumer provides energy to the grid individually and gets paid according to his contribution.

Instead, the prosumers can form coalitions with other prosumers of the same grid which may sometimes result in a higher profit than they would obtain by investing their energy into the grid individually.

4.1.1 System Model

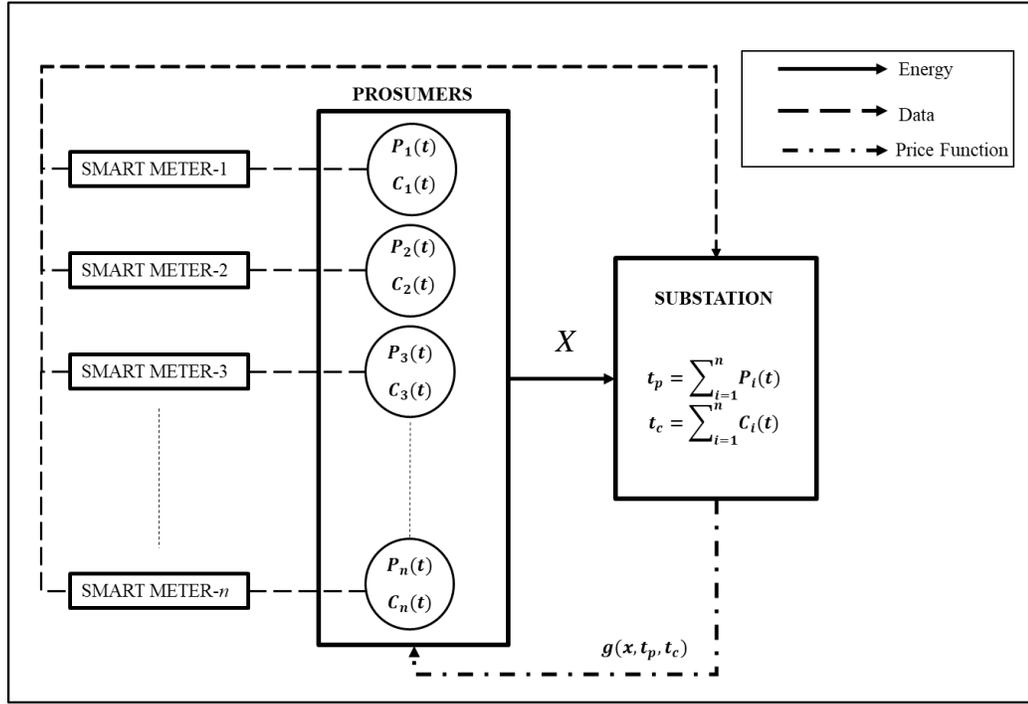


Figure 4.1: Example scenario [2]

In Figure 4.1, we have a set I , where $i=1,2,3,\dots,n$ houses, n smart meters and a substation. $P_i(t)$ is the amount of energy produced and $C_i(t)$ is the amount of energy consumed by i^{th} house. A prosumer consumes his produced energy first and provides excess amount of energy to the grid, that is indicated by X . The data of energy produced and consumed by each prosumer is calculated by smart meters, which are connected to individual houses. The substation uses the data provided by smart meters to calculate the total energy production (t_p) and consumption (t_c) of all the prosumers, and the producers are billed according to the price function $g(X, t_p, t_c)$ for their produced energy.

$$\text{Where, } g(X, t_p, t_c) = \frac{X * q}{e^{(t_p - t_c)^2 / a}}, \text{ (} a \text{ is the scaling factor and } q \text{ is the maximum}$$

cost that is awarded for the injected energy) [2].

4.2 Computing Shapley value

In this section, a simple example is presented to explain the calculation of Shapley value. We test different pricing schemes such as linear, convex and concave to analyze the effect of coalition on gains obtained by prosumers.

4.2.1 Example of Shapley value Calculation for 3 Prosumers

We consider a community of three solar prosumers $P = \{1,2,3\}$, who agree to form a coalition and produces energy as shown in Table 4.1. Here, the number of possible coalitions are $2^3=8$ and the number of ways to build the grand coalition is $3! = 6$.

Mathematical Model

Parameter	Definition	
I	Total number of prosumers, $i = \{1,2,3\}$	
S	Subset of $i = \{\{1\},\{2\},\{3\}, \{1,2\} \dots\dots\dots, \{1,2,3\}\}$	
p_i	Energy produced by i^{th} prosumer	
Total energy (kWh), $P_i = \sum_{i=1}^3 P_i$		(4.1)

Consider, for example, the characteristic function indicated by $v(S)$, which determines the value obtained by coalition of different subsets of prosumers.

$$v(S) = 0.12 * P_i^2 \tag{4.2}$$

Table 4.1: Energy produced by each prosumer

Prosumers	Energy Produced (kWh)
1	100
2	30
3	85

Table 4.2: Possible coalitions of each prosumer

Coalitions		Value of Characteristic function	
1	$S = \{\emptyset\}$	$v(\emptyset)$	0
2	$S = \{1\}$	$v(1)$	1200
3	$S = \{2\}$	$v(2)$	108
4	$S = \{3\}$	$v(3)$	867
5	$S = \{1,2\}$	$v(12)$	2028
6	$S = \{1,3\}$	$v(13)$	4107
7	$S = \{2,3\}$	$v(23)$	1587
8	$S = \{1,2,3\}$	$v(123)$	5547

In Table 4.2, 'S' is the set of each coalition formation, and 'v' is the characteristic function that is calculated for all the 8 possible coalitions.

Table 4.3: Shapley value calculation of Prosumer 1

Ways to build grand coalition	Possible ways to build grand coalition	Marginal Contribution of prosumer 1
3! = 6 ways to build the grand coalition	1 12 123	$v(1) - v(\emptyset) = 1200$
	1 13 132	$v(1) - v(\emptyset) = 1200$
	2 21 213	$v(12) - v(2) = 1920$
	2 23 231	$v(123) - v(23) = 3960$
	3 31 312	$v(13) - v(3) = 3240$
	3 32 321	$v(123) - v(32) = 3960$

After calculating the marginal contribution of prosumer 1 for each case in which the grand coalition can be formed (Table 4.3), we can calculate the Shapley value of prosumer 1 by the equation below:

$$\varphi_i(v) = \frac{1}{N!} \sum_{S \subseteq N} |S|! (n - |S| - 1)! (v(S \cup \{i\}) - v(S)) \quad (4.3)$$

Input Parameter

Notation

Set	Definition
S	$S = [\{1,\}, \{2,\}\{3\}, \{1,2\} \dots, \{1,2,3\}] = 2^3$ coalition combinations

Parameter	Definition
n	Total number of prosumers
$n!$	$3! = 6$ coalition combinations
$(v(SU\{i\}) - v(S))$	Marginal Contribution of i^{th} prosumer when added to set S
$\Phi_i(v)$	Shapley value of i^{th} house for cost function
$ S ! (n - S - 1)!$	Different ways in which the marginal contributions can be formed

$$\varphi_1(v) = \frac{1}{3!} \sum [1200 + 1200 + 1920 + 3960 + 3240 + 3960] \quad (4.4)$$

$$\varphi_1(v) = \frac{1}{3!} \sum [2(1200) + 1920 + 2(3960) + 3240] = \$2580 \quad (4.5)$$

Table 4.4: Marginal Contribution of Prosumer 2

Ways to build grand coalition	Possible ways to build grand coalition	Marginal Contribution of prosumer 2
3! = 6 ways to build the grand coalition	1 12 123	$v(12) - v(1) = 828$
	1 13 132	$v(132) - v(13) = 1440$
	2 21 213	$v(2) - v(\varnothing) = 108$
	2 23 231	$v(2) - v(\varnothing) = 108$
	3 31 312	$v(312) - v(31) = 1440$
	3 32 321	$v(32) - v(3) = 720$

$$\phi_2(v) = \frac{1}{3!} \sum [828 + 1440 + 108 + 108 + 1440 + 720] \quad (4.6)$$

$$\phi_2(v) = \frac{1}{3!} \sum [828 + 2(1440) + 2(108) + 720] = \$774 \quad (4.7)$$

Table 4.5: Marginal Contribution of Prosumer 3

Ways to build grand coalition	Possible ways to build grand coalition	Marginal Contribution of prosumer 3
3! = 6 ways to build the grand coalition	1 12 123	$v(123) - v(12) = 3519$
	1 13 132	$v(13) - v(1) = 2907$
	2 21 213	$v(213) - v(21) = 3519$
	2 23 231	$v(23) - v(2) = 1479$
	3 31 312	$v(3) - v(\varnothing) = 867$
	3 32 321	$v(3) - v(\varnothing) = 867$

$$\phi_3(v) = \frac{1}{3!} \sum [3519 + 2907 + 3519 + 1479 + 867 + 867] \quad (4.8)$$

$$\phi_3(v) = \frac{1}{3!} \sum [2(3519) + 2907 + 1479 + 2(867)] = \$2193 \quad (4.9)$$

The Shapley values of Prosumer 2 and 3 indicated by equations (4.7) and (4.9) are calculated similar to Prosumer 1.

Table 4.6: Shapley values with and without Coalition of each prosumer

Prosumer	Shapley value with coalition	Shapley value without coalition
1	\$2580	\$1200
2	\$774	\$108
3	\$2193	\$867

From Table 4.6, it can be observed that all three solar prosumers receive better benefits by investing their energy in a coalition with characteristic function,

$$v = 0.12 * p(t)^2 \tag{4.10}$$

4.2.2 Comparison of Shapley value for Convex, Linear and Concave

characteristic functions ($Y=X^2, Y=X$, and $Y=\sqrt{X}$)

Table 4.7: Energy produced by individual prosumer

Number of prosumers (N)	Energy produced by the individual Prosumer
2	[100 30]
3	[100 30 85]
4	[100 30 85 70]
5	[100 30 85 70 60]
6	[100 30 85 70 60 45]
7	[100 30 85 70 60 45 20]
8	[100 30 85 70 60 45 20 15]
9	[100 30 85 70 60 45 20 15 90]
10	[10 30 85 70 60 45 20 15 90 110]

Table 4.8: Shapley value calculation with and without coalition for different characteristic functions

<i>Number of prosumers</i>	<i>$Y=X^2$ Average Shapley value</i>		<i>$Y=X$ Average Shapley value</i>		<i>$Y=\sqrt{X}$ Average Shapley value</i>	
	<i>With Coalition</i>	<i>Without Coalition</i>	<i>With Coalition</i>	<i>Without Coalition</i>	<i>With Coalition</i>	<i>Without Coalition</i>
2	1014	654	7.8	7.8	0.6838	0.9286
3	1849	725	8.6	8.6	0.5865	0.9878
4	2436.75	690.75	8.55	8.55	0.5064	0.9919
5	2856.6	639	8.28	8.28	0.4458	0.9794
6	3042	573	7.8	7.8	0.3949	0.9503
7	2881.7	498	7.025	7.025	0.3471	0.8912
8	2709.3	438.7	6.375	6.375	0.3092	0.8379
9	3536.3	498.3	6.866	6.866	0.3025	0.8713
10	3000	300	6	6	0.2683	0.8485

The energy produced by each prosumer and average Shapley values for different number of prosumers ($N=2$ to 10) assuming that all prosumers form grand coalition is listed in the above tables (4.7) (4.8). Shapley value is calculated for two instances: when prosumers form (i) a coalition (ii) without coalition, and is computed for convex, linear and concave characteristic functions.

From the first condition i.e., when $Y = X^2$, we can infer that the prosumers can earn more profits by forming coalitions rather than putting the energy into the grid individually.

In the linear condition, $Y = X$ we can observe that the profits earned by the prosumers are the same when they form coalitions and when they contribute energy individually. From the third condition, $Y = \sqrt{X}$, we can conclude that the profits earned by prosumers by not forming a coalition is greater when compared to the profits they earn by collaborating.

4.3 Analysis and Comparison of Three Codes

The total coalition formations and Shapley values are calculated by a code written in MATLAB. Shapley value can be computed using three different codes, that are analyzed by comparing execution time of the three codes.

