GEOMETRY TEACHING KNOWLEDGE: A COMPARISON BETWEEN PRE-SERVICE AND HIGH SCHOOL GEOMETRY TEACHERS

DISSERTATION

by

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DEDICATION

This dissertation is dedicated to my amazing and supportive husband, Patrick Smith. I could not have done this without you.

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LIST OF ABBREVIATIONS

Abbreviation **Description**

GTK- Geometry Teaching Knowledge

PCK- Pedagogical Content Knowledge

MKT- Mathematical Knowledge for Teaching

CCK- Common Content Knowledge

SCK- Specialized Content Knowledge

KCS- Knowledge of Content and Students

KCT- Knowledge of Content and Teaching

MKT-G- Mathematical Knowledge for Teaching Geometry

GCK- Geometry Content Knowledge

SGK- Specialized Geometry Knowledge

KGS- Knowledge of Geometry and Students

KGT- Knowledge of Geometry and Teaching

ABSTRACT

Geometry is a field in mathematics that every student in the United States is required to study in order to fulfill high school graduation requirements. The literature shows that three possible reasons for poor performance in Geometry and Measurement are: not enough exposure and emphasis in K-12 curriculum implemented by the teacher, challenges associated with implementation of Geometry and Measurement in the classroom, and limited knowledge of the teacher (Steele, 2013). Research is needed to investigate the levels of Geometry Teaching Knowledge of pre-service and high school geometry teachers. This study compares Geometry Teaching Knowledge between pre-service and current high school geometry teachers. Data was collected via an online MKT-G assessment developed by Herbst and Kosko (2014) and a post-assessment survey. Additional data was collected through interviews of three pre-service teachers and four high school teachers. Furthermore, this study also investigates where this knowledge is developed. Pre-service teachers did not perform as well as the high school geometry teachers in all of the domains: Geometry Content Knowledge, Specialized Geometry Knowledge, Knowledge of Geometry and Students, and Knowledge of Geometry and Teaching. When comparisons were made regarding experiences in pre-service teacher mathematics courses, education courses, professional development, current geometry classrooms, and ideal classrooms of both pre-service and current high school teachers, there were statistically significant differences. This study provides insight into the domains of Geometry Teaching Knowledge that could be used in making decisions regarding pre-service teacher education programs and high school geometry teacher professional development.

CHAPTER 1

INTRODUCTION

Background

Geometry is a field in mathematics that every student in the United States is required to study in order to fulfill high school graduation requirements. The Common Core State Standards Initiative (2010) stresses that Geometry is a vital course when preparing students to enter a science, technology, mathematics, or engineering field. According to the National Center for Education Statistics (2012), two content areas in mathematics are consistently behind in performance: Geometry and Measurement. The literature shows that three possible reasons for poor performance in Geometry and Measurement are: not enough exposure and emphasis in K-12 curriculum implemented by the teacher, challenges associated with implementation of Geometry and Measurement in the classroom, and limited knowledge of the teacher (Steele, 2013).

Teachers that have completed a bachelor's degree in mathematics and a traditional teacher-preparation program are considered some of the most qualified teacher candidates. According to No Child Left Behind legislation, a highly qualified teacher holds a bachelor's degree, and has passed a state academic subject test (2010). Even though teachers follow a traditional teacher-preparation program, they may not be prepared to teach the mathematics required of them when they leave the university and enter the secondary school.

In Texas, as a first-year teacher, one is not typically given the choice of what subjects to teach or preferred grade level. In my experience, a first year teacher is usually assigned whatever subject is in need of a teacher. When a teacher receives a Texas teaching certificate in grades 8 through 12, it is understood that this teacher is qualified to teach any of the subjects in those grade levels. The topics include Algebra, Geometry, Trigonometry, Statistics, and Calculus.

TExES certification exams are the required teacher certification exams for the state of Texas. If one is to take the scores that pre-service secondary teachers receive on the TExES certification exam as valid, then pre-service teachers are qualified to teach any level of mathematics offered in grade 8 through grade 12. Figure 1 shows the outline of the TExES teacher exam. The focus of this study is in the subject area of Geometry. According to the figure, only 19% of the TExES test addresses Geometry and Measurement. Figure 2 shows the breakdown of topics addressed in the Geometry and Measurement section of the TExES test. According to the topics addressed in the exam, a pre-service teacher should be prepared to teach Geometry when entering the secondary classroom; however, Mitchell and Barth point out that individuals can pass state certification tests without having to pass all of the domains assessed on the test. If a pre-service teacher does not pass the Geometry and Measurement section of the exam, they could still pass the exam. That pre-service teacher may not have enough content knowledge in Geometry to be a successful Geometry teacher. There is a need to make sure all teachers entering the secondary schools have sufficient knowledge of Geometry.

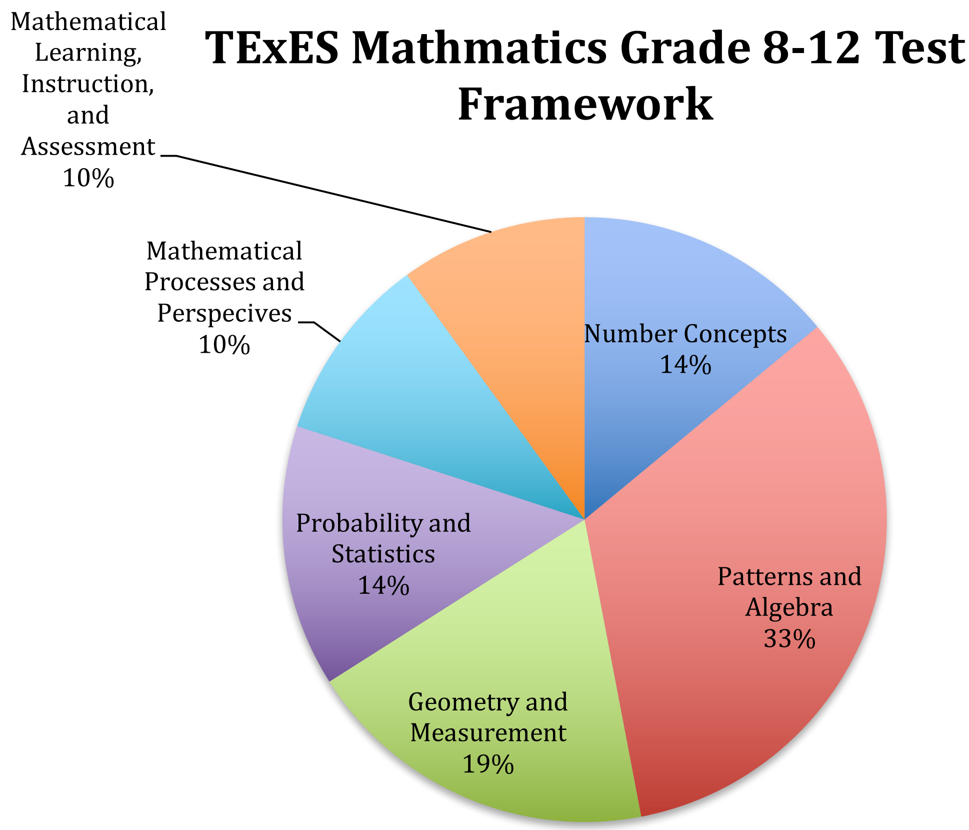


Figure 1. TExES Mathematics Grade 8-12 Test Framework (Texas Education Agency, TExES Preparation Manual Mathematics 8-12, p. 12)

The Texas Education Agency (2010) describes the Geometry and Measurement Competencies to be the following:

Competency 011: The teacher understands measurement as a process. The beginning teacher:

A. Applies dimensional analysis to derive units and formulas in a variety of situations and to find and evaluate solutions to problems.

B. Applies formulas for perimeter, area, surface area, and volume of geometric figures and shapes to solve problems.

C. Recognizes the effects on length, area, or volume when the linear dimensions of plane figures or solids are changed.

D. Applies the Pythagorean Theorem, proportional reasoning, and right triangle trigonometry to solve measurement problems.

E. Relates the concept of area under a curve to the limit of a Riemann sum

F. Uses integral calculus to compute various measurements associated with curves and regions in the plane, and measurements associated with curves, surfaces, and regions in three-space.

Competency 012: The teacher understands geometries, in particular Euclidean Geometry, as axiomatic systems. The beginning teacher:

A. Understands axiomatic systems and their components

B. Uses properties of points, lines, planes, angles, lengths, and distances to solve problems.

C. Applies the properties of parallel and perpendicular lines to solve problems.

D. Uses properties of congruence and similarity to explore geometric relationships, justify conjectures and prove theorems.

E. Describes and justifies geometric constructions made using a compass and straightedge, reflection devices, and other appropriate technologies

F. Demonstrates an understanding of the use of appropriate software to explore attributes of geometric figures and to make and evaluate conjectures about geometric relationships.

G. Compares and contrasts the axioms of Euclidean geometry with those of non-Euclidean geometry (hyperbolic and elliptic geometry).

Competency 013: The teacher understands the results, uses, and applications of Euclidean geometry. The beginning teacher:

A. Analyzes the properties of polygons and their components.
B. Analyzes the properties of circles and the lines that intersect them.
C. Uses geometric patterns and properties to make generalizations about two- and three-dimensional figures and shapes.
D. Computes the perimeter, area, and volume of figures and shapes created by subdividing and combining other figures and shapes.
E. Analyzes cross-sections and nets of three-dimensional shapes.
F. Uses top, front, side, and corner views of three-dimensional shapes to create complete representation and solve problems.
G. Applies properties of two- and three- dimensional shapes to solve problems across the curriculum and in everyday life.

Competency 014: The teacher understands coordinate, transformational, and vector geometry and their connections. The beginning teacher:

A. Identifies transformations and explores their properties.
B. Uses the properties of transformations and their compositions to solve problems
C. Uses transformations to explore and describe reflectional, rotational, and translational symmetry.
D. Applies transformations in the coordinate plane.

E. Applies concepts and properties of slope, midpoint, parallelism, perpendicularity, and distance to explore properties of geometric figures and solve problems in the coordinate plane.

F. Uses coordinate geometry to derive and explore the equations, properties, and applications of conic sections.

G. Relates geometry and algebra by representing transformations as matrices and uses this relationship to solve problems.

H. Explores the relationship between geometric and algebraic representations of vectors and uses this relationship to solve problems. (p. 20-21)

My interest in this topic stems from my own experiences as a first year teacher. When I started my first job as a high school teacher, I was assigned Pre-AP Geometry and Pre-Calculus. As I began planning for my first class, I realized that I had not been in a Geometry classroom since I had been in high school myself. I knew the basics of Geometry, I had passed my certification exams, and I felt comfortable with my knowledge, however, once I began to plan lessons, I realized that I was not as familiar as I would have hoped with the topics. Along with having to plan activities for my students, I was studying the material so that I had at least some understanding of the tasks at hand. A teacher in their first year is already overwhelmed with planning, grading, and developing their own style of teaching, but having to learn material that they may or may not have learned before adds more pressure to the situation. I began asking experienced teachers for advice and help. Many of the teachers I asked gave me materials to help with the classroom activities, grading strategies, and knowledge of student struggles, which allowed me more time to focus on the material I was to present on a daily basis. On top of the lack of Geometry Content Knowledge, I had to develop ways of presenting material to students in a manner that they could understand and retain the content.

Once I completed my first year of teaching Pre-AP Geometry, I realized that I was not as prepared as I would have expected coming from a four-year teacher-training program. This experience made me more aware of new teachers coming into the school as well as student teachers teaching at the school. I went out of my way to help those who needed it. Other teachers had helped me, and it was my turn to lend a hand. During my fifth year, I was a mentor for a student teacher from a local university. I tried to help her any way I could. I would come to school early and stay late in order to address any issues she had with the material being taught and to help her brainstorm different ways to present lessons to the students. This experience helped me to realize that I was not alone in this experience of entering a classroom for the first time. I became interested in changing this aspect of the first year of teaching. As a result, I decided to focus my dissertation on a teacher's mathematical knowledge for teaching Geometry.

Focus of this Study

The focus of this study was two-fold, the first portion of this study investigated the differences between the knowledge high school pre-service and high school Geometry in-service teachers have regarding geometry content knowledge, mathematical knowledge for teaching geometry, and knowledge of geometric techniques and methods used in the geometry classroom. This study focused on the high school pre-service teachers at a four-year university in Texas and high school Geometry teachers from multiple school districts in Texas.

The comparison of the pre-service teachers and the in-service teacher knowledge lead to the second portion of this study that investigated where and how mathematical knowledge for teaching geometry, and knowledge of geometric techniques and methods used in the Geometry classroom are developed.

Purpose of this Study

The purpose of this study was to compare the geometry content knowledge, mathematical knowledge for teaching geometry, and knowledge of Geometric techniques and methods used in the Geometry classroom of pre-service and in-service high school teachers. This study examined the differences in knowledge and where and how this knowledge is developed.

Research Questions

The research questions for this study are:

1. What do high school pre-service teachers and high school Geometry teachers know about *Geometry Teaching Knowledge* which consists of the following:
    a. Mathematical knowledge for teaching geometry?
    b. Geometry techniques and methods used in the high school Geometry classroom?
2. How do pre-service and current high school teachers' *Geometry Teaching Knowledge* compare?
3. What are the sources of the high school teachers *Geometry Teaching Knowledge* that can be transferred to pre-service teachers?
4. What are the sources of the pre-service teachers *Geometry Teaching Knowledge* that can be transferred to high school teachers?

5. What do high school pre-service teachers need to know to be prepared to teach high school Geometry?

Significance of Study

This study shed light on the *Geometry Teaching Knowledge* that high school pre-service and high school Geometry in-service teachers have and where this knowledge originates. This study helps fill in the gap in research regarding the Geometry Content Knowledge, Mathematical Knowledge for Teaching Geometry, and knowledge of geometric techniques and methods used in the high school Geometry classroom that high school pre-service and high school Geometry in-service teachers. The instruments used to address these questions could be used in other pre-service mathematics teacher training programs and in professional development of high school in-service teachers to address any gaps that may exist in their knowledge of Geometry and of teaching Geometry. This may impact future student performance in Geometry and Measurement since the three main reasons for a lag in performance are weak attention in K-12 curriculum, challenges associated with implementation of Geometry and Measurement in the classroom, and limited knowledge of the teacher (Clements, 1999; Steel, 2013).

Definition of Terms

Some terms may have several interpretations. Here are some of the terms used in order to provide clarification.

Geometry Teaching Knowledge (GTK). In this study, Geometry Teaching Knowledge (GTK) is a term developed specifically for this study that combines Geometry content knowledge, Mathematical Knowledge for Teaching Geometry, and Geometry techniques and methods used in the high school Geometry classroom.

Pre-Service teacher. In this study, a pre-service teacher is a student at a university that is currently working on coursework to complete their bachelor's degree or master's degree and is seeking Grades 8-12 Teaching Certification in Mathematics.

High School Geometry teacher. In this study, a high school Geometry teacher is a teacher teaching at a high school in Texas and is currently teaching Geometry.

Pedagogical Content Knowledge (PCK). Pedagogical Content Knowledge (PCK) "represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the divers interests and abilities of learners, and presented for instruction" (Shulman, 1987)

Mathematical Knowledge for Teaching (MKT). Mathematical Knowledge for Teaching (MKT) was developed by Deborah Ball and her colleagues (2008) based on Shulman's Pedagogical Content Knowledge model applied to mathematics. Ball and her colleagues have taken Shulman's idea of subject matter knowledge and pedagogical content knowledge and defined different categories that all define Mathematical Knowledge for Teaching: Comment Content Knowledge, Specialized Content Knowledge, Knowledge of Content and Students, Knowledge of Content and Teaching, Knowledge of Content and Curriculum, and Horizon Content Knowledge, all of which are defined below.

Common Content Knowledge (CCK). Common Content Knowledge is the mathematical knowledge needed to simply calculate the solution or correctly solve the problem. Ball (2008) emphasizes that common does not mean that everyone has this knowledge, but that this knowledge is used in other fields not unique to teaching.

Specialized Content Knowledge (SCK). Specialized Content Knowledge is "mathematical knowledge and skill unique to teaching" (Ball et al., 2008). SCK is the knowledge of mathematics that is not necessarily used in any other field.

Knowledge of Content and Student (KCS). Knowledge of Content and Students is "knowledge that combines knowledge about students and knowing about mathematics" (Ball et al., 2008). KCS is the knowledge teachers need in order to predict how students will react to a new topic, or what misconceptions and confusion will students have going into a lesson.

Knowledge of Content and Teaching (KCT). Knowledge of Content and Teaching is the category that "combines knowing about teaching and knowing about mathematics" (Ball et al., 2008). KCT primarily focuses on the planning of the teacher, the sequencing of topics so that students are the most successful, or what examples the teacher decides to show the students.

Horizon Content Knowledge. Horizon content knowledge is the "awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (Ball et al., 2008). This is not necessarily restricted to the current course the student is participating in, but can extend horizontally to the other courses the students are taking concurrently with their mathematics course as well as vertically, the courses the student has taken or will take in the future.

Knowledge of Content and Curriculum. Knowledge of content and curriculum is the knowledge a teacher needs regarding the content of the course they are teaching and the curriculum used in teaching the course. A teacher will need to have a grasp of the lessons being taught and have an understanding of how the curriculum selected by the department works.

Mathematical Knowledge for Teaching Geometry (MKT-G). Mathematical Knowledge for Teaching Geometry (MKT-G) was developed by Herbst and Kosko (2014) based on Ball's Mathematical Content Knowledge for Teaching (MKT). Herbst and Kosko's Mathematical Knowledge for Teaching Geometry (MKT-G) consists of the following domains: Comment Content Knowledge, Specialized Content Knowledge, Knowledge of Content and Students, Knowledge of Content and Teaching, Knowledge of Content and Curriculum, and Horizon Content Knowledge, all of which are defined above.

Geometry Content Knowledge (GCK). Geometry Content Knowledge is the mathematical knowledge needed to simply calculate the solution or correctly solve a Geometry problem. This is a CCK specific to Geometry.

Specialized Geometry Knowledge (SGK). Specialized Geometry Knowledge is Geometry knowledge and skill unique to teaching. SGK is the knowledge of Geometry that is not necessarily used in any other field. This is SCK specific to Geometry.

Knowledge of Geometry and Students (KGS). Knowledge of Geometry and Students is the combination of knowing about students and knowing about geometry. KGS is the knowledge teachers need in order to predict how students will react to a new Geometry topic, or what misconceptions and confusion will students have going into a Geometry lesson.

Knowledge of Geometry and Teaching (KGT). Knowledge of Geometry and Teaching is the category that combines knowing about teaching and knowing about Geometry. KGT primarily focuses on the planning of the teacher, the sequencing of Geometry topics so that students are the most successful, or what Geometric examples the teacher decides to show the students.

CHAPTER 2

KNOWLEDGE FOR TEACHING

This chapter reviews literature relevant to the teaching of Geometry and the knowledge needed for a teacher. The chapter is divided into seven sections: Pedagogical Content Knowledge, Mathematical Content Knowledge, Mathematical Content Knowledge of Geometry, Assessing Mathematical Knowledge of Teaching Geometry, Teacher Preparation Programs, the Gap in the Literature, and the Theoretical Framework of this study.

Pedagogical Content Knowledge

Lee S. Shulman (1986) and colleagues investigated the conceptions of teacher knowledge. Shulman started with analyzing tests for teachers that were used in the United States at the state and county levels. He found that exams focused primarily on subject matter content and a small portion of the exams dealt with pedagogical knowledge; however, the pedagogical knowledge was not related to the specific content areas. The pedagogical knowledge focused more on general pedagogy, for example; "What course would you pursue to keep up with the progress of teaching? How do you interest lazy and careless pupils?" (Shulman, 1986). "Although knowledge of the theories and methods of teaching is important, it plays a decidedly secondary role in the qualifications of a teacher" (Shulman, 1986). He points out that a primary problem with the assessment of teachers and their knowledge is due to policymakers being the major contributors to the development of assessments. Policymakers base their decisions on current research in education, which is lacking the connection between pedagogy and content knowledge, so "resulting standards and mandates lack any reference to content dimension of teaching"

(Shulman, 1986). Shulman (1987) and his colleagues defined, at a minimum, that a teacher's knowledge base should consist of the following categories: content knowledge; general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners; knowledge of educational contexts, and knowledge of educational ends, purposes, and values. Shulman and colleagues pay particular attention to the two categories, content knowledge and pedagogical content knowledge. Shulman calls for research into pedagogical content knowledge, also known as PCK. PCK "represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction" (Shulman, 1987). Even though Shulman calls for this research in the mid 1980s, the research into pedagogical content knowledge still remains underdeveloped (Ball et al., 2008). There is still much to do in defining what is meant by PCK. The term is "underspecified" and "has lacked definition and empirical foundation, limiting its usefulness" (Ball et al., 2008).

Mathematical Knowledge for Teaching

Shulman highlights the different types of knowledge that teachers should have, however Deborah Ball and her colleagues developed the concept Mathematical Knowledge for Teaching, also known as MKT. Using Shulman's major categories of teacher knowledge, she began testing Shulman's hypothesis about content knowledge and pedagogical content knowledge. Throughout their research, they began to see that "pedagogical content knowledge begins to look as though it includes almost everything a teacher might know in teaching a particular topic" (Ball et al., 2008). Ball begins to focus on mathematics and throughout history, the prevailing assumption of what mathematical knowledge a teacher requires is the mathematics that will be covered in the course they are teaching along with some additional study of mathematics at the college level. This seems to disregard Shulman's idea of pedagogical content knowledge completely. Deborah Ball and her colleagues decided to develop Shulman's model in the field of mathematics. The primary data used for the analysis was a National Science Foundation funded longitudinal study that documented an entire year of mathematics teaching in a third grade public school classroom. They analyzed videotapes, audiotapes, transcripts, copies of student work, teacher plans, teacher notes, and teacher reflections. A second source for data was the experience and disciplinary backgrounds of the research group, and the third source was a set of tools they developed for coordinating mathematical and pedagogical perspectives. Using all of this data, they began to build the model of mathematical knowledge for teaching. This was conducted as a qualitative approach to a large amount of data. The questions that guided their qualitative approach were:

> "1. What are the recurrent tasks and problems of teaching mathematics? What do teachers do to teach mathematics?
>
> 2. What mathematical knowledge, skills, and sensibilities are required to manage these tasks?" (Ball et al., 2008).

Through analysis, they found evidence that "mathematical knowledge needed for teaching is multidimensional" (Ball et al., 2008).

