EVOLVING MATHEMATICAL IDENTITY IN POST-SECONDARY STUDENTS

by

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ABSTRACT

Mathematics encompasses more than formulas, theorems and proofs. For many, mathematics can be a way of life or even a culture and like within any culture, an individual who associates with that culture has an identity: a description of how one knows and sees oneself with respect to the norms and members of the culture. However, it can be argued the mathematical culture has barriers to entry by members of certain groups. These barriers are built upon stereotypes and biases and these biases and stereotypes are now controlling who can enter the discipline and how they do so (Burton, 2009. Students often enter post-secondary education with these stereotypical views of mathematics that they have picked up from their K-12 education. These views place a minimal value on mathematics and in turn making students less inclined to join the mathematics community. Therefore, traditional classrooms may not be the best venues for acculturating the students into mathematics. Traditional pedagogies and procedural views of mathematics combine to produce environments in which most students must surrender agency and thought in order to follow predetermined routines (Boaler, 1997; Schoenfeld, 1988, 1992). That is why investigating how a social environment through a student mathematics seminar for post-secondary students facilitates a student's acculturation into mathematics and perhaps diminishes the stereotypes and biases of the mathematics culture was pursued in this study. By observing four post-secondary student in a series of presentations, it was determined how mathematical identity could be affected within four

particular aspects of identity: position, self-efficacy, perceptions of mathematics, and forms of engagement.

I. INTRODUCTION

Mathematics encompasses more than formulas, theorems and proofs. For many, mathematics can be a way of life or even a culture (Davis,1989). Tyler (1871) defines culture as a "complex whole which includes knowledge, belief, art, morals, law, customs, and any other capabilities and habits acquired by man as a member of society" (p.1). Burton (2009), Bishop (1991), and many others describe how these aspects manifest in mathematics. Historically, the culture of mathematics has been integral to the field of mathematics and seen as part of what students are expected to acquire in the process of becoming mathematicians (Wood et al., 2012).

Within any culture, an individual who associates with that culture has an identity: a description of how one knows and sees oneself with respect to the norms and members of the culture. It is socially constructed and constantly changes as individuals interact within different communities in which they live, learn or work (Holland and Lave, 2001). How one would view oneself within the mathematics culture is one's mathematical identity. Leatham and Hill (2010) define mathematical identity as "an individual's relationship with mathematics" (p 226). That is, the ways a person learns, does, thinks about, retains, or chooses to associate with the subject. Now, that also means the ways a person does not choose to learn, think about or associate with the subject. Identities address multiple aspects including gender, race, and physical weight (Ibrahim, 1993). In mathematical identity, aspects such as gender and race can play a role as well as self-confidence, past experiences and interactions with peers (Nasir and Cobb, 2007; Nasir, 2009).

Statement of Problem

There is a nationwide necessity for more students to belong to the mathematics community (Engineering for Kids, 2016) and with the drastic changes in the economy, the competition for employment has become quite fierce (Carmody, 2016). Also, for higher education to receive additional funding, many universities have started the process of becoming research institutions (Pryor, et al., 2012). In order for students to become more marketable upon graduation and universities to achieve recognition as research institutions, students must develop research skills and the skills to communicate the research. Few students leave the school system with mathematical success as measured by examinations and have a lack of abilities in thinking mathematically (Burton, 1984). Even though, Burton's observation took place over twenty years ago, this problem still exists. For example, as measured by the National Assessment of Educational Progress (Hemphill and Vanneman, 2011), roughly 75 percent of U.S. 8th graders are not proficient in mathematics when they complete 8th grade. Furthermore, 38 percent of students who start college with a STEM major do not graduate with one, which is reflected by the fact that 63 percent of high school graduates are not prepared for college-level mathematics or science (Brody, 2016). The difficult task at hand is engaging students in mathematics and seeing its importance.

In October of 2000, the Texas Higher Education Coordinating Board (THECB) adopted the "Closing the Gaps" plan that called for diminishing the gaps in education in the state of Texas and the differences amongst other states. In this plan, it called for the increase of college graduates with degrees in science, technology, engineering, and mathematics (STEM) from 14,500 to 29,000 by 2015. In February of 2012, the

President's Council of Advisors on Science and Technology (PCAST) published a report titled *Engage to Excel* (PCAST, 2012), which stated that the United States' workforce would suffer a great loss of STEM graduates in the next ten years (Graham, et al., 2013). So, in turn, PCAST called for one million more college graduates with degrees in STEM in the next decade in order for the United States "to maintain its historical preeminence in science and technology" (p.i).

In a 2010 report, Business Higher Education Forum (BHEF) argued that in order for the United States to remain competitive with other countries, the number of STEM graduates must increase in order to "produce a skilled innovative workforce" (p. 2). BHEF launched the Securing America's Leadership in STEM Initiative in 2005 with the goal to double the number of STEM graduates by 2015 (BHEF, 2010). These calls to action seek to increase U.S. competitiveness globally, but while political and economic demand for STEM degrees is high, the actual number of bachelor's degrees awarded has not drastically increased during this time period.

Despite these efforts by national and state programs, the number of STEM majors still falls short. The Closing the Gaps reported that the state of Texas only reached 21,512 of the 28,000 that was to be attained by 2014. A report released by the NSF in 2014, STEM majors have only increased to a little over 500,000, which is only half-way to the goal that should be reached in 2015. Also, other states have reported that STEM majors were increasing, however mathematics and computer science majors were greatly declining (Davies, 2014). Furthermore, we still have a problem of retaining STEM majors. According to the National Math and Science Initiative (NMSI), 38% of those who start a STEM major do not graduate with one.

Mathematics plays an important role for STEM. According to the American Association for the Advancement of Science (1993, para. 3), “mathematics is the science of patterns and relationships.” Mathematics provides the exact language of for science, technology, and engineering (Dugger, 2010). In other words, without mathematics, the other components of STEM would have troubles advancing.

This deficiency in the number of STEM majors may be due to the fact that students fail to make an individualized connection to these subjects. That is their identities in these fields are generally not aligned with those in these fields. For example, in subject of mathematics, Boaler and Greeno (2000) have found that many students who are successful in traditional mathematics courses chose not to pursue studying mathematics because being successful in mathematics does not align with who they identify themselves to be. The reality may be that there are students who could potentially excel in the STEM subjects, but they just feel like they do not belong in those fields due to a lack of connection to the culture of mathematics.

Furthermore, it can be argued the mathematical culture can create barriers to entry by members of certain groups. Mathematics, as any other culture, has developed stereotypes and biases. There exists stereotypes and biases both within and outside the culture of mathematics (Boaler, 2013) and can then affect student performance (Aronson, et. al., 1999). Biases and stereotypes within the culture of mathematics for example would be a gender issue, female mathematicians may be viewed as less capable and less intelligent than male mathematicians (Good, et al., 2008; Dweck 2006). And then there are those stereotypes that are developed by those who are not encultured with mathematics. Many students view mathematics as a collection of pointless and repetitive formulas or even as forms of torture (Zaslavsky, 1994). Some believe that you must be born with the natural ability to do mathematics and consequently develop a fear of failure when attempting try mathematics (Zaslavsky, 1994; Dweck, 2006).

Most of these stereotypes and biases are developed from experiences in the classroom, homework assignments, and test performances. Seymour and Hewitt (1997) found that students in science, mathematics and engineers courses describe their classroom environments as cold, the instructors as unapproachable, and lectures did not welcome discussion. This can lead to students taking on a passive role when attending class (Bressoud, 1994).

These biases and stereotypes are now controlling who can enter the discipline and how they do so (Burton, 2009). Burton further states that it is the mathematical culture that exercises the power over how the culture of mathematics is understood, and thus, it is the mathematical culture that must be addressed if mathematics is to achieve widespread accessibility. Students enter post-secondary education with these stereotypical views of mathematics that they have picked up from their K-12 education. Students believe that understanding mathematics is unnecessary and that the only thing that matters is knowing the rules to get to the correct answer (Mason, 2003; Muis 2004). Thus, when they enroll in an undergraduate program they are "forced" to take pointless mathematics course(s) just to fulfill a requirement for graduation. These views place a minimal value on mathematics and in turn making students less inclined to join the mathematics community.

Therefore, traditional classrooms may not be the best venues for acculturating the students into mathematics. Traditional pedagogies and procedural views of mathematics combine to produce environments in which most students must surrender agency and thought in order to follow predetermined routines (Boaler, 1997; Schoenfeld, 1988, 1992). That is why investigating how a social environment through a student seminar for post-secondary students facilitates a student's acculturation into mathematics and perhaps diminish the stereotypes and biases of the mathematics culture was pursued in this study.

Purpose of Study

The purpose of the study was to investigate the existence of a relationship between the evolution of mathematical identity and involvement in a student mathematics seminar. The factors of identity observed in this study were position, self-efficacy, attitudes and values of mathematics, and forms of mathematical engagement.

Definitions

The following definitions provided are for terms that will be used in the research questions and throughout the rest of the study.

1. Student Math Seminar (SMS) – weekly seminar for graduate and undergraduate students to give presentations on mathematics or mathematics education that range from 10 to 45 minutes in length.
2. Mathematics Culture – the complex whole of mathematics that includes knowledge, beliefs, laws, customs, and any other capabilities and habits of the individuals that belong to the mathematics community (Bishop, 1991).
3. Mathematical Identity – one's relationship with the culture of mathematics. That is, how one learns, engages in, thinks or feels about, or chooses to associate with mathematics (Holland & Lave, 2001; Leatham & Hill, 2010).

4. Mathematics Acculturation—tendency to positively align one's mathematical identity with the norms of the mathematics culture.
5. Audience Perception Survey-survey designed to measure of the mathematical identity of the SMS participants. The survey will be further discussed in Chapter 3.
6. Position – where one views their location or where others view one's location within or outside the mathematical community. Furthermore, if considering their location within the community, what role do they believe they will take on (authoritative/expert or compliant/novice) (Wegner, 1998; Boaler & Greeno, 2000).
7. Self-Efficacy – the personal conviction that an individual has about their own ability to attain a goal or desired outcomes in mathematics (Howard, 2015).
8. Forms of Engagement in Mathematics – activities that demonstrate how one can participate within the mathematical community or how they would enact their identity (Grandgenett, et al., 2009)
9. Mathematical Discourse – any form of communication in mathematics (Miller, 2013).
10. Perceptions of Mathematics – disposition towards aspects of mathematics that has been acquired by an individual through his or her own beliefs and experiences but can be changed (Eshun, 2004) or influenced by factors associated by the individual (self-efficacy, achievement, anxiety, motivation), by instructors or institutions (teacher knowledge, teacher attitudes, classroom management), or by environment (peers within community) (Mohamed & Waheed, 2011).

11. Beginning Mathematician – a student that is a non-mathematics major
12. Advanced Mathematician – a student that is classified mathematics major or related field like physics or engineering.
13. Initial Mathematical Identity – the participant's mathematical identity in the first stage of the study.
14. Intermediate Mathematical Identity – the participant's mathematical identity in the second and third stages of the study.
15. Evolved Mathematical Identity – the participant's mathematical identity in the last stage of the study.

Research Questions

The study investigated how participating in the Student Math Seminar (SMS) is related to a student's mathematical identity. Students participate in the weekly seminar by either attending or presenting. This study attempted to answer the following questions about mathematical identity for the two types of subjects of this study, the presenters and the audience. The questions of this study will first consider the presenters, then the audience and then address the relationship between the presenters and the audience.

Audience

1. What was the mathematical identity of the SMS audience as measured by the Audience Perception Survey?

Presenters

2. How did the student presenters' mathematical identities compare to the mathematical identity of the audience?

3. In what ways did the student presenter's mathematical identity evolve during their participation in SMS?
4. In what ways did the level of mathematics acculturation differ between beginning mathematicians and advanced mathematicians?

Student Math Seminar

Student Math Seminar (SMS) was a weekly seminar that was developed in the fall semester of 2012. Graduate and undergraduate students were invited to give presentations on mathematics or mathematics education. The presentations range from 10 to 45 minutes in length. A talk could have been a formal presentation like one would give at a conference, or a short description of something that is just interesting. The seminar provided an opportunity for the students to gain experience in giving presentations and may encourage more students to do research.

An additional purpose for this seminar was for students to find their own connection to mathematics. That is, to develop a mathematical identity more in tune with the norms of mathematics, or the mathematical activities associated with the mathematics culture (McNeal and Simon, 2000).

Many components of a student's mathematical identity can be observed (Leatham and Hill, 2010). We can see that someone will choose or not choose to study mathematics. We can also see how a student performs on a mathematical assessment. However, there are components that cannot be plainly observed. Those components include a student's perception of the subject and their ability to do it.

Attitudes and perceptions about mathematics affect a student's attitudes toward it (Goodson, 2012). When students have a negative experience in a class or have trouble understanding a concept, they believe it is the subject they hate. Furthermore, students fail to see mathematics, or its importance is other aspects of their lives (Bandura et al., 1996; Gilroy 2002). These negative feelings result in students feeling apathetic to engagement in mathematics activities. Though, students feel mathematics to be important, for many it is overly complex, meaningless and boring. Students believe mathematics "is not 'about' anything, and it creates feelings of fear, feelings of lack of confidence and, indeed, feelings of hatred." (Bishop, 1991, p.2) Students tend to believe that understanding mathematics is unnecessary and knowing the rules is the only important thing about mathematics (Mason, 2003; Muis, 2004). Furthermore, negative students' beliefs about mathematics have a negative impact on their academic performance as well as their educational futures (Gilroy, 2002).

Therefore, if we want to increase and diversify the number of students studying mathematics, we need to eliminate this negative perception of mathematics and its culture. In doing so, methods that will acculturate these students into mathematics need to be considered. Bishop (1991) claimed culture "doesn't just link us to our physical environment" and "therefore we need to define some activities which are more concerned with relating us to each other" (p.23). Students need to be engaged in activities within a social environment to accomplish this task, which is why the seminar, Student Math Seminar (SMS), is proposed to serve as this social environment. In SMS, both presenters and its audience members will see how mathematics can be applicable to their own everyday lives, and perhaps that mathematics can be fun and exciting.

Creation of the Seminar

As I was completing my master's program, I had my first opportunity to attend a professional mathematics conference. At that conference, I realized that I was quite underdeveloped as a "true mathematician." I saw first and second year undergraduate students presenting in-depth theoretical mathematics research, which was far beyond any of the mathematics I have practiced so far. I also learned in a job application workshop, taking place that same conference, I would be greatly under-qualified for any academic or research employment. It was stressed to have experience with research and communicating that research. I needed to have conference presentations and publications in order to be competitive in the job market. Here I was, almost finished with my masters, with no research, no publications, and no presentations. But how was I going to start? How would learn how to research mathematics? And where would I present my research on concepts that I felt that I knew nothing about?

The mathematics department at my university had weekly colloquia but they were hour-long presentations on elaborate mathematics research that professors have been developing for years, even decades. The concept of someone, who had no idea of what mathematics research really was, to be able to develop a presentation to entertain a room full of experts for an hour was extremely overwhelming.

Thus, I asked the department, if a student seminar could be developed. The only presenters at the seminar would be undergraduate and graduate students who would like to be initiated to mathematics research culture. The presentations could last as long as the student needed, and the majority of the audience would be supportive fellow students. The seminar was not designed to make a student into an expert mathematics researcher overnight, but to provide a more casual environment for students to learn how to participate in mathematics research and share their findings from their own research. Furthermore, they could receive constructive feedback in order to better develop their ideas and find other students and perhaps professors to collaborate with. From my own experience of participating in SMS, I feel that I have gained valuable experience in regard to developing as a researcher. By having an informal forum to share my research, even if some may have found my findings insignificant. I received constructive feedback from my peers, which in turn guided me to find results that would be considered significant. I have gained more confidence in my abilities of not only performing mathematics research but gained more confidence in publicly sharing my research with other experts across the nation. Additionally, by having a place to practice and develop a presentation with an informal audience, made the experience of presenting to a professional audience much less daunting.

Additionally, I have received many comments from audience members and other presenters that made me realize that this seminar did not solely provide a practical purpose. Students, who were non-mathematics majors, said that they did not realize that mathematics could be so interesting and wished that class could be more like the seminar. Presenters have come up to me to thank me for the opportunity to speak. For example, one shared that they would have never have realized that the “stuff they learned in class actually applied to stuff they cared about” until they started researching for their presentation. From these comments, I began to see there was more at play than just students giving mathematics presentations.

The audience and the presenters could influence each other's mathematical identity by their interactions during the seminar. The audience's perceptions of mathematics may be affected by the presenters' points of view and representations of mathematics. Frequently, the presenters expose the audience to applications of mathematics in various fields such as communication design, dance, or sports with which the audience members have some personal connection. This in turn can inspire the audience to reflect upon their own interests and how mathematics plays a role in them. The act of presenting mathematical material to an audience, places the presenters in the new role of authority/expert on their topic of presentation. This change from novice/student to expert/instructor can have a profound effect on the identity of the presenter. In addition, the interaction between the presenter and the audience can influence the presenter's identity. The presenter must address questions and comments from the audience and often times the presenter may have to interact with audience in terms of tasks that the audience is asked to perform which can affect the group.

Introduction to Methodology

A qualitative case study of SMS volunteer presenters will be conducted at a large, public university in Texas. The first phase will be a pre-survey, a written autobiography, and an interview, followed by their first presentation. The second phase will be a second presentation followed by an interview. The final phase will be the third presentation, a revision of their autobiography, a post-survey and an exit interview. A quantitative analysis will be conducted on the survey to see if there is any significant difference among the factors of identity. Then a qualitative analysis will be conducted to see if there has been any development throughout the series of presentations and interviews.

Additionally, a narrative analysis will be performed on the autobiographies to see any changes among the factors of identity.

The audience members will be administered an Audience Perception Survey to gauge the overall mathematical identity of those in attendance addressing components of position, self-efficacy, and perceptions of mathematics. An exploratory factor analysis will be performed on the survey to determined which items will be measured together and how well that compares to the proposed framework on the study. Once that has been completed, a confirmatory factor analysis will be performed using structural equation modeling in R Studio.

Significance of Study

Fostering student engagement so that they have a sense of affiliation with or an ownership of the mathematical activity being researched is vital to the success of the design of the experiment and has rarely been a primary focus of research (Cobb et al., 2003). There has been significant evidence that the development of student's mathematical skills has between strongly linked to the development of their mathematical identity. However, most of those studies have taken place within the classroom (Boaler, 2002; Nasir, 2002, Sfard, 2002) and are restricted to only the social context within the classroom (de Abreu, 1995; Martin, 2000). These past studies have not considered the relationships between students' mathematical practices outside the classroom and a social community (Cobb, 2004).

One study that has taken place outside the classroom was Namakshi's (2016). This study looked at the affects a math summer camp had on the participants from economically disadvantaged and/or minority backgrounds of those who attended a university affiliated informal high school mathematics summer program, Riverside Summer Math Camp (RSMC). Riverside Summer Math Camp (RSMC) is an intensive summer program for high school students who are excited about doing mathematics. Students are taught by university faculty and mentored by undergraduate counselors. Courses taught at RSMC include number theory, analysis, and combinatorics. Additionally, students are provided opportunities to conduct original research under the guidance of a faculty member.

As a part of her study, she investigated how this program affected four individuals' mathematics learning identity (MLI) and how it influenced their educational and career trajectories. The study allowed for looking at the long-term effects of a mathematics camp on the educational and career trajectories of the participants in order to gain a better understanding of the role that informal mathematics summer camps such as RSMC play in increasing the participation of women in STEM. Namakshi found that all four participants displayed a narrow pre-MLI in a procedure driven figured world of their school environments. After participating in RSMC, all four participants developed a broad and holistic understanding of the field of mathematics, learning, and themselves as learners of mathematics.

By investigating the unique student seminar, SMS, we hoped to gain insight to students' mathematical identities and how they are affected by participation within SMS. Not only were these activities that took place during the seminar were studied, but the presenters were interviewed about their mathematical practices outside the seminar and the classroom. The presenters shared how they view themselves within the mathematics community prior to their participation and how they were affected by either presenting or just attending in terms of self-efficacy and attitudes towards mathematics. Furthermore, those who were presenting were observed to see how their research methods for preparation for the seminar and forms of engagement were affected by presenting to audience of peers. By determining the effects that took place from this type of non-traditional intervention, action could be taken to change students' relationships with mathematics.

II. LITERATURE REVIEW

This chapter reviews the literature on mathematical identity and presents a theoretical framework that will provide a method to view mathematical identity in the scope of the study.

The proposed aspects of mathematical identity for this study align more with Bishop's (2012) makeup of mathematical identity. Bishop states that mathematical identity is the set of beliefs that one has about who one concerns mathematics or one's position. Identity is dependent on what it is to do mathematics (beliefs about mathematics) and includes ways of talking, acting, being (forms of engagement), and how others position one concerning mathematics (p. 38).

Mathematical Identity

Identity is typically considered how one knows and sees oneself. Identity is what "kind of person" one is believed to be (Gee, 2001). It is socially constructed and constantly changes as individuals interact within different communities in which they live, learn or work (Holland and Lave, 2001). How one would view oneself within the mathematics culture is one's mathematical identity. Leatham and Hill (2010) define mathematical identity as "an individual's relationship with mathematics" (p 226). That is, the ways a person learns, does, thinks about, retains, or chooses to associate with the subject.

Research has proposed a variety of factors that make up one's mathematical identity. Identities can be rooted in self-perception and reflection or what can be learned about one's self through others (Davies and Harre, 1990; Sfard and Prusak, 2005). Identity can be based on one's affiliations with a community (Gee, 2001) or can be contingent on fixed characteristics such as gender, race, ethnicity, or socioeconomic status (Bishop, 2012).

These characteristics are simultaneously present in any enacted identity and are continuously shifting and developing based on exposure to other types of communities and experiences (Markus and Wurf, 1987). These changes in identity depend greatly on history and experiences (Gee, 2001; Wenger, 1998). At one-time identities could have been impressionable but through repeated behaviors and experiences, "solidification" of one's identity is created (Bishop, 2012, p. 38).

Some specific factors that construct mathematical identity include, but not limited to, are forms of engagement, narratives, position, beliefs, and dispositions, social and historical factors. Boaler and Greeno (2000) state that mathematical identity consists one's relationship with the discipline of mathematics through the experiences in the classroom. Sfard and Prusak (2005) claim mathematical identity is a collection of narratives that are "reifying, endorsable, and significant" (p. 16). Martin (2002) defines mathematical identity as one's belief about mathematics ability, the importance of mathematics, the constraints and opportunities provided in local context, and the motivation to obtain mathematical knowledge. The factors that were the most apparent in SMS, as seen in Figure 1, were position, self-efficacy, perceptions of mathematics and forms of engagement.

Theoretical Framework

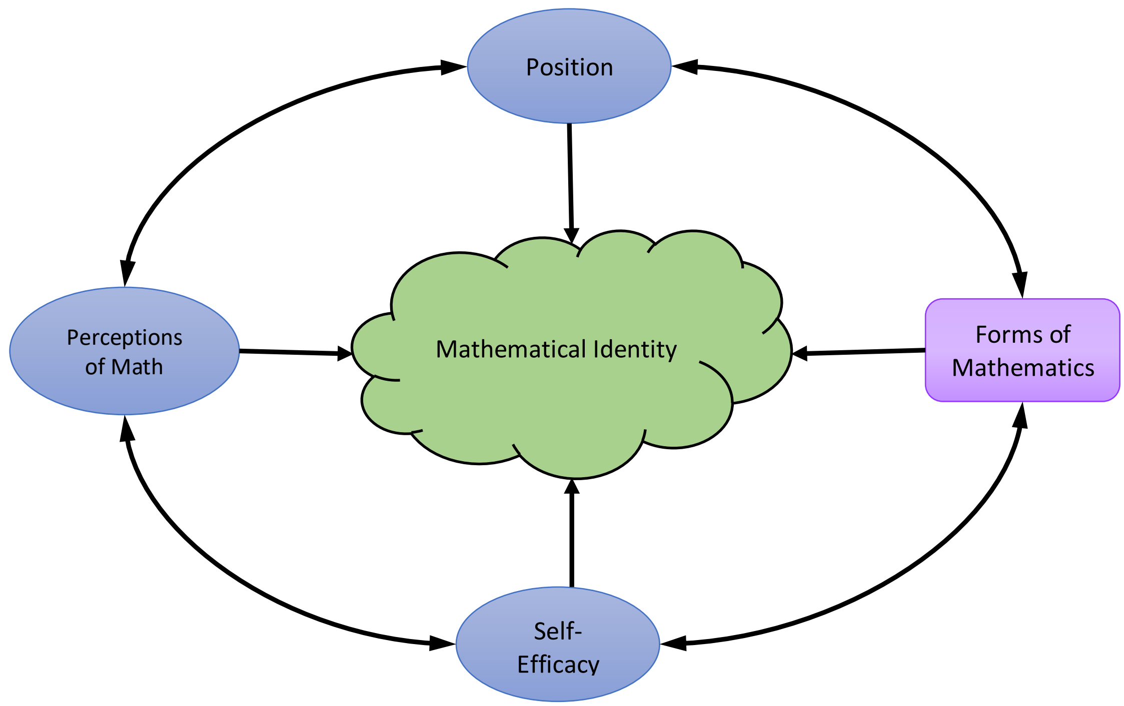

Figure 1. The aspects of mathematical identity. The round blue objects labeled position, perceptions of math, and self-efficacy are internalized factors. The square purple object, forms of engagement, is an external factor.

Mathematical identity can be dependent on a variety of factors. However, for this study, the focus was on those that are apparent and can potentially be affected by participation in SMS. This study considered factors that are internal and external to the individual. The internal factors are self-efficacy, positioning, and perceptions. These factors are known only to the individual until communicated. The external factor addressed in this study was forms of engagement. Forms of engagement are more easily seen or heard by the community. Furthermore, some factors can also influence others as shown in Figure 1.

Self-Efficacy

Self-efficacy is the personal conviction that an individual has about their ability to attain a goal or desired outcomes in mathematics (Howard, 2015). Self-efficacy was first defined in 1977 by Albert Bandura (Maddux and Gosselin, 2011). Bandura defines self-efficacy as one's belief in their ability to successfully perform the task that is necessary for attaining valued goals. A person's perceived self-efficacy influences thought patterns, emotional reactivity, choice behavior and task performance (Bandura, 1986). Bandura's work has been extended to find that self-efficacy affects educational and career choices and differ between men and women (Hackett and Betz, 1981; Bandura, 1982).

Mathematics self-efficacy can be set apart from other measures of attitudes towards mathematics. The reason for saying so is that mathematics self-efficacy is a situational assessment of one's confidence in his or her ability to complete a particular task or problem and is greatly related to one's general confidence for learning mathematics (Hackett and Betz, 1986) rather than an attitude or feeling about the discipline of mathematics.

Self-efficacy is a predictor of one's future performance in mathematics (Bandura, 1986; Betz and Hackett, 1983, 1985; Hackett and Betz, 1986). Hackett (1985) found that self-efficacy contributed more to predicting the choice of mathematics-related college major than sex, years of high school mathematics, ACT mathematics scores, or mathematics anxiety. Hackett and Betz (1986) have found in their study of 262 that students, both male, and female, had inaccurate self-efficacy beliefs than the skills they demonstrated on the given set of mathematics problems. Hackett and Betz's research implied that by strengthening a student's mathematics self-efficacy, that student might be more easily influenced to pursue a mathematics related college major. This suggests that self-efficacy could influence forms of engagement with mathematics and whether or not a student would choose to position themselves in mathematics.

Position

The position is where one views their location or where others view one's location within or outside the mathematical community. Furthermore, if considering their location within the community, what role do they believe they will take on (authoritative/expert or compliant/novice) (Wegner, 1998; Boaler & Greeno, 2000). How one would function in a situation depends on how one is positioned by an authority or how one positioned themselves (Littleton & Howe, 2010).   Sometimes the positions taken up reflect an enacted identity. For example, imagine a classroom in which a teacher consistently refers to her students as "mathematicians." This type of discourse move is a positioning act meant to reflect and encourage students to enact the desired identity (Bishop, 2012). Students make use of positional identities when and how they chose to enter into a discussion with others or participate in the class activity (Graves, 2011) or other forms of engagement.

Position can be given internally or externally. Authority figures, such as instructors, parents, or employers can "assign" a position. Gee (2001) said that one way of defining who you are as an individual is based on your position. It is not defined by nature or not something that can always be achieved by oneself. The source of position is from a "set of authorities." Others who are in power authorize what role is assigned to an individual. For example, a child who is diagnosed with ADHD by a doctor will forever identify himself as someone who has ADHD since an authority, the doctor, said so (Mehan, Hertweck & Lee, 1986). The same situation can be applied to a student's position in a classroom. Based on feedback received from an instructor or classmates, a student can be positioned as either a strong mathematics student or a weak mathematics student. These external reviews suggest a student could develop their mathematical identity as either someone who "belongs" to the mathematical community or does not.

Perceptions of Mathematics

There have been numerous studies that explored the effects of beliefs, values, and attitudes about mathematics. There has also been researching into trying to distinguish the differences among them (Underwood, 2002). Attitudes and values of mathematics are dispositions towards aspects of mathematics that has been acquired by an individual through his or her own beliefs and experiences but can be changed (Eshun, 2004) or influenced by factors associated with the individual (self-efficacy, achievement, anxiety, motivation), by instructors or institutions (teacher knowledge, teacher attitudes, classroom management), or by environment (peers within community) (Mohamed & Waheed, 2011).

Attitudes are a central part of human identity. Everyday people love, hate, like, dislike, favor, oppose, agree, disagree, argue, persuade, etc. All these are evaluative responses to an object. Hence attitudes can be defined as "a summary evaluation of an object of thought" (Bohner & Wänke, 2002). Attitudes and values guide an individual's behavior and can be developed from personal factors such as assessment scores, mathematics anxiety, and self-efficacy (Tahar et al, 2010; Mohamed &Waheed, 2011). Attitudes may have an ego-defensive function in that they can protect one from unflattering or negative feelings towards oneself (Underwood, 2002). Attitudes and values can also be influenced by instructors and instructional methods (Papanastasiou, 2000).

Attitudes and values are factors that greatly influence students' performance in mathematics (Mohd, Mahmood, & Ismail, 2011; Ma & Kishor, 1997). Success in mathematics often has as much to do with attitudes and beliefs about mathematics than about actual mathematics ability (Nolting, 2007). Mathematics educators believe that children learn more effectively when they are interested in what they learn and that they will achieve better in mathematics if they like mathematics (Suydam and Weaver, 1975).

Ma and Kishor (1997) analyzed 113 survey studies from 1993 to 1996 that addressed the relationship between attitude towards mathematics and achievement in mathematics. There was a total of 82, 941 students from grades 1-12 that participated in these surveys. After combining studies according to their sample sizes and the number of primary studies, they found an overall weighted mean effect size of 0.12, with a 95% confidence interval from 0.12 to 0.13. Because the confidence interval did not include zero, they concluded that that the overall mean effect size was significantly different from zero, indicating the relationship between the attitudes towards mathematics and mathematical achievement was positive and reliable, but not strong.

There has been much debate on separating attitudes and perceptions (Underwood, 2002). But it this issue is far more complex and much more than what is necessary for this study. So, therefore, using Underwood's (2002) research on belief and attitude changes, we will assume a simpler theory-beliefs are based on values and attitudes are developed from perceptions. Cognitive theorists assume that perceptions are formed about the attitude object through a cognitive learning process. As people gain information about the attitude object or idea, new or altered perceptions may result from that exposure (Underwood, 2002). If altered or new beliefs develop, they will be followed by changed attitudes. Hence, I will just generally address students' beliefs about mathematics and their attitudes as reflections of those perceptions.

A widespread public image of mathematics is that it is difficult, cold, abstract, theoretical, and ultra-rational, and, also important and largely masculine (Ernest, 2008). It also has the image of being remote and inaccessible to all, but a few extra-ordinary persons with 'mathematical minds' (Buerk, 1982; Buxton, 1981; Ernest, 1996; Seymour & Hewitt, 1997; Picker & Berry, 2000).

Seymour and Hewitt (1997) conducted a three-year ethnographic study on 335 students' reflections of their undergraduate experiences. They found that students in science, mathematics and engineers courses describe their classroom environments as cold, the instructors as unapproachable, and lectures did not welcome discussion. This can lead to students taking on a passive role when attending class (Bressoud, 1994). This belief can be particularly fixed for adult students who have held this belief for many years and may contribute to avoidance of mathematics courses and affect engagement in mathematics classrooms and thus affect learning. The longer a person holds a belief, the more durable it becomes, and it is eventually incorporated into their cultural identity (Swain, et al., 2005). Often this belief that mathematical knowledge is unattainable is sustained even after adult students are successful in mathematics courses (Lawrence, 1988; Wedege & Evans, 2006).

For many people, this negative image of mathematics is also associated with poor self-efficacy. This results in anxiety and failure in mathematics. Many researchers (Ma

& Kishor, 1997; Richardson & Suinn, 1972; Tobias & Weissbrod, 1980) reported the consequences of being anxious toward mathematics. The consequences included weak self-efficacy and avoiding engagements with mathematics. Mathematics anxiety has led to the belief of having the inability to do mathematics, the escaping of mathematics courses, the limitation of students in selecting college mathematics majors and related future careers.

In 1982, data was gathered on adult numeracy for the Cockcroft Report (1982). The Cockcroft report was a proposal for reform for England's education of mathematics. An inquiry was conducted by asking a sample of adults on the street if they would answer some questions. Half of them refused to answer further questions when they realized the questions were about mathematics, suggesting negative attitudes.

In fact, the consensus of mathematics educators is that school mathematics must counter that image (Howson & Wilson, 1986). Instead, mathematics should offer something that is personally engaging and useful or motivating in some other way (NCTM, 1989; Skovsmose, 1994). Belonging or having a more aligned position to coursework has been perceived to be associated with student wellbeing and academic achievement (Cooper et al., 1998). Therefore, "continual attention should be directed towards creating, developing, maintaining and reinforcing positive beliefs and attitudes" (Suydam & Weaver, 1975, p. 45)

Not only is there a consensus that many students have negative views of mathematics, but they also have negatives views of mathematicians. Picker and Berry (2000) found in an investigation of the images students have of mathematicians that students tend to have a negative view of mathematicians. Drawings by 12-13-year-olds depicted mathematicians foolish, nervous, and having special powers.

These views are not only held by 7th and 8th-grade students since it has been observed that student beliefs about mathematics either do not change or tend to become less positive as they make their way through secondary school (Wilkins & Ma, 2003; Goodson 2012). Mathematicians are seen as frightening individuals who intimidate their students into doing their work correctly (Picker & Berry, 2000). Many of these beliefs about mathematicians come from the way society perpetuates stereotypes. Picker and Berry point out that through teachers and the media, students perceive that a privileged few can do mathematics, mathematics is a special language for a selected few, and that mathematics should be done quickly. This leads students to the conclusion that mathematicians are authoritarian figures (Picker & Berry 2000). By understanding how students imagine mathematicians and how we could change those images, we can broaden their "thinking about their roles as mathematicians" (Rock & Shaw, 2000). However, if students' images of mathematicians continue to indicate that they perceive mathematics to be an unattractive field of study, students would not want to position themselves within mathematics. Thus, the decline in enrolment of students in advanced mathematics courses (Garfunkel & Young, 1998) is sure to continue fulfilling predictions of an increasing shortage of mathematicians (Picker & Berry, 2000).

Many students also have misconceptions about the work that mathematicians do and what that work entails. Rock and Shaw (2000) found that elementary students believed that mathematicians do work that is similar to the work that they are doing but with bigger numbers, or simply do problems that other people do not know how to solve.

When middle school students were asked when they would hire a mathematician, many either did not know when someone would hire a mathematician, did not know what a mathematician does, or thought that people do not need mathematicians (Picker & Berry, 2000). When the students did mention jobs for which you would hire a mathematician, the majority only could name teaching. This implied a general lack of knowledge about the work of mathematicians (Picker & Berry, 2000).

Muis (2004) reviewed a study that found about a third of students thought mathematicians worked with symbols rather than ideas. Those same students also believed that discoveries are seldom made. The NCTM Standards for Teaching (1991) state that, "mathematics is a changing and evolving domain, one in which ideas grow and develop over time" (p. 26). However, Muis (2004) claims that students view mathematics as unchanging. Muis' review of research shows that many students believe that mathematics is a set of "fragmented rules and procedures." Students also believe that understanding mathematics is unnecessary and that the only thing that matters is knowing the rules to get to the correct answer (Goodson, 2012). Additionally, students believe that being able to do well in mathematics is a natural ability (Muis 2004). This belief has a negative impact on self-efficacy. When a person who does not believe that he or she is good at mathematics, they will be less inclined to try to understand the subject (Hackett and Betz, 1981; Gilroy, 2002).

Research has also investigated students' perceptions of the usefulness of mathematics. Klooserman et al. (1996) found that in early grade levels, students either believe mathematics is useful as a means of moving on to the next grade level or they express knowing that mathematics is useful but cannot give an example or reason. Even many older students cannot see how mathematics can be useful in their lives (Gilroy, 2002). This failure to see the usefulness of mathematics, coupled with a dislike for mathematics, can affect the number and type of mathematics courses a student takes, or worse, convince a student to take mathematics courses at the last moment in college (Hackett and Betz, 1981; Gilroy, 2002; Goodson, 2012).

Forms of Mathematical Engagement

The way a person participates in mathematics can develop one's identity and can be a determining factor for whether a study would continue to study mathematics (Cobb, 2004). Forms of mathematical engagement are mathematical activities that demonstrate how one can participate within the mathematical community or how they would enact their identity (Grandgenett et al., 2009). The best way for one to learn mathematics would be to have multiple opportunities to practice methods, thus reinforcing certain behaviors (Greeno and MMAP, 1998). However, traditional classroom settings do not always allow for those opportunities. Students who learn in these traditional classrooms can be successful, but many students experience an important conflict between the practices in which they engaged, and their developing identities as people (Boaler, 2002).

In 2000, Boaler and Greeno interviewed eight students taking AP calculus from each of six northern Californian high schools. Four schools used a traditional approach based on teacher demonstration and student practice. The other school required students to work on material individually and then discuss their different ideas in groups. The purpose of Boaler and Greeno's research, as well as the purpose of this study, concern the relationship between the teaching methods and student beliefs.

The students in the traditional classroom were successful but were not connecting their mathematical practices to who they were developing to be as a person. Thus, many of the students talked about their dislike of mathematics and their plans to give the subject up as soon as they were able. They did not want to do mathematics because of the cognitive demand, but because they did not want to be positioned as only a recipient of knowledge that engaged in practices that left no room for their interpretation. The students all talked about the kinds of person they wanted to be - people who used their ideas, engaged in social interaction and exercised their freedom and thought. However, their classroom experiences left them feeling prohibited from doing so. Also, the primary authority lay with the teacher and the textbook.

The disaffected students interviewed were being turned away from mathematics because of pedagogical practices that did not relate to the nature of mathematics. Most of the students shared their rejection of mathematics in the traditional classrooms, even though they were successful, had decided to no longer pursue the discipline because they wanted to participate in subjects that offered opportunities for personal expression. In contrast, those in the traditional classes who remained motivated were those who seemed happy to receive knowledge and to relinquish the requirement to think deeply. The students liked mathematics did so because there were only right and wrong answers, and because they did not have to consider different ideas and methods. They did not need to think about “how or why” mathematics worked, and they seemed to appreciate the passive positions that they adopted about the discipline.

In the discussion-oriented classes, the students had formed very different relationships with mathematics. These relationships did not conflict with the identities they were forming in the rest of their lives. The students in these classes regarded their role to be learning and understanding mathematical relationships; they did not perceive mathematics to be “a ritual of procedure reproduction.” This lack of conflict meant that the students who wanted to do more than receiving knowledge were able to form plans for themselves as continuing mathematics learners, and maybe becoming mathematicians.

Wenger's (1998) depiction of learning as a process of 'becoming' is consistent with Boaler and Greeno’s (2000) findings:

> Because learning transforms who we are and what we can do, it is an experience of identity. It is not just an accumulation of skills and information, but a process of becoming - to become a certain person or, conversely, to avoid becoming a certain person. Even the learning that we do entirely by ourselves eventually contributes to making us into a specific kind of person. We accumulate skills and information, not in the abstract as ends in themselves, but in the service of an identity, (p. 215)

Now the way we learn and how we communicate what we learned in the form of mathematical engagement that would demonstrate how we identify ourselves in the mathematics culture (Grandgenett et al., 2009). “Part of learning mathematics is learning to speak like a mathematician” (Pimm, 1987, p. 76). In the SMS, students will have to communicate the mathematical ideas, which is mathematical activity in itself and will demonstrate mathematical activities in their presentations. Burton (1984) claimed four processes could be shown to be central to mathematical activity. The four processes are (a) specializing, (b) conjecturing, (c) generalizing, and (d) convincing. Specializing is the key to an inductive approach to learning and is observed as natural to learning. Conjecturing is when enough examples have been examined, a conclusion about the relationship that connects concepts is formulated, and a sense of any underlying pattern is explored, expressed, and then substantiated. Generalizing is the statements of recognition of pattern or regularity. These generalized statements appear to be the building blocks used by learners to create order and meaning out of an overwhelming quantity of sense data. Finally, there is convincing. The convincing process is the means by which a generalization moves from being personal to being public. There is an interest to see how these processes are being utilized when students prepare and present their "math talk" for SMS aides these processes of specializing, conjecturing, generalizing and convincing.

Difficulties Changing Mathematical Identity

Mathematics culture often viewed as a "gatekeeper" of those who choose to study mathematics because either students are not welcomed by those who already practice the culture or students perceive that they are not welcomed by those who practice the culture (Cobb, 2004). Students who are successful in traditional mathematics courses choose not to pursue studying mathematics because being successful in mathematics does not align with how they identify themselves to be (Boaler & Greeno, 2000). After all, who we believe ourselves to be influences how we would interact, engage, behave, and learn (Bishop, 2012).

Furthermore, mathematicians and mathematics students view mathematics quite differently. Burton (1999a, 1999b) conducted one of the first studies to give insight into the practices of university mathematicians. Burton interviewed 70 research mathematicians to find out about the nature of their work, as well as their understanding of knowing. She found that the mathematicians emphasized the importance of intuition, uncertainty, and connectivity. They did not talk about the procedural nature of mathematics. Instead, they described the importance of creativity. They spoke about the exhilaration they experienced when solving problems and the fun and excitement of mathematics. Mathematicians claim that their practices involve aesthetic values like elegance, simplicity, and generalizability (Moschkovich, 2004).