Table 4.9: Pseudo code for all 3 codes

Pseudo Code		
Code 1	Code 2	Code 3
<p>Inputs : m = total possible coalition combinations A = matrix with all possible coalitions n = Total number of prosumers</p> <pre> for k=1:n for i=1:m if ith column of each row > 0 K₁ stores 'v' value of that row A₁ Matrix stores the coalitions when its ith column is >0 else end end end A₃ stores coalitions not listed in A₁ for i=1:m for j=1:m if A₃ []==A [] K₂ stores 'v' value of coalitions not listed in K₁ else end end end end K₃ Calculates marginal contribution of kth prosumer K₃ = (K₂ - K₁) K₄ sum of coalitions in set A₂ for i=1:m if K₄(i)>0 'r' calculates sum of all marginal contributions in K₃ else end end end S divides 'r' by 'n' to calculate Shapley value of kth prosumer. end </pre>	<p>Function result =Shapley(v, coalition ,player)</p> <p>Switch nargin (returns number of function input arguments) case 2 case 3 if strcmp (firstplayer, 'left') matrix of original coalition is flipped to left end original coalition is converted to decimal and stored in 'id' & v(id) assigned with values of 'v' end</p> <p>'n' is the total number of prosumers n_factorial= calculates factorial value of 'n' coalitions=de2bi(); (converts coalitions from binary to decimal)</p> <pre> for i=1:n part; coalitions with ith prosumer ex: when n=3 part=[100 101 110 111] not part; coalitions not involved in part part id; part matrix converted to decimal not part id; not part matrix converted to decimal Result; calculates Shapley value of individual prosumer end </pre>	<p>Function Shapley=Shappie</p> <p>n=Total number of prosumers A1=[matr(n) v'] (contains all coalitions with 'v' values) While i<n for M; calculates marginal contributions of coalitions Shapley ; calculates Shapley value by dividing sum of M with n! end</p>

4.3.1 Comparison of Execution Times

We consider four different sets of prosumers (n) such as {2,5,7,10}, and for each set of prosumers, various production and consumption values are considered and studied for 3 different cases:

1. Production > Consumption
2. Production = Consumption
3. Production < Consumption

Table 4.10: Input values when production > consumption

No. of prosumers	Production Values	Consumption Values
2	{35,55}	{15,25}
5	{35,60,85,110,135}	{10,25,60,55,100}
7	{35,60,85,110,135,150,175}	{10,25,60,55,100,75,125}
10	{35,60,85,110,135,150,175,200,215,235}	{10,25,60,55,100,75,125,150,185,175}

Table 4.11: Input values when production = consumption

No. of prosumers	Production Values	Consumption Values
2	{10,20}	{10,20}
5	{10,30,50,70,90}	{10,30,50,70,90}
7	{10,30,50,70,90,110,130}	{10,30,50,70,90,110,130}
10	{10,30,50,70,90,110,130,150,170,190}	{10,30,50,70,90,110,130,150,170,190}

Table 4.12: Input values when production < consumption

No. of prosumers	Production Values	Consumption Values
2	{15,25}	{35,55}
5	{10,25,60,55,100}	{35,60,85,110,135}
7	{10,25,60,55,100,75,125}	{35,60,85,110,135,150,175}
10	{10,25,60,55,100,75,125,150,185,175}	{35,60,85,110,135,150,175,200,215,235}

We consider input values from tables 4.10, 4.11 and 4.12 for 3 different codes, each used to calculate the Shapley value and then calculate the execution time taken by each of the codes.

(a) Case 1: (Code 1)

In this case, the execution time taken by Code 1 is calculated by considering input values from Tables 4.10, 4.11 and 4.12 and studied for 3 different cases ($P>C$, $P=C$ and $P<C$) as shown in Table 4.13.

Table 4.13: Execution time for Code 1

Number of prosumers (n)	Execution Time of Code 1 (sec)		
	P>C	P=C	P<C
2	0.015	0.013	0.015
5	0.036	0.037	0.039
7	0.368	0.413	0.387
10	23.052	22.621	22.879

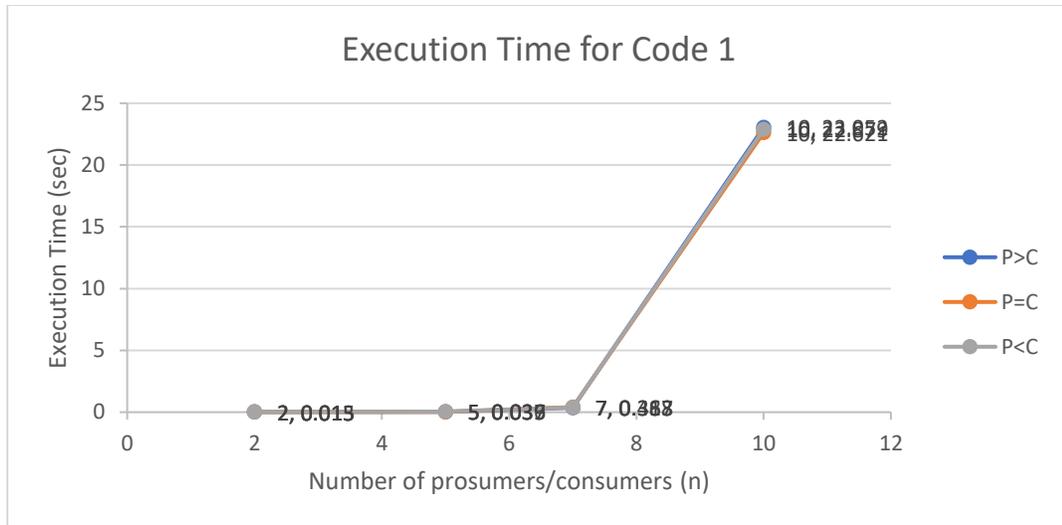


Figure 4.2: Execution time for Code 1 for 3 different cases of production and consumption values

In Figure 4.2, the plot represents the execution time for 3 different cases of input values (n). The plot indicates that time taken by Code 1 to execute the Shapley value is roughly the same for all 3 cases ($P>C$, $P=C$ and $P<C$) as the three plots are overlapping on one another. The execution time is increasing from $n=7$ to $n=10$, as the number of coalition combinations gradually increases from $n=7$ (128 coalition combinations) to $n=10$ (1024 coalition combinations).

(b) Case 2: (Code 2)

In this case, the time taken by Code 2 to execute the Shapley value is obtained for 3 different scenarios of production and consumption values, as mentioned in Table 4.14.

Table 4.14: Execution time for Code 2

Number of prosumers (n)	Execution Time for Code 2 (sec)		
	P>C	P=C	P<C
2	0.030	0.031	0.030
5	0.033	0.033	0.034
7	0.035	0.035	0.037
10	0.053	0.051	0.061

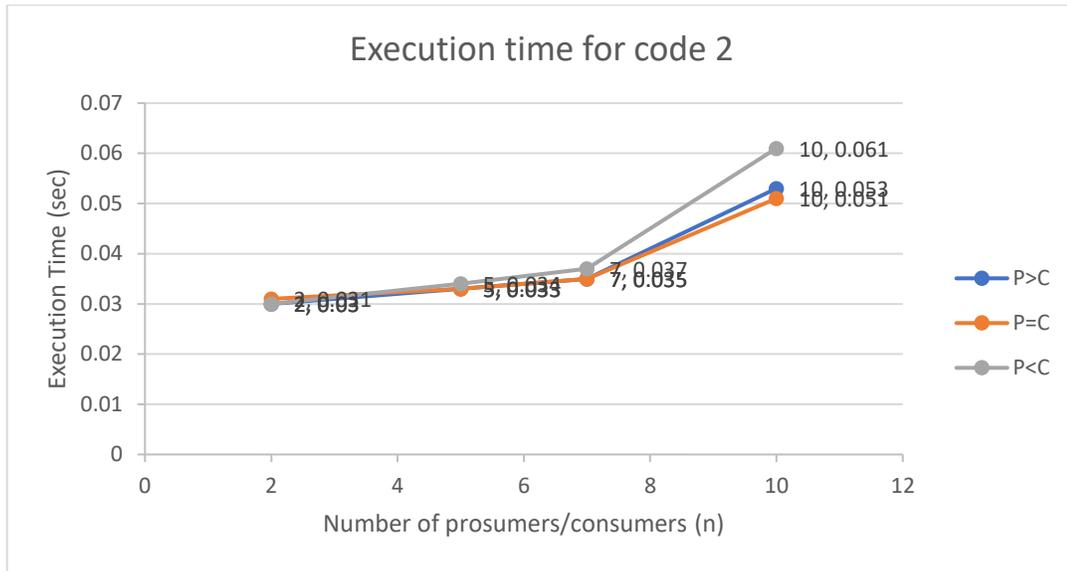


Figure 4.3: Execution time of Code 2 for 3 different cases of production and consumption values

When compared with Code 1, the execution time taken by Code 2 when the number of prosumers (n) increases is much less. Code 2 is best used for the cases when ' n ' value is higher.

Case 3: (Code 3)

In Case 3, the execution time taken by Code 3 is calculated by considering input values from Tables 4.10, 4.11 and 4.12 and studied for 3 different cases as mentioned in Table 4.15.

Table 4.15: Execution time for Code 3

Number of prosumers (n)	Execution Time for Code 3 (sec)		
	P>C	P=C	P<C
2	0.005	0.006	0.005
5	0.011	0.012	0.013
7	0.028	0.023	0.030
10	0.201	0.220	0.216

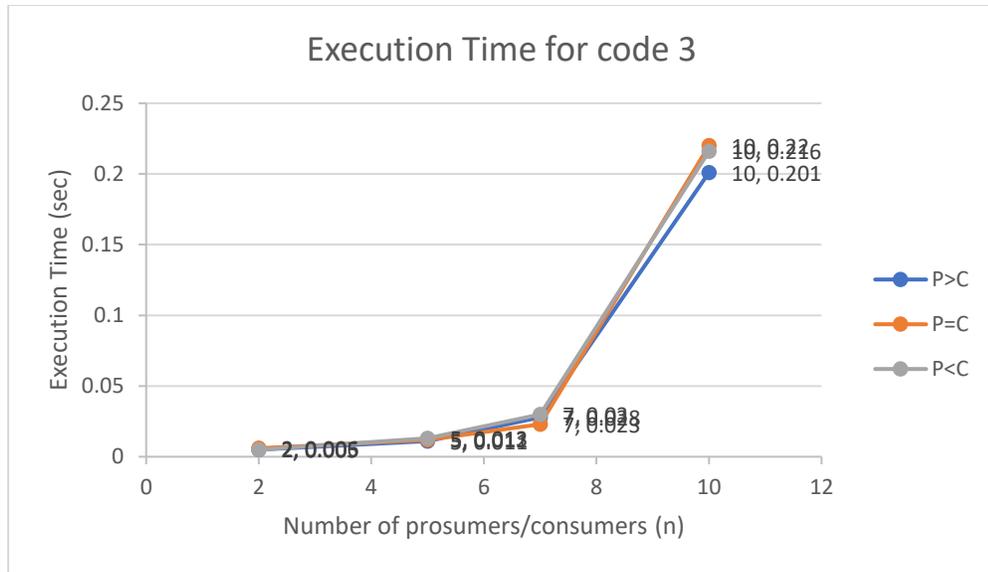


Figure 4.4: Execution time of Code 3 for 3 different cases of production and consumption values

In Figure 3, the time taken by Code 3 to execute the Shapley value is roughly the same for all 3 cases ($P>C$, $P=C$ and $P<C$). Compared to the other 2 codes, the execution time taken by Code 3 when the number of prosumers (n) is less ($n = 2$ to 7) is smaller. Hence, Code 3 is suitable for the cases when $n \leq 7$.