The following is the Domains of Mathematical Knowledge for Teaching that Deborah Ball and her colleagues developed after analyzing the data from this study:

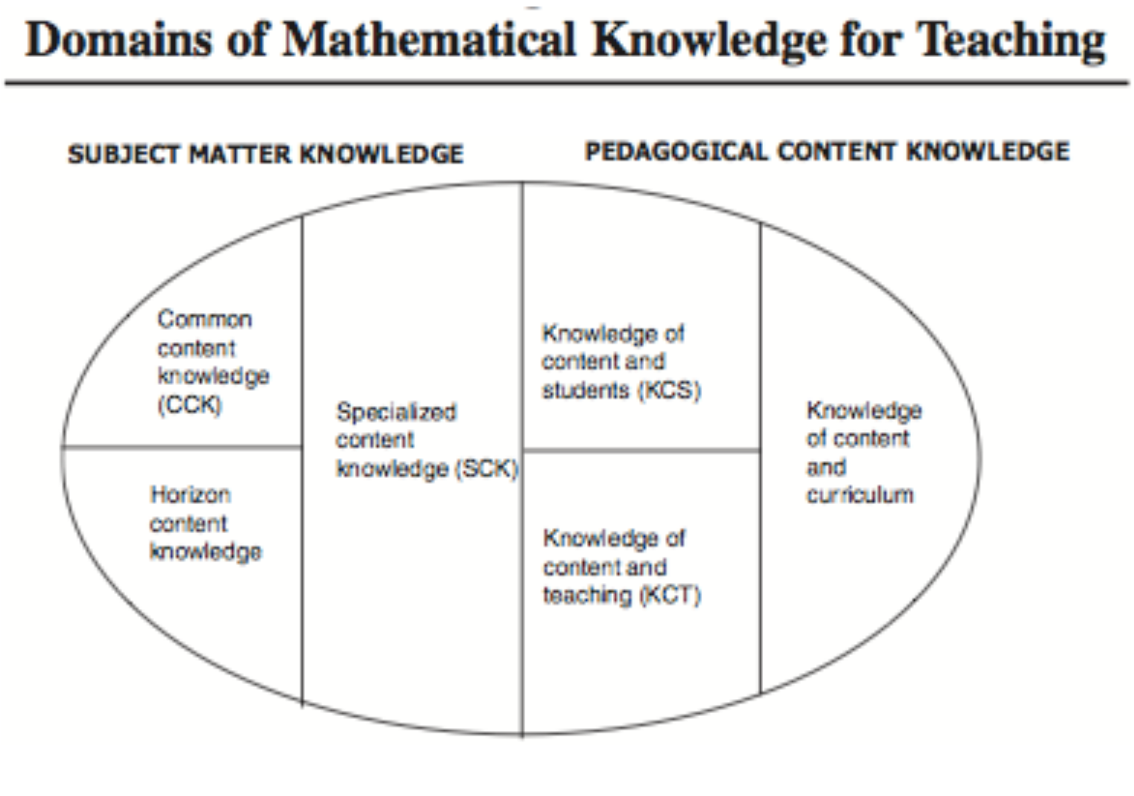


Figure 2. Domains of Mathematical Knowledge for Teaching (Ball et al., 2008)

Ball and her colleagues have taken Shulman's idea of subject matter knowledge and pedagogical content knowledge and defined different categories imbedded in Shulman's original model. Common content knowledge, CCK, is defined as "the mathematical knowledge and skill used in settings other than teaching" (Ball et al., 2008). CCK is the mathematical knowledge needed to simply calculate the solution or correctly solve the problem. Ball emphasizes that common does not mean that everyone has this knowledge, but that this knowledge is used in other fields not unique to teaching. Specialized content knowledge, SCK, is "mathematical knowledge and skill unique to teaching" (Ball et al., 2008). SCK is the knowledge of mathematics that is not necessarily used in any other field. For example, the knowledge needed to see what a student's mistake is when solving a problem incorrectly. Knowledge of content and students, KCS, is "knowledge that combines knowledge about students and knowing about mathematics" (Ball et al., 2008). KCS is the knowledge teachers need in order to predict how students will react to a new topic, or what misconceptions and confusion students will have going into a lesson. Knowledge of content and teaching, KCT, is the category that "combines knowing about teaching and knowing about mathematics" (Ball et al., 2008). KCT primarily focuses on the planning of the teacher, the sequencing of topics so that students are the most successful, or what examples the teacher decides to show the students. Horizon content knowledge is the "awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (Ball et al., 2008). This is not necessarily restricted to the current course in which the student is participating, but can extend horizontally to the other courses the students are taking concurrently with their mathematics course as well as vertically, the courses the student has already taken or will take in the future. Knowledge of content and curriculum is the knowledge a teacher needs regarding the content of the course they are teaching and the curriculum used in teaching the course. A teacher will need to have a grasp of the lessons being taught and have an understanding of how the curriculum selected by the department works.

Ball backs this model with factor analysis, however there is some room for refinement and revisions to the model in order to fully cover all the categories of mathematical knowledge for teaching. There are also problems with the model because it is difficult to distinguish some situations as a specific category. Since multiple categories can be used to describe certain situations in teaching, there is a need for more research into the different categories in order to have situations fall into the correct categories of teacher knowledge.

Hill, Rowan, and Ball (2005) have shown through their work in MKT assessments at the elementary school level that high achievement on the MKT assessments correlates significantly with effective mathematics instructions demonstrated through K-8 standardized test scores (Hill, et. al, 2005). Other research has shown a significant correlation between MKT assessment scores and observation protocols (Hill, Ball, Blunk, Goffney, & Rowan, 2007). These studies have addressed what knowledge is important and how it is displayed in successful teaching practice.

Prior to the work of Ball, Ruhama Even (1993) investigated the subject-matter knowledge and pedagogical content knowledge of pre-service secondary teachers pertaining to the concept of function. This study consisted of 152 pre-service secondary teachers competing an open-ended questionnaire regarding their knowledge of functions and an additional 10 prospective teachers were interviewed after responding to the questionnaire. Analysis of these questionnaires and interviews showed that pre-service teachers did not have a modern conception of functions. Modern conceptions of functions are defined as such: “if there seemed to be some reference to the arbitrary nature of the functions” (Even, 1993, p.103). For example, “A function is a set of ordered pairs (x, y) that have different x values but may or may not have the same y value” (Even, 1993, p.103). Definitions were considered not modern “if some regularity of the function behavior was included” (Even, 1993, p.103). For example, “a function is a relationship between coordinates that meets certain requirements of smoothness” (Even, 1993, p.103) and are considered “nice” (Even, 1993, p.111). The types of definitions pre-service teachers had of functions becomes problematic when explaining functions to students because they had difficulty in using modern terms to describe functions. This caused many of the pre-service teachers to give students a rule for functions without concern for understanding. This study highlighted the problems that could occur when teachers do not have a full understanding of the concepts they are required to teach. “Their pedagogical decisions- questions they ask, activities they design, students’ suggestions they follow- are based, in part, on their subject matter knowledge” (Even, 1993, p.113).

Charalambous, Hill, and Mitchell (2012) examine the Mathematical Knowledge for Teaching (MKT) and curriculum materials that contribute to the implementation of lessons on integer subtraction. They followed three middle school teachers with differing MKT levels using two editions of the same curriculum addressing integer subtraction. The two editions had differing levels of curriculum support for the instructors. They found that teachers with high MKT were able to use the higher levels of curriculum support, but were also able to compensate for the lack of teacher support given in the different edition of curriculum. They were also more likely to use higher-level mathematical language and had the ability to build on established ideas from students. Those who had lower MKT had a difficult time when given curriculum with little support, however when they were given curriculum with a higher level of support, the teachers were able to provide adequate instruction. This study shows that even though some teachers may have limited MKT, instruction can be improved to some degree with curriculum materials that have a high-level of supportive materials.

McCrory, Floden, Ferrini-Mundy, Reckase, and Senk (2012) define the “categories of knowledge and practices of teaching necessary for understanding and assessing teachers’ knowledge for teaching algebra” (McCrory, et al., 2012, p. 584). They acknowledge that the majority of research into MKT has primarily been in the elementary school levels, and saw the need for more research in MKT at the secondary level. In this study, McCrory, Floden, Ferrini-Mundy, Reckase, and Senk (2012) describe a framework that was developed as part of an effort to create a MKT assessment for teaching algebra that is “sensitive to both advanced mathematical knowledge and knowledge closer to teaching” (McCrory, et al., 2012, p. 587), when compared to teacher assessments that primarily measure algebra content knowledge. During the first stage of the study, researchers analyzed sections of four secondary school algebra books. The topics they decided to focus on for item writing were variables, expressions, equations, development of methods to solve equations of the form $ax + b = cx + d$, and logarithmic expressions and equations. The purpose of this portion of the study was to understand how the topics were presented in difference curriculums used in the United States and “to focus our attention on specific ways in which mathematical knowledge needed for teaching might vary depending on the curriculum” (McCrory, et al., 2012, p. 592) and on ways in which teachers’ knowledge could be coherent or at odds with the curriculum they are expected to teach. The second portion of the study consisted of interviews of 17 high school algebra teachers serving as mentors to pre-service teachers. The interviews focused on teachers’ knowledge of algebra and research about student learning. Analysis of the interviews consisted of targeting portions of the interviews where the teachers talked about student misunderstanding, identifying mathematics used to explain the student responses, and the knowledge used to help the students progress in those topics. The third portion of the study pertained to analysis of video taped lessons. Three other studies supplied the videos and these videos were analyzed with the focus on the mathematics they taught. Their findings resulted in the development of framework categories across two dimensions: Mathematics Content Knowledge and Mathematical Uses of Knowledge in Teaching. Mathematics Content Knowledge consists of “knowing what they will teach (school knowledge of algebra), knowing more advanced mathematics that is relevant to what they will teach (advanced knowledge), and knowing mathematics that is particularly relevant for teaching and would not typically be taught in undergraduate mathematics courses (teaching knowledge)” (McCrory, et al., 2012, p. 595). Mathematical Uses of Knowledge in Teaching consists of “trimming, bridging, and decompressing” (McCrory, et al., 2012, p. 595). Trimming requires the teacher to be able to cut down or build up connections to mathematics in order to meet the needs or abilities of the students. Bridging refers the teachers’ ability to connect and link mathematics across topics and courses. The ability for teachers to decompress their knowledge refers to the requirement of upper level mathematics in their training, so teachers need to be able to isolate what is needed for student at the novice level of algebra. The researchers claim that this framework can help with developing teacher preparation programs in order to isolate the topics needed to be addressed for the teachers to be more successful in the high school algebra classroom.

Other contributors to research into MKT have developed their own frameworks that include categories that contribute to MKT. Usiskin’s (2001) framework contains three major categories: “concept analysis- the phenomenology of mathematics concepts;…problem analysis- the extended analysis of related problems;… and the connections and generalizations within and among diverse branches of mathematics” (Usiskin, 2001, p. 3) Silverman and Thompson (2008) based their own framework of MKT on Simon’s (2006) idea that consisted of powerful understanding of concept and the transformation of those concepts from an "understanding having pedagogical potential to an understanding that does have pedagogical power" (Simon, 2006, p. 502).

Wilson and Heid (2010) have attempted to define what they refer to as "mathematical understanding for secondary teaching" (Wilson & Heid, 2010). They define this concept as teachers needing "mathematical understanding for teaching at the secondary level is the mathematical expertise and skill a teacher has and uses for the purpose of promoting students' understanding of, proficiency with, and appreciation for mathematics" (Wilson & Heid, 2010). Their framework was built on mathematical analysis of classroom-based incidents.

The Teacher Education Development Study in Mathematics (TEDS-M) identifies two components to MKT: mathematical content knowledge (MCK) and mathematical pedagogical content knowledge (MPCK) (Tatto et al., 2012). This study developed a framework to measure pre-service teachers MKT through different domains. The domains for MCK included number, geometry, algebra, data, knowing, applying, and reasoning, and the domains for MPCK included mathematics curricular knowledge, knowledge of planning, and knowledge of enacting mathematics (Tatto et al., 2012).

The Germany project, COACTIV, conducted a study of the connections between content knowledge and pedagogical content knowledge in secondary mathematic curriculums (Krauss et al., 2008). They found that content knowledge and pedagogical content knowledge were distinct factors and highly correlated in the entire sample of teachers, however, teachers considered mathematical experts held knowledge that combined the content knowledge and the pedagogical content knowledge, while those that were not experts had two distinct factors of knowledge. They concluded that pedagogical content knowledge may be supported by higher levels of content knowledge in ways that lower levels of content knowledge may not (Krauss et al., 2008).

Mathematical Content Knowledge of Geometry

In Chinnappan and Lawson's (2005) study of teachers' content knowledge of geometry and knowledge for teaching, they interviewed two teachers. These two teachers had at least 15 years of teaching mathematics experience. Each of them was involved in their schools and was considered leaders in their departments. These two teachers were interviewed three times regarding a square and the properties of squares. The first interview the teachers were asked to talk about the concept of a square. In the second interview, the teachers were asked to solve four math problems involving squares, and in the final interview the teachers were asked probing questions to give the teachers the opportunity to access relevant knowledge regarding the square and any other properties of a square that are important geometrically. The researchers then took the information from the interviews and constructed concept maps based on the answers the teachers gave. "Concept mapping techniques have also been argued to be appropriate for representing complex interrelationships among schematized knowledge within and between domains" (Chinnappan & Lawson, 2005). The researchers assessed the quality of the maps based on how many nodes which are used to represent major topics related to the square; an example would be right angles, and how many connections were made to the nodes. Some of the connections were obvious, but the more complex a connection, the greater value it was given in the assessment. They analyzed the data with respect to content knowledge and knowledge for teaching. The researchers found that concept maps were a significant way to assess the interviews. The teachers had different maps and different complexity. One of the teachers was thought to have a more complex understanding of the concept of a square than the other teacher, but both had some interesting connections, however neither of the teachers had any real world examples or application problems involving a square on their concept maps.

Knuth (2002) conducted a study of how teachers understand the concept of proof in Geometry. Knuth interviewed 16 teachers and the interviews were coded to give the researcher data that clearly showed relevant themes with regard to the concept of proof. Knuth highlighted the responses of 4 of the 16 teachers interviewed in his research. All of the teachers had some concept of the importance of proof, but had differing opinions on the values of proof. The teachers involved in the study had differing levels of experience, but knew what a proof was to some extent. Knuth found that teachers had such varying opinions on the value and understanding of proof that this needs to be addressed. The only appropriate place that this could be addressed would be in pre-service teacher preparation programs. There is a push to have students understand the concept of proof and to be comfortable with its use before entering higher level mathematics, but "if teachers are to be successful in enhancing the role of proof in secondary school mathematics classrooms, then their conceptions of proof must be enhanced"(Knuth, 2002).

Mathematical Knowledge for Teaching Geometry

Herbst and Kosko (2014) conducted a study that targeted the mathematical knowledge for teaching geometry, MKT-G. This study was based off Deborah Ball's model of mathematical knowledge for teaching but targeted the knowledge needed for high school geometry. The majority of the work done with MKT using Ball's model has been focused on elementary and middle school levels of mathematics. Herbst and Kosko decided to "follow the theoretical conceptualization of MKT and item development procedures of Ball and Hill's group" (Herbst & Kosko, 2014). Herbst and Kosko developed an assessment targeting the CCK, SCK, KCT, and KCS domains of the model. Their interest in MKT-G stems from the a "long-term agenda that seeks to understand the work of teaching in specific instructional systems such as high school geometry" (Herbst & Kosko, 2014). Their purpose in designing the instrument to measure MKT-G was "concerned not with geometry as a mathematical domain but with high school geometry as a course of studies" (Herbst & Kosko, 2014).

Herbst and Kosko used the approach to measuring MKT developed by Ball and Hill. They did not only use the conceptualization of the domains but also methods of developing items. The assessment contains multiple choice and multiple response items instead of open-ended items. In order to develop the assessment, Herbst and Kosko used the curriculum guidelines from various states to "develop items dealing with definitions, properties, and constructions of plane figures" (Herbst & Kosko, 2014). These items were sufficient to cover the CCK category in the framework. In order to address the SCK category, they listed tasks of teaching in which the teacher may be required to do mathematical work. The list included items such as "designing a problem or task to pose to students, evaluating students' constructed responses, particularly student-created definitions, explanations, arguments, and solutions to problems, creating an answer key or a rubric for a test, and translating students' mathematical statements into conventional vocabulary" (Herbst & Kosko, 2014). These items became more and more complex through the development of assessment items. The difficulty in developing this category of the model with geometry drew attention to the possibility that geometry may be entirely different area of study for the model and may require some restructuring of the organization of the MKT model. The KCT items also used the task in teaching list and the rational behind using similar items came from the notion that a teacher would choose the correct answer based off their knowledge of the material covered in their course. The final category, KCS, was designed to measure the teacher's knowledge of misconceptions of the students. The items did not focus solely on student misconceptions of geometric figures but also on misconceptions of processes inherit to geometry. The initial instrument was composed of 13 CCK, 20 SCK, 26 KCT, and 16 KCS questions.

Herbst and Kosko assessed the category addressed and the degree of interpretation of the questions by interview teachers as they completed the assessment. The teachers were asked to read through the items and then explain to the interviewer what the teacher thought the questions was asking them to do. They used the interview data to examine content validity and also improve the validity. They found that the majority of their items measured what they intended, and some were reworded or restructured to correct for any ambiguity. The interviews gave insight into whether the teacher participating in the assessment saw pedagogy in the question or pure geometry content, and this information was used to inform additional revisions of the items. The instrument was then piloted with in-service secondary teachers from a Midwestern state and a Mid Atlantic state between July 2011 and May 2012. The assessment was administered through the Lesson Sketch online platform (www.lessonsketch.org), and was taken by participants in a computer lab or at home. Combining both samples, the Midwestern state and the Mid Atlantic state, there were 83 participants in the pilot study.

Herbst and Kosko first administered the instrument to a Midwestern state. After this administration, some items were removed due to not performing well. The instrument retained 34 items after removing 46.9% of all items with the intention to increase reliability of the remaining questions. Following this initial pilot, the researchers administered the revised instrument to the Mid Atlantic state. Even though they analyzed the instrument based off the individual categories, the researchers decided they wanted to better understand the MKT-G construct as a whole and how their performance compared to their varying background factors. A question that was posed when analyzing the pilot data was "what is the relationship between mathematical knowledge for teaching geometry and experience teaching high school geometry?" (Herbst & Kosko, 2014). They found that having three or more years of teaching experience of geometry has statistically significant positive effect on the MKT-G scores. Teaching experience in general had a positive effect on the scores, but teaching geometry specifically, had a statistically significant effect on their performance. Previous research in elementary and middle schools show that teachers with years of teaching experience have higher MKT scores (Hill, 2007; 2010). An interesting finding in this study shows that teaching experience in general can affect the MKT-G scores of a teacher, but teaching experience in Geometry is the factor that matters the most. According to the published literature on this instrument, in-service teachers are the only populations that have been studied. Herbst and Kosko (2014) call for more research into the instrument with the addition of pre-service teachers.

Since there is little information regarding pre-service teachers' Geometry knowledge, there is a need to study those who are currently participating in teacher preparation programs. There are many different types of teacher preparation programs and there have been a few studies focusing on how well these programs perform.

Teacher Preparation Programs

Yow (2009) conducted a study on teacher preparation programs and the opinions on two second-year high school mathematics teachers. She wanted to know how secondary mathematics teachers reflect on their experiences in their preparation programs, how do they remember their preparation programs, what is the most valuable part of their preparation programs, and what would they like to see their preparation programs add to their curriculum. The two teachers she selected to interview were from different programs. The interview lasted anywhere from one to two hours. The interviews were recorded and then the degree plans from each of the preparation programs were compared to the answers given in the interviews. Different programs prepared both of the teachers, so the programs focused on different things, but the teachers seemed to be happy with their teacher preparation programs in general. They felt that the programs prepared them for the majority of the tasks they are required to do on a daily basis, but they did recommend that there be more classes and training with regard to everyday life in the high school classroom. In both programs, there was talk about what theoretically happens on a daily basis, but there was never any actual real life experience with it and how to deal with distractions, relationships, and motivation in the high school classroom (Yow, 2009). All teacher preparation programs are going to have faults, but it is necessary to give pre-service teachers all the information they could possibly need in the everyday classroom.

Charalambous, Hill, and Ball (2011) investigate pre-service teachers' learning to provide instructional explanations. Their primary research questions address if pre-service teachers can learn to provide explanations during their teacher preparation, and if they can what does this learning entail and what contributes to it (Charalambous, et. al, 2011, p. 445). They followed sixteen pre-service teachers enrolled in two courses that were part of a 1-year intensive teaching education program that lead to K-8 teacher certification and a Masters of Arts degree in education. In order to provide an in-depth investigation of how this knowledge is developed, they decided to follow four students through the 1-year program. The four students were selected based on the large variation of the abilities to provide explanations, had differing mathematical backgrounds, and different teaching experiences. Using in-class artifacts, their performance on examinations, comments on reflection cards given periodically in class, and explanations to others in the class captured by videotape analyzed four pre-service teachers' ability to explain concepts. The findings of this show that courses that target the ability for pre-service teachers to explain concepts does help pre-service teachers improve their explanations to varying degrees. There is an overall improvement, however one of the teachers did not improve enough to be able to explain concepts at an acceptable level. The researchers call for addition research into what is involved in a successful mathematical explanation in order to address these components in pre-service teacher preparation courses.

How are preparation programs developed in Texas? Preparation programs are based on the standards recommended appropriate for high school mathematics students. Then guidelines delineate the mathematics teacher preparation curriculum that is necessary to provide teachers with the knowledge required to teach that high school mathematics curriculum (Tooke, 1993). What the student needs to know drives most preparation programs and then additional material is added to make sure the pre-service teachers have the pedagogical skills to teach the material the students must know according to state standards. “Teacher educators are continually searching for the best way to educate teachers”. Most programs follow the NCTM standards, but the main focus in Texas preparation programs is the Texas Essential Knowledge and Skills (TEKS) and the standards of the TExES exams.

At a central Texas university, there is a program specifically for a Bachelor of Arts or Bachelor of Science in Mathematics with a grade 8 through 12 teaching certificate. This program differs from the program to prepare middle school teachers and elementary school teachers. There is “no instructional strategy that is appropriate for all age levels” (Brophy, 1979), so it seems intuitive to require students that are teaching higher-level mathematics courses to have taken more hours in that field. Begel (1972) points out that “there is a minimum amount of necessary mathematical knowledge for a teacher, and there might not be a maximum” (Begel, 1972). Research has shown that “student achievement increases as teachers’ knowledge of mathematics increases” (Tooke, 1993). This seems obvious, but it is interesting to note that the increase in number of classes influences the teacher, but not the teacher grade point average. (Tooke, 1993) How a teacher does in math courses does not affect the students’ achievement, but the teacher’s exposure to many different kinds of mathematics shows an increase in student achievement.

The Mathematics Education for Teachers II Report (2012) gives requirements and suggestions for teacher preparation programs in the United States. These requirements are based off the Common Core Standards. According to the Mathematics Education for Teachers II Report (2012), these are some "ingredients" of specialized courses for teachers:

> Geometry and transformations. The approach to geometry in the Common Core State Standards replaces the initial phases of axiomatic Euclidean geometry. In the latter, the triangle congruence and similarity criteria are derived from axioms. The Common Core, on the other hand, uses a treatment based on translations, rotations, reflections, and dilations, whose basic angle and distance preserving properties are taken as axiomatic.
>
> The Pythagorean Theorem is a fundamental topic in school geometry, and students should see a proof of the theorem and its converse.
>
> An understanding of the role played by the parallel postulate in Euclidean geometry is essential for geometry teachers. Knowing where the postulate is hiding underneath the major theorems in plane geometry, from angle sums in polygons to area formulas, helps teachers build a coherent and logical story for their students.