These points of view were quite contrasted to the views of many of the students interviewed in a study conducted by Boaler and Greeno (2000). The students interviewed who enjoyed mathematics in coursework did not relate their enjoyment to the pleasure of problem-solving, but to the structure and limits of the discipline as they experienced it. While the mathematicians Burton interviewed emphasized the uncertainty of their explorations, the students who liked mathematics emphasized the certainty of their work. These contrasting perspectives suggest that narrow mathematical practices within the school are problematic, not only because they disenfranchise many students, but because they encourage forms of knowing and ways of working that are inconsistent with the discipline. Thus, school mathematics, as noted by Burton (1999a) and others, is unlike the mathematics encountered in life (Boaler, 1997).

In addition to the contrasting views pointed out by Boaler and Greeno (2000), there are other difficulties in truly understanding the true meaning of mathematics. Restivo (1993) stated that is difficult to uniformly define mathematical practices since so many autobiographical descriptions of mathematicians' practices differ between the numerous subfields of mathematics. Also, many mathematicians themselves will disagree what it is that mathematicians do. So, if mathematicians do not agree on how to define mathematics is or what mathematical practices are, how can a newcomer like a student begin to understand.

Mathematics is often "depicted as monolithic and as involving the disembodied voice of objectivism and rationalism" (Cobb, 2004, p. 333). When someone would ask you how to define mathematics, what would you say? Or could you determine if mathematics is something to be discovered or something that is created? Many would hesitate when trying to answer these questions. Either they find mathematics to be too complex to define, or they believe it is something so simple like just adding and subtracting numbers. With so many individuals, with such contrasting understandings of mathematics, it can be quite difficult for someone who is just being in introduced to the world of mathematics.

There are also so many ways mathematics is used and viewed. The way a pure mathematician sees mathematics can differ greatly from how an engineer sees it and can be more so from a school mathematics teacher (Cobb, 2004). Therefore, students may only be exposed to one side of the mathematics story in the classroom.

By participating in SMS, students have the opportunity to create their own story of the mathematics. Through their investigations for their presentations, students have the opportunity to see both the mathematics and the application of the mathematics. They can define their mathematical value to the knowledge they can gain.

Structure of Seminar

Even though SMS was not developed with considerations of researching the educational benefits of this seminar, I began to think about what the research would say about a program like this.

First, we can look at how the structure of the seminar makes the participant consider their position. By doing presentations, students learn how to speak in front of an audience, which a broadly applicable skill to any profession. They learn how to prepare material for a public presentation, and practice with feedback can improve their speaking skills (Weimer, 2013). Furthermore, from a pedagogical standpoint, it can help students gain a more holistic view of the content (Baranowski and Weir, 2011). This can possibly affect the students perceived value of mathematics.

Next the structure can be effective in providing different forms of engagement than what is provided in the more traditional classroom. The structure of the seminar falls somewhere between problem-based learning (PBL) projects and programs like REUs (Research Experiences for Undergraduates). PBL is an approach in which students consider and respond to real-world questions and problems. With these PBL projects, students engage in peer collaboration, develop critical thinking and communication skills (Lattimer & Riordan, 2011). PBL was further defined by Adderley et al. (1975, p.1) as

1. Projects involving the solution of a problem; not necessarily set by the student;
2. Involving initiative by the student and necessitate a variety of educational activities;
3. Commonly resulting in an end product (report, model, design plans, etc.);
4. Work involving goes on for an extended period;
5. Teacher(s) are involved in an advisory role rather than an authoritarian role at any or all stages.

The basis of problem-based learning is that a question or problem serves to organize and drive the activities. The activities will then result in a final product that addresses the driving question (Blumenfeld, et al., 1991). The great benefit of PBL projects is that it can be more effective than traditional learning that takes place in the classroom. PBL projects support long-term mathematical knowledge retention and engage students in synthesizing and explaining mathematical concepts (Boaler, 1997).

There are four primary motives for using PBL (Heitman, 1996). Helle et al. (2006) described them as the following:

1. Professional-practice orientation or work-based learning;
2. Humanitarian-service-learning and incorporating humanistic studies;
3. Scientific-foster critical thinking;
4. Pedagogic-foster understanding of subject matter.

Now like problem-based learning projects, the SMS seminar has the student take the initiative to investigate real-world problems and communicate mathematical concepts in their presentations. However, unlike PBL projects, there are no instructors intervening or playing an advisory role. Also, the solutions or mathematical conclusions have not been predetermined by an instructor. The student's position is considered the authority of the knowledge presented and how it is presented. Furthermore, SMS aligns with at least three of the four motivations for PBL. For students who are pursuing careers in academia, they would choose to participate in SMS for the experience/practice of speaking about mathematics to public audiences. Mathematics instructors would encourage students to participate in SMS for scientific and pedagogical motives. Students could potentially improve their critical thinking skills and their understanding of mathematical content by engaging in research for a presentation or even listening to a presentation.

SMS has some similarities to programs like REUs, however, is still not quite up to the caliber of those programs. REUs are National Science Foundation (NSF) funded programs where each student is associated with a specific research project, where he/she works closely with the faculty and other researchers (NSF). Also, there is a rigorous application process for students to become part of a REU project. SMS, again, does not involve any faculty and students that participate are taken on a volunteer basis. No application is necessary. However, there are some similarities to REUs. Students have the opportunity to gain invaluable speaking skills and can be influenced to pursue further studies in mathematics (Long and Monks, 2009).

It can be argued, though, that learning through SMS is a type of experiential learning. In higher education, experiential learning is seen as a way to revitalize the university curriculum for groups of diverse students (Kolb, 2015). Experiential learning is where one "cognitively, affectively, and behaviorally processes knowledge, skills, and attitudes in a learning situation characterized by a high level of active involvement " (Hoover & Whitehead, 1975, p. 1). Students would be directly in contact with the material being studied instead of just merely thinking about the material or just considering the possibility of doing something with it (Keeton & Tate, 1978), which is quite common with students in mathematics courses. They think that mathematics is only needed in a mathematics class and there is no connection to their everyday lives (Bandura et al., 1996; Gilroy 2002).

In SMS, students who present are "experiencing" similar feelings and situations one would during a professional presentation. When the students are presenting, they are imitating the role of a mathematician. Studies done by Pelletier and Shore (2003) and Sriraman (2004) have found that mathematically talented students tend to think about mathematics in ways that are similar to the ways that experts or professional mathematicians do. Thus, it can be argued that if students can start to think about mathematics in ways that professionals operate, they may tend to be more engaged in the subject. Practicing verbal and written communication in mathematics helps engage students' thought processes that resemble those of practicing mathematicians (Gavin, et al., 2007). Jacques Hadamard (1954) and George Polya (1954) believed that the sole difference between the work of a professional mathematician and the work of a student is in the degree of sophistication they possess. Both are capable of being creative and analytical in solving problems and in posing new problems at their respective levels.

Curricula have been designed to test this idea. For example, in the Project M^3: Mentoring Mathematical Minds (Gavin et al., 2007), teachers had approximately 200 middle school students from nine different schools process mathematics like practicing mathematicians. This curriculum resulted in students demonstrating significant mathematical gains in understanding of the mathematical concepts outlined in the curricular units.

In another program, the Enrichment Triad Model (Renzulli, 1977), students from multiple Connecticut school districts were taught with a curriculum that required students to pursue problems of their interests as if they were "practicing professionals." By applying the knowledge from prior classroom instruction, students were able to gain deeper knowledge and acquire stronger thinking skills since the material studied was within the context of real-world problems (Renzulli, 1994). Student learning was also more effective when the student enjoyed what they were doing (Gubbins, 1995).

SMS slightly resembles Project M3 and the Enrichment Triad Model with respects to how students are adopting the position of a mathematician. When students are presenting, they are to consider what content is necessary to include and how a professional mathematics presenter would communicate that content and engage in other mathematical practices. And similar to the Enrichment Triad Model, students have the freedom to choose their topic. Students often choose topics that are related to their majors, current employment, and even hobbies. By discussing topics that they are passionate about, they tend to express how much more appreciation they have for the mathematics.

With having the benefits of programs such as PBL projects, Project M3, and the Enrichment Triad Model, SMS was considered the intervention taken for students to make their own connections to and appreciations of mathematics and in turn evolve their mathematical identity overall. The next chapter will describe the methods that will measure the aspects of mathematical identity that were just previously defined. The methods gauged the mathematical identities of the presenters, the audience, and against each other. Chapters 4 and 5 will describe the results and implications of those methods.

III. METHODOLOGY

The previous chapters presented research about mathematical identity and the theoretical framework that structured this study. This chapter will review the methodologies for how the participation in the Student Math Seminar (SMS) relates to a student's evolving mathematical identity. This chapter presents the methodologies used in the study, the research settings, instruments used, and data collection methods.

As mentioned previously, students participated in the weekly seminar, SMS by either attending or presenting. This stud y attempted to answer the following questions about mathematical identity for two types of subjects, the presenters and the audience. The aspects of mathematical identity consider position, self-efficacy, perceptions of mathematics, and forms of engagement.

Audience

1. What was the mathematical identity of the SMS audience as measured by the Audience Perception Survey?

Presenters

2. How do the student presenters' mathematical identities compare to the mathematical identity of the audience?
3. In what ways did the student presenter's mathematical identity evolve during their participation in SMS?
4. In what ways did the level of mathematics acculturation differ between beginning mathematicians and advanced mathematicians?

Table 1 lists the instruments and the research questions they would potentially answer.

Table 1. Aspects of math identity addressed by the instruments. This table indicates which research questions and aspects of the framework the instruments address.

Instrument	Information on Aspects of Identity	Research Question Addressed
Audience Perception Survey	Self-Efficacy, Position, Forms of Engagement, Perception of Mathematics	1, 2, 3, 4
Math Autobiography	Self-Efficacy, Position, Forms of Engagement, Perception of Mathematics	3, 4
Presenter Interview Protocol (PIP)	Self-Efficacy, Position, Forms of Engagement, Perception of Mathematics	3, 4
Presenter Observation Protocol (POP)	Position, Forms of Engagement	3, 4

A quantitative approach was used to determine the mathematical identity of the audience. Audience members completed a survey developed to address the components of the theoretical framework—position, beliefs about mathematics, self-efficacy, and forms of engagement.

Setting

The Student Math Seminar (SMS) is a student-based seminar that takes place at large, public four-year university in central Texas with an enrollment of approximately 38,000 students. SMS is a weekly seminar that was developed in the Fall semester of 2012. Graduate and undergraduate students are invited to give presentations on mathematics or mathematics education. The presentations range from 10 to 45 minutes in length. The talks can be a formal presentation like one would give at a professional or academic conference, or they can be short a description of a topic of the presenter's interest. SMS provides an opportunity for the students to gain experience in giving presentations and may encourage more students to do research. Additionally, the seminar creates a context outside of the classroom for students to find their own connection to mathematics.

A student can make as many presentations as they would like on as many topics that they would like. Since the seminar's inception, the majority of presenters have given only one presentation; others have given multiple presentations and on different topics. For this study, in order to have a consistent context to measure change in their identity, the presenters were asked to give three presentations on a single topic. The study took place in the spring of 2016 and fall of 2016.

Participants

This study considered two groups of subjects, the audience and the presenters. The audience and presenters will be considered as either Beginning Student Mathematicians (BSM) or Advanced Student Mathematicians (ASM). Beginning Student Mathematician are defined as a student who is not a mathematics major and Advanced Student Mathematician as a student that is a declared mathematics major or related field like physics or engineering.

Four undergraduate participants volunteered to give a series of 3 talks on the topic of their choosing. Two of the students were freshman non-mathematics majors, BSM. One was a sophomore mathematics major, and lastly a senior mathematics major, both considered ASM. All of the presenters and members of the audience were included in the study given their informed consent to participate.

Audience

The audience attending SMS was comprised of a majority of undergraduate students (BSM) and some graduate students (some BSM and some ASM) with diverse backgrounds and mathematical levels. The majority of the BSM were enrolled in either College Algebra or Business Calculus and majoring in fields outside of mathematics such as nursing, business, and finance. These undergraduates were mostly motivated to attend by receiving extra credit from their mathematics professors; however, some of the instructors required to write a reflection paper on the seminar in addition to their attendance to receive the extra points. The spring and fall semesters of 2016, approximately 90 students were in attendance of SMS each week. Each week audience members complete an attendance sheet that asked for their major and current mathematics course(s). To investigate the mathematical identity of the audience, the Audience Perception Survey, which is further described below, was administered every week for first 4 weeks of the semester and again for the last four weeks. This was to try to ensure the maximum number of participants of the survey since the audience changed every week as seen in Figure 2.

When SMS first started, the average size for the audience was 9.8 attendees. By the seventh semester, attendance grew by a 937% in average attendance. For the semester of the study, Fall 2016, the average attendance was 88.2. There is a summary of the average attendance of each semester since the start of SMS in Figure 3.

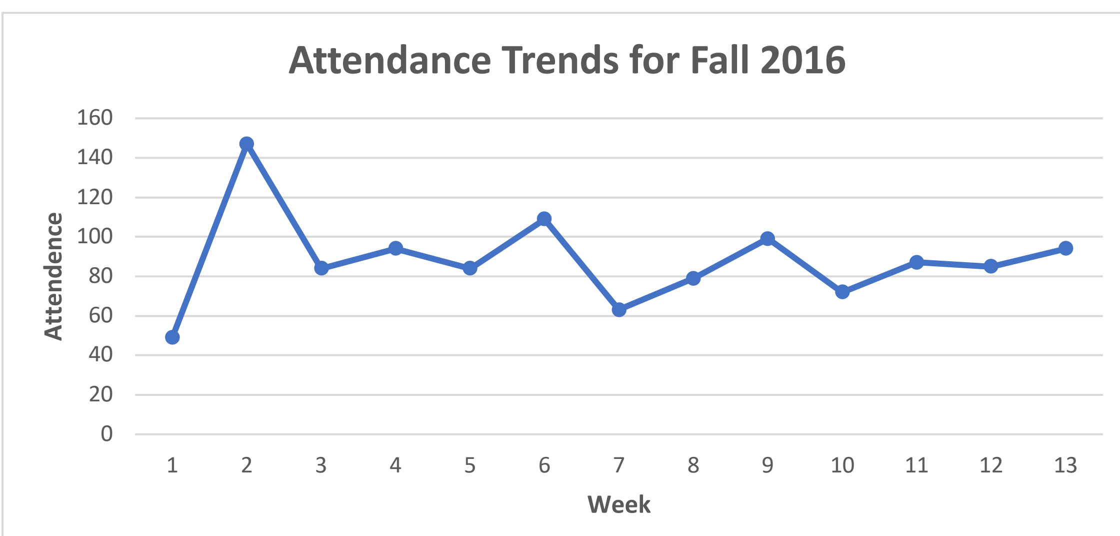

Figure 2. The attendance trends during the study.

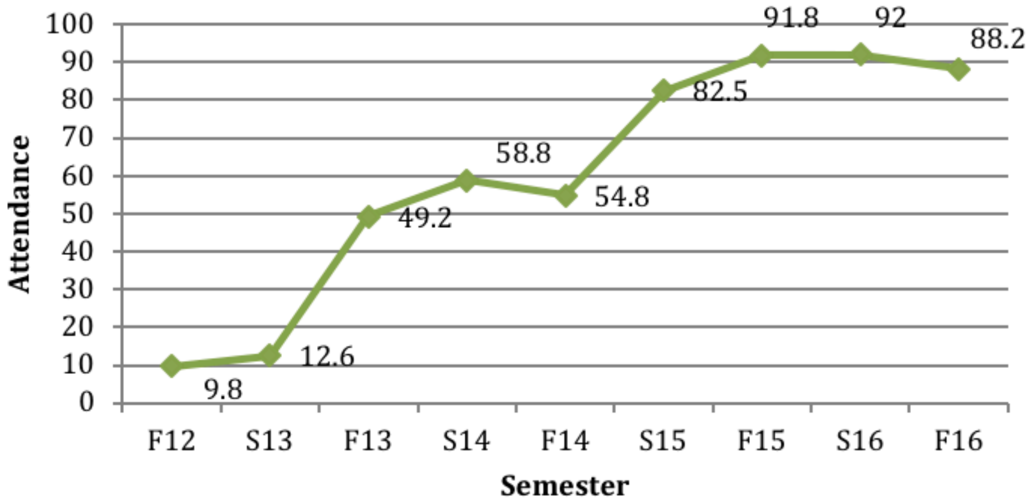

Figure 3. Average attendance per semester.

Presenters

To qualify as a presenter for SMS, the presenter had to be a student, either undergraduate or graduate, and the presentation must be about mathematics. The chosen topic does not necessarily be based solely on mathematics but must address the mathematics involved in that topic. For example, a presentation could be given on the art of dance; however, the student must include mathematics somehow. The presentation could include how choreography includes movements of symmetry or represent a function or how dance could be incorporated into teaching mathematics.

The students were asked to volunteer to participate in the study. Each participant was interviewed and asked to write a math autobiography. They were also required to give three presentations on the topic of their choosing. Even though students typically only gave one presentation, the participants were asked to give a series of presentations in order to see if there was any change in their identity through presenting since one presentation is not adequate for seeing notable change. Four volunteers were selected to be a part of this case study, two ASM and two BSM.

Instruments for Data Collection

This section describes the instruments for data collection from the audience and from the four stages of the study of the presenters. These instruments include the Audience Perception Survey (audience and presenter tasks), Math Autobiography (presenter task), Presenter Interview Protocol (PIP) (used for interview of presenters), and the Presenter Observation Protocol (POP) (used for observing presenters). Table 2 shows a list of the instruments and the data collected at each stage the instrument is used.

Table 2. Summary of instruments. This table is a description of the instruments and the data collected at each stage of the study.

Instrument	Subject (Audience/Presenter)	Data Collected	Stage Collected from Presenters
Audience Perception Survey	Audience and Presenter	Likert-scaled statements that address value of SMS and aspects of mathematical identity	1, 4
Math Autobiography	Presenter	Collect background information, assess initial and evolved math identity	1, 4
Presenter Interview Protocol (PIP)	Presenter	Collect background information, assess overall initial, intermediate, and evolved math identity	1, 2, 3, 4
Presenter Observation Protocol (POP)	Presenter	Assess forms of engagement and position	2, 3, 4

Audience Perception Survey

In the development of the Audience Perception Survey there were two versions prior to the one used in the study. The first survey, Pilot Survey A addressed mathematical identity but used a different theoretical framework that did not fully encompass the entire range of components of mathematical identity addressed in this study. The second survey, Pilot Survey B was then developed from the results of the first survey and the new proposed theoretical framework. Finally, the last survey, Audience Perception Survey, was developed by eliminating problematic items from the previous study.

Pilot Survey A. Pilot Survey A survey, found in Appendix A, was an adapted version of the Fenemma-Sherman Mathematics Attitudes Scales (FSMAS) tests to measure levels of mathematical identity. The FSMAS evaluates nine specific domains using Likert-type scales: attitude toward success, mathematics as a male domain, mother's attitude, father's attitude, teacher's attitude, confidence in learning mathematics, mathematics anxiety, affect motivation and mathematics usefulness (Fenemma & Sherman, 1976). The FSMAS was adapted to address one's attitudes towards success, confidence in learning mathematics, mathematics anxiety, affect motivation, and mathematics usefulness. The survey was administered over a period of 8 weeks in the fall of 2014 within SMS. Every week each audience member received a survey and were asked to complete it to the best of their ability. If anyone had completed a survey in weeks prior, only their previous results were considered. Pilot Survey A was administered with a letter of consent that described the study and informed the students that participation in the study was voluntary. A total of 117 participants completed the survey. There were 19 Likert-type statements as found in Appendix A and the results showed no outliers nor missing data.

An Exploratory Factor Analysis (EFA) was conducted to determine if the survey was adequately measuring components of mathematical identity within the proposed theoretical framework. From the EFA, four factors were determined from 17 items from the survey. Items 3 through 19 were analyzed since those items addressed mathematical identity. The first two items were questions addressing the structure of the seminar.

Based on the results of the pattern matrix, the loadings were all acceptable except for item 9 being loaded with items 3 and 4. This may be because item 9 was misinterpreted by the participants. Therefore item 9 was excluded in the confirmatory factor analysis (CFA) and was rewritten for future surveys. A Confirmatory Factor Analysis (CFA) was carried out to test a theory or a model when the constructs or factors have already been established (Vogt, 2007). This test was appropriate to assess interpretations of "differences in the behavior of person who score high or low on a factor" (Tabaschnick & Fidell, 2007, p.607). A specific type of CFA, structural equation modeling (SEM), was performed using the software AMOS, a package program of SPSS. SEM is "a collection of statistical techniques that allow a set of relationships between one or more independent variables" and one or more dependent variables to be examined (Tabaschnick & Fidell, 2007, p.681).

From these factor loadings and the statements of the items, four obvious themes emerged. The first factor addresses perceptions of mathematical norms (5, 7, 8, 10, 11, 16, 17, 18, 19). The second factor pertains to understanding that not all of mathematics has been discovered (6, 13, 15). The third factor addresses the usefulness of mathematics (3, 4). And the last factor addresses one's ability to practice mathematics (12, 14). These four factors are subcomponents of the pieces of the theoretical framework, which make up one's mathematical identity. This led to the development of Pilot Survey B.

Pilot Survey B. Based on the results of the Pilot Survey A, additional items were added to the survey to create Pilot Survey B. In the spring semester of 2016, Pilot Survey B survey was administered to 160 students addressing 53 items as seen in Appendix B. A new exploratory factor analysis was performed using a Varimax rotation.

From this new factor analysis, it was suggested that there could potentially be four factors—perceptions of mathematics, self-efficacy, position, and forms of engagement. The factor loadings in the rotated matrix showed that 17 items were loaded on the first factor with a high reliability of a Cronbach's alpha of 0.901. The majority of the items dealt with self-efficacy. Other items that loaded on this first factor were items that addressed position. These items were then reexamined to determine their loading with self-efficacy. Since there is an overlap with many components of self-efficacy and position and these items in question could have been considered as either self-efficacy or as position items, it could be accepted that these position items could be loaded with the self-efficacy factor, however they were still considered to be self-efficacy.

The remaining 34 items were then investigated further to determine how those were being loaded. From the rotated factor matrix, the correlation table, and the items themselves, it was concluded that there were some items that may be problematic. The most apparent items that could be an issue, were items about who mathematicians are. These items addressed the likableness of mathematicians and stereotypes of mathematician personalities. These items could be misinterpreted by the participants, so the items were to be considered for removal for future studies.

Audience Perception Survey. The final version of the survey administered for this study was known as the Audience Perception Survey, found in Appendix C. This survey consisted of four parts as seen in Table 3. The first section, Participant Information asked for the following information from the participant: major, classification, gender, intentions for attending graduate school, current mathematics course, and current mathematics instructor. The second section, Participation in Student Math Seminar asked two questions and two statements. The first question addressed the reason for attending the seminar. The choices for reason for attending were for extra credit, required for class, enjoy attending or other with explanation. The participants were also asked about how often they talk about SMS during the week. This question was asked to see if participation in the seminar affected the conversations they had with fellow students. The last two statements addressed if SMS benefited their understandings of mathematics and increased their appreciation of mathematics.

Table 3. Summary of audience perception survey. Data collected in each section and number of items included in each section.

Section Name	**Data Collected**	**Number of Items**
Participant Information	Name, Major, Classification, Gender, Plans to Attend Graduate School	7
Participation in SMS	Reason for Attending, How Often Talks about SMS, Appreciation of SMS, Benefit of SMS	4
Audience Identity	Perceptions, Forms of Engagement, Position and Self-Efficacy	48
Open Ended Questions	Free response on definition of mathematics and mathematics research and addition comments on SMS	4

The next section of the survey was Audience Perception of Mathematics. This section consisted of 48 statements related to mathematical identity were to given responses based on a Likert scale. Of these 48 statements, 15 addressed position, 14 addressed self-efficacy, 14 beliefs about mathematics and 5 forms of engagement. A statement from the survey that would address position would be “I believe my teachers respect my mathematical abilities.” A statement that addresses beliefs about mathematics was “Everything in mathematics has been discovered.” A statement about forms of engagement is “I use diagrams and graphs when solving math problems.” And lastly a statement on self-efficacy would be, “I am sure I can learn math.”

Math Autobiography

Autobiographical storytelling, in general, has been widely used in social sciences (Miller, 2000) and has become a more common methodology in identity research (Ellsworth and Buss, 2000). Mathematics autobiographies have the potential to provide insight to one's mathematical identities (Hobden and Mitchell, 2011; McCulloh et al., 2013). Sfard and Prusak (2005) equate identity with stories that individuals hear and tell about themselves. Mathematics identities include stories related to how one interacts with mathematics both in and out of school. These stories include one's successes and failures, future goals, and beliefs of mathematics ability. Furthermore, research on storytelling in mathematics education can reveal more about one's identity than traditional survey instruments (Ellsworth and Buss, 2000). Hence, the math autobiography was a vital tool to see how the process of presenting in SMS affected the presenters' identity.

To give the presenters the opportunity to share their math story in their own words the participants completed a math autobiography. The autobiography provided insight to their relationship with mathematics prior to the study. The presenters were also asked to re-write or edit their math autobiography at the completion of the study in order to do a comparison of their relationships to mathematics prior to and following the study.

The math autobiography was adapted from versions that were found online (Discovering the Art of Mathematics, 2015) and can be seen in Appendix J. The math autobiography prompt was broken down into five sections as seen in Table 4. A summary table that gives examples questions that address the different components of mathematical identity can be found in Table 5.

Table 4. Summary of data collected in the math autobiography. The type of information collected in each section of the math autobiography instrument.

Section Name	Information Collected
Introduction	Background information such as family, education, hobbies, and interests
Experience in Math	Highest level of mathematics, recollection of past math experiences, relationship with teachers, use of mathematics outside the classroom
Learning a Styles and Habits	Learning style, note taking preferences, level of class participation, and feelings towards group work
The Future	Life goals, career goals, educational goals
After the Study	Reflection on experience in SMS

Table 5. Aspects addressed in the math autobiography. The aspects of the theoretical framework that were addressed within the math autobiography.

Aspect	Question	Section
Position	What do you believe are your responsibilities as a student in a math course?	Learning Styles and Habits
Self-Efficacy	How would you describe your math abilities?	Experiences with Mathematics
Forms of Engagement	Do you take notes? Are they helpful? Are you organized? Do you procrastinate? Do you read the text?	Learn Styles and Habits
Perceptions of Mathematics	What math courses did you like/dislike? Why did you like/dislike them?	Experiences with Mathematics

The first section was "Introduction." This section asked the student to give background information about their education, describe themselves as a person or as a student, and discuss general interests and favorite subjects in school.

The second section, "Experience with Math," focused on mathematics courses the participants had taken, reflections on their own self-efficacy, and descriptions their relationships with past mathematics instructors. Also, in this section, they were asked to reflect on how they used mathematics in everyday life. The motivation for this question was to see how the students valued mathematics and what actions they constituted as doing mathematics. This question addressed the mathematical identity factors forms of engagement and beliefs about mathematics.

The third section, “Learning Styles and Habits,” asked about study habits, learning preferences, and methods used to study mathematics. This was to see how they engage in mathematics in and out of the classroom. Also, in this section the presenter was asked to reflect on their role as a student of mathematics and their responsibilities. Also, they are asked about what expectations they have of their mathematics instructors. This is to gain further insight into how the student positions themselves in mathematics within the classroom setting.

The fourth section of the math autobiography was called “The Future,” which informed on career and academic goals and the role of mathematics in accomplishing these goals. The purpose of this section was to see the presenter’s motivations and if they considered mathematics had value to their education and their future careers.

Autobiographies can serve as a reflection tool by providing a lens through which one can understand themselves personally and professionally (Connelly & Clandinin, 1999; McCulloh et al, 2013). Although autobiographies most often encompass the events of a person’s lifetime (Miller, 2000), they can also focus on small segments of peoples’ lives or experiences in particular arenas. The autobiographies (pre and post) were studied using thematic analysis approach. The autobiographies were coded accordingly to how the student reflects their mathematical identity according to their position, self-efficacy, perceptions about mathematics, and forms of mathematical engagement. The last section, “After the Study” was only to be considered and completed during the last stage of the study. The purpose of this section was for the student presenter to reflect on their overall experience of participating in SMS since this would now be a part of their "math story."

Once the autobiographies were coded, the frequencies of each of the subcategories were recorded into a spreadsheet known as the MIP Summary Table. The frequencies of the pre- and post-autobiographies were then compared focusing primarily on the differences in the factors of identity: position, self-efficacy, perceptions of mathematics, and forms of engagement.

Presenter Interview Protocol

The presenters participated a series of interviews prior to and following each of their presentations using the Presenter Interview Protocol as found in Appendix E. There was a total of four interviews. The interview schedule can be found in Table 6.

The questions the presenters were asked in the first interview addressed background information from their first math autobiography for the purposes of clarifying educational timelines and other lifetime occurrences. They were also asked their choice of topic and how they would prepare for their presentation. In addition, participants were asked how they planned to obtain resources and if they had an outline of their presentation. Also, the presenters were asked about what role they play in the mathematical community and if they could see themselves advancing that role.

Table 6. Interview schedule. The schedule of each interview and what information will be collected in each interview.

Interview Schedule		
Interview Number	Timeline	Purpose
Interview 1	• Follows Pre-Math Autobiography and Pre-Survey • Precedes Presentation 1	• Review Background Information from Autobiography • Gauge Presenter's Initial Mathematical Identity • Gain information on preparation for first presentation
Interview 2	• Follows Presentation 1 • Precedes Presentation 2	• Presenter reflects on Presentation 1 • Gain information on how presenter will adapt for Presentation 2
Interview 3	• Follows Presentation 2 • Precedes Presentation 3	• Presenter reflects on Presentation 1 • Gain information on how presenter will adapt for Presentation 2
Interview 4	• Follows Presentation 3, Post-Math Autobiography, and Post-Survey	• Review Background Information from Autobiographies and Surveys • Gauge Presenter's Mathematical Identity • Presenter reflects on all three presentations

The second and third interviews differed from the first. In both of the second and third interviews, the presenter was requested to reflect on their past performances and how well they engaged with the audience. They were asked if they thought the material was presented clearly and to a level for their audience. Based on these reflections, the presenter is asked how they may change their presentation for improvement.

In the final interview, the presenters were asked to reflect on their last presentation and their overall experience in SMS. They were asked how they compared their performances and how they felt about possibly presenting again in the future. They were also asked again to consider their position in the mathematics community and if by taking on this role as being an expert to their peers affected how they view themselves and their mathematical abilities.

These interviews were then coded the same way as the autobiographies using the Mathematical Identity Coding Protocol, which is described in the later section. First the overall themes from the theoretical framework were identified and then the sub-category codes were assigned. The frequencies of the codes are then recorded in a spreadsheet and used to see if there was a reflection of change in their mathematical identity.

Presentations Observation Protocol and Presentation Observation Rubric

The presentations were observed using the Presentation Observation Protocol, found in Appendix H (POP) with the help of the Presentation Observation Rubric found in Appendix I. The first section of the POP is recording presenter information. The second section was observation notes for the researcher. The final section was a rating scale of different components of the presentation, their abilities as a presenter and audience engagement. These ranking were used to provide a numerical measurement to their presentations. The components of the presentation addressed the forms of engagement the presenter chose to use within the presentation. Their abilities as a presenter addressed components of position and self-efficacy. For example, "The presenter appeared to be relaxed and self-confident" applies to how the presenter is taking on the role of "expert" of that particular topic.

The three presentations given by each participant were observed using the Presentation Observation Protocol (POP). The protocol has a section for field notes to record any comments about the presenter's appearance, mannerisms, and specific statements made by the presenter. Also included in the protocol is a questionnaire for the researcher to reflect on the presenter's performance of imitating a "mathematician."

These statements from the POP were ranked using the Presentation Observation Rubric. These rankings were then recorded in a spreadsheet and then averaged to see an overall comparison of the rankings across the three presentations. If there were any change in their rankings, it was assessed to determine if these changes were reflective of changes in their mathematical identity.

The videos of the presentations were transcribed and coded using the Math Identity Coding Protocol as found in Appendix F. First it was determined what themes appeared within the presentation. Once the themes were assigned, then the sub-category codes were placed on the units of measure.

Stages of Data Collection

The four students who volunteered each picked a topic in mathematics that interested him or her and presented a series of three talks on that topic. There were 11 sessions of SMS in the fall of 2016. These sessions served as opportunities for the participants to present. There were four stages in the data collection process as summarized in Table 7. Each is described below. Prior to the start of the first stage, each subject me with me to schedule each interview and presentation.

Table 7. Stages of data collection. The stages of the study in terms of the stages of math identity and what instruments are used at each stage of the study.

Stage	Presenter Task
Stage1: Initial Math Identity	Pre-Survey, Pre-Math Autobiography, Interview 1
Stage 2: Intermediate Math Identity	Presentation 1, Interview 2
Stage 3: Intermediate Math Identity	Presentation 2, Interview 3
Stage 4: Evolved Math Identity	Presentation 3, Post-Survey, Post-Math Autobiography, Interview 4

Stage 1: Initial Mathematical Identity

To gain background knowledge, prior to the first presentation, the presenter completed a math autobiography and the Audience Perception Survey used for the audience participants. I then interviewed the student to inquire why he or she was interested in that particular topic and to determine his or her accumulated knowledge on the topic. The interview was also video recorded to be transcribed later for coding.

Stage 2: Intermediate Mathematical Identity

The students presented their primary findings and understandings in their first presentation to the SMS. The presentations were videotaped, and transcripts were produced. The Presenter Observation Protocol (POP), described in the next sections, was used to assess the development of the participant's presentation and their presentation skills.

Following the first presentation, in a second interview, which has also video-recorded, the presenter was asked to reflect on the success of their presentation and what changes they should consider for the next presentation. The students reflected on the questions and suggestions from the seminar attendees and shared how they would further research their topics.

Stage 3: Intermediate Mathematical Identity

The student gave their second presentation on the same topic, which included new or better-developed content. These presentations were observed using the Presenter Observation Protocol (POP).

Following the second presentation, a third interview was conducted with the presenter. The presenters were asked how successful they believe the presentation went and how the presentation may be revised. The students reflected on the questions and suggestions from the seminar attendees and share how they would further research their topics.

Stage 4: Evolved Mathematical Identity

The final student presentation included new or revised content. These presentations were observed using the Presenter Observation Protocol (POP). Following the last presentation, the student was asked to add to or revise their math autobiography after their participation in SMS and take the post Audience Participation Survey.

Finally, an exit interview was conducted with the presenter. The presenter was asked how successful they believed the presentation went and how they may change their presentation for a future presentation. The students reflected on the questions and suggestions from the seminar attendees and share how they would further research their topics. Additionally, the presenters were asked about how participating in these series of presentation related to the extent of changes in their beliefs of mathematics and their own abilities.

Methods of Analysis

This section describes how each of these instruments of data collection were analyzed and used to determine the participants initial, intermediate, and evolved mathematical identity. I will discuss the analysis of the Audience Perception Survey and the previously mentioned Math Identity Protocol and how this was used in a summary document called the Subject Dossier to create an image of the evolution of the participants mathematical identity.

Analysis of Survey

A SEM was conducted for the two groups, Beginning and Advanced Student Mathematicians, within this data set. Beginning Student Mathematicians are post-secondary students who were not mathematics majors. Advanced Student Mathematicians were mathematics majors or related majors like physics or engineering. The goal of the analysis was to see if there is any significant difference of mathematical identity within the culture of mathematics between these two groups.

Additionally, reliability for this factor loading was assessed. The composite reliability equation was used to determine reliability of these factor loadings and the internal consistency for indicated factor. The reason for using composite reliability over Cronbach's alpha is due to the being a stricter bound.

The composite reliability is stated below.

$$\frac{\left(\sum L_i\right)^2}{\left(\sum L_i\right)^2 + \sum Var(E_i)}$$

where L_i = the standard factor loadings for the i^{th} factor and

Var (E_i)= the error variance associated factor.

Survey for Presenters

The presenters were given the Audience Perception Survey as found in Appendix C, which was the same survey as the audience. However, their responses were analyzed differently. The presenters were asked to fill out the survey prior to the start of their first interview and prior to the start of their final interview. Their responses were then recorded in a spreadsheet. The results from the pre- and post-surveys were then compared to see if there has been any change in their identity by the change in their responses. These results were also compared to the results from the analysis of the Audience Perception Surveys to see how the presenters may or may not have aligned with the audience.

Math Identity Protocol

A qualitative methods study of the development of mathematical identity of the presenters was conducted using structured interviews, a written math autobiography, presentation observations, and surveys as the primary analysis approach.

The qualitative date collected was coded using the Math Identity Protocol as found in Appendix F. In the development of the Math Identity Protocol, it had to be determined what was considered an extraction. An extraction is considered to be a stand-alone phrase was anything that was a complete thought without having to be a complete sentence. Some examples of a unit are "Even though this was hard for me to sometimes get…" and "…I still had fun." These two statements were from the same sentence but were considered as two different extractions since they addressed different aspects from the theoretical framework. "Even though this was hard for me to sometimes get…" was considered to be self-efficacy, where as "…I still had fun" was considered perceptions about mathematics since it was an attitude about a mathematical task.

The first round of coding determined if the extraction's coding aspect addressed background, position, self-efficacy, beliefs about mathematics, forms of engagement, or addressed nothing. Nothing statements were items like utterances or conversations outside the scope of the study. These themes were based off of the aspects of mathematical identity from the theoretical framework. The extractions were coded to the aspects according to how they aligned to the definitions of the mathematical identity components.

The second round is to determine the sub-categories of each of the coding themes using a method of thematic analysis. Thematic analysis is a method for identifying, analysing, and reporting patterns or themes within data (Braun and Clarke, 2006). The coding of the sub-categories could be found in the Table 8 below.

To check the reliability of the coding instrument, a second individual used the

Table 8. Coding aspects with sub-themes. Sub-themes that emerged within the thematic analysis.

Coding Aspects with Sub-Themes				
Background	Self-Efficacy	Position	Belief about Mathematics	Forms of Engagement
• *Background of Self: Evidence of Effect on Identity vs. No Evidence of effect on Identity* • *Background of Family: Evidence of Effect on Identity vs. No Evidence of effect on Identity* • *Background on Teachers: Evidence of Effect on Identity vs. No Evidence of effect on Identity* • *Background with Seminar* • *Background with Presentation*	• *Belief in Abilities Overall-Positive* • *Belief in Abilities Overall-Negative* • *Belief in Math Abilities-Positive* • *Belief in Math Abilities-Negative*	• *Places Self into Math* • *Places Self outside Math* • *Teachers Place Subject into Math* • *Peers Place Subject into Math* • *Role of Teacher: authority vs. facilitator* • *Role of Peers* • *Role of Self*	• *Attitude-Negative* • *Attitude-Positive* • *Value/Worth-Positive* • *Value/Worth-Negative* • *Beliefs on Definition of Math* • *Value of Seminar-Positive* • *Value of Seminar-Negative*	• *Study Habits* • *Math Stories/Setting Up* • *How subject "does" math/Applications*

protocol to code the same documents. Then the paired documents were compared to determine the percentage of accuracy of the coding. There was an overall 83.3

percentage agreement of the coding. For any codes that were not matched, the coders discussed reasons for disagreement and a consensus was reached.

Dossiers

The entire collection of data was compiled into a dossier for each of the presenters. These dossiers were a way to organize the data and be able to read the data as a whole. Any similarities or differences in the constructs of identity were noted for the presenter.   The dossiers were a structured outline of each instrument for each subject. Appendix G shows how the completed dossiers were structured.

The dossiers included a summary for each phase of the data collection process. In these summaries, there was a synopsis of what discussed or presented and notes of significant statements from the presenters. Additionally, the dossiers provided a numerical count for each aspect that occurred from coding the math autobiographies, presentations, and interviews that will be tallied in the MIP Summary Table included in the subject dossiers. These tallies were then further consolidated by merging those items into the categories, aligned (A), not aligned (N), or neither (na) as seen in Table 9. This was done to make the summarization process simpler to see the change in the subject. Another component of the dossier is the survey summary tables and the POP Summary Tables. These tables are used to compare the initial and the evolved mathematical identity data from the pre- and post-surveys and the initial, intermediate and evolved mathematical identity data from the Presenter Observation Protocol.

Table 9. Coding aspects in with alignment with mathematics culture. This table shows how the extraction's sub-categorization could be consolidated into statements that are aligned (A), not aligned (N), or neither (na).

Coding Aspects with Sub-Themes in Regard to Alignment with Mathematics Culture				
Background	Self-Efficacy	Position	Perceptions of Mathematics	Forms of Engagement
• *Background of Self: A, N, na* • *Background of Family: A, N, na* • *Background on Teachers: A, N, na* • *Background with Seminar: A, N, na* • *Background with Presentation: A, N, na*	• *Belief in Abilities Overall-Positive: A, na* • *Belief in Abilities Overall-Negative: N, na* • *Belief in Math Abilities-Positive: A, na* • *Belief in Math Abilities-Negative: N, na*	• *Places Self into Math: A, na* • *Places Self outside Math: A, na* • *Teachers Place Subject into Math: A, na* • *Teachers Place Subject outside Math: A, na* • *Peers Place Subject into Math: A, na* • *Peers Place Subject outside of Math: N, na* • *Role of Teacher: authority vs. facilitator: A, N, na* • *Role of Peers: A, N, na* • *Role of Self: A, N, na*	• *Attitude-Negative: N, na* • *Attitude-Positive: A, na* • *Value/Worth-Positive: A, na* • *Value/Worth-Negative: A, N, na* • *Beliefs on Definition of Math: A, N, na* • *Value of Seminar-Positive: A, N, na* • *Value of Seminar-Negative: A, N, na*	• *Study Habits: A, N, na* • *Math Stories/Setting Up: A, N, na* • *How subject "does" math/Applications: A, N, na*

The summaries and summary tables provided a transition into the case and cross case analysis to see the evolution of each of the presenter's mathematical identity.