(ii) Comparison among three codes for each of the three cases ($P>C$, $P=C$, $P<C$)

Table 4.16: Comparison among the 3 Codes for the case of production > consumption

Number of prosumers (n)	Production > Consumption		
	Code 1	Code 2	Code 3
2	0.015	0.026	0.005
5	0.036	0.034	0.011
7	0.368	0.099	0.028
10	23.052	0.151	0.201

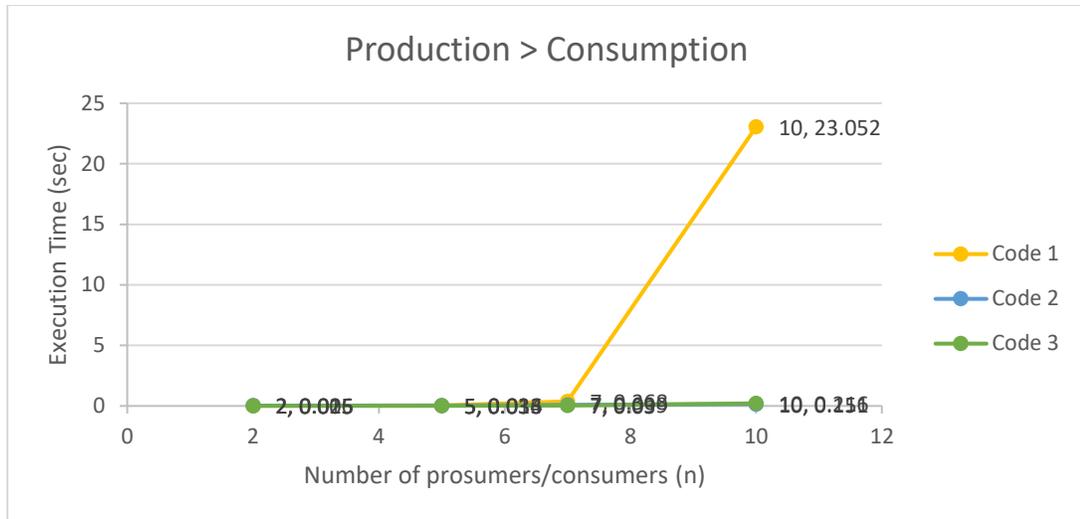


Figure 4.5: Execution time taken by 3 Codes for the case of production > consumption

The table 4.16 provides the execution time taken by the three codes when the production values are greater than the consumption values. From the plot it can be observed that execution time for Code 2 and 3 is almost equal while execution time for Code 1 is very high as the set of input values is increased.

Table 4.17: Comparison among the 3 codes for the case of production = consumption

Number of prosumers (n)	Production = Consumption		
	Code 1	Code 2	Code 3
2	0.013	0.025	0.006
5	0.037	0.036	0.012
7	0.413	0.035	0.023
10	22.621	0.069	0.220

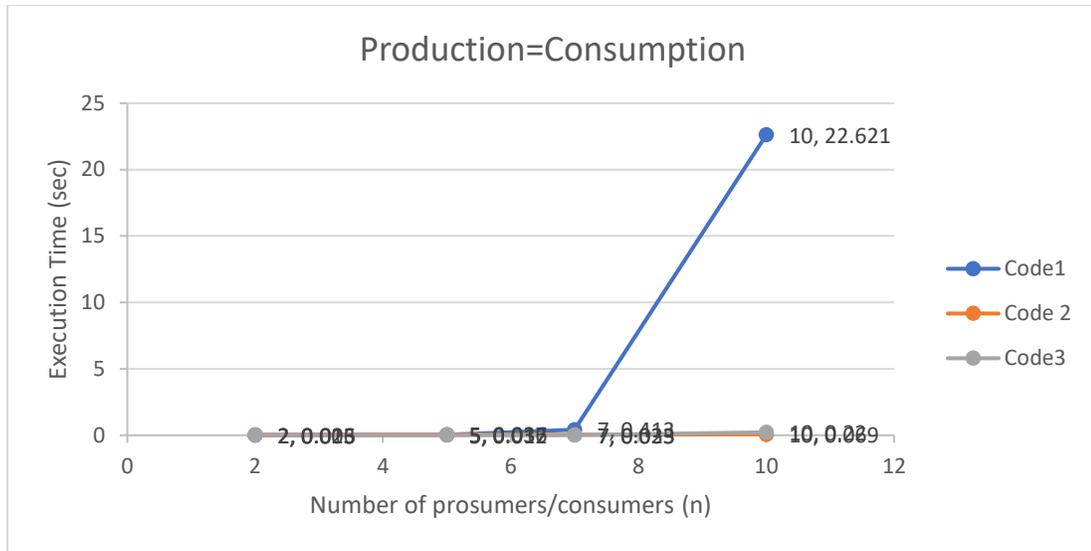


Figure 4.6: Execution time taken by 3 Codes for the case of production = consumption

The table 4.17 provides the execution time taken by the three codes when the production values are equal to the consumption values. As seen in the previous case, execution time for Code 2 and 3 is almost equal when compared with Code 1, whose execution time is very high when the number of prosumers (n=10) is high.

Table 4.18: Comparison among the 3 codes for the case of production < consumption

Number of prosumers (n)	Production < Consumption		
	Code 1	Code 2	Code 3
2	0.015	0.025	0.005
5	0.039	0.035	0.013
7	0.387	0.036	0.030
10	22.879	0.049	0.216

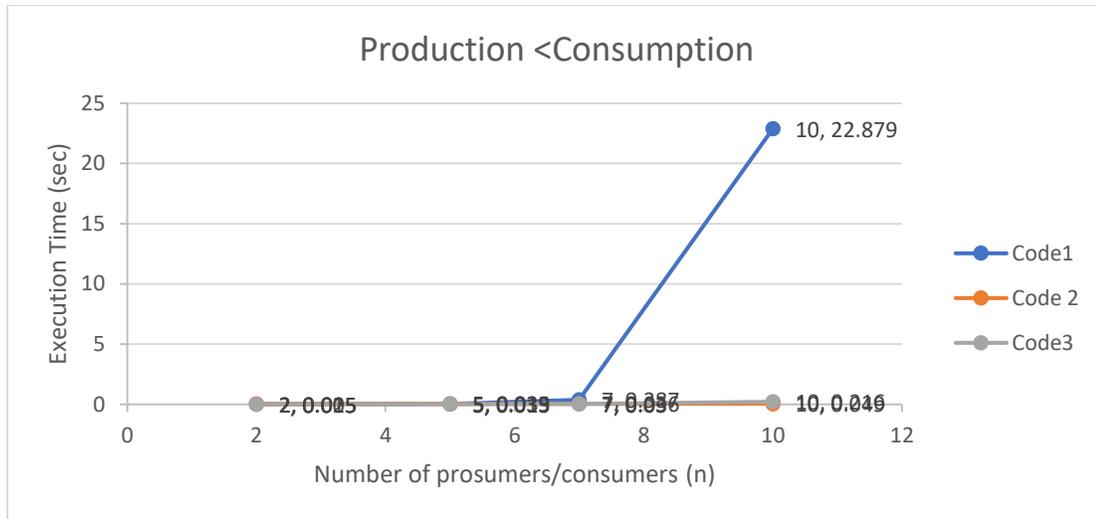


Figure 4.7: Execution time taken by 3 codes for the case of production < consumption

The execution time taken by the 3 codes when the production value is less than the consumption value is provided by table 4.18. As seen in the previous cases ($P > C$ and $P = C$) the time taken by code 1 to execute Shapley value is more when compared with Code 2 and 3.

So, from all the above observations it can be concluded that code 1 takes high execution time as the value of n increases. Code 1 is not preferable for cases where the production/ consumption value (n) is more than 10. Code 2 is suitable for the cases when production is equal to and less than consumption value. The execution time taken by Code 3 when the number of prosumers/ consumers (n) is less ($n = 2$ to 7) is much smaller. So, Code 3 is suitable for the cases when $n \leq 7$.

5. A NETWORK OF PROSUMERS

In this chapter, we consider a network of prosumers who belong to the same grid and agree to form a coalition. In Section 5.1, the production and consumption values of each prosumer are considered hypothetical. In Section 5.2, we collect the data from Pecan Street Inc.[36] to study how Shapley value changes according to the change in production and consumption values.

5.1 A Network of 5 Prosumers with Hypothetical Values

5.1.1 Examination of Multiple Scenarios to find Shapley value using ‘g’ Function

In this section, the network is considered to have 5 prosumers. The function ‘g’ which is the price function for paying producers [2] is considered as the characteristic equation. Multiple scenarios with different hypothetical values of production and consumption are examined.

Mathematical Model

Notation

Set	Definition
I	Total number of prosumers, $i=1,2,3,\dots, n$
S	Coalition combinations of all prosumers, $S= [\{1\},\{2\},\{3\},\{4\},\{5\}, \{1,2\}, \{1,3\} \dots \{1,2,3,4,5\}] = 2^5$ possible ways to form coalitions

Parameter	Definition
q	Maximum cost that is awarded to the prosumer for his produced energy (\$0.1 per kWh)
a	Scaling Factor
P_i	Production of i^{th} prosumer
C_i	Consumption of i^{th} prosumer
X_i	Difference between prosumer production and consumption in i^{th} house
$n!$	$5! = 120$ coalition combinations
$(v(SU\{i\} - v(S)))$	Marginal contribution of i^{th} prosumer when added to S that doesn't contain i
$ S ! (n - S - 1)!$	Different ways in which the marginal contributions can be formed
$\varphi_i(v)$	Shapley value of i^{th} prosumer for characteristic function

Variable	Definition
-----------------	-------------------

t_p	Total energy production of all prosumers
-------	--

t_c	Total energy consumption of all prosumers
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Difference between production and consumption, (kWh)	$X = \sum_{i=1}^n (P_i - C_i)$	(5.1)
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Total energy production of all prosumers, (kWh)	$t_p = \sum_{i=1}^n P_i$	(5.2)
---	--------------------------	-------

Total energy consumption of all prosumers, (kWh)	$t_c = \sum_{i=1}^n C_i$	(5.3)
--	--------------------------	-------

The function ‘g’ is defined as,

$$v = g(x, tp, tc) = \frac{X*q}{e^{(tp-tc)^2/a}} \quad (5.4)$$

5.1.2 Results and Analysis

In this section, we test the behavior of the ‘g’ function that is proposed by NRG-X-change mechanism by analyzing 4 scenarios. Each scenario consists of 5 different cases, where each case represents a network of 5 prosumers with different production and consumption values.

i) Scenario 1:

In the first scenario, in each case we increase the production value of each prosumer from 0 to 100 kWh. The consumption value of all 5 prosumers are constant and equal to [20 20 20 20 20] for each case.