Analytic geometry. Many connections between high school topics and the content of undergraduate mathematics can be highlighted in a course in analytic geometry.

(Conference Board of the Mathematical Sciences, 2012, p. 63-64)

The MET II report suggests that these topics be address in teacher preparation programs and that mathematics educators use these to help guide the development of pre-service and current K-12 teachers. The MET II does draw attention to some Geometry concepts, however there is specific reference to Statistics and Probability, not Geometry. This is interesting because the American Mathematics Society has drawn attention to the weaknesses in mathematical knowledge in some mathematics courses that pre-service teachers are expected to have mastered upon graduation.

Gap in the Literature

Deborah Ball's model has been sited over 1800 times since it was published. Many studies have been conducted to try to solidify this model, and other studies have focused on specific categories in the mathematical knowledge for teaching. For example, Hill, Ball, and Schilling (2008) focused on the knowledge of content and students. They point out that there has been little research in conceptualizing, developing, and measuring teachers' knowledge in each of the domains (Ball et al., 2008). Even though there have been many studies referring to Deborah Ball's MKT model, there is very little research in the secondary level of mathematics. Primary research has been conducted in Elementary levels of Algebra and Number Sense. There are even very few studies in Elementary Geometry. Another study of MKT for Algebra points out that "the University of Michigan's work marks considerable progress in defining and assessing teachers' mathematical knowledge for elementary and, more recently, middle-grades teaching, there is little systematic evidence about whether, or how different types of mathematical knowledge matter for effective teaching of algebra in grades 6-12" (McCrory, et al., 2012, p. 584).

Herbst and Kosko (2014) point out that there is little research into Ball's MKT model in high school specific subjects (Herbst & Kosko, 2014). Herbst and Kosko's MKT-G instrument is still in its beginning stages of development, however it has only been used to assess in-service teachers in the Midwest and Mid-Atlantic regions of the United States. There has not been any quantitative research in MKT-G of pre-service teachers let alone the comparison between pre-service teachers and in-service teachers. The literature calls for more research in pre-service and in-service teacher MKT-G along with an investigation as to where these teachers gain their MKT-G knowledge. Herbst and Kosko (2014) point out that that there is more work to be done to refine the domains of the Ball's MKT model with respect to Geometry and by doing so this "could inform the development of coursework in mathematics or mathematics education for future teachers" (Herbst & Kosko, 2014, p.33)

Theoretical Framework

The theoretical framework used in this study follows the Domains of Mathematical Knowledge for Teaching that Deborah Ball and her colleagues developed and the framework used by Herbst and Kosko, however some of the domains were modified due to the focus of this study. The theoretical framework used by Herbst and Kosko (2014) and the relationship to the theoretical framework used in this study are below.

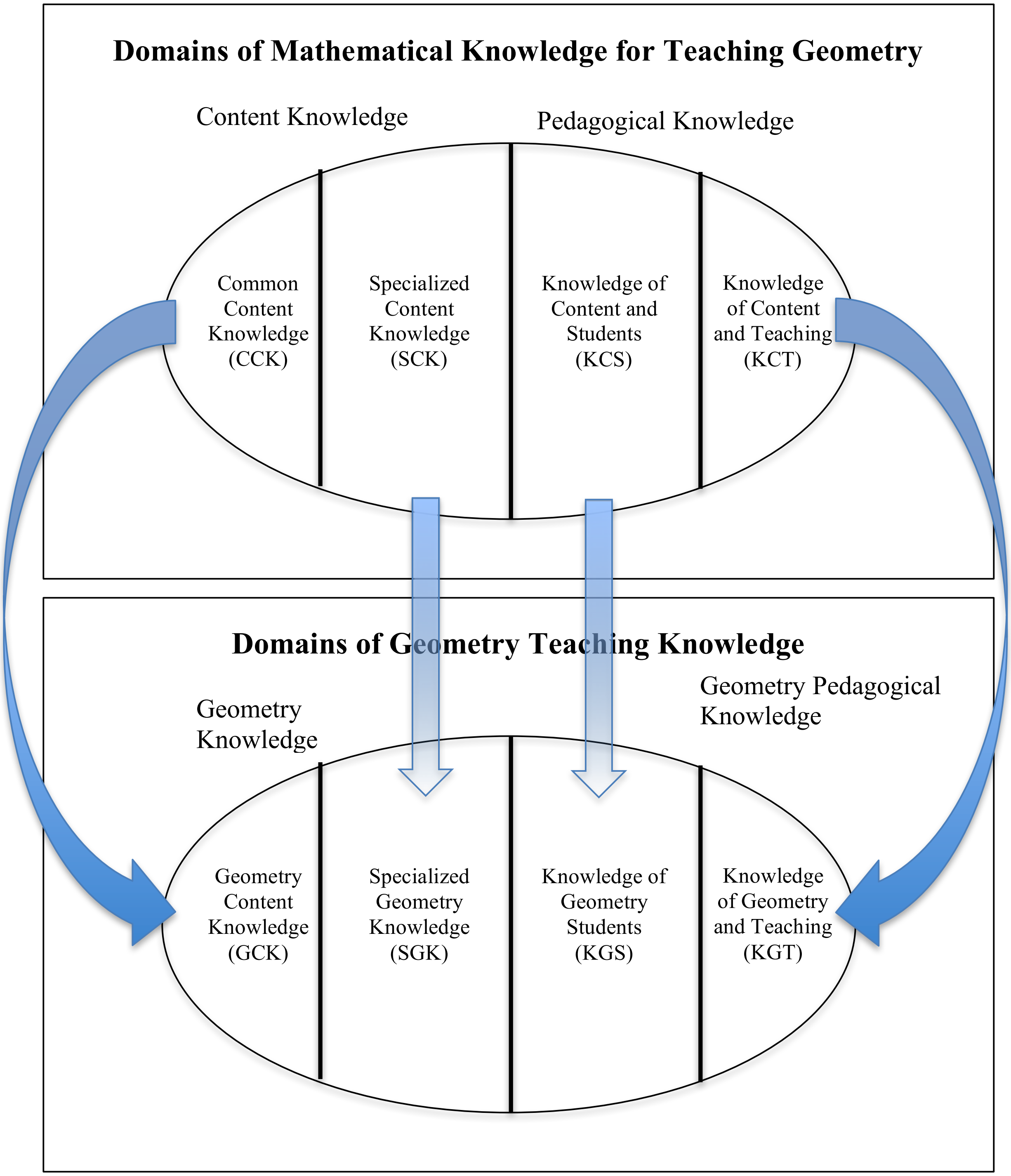


Figure 3. Domains of the Mathematical Knowledge for Teaching Geometry and *Geometry Teaching Knowledge*

Ball and her colleagues took Shulman's idea of subject matter knowledge and pedagogical content knowledge and defined different categories imbedded in Shulman's original model. The framework above is a modification of Ball's model specific to Geometry and this study. Geometry content knowledge, GCK, is defined as the geometry knowledge and skill used in settings other than teaching. GCK is the geometry knowledge needed to simply calculate the solution or correctly solve the problem. Ball emphasizes that common does not mean that everyone has this knowledge, but that this knowledge is used in other fields not unique to teaching. Specialized Geometry Knowledge, SGK, is geometry knowledge and skill unique to teaching. SGK is the knowledge of geometry that is not necessarily used in any other field. For example, the knowledge needed to see what the student's mistake is when solving a problem incorrectly. Knowledge of Geometry and students, KGS, is knowledge that combines knowledge about students and knowing about Geometry. KGS is the knowledge teachers need in order to predict how students will react to a new geometry topic, or what misconceptions and confusion will students have going into a geometry lesson. Knowledge of Geometry and Teaching, KGT, is the category that combines knowing about teaching and knowing about geometry. KGT primarily focuses on the planning of the teacher, the sequencing of geometry topics so that students are the most successful, or what geometry examples the teacher decides to show the students. KGT also includes the knowledge of instructional strategies and methods; investigations/discovery lessons, compass and protractor activities, computer software, and manipulatives and models.

CHAPTER 3

METHODS

Extensive reviews of the literature indicate that mathematical knowledge for teaching is necessary for teachers to be successful in the mathematics classroom environment. Deborah Ball has made vast improvements regarding the assessment of teachers' mathematical knowledge for teaching, however there is little research in the assessment of mathematical knowledge for teaching in geometry. This study answers the following research questions:

1. What do high school pre-service teachers and high school Geometry teachers know about *Geometry Teaching Knowledge* which consists of the following:
    a. Mathematical knowledge for teaching geometry?
    b. Geometry techniques and methods used in the high school Geometry classroom?
2. How do pre-service and current high school teachers' *Geometry Teaching Knowledge* compare?
3. What are the sources of the high school teachers *Geometry Teaching Knowledge* that can be transferred to pre-service teachers?
4. What are the sources of the pre-service teachers *Geometry Teaching Knowledge* that can be transferred to high school teachers?
5. What do high school pre-service teachers need to know to be prepared to teach high school Geometry?

This study will be using a pragmatist paradigm view in which the focus is on the consequences of the research and the research questions, which allows for multiple methods of data collection (Creswell & Clark, 2013). The pragmatic paradigm places the research question as central and applies all approaches to understanding the problem (Creswell, 2013). In this mixed methods study, quantitative data was collected consisting of an assessment instrument and a survey instrument, along with qualitative data in the form of pre-service and in-service teacher interviews and classroom observations. By collecting numerous data types and sources, the researcher gains insight into the *Geometry Teaching Knowledge* of pre-service and high school Geometry teachers.

Design and Conceptual Framework

This study investigates the *Geometry Teaching Knowledge* of pre-service and high school Geometry teachers. The Mathematical Knowledge for Teaching- Geometry Assessment is the primary medium used to measure depth of Mathematical Knowledge for Teaching Geometry of the participants. This assessment was developed using Deborah Ball's Mathematical Knowledge for Teaching Model, however only four of the domains were developed. Below is the conceptual framework used to assess the knowledge of the in-service and pre-service teachers in this study:

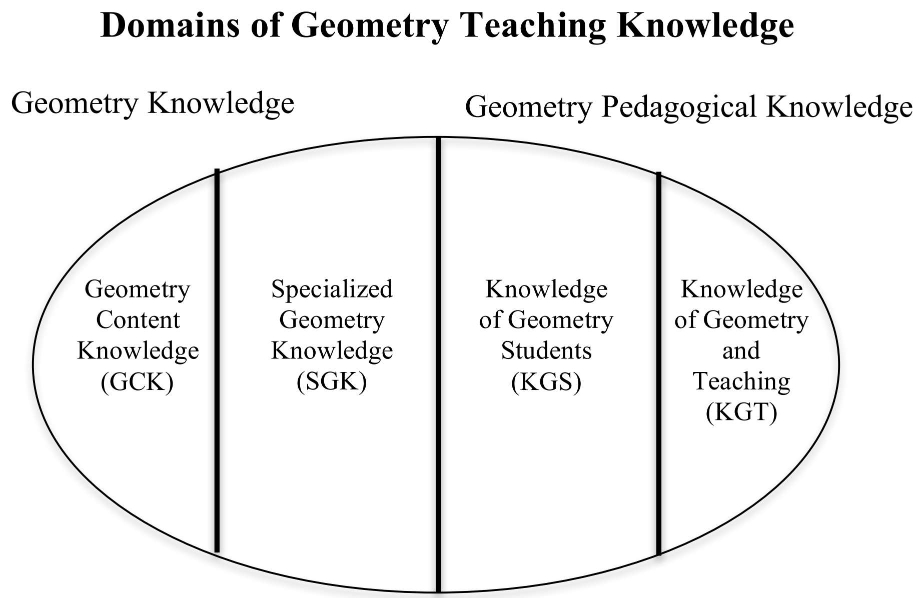


Figure 4: Domains of *Geometry Teaching Knowledge*

The depth of the domains of *Geometry Teaching Knowledge* were measured by this assessment and by interview tasks. Geometry Content Knowledge (GCK) was measured through the assessment given to all participants. The following is an example of a GCK question that will appear on the assessment:

Students in Mr. Wingate's class have been creating nets that they want to be able to fold to create a cube. For each of the student-created nets shown below, identify whether it can be successfully folded into a cube?

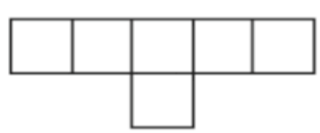
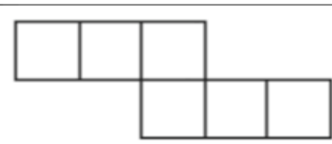
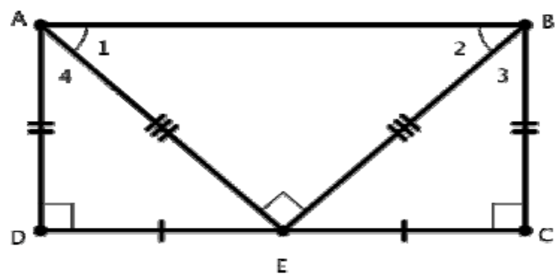
		Yes	**No**
i			
ii			

Figure 5: GCK Example Question (Herbst & Kosko, 2014)

Specialized Geometry Knowledge (SGK) will be assessed when participants take the MKT-G instrument. The following is a released question addressing this domain:

While proving a claim on the board about the figure below, Joe wrote "1 + 2 = 90." Ms. Staples ponders how to correct that statement. Of the following, what is the best alternative?



A	Do nothing. The statement is correct as is.
B	Add the degree symbol so that the statement reads $1 + 2 = 90°$.
C	Replace what Joe wrote; write instead that "$m\angle A + m\angle B = 90°$".
D	Replace what Joe wrote; write instead that "$m\angle EAB + m\angle EBA = 90°$".

Figure 6. SGK Example Question (Herbst & Kosko, 2014)

Knowledge of Geometry Students (KGS) will be addressed in the assessment. The following is a released question that targets this domain:

Ms. Jamison asked students to figure out the number of diagonals in an 11-sided polygon. Of the following erroneous solutions, which one might have resulted from using the number of diagonals through one vertex?

A	11
B	72
C	88
D	99
E	121

Figure 7. KGS Example Question (Herbst & Kosko, 2014)

Knowledge of Geometry and Teaching (KGT) will be addressed in the assessment. The following is a released question from the assessment to address this domain:

After teaching her students the base angles theorem (which states that base angles of an isosceles triangle are congruent), Ms. Wellington is pondering what example she could give to illustrate how this theorem can help find new information. Which of the following would be the best example?

A	If you have a triangle that you know is isosceles and you know two of the sides are 8 and 10, then the third side will be 8 or 10.
B	If you have a triangle that you know the sides have length 8, 10, and 12 you know it does not have two angles that are congruent.
C	If you have an isosceles triangle and two of its angles are 30 you can calculate the third angle by subtracting 60 from 180.
D	If you have a kite and draw its minor diagonal, the kite is divided into two isosceles triangles and those have two congruent angles each.

Figure 8: KGT Example Question (Herbst & Kosko, 2014)

The KGT Domain also includes the knowledge of Geometry techniques and methods used in the high school Geometry classroom. The initial survey and interviews will address the knowledge of Geometry techniques and methods used in the high school Geometry classroom. The types of questions that will address this on the survey are similar to the following example:

1. If you had unlimited access and budget, what instructional techniques would you use in your own Geometry Classroom?

   Read the following techniques and consider which ones you would use in your own Geometry Classroom. You are given a total of 10 points to distribute among 5 techniques however you would like based on what you would think would be best for your students (assign a value between 0 and 10 to all items), with the number of points assigned to the topic reflecting the importance of these techniques in your classroom. You must use all 10 points. Please make sure the points add up to 10 by including a total count at the end.

   a. Investigations (Example: Discovery lessons) ________
   b. The use of a compass and protractor to construct figures ________
   c. Computer Software (Geometer's Sketchpad, Geogebra, etc)________
   d. Manipulatives/Models ________
   e. Other: (please describe) ____________________________________

Total:_______

Figure 9. Example Methods/Technique Problem

This data was verified through observations of the pre-service teacher student teaching and high school Geometry teachers as well as through the interviews of a subset of the participants. To address how the pre-service and high school Geometry teachers compare, the results from the assessment were compared. The interviews with pre-service and high school Geometry teachers is primary source of data to address the origin of the Geometry Content Knowledge and Mathematical Knowledge for Teaching Geometry. Verification of survey, assessment, and interview response was conducted through classroom observations of the one pre-service teacher assigned to teach Geometry as his student teaching assignment and of the current high school Geometry teachers. The research question addressing the origin of theses ideas that can be transferred to pre-service teachers is address through the analysis of the assessment results, interviews, and observations. The study as a whole addresses the research question regarding what pre-service teachers need to know in order to be successful at teaching Geometry.

Population and Sampling

The study was conducted at a central Texas university and other school districts in the state of Texas. The study is composed of 53 pre-service high school teachers at the University and 36 high school geometry teachers in multiple school districts in Texas. Three of the pre-service teachers had not taken informal geometry or modern geometry, so they were excluded from the analysis. One of the high school teachers was excluded from the portion of the data analysis using the survey data due to not completing the survey. The pre-service teachers were chosen based off their enrollment in courses that traditionally have the juniors and seniors in the program. The pre-service teachers were starting their student teaching semester either in Fall 2015, Spring 2016, Fall 2017, or Spring 2017. This guaranteed that the pre-service teachers had completed the majority of their required coursework for their specific graduation plan. The high school teachers were current teachers in multiple school districts in Texas, including San Marcos ISD, Hays ISD, Eanes ISD, Regents School in Austin, Denton ISD, Lake Dallas ISD, and Hurst-Euless-Bedford ISD. The reason for the selection of these Texas school districts was due to the professional connections to these districts. The high school teachers were either currently teaching Geometry, or they had taught Geometry in the past 2 years. Three pre-service teachers and four high school teachers that participated in the MKT-G online assessment were chosen to be interviewed and observed.

Instrumentation

To investigate pre-service and high school teachers' Geometry Content Knowledge, Mathematical Knowledge for Teaching Geometry, and knowledge of geometric techniques and methods used in the high school Geometry classroom, data was gathered by means of an online Mathematical Knowledge for Teaching- Geometry assessment, a Post-Assessment Survey, interviews, and observations. All participants participated in the online Mathematical Knowledge for Teaching- Geometry assessment, and all but one high school Geometry teacher participated in the Post-Assessment Survey. A selection of three pre-service teachers and four in-service teachers were interviewed. The in-service teachers were observed, and the pre-service teacher who was assigned geometry as their student teaching assignment during Fall 2015 was observed.

Mathematical Knowledge for Teaching-Geometry Assessment

Herbst and Kosko (2014) conducted a study that targeted the mathematical knowledge for teaching geometry, MKT-G. This study was based off Deborah Ball's model of mathematical knowledge for teaching but targeted the knowledge needed for high school geometry. The majority of the work done with MKT using Ball's model has been focused on elementary and middle school levels of mathematics. Herbst and Kosko decided to "follow the theoretical conceptualization of MKT and item development procedures of Ball and Hill's group" (Herbst & Kosko, 2014). Herbst and Kosko developed an assessment targeting the CCK, SCK, KCT, and KCS domains of the model. Their interest in MKT-G stems from the a "long-term agenda that seeks to understand the work of teaching in specific instructional systems such as high school geometry" (Herbst & Kosko, 2014). Their purpose in designing the instrument to measure MKT-G was “concerned not with geometry as a mathematical domain but with high school geometry as a course of studies” (Herbst & Kosko, 2014).

Herbst and Kosko used the approach to measuring MKT developed by Ball and Hill. They did not only use the conceptualization of the domains but also methods of developing items. The assessment contains multiple choice and multiple response items instead of open-ended items. In order to develop the assessment, Herbst and Kosko used the curriculum guidelines from various states to “develop items dealing with definitions, properties, and constructions of plane figures” (Herbst & Kosko, 2014). These items were sufficient to cover the CCK domain in the framework. In order to address the SCK domain, they listed tasks of teaching in which the teacher may be required to do mathematical work. The list included items such as “designing a problem or task to pose to students, evaluating students’ constructed responses, particularly student-created definitions, explanations, arguments, and solutions to problems, creating an answer key or a rubric for a test, and translating students’ mathematical statements into conventional vocabulary” (Herbst & Kosko, 2014). These items became more and more complex through the development of assessment items. The difficulty in developing this domain of the model with geometry drew attention to the possibility that geometry may be entirely different area of study for the model and may require some restructuring of the organization of the MKT model. The KCT items also used the task in teaching list and the rational behind using similar items came from the notion that a teacher would choose the correct answer based off their knowledge of the material covered in their course. The final domain, KCS, was designed to measure the teacher’s knowledge of misconceptions of the students. The items did not focus solely on student misconceptions of geometric figures but also on misconceptions of processes inherit to geometry. The initial instrument was composed of 13 CCK, 20 SCK, 26 KCT, and 16 KCS questions.

Herbst and Kosko assessed the category addressed and the degree of interpretation of the questions by interview teachers as they completed the assessment. The teachers were asked to read through the items and then explain to the interviewer what the teacher thought the questions was asking them to do. They used the interview data to examine content validity and also improve the validity. They found that the majority of their items measured what they intended, and some were reworded or restructured to correct for any ambiguity. The interviews gave insight into whether the teacher participating in the assessment saw pedagogy in the question or pure geometry content, and this information was used to inform additional revisions of the items. The instrument was then piloted with in-service secondary teachers from a Midwestern state and a Mid Atlantic state between July 2011 and May 2012. The assessment was administered through the Lesson Sketch online platform (www.lessonsketch.org), and was taken by participants in a computer lab or at home. Combining both samples, the Midwestern state and the Mid Atlantic state, there were 83 participants in the pilot study. Participants were primarily female and Caucasian. Participants also had varying amount of mathematics teaching experience with an average of 11.27 years of experience. Regarding Geometry specifically, 60.2% of participants were classified as "experienced Geometry teachers" (Herbst & Kosko, 2014, p. 11), because they had taught Geometry for three years or more.

Herbst and Kosko first administered the instrument to a Midwestern state. After this administration, some items were removed due to not performing well. The instrument retained 34 items after removing 46.9% of all items with the intention to increase reliability of the remaining questions. Following this initial pilot, the researchers administered the revised instrument to the Mid Atlantic state.

The researchers analyzed the instrument based of the individual domains (CCK, SCK, KCS, KCT). During the initial pilot study, the MKT-G items revised from cognitive pretesting contained 10 questions in each domain. When the researchers examined the statistical reliability of their questions, “multiple- response questions were treated as multiple items, with the number of items per a multiple-response item dependent on the number of accepted responses” (Herbst & Kosko, 2014, p.12). They used biserial correlations to measure item discrimination and these findings were combined with item difficulty and Cronbach’s alpha. The initial examination of the items allowed removal of some items that had zero or negative correlations with the overall pattern of responses per domain. This analysis resulted in the removal of 46.9% of the exam questions, leaving 34 items on the assessment. Items were also removed based off low discrimination power, too long to answer, and response options being too closely related. The tables below show the descriptive statistics by domain. Table 1 shows that the reliability is low for the individual domains. This may be attributed to the difficulty range of questions in each domain and the number of questions in each domain.

Table 1.

Descriptive Statistics by MKT Domain.