Presenter Identity Evolution

Qualitative data were collected from autobiographies, interviews and presentations and surveys for each presenter. The overall analysis compared the quantitative and qualitative data collected and the results as a case study. The survey results (pre and post) and the data of the dossiers were compared for each presenter. The analysis process was as follows:

1. *Individual Case Analysis.*-The first stage of the case study is to look at each individual presenter's initial, intermediate, and evolved mathematical identities.

This data was collected from the math autobiographies, presentations, interviews and surveys and then organized in the subject's dossier. Then looking at the summaries of these dossiers, generalized how each subject's mathematical identity has evolved through the different stages of the study.

2. *Within-Case Analysis*. A cross-case analysis within each Student Mathematician group (BSM and ASM). A comparison the two subjects within the Beginning Student Mathematicians (BSM) (Kendra and Sara) and a comparison the two subjects within the Advanced Student Mathematicians (ASM) (Laurel and Oliver) was performed. The findings from the individual case studies were used to connect their similarities and differences. I did this to see how comparable Kendra was to Sara and Laurel was to Oliver
3. *Cross-Case Analysis.* A cross-case analysis between the Student Mathematician groups (BSM vs. ASM). Using the findings from the within case analysis, a comparison of the evolution of the BSMs and the ASMs mathematical identities through the stages of this study was considered.

You can see the structure for the case study analysis for understanding the evolution of the presenters' mathematical identity in Figure 4 (Creswell, 2007; Yin, 2014). Since this study relies on multiple sources of evidence, with each triangulation of the data shows the evolution of mathematical identity through these series of presentations, a case study would be most appropriate (Yin, 2014). Furthermore, case studies help answer "how" or "why" questions (Yin, 2014; Schramm, 1971). Recall the research questions address how SMS evolves a student's mathematical identity. Thus, with these justifications, a case study was appropriate.

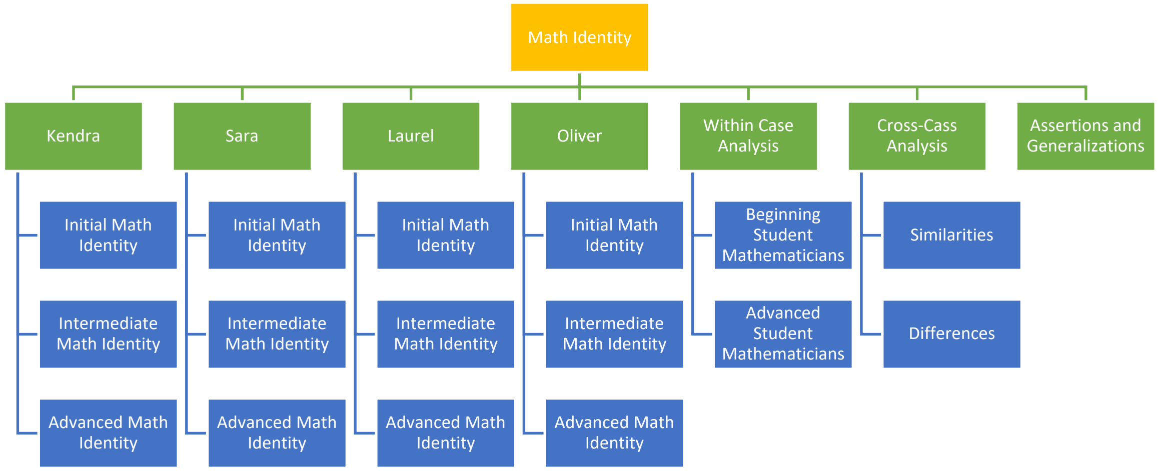

Figure 4. Structure of the case study of analysis of the evolution of the presenters.

From these comparison, generalizations and assertions were made that will be discussed in the next chapter. The next chapter will discuss the results found from the case and cross-case analyses.

IV. RESULTS

This study investigated the effects of a student mathematics seminar had on students' mathematical identity. Both quantitative and qualitative methods were used to determine how the factors of mathematical identity, position, self-efficacy, perceptions about mathematics and forms of engagement were affected by participating by either presenting and/or attending. The audience completed the Audience Perception Survey. The purpose for this survey was to gauge the audience's overall mathematical identity. For the second part of the study, four students who gave a series of presentations were studied to see how these experiences affected their identity.

The study answered the following research questions. The questions address the two different components of this study, the presenters and the audience. The questions consider the presenters as a subject on its own, then the audience and then the audience together.

Audience

1. What was the mathematical identity of the SMS audience as measured by the Audience Perception Survey?

Presenters

2. How did the student presenters' mathematical identities compare to the mathematical identity of the audience?
3. In what ways did the student presenter's mathematical identity evolve during participation in SMS?

4. In what ways did the level of mathematics acculturation differ between beginning mathematicians and advanced mathematicians?

Research Questions 1, 2, 3, and 4 were addressed by data collected from the Audience Perception Survey, Math Autobiographies, interviews, and presentations which was then compiled into each presenter's dossier and then analyzed using cross-case study methods. Research Questions 2, 3, 4 were addressed through the Audience Perception Survey and comparing the findings from the case study analysis.

Audience Perception Survey

After the incomplete and questionable responses were removed, a total of 242 audience members participated in the Audience Perception Survey. As described in Chapter 3, this survey consisted of four parts as seen in Table 3. The first section, Participant Information included demographic information: major, classification, gender, intentions for attending graduate school, current mathematics course, and current mathematics instructor. The second section, Participation in Student Math Seminar asked two questions and two statements addressed if SMS benefited their understandings of mathematics and increased their appreciation of mathematics. The first question addressed the reason for attending the seminar. The choices for reason for attending were for extra credit, required for class, enjoy attending or other with explanation. The participants were also asked about how often they talk about SMS during the week. This question was asked to see if participation in the seminar affected the conversations they had with fellow students.

To analyze the survey initially, an exploratory factor analysis, as described in Chapter 3, was conducted to see if there were any problematic items and to determine if the survey was adequately measuring components of mathematical identity within the proposed theoretical framework. We have found that the items that addressed forms of engagement were not reliably measuring the audience since the item-scale correlations were too low. Those items were removed from the model.

Next, we looked at the Confirmatory Factor Analysis creating the model with the factors defined in Table 10. The items can be found in Appendix K.

id=~ values + se.self + se.others + se.new + p.others + p.self + beliefs + bel.rev

Table 10. Structural equation model. The variables of the structural equation with their definitions and the items that were included from the survey.

Variable	Aspect of Framework	Definition	Items from Survey
id	Mathematical Identity	One's overall alignment with the mathematics culture	
values	Perceptions of Math	Items that addressed the values of mathematics	3.1, 3.2, 3.10, 3.42, 3.43
se.self	Self-Efficacy	Items that addressed one's conviction to practice mathematics on their own	3.16, 3.23, 3.24, 3.25, 3.18
se.others	Self-Efficacy	Items that addressed one's convictions to practice mathematics in front of others	3.21, 3.22, 3.17
se.new	Self-Efficacy	Items that addressed one's conviction to practice new mathematics	3.13, 3.26, 3.14, 3.15, 3.20, 3.27
p.others	Position	Items that addressed how peers and instructors place one within the mathematics culture	3.28, 3.32, 3.36, 3.34, 3.39
p.self	Position	Items that addressed how one places own self within the mathematics culture	3.29, 3.33, 3.35, 3.40, 3.41, 3.30, 3.31
beliefs	Perceptions of Math	Items that addressed beliefs about the definition of mathematics	3.6, 3.5, 3.7, 3.3, 3.4, 3.8, 3.9

From these factors, we can see the reliability of each of these factors by considering the composite reliability, RMSEAs, Tucker-Lewis Index (TLI) and the Comparative Fit Index (CFI) for each of these to show how each of these are influential in the overall model. The structural equation model constructed from the results is found in Appendix K. Overall the mathematical identity of the audience can be seen in Figure 5. This is a density graph that shows the distribution of mathematical identity score calculated from the responses of the survey. The scale generated ranged from -10 to 10. A larger negative score indicated one's mathematical identity is greatly misaligned with those in the mathematics culture. An individual with a negative score can be viewed as someone who disassociates themselves with mathematics and its practices. This individual would also have negative perceptions of mathematics and low self-efficacy. The greater a positive mathematical identity score would suggest the individual positions themselves within the mathematics culture and is highly engaged in mathematical practices. This individual would also have positive perceptions of mathematics and high self-efficacy.

The overall mean was found to be 4.33e-08, implying it was relatively neutral in terms of the alignment of the mathematics culture. That is, it is neither completely aligned nor completely misaligned. However, the median was -0.32 meaning there was a negative skewness to the distribution.

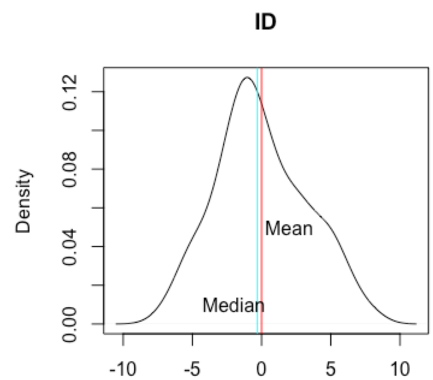

Figure 5. Graph of the mathematical identity scores of the audience. The mean was 4.33e-08 and the median was -0.32.

In the figures on the next page, we can see the distribution of the aspects of mathematical identity (position, self-efficacy, and perceptions of mathematics) that were measured by the survey. Figure 6 shows the graph of the position scores of the audience. Coincidently the scores were almost identical to the overall identity scores. Again, a negative score can be viewed as someone who disassociates themselves with mathematics and its practices and a positive mathematical identity score would suggest the individual positions themselves within the mathematics culture and is highly engaged in mathematical practices. The average position score was 4.33e-08. This meant the position of the audience was relatively neutral since it was approximately zero. However, the median was -0.32 meaning there was a negative skewness to the distribution. Figure 7 shows the graph of the self-efficacy scores of the audience. The average self-efficacy score was 2.77e-07. This meant the self-efficacy of the audience was relatively neutral since it was approximately zero. However, the median was -0.01 meaning there was a slight negative skewness to the distribution. Figure 8 shows the graph of the perceptions of mathematics scores of the audience. The average perception score was -9.30e-09. This meant the position of the audience was relatively neutral since it was approximately zero. However, the median was 0.10 meaning there was a slight positive skewness to the distribution.

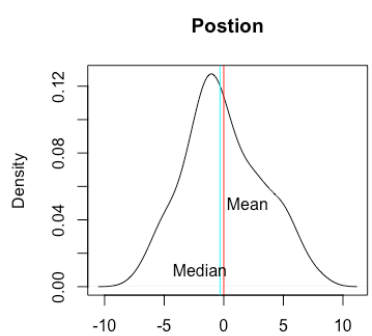

Figure 6. Position scores of the audience. The mean was 4.33e-08 and the median was -0.32.

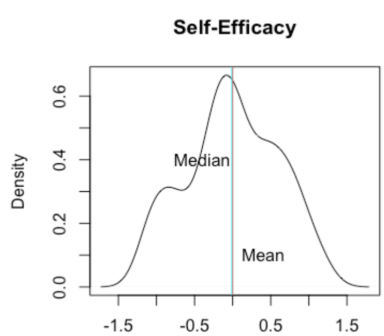

Figure 7. Self-efficacy scores of the audience. The mean was 2.77e-07 and the median was -0.01.

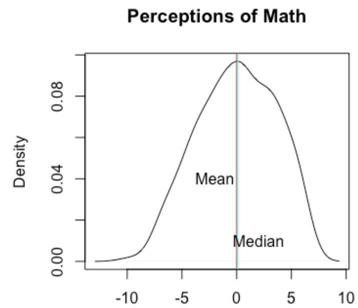

Figure 8. Graph of the perceptions of math scores of the audience. The mean was -9.30e-09 and the median was 0.10.

From these values, we can see that the average scores of position and self-efficacy were slightly positive and the average score of perceptions of mathematics was slightly negative, but they were still so close to zero they could be considered neutral.

Presenter Case Analyses

There were four student volunteers who participated in this study. Two were advanced student mathematicians, Laurel and Oliver, and two were beginning student mathematicians, Kendra and Sara. A case study was conducted to view how each of the participant's mathematical identity evolved through the progression of the study. First, they were looked at individually, then within their respective student mathematician group, and then looked at holistically by comparing the two student mathematician groups as shown in the Figure 3 from Chapter 3.

Individual Case Analyses

In this section, each student mathematician will be introduced, and their evolution of their mathematical identity will be described through each phase of the study and seen in Table 11.

Table 11. Stages of data collection and presenter tasks. This table shows which tasks were completed at each stage of the study.

Stage	Presenter Task
Stage1: Initial Math Identity	Pre-Survey, Pre-Math Autobiography, Interview 1
Stage 2: Intermediate Math Identity	Presentation 1, Interview 2
Stage 3: Intermediate Math Identity	Presentation 2, Interview 3
Stage 4: Evolved Math Identity	Presentation 3, Post-Survey, Post-Math Autobiography, Interview 4

Beginning Student Mathematicians (BSM)

Beginning mathematicians are students whose majors are not affiliated with a mathematics intensive major. There were two BSM who participated, Sara and Kendra. They were considered as such since they were both non-mathematics majors. Sara, an undeclared major, and Kendra, a double major in finance and accounting, will each be looked at individually through each phase of their development of their mathematical identity: Initial, Intermediate, and Evolved.

SARA

Sara's background was quite complex, and she had many obstacles is her past. Sara was an eighteen-year old freshman originally from Utah. She had a very bubbly and upbeat personality and tends to have a very positive and hopeful outlook on life. She spoke with optimistic tones and always has very expressive facial features and large hand gestures when she spoke. With this cheery demeanor, one would not know she had difficult obstacles to overcome in her life.

Growing up she was very fascinated with school and considered herself to do well, particularly in the mathematics and science. She had very supportive teachers and had a strong interest in learning. She had a love for mathematics because she saw how it was relatable to everyday life. She also found enjoyment in the challenges the subject gave her. She felt accomplished and this drove her to keep working. However, when her parents separated in the third grade, her school experience deteriorated when she had to move to a new town, had a hard time keeping up, and felt that everyone was moving on faster than she was.

She continued to feel left behind. But once she met her 7th grade mathematics teacher Ms. Barry, she found her love for mathematics again. Ms. Barry made mathematics exciting for her and created "an easy-learning environment" for her. Sara was happy in school once again.

But then again, at the start of her first year of high school, her mother took a job in Texas and Sara had to move again once more. Sara became frustrated with life and school was not a high priority for her. She became dependent on fellow classmates to get her work done and did not gain a connection with her new teachers. Sara became disinterested in school and particularly mathematics. She did not care to find that enjoyment from the challenges that mathematics would give her and "put it on the back burner." Her lack of interest in her new school and new living situation motivated her to graduate high school a semester early and take time to figure out her life path.

With those extra months, she spent time with her family in Utah and they encouraged her enroll at a university back in Texas. She thought this new environment may help her find a new interest and a fresh start. At the time of this study, Sara had not declared a major yet. She mentioned possibly pursing a nursing degree, or business degree, or perhaps a mathematics degree. Sara was undecided what she is truly interested in and is hoping her introductory courses could help her decide. She was currently enrolled in Basic Math, which is remedial course that provides the basics for understanding operations in different number systems, foundations of basic geometry, and simple application problems. Her enrollment in this course made her feel that she was set back much farther than she should be at this point in her academic career, however she was looking at it as an opportunity to “review and refresh” her skills and it had helped her find confidence in realizing she knew more of the material than she thought she did.

In the following sections are summaries of Sara’s initial, intermediate, and evolved mathematical identities, as well as, graphical representations of their initial and evolved mathematical identities in comparison to the audience’s mathematical identity. Table 12 below is a summary of Sara’s presentations that will be described shortly.

Table 12. Sara’s presentation schedule. Information on Sara’s three presentations with title, date, audience size and media used.

	Present 1	Present 2	Present 3
Title	Introduction to Pythagorean Theorem	Introduction to Pythagorean Theorem	The Pythagorean Theorem
Date	11/4/16	11/11/16	12/9/16
Length	6:54	6:32	7:08
Audience Size	72	87	94
Media Used	PPT	PPT	PPT

Sara’s Initial Mathematical Identity. Sara’s initial mathematical identity was determined from the results of the pre-Audience Perception Survey, pre-math autobiography, and first interview. In her pre-math autobiography, Sara focused primarily on her personal background. She elaborated on describing her personality and her family life. In her pre-autobiography she had strong confidence in her abilities as a student but expressed that her weaknesses in mathematics come from lack of motivation at key moments in her life and her negative relationships with some of her past teachers. Sara also mentioned the value of learning mathematics and the important role it plays in her everyday life very frequently.

In Sara's first interview, Sara mainly discussed her family and her mathematics experiences that were discussed in the pre-autobiography. She also seemed to have less confidence in her mathematics abilities, or that is less self-efficacy, compared to what was expressed in her pre-autobiography. She mentioned that she felt that she did not have her "basics down and that she felt that [she] was placed as someone who lacks mathematical skill since [she] was place in a remedial mathematics course. But Sara decided to look at this remedial course as a chance to get "a refresher." Sara's initial self-efficacy was considered to be low and has been positioned as an underdeveloped student. Sara shared that since was placed in this remedial course that she assumed that she would not ever be positioned as someone belonging to mathematics.

In her first interview, Sara also described her presentation and how she would prepare. Sara decided to give a presentation on then Pythagorean Theorem. In this part of the interview, Sara described her forms of engagement she would incorporate in her presentation first presentation in SMS. She said she would begin her presentation with an introduction to who Pythagoras was and where he came from. She also wanted to cover how Pythagoras had his own religion. Then she would "give an example of the theorem." Sara planned to primarily rely on using the internet to find her resources for her talk. She was aware that she could find information in books, but it was just easier to find everything online.

Sara also mentioned in her interview that she said she was familiar with SMS. She thought SMS "lets you open up to your classmates" and "to new ideas." She also viewed the seminar as an opportunity to "see how others are learning" and she can "teach others about how to learn differently." Sara also was viewing her presentation as a "stepping stone to something bigger." That this "smaller research will help [her] get to the bigger research."

As a summary of Sara's initial mathematical identity, her dossier summary and her pre-Audience Perceptions Survey were considered. In the Figures 9-12, Sara's initial mathematical identity score in relationship to the audience and her aspect scores from the pre-Audience Perceptions Survey can be seen. Again, a negative score can be viewed as someone who disassociates themselves with mathematics and its practices. This individual would also have negative perceptions of mathematics and low self-efficacy. The greater a positive mathematical identity score would suggest the individual positions themselves within the mathematics culture and is highly engaged in mathematical practices. This individual would also have positive perceptions of mathematics and high self-efficacy. Table 13 shows the percentages of the extractions of each aspect in relation to their alignment with the mathematics culture. These percentages are the percentages within respective aspect. Table 14 shows Sara's overall percentages of extractions that align to the mathematics culture.

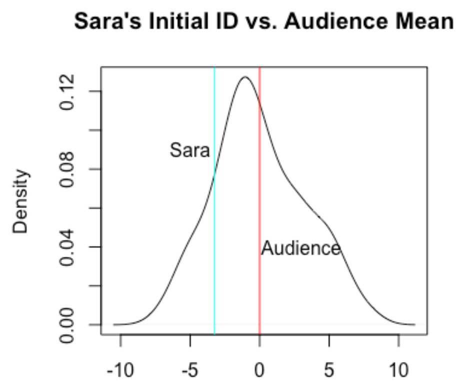

Figure 9. Sara's initial mathematical identity vs the audience. Sara's initial mathematical identity score is -3.24 compared the audiences' mathematical identity mean of 4.33e-08.

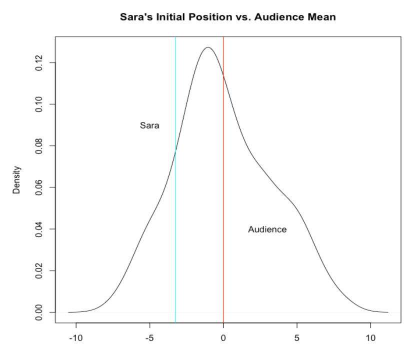

Figure 10. Sara's initial position vs the audience. Sara's initial mathematical identity score is -3.24 compared the audiences' mathematical identity mean of 4.33e-08.

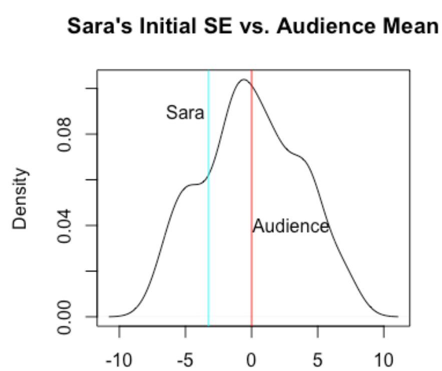

Figure 11. Sara's initial self-efficacy vs the audience. Sara's initial mathematical identity score is -3.26 compared the audiences' mathematical identity mean of 1.69e-08.

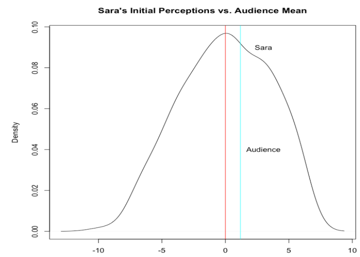

Figure 12. Sara's initial perceptions of math vs the audience. Sara's initial perceptions score is 1.19 compared the audiences' mathematical identity mean of -9.30e-09.

Table 13. Percentages of the extractions of each aspect in Sara's initial math identity. These percentages are in relation to their alignment with the mathematics culture.

	Pre-Auto-biography	1st Int	Average Percentage
Self-Efficacy			
Total Statements	24	19	
Aligned	20	8	83.3
Not Aligned	4	11	16.7
Position			
Total Statements	21	18	
Aligned	13	11	61.6
Not Aligned	8	7	38.4
Beliefs			
Total Statements	28	36	
Aligned	20	31	79.7
Not Aligned	8	5	20.3
Engagement			
Total Statements	28	0	
Aligned	12	0	42.9
Not Aligned	16	0	57.1

Table 14. Sara's initial math identity percentages. These overall percentages of extractions that align to the mathematics culture for the first stage of the study, initial mathematical identity.

	Pre-Auto	**1st Int**	**Average**
Percentage Aligned	37.4	31.0	34.2
Percentage Not Aligned	18.7	19.9	19.3
Percentage na	43.8	49.1	46.5

In Figure 9, we see that Sara's over all identity is negatively aligned with the mathematics culture and that of the audience. Her initial mathematical identity score was -3.24 and the audience's mathematical identity average score is approximately zero. From the graphs in Figures 10-12, it can be seen that Sara's initial position and self-efficacy scores are negatively aligned in comparison to the audience and that her perception scores are higher than that of the audience. In her pre-Math Autobiography and first interview approximately 34.2% of the extractions taken in those instruments aligned with the mathematics culture. This percentage was calculated by finding the ratio of the extractions of those instruments that aligned with the mathematics culture divided by every extraction in taken from those instruments.

Sara's Intermediate Mathematical Identity. Sara's intermediate mathematical identity comprised of data collected from Interviews 2 and 3 and Presentations 1 and 2. Sara's second interview addressed her first presentation. In her first presentation, Sara situated herself in the lecture chair behind the podium throughout the entire presentation and the entire presentation lasted 6 minutes and 54 seconds with 72 students in the audience. Sara stayed behind the podium basically hiding behind the podium and just read off her slides as she constantly swiveled back and forth in her chair.

Sara started her presentation, titled "Introduction to Pythagorean Theorem," with giving background on the mathematician Pythagoras. She talked about where he was born and that when he moved to Egypt, Pythagoras created his own religion. She then stated a quote about numbers but did not state in what context it was important or how it was connected to Pythagoras. It was assumed that the quote was from a Pythagorean.

After providing the history of Pythagoras and his followers, Sara introduced two different proofs of the Pythagorean Theorem. The first of the two was President Garfield's proof using the idea of equaling the area of a trapezoid to the sum of three inscribed right triangles within the trapezoid. Sara broke down how to find the area of the three triangles and the trapezoid. However, she did not explain how to set the two areas equal to each and reduce the equation to the main statement "a squared plus b squared equals c squared." She just stopped and said, "that's it."

She then moved onto her next proof, Bhaskara's proof using similar triangles. She described how the triangles were constructed but she did not clarify what she meant by the "A-A" postulate. She was simply reading steps off her slide but did not seem that she was too sure of what she was explaining.

The audience had the opportunity to ask Sara questions. One member asked where Sara found her proofs. Sara said that she found her examples online and that she had come to realize that there were hundreds of proofs out there. Then the audience member followed and asked if she has attempted to develop her own proof. Sara responded "I'm not there yet. I have a lot to learn."

The second interview required Sara to reflect on how the audience received the information and how well she felt she communicated her material. She felt that "her math", or the forms of engagement, in this presentation was just her stating facts. She said, "it wasn't like a thorough explanation. It was…just…this is it." She wished she had brought in some type of manipulative for the audience or she could have demonstrated solving a problem using the Pythagorean Theorem as opposed to just giving them the formula. Sara said for the next presentation she may want to include some handouts or attempt to show and explain a proof of the Pythagorean Theorem.

In Sara's second presentation, titled "Introduction to the Pythagorean Theorem 2", Sara again sat behind the podium in the lecture chair. But this time she refrained from swiveling back and forth in the chair. Her presentation lasted approximately 6 minutes and a half minutes with 87 audience members in attendance. This group did not include the majority of the 72 previous attendants from her first talk as many different individuals were in attendance.

Similar to the first presentation, Sara began the presentation by sharing the history of Pythagoras and the development of his religion. She also included the same quote as she did in the first presentation, but this time provided the context to how it pertained to the beliefs of the Pythagoreans.

After providing background on Pythagoras, Sara went on to describe proofs to the Pythagorean Theorem. The first proof was President's Garfield. Sara described the method by explaining that the method included using the formula for the area of the trapezoid and equating that to the sum of the area of the three right triangles within the trapezoid. However, unlike the previous presentation she did not go through the steps of solving the problem.

Next, Sara showed a video of the proof using the sum of squares off the legs of the triangle. The video demonstrated the area of the "a" square and the area of the "b" square filling in the area of the "c" square. Sara claimed that this proof "is really easy and just not all that interesting." After the video finished Sara, demonstrated a concrete example for the proof in the video. Sara drew a right triangle with lengths 3, 4, and 5. She then drew squares off the edges with areas 9, 16, and 25. Sara then pointed out to the audience that "25 is 9 plus 16."

Sara then showed her final proof. However, Sara did not explain this one. Her final proof was Bhaskara's proof with using similar triangles. Sara said instead of explaining this proof, she would want the audience to prove this one on their own.

The audience had an opportunity to ask Sara questions. One audience member asked if there were any "real world reasons for us to have to know the Pythagorean Theorem." Sara responded, "Its used in a lot of different ways. But I can't come up with a situation. But I will have to have that for next time." In her following interview we discussed how Sara could find these real-world applications.

In Sara's third interview, Sara compared her second presentation to her first. Sara felt that her second presentation was improved by her inclusion of a video that demonstrated a proof of the Pythagorean Theorem and showed a concrete example of how the proof worked. She thought this provided the audience with stronger content. On the other hand, Sara thought the second presentation was not as engaging for the audience since she felt that is was "a repeat" of the first. She also felt like she rushed through the second presentation because the presentation given before her was longer than anticipated and she felt that she had less time to give hers as a result.

Sara also mentioned that she still felt nervous in front of the audience but was more conscious of her nervous ticks like hiding behind the podium or spinning in the lecturer stool. She thought that using the board helped her from hiding behind the "wall between her and the audience" the podium provided.

She still didn't use the manipulatives in the second presentation because she was not sure how much time the audience would spend on them. She was going to create figures the audience would cut out and be able to manipulate the shapes until they saw a relationship from the theorem but was afraid they would spend more time on cutting paper than being able to listen and work through the proof.

Sara also mentioned that if she was to be more engaging with the audience like "teaching them with the cutouts" or showing a proof on her own, that she would need to be far more prepared than what she was for the previous two presentations. She felt that with these two presentations, she would not have to be so prepared for questions from the audience. But if she was to try to demonstrate different proofs or had the audience be guided to a proof, she would need to be prepared for many different types of questions.

By the end of the interview, Sara decided that if she had the cutouts prepared prior to the presentation, she could save more time within the presentation. She hoped that with the manipulatives, the audience would appreciate the proofs more than just her showing them. Also, she wanted to make her presentation more interesting by providing more information on Pythagoras.

In addition to discussing her presentations, Sara also inquired about writing her next math autobiography. She expressed some enthusiasm for writing about her "new paper for [her] new experiences." She stated that she may look at her past experiences differently and might want to change how she wrote about them. However, she did not reflect much on her past experiences in her post-autobiography.

As a summary of Sara's intermediate mathematical identity, we can consider looking at her dossier summary. We will not be able to have graphical comparisons here since there was no Audience Perceptions Survey in these phases of the study. Table 15 addresses the percentages of the extractions of each aspect in relation to their alignment with the mathematics culture. These percentages are the percentages within the respective aspect. Table 16 addresses her overall percentages of extractions that align to the mathematics culture in her intermediate mathematical identity. This table indicated that Sara's intermediate mathematical identity average extraction percentage that aligned with the mathematics culture is 39.4%. This was a slight increase from her initial mathematical identity average extraction percentage that aligned with the mathematics culture of 34.2%.

Table 15. Percentages of the extractions of each aspect in Sara's intermediate math identity. These percentages are in relation to their alignment with the mathematics culture.

	2nd Int	3rd Int	1st Pres	2nd Pres	Average Percentage
Self-Efficacy					
Total Statements	8	8	2	3	
Aligned	4	3	0	1	38.1
Not Aligned	4	5	2	2	61.9
Position					
Total Statements	10	14	3	3	
Aligned	2	10	3	3	60
Not Aligned	8	4	0	0	40
Beliefs					
Total Statements	10	6	0	4	
Aligned	6	6	0	1	65
Not Aligned	4	0	0	3	35
Engagement					
Total Statements	18	7	118	76	
Aligned	12	6	60	42	54.8
Not Aligned	6	1	58	15	38.8

Table 16. Sara's intermediate math identity percentages. These are the overall percentages of extractions that align to the mathematics culture for the second and third stages of the study, intermediate mathematical identity.

	2nd Int	3rd Int	1st Pres	2nd Pres	Average Percentage
Percentage Aligned	47.8	33.8	24.3	51.6	39.4
Percentage Not Aligned	34.3	23.8	48.5	25.3	33
Percentage na	19.0	42.5	27.2	23.1	28

Sara's Evolved Mathematical Identity. Sara' evolved mathematical is evaluated with the third presentation, fourth interview and the post-Audience Perception Survey. In Sara's third and final presentation, titled "The Pythagorean Theorem", Sara positioned herself seated on a table in front of the podium. She decided to use a clicker in order to keep herself "from hiding behind the wall the podium made." Her presentation again addressed the Pythagorean Theorem and lasted slightly over seven minutes. Sara also made the comment that the audience with 94 members seemed to be a "tough crowd."

Sara began her presentation like her previous two presentations with an overview of Pythagoras' life and the religion that he started. However, she included more detail about the religion and not so much about Pythagoras' life growing up. This time she pointed out why the Pythagoreans were vegetarians and how the mathematical figure, the tetractys was the symbol for their beliefs.

Sara then went into briefly describe Pythagoras' proof for the theorem. She claimed that "It states the sum of the area of two squares on the legs, which these are the legs. Um and then they equal the area of the hypotenuse square. And that's just plugging in the numbers. So, four squared is 16 and 3 squared is 9 and the 5 squared is 25. Well 16 plus 9 is 25."

Then Sara returned to speaking to the beliefs of the Pythagoreans and shared some statements of their beliefs. In the middle of describing their religion, she asked the audience if they could tell that she was nervous because she felt nervous.

After concluding her summary of the Pythagoreans beliefs based on mathematics, Sara went into her explanation of Garfield's proof. She asked the audience to reference the handout she provided in the seminar packet. She asked the audience to try to derive the equation of the Pythagorean Theorem. She gave the audience approximately a minute and a half to work on their own. She then explained how to solve the proof step by step with some details. She then asked the audience to try the other example she provided later at home. Sara then asked the audience if they had any questions. But before anyone had a chance, Sara said "No? Thank you" and walked off to her seat.

In Sara's last interview, Sara reflected on all three presentations and how the completion of the study affected her own mathematical identity. She mentioned that she was nervous in her last presentation and thought it was perhaps the size of the audience that was intimidating her. The audience appeared substantially larger to her than the previous two times she presented. She also felt anxious because she was not relying on reading her PowerPoint slides and was attempting to present differently by interacting with the audience with manipulatives and “trying to teach instead of just reading to them.”

She felt that she explained the material well but that was because the Pythagorean Theorem is pretty “self-explanatory.” She tried to do alternate versions of proving the theorem with the audience using her handouts but didn’t feel like they really tried. But looking back she also felt that she did not provide enough time for them to fully participate since she was rushing due to her nerves.

Sara said that if she was to present for a fourth time, she knows she would perform better. Seeing how she rushed through the manipulatives, she knows she should set aside more time for that piece of her presentation. Also, since she is now familiar with using the manipulatives herself, she is more confident that the next time would go more smoothly.

Sara also regretted that she did not try to develop her own method of proving the Pythagorean Theorem. Even if she found a method that was already discovered, she felt that if she could have shown the audience that she could prove the theorem on her own, the audience could trust her knowledge more. She felt that having to rely on previously done work, that she thought the audience could be skeptical of her. She also wishes that she could have done a comparison of proof methods.

During this interview, Sara also shared how her view of learning mathematics has changed. Her researching for this presentation series made her view multiple online sources and contemplate how all the information was related. At first, she felt it was confusing and slightly overwhelming. Then after taking the time to process the information, she realized that she herself was over complicating the material and it was indeed not all that difficult, which indicated she increased her self-efficacy. Also, that even though her sources were repeating a lot of the same information, she was able to start to see how different approaches were providing new and different information. Things were starting to "click in her head." She was surprised that she was gaining this new understanding. It was not like anything she really experienced in her current class or not even in her overall mathematical history. She started to feel confidence in her abilities to teach herself something, especially "new math."

Sara admitted that her motivation for participating in this study was solely for extra credit in her current mathematics course. But now after the completion of the study, she feels that she gained much more than just her extra credit points. She started to enjoy working through the mathematics and reading up on the history of Pythagoras and his contributions. This meant that she changed her perceptions of mathematics and felt motivated to participate in more forms of engagement. She also gained more confidence in "putting herself out there" and take more risks like speaking in front of large audiences. Since she already spoke in front of a large room of people, she definitely would be able to do it again. She also felt like she has more confidence to speak to professors or other "math geniuses." She wants to have conversations with her instructors. She knows that having these conversations would provide even more resources for her. She wants to take her current knowledge and be able to "see how to relate it to a higher level" which was something she never thought she would be passionate about.

Even though Sara repeatedly highlighted her perceived mistakes, she is still very proud of herself. She said that she was surprised about how much she was able to learn on her own and that she is eager to learn more and that she now knows she can learn more. She was also able learn from her mistakes. Before her work was just about getting it done, but her presentations and her research are something she now wants to keep pursuing since she sees how much more could be done.

In her post-autobiography, Sara reflected more on her experience of the study. In the pre-autobiography, Sara mentions how she uses mathematics only five times but in her post-autobiography, the uses of mathematics were mentioned twelve times. The preparations of the presentations made her think about how her topic of the Pythagorean Theorem was more applicable in everyday life as opposed to what she was learning in class. She also mentioned that when she did mathematics in the past, to be good at mathematics, she was the first to be done with her times tables or first to be done with the worksheet. But after completing her presentations, she realized "there was a lot more to it."

Also, she shared how she valued her experiences in the seminar very frequently. She mentioned how she "enjoyed watching other people share their knowledge" and that she had the opportunity to "expand her math experiences" with "constructive criticism." She also felt that aside from what she gained from the experience of presenting, she hoped the "other students learned something from [her] presentations and [her] understanding of the subject."

As a summary of Sara's evolved mathematical identity, we can consider looking at her dossier summary and her pre-Audience Perceptions Survey. In the Figures 13-16, we can see Sara's evolved mathematical identity score in relationship to the audience and her aspect scores from the post-Audience Perceptions Survey. Again, a negative score can be viewed as someone who disassociates themselves with mathematics and its practices. This individual would also have negative perceptions of mathematics and low self-efficacy. The greater a positive mathematical identity score would suggest the individual positions themselves within the mathematics culture and is highly engaged in mathematical practices. This individual would also have positive perceptions of mathematics and high self-efficacy. Figure 13 indicates that Sara's evolved mathematical identity score is more aligned with the mathematics culture than that of the audience. Figures 14-16 indicate that Sara's position, self-efficacy, and perceptions of mathematics are also more aligned than that of the audience. Table 17 shows the percentages of the extractions of each aspect in relation to their alignment with the mathematics culture in the evolved mathematical identity stage. These percentages are the percentages within respective aspect. Table 18 shows her overall percentages of extractions that align to the mathematics culture.

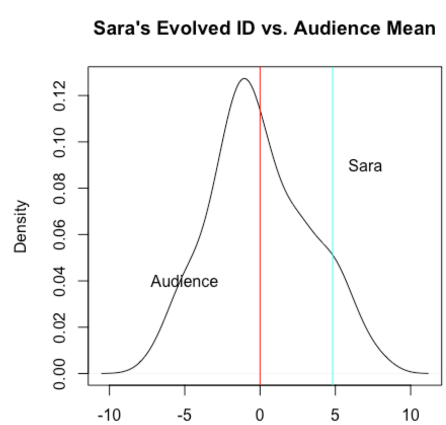

Figure 13. Sara's evolved mathematical identity vs the audience. Sara's evolved mathematical identity score is -4.82 compared the audiences' mathematical identity mean of 4.33e-08.

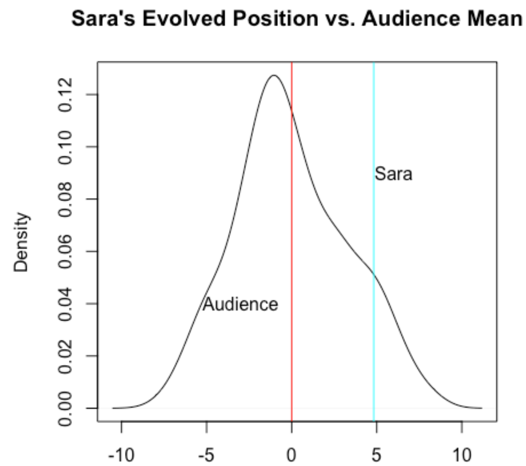

Figure 14. Sara's evolved position vs the audience. Sara's evolved position score is 4.82 compared the audiences' position mean of 4.33e-08.

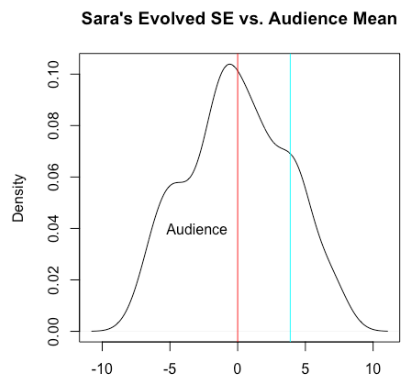

Figure 15. Sara's evolved self-efficacy vs the audience. Sara's evolved self-efficacy score is 3.88 compared the audiences' mathematical identity mean of 1.69e-08.

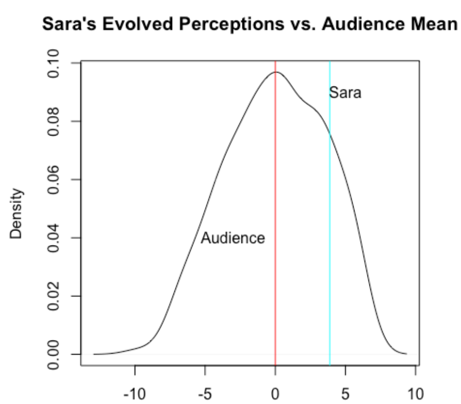

Figure 16. Sara's evolved perceptions of math vs the audience. Sara's evolved perceptions score is 5.54 compared the audiences' mathematical identity mean of -9.30e-09.

Table 17. Percentages of the extractions of each aspect in Sara's evolved math identity. These percentages are in relation to their alignment with the mathematics culture.

	Post-Auto-biography	3rd Pres	4th Int	Average Percentage
Self-Efficacy				
Total Statements	12	2	31	
Aligned	6	1	17	53.3
Not Aligned	6	1	14	46.7
Position				
Total Statements	17	9	45	
Aligned	14	4	24	59.2
Not Aligned	3	0	5	11.3
Beliefs				
Total Statements	21	0	46	
Aligned	21	0	40	91
Not Aligned	0	0	6	9
Engagement				
Total Statements	13	64	28	
Aligned	10	24	15	46.7
Not Aligned	1	5	0	5.7

Table 18. Sara's evolved math identity percentages. These overall percentages of extractions that align to the mathematics culture for the last stage of the study, evolved mathematical identity.

	Post-Auto	3rd Pres	4th Int	Average
Percentage Aligned	67.9	57	69.5	64.8
Percentage Not Aligned	16.7	4.7	16.7	12.7
Percentage na	15.5	38.4	13.8	22.6

Summary of Sara's Evolution of Mathematical Identity. From what was described in at Sara's three stages of mathematical identity we can see that Sara went through some significant changes. To simplify the analysis of each stage, the summary of each instrument was compiled in Sara's dossier. Included in this summary table, the items were then consolidated into whether those items were considered aligned (A) with the mathematics culture, not aligned with the mathematics culture (N), or not applicable (na). Sara's totals can be seen in Table 19.

Table 19. The summary of each instrument for Sara throughout the study. These overall percentages of extractions that align to the mathematics culture for the entirety of the study.