Table 5.1: Varying production value and constant consumption value of each prosumer in different cases

Case	Production Value of each prosumer (P_i) (kWh)					Consumption Value of each prosumer (C_i) (kWh)				
	P_1	P_2	P_3	P_4	P_5	C_1	C_2	C_3	C_4	C_5
1	0	0	0	0	0	20	20	20	20	20
2	10	10	10	10	10	20	20	20	20	20
3	30	30	30	30	30	20	20	20	20	20
4	60	60	60	60	60	20	20	20	20	20
5	100	100	100	100	100	20	20	20	20	20

By considering the production and consumption value of each prosumer from Table 5.1, we calculate the total energy produced and total energy consumed. The total energy offered is calculated by using Equation 5.5. Also, we calculate the Shapley value of all prosumers as shown in Table 5.2.

$$\text{Total Energy offered (kWh)} = \text{Total Energy produced}(t_p) - \text{Total Energy Consumed}(t_c)$$

(5.5)

Table 5.2: Total Shapley value of all prosumers with their total production and consumption values in different cases

<i>Case</i>	<i>Total energy Produced (t_p) (kWh)</i>	<i>Total energy Consumed (t_c) (kWh)</i>	<i>Total energy offered (kWh)</i>	<i>Total Shapley value (Φ_i)</i>
1	0	100	-100	-0.0454
2	50	100	-50	-41.043
3	150	100	50	41.0425
4	300	100	200	0
5	500	100	400	0

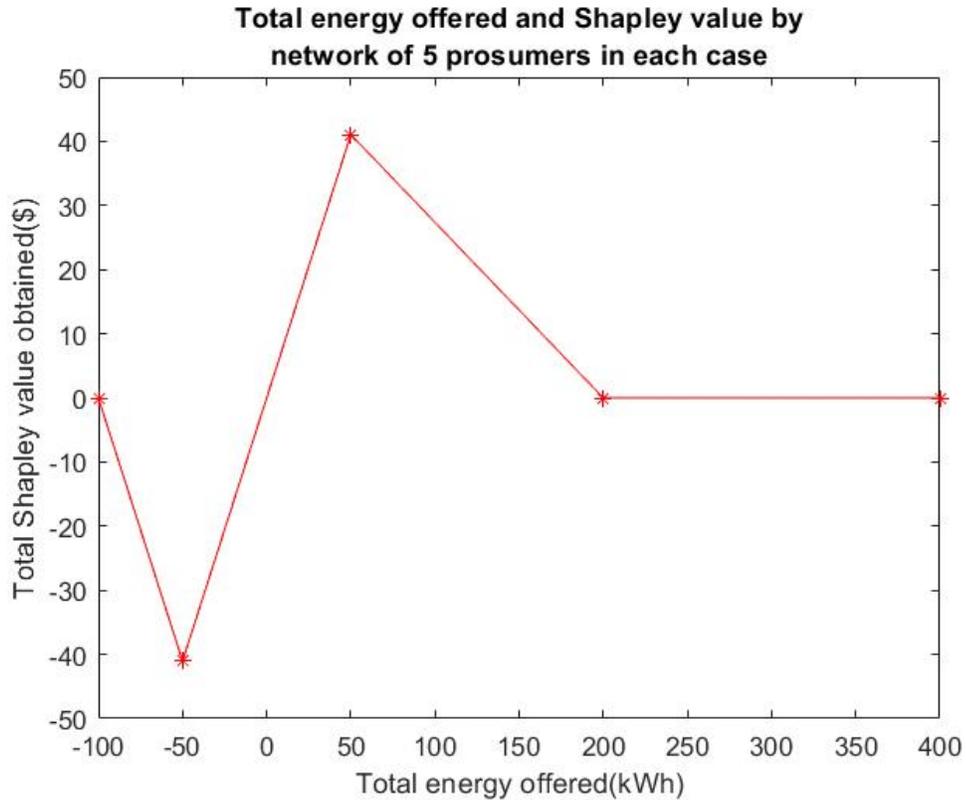


Figure 5.1: Comparison of total offered energy and Shapley value by a network of 5 prosumers in each case

In Case 1, the production value of each prosumer is equal to zero, whereas the consumption value is equal to 20 kWh.

In this case, the total production (t_p) of all prosumers is zero and total consumption (t_c) is 100. As, $t_p \ll t_c$, the total Shapley value that each prosumer receives is almost equal to 0.

In Case 2, even though the individual production value of all prosumers is increased to 10 kWh, the total production remains less than total consumption ($t_p \ll t_c$). Thus, the total Shapley value of all 5 prosumers is negative. The negative Shapley value has a physical meaning of zero.

In Case 3, the production value of each individual prosumer is increased to 30 kWh. Here, the total production is slightly greater than the total consumption ($t_p > t_c$). Thus, the Shapley value is positive.

In Cases 4 and 5, as total production (t_p) is more than twice that of total consumption (t_c) i.e., $t_p \gg t_c$ the Shapley value is zero due to over production.

ii) Scenario 2:

In Scenario 2, we consider the data from Table 5.1 of Scenario 1 and calculate the energy offered by Prosumer 2 with Equation 5.6.

$$\text{Energy offered (kWh)} = \text{Production value (} P_i \text{)} - \text{Consumption value (} C_i \text{)} \quad (5.6)$$

We also plot the graph between energy offered and Shapley value of Prosumer 2 and compare the values in five different cases.

Table 5.3: Energy offered and Shapley value of Prosumer 2 in different cases

<i>Case</i>	<i>Energy offered by prosumer 2 ($P_2 - C_2$) (kWh)</i>	<i>Shapley value of prosumer 2 (Φ_i)</i>
1	-20	-0.0091
2	-10	-8.2085
3	10	8.2085
4	40	0
5	80	0

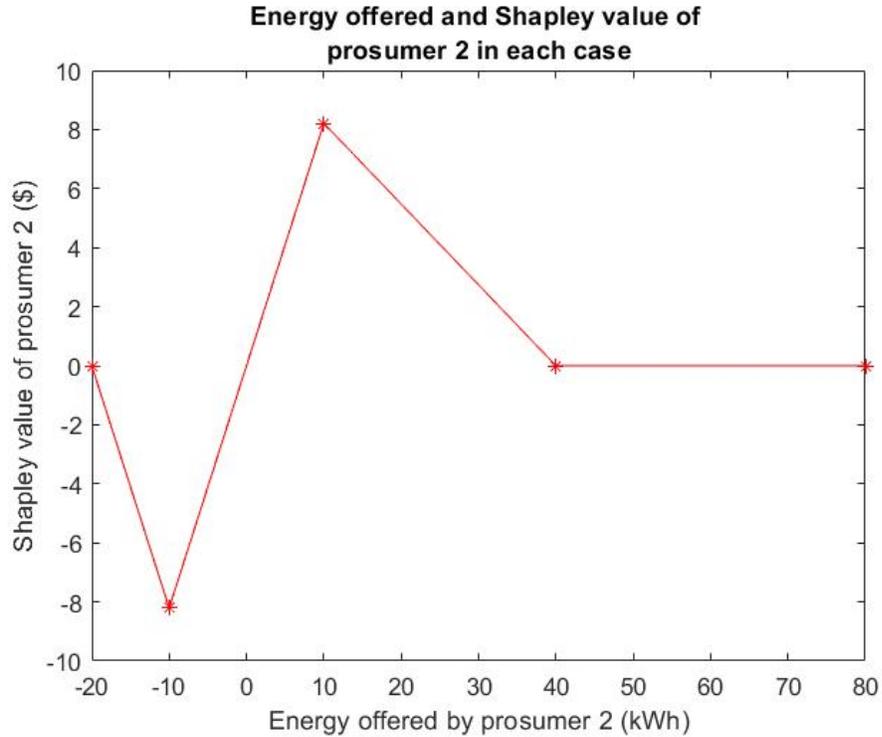


Figure 5.2: Comparison of offered energy and Shapley value of Prosumer 2 in different cases

The energy offered is the difference between production and consumption value of Prosumer 2. In Case 1, the energy offered by Prosumer 2 is negative as consumption is greater than production. The Shapley value of Prosumer 2 decreases for Case 2. In Case 3, as the production almost matches the consumption the Shapley value is positive. In Cases 4 and 5, Shapley value is 0 as the production value of Prosumer 2 is much greater than consumption value.

iii) Scenario 3:

In Scenario 3, we assume different production values for Prosumer 2 in each case while keeping the same production values for other prosumers in all cases. But we maintain the same consumption values for every prosumer in all cases.

Also, we assume high production values and low consumption values for prosumers 1 and 3, when compared to Prosumers 4 and 5 in all cases.

Table 5.4: Constant production and consumption values of every prosumer with varying production value of Prosumer 2 in each case

<i>Case</i>	<i>Production Value of each prosumer (P_i) (kWh)</i>					<i>Consumption Value of each prosumer (C_i) (kWh)</i>				
	P_1	P_2	P_3	P_4	P_5	C_1	C_2	C_3	C_4	C_5
1	100	0	120	30	40	50	30	60	90	75
2	100	10	120	30	40	50	30	60	90	75
3	100	30	120	30	40	50	30	60	90	75
4	100	60	120	30	40	50	30	60	90	75
5	100	100	120	30	40	50	30	60	90	75

By considering production and consumption values of every prosumer from Table 5.4, we calculate the energy offered by each prosumer using equation (5.6). We also compare the energy offered and Shapley value of Prosumer 2 as shown in Table 5.5.

Table 5.5: Energy offered and Shapley value of Prosumer 2

<i>Case</i>	<i>Energy offered by each prosumer (X) (kWh)</i>					<i>Energy offered by prosumer2 (kWh)</i>	<i>Shapley Value of prosumer 2 (Φ_i)</i>
	P_1	P_2	P_3	P_4	P_5		
1	50	-30	60	-60	-35	-30	-239.55
2	50	-20	60	-60	-35	-20	-195.062
3	50	0	60	-60	-35	0	0
4	50	30	60	-60	-35	30	39.59
5	50	70	60	-60	-35	70	0.509

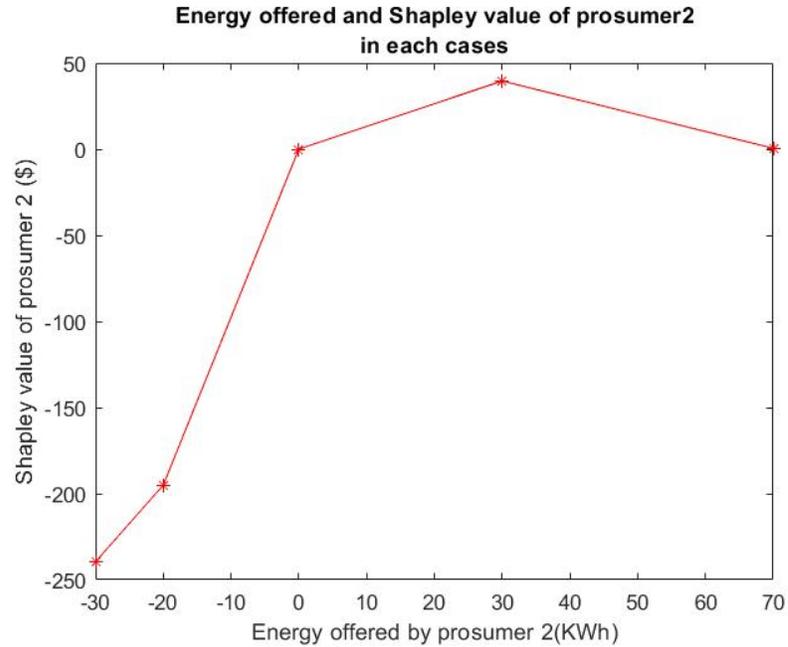


Figure 5.3: Comparison of offered energy and Shapley value of Prosumer 2 in each case

For Case 1 and Case 2, the Shapley value is negative as the production value is less than the consumption value. In Case 3, the Shapley value of Prosumer 2 is 0, as the production value of Prosumer 2 is equal to its consumption value. In Case 4, the production value of Prosumer 2 is twice that of consumption value, hence the Shapley value is positive. In Case 5, the production value is very high when compared to consumption value thus, the Shapley value is very small.

iv) Scenario 4:

In Scenario 4, we increase the production value of each prosumer from 0 to 100 kWh. The consumption values of all 5 prosumers are constant in each case and equal to [30 10 31 25 15] kWh.

Table 5.6: Varying production value and constant consumption value of each prosumer in different cases

<i>Case</i>	<i>Production Value of each prosumer (P_i) (kWh)</i>					<i>Consumption Value of each prosumer (C_i) (kWh)</i>				
	P_1	P_2	P_3	P_4	P_5	C_1	C_2	C_3	C_4	C_5
1	0	0	0	0	0	30	10	31	25	15
2	10	10	10	10	10	30	10	31	25	15
3	30	30	30	30	30	30	10	31	25	15
4	60	60	60	60	60	30	10	31	25	15
5	100	100	100	100	100	30	10	31	25	15

By considering production and consumption value of every prosumer from Table 5.4, we calculate the total energy offered by all prosumers in each case using Equation (5.5). Also, we compare the total offered energy and Shapley value of all prosumers obtained with and without coalition.