Domain	M	SD	N	α
CCK – Geometry	0.66	0.20	83	0.58
SCK – Geometry	0.59	0.21	83	0.65
KCT – Geometry	0.39	0.21	83	0.50
KCS – Geometry	0.38	0.25	83	0.49

(Herbst & Kosko, 2014, p. 14)

The reliability of each of the domains individually does not meet the acceptable rating of .70. SCK has acceptable reliability primarily due to the large range of difficulty and the number of items addressing this domain. KCS, on the other hand, has the lowest reliability rating primarily due to the lack of items addressing this domain.

The correlations between domains are shown in Table 2.

Table 2.

Correlations between MKT-G Domain Scores.

	CCK	SCK	KCT	KCS
CCK	-			
SCK	.40***	-		
KCT	.36**	.54***	-	
KCS	.69***	.48***	.43***	-

**p* < .05, ***p* < .01, ****p*<.001

(Herbst & Kosko, 2014, p. 15)

The correlations between the domains are moderate to strong. This is similar to research findings in other studies conducted by Hill et al. (2004) that suggest that the different domains are interrelated. Herbst and Kosko (2014) decided to analyze the correlation between experience-type and score. Table 3 show the correlations between each domain and the number of years teaching mathematics, number of years teaching Geometry, total number of mathematics courses in college, and total number of Geometry courses taken in college. Years of teaching mathematics, number of mathematics courses, and number of Geometry courses had near-zero correlations. The only significant correlations between individual domains were seen in years of teaching Geometry.

Table 3.

Correlations between Experience-Type and Score.

	Years Experience		Content Coursework	
	Years Teaching Mathematics	Years Teaching Geometry	Total Math Courses	Total Geometry Courses
CCK-G	-.06	.38***	-.05	-.02
SCK-G	.09	.35**	.05	.04
KCT-G	.04	.19[a]	-.00	-.05
KCS-G	-.02	.25*	.10	.08

[a] *p < .10, *p < .05, **p < .01, ***p < .001*

(Herbst & Kosko, 2014, p. 15)

Even though they analyzed the instrument based off the individual categories, the researchers decided they wanted to better understand the MKT-G construct as a whole, and how the teacher's performance compared to their varying background factors. The researchers constructed overall using Item Response Theory (IRT). Through IRT modeling, there is sufficient item reliability (0.96), which is considered acceptable if above 0.90, and person reliability (0.82), which is considered acceptable if above 0.80(Herbst & Kosko, 2014, p. 17). They found that having three or more years of teaching experience of geometry has statistically significant positive effect on the MKT-G scores. Teaching experience in general had a positive effect on the scores, but teaching geometry specifically, had a statistically significant effect on their performance. They also look at the relationship between MKT-G and the total number of mathematics courses and total number of Geometry courses taken in their college coursework. The results showed that neither of these two factors correlated significantly with their MKT-G (Herbst & Kosko, 2014). Possible scores on the assessment range from -5.50 (low MKT-G) and 5.45 (high MKT-G), so a person scoring 0 would have an average score of MKT-G. For the sample, overall scores ranged from -2.27 to 3.43 with M= .19 and SD= 1.03(Herbst & Kosko, 2014, p. 17). The mean being near zero and standard deviation being approximately 1, indicates that the assessment provides good data (Herbst & Kosko, 2014, p. 18).

They then conducted a "multiple regression to examine he effect of being an 'experienced' Geometry teacher, meaning 3 or more years teaching Geometry, on the MKT-G scores" (Herbst & Kosko, 2014, p. 18). The regression equation

$$MKTG = \beta_0 + \beta_1(Experienced\ Geometry) + \beta_2(dMidAtlantic) + e$$

was used where dMidAtlantic is a dummy variable distinguishing the location of the participants from Mid-Atlantic or Midwestern regions (Herbst & Kosko, 2014, p. 18). According to the model, β_0 represent an average score of a teacher with less than 3 years experience teaching Geometry from the Midwest. Analysis showed this model was statistically significant ($F_{df=2} = 9.10, p < .001$) with an $r^2 = .19$ (Herbst & Kosko, 2014, p. 18). The multiple regression analysis showed that teachers with less than 3 years of experience in Geometry and from the Midwest had slightly lower MKT-G scores, but was not statistically significant from the average score of 0. There was a negative effect on MKT-G scores if the teacher was from the Mid-Atlantic regardless of experience. When accounting for location of teachers, having at least 3 years experience teaching high school Geometry was found to have a positive effect on the MKT-G scores and was statistically significant ($\beta_1 = .78, p < .001$) (Herbst & Kosko, 2014, p. 18).

A question that was posed when analyzing the pilot data was “what is the relationship between mathematical knowledge for teaching geometry and experience teaching high school geometry?” (Herbst & Kosko, 2014, p. 20). They found that having 3 or more years teaching Geometry had a positive effect on their MKT-G scores and was statistically significant. Previous research in elementary and middle schools show that teachers with years of teaching experience have higher MKT scores (Hill, 2007; 2010). An interesting finding in this study shows that teaching experience in general can affect the MKT-G scores of a teacher, but teaching experience in Geometry is the factor that matters the most.

This instrument was used to measure MKT-G of pre-service and high school Geometry teachers. This addresses the research question: What do high school pre-service teachers and high school Geometry teachers know about Mathematical Knowledge for Teaching Geometry (MKT-G)? Analysis of these scores will then address the research question: How do these groups compare? I have contacted Patricio Herbst regarding his instrument, and I had a webinar regarding what the instrument addresses and how some of the questions are asked. When I completed the webinar, I was sent a Terms of Use Contract. This is included in Appendix 1. There are only 4 released items in which I am allowed access. I have included them in the Appendix 3. I was also given a selection of 17 geometry topics that are addressed in the MKT-G. These topics were incorporated into the Post-Assessment survey.

Post-Assessment Survey

To answer the research question about knowledge of geometric techniques and methods used in the high school Geometry classroom, the Post-Assessment Survey was administered to those agreeing to participate in the study, after taking the Mathematical Knowledge for Teaching-Geometry assessment. This survey included demographic information along with questions regarding the teaching of geometry. Some of the demographic information was also collected through the Mathematical Knowledge for Teaching- Geometry Assessment. The demographic information included in the MKT-G assessment can be found in Appendix 2. Those who were in-service teachers, answered the questions based on how they were currently teaching their geometry courses, what they had seen in professional development, and how they would teach their ideal classroom. Pre-service teachers answered the questions based on how they had seen geometry taught, what they had seen in their education courses, and how they would plan to teach a geometry course. At the end of the Post-Assessment survey, all participants were asked to assess how they thought they performed on the MKT-G assessment based on the list of geometry topics provided by the developers of the assessment. The participants were given a 5-point Likert Scale in order to self-assess how they felt they performed on the assessment with 1 meaning that they have never seen the topic and 5 meaning they are certain they were correct on the MKT-G assessment.

Post-Assessment Pilot Study

The Pre-Service Teacher Survey was given to 19 students in math course at a central Texas university. This course is a Geometry course that pre-service Elementary and Middle School teachers are required to take. The students were given the survey and an additional page was added to the end of the survey to ask questions regarding clarification of certain topics and any modifications that may be recommended. Some of the modifications suggested do not pertain to the population in which I am primarily interested for my study, for example, since the survey was administered to pre-service Elementary teachers, a few students suggested that there should be a modification to the subjects the teacher would prefer to teach. The subjects that are addressed in the survey are all mathematics topics, and some of the pre-service teachers do not have any interest in teaching mathematics. After reviewing the surveys and the suggested modifications, the following changes were addressed. I clarified my question regarding their tutoring experiences to address what levels of mathematics and what age of students. I included a circle, yes or no, regarding the questions about student teaching, in order for there to be a better flow to answering those questions instead of having the participants write yes or no. I included more description regarding the instructional techniques included in three of the questions on the assessment. There was confusion as to what investigations were and what computer software could mean. The students were also asked to write how long it took them to take the survey, and on average, the survey took approximately 5.5 minutes. This time is also including how long it took them to answer questions about survey modifications, so I do not think that the length of the survey needs to be modified.

The High School Teacher Survey was sent to two current high school Geometry teachers in order for them to assess the face validity of the survey. The two teachers did not give me any modification feedback, but considering the modification feedback of the Pre-Service Teacher Survey, the clarifications regarding the instructional methods will be included as well. The surveys can be found in Appendix D and Appendix E.

Interviews

Originally pre-service and high school Geometry teachers were to be selected for interviews based on their performance on the Mathematical Knowledge for Teaching-Geometry assessment, however due to time constrictions, participants were chosen randomly. Three pre-service teachers were chosen to be interviewed. One of the pre-service teachers (Daniel) was automatically chosen because he was the only pre-service teacher assigned to teach Geometry in the Fall of 2015. The other two pre-service teachers were randomly selected, one preparing to enter student teaching Spring 2016 and the other currently student teaching 8^{th} grade mathematics. The current high school teachers were split into three groups based on the number of years they had taught Geometry. The groups were divided into one to four years experience, five to nine years experience, and 10 or more years experience. The reasoning for splitting them in this way was due to the results of the MKT-G assessment reported by Herbst and Kosko (2014). They found that teachers who had at least three years of experience teaching Geometry performed better on the assessment. Originally only three high school teachers were going to be interviewed, but after having current high school teachers take the MKT-G assessment, a first-year teacher that graduated from a central Texas University decided to be a willing participant in the interviews and the observations. This gave the opportunity to investigate a participant that was a current high school teacher, but a recent graduate of the program that all of the pre-service teachers were completing. The interviews were used to address the research questions pertaining to where their Geometry Content Knowledge and Mathematical Knowledge for Teaching Geometry was developed. All interviews were conducted in the Fall 2015 semester. These interviews were semistructured and included a prompt that evaluated the teacher's mathematical knowledge for teaching geometry. The interviews addressed all four domains of MKT-G. The interview question that focuses on Geometry Content Knowledge assesses the understanding of triangle midsegments. This is a topic that the teachers will be teaching their students at some point in the school year. The other three interview questions were released problems directly from the MKT-G assessment. The goal of interview was to determine why the participant answered the question the way they did, whether right or wrong, and then investigate where the participant had acquired this knowledge. The interview protocol was based on the interview control group protocol used in the Dynamic Geometry Project conducted at Texas State University. The interviews were recorded and the work that each of the participants produced was kept for data analysis. The purpose of these interviews was to investigate where the pre-service and high school teachers' Mathematical Knowledge for Teaching Geometry, and knowledge of geometric techniques and methods used in the high school Geometry classroom were established. The interview protocol has been used prior to this study and slight modifications have been made in order to accommodate the different areas that were addressed. I have personally conducted several of interviews using this protocol and I have modified this protocol to address questions that were not previously included in the Dynamic Geometry Project.

Observations

The pre-service teachers assigned to teach geometry in Fall 2015 and the four current high school Geometry teachers were observed. The pre-service and high school teachers were the same teachers selected for interviews. The observation protocol that was used was based on the control group protocol used in the Dynamic Geometry Project. I have personally used the observation protocol on several occasions while working with the Dynamic Geometry Project. This observation protocol was modified to address the research question regarding what methods and techniques are used in the classroom. The pre-service teacher and three of the high school teachers were observed three times, and the fourth high school teacher was observed twice due to scheduling conflicts. The primary purpose of the observations is to observe the teachers to verify the methods reportedly used in the survey and interviews conducted throughout the semester and to observe their Mathematical Knowledge for Teaching Geometry and knowledge of instructional techniques and methods used in the high school Geometry classroom.

CHAPTER 4

RESULTS

Introduction

In this chapter, the research questions will be addressed based on the MKT-G Assessment results, the Post-Assessment Survey data, interviews, and observations. The Mathematical Knowledge for Teaching Geometry (MKT-G) Assessment results will be presented based on the four domains, Geometry Content Knowledge, Specialized Geometry Knowledge, Knowledge of Geometry and Students, and Knowledge of Geometry and Teaching for all participants. Next, the Post-Assessment Survey results will be provided, followed by a summary of the three pre-service teachers and four high school Geometry teachers interviewed and observed. The research questions that will be addressed are:

1. What do high school pre-service teachers and high school Geometry teachers know about Geometry Teaching Knowledge which consists of the following:
    a. Mathematical knowledge for teaching geometry?
    b. Geometry techniques and methods used in the high school Geometry classroom?
2. How do pre-service and current high school Geometry Teachers' Geometry Teaching Knowledge compare?
3. What are the sources of the high school Geometry teachers' Geometry Teaching Knowledge that can be transferred to high school pre-service teachers?

4. What are the sources of the high school Geometry pre-service teachers Geometry Teaching Knowledge that can be transferred to high school teachers?
5. What do high school pre-service teachers need to know to be prepared to teach high school Geometry?

Mathematical Knowledge for Teaching Geometry Knowledge

The MKT-G Assessment was given to pre-service teachers and high school Geometry teachers to assess their Mathematical Knowledge for Teaching Geometry. The assessment addresses four of domains of mathematical knowledge for teaching, Geometry Content Knowledge (GCK), Specialized Geometry Knowledge (SGK), Knowledge of Geometry and Student (KGS), and Knowledge of Geometry and Teaching (KGT). A description of the MKT-G Assessment can be found in Chapter 2 and Chapter 3. All 87 participants were combined to form the following descriptive statistics over each of the domains and the total score. A lower score indicates lower ability and the higher score indicates higher ability. The results are presented in Table 4, where the domains are abbreviated as follows: Geometry Content Knowledge (GCK), Specialized Geometry Knowledge (SGK), Knowledge of Geometry and Student (KGS), and Knowledge of Geometry and Teaching (KGT). When comparing the means of each of the domains, all of the participants did the best in the Geometry Content Knowledge domain, and did the worst in the Knowledge of Geometry and Teaching.

Table 4.
Descriptive Statistics by MKT-G Domain and Total Score.

Domain	Mean	Standard Deviation	N
GCK	5.18	1.76	87
SGK	12	2.90	87
KGS	2.75	1.39	87
KGT	2.59	1.65	87
Total	22.54	5.70	87

Geometry Content Knowledge (GCK). The analysis in the previous section describes the descriptive statistics using the raw scores of all 87 participants. In order to better understand the differences between pre-service teachers and high school Geometry teachers, a comparison using the raw test scores in the Geometry Content Knowledge (GCK) domain of each group was performed. The box plots in Figure 10 below shows the differences between the two groups.

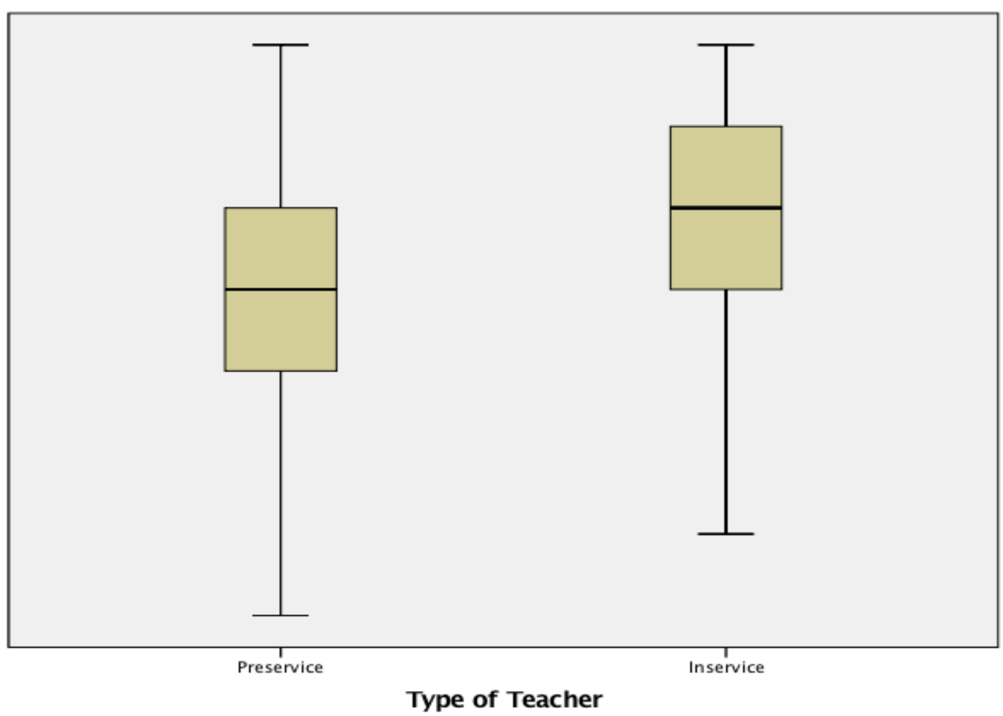


Figure 10. Boxplot Comparing Pre-Service and High School GCK Scores

A t-test for independent groups was performed. Pre-Service teachers had lower GCK Scores (M=4.65, SD=1.66) on the MKT-G Assessment than did those that were current high school Geometry teachers (M=5.94, SD=1.62), t(76.61)=-3.642, p<.001, *d*=-.832. Cohen's effect size (*d*=-.832) suggests a moderate practical significance.

Specialized Geometry Knowledge (SGK). In order to better understand the difference between pre-service teachers and high school Geometry teachers, a comparison using the raw test scores in the Specialized Geometry Knowledge (SCK) domain of each group was performed. The box plot in Figure 11 shows the differences between the two groups.

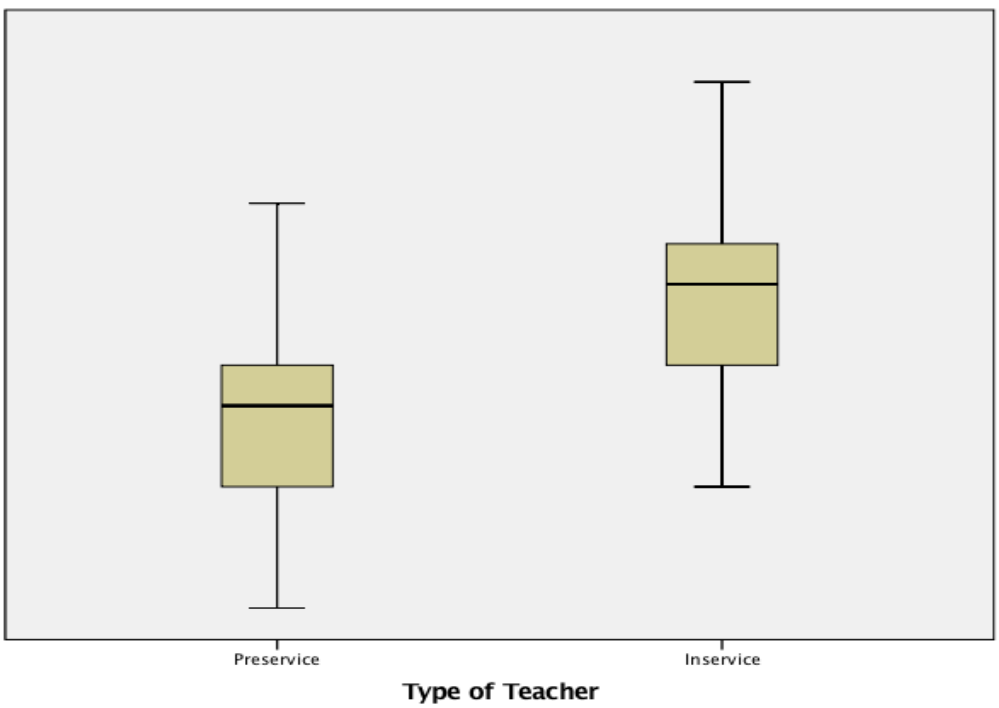


Figure 11. Boxplot Comparing Pre-Service and High School SGK Scores

A t-test for independent groups was performed, and pre-service teachers had lower Specialized Geometry Knowledge (SGK) Scores (M=10.69, SD=2.37) on the MKT-G Assessment than did those that were current high school Geometry teachers (M=13.86, SD=2.55), t(71.899)=-5.882, p<.001, *d*=-1.3873. Cohen's effect size (*d*=-1.387) suggests a large practical significance.

Knowledge of Geometry and Students (KGS). A comparison using the raw test scores in the Knowledge of Geometry and Students (KGS) domain of each group was performed. The box plots in Figure 12 shows the differences between the two groups.

Figure 12. Boxplot Comparing Pre-Service and High School KGS Scores

A t-test for independent groups was performed, and Pre-Service teachers had lower Knowledge of Geometry and Students (KGS) Scores (M=2.35, SD=1.28) on the MKT-G Assessment than did those that were current high school Geometry teachers (M=3.31, SD=1.37), t(72.16)=-3.285, p=.002, *d*=-.773. Cohen's effect size (*d*=-.773) suggests a moderate to large practical significance.

Knowledge of Geometry and Teaching (KGT). A comparison using the raw test scores in the Knowledge of Geometry and Teaching (KGT) domain of each group was performed. The box plots in Figure 13 shows the differences between the two groups.

Figure 13. Boxplot Comparing Pre-Service and High School KGT Scores

A t-test for independent groups was performed, and pre-service teachers had lower Knowledge of Geometry and Teaching (KGT) Scores (M=1.80, SD=1.43) on the MKT-G Assessment than did those that were current high school Geometry teachers (M=3.69, SD=1.26), t(80.76)=-6.516, p<.001, *d*=-1.45. Cohen's effect size (*d*=-1.45) suggests a large practical significance.

Total Score. A comparison using the raw test scores for the Total Score was performed. The box plots in Figure 14 shows the differences between the two groups.

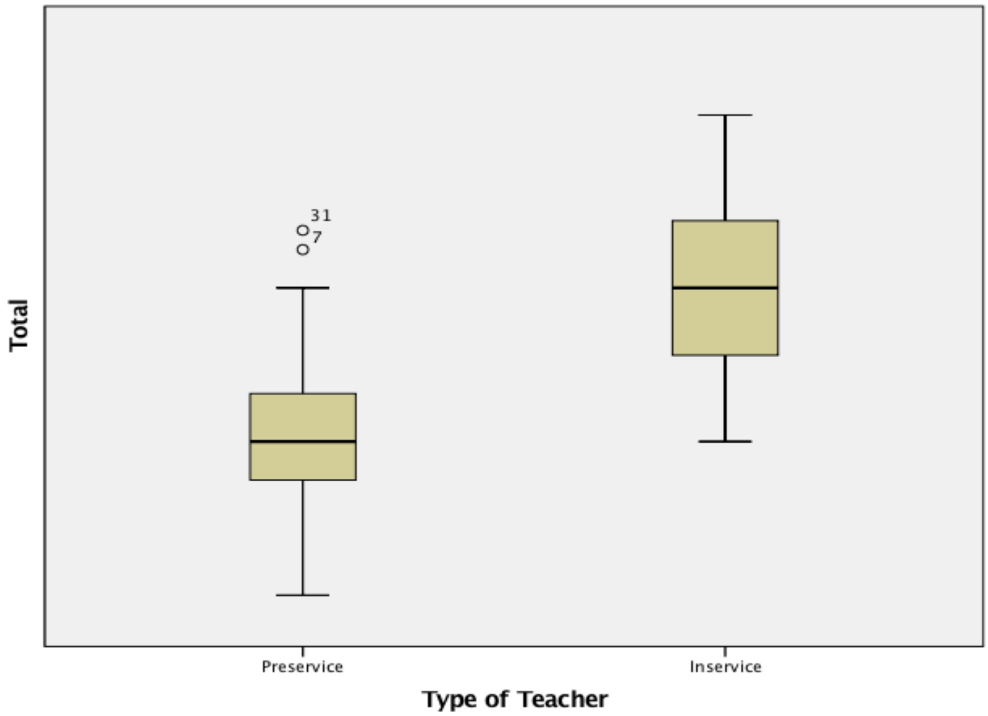


Figure 14. Boxplot Comparing Pre-Service and High School Total Scores

A t-test for independent groups was performed, and pre-service teachers had lower Total Scores (M=19.49, SD=4.20) on the MKT-G Assessment than did those that were current high school Geometry teachers (M=26.86, SD=4.69), t(70.13)=-7.542, p<.001, *d*=-1.80. Cohen's effect size (*d*=-1.80) suggests a large practical significance.