	Pre-Auto	Post-Auto	1st Int	2nd Int	3rd Int	4th Int	1st Pres	2nd Pres	3rd Pres
Percentage Aligned	37.4	67.9	31.0	47.8	33.8	69.5	24.3	52.5	57
Percentage Not Aligned	18.7	16.7	19.9	34.3	23.8	16.7	48.5	32.3	4.7
Percentage na	43.8	15.5	52.4	19.0	42.5	13.8	27.2	15.2	38.4

Table 19 indicates percentages of alignment of the extractions from the instruments were in each stage of mathematical identity. Sara's initial mathematical identity includes the pre-math autobiography, first interview and first presentation. The average of these percentages of being aligned was approximately 30.9%. Sara's intermediate mathematical identity included the second interview, third interview, and second presentation. The average of the percentages of being aligned was 44.7%. Then lastly the items included in her evolved mathematical identity were the post-math autobiography, fourth interview and third presentation. Her overall aligned percentage was 64.8%. This shows that Sara had a 33.9% increase of aligned extractions after completion of the study. This of course does not give an exact measure of her mathematical identity, but it does indicate that Sara's statements were becoming more aligned with those of the mathematical culture at each stage of her evolving mathematical identity. Additionally, we can see an evolution by comparing her pre- and post-Audience Perception Survey. Figure 17 shows the evolution of Sara's mathematical identity in comparison to the audience. Sara's Pre-Audience Perception Survey Predicted Mathematical Identity was -3.24 by using the structural equation. Her Post-Audience Perception Survey Predicted Mathematical Identity was 4.82. This a positive evolution 8.06. Figures 18-20 show the significant evolution of the aspects of Sara's position, self-efficacy, and perceptions of mathematics.

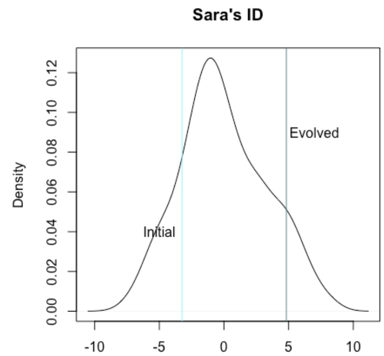

Figure 17. Sara's initial and evolved mathematical identity. At the start of the study Sara's initial mathematical identity was -3.24. At the completion of the study Sara's evolved mathematical identity was 4.82.

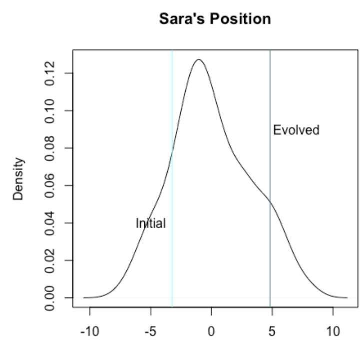

Figure 18. Sara's initial and evolved position. At the start of the study Sara's initial position was -3.24. At the completion of the study Sara's evolved position was 4.82.

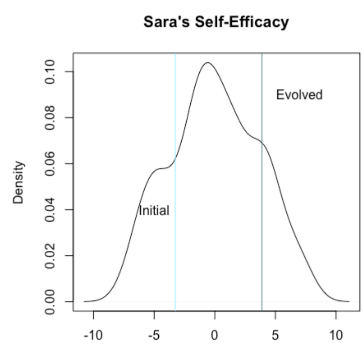

Figure 19. Sara's initial and evolved self-efficacy. At the start of the study Sara's initial self-efficacy was -3.26. At the completion of the study Sara's evolved self-efficacy was 3.88.

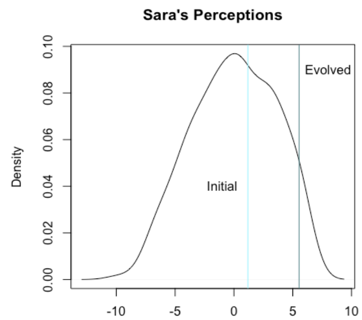

Figure 20. Sara's initial and evolved perceptions of math. At the start of the study Sara's initial perceptions of mathematics was 1.19. At the completion of the study Sara's evolved perceptions of mathematics was 5.54.

Additional evidence supporting the further evolution of Sara's mathematical identity can be found in her summary of her Presenter Observation Protocol as seen in Table 20. The scale ranged from 1, indicating novice practices to 4, indicating distinguished practices. Sara increased her averages in her abilities of presenting, style of presentation, and engaging with the audience at each stage of the study. Sara increased her overall average by 1.25 points. Table 21 shows an overall summary that highlights some of Sara's changes in the aspects of her mathematical identity. The items in her initial mathematical identity are examples of why her mathematical identity scores were so negative in her pre-Audience Perception Survey. The items in here evolved mathematical identity supports her large increase of her mathematical identity scores in her post-Audience Perception Survey.

Table 20. Sara's POP scores. This table displays Sara's average Presenter Observation Protocol scores as determined by the Presenter Observation Protocol Rubric. A 1 indicates a novice performance and the high score of a 4 indicates a distinguished performance.

Average	**Presentation 1**	**Presentation 2**	**Presentation 3**
AVG Presenter	1.33	1.80	2.75
AVG Presentation	2.00	2.14	3.29
AVG Engage	1.00	1.75	2.50
Overall	1.50	1.80	2.75

Table 21. Summary of Sara's changes. This table summarizes a few of the changes in Sara's mathematical identity through her completion of the study.

	Initial Identity	**Evolved Identity**
Position	• Considers her placement in developmental math course as indication of position	• Wants to take on a more active role in math classes • Has a foot in the door to do more advanced math
Self-Efficacy	• Only capable of learning remedial math	• Knows she can teach herself new math
Perceptions of Math	• Math is just something she "had to do"	• "Want[s] to do more" • Proud of herself for knowing more
Forms of Engagement	• Stating proofs • Reciting proof	• Demonstrated multiple proofs

KENDRA

Kendra is a nineteen-year old freshman who is double majoring in finance and accounting. Kendra is originally from a large town in Texas. She has a very quiet and reserved personality and speaks very softly.

Kendra claimed to grow up in a home that stressed the importance of mathematics. Her father was a mathematics major and her mother was an accounting major. Both parents were very active in her educational choices and she greatly depends on them when she needs assistance with her mathematics-based courses.

In high school, she attempted to take every mathematics course offered. However, the calculus course offered was a dual-credit course taught online and she quickly opted to drop the course. She is currently enrolled in a business calculus course. Her hope is to one day either become a financial advisor or a finance lawyer.

In the following sections are summaries of Kendra's initial, intermediate, and evolved mathematical identities, as well as, graphical representations of their initial and evolved mathematical identities in comparison to the audience's mathematical identity. In Table 22 is summary of Kendra's presentations that will be described shortly.

Table 22. Kendra's presentation schedule. Information on Kendra's three presentations with title, date, audience size and media used.

	Present 1	Present 2	Present 3
Title	Introduction to the Rubik's Cube	Running with the Rubik's	Running with the Rubik's 2
Date	11/4/16	12/2/16	12/9/16
Length	13:03	9:37	17:56
Audience Size	72	85	94
Media Used	PPT	PPT	PPT

Kendra's Initial Mathematical Identity. Kendra's initial mathematical identity was determined from the results of the pre-Audience Perception Survey, pre-math autobiography, and first interview. In Kendra's pre-autobiography, she mainly discussed her background information about where she grew up and what her past mathematics experiences were like. She shared that she always liked "math because it almost always has a definite solution and doesn't leave me wondering like science" and that she also preferred subjects that she "can interact with and not just be lectured over." She claimed that when she was in junior high she "began taking classes ahead of [her] grade."

She also addresses her relationship with her parents, which surprisingly involved mathematics. Both of her parents had degrees in mathematics. Her mother went on to become an accountant and her father started his own engineering firm. She also heavily relies on her father for help when she is working on her current college mathematics course. Her parents' involvement in mathematics were inspiration for her deciding to study finance and accounting since they use so much mathematics and they would be "marketable degrees." She also strongly believes that mathematics is important for her success in her degrees and future career, whether that be a corporate lawyer for financial advisor.

Kendra also reflected on her learning style and study habits. She sometimes felt embarrassed to ask questions in class, especially if the class size is large. She also preferred to see how to solve mathematics problems by being shown every step with all the detail because she wanted to "know everything about how the formula works." Kendra also liked to work in groups because she liked to "bounce ideas off of people."

Kendra also believed that "we use math everyday whether we realize it or not." She reflected on an experience where she had a very practical way of using mathematics. She gave an example of where she was in charge of ordering a large number of items for an organization she was in. Her task was to calculate multiple prices from multiple vendors to minimize overall costs for her organization. She knew that her "math skills saved the organization a lot of money."

In Kendra's first interview, she gave background information on her mathematics experiences so far. She reiterated much of what was in her autobiography. She was currently taking business and economics math 2, which primarily addressed calculus applications to business models. She also stated that she that she was placed in algebra in her eighth-grade year, which was "considered early at her school." She took geometry, algebra 2 and pre-calculus in high school. Kendra attempted to take calculus her senior year, however, it was taught online. She felt that she would not be able to perform well in a class designed like that and was afraid of maintaining her GPA, so she dropped it and decided to "take it easy" her senior year. She also restated that she is a double major of finance and accounting. She believed that these degrees would make her more "marketable" when looking for a job and that would always be needed for her career goal of either a lawyer or financial advisor.

We also discussed how her college mathematics classes differed from her previous mathematics experiences. Kendra attended a 6-A school and her class sizes ranged from 35-45 people and she felt intimidated to ask questions in that large of a setting. But her current mathematics course "had far less people" and she felt more comfortable in that smaller environment.

Kendra also believes that she was "raised to have a math brain." She grew up with parents who excelled in mathematics. Her mother was an accountant and her father started his own engineering firm. She felt that she had to excel in her related courses like mathematics and finance in order to appease her parents. This indicated that Kendra has aligned her position within mathematics, at least with the context of her family. Also, Kendra and her father often had talks about mathematics, especially since he was her first call for help in her mathematics course. She said growing up, they often talked about mathematics since "they were both curious about how things worked in real life."

Also, in Kendra's first interview, we discussed her presentation topic. She decided that she wanted to discuss solving the Rubik's cube since it was something she was always "curious about." She shared that "I've always seen people do it and I thought this would be a good opportunity to learn something I wanted to learn, and other people don't know it. It would be something that would keep people interested and I'm interested in it, so it might easier to talk about something you're interested in than something boring."

Her plan to find resources for her presentation was to mainly look online. She had found a couple of websites that had some ideas for her to use. Also, she was acquainted with someone in her dorm that was well versed in solving the Rubik's cube. She was aware that there were multiple methods to solve the Rubik's cube, but she was going to mainly focus on the "basic one because it's a sequence. And that's how the mathematics ties into it more than anything else because it's just memorizing sequences and that's how people can do it so quick, because it's just muscle memory by then so I think it's pretty neat." This showed that Kendra perceived that a part of mathematics was just memorizing sequences of steps.

Kendra also believed that this presentation experience would differ than her classroom experiences. Kendra thought that in mathematics courses she was just learning concepts that were more "related to everyday life and finance stuff." For this presentation, she thought to look at it as "starting a hobby."

As a summary of Kendra's initial mathematical identity, we can consider looking at her dossier summary and her pre-Audience Perceptions Survey. In the Figures 21-24, we can see Kendra's initial mathematical identity score in relationship to the audience and her aspect scores from the pre-Audience Perceptions Survey. In Table 23, we can see the percentages of the extractions of each aspect in relation to their alignment with the mathematics culture. These percentages are the percentages within respective aspect. In the Table 24, we can see her overall percentages of extractions that align to the mathematics culture.

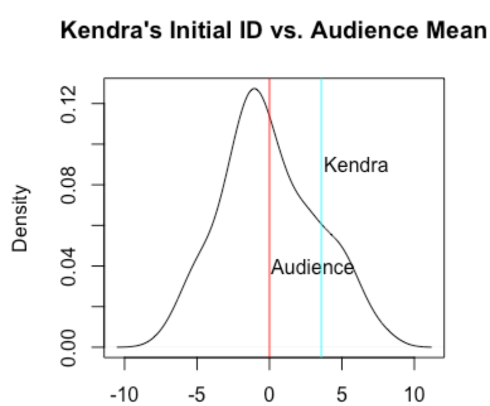

Figure 21. Kendra's initial mathematical identity vs the audience. Kendra's initial mathematical identity score is 3.52 compared the audiences' mathematical identity mean of 4.33e-08.

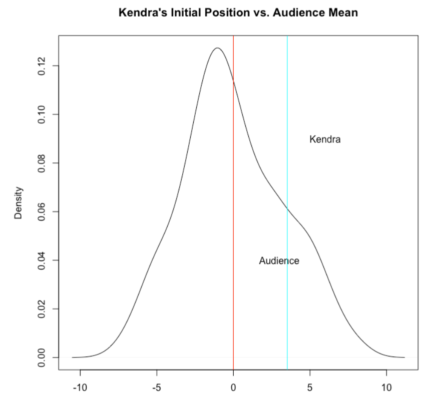

Figure 22. Kendra’s initial position vs the audience. Kendra’s initial mathematical identity score was 3.52 compared the audiences’ mathematical identity mean of 4.33e-08.

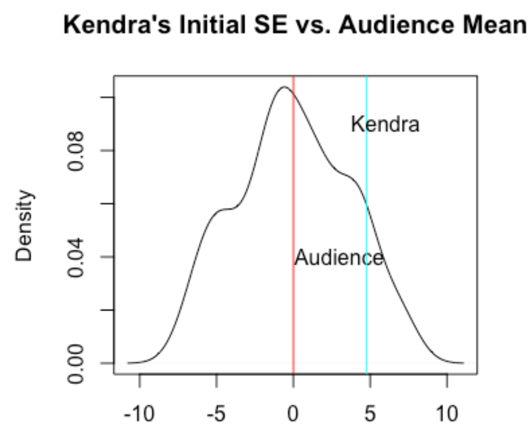

Figure 23. Kendra’s initial self-efficacy vs the audience. Kendra’s initial perceptions score was 4.75 compared the audiences’ perceptions mean of 1.69e-08.

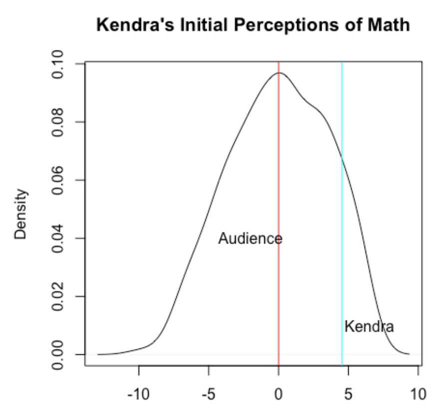

Figure 24. Kendra’s initial perceptions of math vs the audience. Kendra’s initial perceptions score is 4.54 compared the audiences’ mathematical identity mean of -9.30e-09.

Table 23. Percentages of the extractions of each aspect in Kendra's initial math identity. These percentages are in relation to their alignment with the mathematics culture.

	Pre-Auto-biography	1st Int	Average Percentage
Self-Efficacy			
Total Statements	14	5	
Aligned	8	3	57.9
Not Aligned	6	2	42.1
Position			
Total Statements	48	11	
Aligned	20	10	50.1
Not Aligned	28	1	56.9
Beliefs			
Total Statements	23	55	
Aligned	10	41	65.4
Not Aligned	4	10	17.9
Engagement			
Total Statements	30	17	
Aligned	5	3	17.0
Not Aligned	15	6	44.9

Table 24. Kendra's initial math identity percentages. These overall percentages of extractions that align to the mathematics culture for the first stage of the study, initial mathematical identity.

	Pre-Auto	1st Int	Average
Percentage Aligned	23.8	27.5	25.7
Percentage Not Aligned	13.1	15.6	14.4
Percentage na	63.1	56.6	59.9

In Figure 21, we see that Kendra's over all identity is more positively aligned with the mathematics culture than that of the audience. Again, the scale generated ranged from -10 to 10. A larger negative score indicated one's mathematical identity is greatly misaligned with those in the mathematics culture. An individual with a negative score can be viewed as someone who disassociates themselves with mathematics and its practices. This individual would also have negative perceptions of mathematics and low self-efficacy. The greater a positive mathematical identity score would suggest the individual positions themselves within the mathematics culture and is highly engaged in mathematical practices and had positive perceptions of mathematics and high self-efficacy. Kendra's initial mathematical identity score was 3.52 and the audience's mathematical identity average score is approximately zero, which meant that Kendra's overall initial mathematical identity was more aligned to the mathematics culture than the audience. From the graphs in Figures 22-24, it can be seen that Kendra's initial position and self-efficacy scores are more positively aligned in comparison to the audience and that her perception scores are higher than that of the audience. In her pre-Math Autobiography and first interview approximately 25.7% of the extractions taken in those instruments aligned with the mathematics culture. This percentage was calculated by finding the ratio of the extractions of those instruments that aligned with the mathematics culture divided by every extraction in taken from those instruments.

Kendra's Intermediate Mathematical Identity. Kendra's intermediate mathematical identity comprised of data collected from Interviews 2 and 3 and Presentations 1 and 2. Kendra's first presentation was titled "Introduction to the Rubik's Cube" and lasted 13 minutes. She used a PowerPoint presentation as her media for presenting the content. The audience was comprised of 72 students. Her presentation addressed the method to solve the Rubik's cube and her abstract claimed to bring in mathematical connections. She positioned herself behind the media cabinet throughout the presentation.

Kendra started her presentation asking the audience if anyone has solved this puzzle. Three audience members raised their hands. She then continued to say that solving the Rubik's cube "is cool. But it was a lot more complicated that I thought." She acknowledged that were "many solutions" to solving the Rubik's cube but not give any indication to what constituted as "many." Kendra then proceeded to vaguely describe the make-up of the Rubik's cube. She very quickly listed off the number of corners, edges and centers and how they could be maneuvered. Then she continued by quickly listing off each step of the procedure as shown on her slide.

Next Kendra repeated listing the steps again but used the Rubik's cube as she continued through the list, however, she used very plain and vague descriptions of the steps. For example, "So your gonna turn it around and your gonna move it so you already got one here and move this one here so if you keep going you're gonna end up with the orange side fixed." This was difficult to follow, especially since the audience was unable to see how she was manipulating the cube.

Also, the forms of mathematical engagement were not quite aligned with what would be expected of a more "advanced mathematician." The majority of the discourse that Kendra used was very simple and plain language when it came to connecting the mathematics. The only instance that she used any mathematical terminology happened in her last slide of her presentation as she described the process of solving the Rubik's cube as "an algorithm that has different ways."

In both the second and third interviews Kendra was asked to reflect on her performance in the seminar. In her second interview she began commenting with how she thought she rushed through the presentation. She "got all the information in but definitely rushed it." She felt that the audience was not able to understand everything because they did not have enough time to process the information since she "rushed through everything." She thought this was due to her being very nervous and there were far more people in the audience than she anticipated.

Kendra commented that she thought the audience also was not as engaged as she would have liked. Her first reason was that she thought the previous speaker "was very in depth so I felt like some of them checked out. Like some of them were ready to leave because hers was so detailed." However, she continued to say that she wished her presentation was like the previous one that happened in that day's SMS session. "It was really good. I need to learn how to be more in depth like her. She explained it thoroughly. And I didn't." She also said that "people were ready to go since it was Friday afternoon, so I felt like it was my fault that I didn't get them as engaged." She felt that she needed to find a way to overcome audience's desire to spend their Friday afternoons not "doing math."

Kendra also commented on that she felt like her power point had all of the steps for solving the Rubik's cube broken down well, but she did not explain it well. She then debated whether or not she wanted to include different the media for her presentation. She stated

> "I wanna try showing them I think 'cause I think the power point...just looking at a power point versus actually doing it...They'll get more out of the lesson if they see me do it I think. Like the sequences and stuff but I think it would be better if I showed them and gave them a hand out or like let someone else come up and try as I guide them through the steps. So, like they are learning."

We then discussed how she might achieve these methods. She began to consider using the document camera, but the technology would not allow for a split screen of showing the power point and the overhead at the same time. She started to consider making handouts for the audience but was unsure if she wanted to go that route.

We concluded the second interview by discussing her role as a mathematics researcher. Kendra claimed that "it definitely took a lot of time to learn how" to solve the

Rubik's cube and is "still learning" but "actually have it down now so I'm really excited." She also described her research as a "spare time hobby now. So, it's cool." However, she continued on to state that in order to be considered "a true math researcher." She believed that she "need[s] to search more than just one topic…and do it more before I …and do it religiously that just this one time."

Kendra felt like the second presentation was a far better and experience and was excited about her last presentation. "This next one has to be the best. I am very determined about it. I'm gonna be there ahead of time. Be ready to go this time. So, I'm excited!"

Kendra's second presentation was very similar to her first presentation. She was still addressing the Rubik's Cube but had more clarification on the process of solving the cube and introduced a formula to calculation the number of solutions to solving the cube. This presentation lasted 9 minutes and 37 seconds.

Kendra began her presentation by introducing the different components of the Rubik's cube with more detail than her previous presentation. She explained how each piece moved and how many of each type. She also stressed the importance of knowing the make-up of the Rubik's cube. This would be important in understanding the sequences of maneuvers for solving the Rubik's cube.

Kendra introduced the process of solving the Rubik's cube as an algorithm within the first two minutes of the talk as opposed to the very last slide. She also stressed the importance of the orientation of the cube, unlike last time she did not mention this. She then defined a variable representation for describing the operations performed on the cube. However, she did not reference these symbols much. She still used plain and vague terminology. “I have a white piece here in the middle and then you move it to the orange side and the spin it and then it’s on its white side.” This was hard to follow from sitting in the audience. She explained the stages of solving the cube for a little over three minutes but pointed out the minimum number of steps for these stages.

Next Kendra tried to explain the formula for calculating the number of solutions to solving the cube but could not give much detail. “We talked about on this slide where there's large number of possibilities. So, the way that’s broken down is based on the number of sides and it goes into 8 different ways or more like 8 times 7 times 6 times and so on... and there are 3 different rotations which means its 3 to the exponential power of 8.”

She mentioned some of the numbers that were found in the formula but not all. For those in the audience they may not have realized that 8! meant “8 times 7 times 6 and so on” they may not have been able to follow her explanation. Also, during this time Kendra seemed to have lost her place to know where she was going with explaining the formula. She made her recovery by sharing that she knows of individual who could “solve the cube in like 30 seconds” because they could combine certain moves together.

Kendra then cut to her reference slide and closed with “There is a lot of algorithms to solve the Rubik's cube so that's why it includes math so much and that's it.” Once she finished the audience responded by asking questions. Multiple hands went up indicating she had more engagement with the audience compare to her previous talk. Many questions were like how fast she could solve the cube or how fast could her friend solve it. One member asked if she had come across to any applications to other fields of mathematics. Kendra responded, "Um...not yet. But I know that the 3D components about it is supposed to be helpful for things, but I don't know what that's for yet."

Kendra's third interview was structured similarly to the second. Kendra was asked to reflect how her second presentation went. She felt that the second was better than the first "but still needs work." She felt that she did not rush as much as she did in her first presentation and did not feel as nervous, despite the audience being larger. She also felt proud about her presentation because an audience member asked her to email her presentation to him, so he could learn at home and another audience member came up to her who told her that he came to SMS that day because he was interested in her talk. She "was surprised by that." She also felt that the audience was more engaged in the second presentation by receiving more questions from them throughout the presentation and at the end.   She also thought the audience asked more questions because she did not rush through the material as much, so they were able to process the information better.

We also discussed her physical positioning to the audience in the past presentations. In the first presentation, Kendra felt like she was hiding behind the podium and in the second presentation she came out in front of the podium some and tried to make more eye contact with the audience. She also brought along a clicker with a laser pointer. She felt like this allowed her "more freedom to spell things out."

She additionally mentioned how she wished that she used an alternate method to show the steps of solving the Rubik's cube through the PowerPoint. She again wished that she showed the steps of solving the Rubik's cube as she was describing the process of solving it. She decided against using the document camera for the second presentation but considered it for the second.

Kendra also discussed the mathematics she brought into the second presentation. She reflected on her ability of explaining the formula for calculating the possible number of moves for solving a Rubik's cube. She felt that this part of the presentation was done poorly. She got very nervous with explaining the formula and felt that she "got lost" and began to feel like she was showing the wrong version because she kept losing her train of thought. She admitted that she rushed through explaining the formula because she was unsure of "where she was at" within the presentation. This suggests that Kendra may still have a lower level of self-efficacy with the mathematics involved in her presentation.

We discussed how she could better clarify the formula for the audience and maybe reconsider the order of how she presented the material or perhaps add more slides to add more detail for herself without having the previous slides being too crowded and she has more reminders of where she is at within in explaining the mathematics.

We also discussed the possibilities of bringing in real world applications for solving the Rubik's cube since it was asked by an audience member. Kendra said that she was considering looking into find some resources for that. She said she found a research article that addressed Rubik's cubes and has not reviewed but might look at it and plans to look at some more websites.

As a summary of Kendra's intermediate mathematical identity, we can consider looking at her dossier summary. We were not able to have graphical comparisons here since there was no Audience Perceptions Survey in these phases of the study. Table 25 shows the percentages of the extractions of each aspect in relation to their alignment with the mathematics culture. These percentages are the percentages within the respective aspect. Table 26 displays Kendra's overall percentages of extractions that align to the mathematics culture in her intermediate mathematical identity.

Table 25. Percentages of the extractions of each aspect in Kendra's intermediate math identity. These percentages are in relation to their alignment with the mathematics culture.

	2nd Int	3rd Int	1st Pres	2nd Pres	Average Percentage
Self-Efficacy					
Total Statements	15	28	7	24	
Aligned	3	7	5	14	39.2
Not Aligned	12	21	2	10	60.1
Position					
Total Statements	10	14	5	8	
Aligned	2	14	4	1	56.8
Not Aligned	8	0	0	0	21.6
Beliefs					
Total Statements	8	3	1	14	
Aligned	6	3	1	10	76.9
Not Aligned	2	0	0	3	19.2
Engagement					
Total Statements	2	18	76	135	
Aligned	0	15	7	8	11.5
Not Aligned	0	3	0	0	1.1

Table 26. Kendra's intermediate math identity percentages. These overall percentages of extractions that align to the mathematics culture for the second and third stages of the study, intermediate mathematical identity.

	2nd Int	3rd Int	1st Pres	2nd Pres	Average
Percentage Aligned	28.8	22.7	7.8	21.5	20.2
Percentage Not Aligned	30	17.6	8.9	8.4	16.2
Percentage na	41.2	59.7	83.5	70.1	63.6

Kendra's Evolved Mathematical Identity. Kendra's evolved mathematical identity is determined by her third presentation, post-math autobiography, the post-Audience Perception Survey, and fourth interview. Kendra's third presentation was again on solving the Rubik's cube. This presentation was approximately 18 minutes long which more than double her second presentation. Kendra started off her presentation asking how many from the audience have solved the Rubik's cube and three people have rose their hands.

Next, Kendra went into the different pieces of the cube and how each moved, the corner pieces moved to corners, edges to edges, and the center do not move at all. And as she was explaining she walked out towards the audience to rotating it for the front rows to see.

Kendra then moved onto the number of solutions to solving the cube. She showed the formula and explained the formula in far more detail than previously but still slightly rushed through. "…since there are 8 corners they can be arranged in 8 different ways, so that's 8 times 7 times six times five times four times three times two times one. That's 8 factorial." This was more detail than the second presentation since she said that the numbers in the formula are just based on the number of sides.

After explaining the formula, Kendra moved into explaining the procedure to solving the Rubik's cube as "an algorithm." Kendra introduced a variable notation for the different ways to manipulate the cube. Kendra used this terminology as she was manipulating the cube and walked towards different audience members as she did this. She explained these steps with 4 minutes.

Once she explained this process she made a connection to the idea that "algorithm" of solving the Rubik's cube is related to field of computer science. Since the field of computer science is comprised of creating and memorizing algorithms, we could relate the solving the Rubik's cube to memorizing a process. This leads us to believe that Kendra perceives performing and developing algorithms is a process that is just memorizing, which research has also revealed that many students believe that is all mathematics is comprised of.

Kendra concluded her presentation by demonstrating to solve the cube as she referenced the stages from her power point. It took her approximately 9 minutes to do so. Half way through she realized that she made a mistake and showed the audience how to recover. The audience made encouraging remarks about her to continuing through mistakes. For example, “You're doing good girl. I wouldn't have even gotten that far.” This was a comment that suggests that Kendra is accepted to be more an authority than that one audience member considered herself to be. Kendra finally solved the cube and feeling that she “took up too much time,” she rushed off to her seat leaving no place for audience questions.

Kendra’s post-autobiography was very similar to the pre-autobiography except that she added more detail about certain likes and dislikes about mathematics. Her favorite mathematics classes are the ones before they do not exist, or not real numbers came into play. She specifically liked algebra due to the fact that “it was strictly numbers, not all those abstract letters.” This does not exactly align with how many would describe the topic of algebra. Heirstein (1964) defined algebra as the study of mathematical symbols and the rules for manipulating these symbols. She also “did not like geometry as much since it was more shapes and the concepts were a lot different.”

Kendra also reflected on her overall SMS experience. She admitted her initial motivation for participating in SMS was for extra credit. She did not expect to gain much from the experience but was surprised she did. In the end, she taught herself how to solve the Rubik’s cube and more importantly knows now she “is capable of teaching herself something new and it’s math.” She discovered that “she’s capable of being able to present a subject in front of a bunch of strangers, as long as she prepares and researches it.” She described that the experience as “amazing” and “a blessing.”
She still feels intimidated to present in front of “math experts” but feels like this experience “would help her future career as a lawyer even though it’s not math but it

helps." Thus, Kendra's position is aligned with the mathematical culture in the context of her peers but not in the context of professional mathematicians or instructors.

In Kendra's fourth interview we discussed her last presentation, her post-math autobiography and her overall SMS experience. We first reflected on her final presentation. Kendra thought the presentation went "smoother...except for the part I messed up with the Rubik's cube but then I solved it. So, then it was all okay." Her biggest fear was messing up and not being able to fix it. But she then "considered it a success even though [she] messed up 'cause [she] could fix it and I got through the cube." Kendra was also confident that everyone saw they solve it based on the steps that she had taught them.

We then discussed if she would now consider herself an expert on the Rubik's cube and the mathematical concepts behind it. She felt that she would have to put a lot more effort and time on that to be an expert. She "still wants to learn the short cuts and like how to solve different forms of the Rubik's cube [be]cause there are like bigger ones and stuff." She knows that there is much more for her to learn. She would like to learn to solve it in fewer steps. She also wants to spend more time learning more mathematical applications. She thought that when she has "more free time" she would like to "research more of the math." This implied that her attitudes towards engaging in multiple forms of mathematical activities has become more aligned with those in the mathematical community.

After these presentations, Kendra considers herself "like an expert beginner but not an expert in the subject." However, she believes that the audience would consider her "more of an expert than themselves because no one knew except the lady who helps you and so that was pretty cool."   She was sure that by the end of the presentation they felt like they could probably do it because she was the expert because she "could teach them how to do it."

She also talked about if she would do another presentation in SMS, she would "definitely talk more about the math" and she also would want to talk about other applications than just the computer science ones she previously addressed. She would also like to bring in different types of Rubik's cube puzzles and maybe have someone from the audience demonstrate solving the Rubik's cube with using the steps she provided but she needs a lot more time to prepare because she "can't get there yet."

We also reflected on her self-efficacy in both mathematics and presenting have changed throughout her entire SMS experience. She adamantly believed that she has gained confidence. She said, "I knew no one. And when I typically present I at least know a couple of people in the class like I'm in a class of peers I've been with through the whole semester. And so, kind of going into blindsided like not knowing who'd be there and what week, it definitely gave me more confidence to present in front of strangers that I didn't have before." She then continued, "I came across the opportunity to give a presentation, I would say I would be a lot more confident now to be able to do it than I was at the beginning. And so, I would think it would be a great experience to do it now after I did this because now I could do it better. I think the public speaking aspect of it really helped me and the fact that I was researching stuff built my confidence to do stuff on my own that was math related."

We then reviewed her reasons for participating in SMS and if she could summarize her experience. Kendra's closing thoughts were as follows:

> "It started out as extra credit because I needed it to make sure I could get my A and tried to get every chance I could get. But like at the end it was like a community. Like I met you and I got to know you a little bit and I met your assistant and that other girl who helps you and so it became more of a safe place to go than a worrisome place cause like at the beginning I didn't know of y'all. I was very nervous to meet you. And I was like I don't know these people but by the end it was kind of a blessing."

As a summary of Kendra's evolved mathematical identity, we can consider looking at her dossier summary and her pre-Audience Perceptions Survey. In the Figures 25-28, Kendra's evolved mathematical identity score in relationship to the audience and her aspect scores from the post-Audience Perceptions Survey are shown. Again, as mentioned earlier, the scale generated ranged from -10 to 10. A larger negative score indicated one's mathematical identity is greatly misaligned with those in the mathematics culture. The greater a positive mathematical identity score would suggest the individual positions themselves within the mathematics culture and is highly engaged in mathematical practices. Kendra's evolved mathematical identity score was a 5.31, which was far more aligned with the mathematics culture than that of the audience. Table 27 displays the percentages of the extractions of each aspect in relation to their alignment with the mathematics culture. These percentages are the percentages within respective aspect. Table 28 displays Kendra's overall percentages of extractions that align to the mathematics culture.

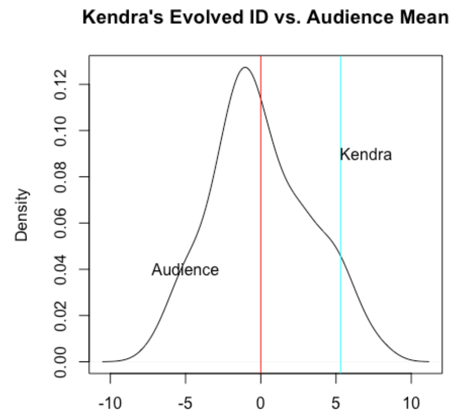

Figure 25. Kendra's evolved mathematical identity vs the audience. Kendra's evolved mathematical identity score is 5.31 compared the audiences' mathematical identity mean of 4.33e-08.

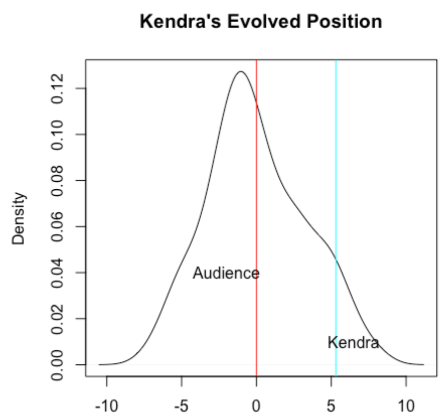

Figure 26. Kendra's evolved position vs the audience. Kendra's evolved mathematical identity score is 5.31 compared the audiences' mathematical identity mean of 4.33e-08.

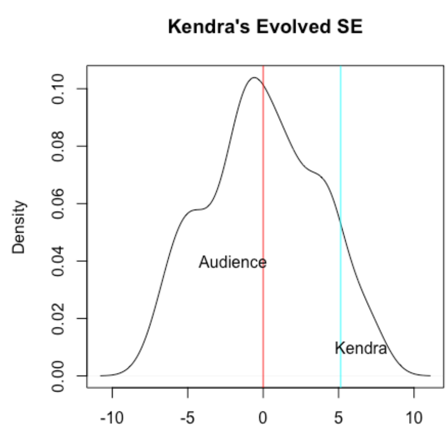

Figure 27. Kendra's evolved self-efficacy vs the audience. Kendra's initial mathematical identity score is 5.14 compared the audiences' mathematical identity mean of 1.69e-08.

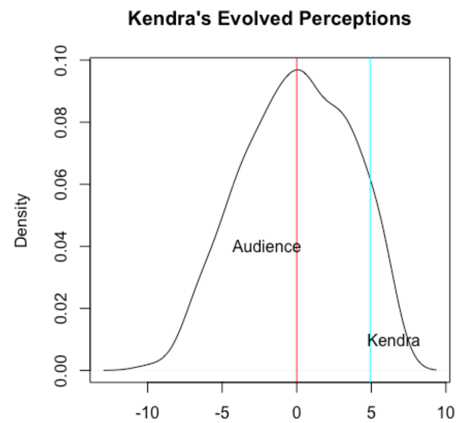

Figure 28. Kendra's evolved perceptions of math vs the audience. Kendra's evolved perceptions score was 5.54 compared the audiences' mathematical identity mean of -9.30e-09.

Table 27. Percentages of the extractions of each aspect in Kendra's evolved math identity. These percentages are in relation to their alignment with the mathematics culture.

	Post-Auto	**4th Int**	**3rd Pres**	**Average**
Percentage Aligned	67.9	69.5	57	64.7
Percentage Not Aligned	16.7	16.7	4.7	12.7
Percentage na	15.5	13.8	38.4	22.5

Table 28. Kendra's evolved math identity percentages. These overall percentages of extractions that align to the mathematics culture for the last stage of the study, evolved mathematical identity.

	Post-Auto	4th Int	3rd Pres	Average Percentage
Self-Efficacy				
Total Statements	33	26	4	
Aligned	28	22	4	81.8
Not Aligned	5	4	0	14.2
Position				
Total Statements	41	29	4	
Aligned	16	23	0	52.7
Not Aligned	9	6	0	20.3
Beliefs				
Total Statements	30	28	6	
Aligned	25	16	3	68.8
Not Aligned	5	2	0	10.9
Engagement				
Total Statements	47	9	166	
Aligned	4	5	78	39.2
Not Aligned	2	2	0	1.8

Summary of Kendra's Evolution of Mathematical Identity. From what was described in at Kendra's three stages of mathematical identity, there is evidence that Kendra went through some changes. To simplify the analysis of each stage, the summary of each instrument was completed in Kendra's dossier. Included in this summary table, the items were then consolidated into whether those items were considered aligned (A) with the mathematics culture, not aligned with the mathematics culture (N), or not applicable (na). Kendra's totals can be seen in Table 29.

Table 29. The summary of each instrument for Kendra throughout the study. These overall percentages of extractions that align to the mathematics culture for the entirety of the study.

	Pre-Auto	Post-Auto	1st Int	2nd Int	3rd Int	4th Int	1st Pres	2nd Pres	3rd Pres
Percentage Aligned	37.4	67.9	31.0	47.8	33.8	69.5	24.3	52.5	57
Percentage Not Aligned	18.7	16.7	19.9	34.3	23.8	16.7	48.5	32.3	4.7
Percentage na	43.8	15.5	52.4	19.0	42.5	13.8	27.2	15.2	38.4

Table 29 shows what percentage the extractions from the instruments were in each stage of mathematical identity. Kendra's initial mathematical identity includes the pre-math autobiography, first interview and first presentation. The average of these percentages of being aligned was approximately 19.7%. Kendra's intermediate mathematical identity included the second interview, third interview, and second presentation. The average of the percentages of being aligned was 24.3%. Then lastly the items included in her evolved mathematical identity were the post-math autobiography, fourth interview and third presentation. Her overall aligned percentage was 41.8%. This shows that Kendra had a 22.1% evolution of her mathematical identity after completion of the study. This of course does not give an exact measure of her mathematical identity, but it does indicate that Kendra's statements were becoming more aligned with those of the mathematical culture at each stage of her evolving mathematical identity. Additionally, we can see an evolution by comparing her pre- and post-Audience Perception Survey. Figure 29 shows the evolution of Kendra's mathematical identity in comparison to the audience. Recall a larger negative score indicated one's mathematical identity is greatly misaligned with those in the mathematics culture. The greater a positive mathematical identity score would suggest the individual positions themselves within the mathematics culture and is highly engaged in mathematical practices. This individual would also have positive perceptions of mathematics and high self-efficacy. Kendra's Pre-Audience Perception Survey Predicted Mathematical Identity was 3.52 by using the structural equation. Her Post-Audience Perception Survey Predicted Mathematical Identity was 5.31. This a positive evolution of 1.79. Furthermore, we can see how Kendra's identity evolved in the aspects of position, self-efficacy and perceptions of mathematics from the pre- and post-Audience Perceptions Surveys in Figures 29-32.

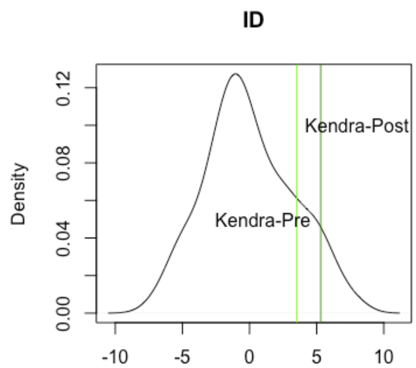

Figure 29. Kendra's initial and evolved mathematical identity. At the start of the study Kendra's initial mathematical identity was 3.52. At the completion of the study Kendra's evolved mathematical identity was 5.31.

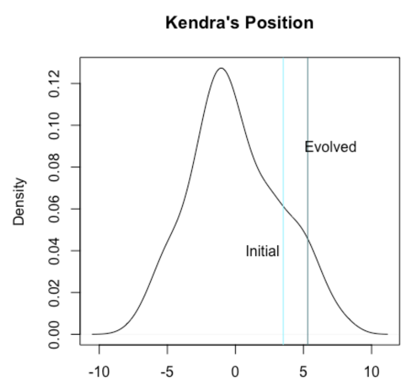

Figure 30. Kendra's initial and evolved position. Kendra's initial position score was 3.52 and evolved position score was a 5.31, which is a positive evolution of 1.79.

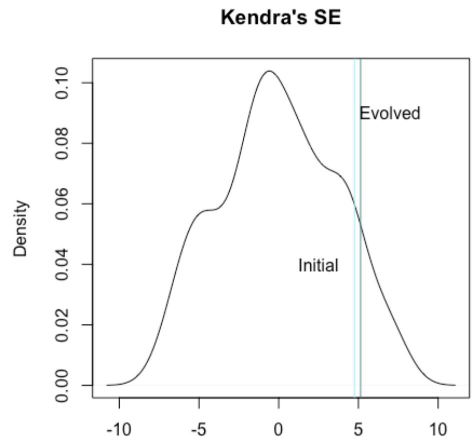

Figure 31. Kendra's initial and evolved self-efficacy. Kendra's initial self-efficacy score was 4.75 and evolved position score was a 5.14, which is a positive evolution of 0.39.

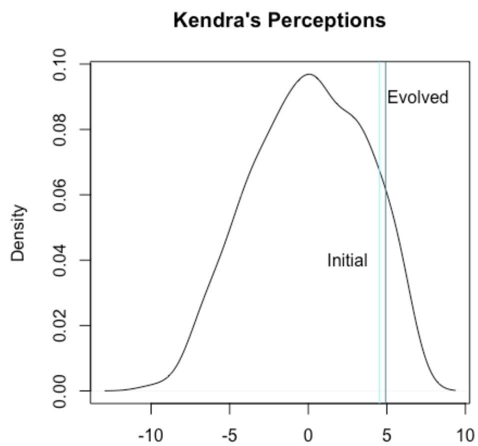

Figure 32. Kendra's initial and evolved perceptions of mathematics. Kendra's initial perception score was 4.54 and evolved position score was a 4.92, which is a positive evolution of 0.38.