Table 5.7: Shapley value (with and without coalition) for total energy offered by all prosumers in each case

<i>Case</i>	<i>Total energy Produced (t_p) (kWh)</i>	<i>Total energy Consumed (t_c) (kWh)</i>	<i>Total energy offered (kWh)</i>	<i>Shapley Value (Φ_i)</i>
1	0	111	-111	-0.04
2	50	111	-61	-41.04
3	150	111	39	41.04
4	300	111	189	0
5	500	111	389	0

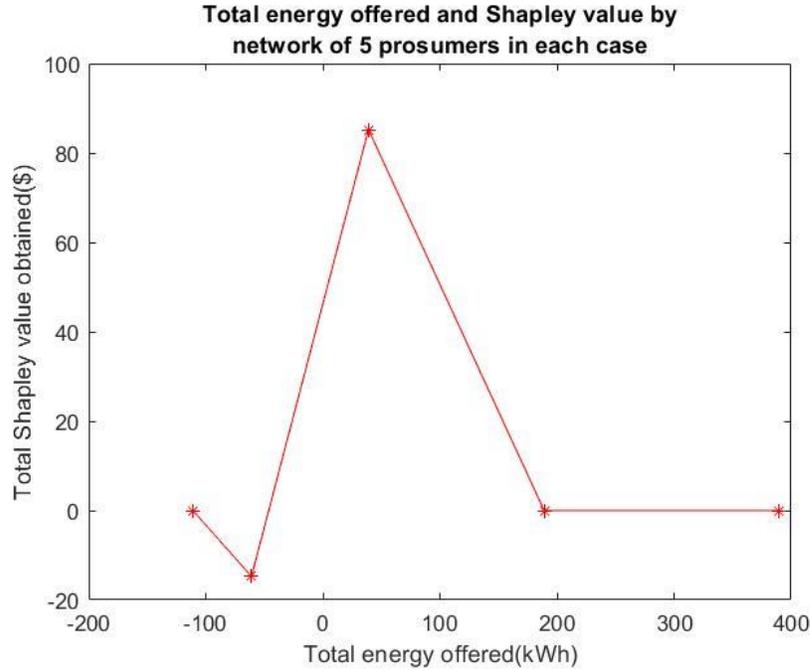


Figure 5.4: Comparison of total offered energy and Shapley value obtained by network of prosumers in each case

In Case 1 and Case 2, as the total produced energy is less than the total consumed energy, the total Shapley value obtained by all the prosumers is negative. In Case 3, the total produced energy is slightly greater than total energy consumed. Thus, the Shapley value is positive. For Case 4 and Case 5, as the total produced energy is more than twice that of total energy consumed, the Shapley value is equal to zero.

From all the Scenarios, we observe that prosumers obtain positive Shapley values when production is slightly greater or twice that of consumption. When production exceeds more than twice of consumption, the Shapley value is negative.

5.2 Pecan Street

Pecan Street is a research and development organization that gathers data from 1,115 active homes, 250 solar homes and 65 electric vehicle owners.

For each house, Pecan Street calculates the energy generation and usage for an interval ranging from one second to one hour [36].

Pecan Street information consists of a metadata file which is a .csv file as shown in Figure 5.5. This file consists of DATA IDs which are numbers unique to a house-resident pair. Each DATA ID is provided with information such as building type, city in which the resident is located, PV system installation, date enrolled with Pecan Street, date withdrawn, power generated by the PV system, power drawn from the electric grid and whole-home electricity usage.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1	dataid	active_rec_building_t	program_579	program_baseline	program_energy_internet_demo	program_l	program_p	program_c	program_c	program_shines	city	state	pv	pv_panel_total_amo	amount_o	amount_o	amo	
2	2836	Single-Family Home		yes							Austin	Texas						
3	2743	Single-Family Home		yes							Austin	Texas						
4	5323	Single-Family Home		yes	yes						Austin	Texas						
5	8560	Single-Family Home		yes	yes						Austin	Texas						
6	3313	Single-Family Home		yes	yes						Austin	Texas						
7	5052	Single-Family Home		yes	yes						Austin	Texas						
8	3936	Single-Family Home		yes	yes						Austin	Texas						
9	1622	Town Home		yes	yes						Austin	Texas						
10	1114	Single-Family Home		yes	yes						Austin	Texas	yes	South	5.25	5.25	0	
11	5433	Single-Family Home		yes	yes				CCET - Control		Austin	Texas	yes					
12	1101	Single-Family Home		yes	yes				CCET - Control		Austin	Texas	yes	South/Wes	6.25	3	3.25	
13	8872	Single-Family Home		yes	yes				CCET - Control		Austin	Texas	yes	South/Wes	5.5		3.25	
14	1464	Single-Family Home		yes	yes				CCET - Control		Austin	Texas	yes	South/Wes	3.3	1.65	1.65	
15	3824	Single-Family Home		yes	yes				CCET - Text Message		Austin	Texas	yes	South/Wes	6.37		3.185	
16	78	Single-Family Home		yes	yes						Austin	Texas	yes					
17	8047	Single-Family Home		yes	yes				CCET - Pricing Trial		Austin	Texas	yes					
18	5839	Single-Family Home		yes	yes						Austin	Texas	yes					
19	1050	Single-Family Home		yes	yes				CCET - Control		Austin	Texas						
20	8597	Single-Family Home		yes	yes				CCET - UT Text		Austin	Texas	yes	West	5.635		5.635	
21	7982	Single-Family Home		yes	yes		yes		CCET - Pricing Trial		Austin	Texas	yes	South/Wes	3.885	1.295	2.59	
22	7057	Single-Family Home		yes	yes						Austin	Texas						
23	3192	Single-Family Home		yes	yes				CCET - Pricing Trial		Austin	Texas	yes	South/Wes	6.105	4.07	2.035	
24	1790	Single-Family Home		yes	yes				CCET - Control		Austin	Texas	yes	South/Wes	6.37	3.43	2.94	
25	3577	Single-Family Home		yes	yes						Austin	Texas						
26	8890	Single-Family Home		yes	yes				CCET - Pricing Trial		Austin	Texas	yes	South/Wes	5.64	3.525	2.115	
27	6830	Single-Family Home		yes	yes						Austin	Texas						
28	2575	Single-Family Home		yes	yes				CCET - Pricing Trial		Austin	Texas	yes	South	6.25	6.25	0	
29	5785	Single-Family Home		yes	yes				CCET - IIT Text		Austin	Texas	yes	South/Wes	6.37	1.96	4.41	

Figure 5.5: Metadata file of Pecan Street [36]

Depending upon our requirements, we select the DATA ID. To access the data of a particular DATA ID (i.e., a house) we need to download PgAdmin. PgAdmin is a tool for PostgreSQL, that connects to the Dataport database server.

To get the required data, first we need to launch the SQL editor and write queries. The results can be seen in the data output window by clicking on the “Execute Query” button. For example, to download hourly electricity data of house (2755) from September 23rd to December 22nd we enter the below query in the SQL editor,
Select *from electricity.eg_realpower_1hr

Where dataid=2755 and localhour>= ‘2015-09-23’ and localhour < ‘2015-12-22’

5.2.1 Computing Shapley value with data collected from Pecan Street

From the metadata file, we have selected 6 single-family homes located in Austin, Texas. Each home was installed with a PV system for the year 2015. For each of the 6 homes, we consider ‘Gen’ and ‘Use’ Column data, that is provided for every hour of the day during Fall, Spring and Winter. ‘Gen’ is the power generated by a solar photovoltaic system, ‘Use’ is the whole electricity usage data and each DATA ID i.e., house is considered as a prosumer.

5.2.1.1 Calculation of Shapley value with Linear Characteristic Function for Fall, Spring and Winter

In this section, we calculate Shapley value (with and without coalition) by considering a linear characteristic function shown in Equation (5.8) and compare the results for Fall, Spring and Winter.

Mathematical Model

Notation

Set	Definition
T	Total number of power generation periods, $t=1, 2, \dots, 2160$
I	Total number of houses, $i=1,2,3,4,5,6$
S	Coalition Combinations $S = [\{1\},\{2\},\{3\},\{4\},\{5\},\{6\}, \{1,2\}, \{1,3\} \dots, \{1,2,3,4,5,6\}] = 2^6$ coalition combinations
Parameter	Definition
Gen_{it}	Power generated by i^{th} house in t^{th} period (kW)
Use_{it}	Power consumed by i^{th} house in t^{th} period (kW)
X	Difference between the average energy production and consumption of each house
q	Maximum cost that is awarded to the prosumers for their injected energy (\$10 per kWh)
a	Scaling Factor (10^6)
n	Total number of prosumers/consumers
$n!$	$6! = 720$ coalition combinations
$\varphi_i(v)$	Shapley value of i^{th} house
Gen_a	Total average energy production by all house (Kwh)
Use_a	Total average energy consumption by all house (Kwh)
Variable	Definition
Gen_{ai}	Monthly average energy production for each house (kWh)
Use_{ai}	Monthly average energy consumption for each house (kWh)

$$\text{Monthly average energy production (kWh), } Gen_{ai} = \frac{\sum_{t=1}^{2160} Gen_{it}}{3} \quad (5.6)$$

$$\text{Monthly average energy consumption (kWh), } Use_{ai} = \frac{\sum_{t=1}^{2160} Use_{it}}{3} \quad (5.7)$$

$$\text{Characteristic Function, } v(i) = \frac{X*q}{e^{\frac{[Gen_a - Use_a]^2}{a}}} \quad (5.8)$$

$$v(i) = \frac{(Gen_{ai} - Use_{ai}) * 10}{e^{\frac{[\sum_{i=1}^6 (Gen_{ai}) - \sum_{i=1}^6 (Use_{ai})]^2}{10^6}}} \quad (5.9)$$

(a) Fall

To calculate the averages of power generation and consumption for each house in fall, we use the generation and consumption data from Pecan Street [36], for each time-period considered from September 23rd to December 21st of 2015.

Table 5.8: Monthly average production and consumption values of each prosumer during Fall with linear characteristic function (X)

<i>i</i> (House)	FALL	
<i>Prosumer</i>	<i>Monthly average Energy production for Fall (kWh)</i> <i>Gen_{ai}</i>	<i>Monthly average Energy Consumption for Fall (kWh)</i> <i>Use_{ai}</i>
1	1473	1523
2	1215	1056
3	1006	367
4	643	970
5	1737	676
6	1518	1098
	<i>Gen_{ai} = 7592</i>	<i>Use_{ai} = 5690</i>

In Table 5.8, we calculate monthly average energy production and consumption values during fall for each of the prosumers with Equations (5.6) and (5.7). We observe that the average production is high for prosumers 2,3,5 and 6 while Prosumers 1 and 4 have high consumption values. The total power generated by all prosumers is 7592 kWh and the total energy consumed is 5690 kWh.

Table 5.9: Shapley values with and without coalition for energy offered by each prosumer during Fall with linear characteristic function (X)

<i>i</i> (House)	<i>Energy offered by each prosumer (kWh) (X)</i>	<i>Shapley Value</i> $\Phi_i(v)$ (\\$)	
<i>Prosumer</i>		<i>With Coalition</i>	<i>Without Coalition</i>
1	-49	-13.42	-13.42
2	+159	42.68	42.68
3	+639	171.55	171.55
4	-327	-87.78	-87.78
5	+1060	284.84	284.84
6	+420	112.75	112.75

By considering average energy production and consumption values of each prosumer from Table 5.8, we calculate the energy offered by each prosumer using Equation (5.5).