All of the results from the analysis above can be found in Table 5. Based on the t-tests performed, pre-service teachers had lower scores in all domains and in total scores. There is also large practical significance to all of the comparisons.

Table 5.
Descriptive Statistics by MKT-G Domain and Total Score of Pre-Service and High School Teachers.

Domain	Pre-Service Mean	Pre-Service Standard Deviation	High School Teachers Mean	High School Teachers Standard Deviation	Cohen's *d*
GCK	4.65	1.66	5.94	1.62	-.832
SGK	10.69	2.37	13.86	2.55	-1.387
KGS	2.35	1.28	3.31	1.37	-.773
KGT	1.80	1.43	3.69	1.26	-1.45
Total	19.49	4.20	26.86	4.69	-1.80

Correlation between Domains. Correlations between the domain scores are presented in Table 6, and suggest a moderate relationship between the different variables. These correlations were examined in order to make sure the results from this study are similar to the correlations reported by Herbst and Kosko (2014). These results show similar trends to those reported by Herbst and Kosko (2014), which suggests that the four domains are interrelated, to a degree.

Table 6.
Correlations between MKT-G Domains.

	CCK	SCK	KCS	KCT
GCK	-			
SGK	.343**	-		
KGS	.391**	.389**	-	
KGT	.361**	.456**	.304**	-

**p<.01

Correlations between Current High School Teachers and MKT-G Scores. Correlations between each of the domains, total score, and the participants being high school Geometry teachers were examined in order to make sure the results from this study are similar to the correlations reported by Herbst and Kosko (2014). The correlation between high school Geometry teachers and Geometry Content Knowledge (GCK) and Knowledge of Geometry and Students (KGS) was moderate, but the correlation between Specialized Geometry Knowledge (SGK), Knowledge of Geometry and Teaching (KGT) and total score were stronger. The correlations between high school Geometry teachers and all of the domains and total scores were significant.

Table 7.
Correlations between Current High School Teachers and Domain and Total Score.

	Current High School Geometry Teacher
GCK	.366**
SGK	.543**
KGS	.339**
KGT	.569**
Total	.640**

**p<.01

Correlations between Years Teaching Mathematics and Geometry and MKT-G scores. Correlations between each of the domains, total score, and the participants' years of teaching mathematics and years of teaching Geometry were examined in order to make sure the results from this study are similar to the correlations reported by Herbst and Kosko (2014). The correlation between the number of years teaching mathematics and Geometry Content Knowledge (GCK) and Knowledge of Geometry and Students (KGS) were statistically significant, but weak. The correlation between Specialized Geometry Knowledge (SGK), Knowledge of Geometry and Teaching (KGT), and Total score were statistically significant, but moderate. The correlation between the number of years teaching Geometry and Knowledge of Geometry and Students (KGS) is statistically significant, but weak. The correlation between Geometry Content Knowledge (GCK), Specialized Geometry Knowledge (SGK), Knowledge of Geometry and Teaching (KGT), Total score were statistically significant, but moderate.

Table 8.
Correlations between Years Experience and Scores.

	Years Teaching Math	Years Teaching Geometry
GCK	.239**	.323**
SGK	.361**	.352**
KGS	.265*	.286**
KGT	.448**	.397**
Total	.465**	.471**

*p<.05, **p<.01

Knowledge of Instructional Techniques and Methods

As part of the Post-Assessment Survey, participants were asked questions regarding their experiences with different Instructional Techniques and Methods that are frequently used in the Geometry classroom. Pre-service teachers and current high school teachers were asked different questions regarding their knowledge. Pre-service teachers were asked: what types of instructional techniques or methods have they seen in their Geometry courses, what types of instructional techniques or methods have they seen in their Education Courses, and what types of instructional techniques or methods would they use in their ideal classroom. An ideal classroom is described as a situation where you would have an unlimited budget and unlimited resources. High school teachers were asked: what types of instructional techniques or methods do they use in their current Geometry classes, what types of instructional techniques or methods have they seen in their Professional Development, and what types of instructional techniques or methods would they use in their ideal classroom. Discussion of the format of these questions can be found in Chapter 3, but the following are comparisons of the responses. Only 86 of the 87 participants were included in this analysis due to one high school teacher not completing the survey. Figure 15 shows the pre-service teacher survey results, specifically the distribution of experience with what types of instructional techniques or methods have they seen in their Geometry courses, what types of instructional techniques or methods have they seen in their Education Courses, and what types of instructional techniques or methods would they use in their ideal classroom.

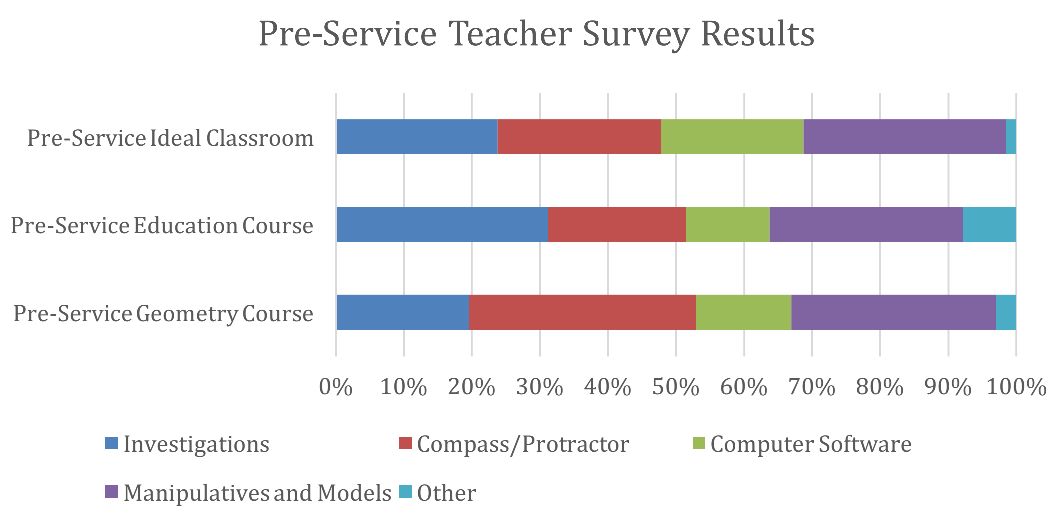


Figure 15. Pre-Service Teacher Survey Results

For pre-service teachers' Geometry courses, participants reported experiencing compass and protractor activities (33.3%) and manipulatives and models (30.1%) the most, and computer software (14.1%) the least. In their education courses, pre-service teachers reported seeing investigations (31.2%) the most and computer software (12.3%) the least. Pre-service teachers would use manipulatives and models (29.7%) the most and computer software (21%) the least in their ideal classrooms.

Figure 16 shows the high school teachers survey results, specifically what types of instructional techniques or methods they use in their current Geometry classes, what types of instructional techniques or methods have they seen in their Professional Development, and what types of instructional techniques or methods would they use in their ideal classroom.

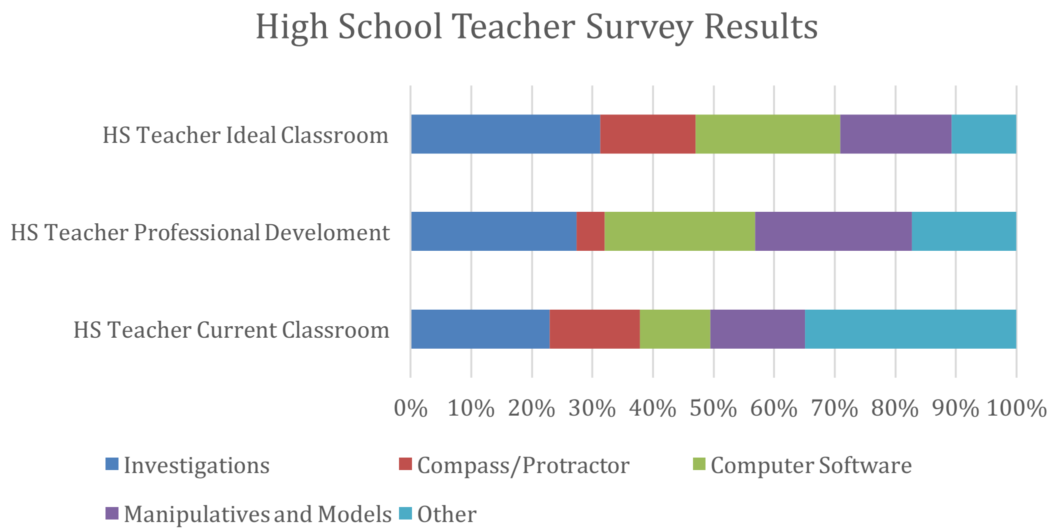


Figure 16. High School Teacher Survey Results

In current Geometry classes, high school teachers report the use of other (35%) the most in their classrooms. Other was defined as Lecture by 80% of the participants. They reported computer software (11.6%) is used the least in their current geometry classes. High school teachers reported seeing investigations (27.3%) the most and compass and protractor activities (4.7%) the least in their professional development. When teachers were asked about their ideal classroom, high school teachers would use investigations (31.3%) the most and compass and protractor activities (15.7%) the least.

Pre-Service Teachers' vs. Current Teachers' Ideal Classroom. Both groups were asked how they would spend time if they had an ideal classroom. An ideal classroom would consist of having unlimited resources, time, and the ideal students. A chi-square test of independence was performed to examine the relation between Pre-Service teachers' ideal classroom and current high school teachers' ideal classroom. There is a significant difference between these variables, $\chi^2(4, N = 86) = 59.93, p < .01$. This shows that there is a statistically significant difference between what high school teachers think would be best for their ideal classroom and what the pre-service teachers think would be best for their ideal classroom. In Figure 17, the strip diagrams show the distribution among the instructional techniques and methods of the pre-service and high school teachers' ideal classrooms.

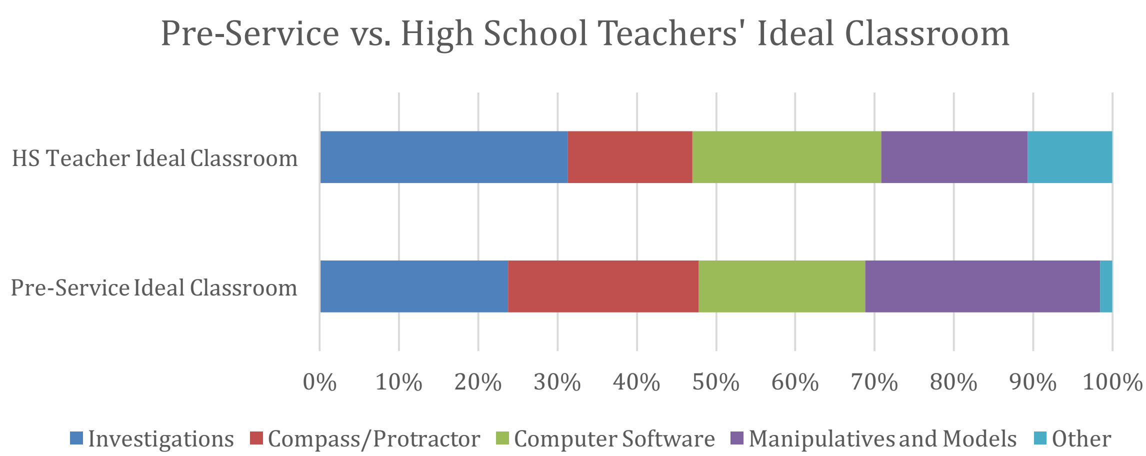


Figure 17. Pre-Service vs. High School Teachers' Ideal Classroom

Pre-service teachers thought that more compass and protractor activities (24% of the time) and manipulatives and models (29.7% of the time) were important to their ideal classes when compared to the high school teachers (15.7% and 18.4% respectively). The high school teachers thought more investigations (31.2%) and computer software (23.9%) would be important to their ideal classrooms, as well as a larger portion dedicated to other (10.7%) when compared to pre-service teachers' distribution of classroom time (23.8%, 20.9%, and 1.6% respectively). Lecture and Direct teach is what 49% of the high school teachers described as other.

Table 9.
Pre-Service vs. High School Teachers' Ideal Classroom.

	Pre-Service Teachers	High School Teachers
Investigations	23.8%	31.3%
Compass and Protractors	24%	15.7%
Computer Software	21%	23.9%
Manipulatives and Models	29.7%	18.4%
Other	1.6%	10.7%

Pre-Service Teachers' Geometry Courses vs. Education Courses. Pre-Service teachers were asked which instructional techniques and methods have they used or seen in their Geometry Courses and Education Courses. A chi-square test of independence was performed to examine the relation between Pre-Service teachers' experiences in their Geometry courses and experiences in their Education courses. There is a significant difference between these variables, $\chi^2(4, N = 51) = 41.56, p < .01$. This suggests that pre-service teachers are given different opportunities in their education courses than they are given in their geometry courses. In Figure 18, the strip diagrams show the distribution among the instructional techniques and methods of the pre-service teacher's Geometry Courses and Education Courses.

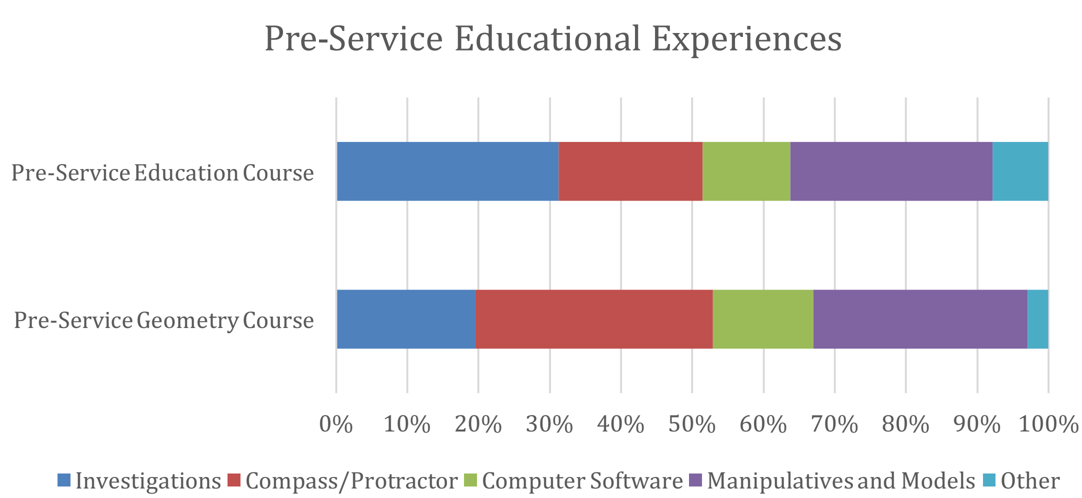


Figure 18. Pre-Service Educational Experiences

When comparing pre-service teachers' Geometry courses and Education courses, they reported that they have seen or experienced more compass and protractor activities (33.4%) and manipulatives and models (30.1%) in their Geometry courses than in their Education courses (20.2% and 28.3% respectively). They reported more investigations (31.2%) and other (7.9%) when compared to their Geometry course (19.6% and 2.9% respectively). The pre-service teachers did not have common examples of other for their Education courses. Other included lesson plans, PowerPoints, projects, and lecture.

Table 10.
Pre-Service Education Courses vs. Geometry Courses Comparison.

	Education Courses	Geometry Courses
Investigations	31.2%	19.6%
Compass and Protractors	20.2%	33.4%
Computer Software	12.3%	14.1%
Manipulatives and Models	28.3%	30.1%
Other	17.9%	2.9%

Pre-Service Teachers' Geometry Courses vs. Ideal Classroom. Pre-Service teachers were asked which instructional techniques and methods have they used or seen in their Geometry Courses and which ones they would use in their ideal classroom. A chi-square test of independence was performed to examine the relation between Pre-Service teachers' experiences in their Geometry courses and their ideal classroom. There is a significant difference between these variables, $\chi^2(4, N = 51) = 30.70, p < .01$. This suggests that there is difference between what pre-service teachers' think would be best in their ideal classrooms and what they have seen in their Geometry courses. In Figure 19, the strip diagrams show the distribution among the instructional techniques and methods of the pre-service teacher's Geometry Courses and what they would use in their ideal classroom.

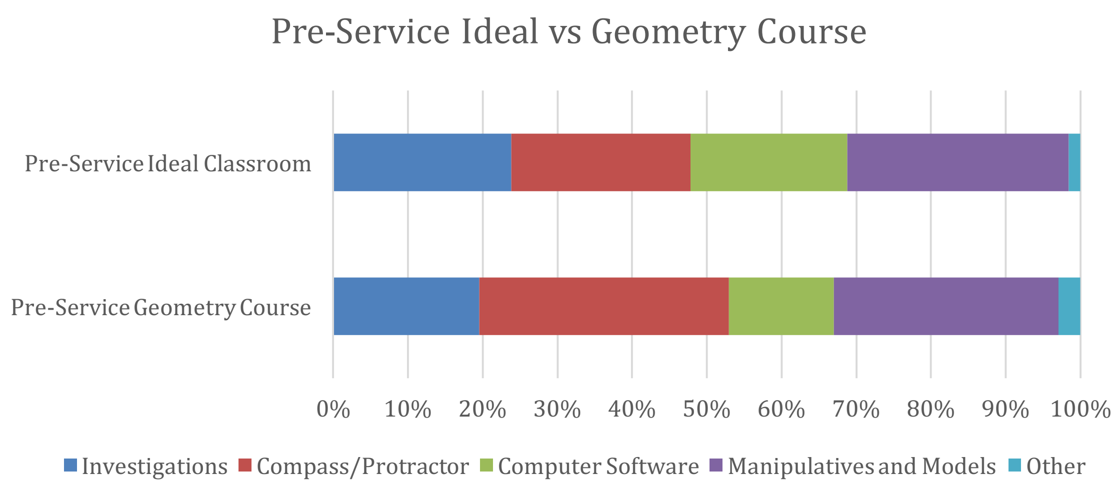


Figure 19. Pre-Service Ideal vs. Geometry Course

Pre-service teachers thought that more investigations (23.8% of the time) and computer software (21% of the time) were important to their ideal classes when compared to their Geometry courses (19.6% and 14.1% respectively). They reported more compass and protractor activities in their Geometry courses (33.4%) than they thought would be important to their ideal classrooms (23.8%). Manipulatives and models were weighted almost the same for their ideal classroom (29.7%) and their Geometry courses (30%).

Table 11.
Pre-Service Ideal Classroom vs. Geometry Courses Comparison.

	Ideal Classroom	Geometry Courses
Investigations	23.8%	19.6%
Compass and Protractors	24%	33.4%
Computer Software	21%	14.1%
Manipulatives and Models	29.7%	30.1%
Other	1.6%	2.9%

Pre-Service Teachers' Education Courses vs. Ideal Classroom. Pre-Service teachers were asked which instructional techniques and methods have they used or seen in their education courses and which ones they would use in their ideal classroom. A chi-square test of independence was performed to examine the relation between Pre-Service teachers' experiences in their education courses and their ideal classroom. There is a significant difference between these variables, $\chi^2(4, N = 51) = 46.30, p < .01$. This suggests that there is difference between what pre-service teachers' think would be best in their ideal classrooms and what they have seen in their education courses. In Figure 20, the strip diagrams show the distribution among the instructional techniques and methods of the pre-service teacher's Geometry Courses and what they would use in their ideal classroom.

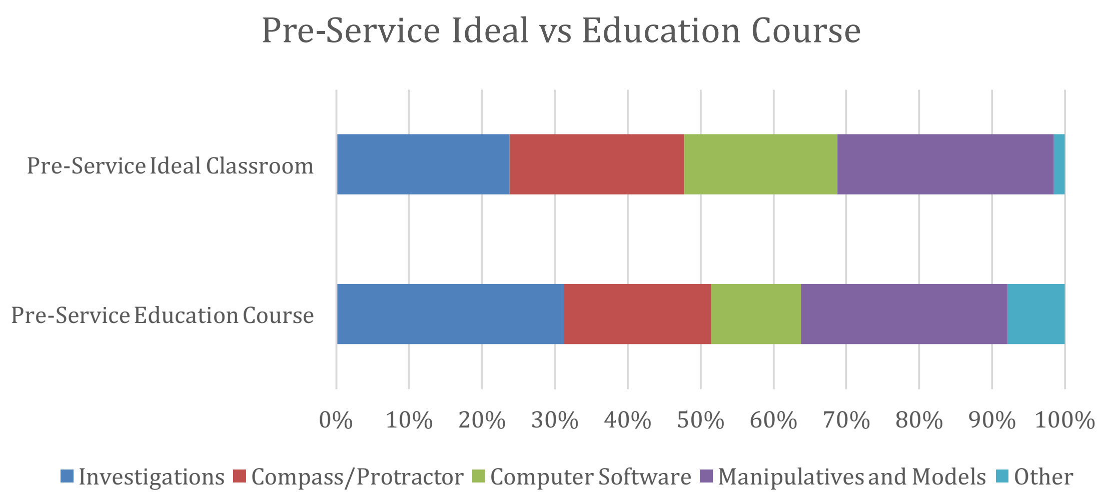


Figure 20. Pre-Service Ideal vs. Education Course

Pre-service teachers thought that more compass and protractor activities (24% of the time) and computer software (21% of the time) were important to their ideal classes when compared to what they have seen in their education courses (20.2% and 12.3% respectively). They reported more investigations in their education courses (31.2%) than they thought would be important to their ideal classrooms (23.8%). Manipulatives and models were weighted almost the same for their ideal classroom (28.3%) and their education courses (29.7%).

Table 12.
Pre-Service Ideal Classroom vs. Geometry Courses Comparison.

	Ideal Classroom	Education Courses
Investigations	23.8%	31.2%
Compass and Protractors	24%	20.2%
Computer Software	21%	12.3%
Manipulatives and Models	29.7%	28.3%
Other	1.6%	17.9%

Current Teachers' Ideal Classroom vs. Current Classroom. Current teachers were asked which instructional techniques and methods have they used in their current classrooms and which ones they would use in their ideal classroom. A chi-square test of independence was performed to examine the relation between current high school teachers' ideal classroom and actual classrooms. There is a significant difference between these variables, $\chi^2(4, N = 35) = 121.32, p < .01$. This suggests that there is difference between what high school teachers' think would be best in their ideal classrooms and what they actually do in their current Geometry courses. In Figure 21, the strip diagrams show the distribution among the instructional techniques and methods of the pre-service teacher's Geometry Courses and what they would use in their ideal classroom.