We also can see a change in Kendra's summary of her Presenter Observation Protocol as seen in Table 30. Kendra increased her averages in her abilities of presenting, style of presentation, and engaging with the audience at each stage of the study. Kendra increased her overall average by 0.95 points. Table 31 shows an overall summary that highlights some of Kendra's positive changes in the aspects of her mathematical identity.

Table 30. Kendra's POP scores. This table displays Kendra's average Presenter Observation Protocol scores as determined by the Presenter Observation Protocol Rubric. A 1 indicates a novice performance and the high score of a 4 indicates a distinguished performance.

Average	Presentation 1	Presentation 2	Presentation 3
AVG Presenter	2.22	2.44	2.67
AVG Presentation	1.86	2.71	2.86
AVG Engage	1.25	2.00	3.25
Overall	1.90	2.45	2.85

Table 31. Summary of Kendra's changes. This table summarizes a few of the changes in Kendra's mathematical identity through her completion of the study.

	Initial Identity	Evolved Identity
Position	• Did not think she could belong to SMS forum	• Is an "expert" beginner
Self-Efficacy	• Sometimes embarrassed to ask questions or admit she does not understand	• Knows she can teach herself new math
Perceptions of Math	• Algebra is strictly numbers	• There is much more to math than just formulas
Forms of Engagement	• Vaguely uses terminology • States "there is a formula"	• Defines variables and explains parts of the formula

Advanced Student Mathematicians

The advanced student mathematicians are considered to be students who are mathematics majors or majors in a related field such as physics or engineering. These other majors are considered to be "advanced" since these majors include higher level mathematics courses. The advanced mathematicians for this study are Oliver and Laurel. In this section, we introduce Oliver and Laurel and provide some of their mathematical background prior to the study. Also, in this section we summarize their autobiographies, interviews and presentations.

OLIVER

Oliver is a nineteen-year old sophomore mathematics major. He grew up only thirty minutes away from the university. Oliver is currently enrolled in differential equations. Oliver says he greatly enjoys mathematics but does not care for computational courses. He prefers more of the theoretical and proof-based courses and thinks computations are “boring.”

Growing up, Oliver did not care much for mathematics or just school in general. He felt that his priorities laid mostly with his social life and when it came to schoolwork, he felt lazy. That did not change until his pre-calculus teacher made his class do a research project. The project required each student to put together a research presentation on more advanced concepts than what they were currently studying. Oliver felt inspired by this project since he was able to find his own connections to the mathematics. He excelled on this project and made the highest score in his class. This experience let Oliver find a new motivation to continue to study mathematics.

Once Oliver enrolled in university courses, he wanted to “correct his laziness” and fully engulfed himself in his studies and took on employment that supported his academics. He works in a mathematics tutoring lab and does private tutoring. Additionally, he works as an undergraduate instructional assistant teaching a calculus I lab. Oliver truly enjoys teaching mathematics and helping other students. He plans to get a Ph.D. in mathematics and would like to one day be a professor at a university.

In the following sections are summaries of Oliver’s initial, intermediate, and evolved mathematical identities, as well as, graphical representations of their initial and evolved mathematical identities in comparison to the audience’s mathematical identity. Table 32 is a summary of Oliver’s presentations that will be described shortly.

Table 32. Oliver’s presentation schedule. Information on Oliver’s three presentations with title, date, audience size and media used.

	Present 1	Present 2	Present 3
Title	Zeno's Paradox	Zeno's Paradox 2	Zeno's Paradox 3
Date	10/14/16	11/11/16	12/2/16
Length	16:50	18:07	21:38
Audience Size	63	87	85
Media Used	Elmo	Elmo	Elmo

Oliver's Initial Mathematical Identity. Oliver's initial mathematical identity was determined from the results of the pre-Audience Perception Survey, pre-math autobiography, and first interview. In Oliver's Pre-Math Autobiography, he provided background information on where he grew up and went to school. He is a first-generation college student and has always been passionate about mathematics.

When Oliver was in high school, he was placed in Pre-AP mathematics courses, but he did not identify himself as a "great student." He would rarely complete homework assignments and his "grades were not the best." Oliver did not value his assignments as a part of his learning. He "thought that learning information and taking something away from class was far more important than grades." Despite his lack of desire to do homework, Oliver was able to perform well on exams. Oliver also shared that spending time with friends was far more important than school.

His view on school began to change when he took Pre-Calculus. Despite him being "an awful student," he felt that his teacher cared about him and his potential. This teacher assigned a final project where the students were to give presentations about "advanced mathematics concepts." Oliver claimed that this was his "first exposure to advanced math" and "thoroughly enjoyed the project." Oliver excelled on the project and received the highest score. He attributed this experience and teacher as the reasons why he decided to study mathematics. His success with this project strengthened his self-efficacy and aligned him to a position in mathematics.

When Oliver enrolled in college he "came in with the goal of redeeming [him]self from [his] previous laziness." He had felt that all of his post-secondary mathematics experiences "have been wonderful." He claimed that his successes in mathematics have been due to his “committed and understanding professors.” He has enjoyed all three of his calculus courses and his introduction to advanced mathematics course. However, he did not enjoy Differential Equations because to him it seemed “monotonous and uninteresting.” Oliver did not enjoy brute calculations and preferred more of the abstract and theoretical side of mathematics. He valued learning about the concepts behind mathematics and not so much the applications.

As Oliver has progressed through his mathematics course, he began to find a “competitive nature of mathematics.” Despite having success in his mathematics courses, he was afraid that the content will become difficult and “struggle deeply.” He also had noticed that there were more encounters with others who are “so far ahead of [him] and that they have started learning and “proving their own theorems before [he] even got to college.” He feared that he would not be able to “compete with others when it comes to math and this, at times, discourages me from doing math.” Notwithstanding the occasional discouragements, he hoped to one day be a mathematics professor.

Other instruments used in measuring Oliver’s initial mathematical identity were the first interview. In Oliver’s first interview, he reviewed many of the items discussed in the pre-Math Autobiography such as where he grew up and mathematics courses he has taken. Oliver also mentioned that he possessed a positive attitude towards mathematics. He claimed that mathematics made him happy and greatly enjoyed it. He also held mathematics in high value, not in terms of its applications, but as an outlet for his anxiousness. Oliver described himself as someone who always overthinks things but with mathematics, he finds himself being permitted to do so due to the “analytic nature of mathematics.” He continued to go on about not caring about the applications of mathematics. He understood that applications are a good thing about mathematics, but that he cared more about "doing math for math's sake." He highly enjoyed independently researching mathematical ideas. He also said in order to be successful in mathematics, you have to have "resilience." You have to be willing to "stare at a problem for like a week non-stop thinking about it" until you come to a conclusion from multiple points of view. This is an idea that is highly stressed in teaching mathematics (Polya, 1954; Beckman, 2018). Thus, Oliver considered multiple forms of engagement and the role or position he took on when practicing mathematics.

Oliver continued to say that knowing mathematics is not something you are born with and there are no pre-determined time frames to understanding mathematics. It is more important to be determined and work hard than to be quick in finding a solution. This is very important if one hopes to be a mathematician. Also, Oliver stressed the importance of using the proper terminology when one participates in mathematics discourse. He said this is something he reflected much on and that it is imperative to be able to "articulate yourself" when applying terminology to "do math." Oliver also mentioned that there is also an artfulness to practicing mathematics. One needs an open imagination and creativity is important to learn new mathematics and there is an "artistic style" that goes into practicing mathematics, particularly when "writing elegant proofs." Oliver's responses here greatly aligned with the perceptions of mathematics that many mathematicians have such as in the studies of Boaler and Greeno (2000) and Boaler (2013).

When Oliver was asked if he would consider himself as a mathematician, he said that he said he is capable of becoming one, but he has "not encountered enough mathematics" yet. The word "mathematician" is a "big title." He still felt intimidated to engage in mathematics research. He was still "at the beginning level." He still was figuring out "what types of mathematics" he is "good at" or "what types of mathematics" he even liked. He felt that the mathematics he has been exposed to so far has been fairly elementary and once he started to pursue researching mathematics, the content would become exponentially more difficult and the field would become much more competitive. However, one of his ultimate goals was to become a mathematician.

After discussing Oliver's background, we discussed his first presentation. He chose to create a presentation on Zeno's Paradox because he found it "philosophically interesting" and it is not "just brute mathematics." He thought this topic would be something the audience could "wrap their head around." Oliver was considering sharing a YouTube video and showing some examples of limits and series as part of the presentation.

As a summary of Oliver's initial mathematical identity, his dossier summary and his pre-Audience Perceptions Survey were considered. Figures 33-36 compare Oliver's initial mathematical identity score in relationship to the audience and his aspect scores from the pre-Audience Perceptions Survey. Again, a larger negative score indicated one's mathematical identity is greatly misaligned with those in the mathematics culture. The greater a positive mathematical identity score would suggest the individual's mathematical identity aligns with the mathematics culture. Table 33 contains the percentages of the extractions of each aspect in relation to their alignment with the mathematics culture. These percentages are the percentages within respective aspect.

Table 34 contains Oliver's overall percentages of extractions that align to the mathematics culture.

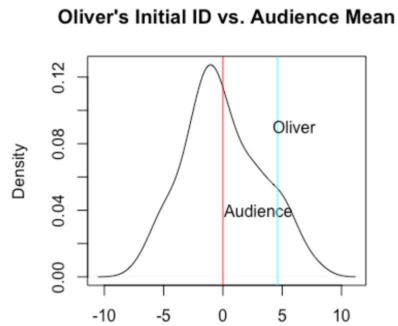

Figure 33. Oliver's initial mathematical identity vs the audience. Oliver's initial mathematical identity score is 4.61 compared the audiences' mathematical identity mean of 4.33e-08.

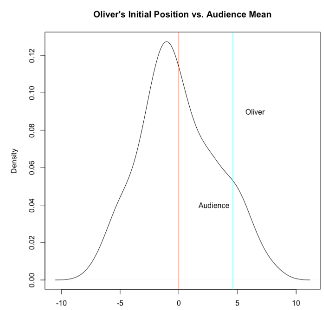

Figure 34. Oliver's initial position vs the audience. Oliver's initial mathematical identity score was 4.61 compared the audiences' mathematical identity mean of 4.33e-08.

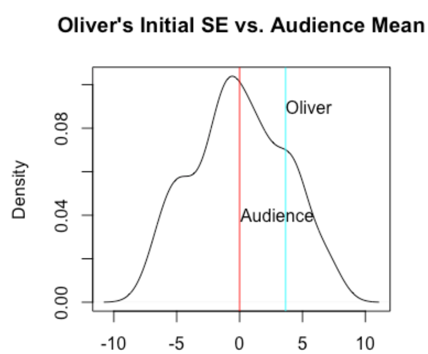

Figure 35. Oliver's initial self-efficacy vs the audience. Oliver's initial self-efficacy score was 3.66 compared the audiences' perceptions mean of 1.69e-08.

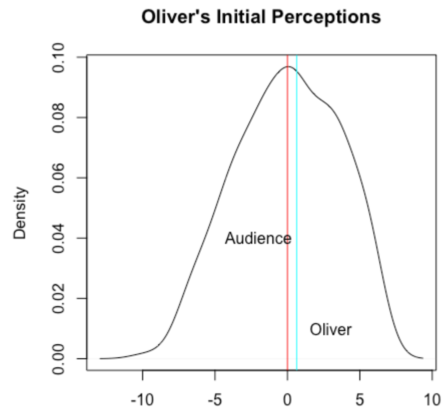

Figure 36. Oliver's initial perceptions of math vs the audience. Oliver's initial perceptions score is 0.638 compared the audiences' mathematical identity mean of -9.30e-09.

Table 33. Percentages of the extractions of each aspect in Oliver's initial math identity. These percentages are in relation to their alignment with the mathematics culture.

	Pre-Auto-biography	1st Int	Average Percentage
Self-Efficacy			
Total Statements	14	53	
Aligned	6	19	37.3
Not Aligned	8	34	62.7
Position			
Total Statements	18	65	
Aligned	8	19	32.5
Not Aligned	10	10	24.1
Beliefs			
Total Statements	28	120	
Aligned	14	75	60.1
Not Aligned	7	10	11.5
Engagement			
Total Statements	24	11	
Aligned	1	6	20
Not Aligned	0	5	14.3

Table 34. Oliver's initial math identity percentages. These overall percentages of extractions that align to the mathematics culture for the first stage of the study, initial mathematical identity.

	Pre-Auto	1st Int	Average
Percentage Aligned	28.7	47	37.9
Percentage Not Aligned	21.1	18	19.6
Percentage na	45.2	35	40.1

In Figure 33, we see that Oliver's over all identity is more positively aligned with the mathematics culture than that of the audience. His initial mathematical identity score was 4.61 and the audience's mathematical identity average score is approximately zero. From the graphs in Figures 34-36, it can be seen that Oliver's initial position, self-efficacy, and perception scores are more positively aligned in comparison to the audience. In his pre-Math Autobiography and first interview approximately 37.9% of the extractions taken in those instruments aligned with the mathematics culture. This percentage was calculated by finding the ratio of the extractions of those instruments that aligned with the mathematics culture divided by every extraction in taken from those instruments.

Oliver's Intermediate Mathematical Identity. Oliver's intermediate mathematical identity comprised of data collected from Presentations 1 and 2 and Interviews 2 and 3. Oliver's first presentation lasted nearly 17 minutes. Oliver's chosen media for his presentation, "Zeno's Paradox," was using a document camera. Oliver started the presentation by thanking the audience for coming to the seminar that day since they may have felt that they may had more enjoyable things to do that day. He also informed the audience should they have any questions they should stop him at any point. This showed that Oliver was 1) understanding that the audience may have a low value on or possibly negative attitudes on mathematics, and 2) that he was willing to take on an authoritative position by being able to or responsible for answering any audience questions.

Oliver then continued into the talk by providing historical information on Zeno and then giving an anecdote of Zeno walking to the park starting one mile away and walking in a pattern where he walks lengths that would half his distance and then stop. Then walk half the distance and then stop. He drew a stick figure man with a beard, walking along a path to a park and labeled each of those distances walked and where Zeno would pause. From his drawing he led the audience to believe that Zeno may never reach the park. That was when he introduced the concept of summations. He described what would happen when introducing summation notation and summing the powers of one-half. Using the concept of limits but without defining limits, Oliver showed how to derive the solution for summing these powers of one-half to show that sum is indeed 1. Therefore, Zeno would reach the park. Oliver described this method as "wizardry" since this required "knowing tricks" and "being clever." This comment implied that these calculations are perceived as magic and that there is not a concrete reason for finding the solution.

Oliver then introduced the formula for finding the solution to Zeno's story by using the geometric series summation formula to find the same answer he did prior. He explained which values were to be substituted into the formula and computed the answer. He then went on to say that this was a fun and exciting problem since the concepts of limits and infinity and those ideas are important to know in mathematics, which further implied he found value in these concepts of mathematics.

Oliver was also accepting of questions from the audience and responded very confidently and satisfactorily to most of the audience questions. However, one member asked if he knew any applications to Zeno's Paradox or summations. His response was "I don't know, and I really don't care about the applications. I do this stuff just for fun." This showed that Oliver may not share all the values that are shared with the mathematical community.

In Oliver's second interview, he reflected on his first presentation. He recalled how well he engaged with the audience and how the audience reciprocated. He stated that he enjoyed presenting his topic for the sole purpose "that it was fun." However, he did not expect the audience to feel the same. He did not believe the audience of SMS had the same passion for mathematics and his topic, “Zeno’s Paradox,” as he had. Or in other words, the audience and he do not share the same perceptions of mathematics. He described the audience of SMS as having a mathematical identity that was not quite aligned with his. But he had hopes that his presentation would inspire some to be more reflective of mathematics in the future.

Also, in this second interview, Oliver gave a summary of his first talk and how he would adjust for the second talk. He did mention he had a concern that audience may be less engaged and “bored next time” since they already heard this talk. However, he wanted to include more examples of calculations and is contemplating finding applications, even though he “[did] not care about applications.”

Overall, Oliver felt that his first presentation went smoothly and was well organized. His description of his forms of engagement reflected he had high self-efficacy when it came to the topic of limits and summations. He did wish he had better eye-contact with the audience since he was focusing on what he was writing as he was speaking. He said if he had more eye-contact he could gage the audience’s reception of his information.

However, in Oliver’s second presentation, Oliver did not have more eye-contact with the audience. He actually had less. His presentation had less flow and organization compared to his first. This was reflected in the interview following his presentation. In Oliver’s third interview, we discussed his second presentation. Oliver’s overall assessment of his second presentation was not held in high regards. He felt that it was “not good…actually, it sucked.” He thought it went so “terribly” because he felt “so nervous” for this talked. He was asked if it was because the size of the audience and said that was not it. He assumed that his nervousness was caused by his significant other attending his second talk. He said, “I felt like she was watching me, and I was thinking about what she might be thinking, and it got me off track in my thinking about the presentation sometimes. I might not have her come next time.”

Oliver said that for the next presentation he may need to practice more before the last presentation and that he may need to work more examples for the audience to better explain the summations and limits. He would also look more into the applications to add more to the talk.

Oliver’s second presentation lasted right around 18 minutes. Oliver spoke with a fast tempo the entire presentation and rarely looked up from the document camera. There also times where he had small pauses where he seemed to be regathering his thoughts. This implied Oliver had less self-efficacy in his abilities of presenting.

He started his second presentation by telling the audience that this talk was going to be like the first, but he was going to elaborate more on certain topics. Unlike the first presentation, Oliver did not immediately invite the audience to stop him to ask questions throughout the presentation. Oliver’s form of engagement had no change from his first presentation. He shared the anecdote of Zeno, a bearded stick figure, walking to a park in a “more mathematically interesting” way. Zeno walked to the park starting one mile away and walking in a pattern where he walks lengths that would half his distance and then stop. Then walk half the distance and then stop. With this pattern, Oliver asked the audience if Zeno would ever reach the park.

This led Oliver into deriving the solution as he did last time but with far less detail. However, Oliver did make some minor mistakes in his calculations, but he was able to catch those as he went along. After completing the problem with this method, Oliver also showed how the solution could be found using the formula for geometric series.

Oliver did not fully address the application questions he had from the previous presentation. He mentioned an example of where summations could be used but only vaguely described that application. But Oliver did take questions from the audience and only one asked what would happen if Zeno would walk by only fourthing his distance. Oliver attempted to answer the question by writing out the distances but made some mathematical errors. This led Oliver to think he wasn't sure if he could finish the calculations to find the answer, but he would like to think about it.

As a summary of Oliver's intermediate mathematical identity, we can consider looking at his dossier summary. We will not be able to have graphical comparisons here since there was no Audience Perceptions Survey in these phases of the study Table 35 shows the percentages of the extractions of each aspect in relation to their alignment with the mathematics culture. These percentages are the percentages within the respective aspect. Table 36 shows Oliver's overall percentages of extractions that align to the mathematics culture in his intermediate mathematical identity.

Table 35. Percentages of the extractions of each aspect in Oliver's intermediate math identity. These percentages are in relation to their alignment with the mathematics culture.

	2nd Int	3rd Int	1st Pres	2nd Pres	Average Percentage
Self-Efficacy					
Total Statements	29	43	1	3	
Aligned	15	3	0	1	25
Not Aligned	14	40	1	2	75
Position					
Total Statements	24	12	9	13	
Aligned	5	6	6	7	41.4
Not Aligned	9	0	0	0	15.6
Beliefs					
Total Statements	33	21	43	32	
Aligned	19	13	24	17	56.6
Not Aligned	10	2	13	4	22.5
Engagement					
Total Statements	85	40	223	258	
Aligned	60	35	158	191	73.3
Not Aligned	0	0	0	0	0

Table 36. Oliver's intermediate math identity percentages. These overall percentages of extractions that align to the mathematics culture for the second and third stages of the study, intermediate mathematical identity.

	2nd Int	3rd Int	1st Pres	2nd Pres	Average
Percentage Aligned	15.6	16.7	39.8	52	31.0
Percentage Not Aligned	21.2	31.6	4.7	1.4	14.7
Percentage na	63.2	51.7	55.5	46.6	54.3

Oliver's Evolved Mathematical Identity. Oliver's evolved mathematical identity is determined by his third presentation, post-math autobiography, and the post-Audience Perception Survey. Oliver started his last presentation, which lasted nearly 22 minutes by first writing the title of his last talk, "Zeno's Paradox" on a paper from the document camera and then moved out in front of the media cabinet and pulled a chair up next to him. He then thanked the audience for coming to his talk, despite that it was reading day and he was sure they had finals to be studying for.

He continued by recalling that he had spoken about Zeno is his previous talks and said that Zeno had many other paradoxes besides the one he would be speaking about that day. He then described the story of "Achilles and the Tortoise." He said that in this story, Achilles and the tortoise are in a race, and with Achilles being so much faster than the tortoise, the tortoise was allowed a head start. As the race started, Achilles would then run to the position the tortoise started, however, the tortoise would have then moved forward. Then Achilles would have ran to the tortoise's new position, but the tortoise had moved even further. Each time this happened, Oliver moved the chair forward, which represented the tortoise and he would move forward demonstrating the movement of Achilles. With this pattern of movement, it would be assumed that Achilles would never catch up to the tortoise, however, "we all live in the real world and we all know Achilles would beat the tortoise."

Oliver said that this was only one example of the paradoxes Zeno had developed but since this was not the subject of his talk, he did not want to focus on the race between Achilles and the Tortoise. That is when he returned to behind the media cabinet to continue using the document camera to describe Zeno's paradox. He draws a stick figure with a beard and the figure is stroking its beard. Zeno is thinking about walking to the park and draws an area labeled "Park."

Oliver then steps out in front the podium again and moves a chair to the opposite side of the room and asks the audience to let the chair represent the park and Oliver would represent Zeno and Oliver walks to the other side of the room. Now Zeno would want to walk to the park but does not want to make a direct shot for the park since that "would not be very interesting." Zeno would like to walk to the park in the following manner: walk half the distance, and then walk half the remaining distance, and then walk half of that remaining distance, and so on. This led Oliver to ask if Zeno would ever reach the park. He then wrote out mathematically how calculate the overall distance Zeno actually walked.

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$$

Oliver pointed out to the audience that denominators were all powers of two which made it convenient to write the sum of the distances in summation notation. That is when Oliver defined summation notation and provided multiple examples of summations. After this, he continued on to derive the solution for the summation of one over 2 to the *k*. Through each stage of the solving, Oliver active engaged with the audience asking them for suggestions on what to do for the next step. The audience often provided input. If those suggestions were incorrect, Oliver made corrections very respectfully. Also, Oliver used multiple instruments such as tables and graphs to aide in explanation that was occurring in the later terms. This led him to find that Zeno was then able to conclude that Zeno would then be able to reach the park.

Oliver then showed how this solution could have been found by using the formula for geometric series. He defined geometric series and related how this formula relates to the method he derived.

Oliver also included a real-life application how geometric series could related to interest rates on bank accounts. Overall, Oliver spoke in an appropriate tone and often engaged with the audience. He frequently, walked out from behind the podium and maintained connections with the audience. He was also very open for questions and was able to answer each question in detail.

In both Oliver's Post-Math Autobiography and final interview, he reflected on his experience in SMS. He claimed that the seminar "has certainly been a positive force in [his] academic career." He had felt that it has helped him with speaking in front of a large audience. He considered the environment of SMS to "that of a large research conference." He followed up that statement with, "I have since fantasized about what it would be like to give a presentation to a similarly sized group, but instead of giving a presentation on a topic that is well established mathematics, I would be talking about what mathematical research I have done. The experience of giving my 3 talks felt similar to that of mathematical research in the sense that I was having to really work out all the details of my topic and prepare to explain it to a wider audience who may not know, fully, all the aspects that it contains." This experience in SMS allowed him to take on the position of belonging to the mathematically community in a "'low stakes' environment."

Oliver also addressed how this experience had helped in his teaching as an undergraduate instructional assistant within the Department of Mathematics. It made him realize that he "cannot at all times work under the assumption that [his] students understand all the intricacies of the mathematics at hand. So, explaining higher – level mathematics to an audience who may know very little mathematics at SMS has helped me to bridge the gap between [his] knowledge and the student's level of understanding." This statement addressed how Oliver positions himself with the students in the seminar and in the courses that he taught. Oliver began to consider himself as an authority amongst his peers. He was more aware of what students might be thinking. Thus, these experiences made him consider is position and the role he plays in teaching other students.

As a summary of Oliver's evolved mathematical identity, his dossier summary and his post-Audience Perceptions Survey was considered. Figures 37-40 show Oliver's evolved mathematical identity score in relationship to the audience and his aspect scores from the post-Audience Perceptions Survey. Again, to interpret the scores, a larger negative score indicated one's mathematical identity is greatly misaligned with those in the mathematics culture. The greater a positive mathematical identity score would suggest the individual's mathematical identity aligns more with the mathematics culture. Table 37 contains the percentages of the extractions of each aspect in relation to their alignment with the mathematics culture. These percentages are the percentages within respective aspect. Table 38 contains his overall percentages of extractions that align to the mathematics culture.

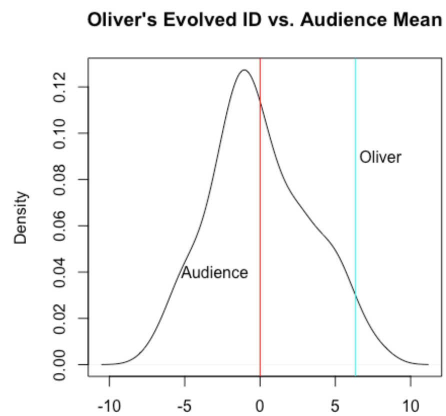

Figure 37. Oliver's evolved mathematical identity vs the audience. Oliver's evolved mathematical identity score was 6.34 compared the audiences' mathematical identity mean of 4.33e-08.

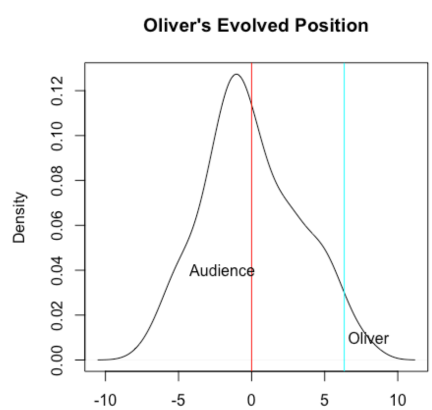

Figure 38. Oliver's evolved position vs the audience. Oliver's evolved position score was 6.34 compared the audiences' mathematical identity mean of 4.33e-08.

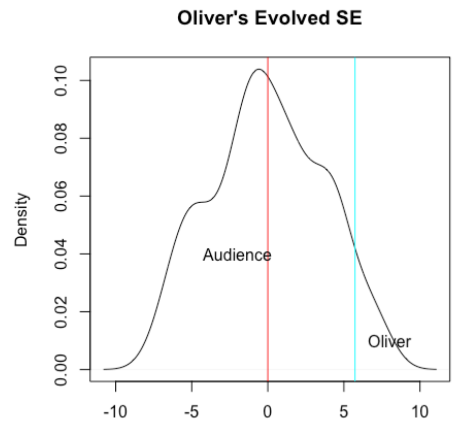

Figure 39. Oliver's evolved self-efficacy vs the audience. Oliver's evolved self-efficacy score was 3.66 compared the audiences' mathematical identity mean of 1.69e-08.

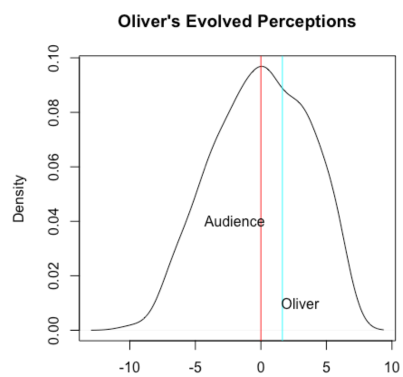

Figure 40. Oliver's evolved perceptions of math vs the audience. Oliver's evolved perceptions score was 1.64 compared the audiences' mathematical identity mean of -9.30e-09.

Table 37. Percentages of the extractions of each aspect in Oliver's evolved math identity. These percentages are in relation to their alignment with the mathematics culture.

	Post-Auto	**4th Int**	**3rd Pres**	**Average**
Percentage Aligned	47.5	49.4	42.5	46.5
Percentage Not Aligned	3.3	6.6	1.7	3.9
Percentage na	49.2	44	55.8	49.7

Table 38. Oliver's evolved math identity percentages. These overall percentages of extractions that align to the mathematics culture for the last stage of the study, evolved mathematical identity.

	Post-Auto	4th Int	3rd Pres	Average Percentage
Self-Efficacy				
Total Statements	2	21	0	
Aligned	2	13	0	65.2
Not Aligned	0	8	0	34.8
Position				
Total Statements	14	34	19	
Aligned	14	27	8	73.1
Not Aligned	0	0	0	0
Beliefs				
Total Statements	10	45	43	
Aligned	10	41	37	89.8
Not Aligned	0	4	2	6.1
Engagement				
Total Statements	6	0	350	
Aligned	3	0	206	58.7
Not Aligned	0	0	0	0

Summary of Oliver's Evolution of Mathematical Identity. From what was described in at Oliver's three stages of mathematical identity there is evidence that Oliver went through some positive changes. To simplify the analysis of each stage, the summary of each instrument was compiled in Oliver's dossier. The items were then consolidated into whether those items were considered aligned (A) with the mathematics culture, not aligned with the mathematics culture (N), or not applicable (na). Oliver's totals can be seen in Table 38.

Table 39. The summary of each instrument for Oliver throughout the study. These overall percentages of extractions that align to the mathematics culture for the entirety of the study.

	Pre-Auto	Post-Auto	1st Int	2nd Int	3rd Int	4th Int	1st Pres	2nd Pres	3rd Pres
Percentage Aligned	28.7	47.5	47	15.6	16.7	49.4	39.8	52	42.5
Percentage Not Aligned	21.1	3.3	18	21.2	31.6	6.6	4.7	1.4	1.7
Percentage na	45.2	49.2	35	63.2	51.7	44	55.5	46.6	55.8

The information in Table 39 shows percentage the extractions from the instruments were in each stage of mathematical identity. Oliver's initial mathematical identity includes the pre-math autobiography, first interview and first presentation. The average of these percentages of being aligned was approximately 38.5%. Oliver's intermediate mathematical identity included the second interview, third interview, and second presentation. The average of the percentages of being aligned, which dropped from his initial identity was 28.1%. Then lastly the items included in his evolved mathematical identity were the post-math autobiography, fourth interview and third presentation. His overall aligned percentage was 46.5%. This shows that Oliver had an 8% evolution of his mathematical identity after completion of the study. Additionally, we can see an evolution by comparing his pre- and post-Audience Perception Survey. Figures 41-44 show the evolution of Oliver's mathematical identity, position, self-efficacy, and perception scores in comparison to the audience. Oliver's Pre-Audience Perception Survey Predicted Mathematical Identity was 4.61 by using the structural equation. His Post-Audience Perception Survey Predicted Mathematical Identity score was 6.34. This was a positive evolution of 1.73.

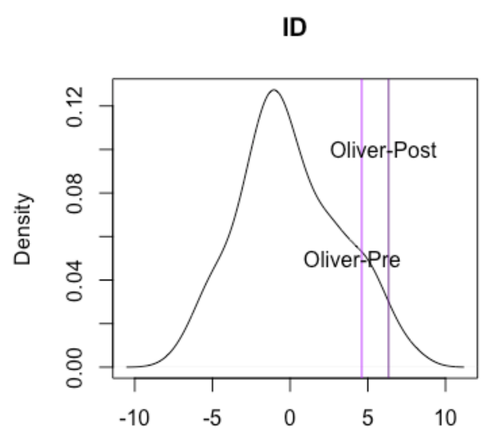

Figure 41. Oliver's initial and evolved mathematical identity. At the start of the study Oliver's initial mathematical identity was 4.61. At the completion of the study Oliver's evolved mathematical identity was 6.34.

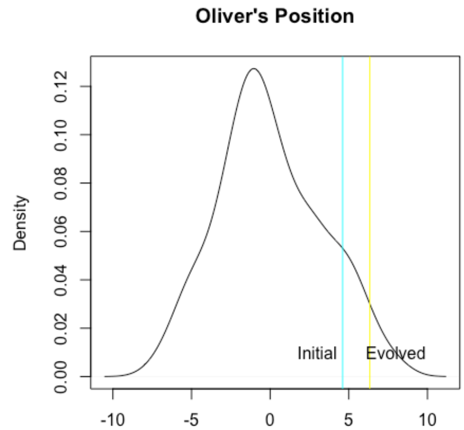

Figure 42. Oliver's initial and evolved position. Oliver's initial position score was 4.61 and evolved position Score was a 6.34, which was a positive evolution of 1.73.

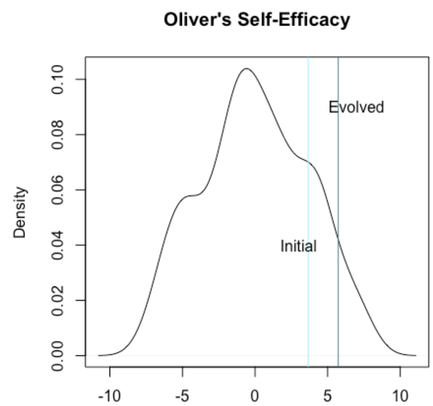

Figure 43. Oliver's initial and evolved self-efficacy. Oliver's initial self-efficacy score was 3.67 and evolved position score was a 5.73, which was a positive evolution of 2.06.

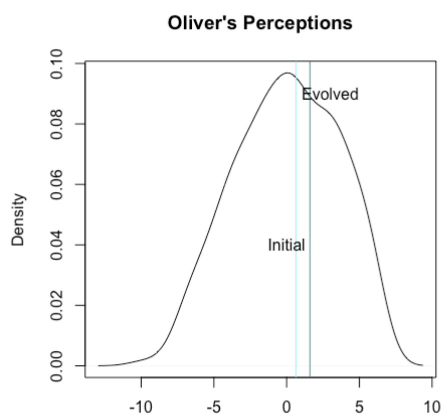

Figure 44. Oliver's initial and evolved perceptions of mathematics. Oliver's initial perception score was 0.64 and evolved perception score was a 1.64, which is a positive evolution of 1.00.

Additionally, there was positive changes in Oliver's summary of his Presenter Observation Protocol as seen in Table 40. Oliver increased her averages in her abilities of presenting, style of presentation, and engaging with the audience from the first presentation to the third, however his second presentation scored lower which was something Oliver acknowledged in his interviews. Oliver increased his overall average from the first presentation to the third by 0.90 points. Table 41 shows an overall summary that highlights some of Oliver's positive changes in the aspects of his mathematical identity.

Table 40. Oliver's POP scores. This table displays Oliver's average Presenter Observation Protocol scores as determined by the Presenter Observation Protocol Rubric. A 1 indicates a novice performance and the high score of a 4 indicates a distinguished performance.

Average	Presentation 1	Presentation 2	Presentation 3
AVG Presenter	2.78	2.33	3.22
AVG Presentation	2.14	2.43	3.71
AVG Engage	2.33	2.33	3.33
Overall	2.55	2.45	3.45

Table 41. Summary of Oliver's changes. This table summarizes a few of the changes in Oliver's mathematical identity through her completion of the study.

	Initial Identity	Evolved Identity
Position	• Considers himself a part of the math community but not in the context of math experts	• Starts to envision himself as a math expert, but still has lots to prove
Self-Efficacy	• Moderate levels within communicating mathematics and performing math tasks in a small setting	• High levels of confidence presenting to large audiences
Perceptions of Math	• Applications are not necessary to understand mathematics	• Applications can strengthen understanding
Forms of Engagement	• High levels of engaging in math	• Moved from doing to teaching

LAUREL

Laurel was a twenty-year old senior from Texas. She was a senior mathematics major planning to complete her degree within three years. She had an identity that was more strongly rooted in mathematics compared to the other participants.

Laurel was currently taking modern algebra and analysis. She had taken multiple mathematics courses including graph theory and combinatorics. She preferred mathematics courses that were rooted more in computation than the abstract courses that require proof writing. Laurel was also a student worker in the mathematics department and had experience as a department paper grader and a classroom assistant. At the time of the study, Laurel was working as an undergraduate instructional assistant. This position required her to teach a calculus II laboratory.

Laurel did not particularly enjoy teaching and preferred to work on her own studies independently. She felt that when she did group work, she ended up taking on the role of teaching everyone how to do the homework and essentially did everything herself.

Laurel planned to continue her studies in mathematics by attaining her masters. She was undecided on whether she would like to pursue a Ph.D. in mathematics but hoped to find a job working in industry as opposed to working in academia.

In the following sections are summaries of Laurel's initial, intermediate, and evolved mathematical identities, as well as, graphical representations of their initial and evolved mathematical identities in comparison to the audience's mathematical identity. Table 42 provides a summary of Laurel's presentations that will be described shortly.

Table 42. Laurel's presentation schedule. Information on Laurel's three presentations with title, date, audience size and media used.

	Present 1	Present 2	Present 3
Title	The Puzzles of Chess	The Puzzles of Chess 2	The Puzzles of Chess 3
Date	9/30/16	10/14/16	11/4/16
Length	19:36	18:45	24:56
Audience Size	84	63	72
Media Used	PPt	PPt	PPt

Laurel's Initial Mathematical Identity. In Laurel's Pre-Autobiography, she gave background information such as where she grew up, her major of Applied Mathematics and that she came to the university due to scholarships that she had received. Laurel also shared that she also worked on campus in a variety of roles within the mathematics department. She worked in the department office, as a grader and classroom assistant. At the time of the study, Laurel was working as a laboratory assistant for calculus 2.

Laurel claimed that mathematics had always been her favorite subject and "had a blast in all of them." She had always thought she had done well because the "grades in these classes have all been excellent, and [she] gotten an A in all of them." She felt that all her "experiences [her] math classes have all been amazing." She has "always had phenomenal professors who always inspired [her] to do my best" and "feels very comfortable in math classes." However, there have been times in her recent mathematics courses where she felt "slightly uncertain because the material may be difficult or confusing," but with help from her professors she still "ended up being successful."

Laurel also addressed about how she engaged in mathematics in her everyday life. Her example was trying to win the Powerball Lottery. In order to increase her family's chances of winning, she entered all of the numbers from the bought tickets into an excel sheet to determine which numbers have not been chosen and then with that information she was able to come up with ten more combinations of numbers so that they had at least one of every number in some sequence and at least one of every Powerball number. However, even by using her "magical mathematical power," she could not predict the random winning numbers.

Laurel then described her study habits. Laurel learned more by "doing and practicing problems instead of just reading and listening." And she worked the problem repeatedly until she understood everything she needed to and hesitated to ask for help. She felt this was the only way she could truly understand the material. If she "just absolutely [did] not understand" she would ask the professor for help during their office hours. Laurel also preferred to work independently than working with a group. Laurel believed that her role as a mathematics student was to study her notes and the examples covered in class, and the homework that she had completed. She also believed that high levels of participation and asking questions in class was important for her learning.

Laurel also addressed the role her mathematics professors play in her studies of mathematics. She expected instructors "to be fair and kind to all students and to teach the material to the best of their abilities while also making sure that we actually understand the information and are not just memorizing it."

Laurel concluded her pre-autobiography with remarks on her academic goals. She planned to achieve her bachelor's in less than four years and then attend graduate school to get a master's degree and "then find a job." She said "this fits into my educational goals because getting into graduate school will be an extension of my education and my life goals are dependent on my education goals. So, if I can't get through my educational goals then I cannot accomplish any of my life goals."

In Laurel's first interview, we began with discussing her mathematical background. I asked her how her modern algebra and analysis courses were going. Laurel said that she thought they were going "surprisingly well." She shared that before her first "real proof course", Introduction to Advanced Mathematics, she hated proofs. But she was “now for some reason, really enjoying it.” She then reflected on her past courses and claimed that she “had a blast in all of them.”

I then asked her about how she felt about the computational type courses like statistics and combinatorics compared to the more theoretical courses like modern algebra and analysis. Laurel claimed that she found that “the theoretical courses were always a little bit harder.” This led her into recalling an account from her high school physics course. She “was always good at the mechanical side of physics versus the electrical,” since the electrical portion addressed more theoretical concepts instead of mainly computations which were found in mechanical physics. Laurel had low self-efficacy when it came to abstract concepts. She “could see the computational stuff but the theorems were a little bit tougher because they're very abstract and it’s hard for [her] to think abstract[ly].” Her affinity to computations led to her chose applied mathematics major.

We then discussed her presentation topic and how she was preparing for it. Laurel chose to discuss the “The Puzzles of Chess.” She researched this topic previously for an honors graph theory course. She always liked puzzles and that she was fascinated by the “mathematics behind these puzzles.” Her plan was to take her past presentation and elaborate on topics by researching on the internet.

I then asked Laurel how her learning experiences from researching for the presentation differed from her learning experiences in the classroom. Laurel said that when she was given assignments from courses, there was “a structure” and “guidelines.” Unlike these assignments, she “didn’t have to worry about constraints” when presenting in SMS. I then asked her if the research she was doing for this presentation could be considered as “math research.” Laurel said prior to her SMS experiences her beliefs about mathematics research was that she “thought research was you had to come up with something on your own. And it had to be important and like new.” Then she saw a graduate student’s thesis research in SMS. Laurel believes that “now research is just you research a topic that somebody may have already figured out, might not, just putting all that information in one thing, one source.” Her perceptions of mathematics research had changed due to her exposure in SMS. I then asked her if she would think she would be capable of doing more elaborate and publishable mathematics research. Laurel claimed that she could if she became “invested enough.” She “cannot find it boring” and needs to be interested, otherwise she would not perform well.