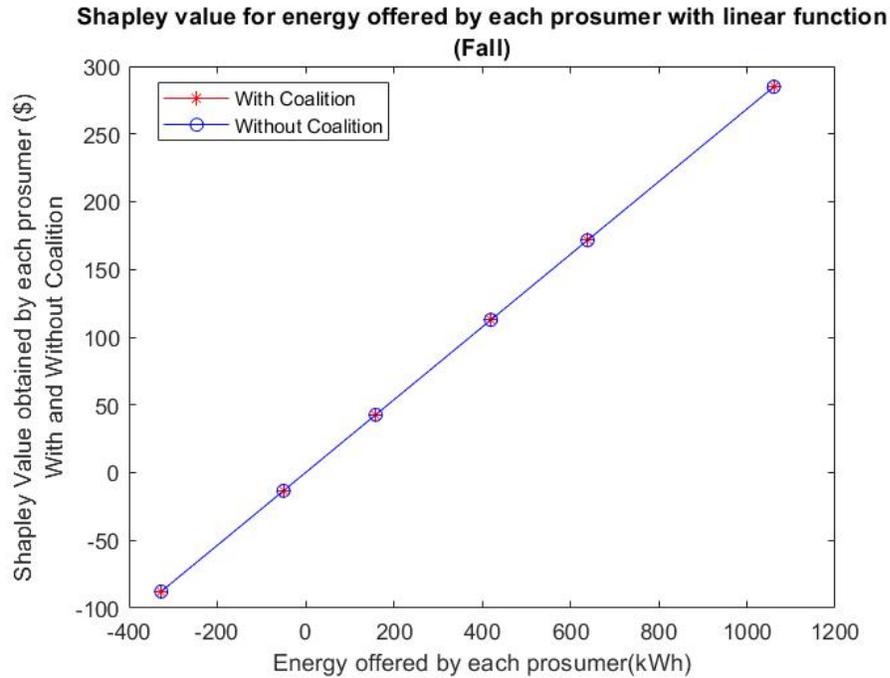


Figure 5.6: Shapley value (with and without coalition) obtained for energy offered during Fall with linear characteristic function (X)

The blue plot is the energy offered by each prosumer with the Shapley value, that is obtained by coalition of all prosumers. Whereas, the red dots plot total offered energy by each prosumer with Shapley value obtained by not forming coalitions. Both the plots overlap as the Shapley value with and without coalitions of prosumers is same. This is due to linear characteristic Equation (5.8) in which only variable X is varying.

(b) Spring:

The average power generation and consumption for Spring is calculated by considering Equations (5.6) and (5.7), with time period considered from March 20th to June 21st of 2015.

Table 5.10: Monthly average production and consumption values of each prosumer during Spring with linear characteristic function (X)

<i>i</i> (House)	SPRING	
<i>Prosumer</i>	<i>Monthly average Energy production for Spring (kWh)</i> Gen_{ai}	<i>Monthly average Energy Consumption for Spring (kWh)</i> Use_{ai}
1	1514	1431
2	1937	1279
3	1337	378
4	903	1087
5	1996	749
6	1560	1346
	$\sum_{i=1}^6 Gen_{ai} = \mathbf{9247}$	$\sum_{i=1}^6 Use_{ai} = \mathbf{6270}$

The average production and consumption for each prosumer are calculated using Equations (5.6) and (5.7). The average production is high for all the prosumers except Prosumer 4, whose average consumption is high. The total average production of all prosumers is 9247 kWh, while the total average consumption of all 6 prosumers is 6270 kWh.

Table 5.11: Shapley values with and without coalition for energy offered by each prosumer during Spring with linear characteristic function (X)

<i>i</i> (House)	Energy offered by each prosumer (kWh) (X)	Shapley Value $\Phi_i(v)$ (\$)	
		With Coalition	Without Coalition
1	+84	0.1175	0.1175
2	+658	0.9317	0.9317
3	+958	1.3579	1.3579
4	-184	-0.2605	-0.2605
5	+1247	1.7657	1.7657
6	+214	0.3030	0.3030

The Shapley values obtained with and without coalitions are very small. As the total energy offered by all prosumers is 2977 kWh, we get a high value in the denominator, resulting in small Shapley values.

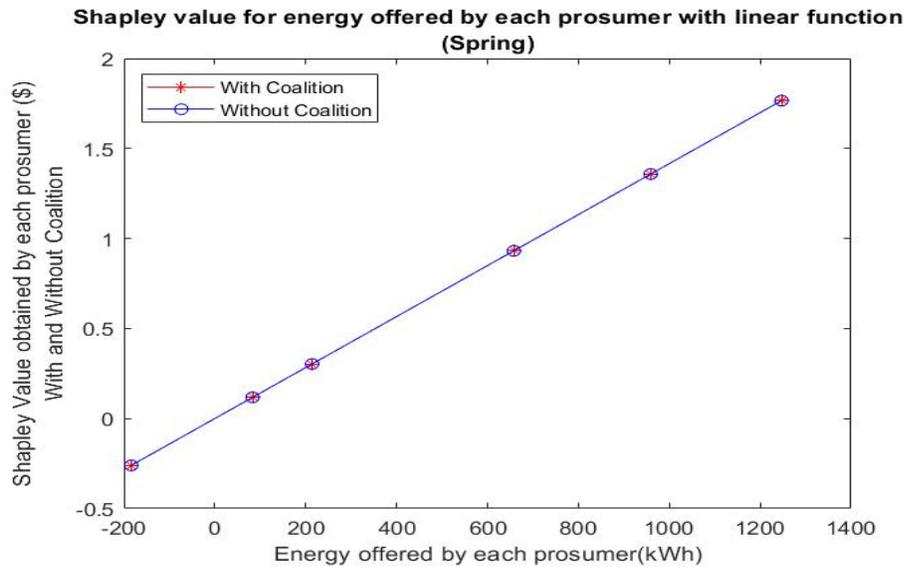


Figure 5.7: Shapley value (with and without coalition) obtained for energy offered during Spring with Linear Characteristic function (X)

We get a linear plot for spring, as we are calculating the Shapley value of each prosumer with and without forming coalitions using characteristic function (X) Equation (5.8).

(c) Winter:

The average power generation and consumption for winter is calculated as per equations (5.6) and (5.7), with time period considered from December 21st to March 19th.

Table 5.12: Monthly average production and consumption values of each prosumer during Winter with linear characteristic function (X)

<i>i</i> (House)	WINTER	
Prosumer	<i>Monthly average Energy production for Winter (kWh)</i> <i>Gen_{ai}</i>	<i>Monthly average Energy Consumption for Winter (kWh)</i> <i>Use_{ai}</i>
1	1239	1138
2	1045	903
3	871	288
4	575	810
5	1447	406
6	1273	677
	$\sum_{i=1}^6 Gen_{ai} = 6450$	$\sum_{i=1}^6 Use_{ai} = 4222$

The average production is high for all prosumers except 4, whose average consumption is high. The total energy produced by all prosumers is 6450 kWh and total energy consumption is 4222 kWh.

Table 5.13: Shapley values with and without coalition for energy offered by each prosumer during winter with linear characteristic function (X)

<i>i</i> (House)	<i>Energy offered by each prosumer (kWh) (X)</i>	<i>Shapley Value</i> $\Phi_i(v)$ (\$)	
		<i>With Coalition</i>	<i>Without Coalition</i>
1	+101	7.054	7.054
2	+142	9.918	9.918
3	+583	40.72	40.72
4	-235	-16.41	-16.41
5	+1041	72.71	72.71
6	+596	41.63	41.63

The Shapley value of Prosumer 4 is negative as energy produced is less than the consumption, the negative Shapley value has a physical meaning of zero. The Shapley values of Prosumers 1 and 2 are less when compared to the values of Prosumers 3,5 and 6 because the individual energy offered by Prosumers 3,5 and 6 is more when compared to the energy offered by Prosumers 1 and 2.

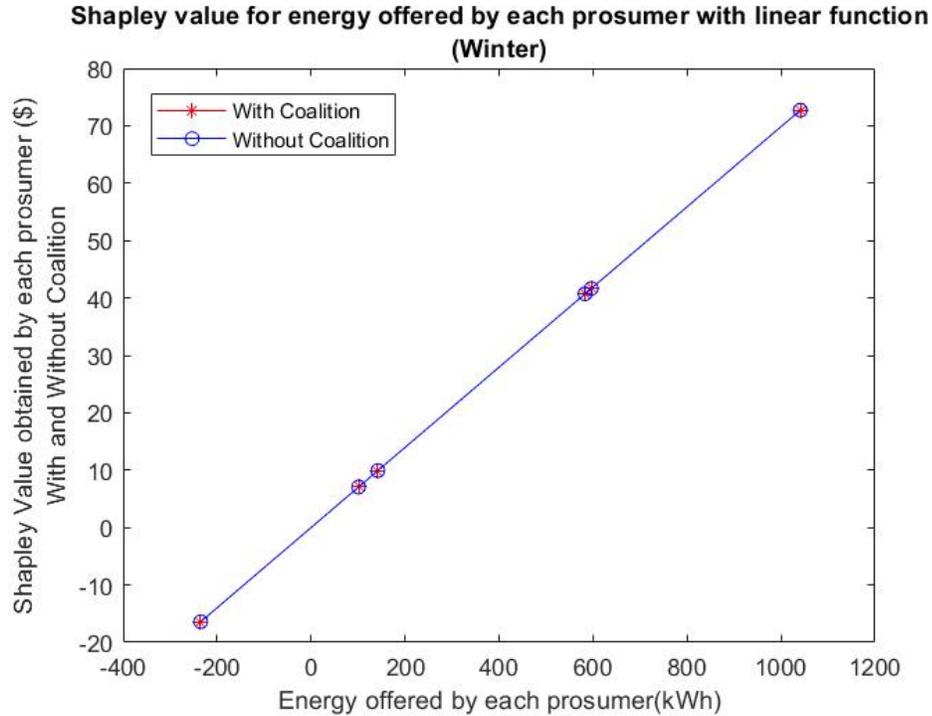


Figure 5.8: Shapley value (with and without coalition) obtained for energy offered during Winter with Linear Characteristic function (X)

The plot is linear, and the Shapley values with and without coalitions are overlapping due to Equation (5.8) which is a linear characteristic function. In each season, we observe that the Shapley value is positive for prosumers whose monthly average energy production is greater than average consumption. The highest payoffs are received by prosumers, whose energy offered value is high.

5.2.1.2 Calculation of Shapley value with Convex Characteristic Function ($X^{1.5}$) for Fall, Spring and Winter

In the previous section, the Shapley value received by few prosumers was negative. Shapley value cannot be negative as it is used to make a fair distribution of gains in a coalition. Negative Shapley value means that the prosumer owes to a coalition. But as Shapley value is not used to define how much money we owe.

We ignore the prosumers with negative Shapley values and consider only those with positive Shapley values. In this section, we consider a convex characteristic function (5.10) and calculate the Shapley value with data from the previous section excluding prosumers whose average production values are less than consumption values, to observe if the prosumer receives higher profits by forming coalitions.

$$\text{Cost Function, } v(i) = \frac{X^n * q}{e^{\frac{[Gen_a - Use_a]^2}{a}}} \quad (5.10)$$

$$v(i) = \frac{(Gen_{ai} - Use_{ai})^n * 10}{e^{\frac{[\sum_{i=1}^5 (Gen_{ai}) - \sum_{i=1}^5 (Use_{ai})]^2}{10^6}}} \quad (5.11)$$

(a) Fall:

We use the same data as in the previous section, excluding production and consumption values of Prosumer 1 and 4.

Table 5.14: Monthly average production and consumption values of each prosumer during Fall with convex characteristic function (X^{1.5})

<i>i</i> (House)	FALL	
Prosumer	Monthly average Energy production for Fall (kWh) <i>Gen_{ai}</i>	Monthly average Energy Consumption for Fall(kWh) <i>Use_{ai}</i>
1	1215	1056
2	1518	1098
3	1006	367
4	1737	676
	$\sum_{i=1}^4 Gen_{ai} = 5476$	$\sum_{i=1}^4 Use_{ai} = 3197$

The total average energy production by all prosumers is 5476 kWh and total consumption value is 3197 kWh.

Table 5.15: Shapley values with and without coalition for energy offered by each prosumer during Fall with convex characteristic function ($X^{1.5}$)

<i>i</i> (Houses)	Energy offered by each prosumer (kWh) (<i>X</i>)	Shapley Value $\Phi_i(v)$ (\$)	
		With Coalition	Without Coalition
1	159	401.79	111.29
2	420	1090.3	477.77
3	639	1684.9	896.59
4	1061	2861.9	1918.3

The Shapley value obtained when prosumers form a coalition is greater as the characteristic function Equation (5.10) is convex. The highest Shapley value is obtained by Prosumer 5 who has offered high energy.