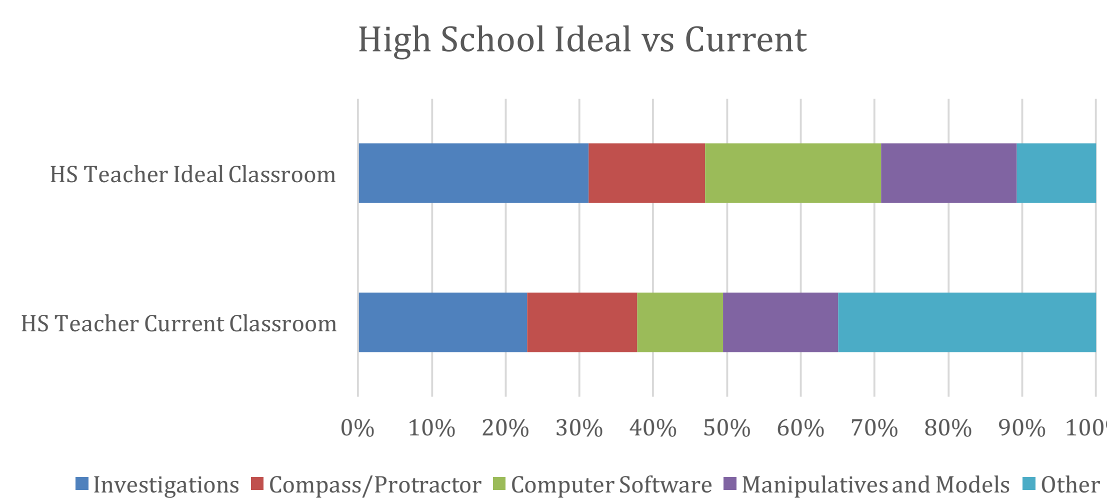


Figure 21. High School Ideal vs. Current

High school teachers thought that more investigations (31.3% of the time) and computer software (23.9% of the time) were important to their ideal classes when compared to their current classroom (22.9% and 11.6% respectively). They reported more other (35%) in their current Geometry course than when in their ideal Geometry course (10.7%). Lecture and Direct teach is what 49% of the high school teachers described as other. Manipulatives and models were weighted almost the same for their ideal classroom (29.7%) and their Geometry courses (30%), as well as, compass and protractor activities (18.4% and 15.6% respectively).

Table 13.
High School Teachers' Ideal Classroom vs. Current Classroom.

	Ideal Classroom	Current Classroom
Investigations	31.3%	22.9%
Compass and Protractors	15.7%	14.9%
Computer Software	23.9%	11.6%
Manipulatives and Models	18.4%	15.6%
Other	10.7%	35%

Current Teachers' Ideal Classroom vs. Professional Development. Current teachers were asked which instructional techniques and methods have they used or seen in their professional development and which ones they would use in their ideal classroom. A chi-square test of independence was performed to examine the relation between current high school teachers' ideal classroom and their experiences in professional development. There is a significant difference between these variables, $\chi^2(4, N = 35) = 118.04, p < .01$. This suggests that there is difference between what high school teachers' think would be best in their ideal classrooms and what they have seen in their professional development opportunities. In Figure 22, the strip diagrams show the distribution among the instructional techniques and methods of the pre-service teacher's Geometry Courses and what they would use in their ideal classroom.

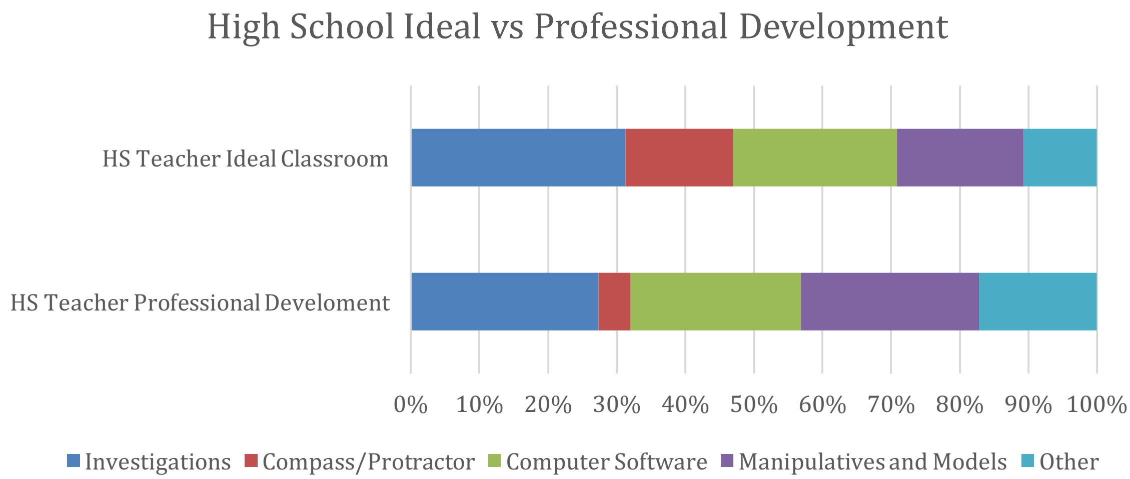


Figure 22. High School Ideal vs. Professional Development

High school teachers thought that more investigations (31.3% of the time) and compass and protractor activities (15.7% of the time) were important to their ideal classes when compared to their professional development experiences (27.3% and 4.7% respectively). They reported more manipulative and models (25.9%) and other (17.2%) in their professional development than in their ideal Geometry course (18.4% and 10.7% respectively). The responses for other in professional development included teaching strategies, classroom management, project based instruction, and direct teach/lecture. Computer software was given almost the same weight in the teachers' ideal classroom (23.9%) and their professional development (24.8%).

Table 14.
High School Teachers' Ideal Classroom vs. Professional Development.

	Ideal Classroom	Professional Development
Investigations	31.3%	27.3%
Compass and Protractors	15.7%	4.7%
Computer Software	23.9%	24.8%
Manipulatives and Models	18.4%	25.9%
Other	10.7%	17.2%

Current Teachers' Current Classroom vs. Professional Development. Current teachers were asked which instructional techniques and methods have they used or seen in their professional development and which ones they use in their current classroom. A chi-square test of independence was performed to examine the relation between current high school teachers' actual classroom and experiences with professional development. There is a significant difference between these variables, $\chi^2(4, N = 35) = 160.12, p < .01$. This suggests that there is difference between what high school teachers' use in their current classrooms and what they have seen in their professional development opportunities. In Figure 23, the strip diagrams show the distribution among the instructional techniques and methods of the pre-service teacher's Geometry Courses and what they would use in their ideal classroom.

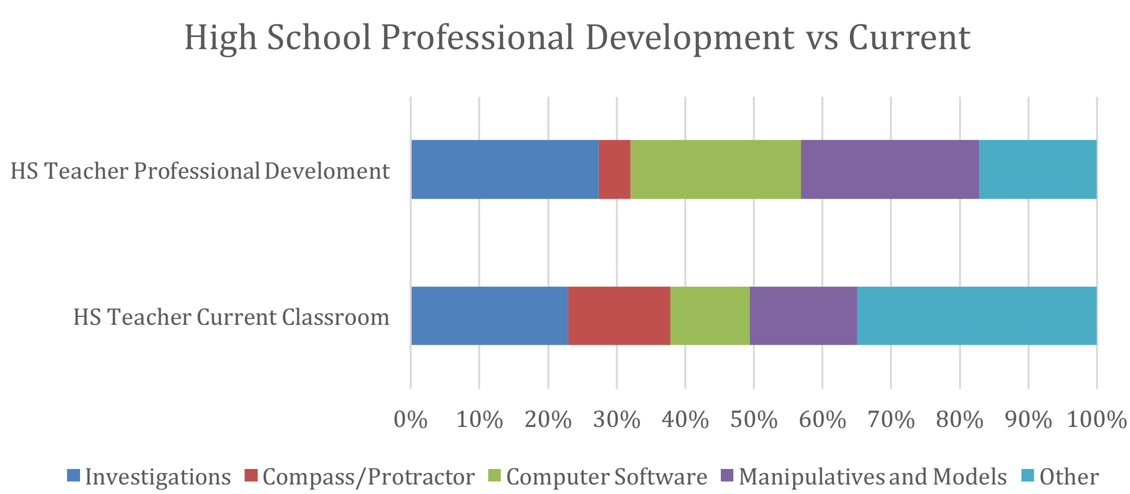


Figure 23. High School Professional Development vs. Current

High school teachers thought that more investigations (27.3% of the time), more computer software (24.8%), and manipulatives and models (25.9%) were seen in their professional development when compared to their current classroom (22.9%, 11.6%, and 15.6% respectively). They reported more compass and protractor activities (14.9%) and other (35%) in their current Geometry course than in their professional development experiences (4.7% and 17.2% respectively). Lecture and Direct teach is what 49% of the high school teachers described as other. The responses for other in professional development included teaching strategies, classroom management, project based instruction, and direct teach/lecture.

Table 15.
High School Teachers' Ideal Classroom vs. Current Classroom.

	Professional Development	Current Classroom
Investigations	27.3%	22.9%
Compass and Protractors	4.7%	14.9%
Computer Software	24.8%	11.6%
Manipulatives and Models	25.9%	15.6%
Other	17.2%	35%

Pre-Service Teachers' Geometry and Education Courses vs. Current Teachers' Professional Development. Pre-service teachers were asked which instructional techniques and methods they have used or seen in their Geometry and Education Courses and current teachers were asked which instructional techniques and methods have they used or seen in their professional development. A chi-square test of independence was performed to examine the relation between pre-service teachers' experience in their Geometry and Education courses to the current high school teachers' professional development. There is a significant difference between these variables, $\chi^2(4, N = 86) = 123.84, p < .01$. This suggests that there is difference between what pre-service teachers' see in their geometry and education courses and what high school teachers have seen in their professional development. In Figure 24, the strip diagrams show the distribution among the instructional techniques and methods of the pre-service teacher's Geometry Courses and what they would use in their ideal classroom.

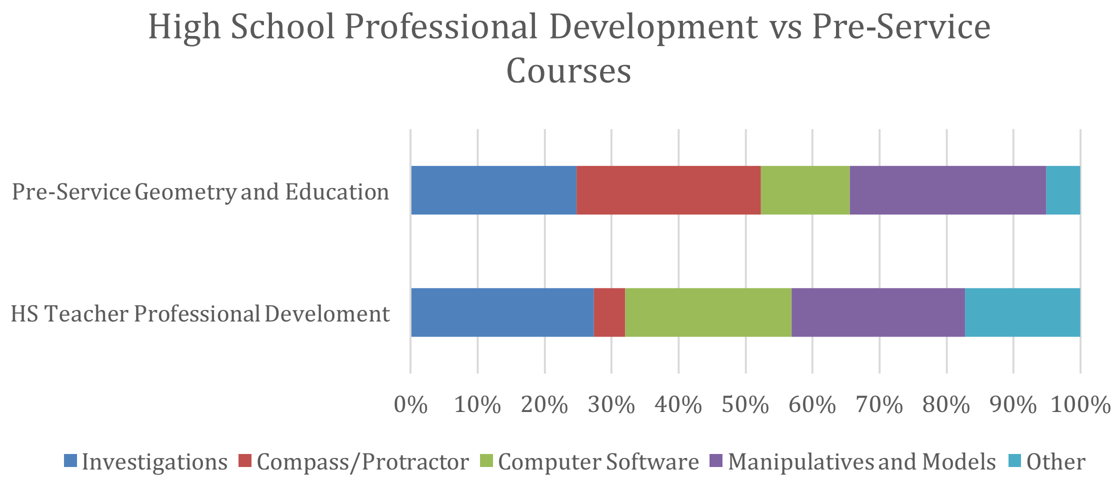


Figure 24. High School Professional Development vs. Pre-Service Courses

Pre-Service teachers have seen more compass and protractor activities (27.5% of the time), and more manipulatives and models (29.3%) in their geometry and education courses when compared to high school teacher's professional development (4.7% and 25.9% respectively). High school teachers reported more investigations (27.3%), computer software (24.8%), and other (17.2%) in their professional development than pre-service teachers have seen in their geometry and education courses (24.7%, 13.3% and 5.1% respectively). The responses for other in professional development included teaching strategies, classroom management, project based instruction, and direct teach/lecture, and the responses for other in their geometry and education courses included lesson plans, PowerPoints, projects, and lecture.

Table 16.
Pre-Service Courses vs. High School Teachers' Professional Development.

	Courses	Professional Development
Investigations	24.7%	27.3%
Compass and Protractors	27.5%	4.7%
Computer Software	13.3%	24.8%
Manipulatives and Models	29.3%	25.9%
Other	5.1%	17.2%

Pre-Service Teachers' Geometry and Education Courses vs. Current Teachers' Current Classroom. Pre-service teachers were asked which instructional techniques and methods they have used or seen in their Geometry and Education Courses and current teachers were asked which instructional techniques and methods they used in their current classroom. A chi-square test of independence was performed to examine the relation between pre-service teachers' experience in their Geometry and Education courses to the current high school teachers' Geometry classes. There is a significant difference between these variables, $\chi^2(4, N = 86) = 196.19, p < .01$. This suggests that there is difference between what pre-service teachers' see in their geometry and education courses and what high school teachers are using in their current geometry classes. In Figure 25, the strip diagrams show the distribution among the instructional techniques and methods of the pre-service teacher's Geometry Courses and what they would use in their ideal classroom.

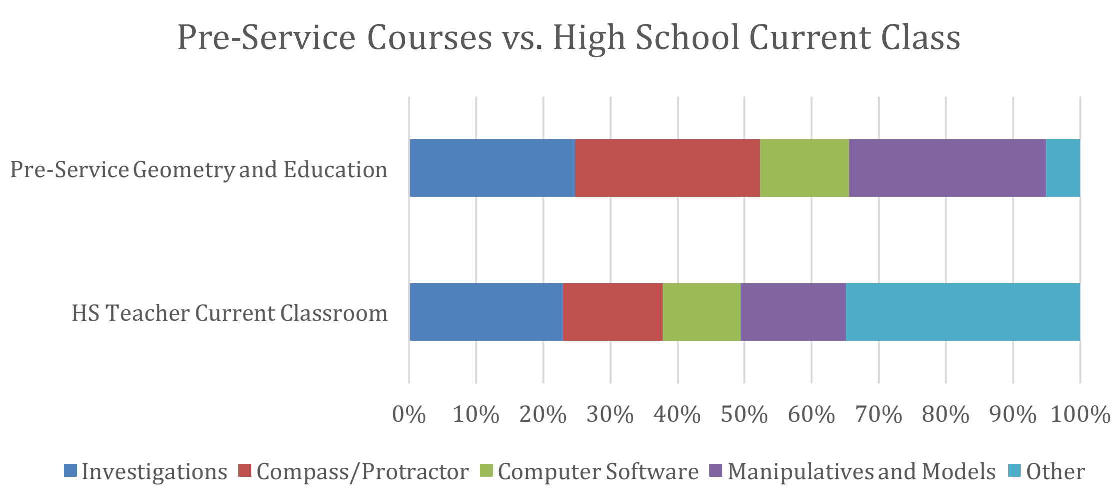


Figure 25. Pre-Service Courses vs. High School Current Class

Pre-service teachers reported more experience with compass and protractor activities (27.5%) and manipulatives and models (29.3%) than high school teachers report time spend in their current classrooms (14.9% and 15.6% respectively). High school teachers report more time spent on other (35%) than pre-service teachers claim in their Geometry and Education courses (5.1%). Lecture and Direct teach is what 49% of the high school teachers described as other. Pre-service and high school teachers distributed points similarly to the investigations (24.7% and 22.9% respectively) and computer software (13.3% and 11.6% respectively).

Table 17.
Pre-Service Courses vs. High School Teachers' Current Class.

	Courses	Current Classroom
Investigations	24.7%	22.9%
Compass and Protractors	27.5%	14.9%
Computer Software	13.3%	11.6%
Manipulatives and Models	29.3%	15.6%
Other	5.1%	35%

Self-Assessment of Performance on MKT-G

As part of the Post-Assessment Survey, participants were asked to self-assess how they performed on the MKT-G Assessment. This was included in the survey in order to possibly pinpoint what topics the pre-service and high school teachers report not to understand or are familiar with and compare that to their actual performance on those types of questions on the MKT-G assessment. Since the MKT-G assessment is not available to the public, the developers of the MKT-G Assessment provided a list of 17 Geometry topics addressed in the assessment. Participants were asked to assess their performance though a 5-point Likert Scale, (1 being that they have never seen the geometry topic before and 5 being they felt they successfully answered questions on the assessment over that specific Geometry topic). The number of questions on the MKT-G

Assessment differs between Geometry topics, but the Likert responses were scaled to match the number of questions of the specific topic. For example, if there are 4 assessment questions over topic A, if a participant answered 1 on the Likert Scale for question A, the corresponding number correct would be 0, if they answered 2, the corresponding number correct would be 1, if they answered 3, the corresponding number correct would be 2, and so on. Only 86 of the 87 participants were included in this analysis due to one high school teacher not completing the survey. The following analysis discusses the differences between the groups of pre-service teachers and high school teachers.

Angle Bisectors. In order to compare how pre-service teachers thought they performed on the angle bisectors questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.066 with p=.650 and Kappa=0.030 with p=0.369, since these are not statistically significant, it can be inferred that there is no reliable relationship between how the pre-service teachers self-assessed their knowledge and how they actually performed in angle bisectors. To compare how in-service teachers thought they performed on the angle bisectors questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.333 with p=.05 and Kappa=0.085 with p=0.056, while not quite statistically significant, there is poor agreement between the in-service teachers' self-evaluation and how they actually performed on the assessment with regard to angle bisectors.

Angle Relations. In order to compare how pre-service teachers thought they performed on the angle relations questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.317 with p=.025 and Kappa=0.079 with p=0.015, since these are statistically significant, it can be said that pre-service teachers' self-assessment and actual performance are related, and there is poor correspondence between the two. This is one of the only Geometry topic in which Kappa is statistically significant, so even though there is a poor correspondence between how the pre-service teachers thought they performed and how they actually performed, there was a statistically significant relationship between the two. To compare how in-service teachers thought they performed on the angle relations questions and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=-.043 with p=.807 and Kappa=0.035 with p=0.599, while not quite statistically significant, there is poor agreement between the in-service teachers' self-evaluation and how they actually performed on the assessment with regard to angle relations.

Altitudes of Triangles. To compare how pre-service teachers thought they performed on the altitudes of triangles questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.260 with p=0.068 and Kappa=0.093 with p=0.051, while statistically significant, there is poor agreement between the pre-service teachers' self-evaluation and how they actually performed on the assessment with regard to altitudes of triangles. In order to compare how in-service teachers thought they performed on the altitudes of a triangle questions on the assessment and how they actually performed, I conducted a consistency estimate using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.136 with p=.437 and Kappa=0.050 with p=0.502, since these are not statistically significant, it can be said that there is no reliable relationship between how the in-service teachers self-assessed their knowledge and how they actually performed in altitudes of triangles.

Isosceles Triangle Properties. In order to compare how pre-service teachers thought they performed on the Isosceles Triangle Property questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.197 with p=0.171 and Kappa=-.031 with p=.239, since these are not statistically significant, it can be said that there is no reliable relationship between how the pre-service teachers self-assessed their knowledge and how they actually performed on isosceles triangle properties. To compare how in-service teachers thought they performed on the isosceles triangle questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=-.031 with p=.858 and Kappa=0.001 with p=0.973, since these are not statistically significant, it can be said that there is no reliable relationship between how the in-service teachers self-assessed their knowledge and how they actually performed in isosceles triangles.

Corresponding Parts of Congruent Triangles are Congruent (CPCTC). Participants were asked about their knowledge of CPCTC (Corresponding Parts of Congruent Triangles are Congruent). In order to compare how pre-service teachers thought they performed on the CPCTC questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.425 with p=0.002 and Kappa=.128 with p=.011, since these are statistically significant, it can be said that pre-service teachers' self-assessment and actual performance are related, and there is poor correspondence between the two. To compare how in-service teachers thought they performed on the CPCTC questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=-.079 with p=.653 and Kappa=-.050 with p=0.623, since these are not statistically significant, it can be said that there is no reliable relationship between how the in-service teachers self-assessed their knowledge and how they actually performed on CPCTC questions.

Quadrilateral Properties. In order to compare how pre-service teachers thought they performed on the quadrilateral properties questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.159 with p=0.271 and Kappa=-.031 with p=.476, since these are not statistically significant, it can be said that there is no reliable relationship between how the pre-service teachers self-assessed their knowledge and how they actually performed on quadrilateral properties. To compare how in-service teachers thought they performed on the quadrilateral properties questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=-.019 with p=.916 and Kappa=0.012 with p=0.788, since these are not statistically significant, it can be said that there is no reliable relationship between how the in-service teachers self-assessed their knowledge and how they actually performed in quadrilateral properties.

Diagonals of Quadrilaterals and Rectangles. To compare how pre-service teachers thought they performed on the diagonals of quadrilaterals and rectangles questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=-.0919 with p=.529 and Kappa=.057 with p=.2276, since these are not statistically significant, it can be said that there is no reliable relationship between how the pre-service teachers self-assessed their knowledge and how they actually performed on diagonals of quadrilaterals and rectangles. To compare how in-service teachers thought they performed on the quadrilateral properties questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=-.019 with p=.916 and Kappa=0.012 with p=0.788, since these are not statistically significant, it can be said that there is no reliable relationship between how the in-service teachers self-assessed their knowledge and how they actually performed on diagonals of quadrilaterals and rectangles.

Rectangle Properties. To compare how pre-service teachers thought they performed on the rectangle properties questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=-.144 with p=.318 and Kappa=.014 with p=.806, since these are not statistically significant, it can be said that there is no reliable relationship between how the pre-service teachers self-assessed their knowledge and how they actually performed on rectangle properties. To compare how in-service teachers thought they performed on the quadrilateral properties questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.000 with p=.998 and Kappa=-.005 with p=0.941, since these are not statistically significant, it can be said that there is no reliable relationship between how the in-service teachers self-assessed their knowledge and how they actually performed on rectangle properties.

Quadrilateral Similarity. To compare how pre-service teachers thought they performed on the quadrilateral similarity questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.193 with p=.180 and Kappa=.055with p=.136, since these are not statistically significant, it can be said that there is no reliable relationship between how the pre-service teachers self-assessed their knowledge and how they actually performed on quadrilateral similarity. To compare how in-service teachers thought they performed on the quadrilateral similarity and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.125 with p=.474 and Kappa=.037 with p=0.568, since these are not statistically significant, it can be said that there is no reliable relationship between how the in-service teachers self-assessed their knowledge and how they actually performed on rectangle properties.

Cube Properties. To compare how pre-service teachers thought they performed on the cube properties questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=-.227 with p=.113 and Kappa=.022 with p=.548, since these are not statistically significant, it can be said that there is no reliable relationship between how the pre-service teachers self-assessed their knowledge and how they actually performed on rectangle properties. To compare how in-service teachers thought they performed on the cube properties questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.310 with p=.070 and Kappa=-.145 with p=0.147, since these are not statistically significant, it can be said that there is no reliable relationship between how the in-service teachers self-assessed their knowledge and how they actually performed on cube properties.