Laurel was then asked about her position in the mathematics community. Laurel believes that she belongs in the mathematics community, “at least in the department.” Laurel believed that by her taking on various roles within the mathematics department such as an office worker, classroom assistant and tutor, she belongs to the mathematics community at her department. However, Laurel seemed hesitant about the mathematical community outside the university. Laurel believed that she “has not made enough contributions.” I then asked her to elaborate on this. I asked her if it was necessary to make contributions to mathematical community in order to be a part of the mathematical community. Laurel responded, “No. I feel like I could belong to it be I don't know if I could be recognized. Because anyone could belong it. A college algebra student could belong in it. Not they'd want to.” Laurel was considering how she would be positioned by others in the mathematical community. Later on, in the interview, Laurel mentioned how her position affected her forms of engagement in her courses. At first, when she was new to the professors, Laurel was afraid to participate and ask questions. Then when she became involved in the department through various forms of employment and mathematics organizations, she changed her level of engagement. She began to speak up in her courses and had conversations with her instructors. Because of these interactions she believed that her professors would position her within mathematics. We concluded her interview by discussing different parts of her autobiography and her academic and career goals.

As a summary of Laurel's initial mathematical identity, her dossier summary and her pre-Audience Perceptions Survey were considered. Figures 45-48 compare Laurel's initial mathematical identity score in relationship to the audience and her aspect scores from the pre-Audience Perceptions Survey. Again, a larger negative score indicated one's mathematical identity is greatly misaligned with those in the mathematics culture. The greater a positive mathematical identity score would suggest the individual's mathematical identity aligns with the mathematics culture. Table 43 contains the percentages of the extractions of each aspect in relation to their alignment with the mathematics culture. These percentages are the percentages within respective aspect. Table 44 contains Laurel's overall percentages of extractions that align to the mathematics culture.

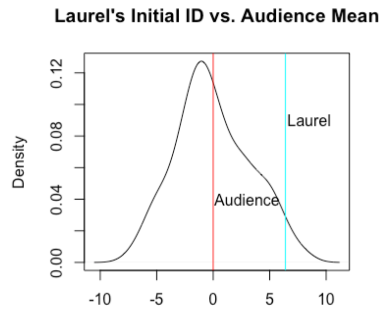

Figure 45. Laurel's initial mathematical identity vs the audience. Laurel's initial mathematical identity score was 6.40 compared the audiences' mathematical identity mean of 4.33e-08.

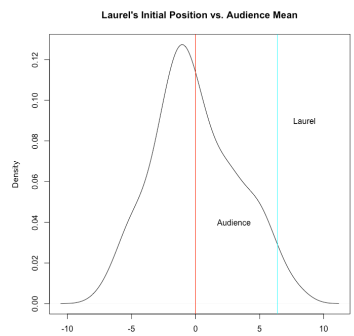

Figure 46. Laurel's initial position vs the audience. Laurel's initial position score was 6.40 compared the audiences' position mean of 4.33e-08.

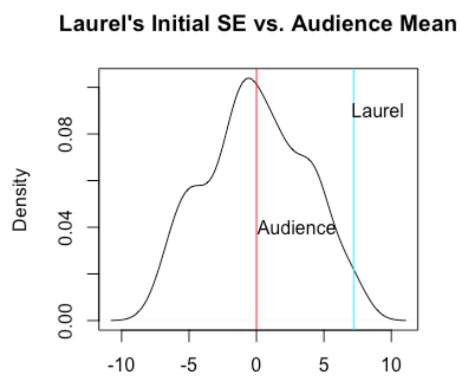

Figure 47. Laurel's initial self-efficacy vs the audience. Laurel's initial self-efficacy score was 7.20 compared the audiences' perceptions mean of 1.69e-08.

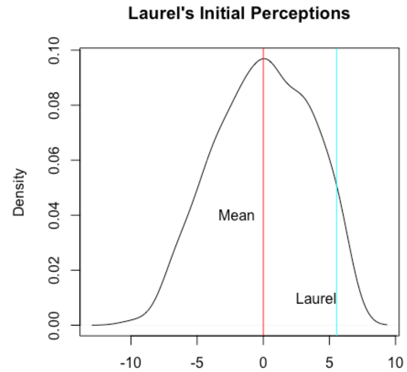

Figure 48. Laurel's initial perceptions of math vs the audience. Laurel's initial perceptions score was 5.54 compared to the audiences' mathematical identity mean of -9.30e-09.

Table 43. Percentages of the extractions of each aspect in Laurel's initial math identity. These percentages are in relation to their alignment with the mathematics culture.

	Pre-Auto-biography	1st Int	Average Percentage
Self-Efficacy			
Total Statements	8	41	
Aligned	5	20	51
Not Aligned	3	21	49
Position			
Total Statements	12	59	
Aligned	4	42	64.8
Not Aligned	0	4	5.6
Beliefs			
Total Statements	15	54	
Aligned	13	32	65.2
Not Aligned	1	10	15.9
Engagement			
Total Statements	49	30	
Aligned	16	4	25.3
Not Aligned	5	6	13.9

Table 44. Laurel's initial math identity percentages. These overall percentages of extractions that align to the mathematics culture for the first stage of the study, initial mathematical identity.

	Pre-Auto	1st Int	Average
Percentage Aligned	49.4	30.9	40.2
Percentage Not Aligned	2.6	13.4	8
Percentage na	48	55.7	51.9

Figure 45 shows that Laurel's over all identity is more positively aligned with the mathematics culture than that of the audience. Her initial mathematical identity score was 6.40 and the audience's mathematical identity average score was approximately zero. From the graphs in Figures 46-48, it can be seen that Laurel's initial position, self-efficacy, and perception scores are more positively aligned in comparison to the audience. In her pre-Math Autobiography and first interview, approximately 40.2% of the extractions taken in those instruments aligned with the mathematics culture. This percentage was calculated by finding the ratio of the extractions of those instruments that aligned with the mathematics culture divided by every extraction in taken from those instruments.

Laurel's Intermediate Mathematical Identity. In Laurel's second interview, she was asked to reflect on how her first presentation went. She believed "it went well" but her "speaking was not the best." She thought she repeated herself unnecessarily. She felt the presentation was still well organized and the material was clearly explained. Laurel said that her explanation of the dominating queen puzzle "was not the easiest" because "that was very math heavy and like abstract heavy." She did not want to explain it in any more detail because she was afraid the audience wouldn't understand. She believed the mathematics involved was far too difficult for the audience. Laurel also found that giving this presentation "was not easy but it wasn't terrible." She found the audience size to be intimidating but "it wasn't that big of a deal." She had moderate self-efficacy since she believed that she was more of an expert on the material than the audience.

Laurel also expressed disappointment about how the audience did not participate in her knights' tour activity. I then asked her if she thought there was a way for the audience to engage more with the activity. We discussed different methods to guide the audience through the activity, but Laurel was then considering to not include the activity for her next presentation. Laurel was considering the difficulty of the puzzle and thought they may not have enough time to complete it during her presentation but "it's still a fun puzzle." She felt the audience found her presentation interesting and that they trusted her information, they just did not care for the activity.

I then asked Laurel what adjustments she would make for the second presentation. Laurel wanted to include more history behind the puzzles and would want to include some real-world applications. We also discussed how she plans to address the mathematics within the presentation. I asked her if she thought there would be a way to improve how she introduced the formulas. Laurel was not too sure "where the line is where [she] stop explaining things. It's just gonna go over their heads." We discussed the importance of gauging the audience and how she could connect the material to the audience.

We then discussed her position as a mathematics researcher after this presentation. Laurel said, "Yeah. I did a talk. I did a presentation that I've never done before other than a class. I think it makes me feel like a math researcher." I asked her if there was anything else that needed to be accomplish before she could say she *is* a mathematics researcher. Laurel said she needed to publish some work, perhaps a master's thesis would suffice. She needed a physical contribution to prove she was a mathematics researcher.

In Laurel's third interview, we discussed how she felt about her second presentation, "The Puzzles of Chess 2." She felt that her overall performance was better because she was more comfortable. She wasn't sure that she explained the material better, but she spoke better. She also felt that she did not do any unnecessary repeating of statements that she believed that she did in the first presentation.

We then discussed what she could improve on for her last presentation. Laurel said that she discussed this previously with a fellow mathematics student. They thought that she could have iterated how the chess pieces moved on the board more since not everyone was familiar with the game. She thought to review how each piece moved when describing the different puzzles. Also, they thought she could find more historical references for these puzzles and maybe another example for real world applications.

We also discussed how she provided more detail in explaining the formulas involved in her presentation and gave concrete examples on how to use the formulas. There were still some formulas that she did not explain as fully, and we discussed if there was a reason she did not cover those. She claimed she "forgot to speak out what she was thinking."

We again reviewed her knight's tour activity. She decided not to include the activity since she believed that the audience would not have enough time to complete the activity within the duration of the presentation and if they would they may not have paid attention to her explanations.

We also discussed her inclusion of a real-world application in her second presentation. I asked how she researched it. She claimed to have "googled chess puzzles and real-world applications and it popped up with the backtracking algorithm." She believed that this was a great example of how these chess puzzles were used in real life "because that's what that puzzle needs to solve it." I also asked her if there was more of an appreciation for understanding chess puzzles by making connections to applications. She believes so, "at least for computer science" since this can be used in programming, but she wasn't sure if it was found interesting for others. So, Laurel believed that there was some value to understanding the mathematical applications for certain individuals, but that value may not be shared by the general public.

Laurel's first presentation, "The Puzzles of Chess," lasted approximately 19 and a half minutes. She began her presentation by introducing herself and claiming that chess puzzles are popular and very fun. She then described the three pieces that she would be using in her puzzles, the knight, king and queen. Next Laurel gave a brief outline of her presentation to describe the structure of talk.

Laurel then proceeded to describe her first puzzle type, the knight's tour. She explained the two different types of tours, the open and closed, and their differences. After this she listed out what types of tours are attainable on boards sized from a 1x1 up to an 8x8. Laurel then asked the audience to find their own knight's tours on the 6x6 boards she provided as handouts while she made her way through the remainder of the presentation.

Laurel then went into describing non-attacking tours for knights, kings, and queens. She described how these tours differed from the previous tours and showed examples of how to arrange these pieces on boards ranging from 1x1 to 8x8. She then showed the results for the maximum number of pieces that could be placed on these boards to see if there were any patterns between the number of pieces and the size of the board. She explained to the audience that "the whole goal of mathematicians trying to do this is we want to find some kind of pattern or relationship to that. So, we want to find some pattern using these handable boards that we can do by hand and see if we can apply a pattern that we find to higher level boards." Laurel positioned herself with “mathematicians” by using “we” repeatedly. Laurel then stated the formulas for each of these puzzles as the patterns were found.

For her last puzzle, she explained domination. For this puzzle, goal is to find the minimum number of pieces. Laurel listed out the results similarly as she did for non-attacking tours. She pointed out that the king has a pattern, so we are able to define a formula, but that the knight still is considered an open problem and suggested the audience to look into it.

She then went into domination for queens and explained that the best that “we” could do is find an upper and lower bound for the minimum number of dominating queens. She described this puzzle as “mathy” and “more difficult” and “that it was fine if [the audience] don’t understand.” She did not want to explain these bounds because “it’s too difficult to explain…but basically this whole thing means we would get a range of numbers to try.”

Laurel then asked the audience if anyone was able to find a knight’s tour. The audience responded with laughter. No one appeared to attempt the puzzle. Laurel shared a tour that she had found and then shared a theorem, Schwenk’s Theorem, that determines if an mxn chess board would have a knight’s tour and if that tour is open or closed. She explained the three conditions of the theorem and then related those conditions to the results she previously discussed.

Laurel then concluded her presentation by sharing her resources and thanked the audience and allowed for questions. An audience member asked if there were any real-world applications to these puzzles. Laurel said she was unaware of any applications but would look into it for her next presentation.

Laurel's second presentation, titled "The Puzzles of Chess 2," lasted 18 minutes and 45 seconds which was slightly shorter than her first presentation. Her second presentation address the same content as her first presentation with some moderate modifications.

Laurel did not include the activity, finding a knight's tour, for the audience. Also, she moved describing Schwenk's Theorem to when she described the knight's tour puzzle as opposed to the end as she did in the first presentation. She also did multiple connections to the resulting tours to each condition of Schwenk's Theorem. Unlike the first presentation where she only did one brief connection to each condition of the theorem. In describing the formulas for non-attacking tours, Laurel explained the formulas in more detail by describing each piece of the formula. For example, before in her first presentation, she just told the audience that she uses this function called the floor function but did let the audience know how the function operated. In her second presentation she described how the function worked but did not provide concrete examples and did not explicitly check the results from her tables.

Also, in her second presentation, Laurel decided to explain finding the upper and lower bounds for dominating queens. She described in detail the processes for both bounds and gave examples on how to calculate the bounds. She then checked to see if the bounds agreed with the results in her tables.

Laurel concluded her second presentation with how non-attacking queens is a puzzle that has a computer science application called the backtracking algorithm. This algorithm "keeps going down a path until something goes wrong. And if something goes wrong it backtracks to the last point where everything was okay. And then it goes down a different path." Laurel also made this connection to mappings on a tree graph. She then showed a brief video how the backtracking algorithm would be used to place queens on an 8x8 chess board for the non-attacking queens puzzle. Laurel then asked the audience, if anyone had any questions and seeing none, she thanked them for listening.

As a summary of Laurel's intermediate mathematical identity, her dossier summary was considered. Since there was no Audience Perceptions Survey in these phases of the study, there are no graphical comparisons here. Table 45 shows the percentages of the extractions of each aspect in relation to their alignment with the mathematics culture. These percentages are the percentages within the respective aspect. Table 46 shows Laurel's overall percentages of extractions that align to the mathematics culture in her intermediate mathematical identity.

Table 45. Percentages of the extractions of each aspect in Laurel's intermediate math identity. These percentages are in relation to their alignment with the mathematics culture.

	2^{nd} Int	3^{rd} Int	1st Pres	2^{nd} Pres	Average Percentage
Self-Efficacy					
Total Statements	32	10	3	2	
Aligned	12	9	1	2	51
Not Aligned	19	1	2	0	46.8
Position					
Total Statements	10	7	24	4	
Aligned	8	0	17	1	57.8
Not Aligned	2	0	0	0	4.4
Beliefs					
Total Statements	13	8	17	17	
Aligned	5	7	8	13	60
Not Aligned	6	0	7	0	23.6
Engagement					
Total Statements	0	0	366	322	
Aligned	0	0	89	147	60.8
Not Aligned	0	0	14	4	4.6

Table 46. Laurel's intermediate math identity percentages. These overall percentages of extractions that align to the mathematics culture for the second and third stages of the study, intermediate mathematical identity.

	2nd Int	3rd Int	1st Pres	2^{nd} Pres	Average
Percentage Aligned	29.2	18.6	21.4	39.3	27.3
Percentage Not Aligned	29.2	1.2	3.3	1.9	8.9
Percentage na	41.6	80.2	75.5	58.8	64

Laurel's Evolved Mathematical Identity. Laurel's post-autobiography was identical to her pre-autobiography except for her adding a paragraph about her experiences in SMS. She believed that she had learned some "valuable information" about herself. She had learned to "be more comfortable talking in front of a large group of people" and "increasingly got more relaxed which made [her] presentations better."

Laurel also changed her understanding of what it means to do "math research." She "learned that math research did not have to be completely new." She believed that mathematics research had to be over open and unsolved problems. "However, going through this program, [she] now understands that not all research is brand new." She concluded saying that she enjoyed her experiences in SMS and looks forward to doing similar activities in the future.

In Laurel's final interview, we discussed her last presentation, "The Puzzles of Chess 3." Laurel believed that it was her best performance, even though it went longer than she anticipated. She believed that she included better examples and that her new application of chess puzzles connected well with the audience. I then asked her if she was to give her presentation for a fourth time, what would she have done differently. She claimed that she would have demonstrated one of the new and more simple chess puzzles to the audience and provided more detail. She felt that she rushed through that puzzle.

We then discussed a comment that she made in her post-math autobiography. She stated, "I also learned that math research did not have to be completely new. For the longest time, I believed that math research had to be over open problems and I had to work on solving them. However, going through this program, I now understand that not all research is brand new." She believes that doing mathematics research can be repeating past results. By understanding past results we can attain new ones. Also bringing together and summarizing past research "can bring in new connections." This experience made her "feel better that [she] could do this." This realization had increased her self-efficacy in participating in mathematics research. I asked her if she considered herself as a mathematics researcher and she said, "a little bit now…not so much as the unique side but I'm good at explaining math things."

I then asked her if she thought this presentation would be fine to give in a more professional setting, like a research conference or in front of an audience of other mathematics researchers. Laurel said she wouldn't think that would be okay to do because "they would be more picky. They would ask more difficult questions to things like I don't know." This presentation was fine for an undergraduate audience. Laurel's position was that she was an authority or an expert to the audience in SMS, but she still has not aligned herself as a "mathematician."

We then discussed her overall experiences with SMS. Laurel shared that she valued these experiences. She said, "It wasn't easy, but it wasn't stressful. It was moderate in the actual research of it since this wasn't for an actual grade, so I would have to worry about everything being so technical." She was not so concerned about being assessed for a grade and she was doing these presentations more for her own benefit. She continued on by say that giving these presentations "looks good on a resume…it is helpful to your career or whatever it is that you want to go to cause you get that teaching aspect, you the get speaking aspect you need wherever you go."

Laurel's third presentation, "The Puzzles of Chess 3," was structured very similarly to her first two presentations, however, included much more information. Her presentation lasted approximately 25 minutes and used power point as her media for giving her talk. Laurel positioned herself in front of the podium and used a clicker to change slides. Laurel maintained strong eye contact with the audience and used appropriate voice fluctuations throughout her presentation.

She began her presentation by introducing the three chess pieces used in her puzzles, the knight, king, and queen. She described how each piece maneuvered about the board and then provided an outline of how she was presenting her information.

She continued with acquainting the audience with a new chess puzzle called, Guarini's Problem. This problem addressed how two black knights in the top corners of the board and white knights on the bottom corner of the boards on a four by four chess board interchange sides but still have to acknowledge how the knight moves. She said this puzzle is difficult but would not go into any detail. She only wanted to give a brief overview.

Laurel then introduced the knight's tours. She again described the difference between open and closed tours. She showed a closed tour for an 8x8 chess board and an open tour for a 5x5 board and showed how these two puzzles were different. This led her into discussing Schwenk's Theorem that determines if a closed or open tour exists on a *p*x*n* chess board depending on the criteria provided by the theorem. She then connected this theorem to the tours found on every sized chessboard from a 1x1 to an 8x8 chessboard.

Laurel then provided the history of the knight's tour and pointed out that a "very, very famous" mathematician, Leonhard Euler, had written a paper in 1759 on solving knight's tour puzzles. She mentioned what contributions he made to the fields of physics and applied mathematics. She thought that Euler's work in his puzzles paper was important for mathematical findings in chess puzzles. By Laurel identifying Euler as famous and his finding as "important," she was positioning herself within the mathematical community.

After the knight's tours was concluded, Laurel moved to non-attacking puzzles and explained that the goal of non-attacking is to find the maximum number of pieces such that no piece on the board will attack each other. She reviewed again how each piece moved and then listed out all the results for each piece for all boards 1x1 to 8x8 in separate tables. By pointing out the patterns in each puzzle, she was able to show the formula for each puzzle. She then introduced a special non-attacking queen puzzle called the Eight Queens. Laurel said that this puzzle is commonly found in puzzle books and brain teasers and this puzzle was a lot of fun. She said she liked finding these patterns.

Laurel then explained how each formula worked and verified the formulas with an example from her tables by asking the audience to check on their own. After giving the audience time to work through checking the formulas, she showed the results herself. By doing this she was taking on the role of an authority or expert.

The last chess puzzle Laurel introduced was domination. She began with dominating knights. She showed boards for a 1x1, 2x2, 3x3, and 4x4. She then listed a table of results for all boards 1x1 to 8x8. Laurel said that there were no obvious patterns. For dominating knights, there is no known pattern. It has not been solved and if the audience wanted an open problem to work on, she suggested this one.

Laurel then explained dominating kings. She shared a table that listed all found solution for chessboards of 1x1 to 8x8. From these numbers, Laurel showed how to determine a formula for dominating kings. She then explained in detail how the formula worked and asked the audience to try to evaluate the formula for practice. She then had the audience verify the formula by checking the solution to the table.

The final puzzle Laurel talked about was the dominating queens. She told the audience that finding the formula was “tricky;” that even though that the table of results looked like a pattern, it could be misleading. Laurel explained that the solution to the dominating queens has upper and lower bounds. Laurel explained in clear detail how the upper and lower bounds and then asked the audience to try the example of when there was a 7x7 board. She gave the audience a few minutes and then worked through the formulas for the lower and upper bounds to verify their findings. She engaged the audience to have them practice her field of mathematics.

She then asked the why would they care about domination. How could this relate to everyday life? She shared that domination is very important to applied graph theory and a real-life application that the audience could connect to. She provided the example of how cell-towers would be positioned to maximize coverage using the minimum number of towers.

Laurel also reintroduced how the non-attacking queens and kings’ problem is used in a computer science algorithm called backtracking. She explained that “it starts with a problem and it goes down each possible solution and if something doesn't work it backtracks to last spot where everything was okay.” She then shared a video that demonstrated the backtracking algorithm by placing non-attacking queens on an 8x8 board.

Laurel said that was the end of her presentation and asked if there were any questions. No one asked any questions and she thanked the audience for their time.

As a summary of Laurel's evolved mathematical identity, her dossier summary and his post-Audience Perceptions Survey was considered. Figures 49-52 show Laurel's evolved mathematical identity score in relationship to the audience and her aspect scores from the post-Audience Perceptions Survey. Again, to interpret the scores, a larger negative score indicated one's mathematical identity is greatly misaligned with those in the mathematics culture. The greater a positive mathematical identity score would suggest the individual's mathematical identity aligns more with the mathematics culture. Table 47 contains the percentages of the extractions of each aspect in relation to their alignment with the mathematics culture. These percentages are the percentages within respective aspect. Table 48 contains her overall percentages of extractions that align to the mathematics culture.

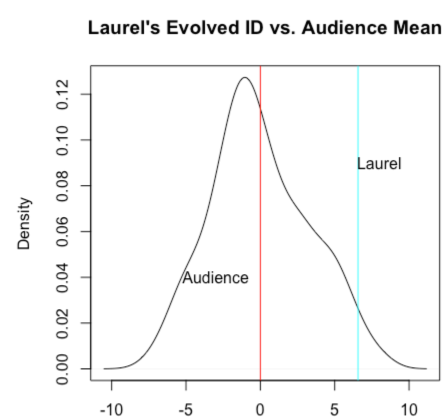

Figure 49. Laurel's evolved mathematical identity vs the audience. Laurel's evolved mathematical identity score was 6.57 compared the audiences' mathematical identity mean of 4.33e-08.

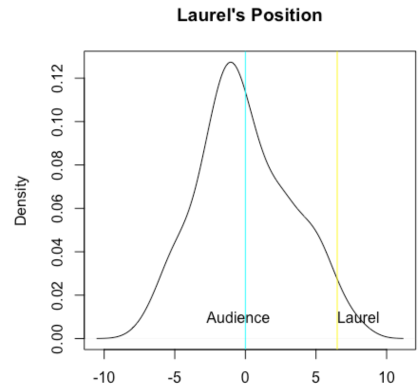

Figure 50. Laurel's evolved position vs the audience. Laurel's evolved position score was 6.57 compared the audiences' mathematical identity mean of 4.33e-08.

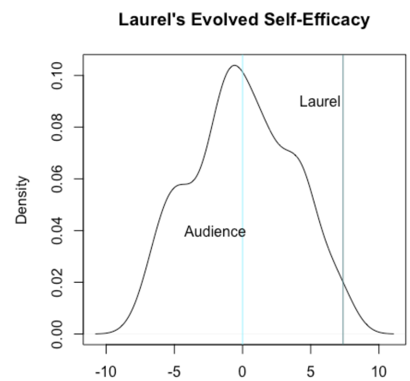

Figure 51. Laurel's evolved self-efficacy vs the audience. Laurel's evolved self-efficacy score was 7.37 compared the audiences' mathematical identity mean of 1.69e-08.

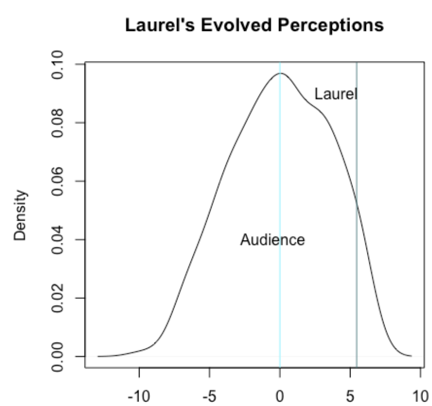

Figure 52. Laurel's evolved perceptions of math vs the audience. Laurel's evolved perceptions score was 5.46 compared the audiences' mathematical identity mean of -9.30e-09.

Table 47. Percentages of the extractions of each aspect in Laurel's evolved math identity. These percentages are in relation to their alignment with the mathematics culture.

	Post-Auto	4th Int	3rd Pres	Average
Percentage Aligned	52.5	48.3	49.3	50
Percentage Not Aligned	2.8	5.7	0.4	3
Percentage na	44.7	46	50.3	47

Table 48. Laurel's evolved math identity percentages. These overall percentages of extractions that align to the mathematics culture for the last stage of the study, evolved mathematical identity.

	Post-Auto	4th Int	3rd Pres	Average Percentage
Self-Efficacy				
Total Statements	8	17	5	
Aligned	5	13	5	76.7
Not Aligned	3	4	0	23.3
Position				
Total Statements	15	4	34	
Aligned	15	2	28	84.9
Not Aligned	0	2	0	3.7
Beliefs				
Total Statements	28	28	47	
Aligned	26	23	29	75.7
Not Aligned	2	1	4	6.8
Engagement				
Total Statements	50	3	434	
Aligned	8	3	342	72.5
Not Aligned	4	0	12	3.3

Summary of Laurel's Evolution of Mathematical Identity. From what was described in at Laurel's three stages of mathematical identity, there is evidence that Laurel went through some changes. To simplify the analysis of each stage, the summary of each instrument was compiled in Laurel's dossier. Included in this summary table, the items were then consolidated into whether those items were considered aligned (A) with the mathematics culture, not aligned with the mathematics culture (N), or not applicable (na). Laurel's totals can be seen in Table 49.

Table 49. The summary of each instrument for Laurel throughout the study. These overall percentages of extractions that align to the mathematics culture for the entirety of the study.

	Pre-Auto	Post-Auto	1st Int	2nd Int	3rd Int	4th Int	1st Pres	2nd Pres	3rd Pres
Percentage Aligned	49.4	52.5	30.9	29.2	18.6	48.3	21.4	39.3	49.3
Percentage Not Aligned	2.6	2.8	13.4	29.2	1.2	5.7	3.3	1.9	0.4
Percentage na	48	44.7	55.7	41.6	80.2	46	75.5	58.8	50.3

Table 49 shows the percentage the extractions from the instruments were in each stage of mathematical identity. Laurel's initial mathematical identity includes the pre-math autobiography, first interview and first presentation. The average of these percentages of being aligned was approximately 40.9%. Laurel's intermediate mathematical identity included the second interview, third interview, and second presentation. The average of the percentages of being aligned was approximately 29%. Then lastly the items included in her evolved mathematical identity were the post-math autobiography, fourth interview and third presentation. Her overall aligned percentage was 50%. This shows that Laurel had a 9.1% evolution of her mathematical identity after completion of the study. Additionally, we can see an evolution by comparing her pre- and post-Audience Perception Survey. Figure 53 shows the evolution of Laurel's mathematical identity in comparison to the audience. Laurel's Pre-Audience Perception Survey Predicted Mathematical Identity was 6.40 by using the structural equation. Her Post-Audience Perception Survey Predicted Mathematical Identity was 6.57. This a positive evolution of 0.17. Again, a larger negative score indicated one's mathematical identity is greatly misaligned with those in the mathematics culture. The greater a positive mathematical identity score would suggest the individual positions themselves within the mathematics culture and is highly engaged in mathematical practices. This individual would also have positive perceptions of mathematics and high self-efficacy. This score indicates that Laurel's initial and evolved-mathematical identity scores were aligned with the mathematics culture.

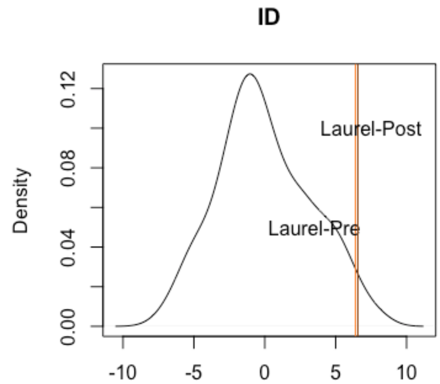

Figure 53. Laurel's initial and evolved mathematical identity. At the start of the study Laurel's initial mathematical identity was 6.40. At the completion of the study Laurel's evolved mathematical identity was 6.57.

There is evidence of a positive change in Laurel's summary of her Presenter Observation Protocol as seen in Table 50. Laurel increased her averages in her abilities of presenting, style of presentation, and engaging with the audience at each stage of the study. Laurel increased her overall average by 1.05 points. Table 51 shows an overall summary that highlights some of Laurel's positive changes in the aspects of her mathematical identity.

Table 50. Laurel's POP scores. This table displays Laurel's average Presenter Observation Protocol scores as determined by the Presenter Observation Protocol Rubric. A 1 indicates a novice performance and the high score of a 4 indicates a distinguished performance.

Average	Presentation 1	Presentation 2	Presentation 3
AVG Presenter	2.65	3.1	3.7
AVG Presentation	2.57142857	3.28571429	4
AVG Engage	2.25	3	3.75
Overall	2.65	3.1	3.7

Table 51. Summary of Laurel's changes. This table summarizes a few of the changes in Laurel's mathematical identity through her completion of the study.

	Initial Identity	**Evolved Identity**
Position	• Considers herself a part of the math community but not in the context of math experts	• Starts to envision herself as a math expert, but still has lots to prove
Self-Efficacy	• Moderate levels within communicating mathematics and performing math tasks in a small setting	• High levels of confidence presenting to large audiences
Perceptions of Math	• Research has to be new	• Research does not have to be new
Forms of Engagement	• High levels of engaging in math	• Moved from doing to teaching

Cross-Case Analyses Within Groups

In this section, a cross-case comparison will be conducted between the student mathematician groups. Kendra's progression of her mathematical identity will be compared to Sara's progression. Laurel's progression of her mathematical identity will be compare to Oliver's progression.

Beginning Mathematicians: Kendra vs. Sara

By comparing the entries in the table below and Figure 54, we can see how differently Kendra and Sara evolved. In Table 52, we see that form the instrument percentages that Sara had higher ratios throughout each stage of the study, however, Kendra's POP overall averages are higher at each stage. Also, from the figures we see that Kendra was placed with a higher initial and evolved mathematical identity from the Pre- and Post-Audience Perception Surveys, but Sara had the most significant positive change in her mathematical identity.

From the dossier summaries and descriptions of the stages of the study it can be seen that the BSMs' initial mathematical identities had low levels self-efficacy, novice levels of engagement, positioned themselves outside the mathematics community, and narrow views of mathematics. At the completion of the study, the BSMs' evolved mathematical identities had improved levels of self-efficacy. That is, they felt that they "had more hope," and realized they were capable of teaching themselves. Sara moved from example to proof and Kendra had more details in her descriptions of her calculations. Both shared that they felt more comfortable discussing mathematics in class and with "mathy people." Also, they realized there was so much more to mathematics than what they saw in class and more than just doing "big calculations."

Table 52. BSMs' stages of math identity. This table compares the BSMs' stages of math identity using extraction alignments and POP scores.

BSM	Initial Math ID		Intermediate Math Id		Evolved Math ID	
	Instrument Percentage	POP Overall	Instrument Percentage	POP Overall	Instrument Percentage	POP Overall
Sara	30.9	1.5	44.7	1.8	64.8	2.75
Kendra	19.7	1.9	24.3	2.45	41.8	2.85

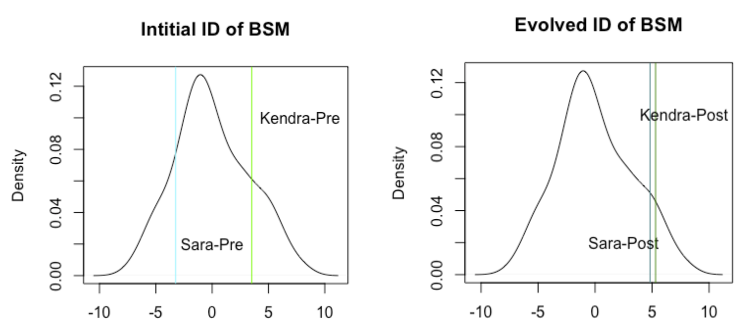

Figure 54. BSM's initial and evolved mathematical identities. This shows a comparison of Sara's and Kendra's evolution of math identities.

Advanced Mathematicians: Laurel vs. Oliver

By comparing the entries in the Table 53 and Figure 55, we can see how differently Oliver and Laurel evolved. In Table 53, we see that form the instrument percentages that Laurel had the most change in her instrument ratios compared to Oliver. Also, Laurel received higher rankings on the POP and had a slightly greater change in those scores from her initial mathematical identity to her evolved mathematical identity. We also see from Figure 56 that Laurel had a higher initial and evolved mathematical identity measured by the survey, but we see that Oliver had much more of an increase of alignment of his mathematical identity.

From the dossier summaries and descriptions of the stages of the study that we can see that the ASMs' initial mathematical identities had moderate levels self-efficacy, high levels of engagement, positioned themselves within the mathematics community in certain contexts, and had broad views of mathematics. After the completion of the study, the ASMs' evolved mathematical identities had improved self-efficacy-more confidence in their abilities to communicate, both moved from demonstrating proof to explanation to novice audience and considered audience views of mathematics. Both Laurel and Oliver felt they have the potential to belong to groups of expert mathematicians but still have a lot to prove. Even though Oliver and Laurel had similar views of mathematics at the end stage of the study, they had expanded their views on what is involved in mathematics research. Both initially felt that the mathematics involved in their presentations could never be considered as mathematics research. In their final interview, both shared that they started to feel their work could be valued as resources or a starting point to build onto past and new research.

Table 53. ASMs' stages of math identity. This table compares the ASMs' stages of math identity using extraction alignments and POP scores.

ASM	Initial Math ID		Intermediate Math Id		Evolved Math ID	
	Instrument Percentage	POP Overall	Instrument Percentage	POP Overall	Instrument Percentage	POP Overall
Oliver	38.5	2.55	28.1	2.45	46.5	3.45
Laurel	33.9	2.65	29	3.1	50	3.7

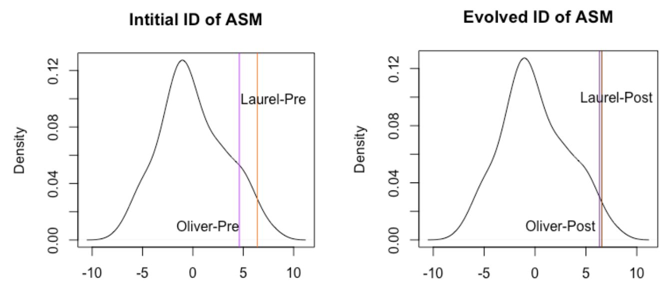

Figure 55. ASM's initial and evolved mathematical identities. This shows a comparison of Oliver's and Laurel's evolution of math identities.

Cross-Case Analysis Between Groups

In this section, the participants are considered in a holistic view. From Figure 56, it can be seen that that the ASMs have more aligned mathematical identities compared to the BSMs, however Kendra is not too far behind. Also, according to the survey, at the completion of the study, all the participants are positively aligned and that one of the BSMs, Kendra, was more aligned, than one of the ASMs, Oliver. Also, this more positive alignment can be seen in the summaries of each of the participant's dossiers as well as from Table 54.

From the dossier summaries and descriptions of the stages of the study that we can see how the BSMs' and the ASMs' initial mathematical identities and the BSMs' and the ASMs' evolved mathematical identities compared. At the start of the study, BSMs had low levels self-efficacy, novice levels of engagement, positioned themselves outside the mathematics community, and narrow views of mathematics. The ASMs had moderate levels self-efficacy, high levels of engagement, positioned themselves within the mathematics community in certain contexts, and had broad views of mathematics. At the completion of the study, the BSMs' evolved mathematical identities had improved levels of self-efficacy. Also, they realized there was so much more to mathematics than what they saw in class and more than just doing "big calculations." With the ASMs, they both evolved from an informational point of view to more of an instructional point of view in their presentations as they were considering the comprehension level of the SMS audience. Also, since Laurel and Oliver already had a broad view of mathematics, they did not change their perceptions on those topics, however, they had developed a broader understanding of engaging in "more true mathematics research."

Table 54. BSMs' and ASMs' stages of math identity. This table compares the BSMs' and the ASMs' stages of math identity using extraction alignments and POP scores.

Participant	Initial Math ID		Intermediate Math Id		Evolved Math ID	
	Instrument Percentage	POP Overall	Instrument Percentage	POP Overall	Instrument Percentage	POP Overall
Sara	30.9	1.5	44.7	1.8	64.8	2.75
Kendra	19.7	1.9	24.3	2.45	41.8	2.85
Oliver	38.5	2.55	28.1	2.45	46.5	3.45
Laurel	33.9	2.65	29	3.1	50	3.7

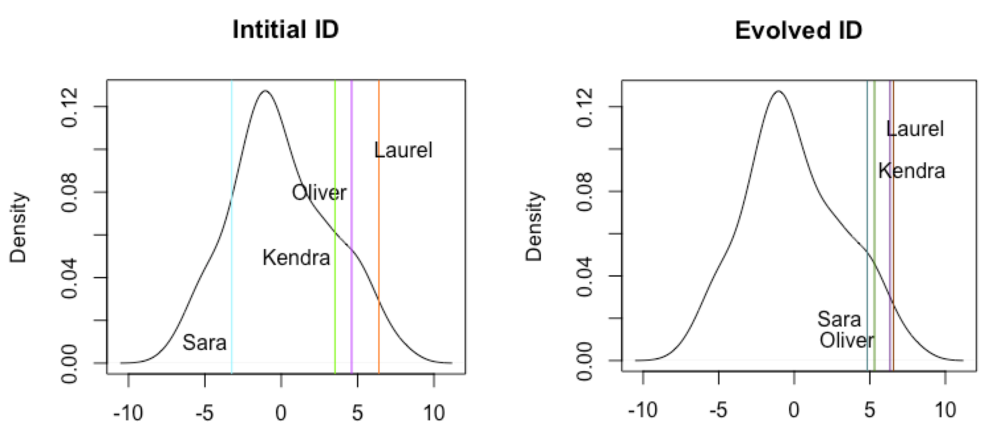

Figure 56. BSMs' and ASMs' initial and evolved mathematical identities. This shows a comparison of presenters' evolution of math identities.

Answering the Research Questions

For this study, I had four research questions that considered the audience, the presenters, and the relationship between the presenters and the audience.

Audience

1. What was the mathematical identity of the SMS audience as measured by the Audience Perception Survey?

Presenters

2. How did the student presenters' mathematical identities compare to the mathematical identity of the audience?
3. In what ways did the student presenter's mathematical identity evolve during their participation in SMS?
4. In what ways did the level of mathematics acculturation differ between beginning mathematicians and advanced mathematicians?

The first question, "What is the mathematical identity of the SMS audience as measured by the Audience Perception Survey?" was answered from the SEM model developed from the data collected in the Audience Perception Survey. This survey was distributed throughout the entire semester of the study. From the 242 respondents we found that the overall mathematical identity seemed relatively neutral. That is the overall mean was found to be 4.33e-08, implying it was relatively neutral in terms of the alignment of the mathematics culture. That is, it is neither completely aligned nor completely misaligned.

The second question, "How do the student presenters' mathematical identities compare to the mathematical identity of the audience?" was answered by comparing the presenters' pre- and post-Audience Perception survey identity values to that of the audience. We saw that Sara's Initial mathematical identity was negatively aligned compared to that of the audience and the Kendra's, Oliver's, and Laurel's mathematical identities were more positively aligned that to the audience. After the completion of the study, all four participants were found to have a more positively aligned mathematical identity compared to the audience as seen in Figure 56.

As seen in all the participants, there was an apparent solution to the last two questions, “In what ways does the student presenter’s mathematical identity evolve during their participation in SMS?” and “In what ways does the level of mathematics acculturation differ between beginning mathematicians and advanced mathematicians?” After completing all four stages, we saw that every presenters’ mathematical identities have become more positively aligned with the mathematics culture. From the dossier summaries and descriptions of the stages of the study that we can see how the BSMs’ and the ASMs’ initial mathematical identities and the BSMs’ and the ASMs’ evolved mathematical identities compared. At the start of the study, the BSMs had low levels self-efficacy, novice levels of engagement, positioned themselves outside the math community, and narrow views of mathematics. The ASMs had moderate levels self-efficacy, high levels of engagement, positioned themselves within the math community in certain contexts, and had broad views of mathematics. At the completion of the study, the BSMs’ evolved mathematical identities had improved levels of self-efficacy. Also, they realized there was so much more to math than what they saw in class and more than just doing “big calculations.” With the ASMs, they both evolved from an informational point of view to more of an instructional point of view in their presentations as they were considering the comprehension level of the SMS audience. Also, since Laurel and Oliver already had a broad view of mathematics, they did not change their perceptions on those topics, however, they had developed a broader understanding of engaging in “more true mathematics research.”

V. DISCUSSION

Mathematics encompasses more than formulas, theorems and proofs. For many, mathematics can be a way of life or even a culture (Davis, 1989). Tyler (1871) defines culture as a "complex whole which includes knowledge, belief, art, morals, law, customs, and any other capabilities and habits acquired by man as a member of society" (p.1). Burton (2009), Bishop (1991), and many others describe how these aspects manifest in mathematics. Historically, the culture of mathematics has been integral to the field of mathematics and seen as part of what students are expected to acquire in the process of becoming mathematicians (Wood et al., 2012).

With a nationwide necessity to increase the mathematics community (Engineering for Kids, 2016), students must develop mathematical skills and the skills to communicate the mathematics. Burton (1984) found that few students leave the school system with mathematical achievement and understanding and most demonstrate a lack of skill in thinking mathematically. Even though Burton's observation took place over twenty years ago, this problem still exists (Hemphill and Vanneman, 2011).

This deficiency in the number of mathematics majors may be because students fail to make an individualized connection to the subject. That is their identities are not aligned with those in the mathematical community. For example, Boaler and Greeno (2000) have found that many students who are successful in traditional mathematics courses chose not to pursue studying mathematics in college because being successful in mathematics does not align with who they identify themselves to be. This suggests that there are students who could potentially excel in the STEM subjects, but who feel that they do not belong in those fields because they lack a sense of connection to the culture of mathematics.