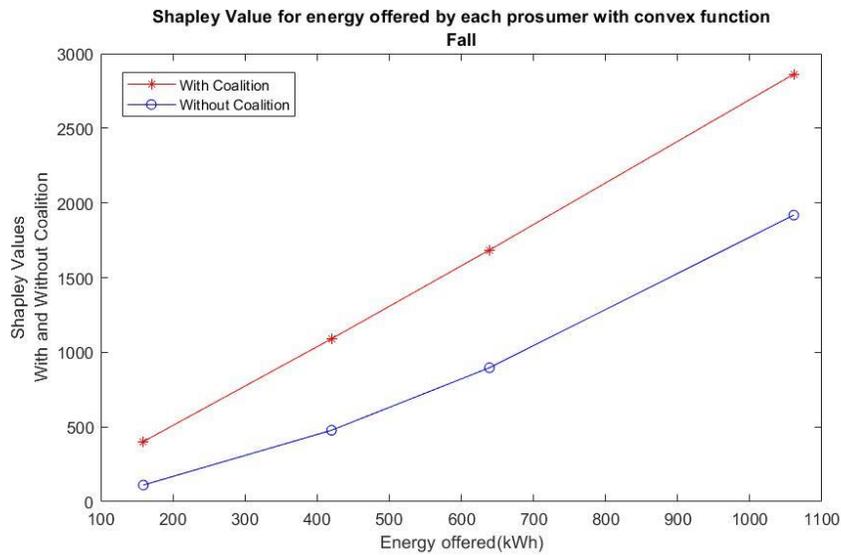


Figure 5.9: Shapley value (with and without coalition) obtained for energy offered during Fall with Convex Characteristic function ($X^{1.5}$)

The red plot is the Shapley value with coalition for energy obtained by all prosumers and the blue plot is the Shapley value without coalition for characteristic function from Equation (5.10). We observe that the Shapley value is large when all prosumers form a coalition because we calculate the Shapley value by Equation (5.10) which is convex.

(b) Spring:

Here, we don't include the average production and consumption value of Prosumer 4 as the energy offered is negative.

Table 5.16: Monthly average production and consumption values of each prosumer during Spring with convex characteristic function ($X^{1.5}$)

<i>i</i> (Houses)	SPRING	
<i>Prosumers</i>	<i>Monthly average Energy production for Spring (kWh)</i> Gen_{ai}	<i>Monthly average Energy Consumption for Spring (kWh)</i> Use_{ai}
1	1514	1431
2	1560	1346
3	1937	1279
4	1337	378
5	1996	749
	$\sum_{i=1}^5 Gen_{ai} = 8344$	$\sum_{i=1}^5 Use_{ai} = 5183$

The total production value of all prosumers is 8344 kWh and the total consumption is 5183 kWh.

Table 5.17: Shapley values with and without coalition for energy offered by each prosumer during Spring with convex characteristic function ($X^{1.5}$)

<i>i</i> (Houses)	Energy offered by each prosumer (kWh) (<i>X</i>)	Shapley Value $\Phi_i(v)$ (\$)	
		With Coalition	Without Coalition
1	83	2.01	0.34
2	214	5.28	1.43
3	658	16.74	7.72
4	959	24.74	13.59
5	1247	32.54	20.15

The Shapley values obtained with coalitions is greater than without coalitions. But the values are very small, as the total energy offered by all prosumers is large (3161 kWh). The value in the denominator increases, resulting in small Shapley values.

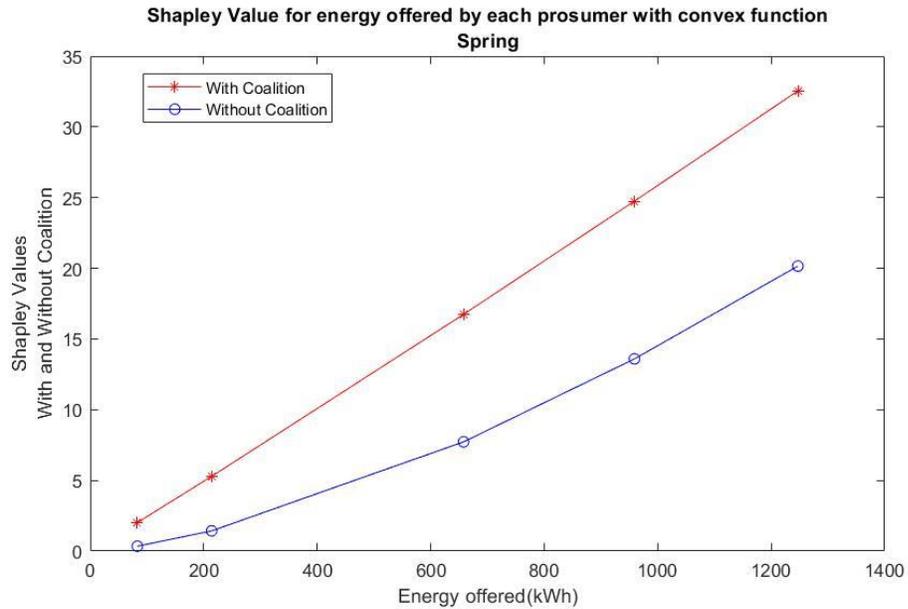


Figure 5.10: Shapley value (with and without coalition) obtained for energy offered during Spring with convex characteristic function ($X^{1.5}$)

Due to the convex characteristic function, the Shapley value is larger when prosumers form a coalition.

(c) Winter:

The same data as in the previous section is considered, excluding the average production and consumption value of Prosumer 4 as the energy offered is negative.

Table 5.18: Monthly average production and consumption values of each prosumer during Winter with convex characteristic function (X^{1-5})

<i>i</i> (Houses)	WINTER	
<i>Prosumers</i>	<i>Monthly average Energy production for Winter (kWh)</i> <i>Gen_{ai}</i>	<i>Monthly average Energy Consumption for Winter (kWh)</i> <i>Use_{ai}</i>
1	1239	1138
2	1045	903
3	871	288
4	1273	677
5	1447	406
	$\sum_{i=1}^5 Gen_{ai}=5875$	$\sum_{i=1}^5 Use_{ai}=3412$

The total average energy produced by all prosumers is 5875 kWh and the total consumption value is 3412 kWh.

Table 5.19: Shapley values with and without coalition for energy offered by each prosumer during Winter with convex characteristic function ($X^{1.5}$)

<i>i</i> (Houses)	Energy offered by each prosumer (kWh) (X)	Shapley Value $\Phi_i(v)$ (\$)	
		With Coalition	Without Coalition
1	101	110.61	23.54
2	142	156.43	39.25
3	583	666.83	326.52
4	596	682.24	337.5
5	1041	1219.2	779.09

The individual Shapley values of all prosumers with coalitions is greater. The highest Shapley value is obtained by Prosumer 5 as its energy offered is high.

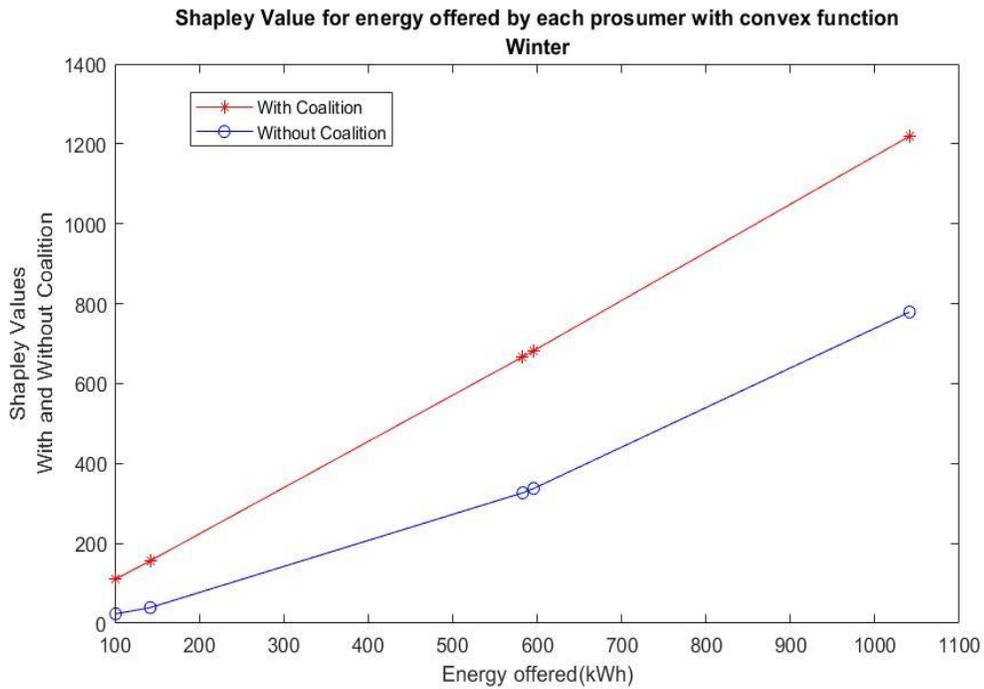


Figure 5.11: Shapley values with and without coalition for energy offered by each prosumer during Winter with convex characteristic function ($X^{1.5}$)

In the case of the linear characteristic function mentioned in Equation (5.9), we observed that the prosumers didn't gain any profits by forming a coalition. Thus, in this section we consider the function to be convex.

The variable X in Equation (5.9) is raised to the power 1.5 ($X^{1.5}$) to compute the Shapley value. For each season i.e., Fall, Spring and Winter, the Shapley value with coalition is greater than the Shapley value obtained by not forming a coalition.

For Spring, even though the Shapley value is greater with coalition. The payoff received is less when compared with fall and winter due to high total offered energy.

5.2.1.3 Calculation of Shapley value with Convex Characteristic (X^2) Function for Fall, Spring and Winter

In this section, we consider data from (Section 5.2.1.2) and compute the Shapley value for larger convex function.

$$\text{Cost Function, } v(i) = \frac{X^2 * q}{\frac{e[Gen_a - Use_a]^2}{a}} \quad (5.12)$$

$$v(i) = \frac{(Gen_{ai} - Use_{ai})^n * 10}{e \frac{[\sum_{i=1}^5 (Gen_{ai}) - \sum_{i=1}^5 (Use_{ai})]^2}{10^6}} \quad (5.13)$$

(a) Fall:

Table 5.20: Monthly average production and consumption values of each prosumer during Fall with convex characteristic function (X^2)

<i>i</i> (Houses)	FALL	
<i>Prosumers</i>	<i>Monthly average Energy production for Fall (kWh)</i> Gen_{ai}	<i>Monthly average Energy Consumption for Fall (kWh)</i> Use_{ai}
1	1215	1056
2	1518	1098
3	1006	367
4	1737	676
	$\sum_{i=1}^4 Gen_{ai}$ 5476	$\sum_{i=1}^4 Use_{ai}$ 3197

The total average energy production is 5476 kWh and total consumption value is 3197 kWh for all 4 prosumers.

Table 5.21: Shapley values with and without coalition for energy offered by each prosumer during Fall with convex characteristic function (X^2)

<i>i</i> (Houses)	<i>Energy offered by each prosumer (kWh)</i> (X)	<i>Shapley Value</i> $\Phi_i(v)$ ($\$$)	
<i>Prosumers</i>		<i>With Coalition</i>	<i>Without Coalition</i>
1	159	20113	1403.3
2	420	53130	9791.3
3	639	80833	22664
4	1061	134220	62485

The Shapley value with coalition is greater than without coalition for all prosumers, as the characteristic function in Equation (5.12) is convex. The Shapley values received by individual prosumers are greater compared to previous section, as the convex function is larger.