Kite Properties. To compare how pre-service teachers thought they performed on the definition of a kite questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=-.45 with p=.756 and Kappa=-.004 with p=.880, since these are not statistically significant, it can be said that there is no reliable relationship between how the pre-service teachers self-assessed their knowledge and how they actually performed on the definition of a kite. To compare how in-service teachers thought they performed on the definition of a kite questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=-.010 with p=.954 and Kappa=-.021 with p=0.678, since these are not statistically significant, it can be said that there is no reliable relationship between how the in-service teachers self-assessed their knowledge and how they actually performed on definition of a kite.

Polygon Diagonals. To compare how pre-service teachers thought they performed on the polygon diagonals questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.150 with p=.297 and Kappa=.010 with p=.513, since these are not statistically significant, it can be said that there is no reliable relationship between how the pre-service teachers self-assessed their knowledge and how they actually performed on polygon diagonals. To compare how in-service teachers thought they performed on the polygon diagonals questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=-.148with p=.395 and Kappa=-.046 with p=0.310, since these are not statistically significant, it can be said that there is no reliable relationship between how the in-service teachers self-assessed their knowledge and how they actually performed on rectangle properties.

Distance Formula. To compare how pre-service teachers thought they performed on the questions addressing the distance formula and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.211 with p=.142 and Kappa=.078 with p=.148, since these are not statistically significant, it can be said that there is no reliable relationship between how the pre-service teachers self-assessed their knowledge and how they actually performed on the distance formula. To compare how in-service teachers thought they performed on the distance formula questions and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.264 with p=.126 and Kappa=.099 with p=0.212, since these are not statistically significant, it can be said that there is no reliable relationship between how the in-service teachers self-assessed their knowledge and how they actually performed on rectangle properties.

Circle Properties. To compare how pre-service teachers thought they performed on the circle properties questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=-.037 with p=.799 and Kappa=.029 with p=.524, since these are not statistically significant, it can be said that there is no reliable relationship between how the pre-service teachers self-assessed their knowledge and how they actually performed on circle properties. To compare how in-service teachers thought they performed on the circle properties questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=-.134 with p=.444 and Kappa=-.011 with p=0.872, since these are not statistically significant, it can be said that there is no reliable relationship between how the in-service teachers self-assessed their knowledge and how they actually performed on circle properties.

Construction of a Tangent. To compare how pre-service teachers thought they performed on the construction of a tangent questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.252 with p=.077 and Kappa=.073 with p=.347, since these are not statistically significant, it can be said that there is no reliable relationship between how the pre-service teachers self-assessed their knowledge and how they actually performed on construction of a tangent. To compare how in-service teachers thought they performed on the construction of a tangent questions and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.236 with p=.172 and Kappa=.042 with p=0.681, since these are not statistically significant, it can be said that there is no reliable relationship between how the in-service teachers self-assessed their knowledge and how they actually performed on construction of a tangent problems.

Inscribed Angles. To compare how pre-service teachers thought they performed on the inscribed angle questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.193 with p=.180 and Kappa=.039 with p=.184, since these are not statistically significant, it can be said that there is no reliable relationship between how the pre-service teachers self-assessed their knowledge and how they actually performed on inscribed angles. To compare how in-service teachers thought they performed on the inscribed angle questions and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.211 with p=.224 and Kappa=.081 with p=0.116, since these are not statistically significant, it can be said that there is no reliable relationship between how the in-service teachers self-assessed their knowledge and how they actually performed on inscribed angle problems.

Proof Validity. To compare how pre-service teachers thought they performed on the proof validity questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.111 with p=.443 and Kappa=.011 with p=.617, since these are not statistically significant, it can be said that there is no reliable relationship between how the pre-service teachers self-assessed their knowledge and how they actually performed on proof validity. To compare how in-service teachers thought they performed on the proof validity questions on the assessment and how they actually performed, a consistency estimate was conducted using interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.257 with p=.137 and Kappa=.022 with p=0.534, since these are not statistically significant, it can be said that there is no reliable relationship between how the in-service teachers self-assessed their knowledge and how they actually performed on proof validity.

Table 18 shows the interrater reliability of pre-service and high school teachers by mathematical topic. There were only two occasions where the Kappa was significant, and when this occurred there was slight agreement between the self-evaluation and actual performance on the MKT-G. Otherwise Kappa was insignificant, which means there is no reliable relationship between the participant's self-evaluation and actual performance on the MKT-G, which means the participants' self-evaluation did not correspond to their performance on the assessment. The participants were not able to reliably evaluate their performance.

Table 18.

Interrater Reliability of Pre-Service and High School Teachers by Mathematical Topic.

	Pre-Service		High School	
	Kappa	p-value	Kappa	p-value
Angle Bisectors	.030	.369	.085	.056
Angle Relations	.079	.015*	.035	.599
Altitude of Triangles	.093	.051	.050	.502
Isosceles Triangle Properties	-.031	.239	.001	.973
CPCTC	.128	.011*	-.050	.623
Quadrilateral Properties	-.031	.476	.012	.788
Diagonals of Quadrilaterals	.057	.227	.051	.626
Rectangle Properties	.014	.806	-.005	.941
Quadrilateral Similarity	.055	.136	.037	.568
Cube Properties	.022	.548	.145	.147
Definition and Properties of a Kite	-.004	.880	.021	.678
Polygon Diagonals	.010	.513	-.046	.310
Distance Formula	.078	.148	.099	.212
Circle Properties	.029	.524	-.011	.872
Construction of a Tangent	.073	.347	.042	.681
Inscribed Angle Theorem	.039	.184	.081	.116
Proof Validity	.011	.617	.022	.534

*p<.05

In-depth Investigation into Teachers' Knowledge

To investigate the sources of Geometry Teaching Knowledge, three pre-service teachers and four high school Geometry teachers were selected to be interviewed. Along with interviews, observations were also conducted. One of the pre-service teachers, Daniel, was observed three times, three of the high school Geometry teachers were observed three times, and due to scheduling conflicts the fourth high school Geometry teacher, Mrs. Evans, was observed twice. The next section provides a summary of evidence from the interviews and observations within the four Geometry Teaching Knowledge domains and knowledge of instructional techniques and methods.

Case 1: Maria. Maria was a senior in the student teacher preparation program at a central Texas university. She is a student teacher during the Spring semester of 2016. She has tutoring experience with middle school mathematics, and would prefer to teach middle and high school when she graduates. She would like to teach Pre-Algebra, Algebra 1, Geometry, Algebra 2, and Statistics when she becomes a teacher at a school district. Maria has already completed and passed Modern Geometry. Maria was selected to be part of the study because she volunteered to participate in the interview process and had not yet started student teaching.

Summary of Geometry Teaching Knowledge.

<u>**MKT-G Assessment Results**</u>**.** Maria took the MKT-G Assessment at the end of the Summer session of 2015. According to the MKT-G Assessment results supplied from the University of Michigan, Table 19 shows Maria's percentile within the pre-service teachers in each of the domains.

Table 19.
Maria's MKT-G Percentile within Pre-Service Teachers.

	MKT-G percentile within Pre-Service
GCK	93.14
SGK	50.00
KGS	44.12
KGT	76.47
Total	81.37

MKT-G Assessment Self-Evaluation. After Maria finished the MKT-G Assessment, she was given a survey. A portion of the survey asked her to rate how she thought she performed on the assessment by giving her a 1 to 5 Likert Scale. The Likert Scale questions only addressed seventeen mathematical concepts that were covered in the assessment. In order to compare how Maria thought she performed on the assessment and how she actually performed, I conducted a consistency estimate using Interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.934 with p<0.001 and Kappa=0.206 with p=0.003, while statistically significant, there is fair agreement between Maria's self-evaluation and how she actually performed on the assessment.

Geometry Content Knowledge. During the interview, Maria was asked to investigate the midsegment of a triangle and then prove that the midsegment of a triangle was half the length and parallel to the third side of a triangle. The investigation started with having Maria draw and arbitrary triangle ABC and find the midpoints of two sides of the triangle, $\overline{AB}$ and $\overline{AC}$, using a compass and a straight edge (protractor) and label those midpoints D and E. She was timid when she started the interview and became very nervous when asked to use a compass and protractor. She had some trouble remembering how to use the compass to find the midpoint of a side, but I helped her recall the process.

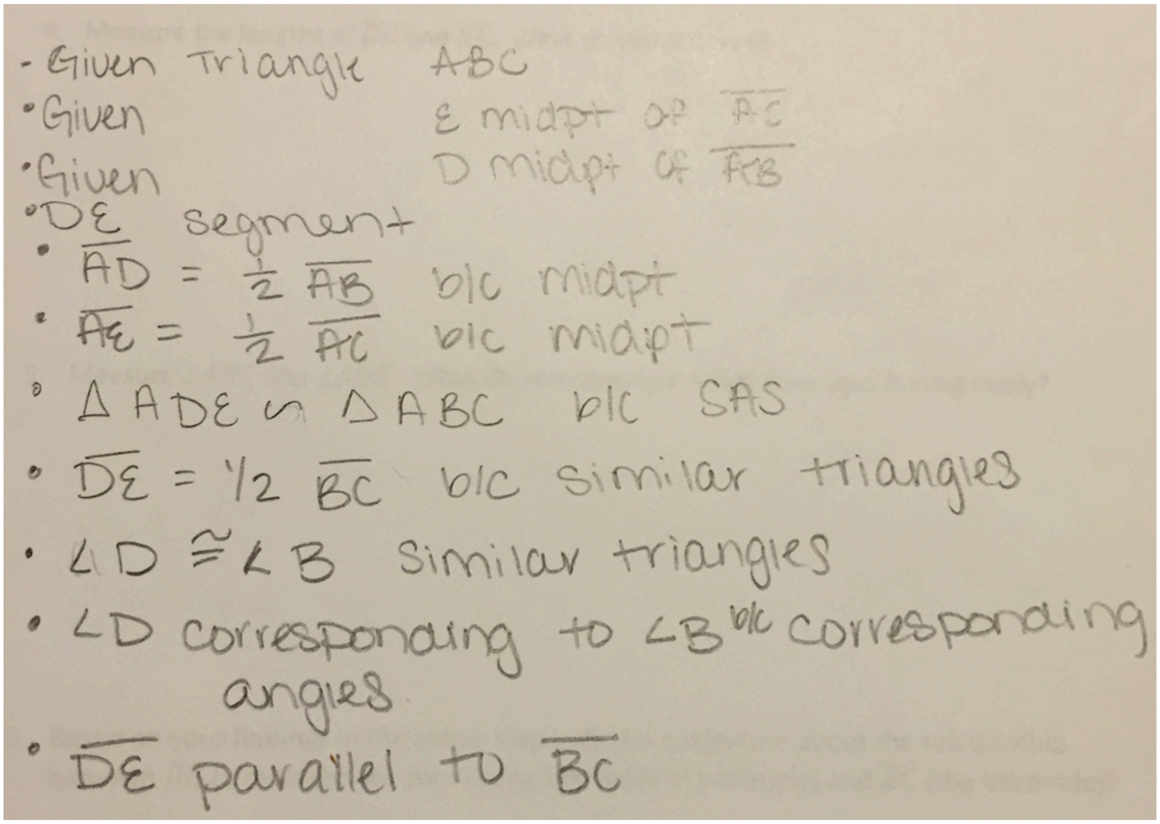
She was then asked to conjecture about what properties she noticed regarding the midsegment. She recalled something about being half as long, and she mentioned ratios. She also mentioned that she saw similar triangles, but she was not sure what that would tell her about the situation. She was prompted to measure the length of $\overline{DE}$ and $\overline{BC}$ and the measure of $\angle ABC$ and $\angle ADE$. She was prompted to talk about what she observed and noticed about her construction. After some discussion, she came to the conclusion that the midsegment was half the length of the third side and that $\angle ABC$ and $\angle ADE$ are congruent. I asked her what $\angle ABC$ and $\angle ADE$ being congruent would mean and she said it was because the triangles were similar. When asked about why similarity would have any relationship, Maria responded, "Okay, well basically, once we prove that theses are the same [pointing at $\angle ABC$ and $\angle ADE$], we can prove that that's [points at $\overline{DE}$ and $\overline{BC}$] the same because they share angle A." She said that $\overline{DE}$ and $\overline{BC}$ were congruent. I questioned her to make sure she meant to say congruent, and she says, "Isn't it like congruent parts of congruent triangles are congruent?". I reminded her that we just measured the sides and that $\overline{DE}$ is half of $\overline{BC}$. When asked about what we knew about $\overline{DE}$ and $\overline{BC}$, she made the connection that $\overline{DE}$ is half of $\overline{BC}$. We moved onto the conjecture that was given in the prompt: "A midsegment connecting two sides of a triangle is parallel and half the length of the third side". Maria was unsure about the parallel portion of the conjecture. I asked, "Do you think $\overline{DE}$ and $\overline{BC}$ are parallel?". She stated the following:

> Just looking at it, to me, it seems like, because whenever you do parallel, you have to have, like, they're never going to touch, and I feel like if I ever extend this even longer, if we go the right way, it won't ever touch, but I don't know how to

make it to make, like, a 90-degree angle so we can come from the other parallel, you know?

I proceeded to try to get her to look at the situation a different way by turning the paper, so the midsegment and the third side were horizontal to see if she can recognize the situation from previous experiences. She connected this situation to having "exterior angles", I asked her to keep thinking along those lines. She was unable to come up with the term used for the types of angles $\angle ABC$ and $\angle ADE$ would be in reference to $\overline{DE}$ and $\overline{BC}$. I gave her some different names of angles, and after saying corresponding angles, she selected corresponding angles. After this interaction, she believed the entire conjecture and was then prompted to prove the conjecture.

Maria was given the choice of a paragraph proof and a two-column proof, and she chose a two-column proof. She then asked me, "For the measurement, do you want me to do, like, the ratio, or I don't know, because technically we measured it so can I really sue that in the proof?". I reminded her that she wanted to make sure that she was proving the conjecture for an arbitrary triangle, not just proving it for the triangle she drew. She started to have difficulty once she wrote the given information, and wanted to jump straight to the length being half as long, but I asked questions that would help her to recall all the information we had already discussed. She recalled that the triangles were similar, but she could not remember the theorem used to prove similarity. I asked her leading questions regarding what information she knew about the triangle, and she came to the conclusion of using Side-Angle-Side Similarity. She kept accidentally saying congruent instead of similar, but every time I questioned her about it, she corrected herself. Below is a picture of the proof Maria gave of the midsegment of a triangle:



- Given Triangle ABC
- Given E midpt of $\overline{AC}$
- Given D midpt of $\overline{AB}$
- DE segment
- $\overline{AD} = \frac{1}{2}\overline{AB}$ b/c midpt
- $\overline{AE} = \frac{1}{2}\overline{AC}$ b/c midpt
- $\triangle ADE \sim \triangle ABC$ b/c SAS
- $\overline{DE} = 1/2\ \overline{BC}$ b/c similar triangles
- $\angle D \cong \angle B$ similar triangles
- $\angle D$ corresponding to $\angle B$ b/c corresponding angles
- $\overline{DE}$ parallel to $\overline{BC}$

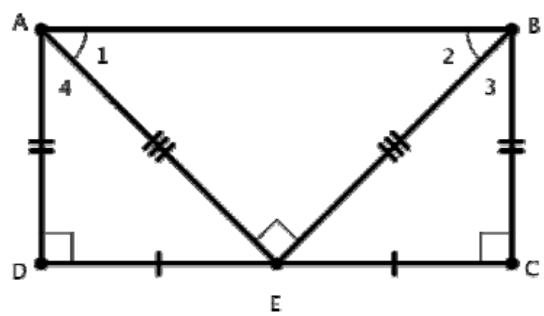
Figure 26. Maria's Midsegment Proof

After Maria finished the proof, she self-assessed that she did not write a very good proof, but she felt good about the problem. When asked if she believed what she had just proved for all triangles, she said, "Yes, I didn't really write it pretty, but now I do".

<u>Specialized Geometry Knowledge</u>. During the interview, Maria was asked to recall the assessment she took at the end of the summer semester. She was asked a question directly from the MKT-G assessment that address the Specialized Geometry Knowledge domain.

The following is the question asked in the MKT-G assessment and the interview:

While proving a claim on the board about the figure below, Joe wrote “1 + 2 = 90.” Ms. Staples ponders how to correct that statement. Of the following, what is the best alternative?



A	Do nothing. The statement is correct as is.
B	Add the degree symbol so that the statement reads 1 + 2 = 90°.
C	Replace what Joe wrote; write instead that “m∠A + m∠B = 90°”.
D	Replace what Joe wrote; write instead that “m∠EAB + m∠EBA = 90°”.

Figure 27. SGK Question (Herbst & Kosko, 2014)

She was asked to recall what she answered on the MKT-G assessment, and she recalled answering D. When asked if she understood what the student originally wrote on the board, she said, “Yes, I would say the correct way to write it would be this [points at D]”. She had a problem with the student just writing 1+2=90 because she did not know what they were measuring. She had had this problem with some of the students she had tutored in the past. They would write answers down, but they would not label what their answer meant to the problem. When asked about how important it is for the student to use the degree symbol, she saw it as a good habit to have, and would like to have seen students use the degree symbol. She revealed that she would also accept if a students answered with C. When asked if she thought it would be an issue if the student answered C because they are only supposed to be talking about part of angle A and angle B, she said, “That’s where my concern comes in…I would write a note on there saying it's technically written this way". If a student went up to the board and wrote 1+2=90, like in the problem, Maria would correct them but be careful not to intimidate them. She would explain that what they wrote is unclear and she would give alternatives. If a student were to answer this way on an exam, she said, "I'm a big person on partial credit… but I would give partial credit on it. I would be like, technically you are right, but I don't like to see it this way".

Knowledge of Geometry and Students. During the interview, Maria was asked to recall the assessment she took at the end of the summer semester. She was asked a question directly from the MKT-G assessment that address the Knowledge of Geometry and Students domain. The following is the question asked in the MKT-G assessment and the interview:

Ms. Jamison asked students to figure out the number of diagonals in an 11-sided polygon. Of the following erroneous solutions, which one might have resulted from using the number of diagonals through one vertex?

A 11

B 72

C 88

D 99

E 121

Figure 28. KGS Question (Herbst & Kosko, 2014)

She was asked to recall what she answered on the MKT-G assessment, and she recalled answering A. She did not change her answer during the interview. She thought students would answer 11 because they would automatically think 11-sided polygon, so the answer must be 11. She talked about not wanting to answer this question on the MKT-G assessment, and just going with answer choice A. She was then prompted to discuss how the students may have obtained the other erroneous solutions given in the problem. The only answer she was unable to see where students would get that number was for answer choice B. She had no idea how they got 72 from the 11-sided polygon, but the other choices were multiples of 11, so she could see where the students could have come up with the other choices.

Knowledge of Geometry and Teaching. During the interview, Maria was asked a question directly from the MKT-G assessment that address the Knowledge of Geometry and Teaching domain. The following is the question asked in the MKT-G assessment and the interview:

After teaching her students the base angles theorem (which states that base angles of an isosceles triangle are congruent), Ms. Wellington is pondering what example she could give to illustrate how this theorem can help find new information. Which of the following would be the best example?

A	If you have a triangle that you know is isosceles and you know two of the sides are 8 and 10, then the third side will be 8 or 10.
B	If you have a triangle that you know the sides have length 8, 10, and 12 you know it does not have two angles that are congruent.
C	If you have an isosceles triangle and two of its angles are 30 you can calculate the third angle by subtracting 60 from 180.
D	If you have a kite and draw its minor diagonal, the kite is divided into two isosceles triangles and those have two congruent angles each.

Figure 29. KGT Question (Herbst & Kosko, 2014)

She was asked to recall what she answered on the MKT-G assessment, and she recalled answering D. She did not change her answer during the interview. She initially showed dislike for choice A because students would be confused about which side is 8 or 10. She did not like choice B and C because she is not sure what they have already gone over in their Geometry class already. She was not as sure of herself, but she chose D because she is a visual learner and the kite would make a lot more sense. She based her decisions on not liking the other choices. When prompted whether she recalled the order of material in Geometry and if students would have covered kites in the curriculum when coving the base angles theorem, she did not think they had gone over kites yet, so this would be new information for the students.

Knowledge of Instructional Techniques and Methods. Maria was given a survey after completing the MKT-G assessment at the end of the summer semester. She was given 10 points to distribute among four instructional methods or techniques and other techniques which she could explain on the survey, giving the most points to the instructional method or technique used most often. She was asked to distribute the points in regards to what she has seen in the Geometry courses she has taken, the education courses she has taken, and how she would teach Geometry in an ideal classroom situation. Table 20 shows how Maria distributed the points.

Table 20.
Maria's Distribution of Points among Instructional Methods and Techniques from Survey.

	Seen in Geometry Courses	Seen in Education Courses	Ideal Classroom
Investigations	3	0	1
Compass/Protractor	1	0	1
Computer Software	4	0	4
Manipulatives/Models	2	0	4
Other	0	10 (None)	0

According to self-reported data given in the survey, Maria saw in her Geometry courses Geometry software 40% of the time, Investigations 30% of the time, Manipulative and Models 20% of the time, and used a compass and protractor 10% of the time. Maria reported that she had not seen any other these instructional techniques or methods in her education courses. In Maria's ideal Geometry classroom, she would spend 40% of the time using manipulatives and models, 40% of the time using computer software, 10% of the time using a compass and protractor, and 10% of the time using investigations.

Sources of Knowledge. Throughout the interview with Maria, she referred to her knowledge of student understanding stemming from her experiences tutoring. When asked if she were to have difficulty understanding a concept that she was required to teach, she would go to the textbook to look up how to work problems or what material she may not remember when teaching certain topics. When asked what she would do if she did not know how to present material to her class, she would go to her mentor teacher or another teacher to ask how she presents the material since she would be the best resource. After she asked her mentor teacher, she would then turn to the internet and look up different ways to present material on Pinterest. Since she was about to start student teaching, she expressed concern over not feeling prepared to handle situations that would arise in the classroom. Her primary concerns were disciplinary concerns, parent interactions, and talking to principals. She was also concerned with the students not liking her and respecting her as a teacher.

Summary of Maria. Maria performed well on the MKT-G assessment. She was high in the Common Content Knowledge domain when compared to the other pre-service teachers. Her lowest domain was Knowledge of Geometry and Students (KGS). Her self-evaluation of her performance was statistically significant and she had fair agreement. Maria's Geometry Content Knowledge (GCK) was not as strong as her score in the GCK domain would seem to imply. She had a difficult time formulating a proof of the properties of the midsegment of a triangle. She did not provide all the steps in the proof and it was not a paragraph proof or a two-column proof. She attempted a two-column proof, but it ended up being a list of steps. Maria was correct during the interview on the Specialized Geometry Knowledge question. She did have some confusion with regard to the correct way for labeling an angle. On the interview question addressing Knowledge of Geometry and Students, Maria was incorrect. She chose 11 diagonals, but did not give much reasoning as to why a student would choose 11 besides them thinking about an 11-sided polygon. She recalled giving up on this question when taking it during the MKT-G assessment. When asked the question from the MKT-G question addressing Knowledge of Geometry and Teaching, she was correct to choose the example of a kite. From her knowledge of the order of the curriculum she did not think that students had addressed kites yet, so that would be new material. Maria reported seeing Geometry software the most in her Geometry courses, seeing none of the techniques and methods in her education courses, and she would spend most of the time in her ideal classroom using computer software and manipulatives and models because she is a visual learner. Maria uses her knowledge she has acquired from tutoring. The resources Maria would use if she needed assistance would first be the textbook, then a cooperating teacher, and then she would turn to the internet.