Furthermore, the mathematical culture can create barriers to entry by members of certain groups. Mathematics, like any other culture, has developed stereotypes and biases. There exists stereotypes and biases both within and outside the culture of mathematics and can then affect student performance (Aronson, et. al., 1999; Boaler, 2013). Most of these stereotypes and biases are developed from experiences in the classroom, homework assignments, and test performances. Seymour and Hewitt (1997) found that students in science, mathematics and engineering courses describe their classroom environments as cold, the instructors as unapproachable, and the lectures as not welcoming discussion. This can lead to students taking on a passive role when attending class (Bressoud, 1994).

These biases and stereotypes are now controlling who can enter the discipline and how they do so (Burton, 2009). Burton further states that it is the mathematical culture that exercises power over how the culture of mathematics is understood, and thus, it is the mathematical culture that must be addressed if mathematics is to achieve widespread accessibility. Students enter post-secondary education with these stereotypical views of mathematics that they have picked up from their K-12 education. Students believe that understanding mathematics is unnecessary and that the only thing that matters is knowing the rules to get to the correct answer (Mason, 2003; Muis 2004). Thus, when they enroll in an undergraduate program, they are “forced” to take pointless mathematics course(s) just to fulfill a requirement for graduation. These views place a minimal value on mathematics and in turn makes students less inclined to join the mathematics community.

Therefore, traditional venues, like the classroom, for acculturating students into mathematics may not be best. Traditional pedagogies and procedural views of mathematics combine to produce environments in which most students must surrender agency in order to follow predetermined routines (Boaler, 1997; Schoenfeld, 1988, 1992).

The purpose of this study was to investigate how creating a social environment through a weekly student seminar for post-secondary students can help facilitate students' acculturation into mathematics, and how students' mathematical identity evolves by presenting and attending these presentations. This chapter will present a discussion of the results of the study, the corresponding implications, and recommendations for future research.

Discussion

This study attempted to determine if and how participation in an informal environment like SMS affected four students' mathematical identity. Using a collection of instruments, these students were observed, interviewed and compared to the audience regarding four aspects of mathematical identity (position, self-efficacy, perceptions of mathematics, and forms of engagement) in context of the following research questions:

1. What is the mathematical identity of the SMS audience as measured by the Audience Perception Survey?
2. How do the student presenters' mathematical identities compare to the mathematical identity of the audience?
3. In what ways does the student presenter's mathematical identity evolve during their participation in SMS?

4. In what ways does the level of mathematics acculturation differ between beginning mathematicians and advanced mathematicians?

The beginning student mathematicians were students who were not mathematics majors and the advanced student mathematicians were students that were classified as mathematics majors. Data was collected and compiled from a series of interviews, presentations, autobiographies into the Subject's Dossier to help answer the last three research questions.

A structural equation model was developed from the data collected in the Audience Perception Survey to answer Research Questions 1 and 2. This survey was distributed throughout the entire semester of the study. The scale generated ranged from -10 to 10. A larger negative score indicated one's mathematical identity is greatly misaligned with those in the mathematics culture. An individual with a negative score can be viewed as someone who disassociates themselves with mathematics and its practices. This individual would also have negative perceptions of mathematics and low self-efficacy. The greater a positive mathematical identity score would suggest the individual positions themselves within the mathematics culture and is highly engaged in mathematical practices. This individual would also have positive perceptions of mathematics and high self-efficacy. From the 242 respondents it was found that the overall mathematical identity seemed relatively neutral. The overall mean was found to be 4.33e-08, implying it was relatively neutral in terms of the alignment of the mathematics culture. That was, either completely aligned nor completely misaligned. Comparing the presenters' pre- and post-Audience Perception survey identity values to that of the audience, it was determined that Sara's Initial mathematical identity was negatively aligned, and that Kendra's, Oliver's, and Laurel's mathematical identities were more positively aligned than that of the audience. After the completion of the study, all four participants were found to have a more positively aligned mathematical identity compared to the audience as seen in Figure 56 which helped to answer Research Questions 2, 3, and 4.

Generalizing Figure 56, along with the results found in Chapter 4, it could be assumed SMS is an alternative intervention to evolve mathematical identity and could motivate students to pursue more mathematical endeavors. This generalization can be elaborated by considering each of the four aspects of mathematical identity for each presenter, namely, position, self-efficacy, perceptions of mathematics and forms of engagement.

Position

Position is defined as where one views their location or where others view one's location within or outside the mathematical community. Furthermore, if considering their location within the community, what role do they believe they will take on (authoritative/expert or compliant/novice) (Wegner, 1998; Boaler & Greeno, 2000). Sara stated in her last interview and post-autobiography that SMS was "a foot in the door to bigger things." These presentation experiences made her more willing to take a more expert role. She said, "You know it makes me wanna go up in front of a lot of people and be like this what the theorem is, and this my subject and it ties to this subject I could know…". Sara was claiming her presentation topic as *her* topic, taking on an expert role. Kendra shared similar statements. Doing presentations gave Kendra an "authoritative feeling" and felt that the audience "would consider [her] more of an expert than themselves because no one knew it and so that was pretty cool cause like by the end they felt like they could probably do it because [she is] an expert cause [she] could teach them how to do it." Laurel and Oliver considered their positions in more of the terms of that of a mathematics researcher. Laurel felt like she was "a little bit more" of a researcher since this was an opportunity to show her abilities to communicate mathematics. Oliver pointed out in his interviews that he already "had a good sense of belonging" in the mathematics community prior to the study but "strengthened that sense of belonging." He also shared that his SMS experiences led him to "fantasized about what it would be like to give a presentation to a similarly sized group, but instead of giving a presentation on a topic that is well established mathematics." He was reflecting on his future role in mathematics. Furthermore, Oliver claimed, "The experience of giving my 3 talks felt similar to that of mathematical research in the sense that I was having to really work out all the details of my topic and prepare to explain it to a wider audience who may not know, fully, all the aspects that it contains." This implied that Oliver was considering his role in respect to the audience in SMS.

In SMS, the presenter takes on the role of playing "the expert" or the authority within that presentation. From the statements made by the presenters and what was seen the results of the Audience Perception Survey, the participants can be assumed that they are considering this more authoritative position. This is similar to findings from the research literature. For example, imagine a classroom in which a teacher consistently refers to her students as "mathematicians." This type of discourse move is a positioning act meant to reflect and encourage students to enact the desired identity (Bishop, 2012). Students make use of positional identities when and how they chose to enter into a discussion with others or participate in the class activity (Graves, 2011) or other forms of engagement.

Self-Efficacy

Howard (2015) defined self-efficacy as the personal conviction that an individual has about their ability to attain a goal or desired outcomes in mathematics. After completing the study, Sara shared how her confidence and goals in mathematics have changed. She stated as she was researching for her second presentation that she "started to see a lot more" and "things were clicking in [her] head." She was "starting to…have this understanding which was not something [she] had before." This experience "surprised" her since this was something that was never experienced "in math class...or just like doing math." Kendra shared that these presentation experience gave her "more confidence to present in front of strangers that [she] didn't have before." She "still struggle[d] with speaking in front of people who are professionals on the subject, but still feel[s] more confident even in this aspect. Each time [she] presented [she] learned how to be more confident and added on to each presentation." Laurel said her participation in study realize her ability to present to a "complex topic to a large audience." Oliver shared a similar comment. He said that "the talks…helped [him] tremendously when it comes to public speaking" and he is more prepared for his academic career since this is "something [he] will have to do in academia."

One of the purposes for SMS was to provide a more casual environment for students to learn how to participate in mathematics research and share their findings from their research. Both Oliver and Laurel shared that they felt that SMS gave them the "opportunity to explain topics in mathematics in a 'low stakes' environment" without the pressure for “doing the presentation for a grade”. This and the results from the Audience Perceptions Survey suggests that students may be more inclined to pursue personal investigations of mathematics in SMS without the stress of completing an assignment for a grade and develop the confidence to take on more complex forms of mathematical engagement. This is similar to what past research has found about fostering self-efficacy in less formal environment could further mathematical engagements.

Hackett and Betz's (1986) research implied that by strengthening a student's mathematics self-efficacy, that student might be more easily influenced to pursue a mathematics related college major. This suggests that self-efficacy could influence forms of engagement with mathematics and whether or not a student would choose to position themselves in mathematics. A person’s perceived self-efficacy influences thought patterns, emotional reactivity, choice behavior and task performance (Bandura, 1986). Bandura’s work has been extended to find that self-efficacy affects educational and career choices and differ between men and women (Hackett and Betz, 1981; Bandura, 1982). Seymour and Hewitt (1997) found that students in science, mathematics and engineers courses describe their classroom environments as cold, the instructors as unapproachable, and lectures did not welcome discussion. This can lead to students taking on a passive role in class (Bressoud, 1994) and contribute to avoidance of mathematics courses and affect engagement in mathematics classrooms and thus affect learning.

Perceptions of Mathematics

Perceptions of mathematics is defined by disposition towards aspects of mathematics that has been acquired by an individual through his or her own beliefs and experiences but can be changed (Eshun, 2004) or influenced by factors associated by the individual (self-efficacy, achievement, anxiety, motivation), by instructors or institutions (teacher knowledge, teacher attitudes, classroom management), or by environment (peers within community) (Mohamed & Waheed, 2011). Throughout the study, there was evidence of the student mathematicians' perceptions changing. Sara's initial motivations to participate in the study were solely for extra credit and viewed mathematics as "just like something [she] had to do." But through the study, her view of mathematics changed. She started to see mathematics as "exciting and made [her] happy or excited to do it." The process of research and presenting made her "want to know more" and "want to do more." She also compared this learning to what took place in her current math course.

> "I want to know more expand more. Before like in class I don't think I would be excited to do this. Like I know this. I can teach myself things. New things. Math things! This makes me want to know more...be proud of myself that I know more or could know more."

Sara's initial perceptions were like what Picker and Berry (2000) stated about mathematics. The widespread public image of mathematics is that it is difficult, cold, abstract, theoretical, and ultra-rational (Ernest, 2008). It also has the image of being remote and inaccessible to all, but a few extra-ordinary persons with 'mathematical minds.' From her experiences in SMS, she is now inspired "to do more."

Kendra's motivation to participate in the study was similar to Sara's. Initially, Kendra was participating for extra credit. But as she progressed, she found there were other benefits to participating. She mentioned in her first interview that she always wanted to learn to solve the Rubik's cube but never took the time to. In this study, she “not only learned a lot about the Rubik’s cube and how it functions” but “how it relates to everyday life.” She also started to see the seminar as “a community.” Kendra thought of SMS as more of a safe place to go than a worrisome place.”

Laurel already had a broad view of mathematics, but her perceptions of mathematics research have changed. Laurel thought mathematics research was restricted to only new and undiscovered mathematics. From her research for her presentation and other presentations in SMS, she believes “math research can like be unique and new, or it could be like all done research put into one presentation.”

Oliver believed SMS was a “positive force in my academic career.” Oliver perceived “the environment at [SMS] resembles that of a large research conference.” This made him think about what researching and presenting his future work would be like in a more professional setting. His perceptions of research however still did not change. He viewed his presentation as “nothing new” or noteworthy. He defined mathematics research as “at least going into at least a little bit of uncharted territory but then conveying that your research is a part of it.” Even though he did not have much room to evolve his perceptions in SMS, he said that this process made him reflect more on his own beliefs of mathematics and mathematics research.

Factors included in SMS was the opportunity for students to find their connection to mathematics and bring mathematics and their hobbies and interests together. Also, SMS allowed students to change their perceptions and find new values of mathematics. This was evident in all four presenters. These factors found in SMS and the results from the Audience Perception Survey align with the research on perceptions of mathematics. Mathematics should offer something that is personally engaging and useful or motivating in some other way (NCTM, 1989; Skovsmose, 1994). Mathematics educators believe that children learn more effectively when they are interested in what they learn and that they will achieve better in mathematics if they like mathematics (Suydam and Weaver, 1975).

Forms of Engagement

Forms of engagement in mathematics are activities that demonstrate how one can participate within the mathematical community or how they would enact their identity (Grandgenett, et al., 2009). In this study, there were evident developments in the presenters' forms of engagement. In Sara's interviews and post autobiography, she discussed how she changed her interactions with mathematics in her mathematics class. She said,

> "…like before I didn't think to always think to ask questions. But like now like after doing this I think like I think about questions more. Like before like when I would be in class I would just listen or just like write stuff down but now really think about it. Now like I think about what I need to know."

Also, these changes were seen in her presentations. In Sara's first presentation, Sara only gave background on Pythagoras, stated the Pythagorean Theorem briefly overviewed a few proofs and showed a numerical example. By her last presentation, Sara included a more extensive background of Pythagoras and his contributions to mathematics and demonstrated multiple proofs of the Pythagorean Theorem. She also mentioned that she was contemplating to attempt to find her proof. The presence of these changes and the statements she made about her engagements in mathematics, especially as a developmental mathematics student, suggest that these experiences in SMS played a significant role.

Kendra's presentations also showed that she was evolving her forms of engagement. In Kendra's first presentation, she only reviewed the process of solving the Rubik's cube as an algorithm. In her last presentation, she explained the formula for determining the number ways to solve a Rubik's cube and shared applications for the algorithm for solving the Rubik's cube. The move from stating math phrases to explaining a formula and understanding application demonstrates a change in engagement. Kendra also shared from her last interview that, "researching stuff built [her] confidence to do stuff on my own that was math related."

Laurel and Oliver's presentations both had advanced forms of engagement, however, there were significant changes in how the material presented. Laurel and Oliver were beginning to think about their presentation from a pedagogical point of view. From their interviews, both shared that they reflected on the audience's comprehension of the material. They both considered how communication and presentation affects understanding. Laurel said that by presenting her talk she was considering the "teaching aspect." Oliver stated that these presentations helped in his position as an undergraduate instructional assistant. He said,

> "As an educator, I have had to explain topics thoroughly. I cannot at all times work under the assumption that my students understand all the intricacies of the mathematics at hand. So, explaining higher – level mathematics to an audience who may know very little mathematics at [SMS] has helped me to bridge the gap between my knowledge and the student's level of understanding."

Kendra and Sara's experiences matched up to the purposes of SMS. SMS provided an opportunity for the students to gain experience in giving presentations and may encourage more students to do research. Furthermore, they could receive constructive feedback to better develop their ideas and find other students and perhaps professors to collaborate with, and Oliver and Laurel's experiences extended those purposes. The way a person participates in mathematics can develop one's identity and can be a determining factor for whether a study would continue to study mathematics (Cobb, 2004). The best way for one to learn mathematics would be to have multiple opportunities to practice methods like mathematical discourse, thus reinforcing certain behaviors (Greeno and MMAP, 1998). Have the presenters repeating presentations can reinforce their mathematical practices.

Furthermore, traditional classroom settings do not always allow for multiple opportunities for practicing. Students who learn in these traditional classrooms can be successful, but many students experience an important conflict between the practices in which they engaged, and their developing identities as people (Boaler, 2002). In the discussion-oriented classes, the students had formed very different relationships with mathematics. In SMS, students could have opportunities engaging in mathematical discussions and form their relationships with mathematics. Or in other words, by providing an informal environment for students to express themselves mathematically, there are substantial opportunities to positively change students' mathematical identities.

Implications

The Student Math Seminar had multiple opportunities for a student to enact their mathematical identity and to evolve that mathematical identity. In this study, the student presenters gave a series of three presentations. For general presenters, it is not required to present multiple times, but there is a marked difference in those that do. Just as practicing the same mathematical proof multiple times, knowledge is in reinforced and methods can become more efficient. Oliver said that giving the three presentations “felt similar to that of mathematical research in the sense that [he] was having to really work out all the details of [the] topic and prepare to explain it to a wider audience who may not know, fully, all the aspects that it contains.”

Additionally, SMS was an intervention that took place outside the classroom. Sara’s experiences in her current math class did not excite her to pursue deeper understandings of mathematical concepts nor did she have the confidence to pursue those deeper understanding on her own. But with this opportunity, she learned much about herself and her capabilities of learning mathematics. “... I want to know more...expand more. Before like in class I don't think I would be excited to do this.” This suggests that by providing this type of informal environment for students to express themselves mathematically, there are substantial opportunities to positively change students’ mathematical identities.

Furthermore, the organizing and planning of SMS created an advisory position that the student presenter could turn to for help. As the coordinator of SMS, I was playing the role of a person who could assist and advise students on their presentations, even the presenters who were not in the study. Many students who considered presenting, corresponded with me through email and met in my department office multiple times prior to presenting to ensure they were prepared to give a well-delivered talk to the type of audience attending SMS. Often times students would meet with me after the talk to get my feedback on their presentation and would discuss ideas for future topics. This was not a requirement for the other presenters, but frequently happened nonetheless. These interactions with other student presenters sustained the continuation of SMS and encouraged many to present.

In SMS, there was a need for a variety of topics to allow for students to observe the many applications and values of mathematics. As stated in earlier chapters, the chosen topic does not necessarily be based solely on mathematics but must address the mathematics involved in that topic. For example, a presentation could be given on the art of dance; however, the student must include mathematics somehow. The presentation could include how choreography includes movements of symmetry or represent a function or how dance could be incorporated into teaching mathematics. Students often choose topics that are related to their majors, current employment, and even hobbies. By discussing topics that they are passionate about, they tend to express how much more appreciation they have for the mathematics. It was important to broaden the scope of SMS to include any and all fields. By not restricting the type of students who participate or limiting the subjects and types of presentation, more students, particularly non-mathematics majors like Sara, will feel more invited to attend and even more so to present. Again, this reiterates the notion that an informal environment for students to express themselves mathematically could to positively change students' mathematical identities.

Limitations of the Study

There were some limitations and difficulties that occurred during the completion of this study. These included implementations of particular instruments, levels of participation, and possible researcher relationship impacts.

First off, there was a missed opportunity to implement a Post-Audience Perception Survey. There was a sufficient sample size to execute the structural equation modeling for the data collected from the Audience Perception Survey, however, this data, was only enough to provide a descriptive image of what the audience's mathematical identity was in that given semester. This did not provide any inclination to how those in the audience were evolving their mathematical identities by attending over a period. Additionally, a pre- and post- survey could help to determine if SMS provides an environment of learning about and understanding mathematics culture. Since the audience consists of multiple different students each week with most not attending on a regular schedule, it was difficult to track and measure a change of one individual's mathematical identity within one semester.

Another issue was that the presenters were a sample of convenience. It was quite often difficult to schedule a student to give one presentation, let alone give three presentations. This led to dependence on those who were just willing to dedicate themselves to a demanding schedule of meeting for interviews, preparing the autobiography documents prior to the interviews, and setting aside hours to create, reflect on, and present their math talks. The most obvious criticism about convenience sampling is sampling bias and that the sample is not representative of the entire population. The two BSMs may not be what could be considered the typical non-math major. These two students could have been what are considered an over-achieving student, or they may have had some other exterior motives, such as a great deal of extra credit to participate in a mathematics seminar or being rewarded for doing well. The BSMs' instructors shared that they were offering extra credit; however, the compensation that the participants were given did not seem to merit the amount of time and work these students were asked to do. Kendra's professor only added a percentage of one full quiz grade based on the percentage of tasks she completed for the entire study. Sara's instructor, only provided up to 5 quiz points for each presentation the student gave, and those points were dependent on the amount of work it appeared that she put into the presentation and the level of the forms of engagement in the presentation. Based on the instructor's grade scale for the course, each presentation a half of a point of her overall grade. The two ASMs, Oliver and Laurel, claimed to have not been offered extra credit.

Also, for the two ASMs, it may have been the situation where these students were trying to provide support to the researcher and not necessarily to gain mathematics presentation experience. There was a specific comment made by Oliver that stated that he was aware that data was being collected on his presentations and he sometimes was concerned how his presentations were going to influence the research. This could have affected how Oliver developed his presentations and which forms of engagement he decided to include at each stage of the study to provide the results that were hoped for. Also, this could have been the case in the other three participants. Kendra mentioned that through the course of the study, she began to see me as someone she could trust and ask for help. She also mentioned she started to see me as friend in the means that she thought

I was supporting her efforts to do well in the seminar and in her classes. This could have meant she could have put in extra efforts, to make sure she would not let me down.

Furthermore, based on my five years of experience, it was difficult to get many mathematics majors to present in SMS. Many of those students said that their course schedules were far too demanding to invest the time to needed to prepare a presentation, especially if it was not grade dependent for a course. Thus, in future studies, specific motivations for the participants, particularly if they are willing to present multiple times, should be investigated.

Another limitation to this research is that this is the only study on the effects of a student math seminar. There have been no outside resources that could have supported the findings found here. This could lead to the question, was the student population already predisposed to the idea of mathematics culture before participating in the seminar, meaning that the audience's mathematically identity for this particular seminar was unique to other audiences that may be in future attendance or that this mathematical identity is unique to the other audiences at other universities. Additionally, with the presenters, this may have been an exceptional group of students. It will need to be confirmed that these results could be established by other students with similar demographics. Therefore, this seminar could be implemented at another university to see if these findings could be replicated.

Future Research

Some means of future research could include addressing the limitations that occurred in this study. One possible investigation would be to conduct a pre- and post-Audience Perception Survey. Based on my review of multiple past semesters' sign-in sheets, it could be feasible to conduct this type of pre- and post-survey over a collection of semesters. There were a large number of students who did attend at least two consecutive semesters. With these type of "dedicated" attendants, it could potentially be determined if simply attending the presentations could evolve their mathematical identity.

Additionally, this survey could have been adapted to be distributed campus wide to see measure the mathematical identity of the entire campus and compare that to those who attended the seminar. This could lead to answering a new research question, how does SMS provide an environment of learning about and understanding mathematics culture? An environment of learning is defined as a location or facility that learning can take place like a classroom, tutoring lab, seminar, etc. This could indicate this seminar has an influence on mathematical identity campus wide.

Moreover, Oliver mentioned how he connected this experience to teaching. He felt that doing these presentations made him feel less intimidated when teaching a class as a UIA and made him more aware of what students might be thinking. Thus, these experiences made him consider his position and the role he plays in teaching other students. This implies that we could look into how presenting these types of talks could influence pre-service teachers. These types of presentation could be incorporated in the classroom with pre-service teachers. By including research and engagements of mathematics discourse, perceptions of future mathematics teachers could be affected. Particularly since many teachers' perceptions of mathematics would influence their future students' perceptions of mathematics. This could help lessen stereotypes of mathematics like the ones mentioned in Chapter 2. By changing teachers' definitions of mathematics and their understanding of how mathematics is used, they may be able to have students better aligned perceptions of mathematics.

Furthermore, the concepts of mathematical identity and mathematics culture could be incorporated in methods courses for pre-service teachers and even in the classrooms of in-service teachers. This could help their future students have better aligned perceptions of mathematics. By understanding how students picture mathematicians and how we could change those images, we can broaden their "thinking about their roles as mathematicians" (Rock & Shaw, 2000). Furthermore, the positions taken up by students reflect an enacted identity. For example, imagine a classroom in which a teacher consistently refers to her students as "mathematicians." If teachers could enact these positional identities, they could encourage students to enact the desired identity (Bishop, 2012). This could lead to more students having more aligned mathematical identities. Thus, by incorporating more non-traditional practices, such as having students take on more of an authoritative position as they would with giving presentations, students could develop more aligned mathematical identities.

Another study that could be considered is to see how this seminar aides in the retention of mathematics majors. Kendra mentioned that SMS became a place where she saw her friends and to make new friends. She started to see the organizers and regular attendants as friends and colleagues. SMS was now being viewed as a community and the more integrated a student becomes in the community, the more likely he or she is to remain (Tinto, 1987; Tennant, 2012). Using the model proposed by Tinto (1987) and used by Tennant (2012), the role SMS plays in the student's perceptions of their mathematics experience and the decision to persist in or to leave the field could be examined.

Tennant (2012) presented a model for Tinto's (1987) understanding of undergraduate attrition by looking at two main factors that influence departure. That is

1. The entering student brings in the intentions and commitments.
2. The interactions with the institution, adjustment to the campus community, academic difficulties, and personal feelings of isolation may affect a student's decision to leave college.

Both these social and academic integrations are the leading influences on student persistence. This integration occurs over time and how well the student can incorporate themselves into the college community. It could be considered to look at the seminar in this context by regarding the seminar as an influence on student persistence to remain in mathematics as opposed to just remaining in college. The model as seen in Figure 57 describes the lens this study could be viewed through.

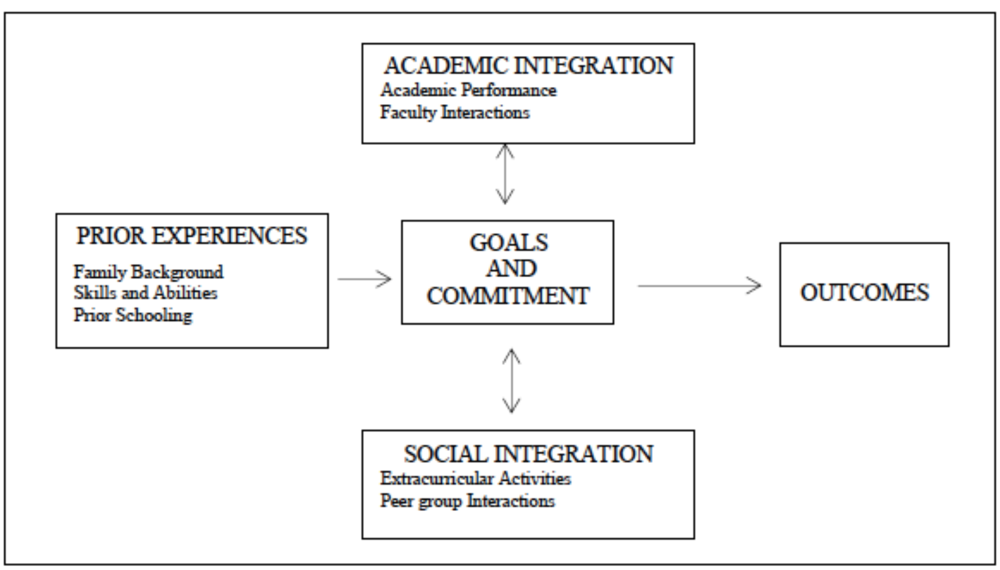

Figure 57. Tennant's (2012) model of Student Outcomes.

Just as in SMS, students bring in prior experiences into the seminar and can reflect on those experience when attending and presenting. There is also an academic integration by including forms of engagement in their presentations and from feedback from faculty and researcher members. Additionally, there are social integration in the seminar by the presenter engaging with the audience by asking them questions and including them in activities taking place in the presentation. From this, it could be seen that the participants have their own goals and levels of commitment that being affected by these interactions. Then of course the outcome would be their willingness to remain within the culture of mathematics and its practices.

Another future study to consider is the mentor/mentee relationship that was developed between the participants and myself. The mentor relationship can significantly enhance development in early adulthood (Kram, 1983). Throughout the study and even including the time following the study, the participants shared that they were beginning to see me playing an advisory and mentoring role in their academic careers. Some have asked advice on applying for graduate school, letters of recommendations for scholarships and even which courses they may want to take in the upcoming semesters. Never once did I tell them that I was one to take on this role. But since they were viewing our interactions in the study as a means of getting to know each other, they began to see me as a mentor and friend. These mentoring relationships help shape not only intellectual interests, but also how we learn and what we do with what we learn (McKinsey 2016; Lechuga 2011). From my own experiences and McKinsey's research, I would like to further investigate how working with students in SMS is taking on a mentoring role and how that role is affecting both the mentor and student.

Another observation of the seminar is the trend in attendance as seen in Figure 58. When SMS first started, the average size of the audience was 9.8 attendees. By the seventh semester, attendance grew by a 937% in average attendance. There has been a definite change in the mathematics culture within the post-secondary community, and I believe it is worthwhile to investigate the reason for this change.

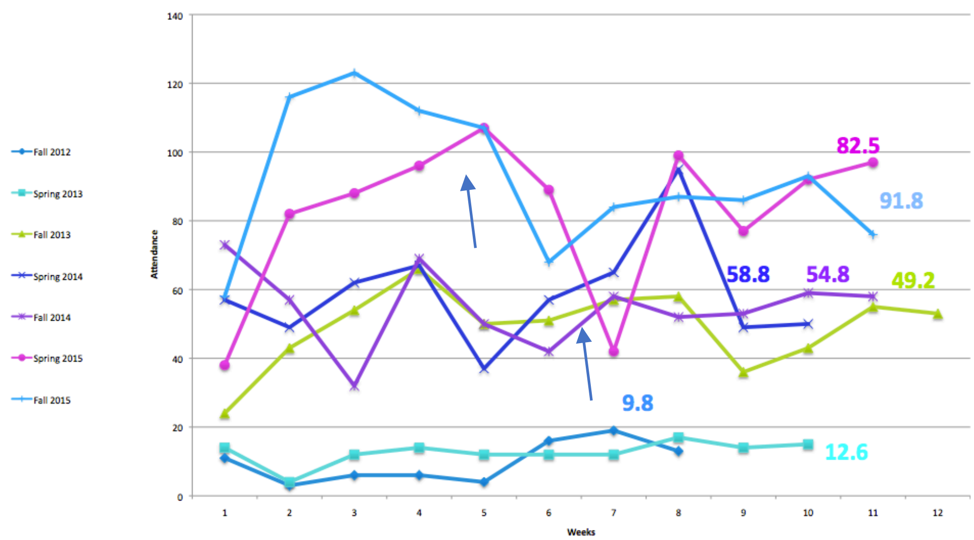

Figure 58. Attendance trends in SMS over the first seven semesters. This shows a dramatic change in student attendance over the semesters of its implementation.

From these trends, I would like to investigate what type of students are being drawn into this seminar, and how, particularly after the periods of drastic increases. These periods include the time between Spring 2013 and Fall 2013 and the time between Fall 2014 and Spring 2015. Furthermore, I would like to see why certain types of majors are actively participating in the seminar by presenting or attending. From an initial glance, the large majority of those in attendance are business-type majors (Finance, Marketing, etc.) and nursing. This population of students was not expected to the greater part of the audience. A study could be done by investigating the mathematics courses that they are currently enrolled and possibly interview a selection of students, particularly those who regularly attend to understand their motivations for participating. Additionally, I would like to see how those students are using their experiences in SMS in their current math courses and perhaps any personal reflections they have done in regard to their academic and career goals.

APPENDIX SECTION

APPENDIX A

TEXAS STATE UNIVERSITY
SAN MARCOS
The rising STAR *of Texas*

Research Participation Consent Form

Dear [Student Math Seminar] Participant,

You are asked to participate in a research study conducted by Joni Schneider (js1824@txstate.edu) from the Mathematics Department of Texas State University. Your participation in this study is entirely voluntary. You have been asked to participate because of your attendance in the Talk Math 2 Me seminar. Please read the information below and ask questions about anything you do not understand before deciding whether or not to participate.

The purpose of the study is to investigate how the seminar [Student Math Seminar] can influence an individual's perception of mathematics. If you volunteer to participate in this study, you may be asked to participate in interview(s) with the researcher to get further feedback on this seminar.

All of your responses will be held in confidence. Only the researcher involved in this study and those responsible for research oversight will have access to any information that could identify your responses. When I publish any results from this study, I will do so in a way that does not identify you unless I get your specific permission to do so. I may also share the data with other researchers to check the accuracy of my conclusions but will only do so if I am certain that your confidentiality is protected.

If you have any questions concerning the nature of the research, please contact me at js1824@txstate.edu. This project, EXP2014E151852D, was approved by the Texas State IRB on September 30, 2014. Pertinent questions or concerns about the research, research participants' rights, and/or research-related injuries to participants should be directed to the IRB chair, Dr. Jon Lasser (512-245-3413 - lasser@txstate.edu) and to Becky Northcut, Director, Research Integrity & Compliance (512-245-2314 - bnorthcut@txstate.edu).

Thank you for taking the time to help me with my research. It is greatly appreciated. If you have no further questions, please sign the Agreement to Participate below acknowledging your consent to participate in this research project.

Sincerely,

Joni Schneider

Agreement to Participate:

I have read the above information, have had the opportunity to have any questions about this study answered and agree to participate in this study.

________________________________ ______________

(printed name) (date)

____

(signature)

Pilot Survey A

Audience Participant Survey

Texas State University

I. Participant Information

Name: ______________________________ Net ID: ______________________________

Major: ______________________________ Classification: ______________________

(Freshman, Sophomore, Junior, Senior, Graduate Student, Faculty, Other (specify))

Gender: ____________________________ Do you plan to attend graduate school?________

(Male or Female) (Yes or No)

II. Participation in [Student Math Seminar] (SMS)

For the following questions circle your answer.

1) Why do you attend [SMS]? (Circle all that apply.)

Extra Credit Required for Class You enjoy coming

Other: ____

2) How often do you talk about [SMS] during the week?

Never 1-3 times 4-6 times More than 6

III. Audience Perception of Mathematics

	Strongly Disagree		Neutral		Strongly Agree
1) [SMS] is beneficial to your understanding of math.	1	2	3	4	5
2) [SMS} increases your appreciation of math.	1	2	3	4	5
3) Even though a math course is required in all majors, you still think math is important to your major.	1	2	3	4	5
4) Math is useful in everyday life.	1	2	3	4	5
5) Math is just memorizing formulas.	1	2	3	4	5
6) When exploring in math, you can only discover something already known.	1	2	3	4	5
7) Mathematicians don't care about other fields of study.	1	2	3	4	5

	Strongly Disagree		Neutral		Strongly Agree
8) If you have questions about a math concept, you feel that you don't understand anything about that topic.	1	2	3	4	5
9) If there are unsolved problems in math, one day you could be capable of solving one of them.	1	2	3	4	5
10) Concepts in one topic of math (i.e. algebra) will not be useful in other math topics (i.e. geometry).	1	2	3	4	5
11) Using formulas well is enough to understand the math concept behind the formula.	1	2	3	4	5
12) You use diagrams and graphs when solving math problems.	1	2	3	4	5
13) Everything in math has been discovered.	1	2	3	4	5
14) If you cannot solve a problem quickly, then spending more time won't help.	1	2	3	4	5
15) Students cannot make new math discoveries. They can only study discoveries by mathematicians.	1	2	3	4	5
16) If you knew every single formula, you could easily solve any math problem.	1	2	3	4	5
17) Math for the most part is made up of procedures and facts.	1	2	3	4	5
18) Mathematicians do what students do, just with bigger numbers.	1	2	3	4	5
19) Mathematicians are hired to make precise measurements and calculations for scientists.	1	2	3	4	5

IV. Open Response Question

Please respond to the following questions.

1) How would you define mathematics research?

2) Do you consider yourself a "math" person? Why or Why not?

3) Please give any overall comments about the seminar, “[Student Math Seminar],” or any general comments about mathematics.

Thank you for your participation in this survey. If you would like to be interviewed about your experiences in this seminar, please leave your email or phone number.

Email:___________________________________

Phone: __________________________________

APPENDIX B

TEXAS STATE UNIVERSITY
SAN MARCOS
The rising STAR *of Texas*

Research Participation Consent Form

Dear [Student Math Seminar] Participant,

You are asked to participate in a research study conducted by Joni Schneider (js1824@txstate.edu) from the Mathematics Department of Texas State University. Your participation in this study is entirely voluntary. You have been asked to participate because of your attendance in the [Student Math Seminar]. Please read the information below and ask questions about anything you do not understand before deciding whether or not to participate.

The purpose of the study is to investigate how the seminar [Student Math Seminar] can influence an individual's perception of mathematics. If you volunteer to participate in this study, you may be asked to participate in interview(s) with the researcher to get further feedback on this seminar.

All of your responses will be held in confidence. Only the researcher involved in this study and those responsible for research oversight will have access to any information that could identify your responses. When I publish any results from this study, I will do so in a way that does not identify you unless I get your specific permission to do so. I may also share the data with other researchers to check the accuracy of my conclusions but will only do so if I am certain that your confidentiality is protected.

If you have any questions concerning the nature of the research, please contact me at js1824@txstate.edu. This project, EXP2014E151852D, was approved by the Texas State IRB on September 30, 2014. Pertinent questions or concerns about the research, research participants' rights, and/or research-related injuries to participants should be directed to the IRB chair, Dr. Jon Lasser (512-245-3413 - lasser@txstate.edu) and to Becky Northcut, Director, Research Integrity & Compliance (512-245-2314 - bnorthcut@txstate.edu).

Thank you for taking the time to help me with my research. It is greatly appreciated. If you have no further questions, please sign the Agreement to Participate below acknowledging your consent to participate in this research project.

Sincerely,

Joni Schneider

Agreement to Participate:

I have read the above information, have had the opportunity to have any questions about this study answered and agree to participate in this study.

____________________________________ _______________

(printed name) (date)

____

(signature)

Pilot Survey B
Presenter Participant Survey

I. Participant Information

Name: ______________________________ Net ID: ______________________________

Major: ______________________________ Classification: ________________________
(Freshman, Sophomore, Junior, Senior, Graduate Student, Faculty, Other (specify))

Gender: ____________________________ (Male or Female)

Do you plan to attend graduate school?_________ (Yes or No)

II. Participation in Talk Math 2 Me (TM2M)

For the following questions circle your answer.

1) Why do you attend TM2M? (Circle all that apply.)
Extra Credit Required for Class You enjoy coming
Other: ____

2) How often do you talk about TM2M during the week?
Never 1-3 times 4-6 times More than 6

	Strongly Disagree		Neutral		Strongly Agree
3) TM2M is beneficial to your understanding of math.	1	2	3	4	5
4) TM2M increases your appreciation of math.	1	2	3	4	5

III. Audience Perception of Mathematics

Circle your response to each statement.

Beliefs	Strongly Disagree		Neutral		Strongly Agree
3) Even though a math course is required in all majors, I still think math is important to my major.	1	2	3	4	5
4) Math is useful in everyday life.	1	2	3	4	5
5) When exploring in math, I can only discover something already known.	1	2	3	4	5
6) Mathematicians don't care about other fields of study.	1	2	3	4	5
7) Mathematics is a type of language.	1	2	3	4	5
8) Mathematics is a type of science.	1	2	3	4	5
9) Mathematics is art.	1	2	3	4	5

	Strongly Disagree		Neutral		Strongly Agree
10) Everything in math has been discovered.	1	2	3	4	5
11) Concepts in one topic of math (i.e. algebra) will not be useful in other math topics (i.e. geometry).	1	2	3	4	5
12) I study math because I know how useful it is.	1	2	3	4	5
13) Knowing math will help me earn a living.	1	2	3	4	5
14) Math is exciting.	1	2	3	4	5
15) Most typical real-world problems do not always have a correct solution.	1	2	3	4	5
16) Mathematicians do what students do, just with bigger numbers.	1	2	3	4	5
17) Mathematicians are hired to make precise measurements and calculations for scientists.	1	2	3	4	5
18) I would never think about math if I didn't have to.	1	2	3	4	5
19) Studying math for any other purpose than for a class would be a waste of my time.	1	2	3	4	5
Self-Efficacy					
20) I am sure that I can learn math.	1	2	3	4	5
21) If I have questions about a math concept, I feel that I don't understand anything about that topic.	1	2	3	4	5
22) I don't think I could do advanced math.	1	2	3	4	5
23) I trust myself when I do math.	1	2	3	4	5
24) I believe that the only way to know if my answer is right or that a statement in math is true is if my teacher tells me so or if I find it in a book.	1	2	3	4	5
25) If I knew every single formula, I could easily solve any math problem.	1	2	3	4	5
26) Math scares me.	1	2	3	4	5
27) I feel comfortable giving a math presentation to my peers.	1	2	3	4	5
28) I feel comfortable giving a math presentation to experts.	1	2	3	4	5

	Strongly Disagree		Neutral		Strongly Agree
29) I am good at performing calculations.	1	2	3	4	5
30) I am good at proving mathematics.	1	2	3	4	5
31) I am good at solving math problems.	1	2	3	4	5
32) I am capable of discovering new mathematics.	1	2	3	4	5
33) The only way I can show that I know math is by getting good grades in my math classes.	1	2	3	4	5
34) Math is my worst subject.	1	2	3	4	5
Position					
35) People who study math usually have weird or strange personalities.	1	2	3	4	5
36) Others would take me seriously if I told them that that I would be interested in pursuing a career in science or mathematics.	1	2	3	4	5
37) My friends trust me for help when doing math homework or studying for a math exam.	1	2	3	4	5
38) It would make me happy to be recognized as an excellent student in math.	1	2	3	4	5
39) I believe my math teachers respect my abilities.	1	2	3	4	5
40) Students cannot make new math discoveries. They can only study discoveries by mathematicians.	1	2	3	4	5
41) I feel comfortable talking about mathematics with those who are good at math.	1	2	3	4	5
42) My teachers think I can excel in math.	1	2	3	4	5
43) If others saw me as someone who is good at math, I would be afraid that they might think I would have a weird or strange personality.	1	2	3	4	5
44) When exploring in math, I can only discover something already known.	1	2	3	4	5
45) My teacher wants to know how I solve a problem.	1	2	3	4	5
46) It doesn't matter what grade I get in a math course, as long as I understand the material.	1	2	3	4	5
47) Seeing my peers give presentations encourages me to give one myself.	1	2	3	4	5

Engagement	Strongly Disagree		Neutral		Strongly Agree
48) Math for the most part is made up of procedures and facts.	1	2	3	4	5
49) Math is just memorizing formulas.	1	2	3	4	5
50) If I cannot solve a problem quickly, then spending more time won't help.	1	2	3	4	5
51) When a question is left unanswered from a math class, I continue to think about it afterwards	1	2	3	4	5
52) Using formulas well is enough to understand the math concept behind the formula.	1	2	3	4	5
53) Understanding the process of finding a solution is more important than finding the correct solution.	1	2	3	4	5
54) I use diagrams and graphs when solving math problems.	1	2	3	4	5
55) I would rather someone give me the solution to a difficult math problem than work it out for myself.	1	2	3	4	5
56) Finding a solution to a problem is more important than the process.	1	2	3	4	5

IV. Open Response Questions

Please respond to the following questions to the best of your ability.

1) How would you define mathematics research?

2) Do you consider yourself a "math" person? Why or Why not?

3) Describe what comes to your mind when you think of the word *mathematician.*

4) Please give any overall comments about the seminar, "[Student Math Seminar]."