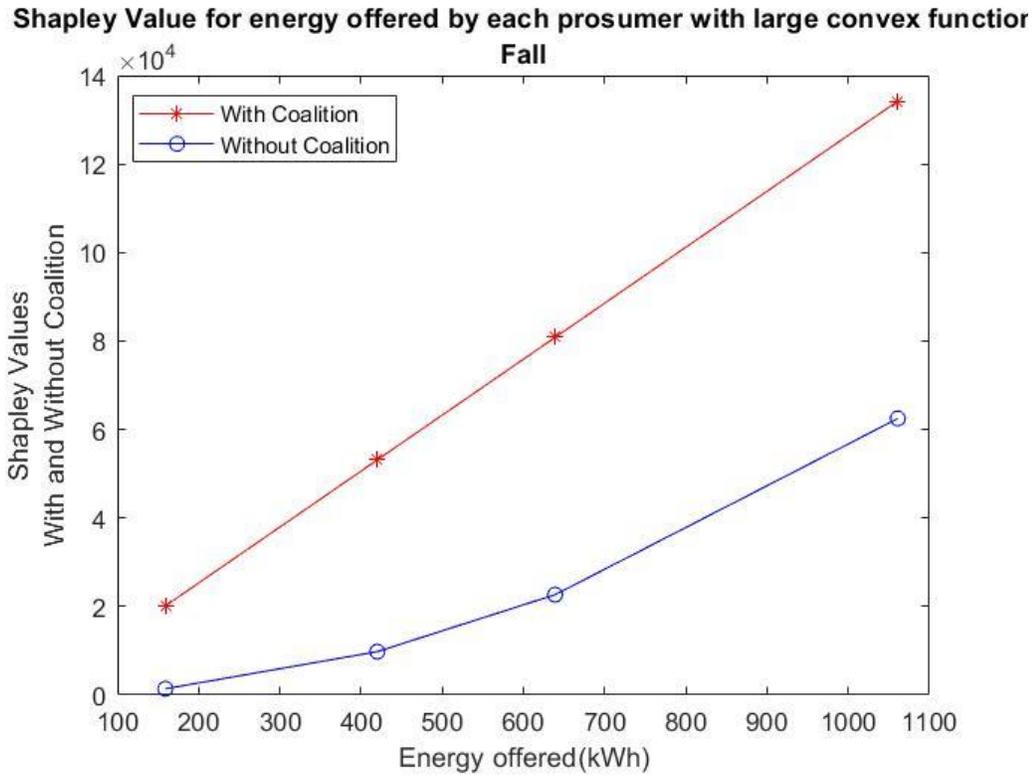


Figure 5.12: Shapley values with and without coalition for energy offered by each prosumer during Fall with convex characteristic function (X^2)

The red plot is the Shapley value with coalition for energy obtained by all prosumers and the blue plot is the Shapley value without coalition for the convex characteristic function from Equation (5.12). We observe that the Shapley value is large when all prosumers form a coalition. Since, we calculate the Shapley value by Equation (5.12) which is larger convex function when compared with previous section.

(b) Spring

Table 5.22: Monthly average production and consumption values of each prosumer during Spring with convex characteristic function (X^2)

<i>i</i> (Houses)	SPRING	
<i>Prosumers</i>	<i>Monthly average Energy production for Spring (kWh)</i> Gen_{ai}	<i>Monthly average Energy Consumption for Spring (kWh)</i> Use_{ai}
1	1514	1431
2	1560	1346
3	1937	1279
4	1337	378
5	1996	749
	$\sum_{i=1}^5 Gen_{ai} = \mathbf{8344}$	$\sum_{i=1}^5 Use_{ai} = \mathbf{5183}$

The highest average energy is produced by Prosumer 3 and consumed by Prosumer 1. The total average energy produced is 8344 kWh and consumed is 5183 kWh.

Table 5.23: Shapley values with and without coalition for energy offered by each prosumer during Spring with convex characteristic function (X^2)

i (Houses)	Energy offered by each prosumer (kWh) (X)	Shapley Value $\Phi_i(v)$ (\$)	
		With Coalition	Without Coalition
1	83	2.01	0.34
2	214	5.28	1.43
3	658	16.74	7.72
4	959	24.74	13.59
5	1247	32.54	20.15

Compared to the previous section, the Shapley value received by prosumers is greater. The highest Shapley value is obtained by Prosumer 4 as he is offering the greatest energy (1247 kWh) to the grid.

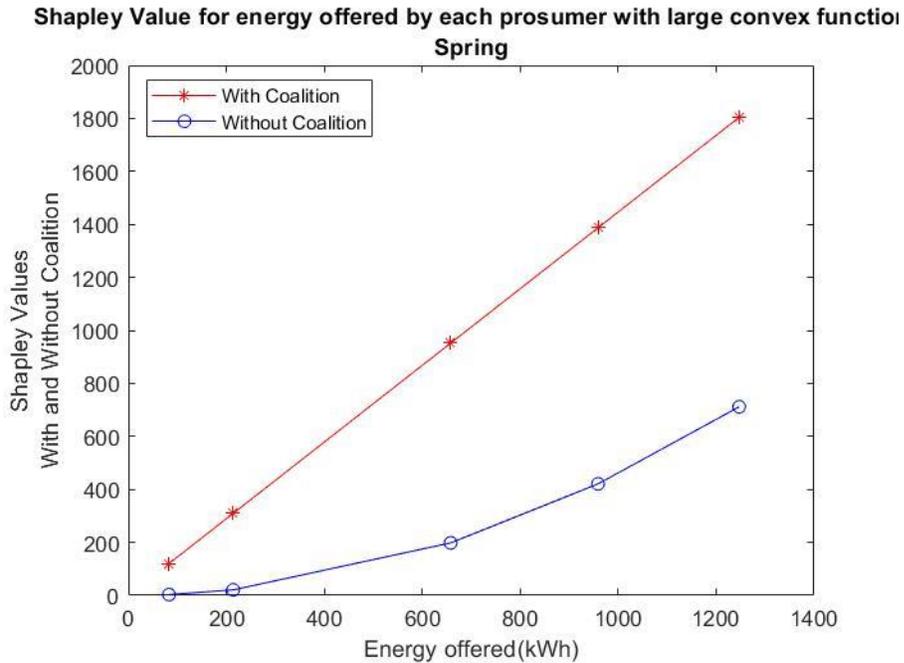


Figure 5.13: Shapley values with and without coalition for energy offered by each prosumer during Spring with convex characteristic function (X^2)

Due to the larger convex function, the Shapley value obtained by all prosumers in a coalition is greater. The Shapley value increase with an increase in the energy offered by each prosumer.

(c) Winter:

Table 5.24: Monthly average production and consumption values of each prosumer during Winter with convex characteristic function (X^2)

<i>i</i> (Houses)	WINTER	
<i>Prosumers</i>	<i>Monthly average Energy production For Winter (kWh)</i> Gen_{ai}	<i>Monthly average Energy Consumption for Winter (kWh)</i> Use_{ai}
1	1239	1138
2	1045	903
3	871	288
4	1273	677
5	1447	406
	$\sum_{i=1}^5 Gen_{ai} = 5875$	$\sum_{i=1}^5 Use_{ai} = 3412$

The highest average energy is produced by Prosumer 5 and consumed by Prosumer 1. The total average energy produced is 5875 kWh and consumed is 3412 kWh.

Table 5.25: Shapley values with and without coalition for energy offered by each prosumer during Winter with convex characteristic function (X^2)

<i>i</i> (Houses)	<i>Energy offered by each prosumer (kWh)</i> (<i>X</i>)	<i>Shapley Value</i> $\Phi_i(v)$ (\$)	
		<i>With Coalition</i>	<i>Without Coalition</i>
1	101	5770.3	236.62
2	142	8112.6	467.72
3	583	33308	7884
4	596	34050	8239.5
5	1041	59474	25137

The Shapley value with and without coalition is computed by calculating the energy offered by each prosumer. The Shapley value is highest for Prosumer 4 as he is offering the greatest (1041 kWh) energy in the coalition.

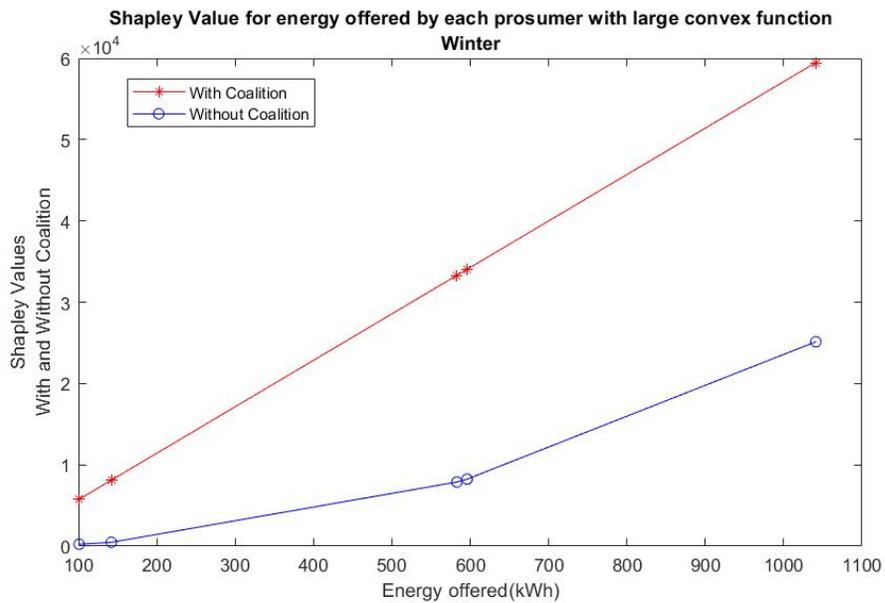


Figure 5.14: Shapley values with and without coalition for energy offered by each prosumer during Winter with convex characteristic function (X^2)

In this section, when we compute the Shapley value with larger convex function, we observe that the Shapley value received by prosumers in a coalition is larger when compared with Section 5.2.1.2 because of the large convex function.

6. CONCLUSION AND FUTURE WORK

The focus of my thesis is to analyze how pricing functions affect the coalition formation of prosumers and the benefits obtained due to coalitions. The results are studied by calculating Shapley value.

The behavior of the 'g' function that is used in NRG-X-Change mechanism is tested in the first section of Chapter 5. Here, we considered four different scenarios, in each scenario, we consider 5 cases, where each case is a network of 5 prosumers.

As the 'g' function is the price function for paying prosumers, in each case, every prosumer is considered to have a varied production value and constant consumption value. Each case has 5 prosumers, and for each group of prosumers we calculate the Shapley value.

The results obtained from different cases are compared and the case with highest Shapley value gains maximum profit. The results are plotted by comparing the total energy offered and total Shapley value of all 5 prosumers in each case for every scenario.

In order to test the effect of coalition, we first considered some hypothetical values for three different pricing functions such as concave, linear and convex and studied the Shapley value with and without coalitions.

We also took into account, the real-time data obtained from Pecan Street Inc. where, the houses selected had solar panels installed. For each of the selected houses, the average production and consumption values generated during fall, spring and winter were considered. At the end the Shapley value with and without forming coalitions were examined during each season by considering linear and convex functions.

From the four scenarios tested to study the behavior of ‘g’ function, we have concluded that when the total energy production is less than the total energy consumption ($t_p < t_c$), the Shapley value is negative. When the production exceeds the consumption ($t_p > t_c$), the Shapley value is observed to be positive. When the total energy production exceeds more than twice that of total consumption ($t_p >> t_c$), the Shapley value obtained is positive but negligible. Hence, we conclude that ‘g’ is maximized when production is twice as great as consumption.

In the investigation of the three different pricing functions, we have found that for the linear function, no payoff was achieved by forming a coalition. In case of concave functions, the overall payoffs are reduced by making coalitions and high payoffs are attained by forming a coalition only when we have a convex function.

From the Pecan Street data, we observed that when the function is linear there was no point of forming a coalition, and the result is the same as observed from hypothetical values. When the function is convex, the total gains are increased by joining coalitions.

For the future work, a greater number of prosumers can be considered, and instead of investigating the Shapley value, we can create an optimization algorithm to optimize the coalition formation and define the members that can be a part of a coalition.

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