Thank you for your participation in this survey. If you would like to be interviewed about your experiences in this seminar, please leave your email or phone number.

Email:____________________________________

Phone: ___________________________________

APPENDIX C

CONSENT FORM – SURVEY

Developing Mathematical Identity in Post-Secondary Students

Investigator: Ms. Joni Schneider js1824@txstate.edu (512) 245-6925
Texas State University, Department of Mathematics, 601 University Dr., San Marcos, TX 78666

You are invited to participate in a research study. The purpose of this study is to investigate how participation in the Student Math Seminar program changes the way students view themselves and their mathematics abilities. To learn about this, we are asking you to complete a brief survey. It should take about 20 minutes to finish the survey.

The survey questions ask about your interests and confidence in mathematics, and how you and others see yourself – for example, "Do your parents/relatives/friends see you as a math person?" We do not think there are any serious risks to taking the survey, but some participants may feel uneasy responding to personal or introspective questions. You may choose not to answer any question(s) for any reason.

There are no direct benefits to you for participating in this research. You will not receive anything for participating. However, society may benefit: as a result of this project, investigators will have a better understanding of how participation in a Student Math Seminar changes a student's "mathematical identity". These results will be used to help other mathematics departments develop more effective undergraduate programs.

We will ask for your name on the survey so that we can match responses over time. We will keep the surveys in a locked file cabinet at Texas State University during the study and for up to five years following the end of the study. Only researchers involved in this project will have access to the surveys. Your name will never be used when the researchers report study results.

Your participation is voluntary. You can decide not to participate, and there is no penalty. If you decide to participate, you may change your mind and stop participating at any time without penalty. If you change your mind, you will not lose any benefits to which you are otherwise entitled.

This project 201603 was approved by the Texas State IRB on October 31, 2016 Pertinent questions or concerns about the research, research participants' rights, and/or research-related injuries to participants should be directed to the IRB chair, Dr. Jon Lasser (512-245-3413 - lasser@txstate.edu) and to Becky Northcut, Director, Research Integrity & Compliance (512-245-2314 - bnorthcut@txstate.edu). Participants in the project may obtain a summary of the results of the project by contacting the investigator, Joni Schneider, Department of Mathematics, Texas State University-San Marcos, 601 University Dr., San Marcos, TX 78666.

CONSENT

Your signature on this form shows that you understand the information about participating in this research project, and you agree to participate. This does not waive your legal rights. This does not release the investigators, sponsors, or involved institutions from their legal and professional responsibilities.

I have read the information above and I agree to participate in this study. I have received a copy of this form.

Participant name (print): ____

Participant signature: ____

Date: ____________________________________

Student Math Seminar
Audience Perception Survey
Texas State University

I. Participant Information

Name: ______________________________ Net ID: ______________________________

Major: ______________________________ Classification: ______________________
(Freshman, Sophomore, Junior, Senior, Graduate Student, Faculty, Other (specify))

Gender: ______________________________ (Male or Female)

Do you plan to attend graduate school?________ (Yes or No)

Current Math Course(s): ____

II. Participation in Student Math Seminar

For the following questions circle your answer.

1) Why do you attend [SMS]? (Circle all that apply.)
   Extra Credit   Required for Class   You enjoy coming
   Other: ____

2) How often do you talk about [SMS] during the week?
   Never   1-3 times   4-6 times   More than 6

	Strongly Disagree		Neutral		Strongly Agree
3) SMS is beneficial to your understanding of math.	1	2	3	4	5
4) SMS increases your appreciation of math.	1	2	3	4	5

III. Audience Perception of Mathematics

Circle your response to each statement.	Strongly Disagree		Neutral		Strongly Agree
1) Even though a math course is required in all majors, I still think math is important to my major.	1	2	3	4	5
2) Math is useful in everyday life.	1	2	3	4	5
3) Math is just memorizing formulas.	1	2	3	4	5
4) When exploring in math, I can only discover something already known.	1	2	3	4	5
5) Mathematics is a type of language.	1	2	3	4	5

	Strongly Disagree		Neutral		Strongly Agree
6) Mathematics is a type of science.	1	2	3	4	5
7) Mathematics is art.	1	2	3	4	5
8) Everything in math has been discovered.	1	2	3	4	5
9) Concepts in one topic of math (i.e. algebra) will not be useful in other math topics (i.e. geometry).	1	2	3	4	5
10) I study math because I know how useful it is.	1	2	3	4	5
11) People who study math usually have weird or strange Personalities.	1	2	3	4	5
12) Math for the most part is made up of procedures and facts.	1	2	3	4	5
13) I am sure that I can learn math.	1	2	3	4	5
14) If I have questions about a math concept, I feel that I don't understand anything about that topic.	1	2	3	4	5
15) I don't think I could do advanced math.	1	2	3	4	5
16) I trust myself when I do math.	1	2	3	4	5
17) I believe that the only way to know if my answer is right or that a statement in math is true is if my teacher tells me so or if I find it in a book.	1	2	3	4	5
18) If I cannot solve a problem quickly, then spending more time won't help.	1	2	3	4	5
19) Others would take me seriously if I told them that that I would be interested in pursuing a career in science or mathematics.	1	2	3	4	5
20) Math scares me.	1	2	3	4	5
21) I feel comfortable giving a math presentation to my peers.	1	2	3	4	5
22) I feel comfortable giving a math presentation to experts.	1	2	3	4	5
23) I am good at performing calculations.	1	2	3	4	5
24) I am good at proving mathematics.	1	2	3	4	5
25) I am good at solving math problems.	1	2	3	4	5
26) I am capable of discovering new mathematics.	1	2	3	4	5

	Strongly Disagree		Neutral		Strongly Agree
27) Math is my worst subject.	1	2	3	4	5
28) My friends trust me for help when doing math homework or studying for a math exam.	1	2	3	4	5
29) When a question is left unanswered from a math class, I continue to think about it afterwards.	1	2	3	4	5
30) The only way I can show that I know math is by getting good grades in my math classes.	1	2	3	4	5
31) I would never think about math if I didn't have to.	1	2	3	4	5
32) It would make me happy to be recognized as an excellent student in math.	1	2	3	4	5
33) I believe my math teachers respect my abilities.	1	2	3	4	5
34) Students cannot make new math discoveries. They can only study discoveries by mathematicians.	1	2	3	4	5
35) I feel comfortable talking about mathematics with those who are good at math.	1	2	3	4	5
36) My teachers think I can excel in math.	1	2	3	4	5
37) If others saw me as someone who is good at math, I would be afraid that they might think I would have a weird or strange personality.	1	2	3	4	5
38) When exploring in math, I can only discover something already known.	1	2	3	4	5
39) My teacher wants to know how I solve a problem.	1	2	3	4	5
40) It doesn't matter what grade I get in a math course, as long as I understand the material.	1	2	3	4	5
41) Seeing my peers give presentations encourages me to give one myself.	1	2	3	4	5
42) It doesn't matter what grade I get in a math course, as long as I understand the material.	1	2	3	4	5
43) Math is exciting.	1	2	3	4	5
44) Using formulas well is enough to understand the math concept behind the formula.	1	2	3	4	5
45) I use diagrams and graphs when solving math problems.	1	2	3	4	5

	Strongly Disagree		Neutral		Strongly Agree
46) I would rather someone give me the solution to a difficult math problem than work it out for myself.	1	2	3	4	5
47) Me studying math for any other purpose than for a class would be a waste of my time.	1	2	3	4	5
48) If I knew every single formula, I could easily solve any math problem.	1	2	3	4	5

IV. Open Response Questions

Please respond to the following questions to the best of your ability.

1) How would you define mathematics research?

2) Do you consider yourself a "math" person? Why or Why not?

3) Describe what comes to your mind when you think of the word *mathematician.*

4) Please give any overall comments about the seminar, "[Student Math Seminar]."

Thank you for your participation in this survey. If you would like to be interviewed about your experiences in this seminar, please leave your email or phone number.

Email:___________________________________

Phone: ___________________________________

APPENDIX D

CONSENT FORM – INTERVIEW
Developing Mathematical Identity in Post-Secondary Students

Investigator: Ms. Joni Schneider js1824@txstate.edu (512) 245-6925
Texas State University, Department of Mathematics, 601 University Dr., San Marcos, TX 78666

PURPOSE AND BENEFITS

You are invited to participate in a research study. The purpose of this study is to investigate how an environment like the one provided in Student Math Seminar (SMS) can affect one's mathematical identity by presenting and by attending. The factors of identity that will be observed in this study will be position, self-efficacy, attitudes and values of mathematics, and forms of mathematical engagement. To learn about this, we are asking to use video-recorded interviews with you for research. The video records will be used to assess how your view of yourself and your ways of interacting in the mathematics community change through participation in SMS. We will also look at other records from your participation in SMS (for example, math autobiography or survey responses) to try to understand your experience as well as we can. The researchers will use these records to do their research.

PROCEDURES

If you agree to participate in this study, you will be video-recorded while being interviewed about your experience as a presenter in SMS. The interview will last about 60 minutes. Examples of questions you might be asked are: 1) What have your classroom experiences in math been like? 2) How do you prepare for a math presentation for an audience of your peers? The video records will be used to study how students' visions about themselves and their ability to participate in a community of mathematicians change in the course of their participation in SMS. You also allow us to examine your math autobiography and surveys, and to use these as part of our research.

The video recordings, math autobiography, and survey data will be stored indefinitely in a password protected electronic database. The data will not be destroyed. Access to these materials will be restricted to science education researchers and teacher educators who can prove they are using the information for research and/or professional development purposes and who promise to abide by the privacy requirements. Professional articles that will be written about this research will never include your name or reference anything that indicates your identity.

RISKS AND DISCOMFORTS

Some people feel uneasy when being video-recorded. Some people may feel uncomfortable answering introspective or social/familial questions.

BENEFITS

As a result of this project, investigators will have a better understanding of how participation in a student math seminar changes a student's "mathematical identity". These results will be used to help other mathematics department develop more effective undergraduate programs. Furthermore, by being interviewed, you are participating in a reflective process about your presentations which could improve future presentations and may improve communications skills.

CONFIDENTIALITY

Your name will not be collected or used on camera. The information in the study videos may be transcribed; if so, there will be no link between your name and the transcript. Information you submitted in surveys, your math autobiography, and presentations may also be used for research. The study information will be kept safe. Your name and other unique identifying information will never be used when the researchers report study results.

In some cases, video data from the project, transcripts of the video, or quotes from other written documents you have submitted will appear in presentations and publications for researchers and educators, or in materials about teaching that will be viewed by teachers and by other researchers. The researchers will keep data from this project in locked (or encrypted and password-protected) storage. The only time anyone other than the researchers will see this data will be in presentations and publications for researchers and educators.

If you choose to participate, you will not be identified by name in any publication or presentation. Still, giving consent involves this minimal risk: When you appear in a video episode used in a presentation or publication, it is possible that someone you know will recognize you.

The study data may be used in future research, in presentations, or for teaching.

CONTACT

This project 2016031 was approved by the Texas State IRB on October 31, 2016. Pertinent questions or concerns about the research, research participants' rights, and/or research-related injuries to participants should be directed to the IRB chair, Dr. Jon Lasser (512-245-3413 - lasser@txstate.edu) and to Becky Northcut, Director, Research Integrity & Compliance (512-245-2314 - bnorthcut@txstate.edu). Participants in the project may obtain a summary of the results of the project by contacting the investigator, Joni Schneider, Department of Mathematics , Texas State University-San Marcos, 601 University Dr., San Marcos, TX 78666.

PARTICIPATION AND ALTERNATIVES TO PARTICIPATION

Your participation in this project is voluntary. You can decide not to participate, and there is no penalty. If you decide to participate, you may change your mind at any time without penalty. If you change your mind, you will not lose any other benefits (that aren't related to the study). If you decide to stop participating in the study before data collection is completed, your data will be discarded, and will not be used in the study. Likewise, the Researcher may end your participation in the study at any time.

CONSENT

Your signature on this form shows that you understand the information about participating in this research project, and you agree to participate. This does not waive your legal rights. This does not release the investigators, sponsors, or involved institutions from their legal and professional responsibilities.

I have read the information above and I agree to participate in this study. I have received a copy of this form.

Participant name (print): ____

Participant signature: ____

Date: ___________________________________

APPENDIX E

Interview Protocols

Presenter Interview Protocol (PIP)

1. What math courses have you taken? Taking?
2. Year? Major? How did you choose your major?
3. What is the topic of your presentation?
4. How did you decide your topic?
5. Tell me how you went about researching your topic.
6. Tell me about how your learning experiences when researching your topic differ/don't differ from your learning experiences in the classroom.
7. Do you feel that there are any benefits to participating in the seminar?
8. Do you feel that you could be capable of presenting more "elaborate" research one day?
9. Do you feel that you belong to the "math world?" Or that one day you might?

Presenter Interview Protocol After Presentation

10. How do you feel about your presentation?
11. Do you think it was well organized?
12. Do you think you explained your information clearly?
13. Do you think the audience found the presentation stimulating? How could you tell?
14. What would you do differently if you were to give the presentation again?
15. Do you think that by presenting, you can call yourself a math researcher?
16. Did you find giving the presentation a stressful experience? Or did you find it to be easy?

APPENDIX F

Math Identity Protocol

This protocol will be used to structure the analysis for each data set.

Stage 1:
Familiarize self with definitions.

Definitions from Data Collection:

- *Data Set*- math autobiography or the transcriptions from an interview or presentation
- *Extraction*- a stand-alone phrase was anything that was a complete thought without having to be a complete sentence.
  Ex:
    - "Even though this was hard for me to sometimes get,…"
    - "…I still had fun."

  Note: These two statements were from the same sentence but were considered as two different extractions since they addressed different aspects from the theoretical framework.
- *Aspect*-a component from the theoretical framework. Defined below.
    - *Position*
    - *Self-Efficacy*
    - *Perceptions of Mathematics*
    - *Forms of Engagement*
- *Nothing Aspect*- items like utterances or conversations outside the scope of the study
  Ex:
    - "…it's like…"
    - "I'm going home this weekend."
- Background Aspect-extractions that address subject's self-descriptors, family life, etc.
  Ex:
    - "…my dad is an engineer."
    - "…I went to a small high school."
    - "I always liked to read."

Aspects from Framework

- *Position* – where one views their location or where others view one's location within or outside the mathematical community. Furthermore, if considering their location within the community, what role do they believe they will take on (authoritative/expert or compliant/novice) (Wegner, 1998; Boaler & Greeno, 2000).
  Ex:
    - "My friends always asked me for help on their homework."
    - "My teachers placed me into the advanced class."

- *Self-Efficacy* – the personal conviction that an individual has about their own ability to attain a goal or desired outcomes in mathematics (Howard, 2015).
  Ex:
    - "Even though this was hard for me to sometimes get,"
    - "…so I can say that I feel very comfortable in math classes"
- *Forms of Engagement in Mathematics* – activities that demonstrate how one can participate within the mathematical community or how they would enact their identity (Grandgenett, et al., 2009)
  Ex:
    - "We did a lot proofs,"
    - "Now plug the numbers into the formula."
- *Perceptions of Mathematics* – disposition towards aspects of mathematics that has been acquired by an individual through his or her own beliefs and experiences but can be changed (Eshun, 2004) or influenced by factors associated by the individual (self-efficacy, achievement, anxiety, motivation), by instructors or institutions (teacher knowledge, teacher attitudes, classroom management), or by environment (peers within community) (Mohamed & Waheed, 2011).
  Ex:
    - "…I still had fun."
    - "…knowing math will help me get a job."

Stage 2:
Highlight each extraction as it aligns with each theoretical aspect of the framework with the color scheme:

Self-Efficacy-Pink
Position-Green
Perceptions of Mathematics-Blue
Forms of Engagement-Yellow
Nothing-Orange

Note: In this stage, you may highlight each extraction with multiple colors if it possibly aligns with multiple aspects.

Ex: "…so I can say that I feel very comfortable in math classes"
This could be an extraction that could be considered as either position or as self-efficacy.

- Position by subject's view of place in math class
- Self-efficacy by "comfort level" in math class

Stage 3:
Determine exactly one color for each extraction for those that are multi-colored. This is determined by the context of the extraction by considering a holistic view. That is considering the discussion around the extraction.

Ex: "so I can say that I feel very comfortable in math classes"
This could be an extraction that could be considered as either position or as self-efficacy but is determined to be self-efficacy since the subject of the extraction is feeling comfortable.

After instrument is finalized with each extraction categorized into exactly one aspect, compare color schemes with at least two other raters. Discuss any extractions that have conflicting aspects. The goal is to have at least an agreement of 80%. In cases where agreement is less than 80%, the definitions of each aspect are clarified until the raters reach consensus. In the case where the agreement is greater than 80%, the researcher must make ultimate decision on the classification of extractions where there was disagreement.

Stage 4:
Separate and regroup extractions by aspect color.

Stage 5:
Using thematic coding, determine sub-themes from each aspect (Braun and Clarke, 2006). Then compare to other documents from other subjects to determine if the sub-themes can be generalized. In the table below is a list of sub-themes found in subjects of this study.

Coding Aspects with Sub-Themes

Background	Self-Efficacy	Position	Belief about Mathematics	Forms of Engagement
• *Background of Self: Evidence of Effect on Identity vs. No Evidence of effect on Identity* • *Background of Family: Evidence of Effect on Identity vs. No Evidence of effect on Identity* • *Background on Teachers: Evidence of Effect on Identity vs. No Evidence of effect on Identity* • *Background with Seminar* • *Background with Presentation*	• *Belief in Abilities Overall-Positive* • *Belief in Abilities Overall-Negative* • *Belief in Math Abilities-Positive* • *Belief in Math Abilities-Negative*	• *Places Self into Math* • *Places Self outside Math* • *Teachers Place Subject into Math* • *Peers Place Subject into Math* • *Role of Teacher: authority vs. facilitator* • *Role of Peers* • *Role of Self*	• *Attitude-Negative* • *Attitude-Positive* • *Value/Worth-Positive* • *Value/Worth-Negative* • *Beliefs on Definition of Math* • *Value of Seminar-Positive* • *Value of Seminar-Negative*	• *Study Habits* • *Math Stories/Setting Up* • *How subject "does" math/Applications*

Stage 6:
Enter each extraction into Subject's Dossier. The Subject Dossier is a way to organize the data and be able to read the data as a whole. Any similarities or differences in the constructs of identity will be noted for the presenter. The dossiers are a structured outline of each instrument for each subject. An example can be found below. The extractions will be put into the outline categorized by aspect and then sub-theme. The extractions are listed as a numerical list that will then be put into the Dossier Summary

Subject's Dossier

Overall summary
Put in any highlighting factors on background of subject and any outstanding remarks that came out in any interviews, presentations, or autobiographies

Ex:

- 19 years old
- Freshman
- Double major-finance and accounting
- Father is an engineer
- Mother is an accountant
- Does not procrastinate

Pre-autobiography
Overall Comments:

Background:
Background of Self

1. My name is Subject
2. I was born and raised in Houston, Texas.
3. I have only moved once in my life

Background of Family

1. Both of my parents were math majors

Background on Teachers

1.

Background with Seminar

1.

Background with Presentation

1.

Self-Efficacy
Belief in Abilities Overall-Positive

1.

Belief in Abilities Overall-Negative

.
.
.
.

Tables to perform a quantifiable comparison of factors at each stage of the study.

APPENDIX G

Subject Dossier Outline

Overall summary

Put in any highlighting factors on background of subject and any outstanding remarks that came out in any interviews, presentations, or autobiographies

Ex:

- 19 years old
- Freshman
- Double major-finance and accounting
- Father is an engineer
- Does not procrastinate
- Seems to be very academically driven
- Enrolled in 1329
- Considers SMS experience "a blessing"
- Views her finance and accounting degrees as "math"

Surveys

Comparison of pre- and post- Audience Perception Survey with the Math Identity Density Function

Pre-autobiography

Overall Comments:

Background:
Background of Self

1. My name is Subject
2. I was born and raised in Houston, Texas.
3. I have only moved once in my life

Background of Family

1. Both of my parents were math majors

Background on Teachers

1.

Background with Seminar

1.

Background with Presentation

1.

Self-Efficacy

Belief in Abilities Overall-Positive

1.

Belief in Abilities Overall-Negative

1.

Belief in Math Abilities-Positive

1.

Belief in Math Abilities-Negative

1.

Position

Places Self into Math

1.

Places Self outside Math

1.

Teachers Place Subject into Math

1.

Teachers Place Subject outside Math

1.

Peers Place Subject into Math

1.

Peers Place Subject outside Math

1.

Role of Teacher/Authority

1.

Role of Peers

1.

Role of Self

1.

Beliefs

Attitude-Negative

1.

Attitude-Positive

1.

Value/Worth-Positive

1.

Value/Worth-Negative

1.

Beliefs on Definition of Math

1.

Value of Seminar-Positive

1.

Value of Seminar-Negative

1.

Value Presentation Experience

1.

Engagement

Study Habits

1.

Math Stories/Setting Up

1.

How subject "does" math/Applications

1.

Second Autobiography

Overall Comments:

Background:

Background of Self

1. My name is Subject
2. I was born and raised in Houston, Texas.
3. I have only moved once in my life

Background of Family

1. Both of my parents were math majors

Background on Teachers

1.

Background with Seminar

1.

Background with Presentation

1.

Self-Efficacy

Belief in Abilities Overall-Positive

1.

Belief in Abilities Overall-Negative

1.

Belief in Math Abilities-Positive

1.

Belief in Math Abilities-Negative

1.

Position

Places Self into Math

1.

Places Self outside Math

1.

Teachers Place Subject into Math

1.

Teachers Place Subject outside Math

1.

Peers Place Subject into Math

1.

Peers Place Subject outside Math

1.

Role of Teacher/Authority

1.

Role of Peers

1.

Role of Self

1.

Beliefs

Attitude-Negative

1.

Attitude-Positive

1.

Value/Worth-Positive

1.

Value/Worth-Negative

1.

Beliefs on Definition of Math

1.

Value of Seminar-Positive

1.

Value of Seminar-Negative

1.

Engagement

Study Habits

1.

Math Stories/Setting Up

1.

How subject "does" math/Applications

1.

First Interview

Overall Comments:

Background:

Background of Self

1.

Background of Family

1.

Background on Teachers

1.

Background with Seminar

1.

Background with Presentation

1.

Self-Efficacy

Belief in Abilities Overall-Positive

1.

Belief in Abilities Overall-Negative

1.

Belief in Math Abilities-Positive

1.

Belief in Math Abilities-Negative

1.

Position

Places Self into Math

1.

Places Self outside Math

1.

Teachers Place Subject into Math

1.

Teachers Place Subject outside Math

1.

Peers Place Subject into Math

1.

Peers Place Subject outside Math

1.

Role of Teacher/Authority

1.

Role of Peers

1.

Role of Self

1.

Beliefs
Attitude-Negative
1.

Attitude-Positive
1.

Value/Worth-Positive
1.

Value/Worth-Negative
1.

Beliefs on Definition of Math
1.

Value of Seminar-Positive
1.

Value of Seminar-Negative
1.

Engagement
Study Habits
1.

Math Stories/Setting Up
1.

How subject "does" math/Applications
1.

Second Interview
Overall Comments:

Background:
Background of Self
1.

Background of Family
1.

Background on Teachers
1.

Background with Seminar

1.

Background with Presentation

1.

Self-Efficacy

Belief in Abilities Overall-Positive

1.

Belief in Abilities Overall-Negative

1.

Belief in Math Abilities-Positive

1.

Belief in Math Abilities-Negative

1.

Position

Places Self into Math

1.

Places Self outside Math

1.

Teachers Place Subject into Math

1.

Teachers Place Subject outside Math

1.

Peers Place Subject into Math

1.

Peers Place Subject outside Math

1.

Role of Teacher/Authority

1.

Role of Peers

1.

Role of Self

1.

Beliefs
Attitude-Negative
1.

Attitude-Positive
1.

Value/Worth-Positive
1.

Value/Worth-Negative
1.

Beliefs on Definition of Math
1.

Value of Seminar-Positive
1.

Value of Seminar-Negative
1.

Engagement
Study Habits
1.

Math Stories/Setting Up
1.

How subject "does" math/Applications
1.

Third Interview
Overall Comments:

Background:
Background of Self
1.

Background of Family
1.

Background on Teachers
1.

Background with Seminar
1.

Background with Presentation
1.

Self-Efficacy
Belief in Abilities Overall-Positive
1.

Belief in Abilities Overall-Negative
1.

Belief in Math Abilities-Positive
1.

Belief in Math Abilities-Negative
1.

Position
Places Self into Math
1.

Places Self outside Math
1.

Teachers Place Subject into Math
1.

Teachers Place Subject outside Math
1.

Peers Place Subject into Math
1.

Peers Place Subject outside Math
1.

Role of Teacher/Authority
1.

Role of Peers
1.

Role of Self
1.

Beliefs
Attitude-Negative
1.

Attitude-Positive
1.

Value/Worth-Positive
1.

Value/Worth-Negative
1.

Beliefs on Definition of Math
1.

Value of Seminar-Positive
1.

Value of Seminar-Negative
1.

Engagement
Study Habits
1.

Math Stories/Setting Up
1.

How subject "does" math/Applications
1.

Fourth Interview
Overall Comments:

Background:
Background of Self
1.

Background of Family
1.

Background on Teachers
1.

Background with Seminar

1.

Background with Presentation

1.

Self-Efficacy

Belief in Abilities Overall-Positive

1.

Belief in Abilities Overall-Negative

1.

Belief in Math Abilities-Positive

1.

Belief in Math Abilities-Negative

1.

Position

Places Self into Math

1.

Places Self outside Math

1.

Teachers Place Subject into Math

1.

Teachers Place Subject outside Math

1.

Peers Place Subject into Math

1.

Peers Place Subject outside Math

1.

Role of Teacher/Authority

1.

Role of Peers

1.

Role of Self

1.

Beliefs
Attitude-Negative
1.

Attitude-Positive
1.

Value/Worth-Positive
1.

Value/Worth-Negative
1.

Beliefs on Definition of Math
1.

Value of Seminar-Positive
1.

Value of Seminar-Negative
1.

Engagement
Study Habits
1.

Math Stories/Setting Up
1.

How subject "does" math/Applications
1.

First Presentation
Overall Comments:

Presentation Length:
13:03

Background:
Background of Self
1.

Background of Family
1.

Background on Teachers

1.

Background with Seminar

1.

Background with Presentation

1.

Self-Efficacy

Belief in Abilities Overall-Positive

1.

Belief in Abilities Overall-Negative

1.

Belief in Math Abilities-Positive

1.

Belief in Math Abilities-Negative

1.

Position

Places Self into Math

1.

Places Self outside Math

1.

Teachers Place Subject into Math

1.

Teachers Place Subject outside Math

1.

Peers Place Subject into Math

1.

Peers Place Subject outside Math

1.

Role of Teacher/Authority

1.

Role of Peers

1.

Role of Self
1.

Beliefs
Attitude-Negative
1.

Attitude-Positive
1.

Value/Worth-Positive
1.

Value/Worth-Negative
1.

Beliefs on Definition of Math
1.

Value of Seminar-Positive
1.

Value of Seminar-Negative
1.

Engagement
Study Habits
1.

Math Stories/Setting Up
1.

How subject "does" math/Applications
1.

Second Presentation
Overall Comments:

Presentation Length:
13:03

Background:
Background of Self
1.

Background of Family
1.

Background on Teachers
1.

Background with Seminar
1.

Background with Presentation
1.

Self-Efficacy
Belief in Abilities Overall-Positive
1.

Belief in Abilities Overall-Negative
1.

Belief in Math Abilities-Positive
1.

Belief in Math Abilities-Negative
1.

Position
Places Self into Math
1.

Places Self outside Math
1.

Teachers Place Subject into Math
1.

Teachers Place Subject outside Math
1.

Peers Place Subject into Math
1.

Peers Place Subject outside Math
1.

Role of Teacher/Authority
1.

Role of Peers
1.

Role of Self
1.

Beliefs
Attitude-Negative
1.

Attitude-Positive
1.

Value/Worth-Positive
1.

Value/Worth-Negative
1.

Beliefs on Definition of Math
1.

Value of Seminar-Positive
1.

Value of Seminar-Negative
1.

Engagement
Study Habits
1.

Math Stories/Setting Up
1.

How subject "does" math/Applications
1.

Third Presentation
Overall Comments:

Presentation Length:
13:03

Background:

Background of Self

1.

Background of Family

1.

Background on Teachers

1.

Background with Seminar

1.

Background with Presentation

1.

Self-Efficacy

Belief in Abilities Overall-Positive

1.

Belief in Abilities Overall-Negative

1.

Belief in Math Abilities-Positive

1.

Belief in Math Abilities-Negative

1.

Position

Places Self into Math

1.

Places Self outside Math

1.

Teachers Place Subject into Math

1.

Teachers Place Subject outside Math

1.

Peers Place Subject into Math

1.

Peers Place Subject outside Math
1.

Role of Teacher/Authority
1.

Role of Peers
1.

Role of Self
1.

Beliefs
Attitude-Negative
1.

Attitude-Positive
1.

Value/Worth-Positive
1.

Value/Worth-Negative
1.

Beliefs on Definition of Math
1.

Value of Seminar-Positive
1.

Value of Seminar-Negative
1.

Engagement
Study Habits
1.

Math Stories/Setting Up
1.

How subject "does" math/Applications
1.

Dossier Summary

	Pre-Auto	Post-Auto	1st Int	2nd Int	3rd Int	4th Int	1st Pres	2nd Pres	3rd Pres
Background									
Background of Self									
Background of Family									
Background on Teachers									
Background with Seminar									
Background with Presentation									
Self-Efficacy									
Belief in Abilities Overall-Positive									
Belief in Abilities Overall-Negative									
Belief in Math Abilities-Positive									
Belief in Math Abilities-Negative									
Position									
Places Self into Math									
Places Self outside Math									
Teachers Place Subject into Math									
Teachers Place Subject outside Math									
Peers Place Subject into Math									
Peers Place Subject outside Math									
Role of Teacher									
Role of Peers									
Role of Self									
Beliefs									
Attitude-Negative									
Attitude-Positive									
Value/Worth-Positive									
Value/Worth-Negative									
Beliefs on Definition of Math									
Value of Seminar-Positive									
Value of Seminar-Negative									
Value of Presentation Experience									
Engagement									
Study Habits									
Math Stories/Setting Up									
How subject "does" math/Applications									
Nothing									
Aligned									
Not Aligned									
na									
Totals									

POP Summary

Item	Pres 1	Pres 2	Pres 3
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
AVG			

APPENDIX H

Presenter Observation Protocol (POP)
Student Math Seminar

V. PRESENTER INFORMATION

Name of Presenter ___________________________________ Presentation Number________

Title of Presentation____

Classification ____________________ Current Math Course(s)____________________________

Observer ____________________________ Date of observation __________________________

Start time ____________________________ End time __________________________________

Audience Size ________________________ Media Used ________________________________
(Powerpoint, Elmo, Chalkboard, Other)

VI. DESCRIPTION OF PRESENTATION

Record observation notes which may help in documenting the ratings.

Time	Notes

Record observation notes which may help in documenting the ratings.

Time	Description of Events

VII. DESCRIPTION OF PRESENTER

	Novice			Distinguished
1. The presenter appeared to be well prepared.	1	2	3	4
2. The presenter appeared to be relaxed and self-confident.	1	2	3	4
3. The presenter is familiar with media used.	1	2	3	4
4. The presenter is appropriately dressed for the audience.	1	2	3	4
5. The presenter used appropriate fluctuation of volume.	1	2	3	4
6. The presenter identifies self as well versed in mathematical practices.	1	2	3	4
7. The presenter claims to be passionate about mathematics.	1	2	3	4
8. The presenter claims mathematics is necessary for personal and academic success.	1	2	3	4
9. The presenter made minimal mistakes and/or easily recovered from mistakes.	1	2	3	4

VIII. DESCRIPTION OF PRESENTATION

	Novice			Distinguished
10. Presentation aligned with provided abstract.	1	2	3	4
11. Presentation had a clear objective and purpose.	1	2	3	4
12. Presentation is well structured and organized.	1	2	3	4
13. Presentation has pertinent examples, counterexamples, and/or data with justification.	1	2	3	4
14. Presentation has strong explanations/proofs appropriate for the audience.	1	2	3	4
15. Demonstrates multiple problem-solving methods.	1	2	3	4
16. Major ideas are well summarized.	1	2	3	4

IX. AUDIENCE ENGAGEMENT

	Novice			Distinguished
17. Presenter increased audience understanding and knowledge.	1	2	3	4
18. Presenter convinces audience to recognize validity.	1	2	3	4
19. Maintains appropriate eye contact with audience.	1	2	3	4

	Novice			Distinguished
20. Welcomes questions and comments.	1	2	3	4

Additional comments about this presentation.

APPENDIX I

Presenter Observation Protocol Rubric (POPR)

Performance Element	Distinguished	Proficient	Apprentice	Novice
Presenter	• Relaxed and self-confident • Appropriately dressed for audience • Fluctuation of volume and inflection to emphasize key points	• Recovers quickly from minor mistakes • Appropriately dressed • Fairly consistent eye contact • Satisfactory variation of volume and inflection	• Some tension exhibited • Somewhat inappropriate dress for audience • Occasional eye contact • Uneven volume with little or no inflection	• Obviously nervous • Inappropriately dressed for audience • No effort to make eye contact • Low volume and monotonous tones
Presentation	• Provides a clear objective and purpose • Is well structured and organized • Provides pertinent examples counter-examples, and data with justification • Provides strong explanations/proof • Demonstrates multiple problem-solving methods • Provides explanation or interpretation of knowledge for audience • Major ideas are well summarized	• Has some success in providing objective and purpose • Has structure and is mostly organized • Provides examples and counter-examples with some justification • Demonstrates some problem-solving skills • Provides some interpretation of knowledge for audience • May need to refine summary of main ideas	• Attempts to provide objective and purpose • Attempts to lead a structured presentation but is somewhat organized • Provides examples without justification • Attempts to demonstrate some problem-solving skills • Provides inadequate interpretation of knowledge • Major ideas are vaguely summarized	• Objective and purpose not clearly defined • Presentation is not well structured and is unorganized • Provides inconsistent examples or incomplete/incorrect justification • Fails to demonstrate knowledge of problem-solving • Gives basic information and/or provides incorrect interpretations • Major ideas are unclear and insufficient support for conclusions
Audience Engagement	• Significantly increases audience understanding and knowledge • Effectively convinces audience to recognize validity of point of view • Welcomes questions and	• Raises audience understanding and awareness of most points • Shows clear point of view, with some inconclusive or incomplete support • Acknowledges questions and	• Slightly raises audience's awareness of most points • Points of view are clear but lack support • Slightly hesitant about addressing questions and comments	• Fails to increase audience awareness and knowledge of subject. • Does not convince audience • Does not acknowledge questions and comments from the audience

	comments of the audience	comments from the audience	from the audience.	
Position	• Identifies self as well versed in mathematical practices • Claims to be passionate about mathematics and its applications	• Identifies self as familiar with mathematical practices • Claims to appreciate mathematics and its applications	• Identifies self as beginning to becoming familiar with mathematical practices • Claims to acknowledge some of the importance of mathematics and its applications	• Identifies self as unknowledgeable of mathematical practices • Does not find mathematics appealing
Belief	• States that mathematics is absolutely necessary to personal and academic successes	• State mathematics is useful for personal and academic success	• Acknowledges the possibility of the usefulness of mathematics in personal and academic successes	• Does not find mathematics useful.

APPENDIX J

Math Autobiography

Content: Your autobiography should address the four sections listed below. I've listed some questions to help guide you, but please don't just go through and answer each question separately. The questions are just to help get you thinking. Write about the things that will give me a picture of you. The key to writing a good autobiography is to give lots of detail. See the example below:

> *Not enough detail:* I hated math in fourth grade, but it got better in sixth grade.
>
> *Good detail:* I hated math in fourth grade because I had trouble learning my multiplication tables. I was really slow at doing problems, and I was always the last one to finish the timed tests. It was really embarrassing. …

Section 1: Introduction

- How would you describe yourself?
- Where are you from?
- What is your educational background? Did you just graduate from high school? Have you been out of school for a few years? If so, what have you been doing since then?
- General interests: favorite subjects in school, favorite activities or hobbies.

Section 2: Experience with Math

- What math classes have you taken and when?
- What have your experiences in math classes been like?
- How do you feel about math?
- How would you describe your math abilities?
- What were previous math grades like?
- What factors contributed to your success or non-success in math?
- What math courses did you like/dislike? Why did you like/dislike them?
- What did you like/dislike about your previous math instructors? Did any affect your beliefs about math or your abilities in math?
- In what ways have you used math outside of school?

Section 3: Learning Styles and Habits (specifically for math)

- Do you learn best from reading, listening or doing?
- Do you prefer to work alone or in groups?
- What do you do when you get "stuck"?
- Do you ask for help? From whom?
- Describe some of your study habits. For example: Do you take notes? Are they helpful? Are you organized? Do you procrastinate? Do you read the text?

- What do you believe are your responsibilities as a student in a math course? What do you expect from your instructor?

Section 4: The Future

- What are your educational and life goals?
- How does math fit into your educational goals and life goals?

Section 5: After Completing the Study

- How did presenting in the Student Math Seminar make you feel?
- How do you think you will use this experience?
- Were there any benefits to participating in SMS?

APPENDIX K

Item Reliabilities from Confirmatory Factor Analysis

id=~ values + se.self + se.others + se.new + p.others + p.self + beliefs

Table()

Variable	Aspect of Framework	Definition	Items from Survey
id	Mathematical Identity	One's overall alignment with the mathematics culture	
values	Perceptions of Math	Items that addressed the values of mathematics	3.1, 3.2, 3.10, 3.42, 3.43
se.self	Self-Efficacy	Items that addressed one's conviction to practice mathematics on their own	3.16, 3.23, 3.24, 3.25, 3.18
se.others	Self-Efficacy	Items that addressed one's convictions to practice mathematics in front of others	3.21, 3.22, 3.17
se.new	Self-Efficacy	Items that addressed one's conviction to practice new mathematics	3.13, 3.26, 3.14, 3.15, 3.20, 3.27
p.others	Position	Items that addressed how peers and instructors place one within the mathematics culture	3.28, 3.32, 3.36, 3.34, 3.39
p.self	Position	Items that addressed how one places own self within the mathematics culture	3.29, 3.33, 3.35, 3.40, 3.41, 3.30, 3.31
beliefs	Perceptions of Math	Items that addressed beliefs about the definition of mathematics	3.6, 3.5, 3.7, 3.3, 3.4, 3.8, 3.9

values
Composite Reliability: 0.7751
RMSEA: 0.071
TLI: 0.880
CFI: 0.940

se.self
Composite Reliability: 0.7378
RMSEA: 0.049
TLI: 0.991
CFI: 0.996

se.others
Composite Reliability: 0.8091
RMSEA: 0.000
TLI: 1.000
CFI: 1.000

se.new
Composite Reliability: 0.8116
RMSEA: 0.078
TLI: 0.955
CFI: 0.973

p.others
Composite Reliability: 0.7751
RMSEA: 0.071
TLI: 0.880
CFI: 0.940

p.self
Composite Reliability: 0.8407
RMSEA: 0.084
TLI: 0.825
CFI: 0.883

beliefs
Composite Reliability: 0.8329
RMSEA: 0.162
TLI: 0.671
CFI: 0.507

Overall CFA	
Number of observations	242
Number of missing patterns	8
Estimator	ML
Minimum Function Test Statistic	21.618
Degrees of freedom	15
P-value (Chi-square)	0.118
Model test baseline model:	
Minimum Function Test Statistic	1036.641
Degrees of freedom	28
P-value	0.000
User model versus baseline model:	
Comparative Fit Index (CFI)	0.993
Tucker-Lewis Index (TLI)	0.988
Loglikelihood and Information Criteria:	
Loglikelihood user model (H0)	-4682.423
Loglikelihood unrestricted model (H1)	-4671.614
Number of free parameters	29
Akaike (AIC)	9422.846
Bayesian (BIC)	9524.025
Sample-size adjusted Bayesian (BIC)	9432.101

Root Mean Square Error of Approximation:

RMSEA	0.043
90 Percent Confidence Interval	0.000 0.080
P-value RMSEA <= 0.05	0.581

Standardized Root Mean Square Residual:

SRMR	0.024

Parameter Estimates:

Information	Observed
Standard Errors	Standard

Latent Variables:

	Estimate	Std.Err	z-value	P(>|z|)
id =~				
values	1.000			
se.self	1.013	0.077	13.228	0.000
se.others	0.479	0.048	9.951	0.000
se.new	1.290	0.095	13.637	0.000
p.others	0.648	0.053	12.210	0.000
p.self	1.141	0.083	13.743	0.000
beliefs	0.324	0.043	7.585	0.000
bel.rev	0.327	0.055	5.920	0.000

Covariances:

	Estimate	Std.Err	z-value	P(>|z|)
.values ~~				
.beliefs	2.185	0.522	4.187	0.000
.se.self ~~				
.se.new	2.642	0.788	3.354	0.001
.beliefs	-1.718	0.428	-4.012	0.000
.se.new ~~				
.beliefs	-1.574	0.514	-3.065	0.002
.p.others ~~				
.bel.rev	1.215	0.400	3.037	0.002

Intercepts:

	Estimate	Std.Err	z-value	P(>|z|)
.values	18.004	0.284	63.318	0.000
.se.self	15.095	0.280	53.961	0.000
.se.others	6.806	0.170	40.148	0.000
.se.new	17.750	0.349	50.826	0.000
.p.others	17.911	0.196	91.192	0.000
.p.self	20.553	0.311	66.068	0.000
.beliefs	11.082	0.161	68.857	0.000
.bel.rev	15.504	0.188	82.542	0.000
id	0.000			

Variances:

	Estimate	Std.Err	z-value	P(>|z|)
.values	7.356	0.835	8.811	0.000
.se.self	6.411	0.803	7.983	0.000
.se.others	4.095	0.407	10.072	0.000
.se.new	9.108	1.185	7.685	0.000
.p.others	4.190	0.444	9.427	0.000
.p.self	7.379	0.916	8.059	0.000
.beliefs	4.940	0.480	10.289	0.000
.bel.rev	7.091	0.667	10.631	0.000
id	12.210	1.722	7.089	0.000

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