Case 2: Jason. Jason was student teaching at a central Texas university when he took the MKT-G assessment, and had completed his student teaching in the Fall semester of 2015 when he was interviewed. His student teaching assignment was to teach 8^{th} grade mathematics, and he has experience tutoring 8^{th} grade mathematics and Algebra 1. Jason would prefer to teach middle school and high school levels of mathematics, specifically Pre-Algebra, Algebra 1, Algebra 2, Calculus, and Statistics. He has completed and passed Modern Geometry. Jason was selected to be interviewed because he volunteered to participate in the interview process, and he was student teaching when he took the MKT-G assessment.

Summary of Geometry Teaching Knowledge.

MKT-G Assessment Results. Jason took the MKT-G Assessment during the Fall semester of 2015. According to the MKT-G Assessment results supplied from the University of Michigan, Table 21 shows Jason's percentile within the pre-service teachers in each of the domains.

Table 21.
Jason's MKT-G Percentile within Pre-Service Teachers.

	MKT-G percentile within Pre-Service
GCK	54.90
SGK	86.27
KGS	88.24
KGT	91.18
Total	92.16

MKT-G Assessment Self-Evaluation. After Jason finished the MKT-G Assessment, he was given a survey. A portion of the survey asked him to rate how he thought he performed on the assessment by giving him a 1 to 5 Likert Scale. The Likert Scale questions only addressed seventeen mathematical concepts that were covered in the assessment. In order to compare how Jason thought he performed on the assessment and how he actually performed, I conducted a consistency estimate using Interrater reliability through SPSS. The results of interrater analysis are Pearson's R=.946 with $p<0.001$ and Kappa=0.164 with $p=0.049$, while statistically significant, there is slight agreement between Jason's self-evaluation and how he actually performed on the assessment.

Geometry Content Knowledge. During the interview, Jason was asked to investigate the midsegment of a triangle and then prove that the midsegment of a triangle was half the length and parallel to the third side of a triangle. He started by drawing an arbitrary triangle ABC and found the midpoints of two sides of the triangle, $\overline{AB}$ and $\overline{AC}$, using a compass and a straight edge (protractor) and labeled those midpoints D and E. Jason did not have any difficult using the compass to find the midpoint of the two sides of the triangle. The first thing he noticed once he had finished constructing his midsegment was that he had similar triangles. He also noticed that $\overline{DE}$ and $\overline{BC}$ were parallel. When prompted about him seeing any other relationship between $\overline{DE}$ and $\overline{BC}$ he said they were congruent, but then after questioning his term congruent, changed his description of $\overline{DE}$ and $\overline{BC}$ to be proportional to one another. He understood that they were proportional, but he was unsure about the length of midsegment $\overline{DE}$ being half the length of $\overline{BC}$. He measured the lengths and decided that the midsegment was half as long as the third side, but was still unsure about always being half as long. However, he noticed quickly that $\angle ABC$ and $\angle ADE$ were congruent because of properties of similar triangles. He moved on to the statement of the conjecture, and began to prove the conjecture, even though he was not certain about $\overline{DE}$ being half of $\overline{BC}$.

He was given the choice of writing a paragraph proof or a two-column proof and he chose a paragraph proof. When he began the proof, he was unsure of where to start and began to bring in the exact measurements that were found in the triangle he drew during the investigation portion of the prompt. He brought up Law of Sines and Law of Cosines since he knew the angle measures, but then he reminded himself he is trying to be general in his proof. Once he was reminded that he noticed they were similar triangles at the beginning of the construction, he tried to reason why the triangles were congruent. At first he used that all the corresponding angles were congruent, but the reasoning behind all the corresponding angles being congruent was based on $\overline{DE}$ and $\overline{BC}$ being parallel. Pointing out that he did not know for sure that $\overline{DE}$ and $\overline{BC}$ were parallel, he got stuck in how he knew the triangles were similar. After asking him about triangle similarity theorems, he came up with Side-Angle-Side Theorem after brainstorming all of the other theorems. From there, Jason was able to set up the proportions for the corresponding sides of the triangles and prove that the triangles were similar. Once he had that the triangles were similar, he then tried to prove that $\overline{DE}$ was half of $\overline{BC}$, but he tried to measure the lengths of the sides to prove it. After being reminded that he must use an arbitrary triangle and what it means for triangles to be similar, he set up the ratios using that D and E were midpoints of sides $\overline{AB}$ and $\overline{AC}$ and then proved that $\overline{DE}$ was half of $\overline{BC}$. The final portion of his proof addressed $\overline{DE}$ being parallel to $\overline{BC}$, which he asked for guidance in proving this portion. He knew that if two lines are parallel, then their corresponding angles are congruent, but he was unsure of using the converse of this statement. Once he was asked guiding questions to get to the point where he believed that the converse was true, he finished the proof. Below is a picture of his finished proof:

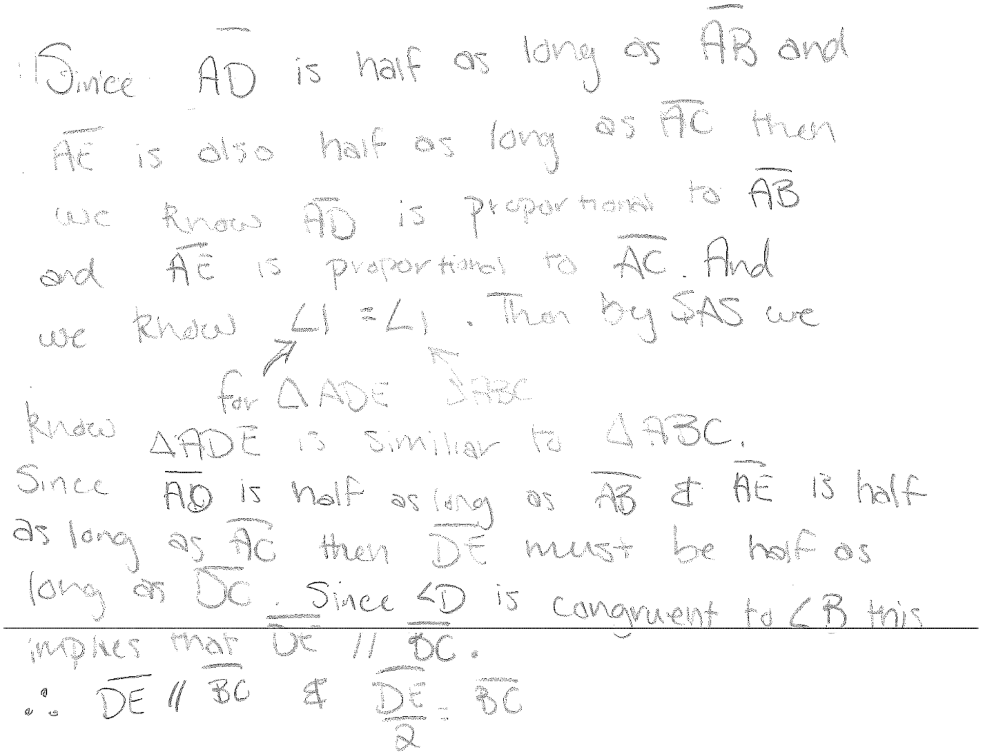
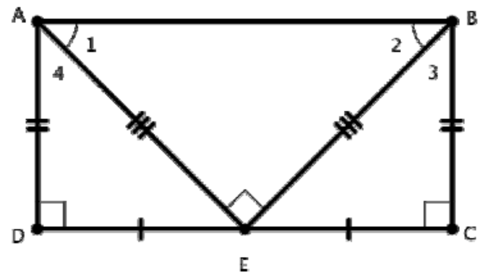

Since $\overline{AD}$ is half as long as $\overline{AB}$ and
$\overline{AE}$ is also half as long as $\overline{AC}$ then
we know $\overline{AD}$ is proportional to $\overline{AB}$
and $\overline{AE}$ is proportional to $\overline{AC}$. And
we know $\angle 1 \cong \angle 1$. Then by SAS we
for $\triangle ADE$ $\triangle ABC$
know $\triangle ADE$ is similiar to $\triangle ABC$.
Since $\overline{AD}$ is half as long as $\overline{AB}$ & $\overline{AE}$ is half
as long as $\overline{AC}$ then $\overline{DE}$ must be half as
long as $\overline{DC}$. Since $\angle D$ is congruent to $\angle B$ this
implies that $\overline{DE} \parallel \overline{BC}$.
$\therefore$ $\overline{DE} \parallel \overline{BC}$ & $\frac{\overline{DE}}{2} = \overline{BC}$

Figure 30. Jason’s Midsegment Proof

After Jason finished his proof, he then believed that the properties of a midsegment of a triangle hold true for all triangles.

Specialized Geometry Knowledge. During the interview, Jason was asked to recall the assessment he took during the Fall semester. He was asked a question directly from the MKT-G assessment that address the Specialized Geometry Knowledge domain. The following is the question asked in the MKT-G assessment and the interview:

While proving a claim on the board about the figure below, Joe wrote "1 + 2 = 90." Ms. Staples ponders how to correct that statement. Of the following, what is the best alternative?



A	Do nothing. The statement is correct as is.
B	Add the degree symbol so that the statement reads 1 + 2 = 90°.
C	Replace what Joe wrote; write instead that "m∠A + m∠B = 90°".
D	Replace what Joe wrote; write instead that "m∠EAB + m∠EBA = 90°".

Figure 31. SGK Question (Herbst & Kosko, 2014)

He was asked to recall what he answered on the MKT-G assessment, and he recalled answering D. Since 1+2=3, then A would not be the correct answer, and just adding the degree symbol does not fix that problem either. Jason did not like choice C because $\angle A$ could mean many different angles. He would stick to choice D, but he would probably have written $m\angle 1 + m\angle 2 = 90°$. Jason understood what the student meant when they wrote 1+2=90, but he thought the student was being too nonchalant about his notation. If a student were to write this on an exam, he would give partial credit because the student does have an understanding of the concept, but he would write the student a note explaining that 1+2=3, just so he can let them know that their notation is not correct.

Knowledge of Geometry and Students. During the interview, Jason was asked to recall a question directly from the MKT-G assessment that addressed the Knowledge of Geometry and Students domain. The following is the question asked in the MKT-G assessment and the interview:

Ms. Jamison asked students to figure out the number of diagonals in an 11-sided polygon. Of the following erroneous solutions, which one might have resulted from using the number of diagonals through one vertex?

A	11
B	72
C	88
D	99
E	121

Figure 32. KGS Question (Herbst & Kosko, 2014)

He was asked to recall what he answered on the MKT-G assessment, and he could not remember what he had chosen. In the interview, he tried to figure out how many diagonals there would be by drawing a quadrilateral and there being 1 diagonal. He then drew a pentagon and drew all the diagonals. After some time, he decided on answer choice A. He did not think any of the choices were very good for this situation, but through his exploration with a quadrilateral and a pentagon, he came up with the number 8 (how many diagonals of an 11-sided polygon through one vertex). He did not make any connections between 8 and the other answer choices.

Knowledge of Geometry and Teaching. During the interview, Jason was asked another question directly from the MKT-G assessment that addressed the Knowledge of Geometry and Teaching domain. The following is the question asked in the MKT-G assessment and the interview:

After teaching her students the base angles theorem (which states that base angles of an isosceles triangle are congruent), Ms. Wellington is pondering what example she could give to illustrate how this theorem can help find new information. Which of the following would be the best example?

A	If you have a triangle that you know is isosceles and you know two of the sides are 8 and 10, then the third side will be 8 or 10.
B	If you have a triangle that you know the sides have length 8, 10, and 12 you know it does not have two angles that are congruent.
C	If you have an isosceles triangle and two of its angles are 30 you can calculate the third angle by subtracting 60 from 180.
D	If you have a kite and draw its minor diagonal, the kite is divided into two isosceles triangles and those have two congruent angles each.

Figure 33. KGT Question (Herbst & Kosko, 2014)

He was asked to recall what he answered on the MKT-G assessment, and he recalled answering either A or D. Jason thought that A was getting into the properties of an isosceles triangle, and it was new information, but not as much new information as choice D. He settled on choice D because it was extending the theorem to more than one triangle to make up a quadrilateral. He did not like choice B because it did not utilize the theorem, and he did not like choice C because this would not be the next step for a student to "expand their learning".

Knowledge of Instructional Techniques and Methods. Jason was given a survey after completing the MKT-G assessment during the Fall Semester. He was given 10 points to distribute among four instructional methods or techniques and other techniques which he could explain on the survey, giving the most points to the instructional method or technique used most often. He was asked to distribute the points in regards to what he has seen in the Geometry courses he has taken, the education courses he has taken, and how he would teach Geometry in an ideal classroom situation. Table 22 shows how Jason distributed the points.

Table 22.
Jason's Distribution of Points among Instructional Methods and Techniques from Survey.

	Seen in Geometry Courses	Seen in Education Courses	Ideal Classroom
Investigations	0	0	2.4
Compass/Protractor	4.3	0	2.4
Computer Software	2.2	0	2.2
Manipulatives/Models	3.5	0	3
Other	0	0 (None)	0

When Jason completed the survey, he did not understand the directions to distribute 10 points to the different methods/techniques. Instead, for the question regarding what he has seen in his Geometry courses, he gave a total of 23 points, so by scaling his answers to be compared to 10 points, he has spent 43% of his time in Geometry courses using a compass and protractor, 35% of his time using manipulatives and models, and 22% of his time using computer software. For the question regarding what he has seen in his Education courses, he has not seen any of these methods. When asked to distribute points to the methods and techniques he would use in his ideal Geometry classroom, he used a total of 33 points, so by scaling his answers to be compared to 10 points, he would spend 30% of classroom time using manipulative and models, 24% of his time using compasses and protractors, 24% of his time on investigations, and 22% of his time using computer software. During the interview, Jason gave more reasoning behind his experiences with the different instructional methods and techniques. He did not have a lot of experience with Computer Software, so if he were to use it in his classroom, he would have to be more familiar with the program. He saw using compasses and protractors in his class, but he thought he would have to invest in a classroom set because students would not always have those tools. He would like to avoid straight lecture as much as possible, and he would like to use different foldables because Geometry has so many different definitions, theorems, and formulas. He saw himself using blocks and different models, as well as, having students come up with their own conjectures through investigations.

Sources of Knowledge. Throughout the interview with Jason, he referred to his knowledge of student understand from his experiences student teaching. When asked what he would do if he did not understand material being covered in the course, he first said he would consult his cooperating teacher, but then changed his answer to consulting the class textbook and any resources supplied with the textbook. He would then go to his cooperating teacher for help, but if that still did not help him, he would go to online resources like Kahn Academy. He explained that of all the mathematics courses he would be expected to be able to teach once he graduates, Geometry would be the one that is of most concern. He thought that the program he was about to graduate from had prepared him for the classroom with all the foundation that he would need to teach any of the mathematics topics, but he would need to study up on Geometry topics if that was his teaching assignment after graduation.

Summary for Jason. Jason did very well on the MKT-G. He did the best in the Knowledge of Geometry and Teaching portion of the exam. His worst domain was Geometry Content Knowledge. His self-evaluation of performance and his actual performance were statistically significant, but he had slight agreement. Jason's performance during the interview question addressing Geometry Content Knowledge was stronger than one would assume from his GCK score. At first he did not believe the conjecture was true for all triangles. He had a hard time starting his proof of the properties of a midsegment. He chose to write a paragraph proof, and he knew that he had similar triangles, but he was unable to prove similarity without help and he had difficulty with proving for an arbitrary triangle. He also had difficulty proving that the midsegment was parallel to the third side, so he needed help on that portion of the proof because he was not able to use the converse of the parallel lines theorem. Jason was correct during the interview on the Specialized Geometry Knowledge question. He did have some doubt in what he might have answered on the MKT-G exam, but he was confident in his answer during the interview. He thought the students were being “nonchalant” about things and not being specific enough. On the interview question addressing Knowledge of Geometry and Students, Jason was incorrect. He chose 11 diagonals, but he did not know where the students would get any of the erroneous solutions. He was able to come up with the number 8, but did not connect that with the answer choice of 88. When asked the question from the MKT-G question addressing Knowledge of Geometry and Teaching, he was correct to choose the example of a kite. From his knowledge of the order of the curriculum he did not think that students had addressed kites yet, so that would be new material. Jason reported seeing compass and protractor activities the most in his Geometry courses, seeing none of the techniques and methods in his education courses, and he would spend most of the time in his ideal classroom using manipulatives and models and foldables so that students can come up with their own conjectures. Jason uses his knowledge he has acquired from student teaching. The resources Jason would use if he needed assistance would first be the textbook, then a cooperating teacher, and then he would turn to the internet.

Case 3: Daniel. Daniel is currently in the student teacher program at a central Texas university, and completed his student teaching in the Fall semester of 2015. He was assigned to teach Geometry at a local high school, and has experience tutoring mathematics from grades 4-12. He prefers to teach middle school and high school levels of mathematics, specifically Pre-Algebra, Algebra 1, Geometry, Algebra 2, and Pre-Calculus. He has passed Informal Geometry and Modern Geometry. Daniel was selected to be interviewed and observed for this study because he is currently student teaching and was assigned to teach Geometry. Due to technical difficulties, only a portion of Daniel's interview was transcribed. The primary source of this information comes from field notes, the interview protocol, and observation notes. Daniel was observed three times.

Summary of Geometry Teaching Knowledge.

MKT-G Assessment Results. Daniel took the MKT-G Assessment during the Fall semester of 2015. According to the MKT-G Assessment results supplied from the University of Michigan, Table 23 shows Daniel's percentile within the pre-service teachers in each of the domains.

Table 23.
Daniel's MKT-G Percentile within Pre-Service Teachers.

	MKT-G percentile within Pre-Service
GCK	34.31
SGK	71.57
KGS	44.12
KGT	76.47
Total	69.61

MKT-G Self Evaluation Results. After Daniel finished the MKT-G Assessment, he was given a survey. A portion of the survey asked him to rate how he thought he performed on the assessment by giving him a 1 to 5 Likert Scale. The Likert Scale questions only addressed seventeen mathematical concepts that were covered in the assessment. In order to compare how Daniel thought he performed on the assessment and how he actually performed, I conducted a consistency estimate using Interrater reliability through SPSS. The results of interrater analysis of the original data were Pearson's R=.915 with p<.001 and Kappa=0.008 with p=0.893. Due to Kappa not being statistically significant, further investigation showed that there was an outlier with one of the mathematical concepts, isosceles triangles. Once that outlier was removed, Pearson's R=.362 with p=.134, which is not significant, so Daniels self evaluation did not correspond with how he actually performed on the assessment.

Geometry Content Knowledge. During the interview, Daniel was asked to investigate the midsegment of a triangle and then prove that the midsegment of a triangle was half the length and parallel to the third side of a triangle. He started by drawing an arbitrary triangle ABC and found the midpoints of two sides of the triangle, $\overline{AB}$ and $\overline{AC}$, using a compass and a straight edge (protractor) and labeled those midpoints D and E. He did not have any difficulty remembering how to find the midpoint of a segment using a compass. When asked about the what relationship he observes between $\overline{DE}$ and $\overline{BC}$ he said that they appear to be parallel. When asked about the lengths of $\overline{DE}$ and $\overline{BC}$, he used the compass to show that $\overline{DE}$ is half of $\overline{BC}$. He observed that $\angle ABC$ and $\angle ADE$ were congruent because $\overline{DE}$ and $\overline{BC}$ were parallel. He recalled the properties of a midsegment, that $\overline{DE}$ was half of $\overline{BC}$ and $\overline{DE}$ and $\overline{BC}$ were parallel. He was then prompted to prove the conjecture, and he was given the choice of writing a paragraph proof or a two-column proof. He chose to write a paragraph proof.

He started out by stating what was given in the problem, but then he was unable to write the proof on his own. Below is the proof that Daniel provided:

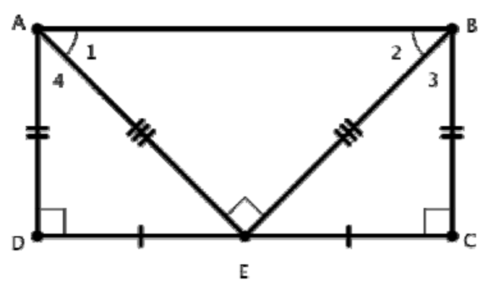
By constructing $\triangle ABC$ and constructing the midpoints of two sides, $\overline{BA}$ and $\overline{AC}$; the segment $\overline{DE}$ appeared to be $\parallel$ to the 3rd side $\overline{BC}$ and half the distance. Measurement of $\overline{DE}$ and $\overline{BC}$ showed that $\overline{DE} = \frac{1}{2}\overline{BC}$, and $\overline{DE} \parallel \overline{BC}$. In addition use of properties of $\parallel$ lines and transversals we can use corresponding angles thm to show congruency of angle $\angle ADE$ and $\angle ABC$.

Figure 34. Daniel's Midsegment Proof

When asked about how triangle ABC and triangle ADE were related, he observed that they were similar triangles. When asked what is known about similar triangles, Daniel stated, "They are similar". When asked about the angles of similar triangles, Daniel showed that he knew the angles were congruent using Angle-Angle-Angle Similarity. Daniel was then asked questions about what could then be said about the midsegment if the corresponding angles of the triangle were congruent, he reasoned that the corresponding angles being congruent made sense because $\overline{DE}$ and $\overline{BC}$ were parallel. He was asked which property would have to come first in the proof, if there were parallel lines, then the corresponding angles were congruent, or if angles were congruent, then that implies there were parallel lines. He was still unable to see the use of the converse in the situation. When questioned about how we knew that the midsegment was half the length of the third side, at first he did not know how to set that up, but then he realized that ratios of the corresponding sides could be used. He knew that the ratio of the sides was one half, but he was not able to write this in a proof. After finishing the discussion, Daniel said that he would not have been able to work through the problem without help.

Specialized Geometry Knowledge. During the interview, Daniel was asked to recall the assessment he took during the Fall semester. He was asked a question directly from the MKT-G assessment that address the Specialized Geometry Knowledge domain. The following is the question asked in the MKT-G assessment and the interview:

While proving a claim on the board about the figure below, Joe wrote "1 + 2 = 90." Ms. Staples ponders how to correct that statement. Of the following, what is the best alternative?



A	Do nothing. The statement is correct as is.
B	Add the degree symbol so that the statement reads 1 + 2 = 90°.
C	Replace what Joe wrote; write instead that "m∠A + m∠B = 90°".
D	Replace what Joe wrote; write instead that "m∠EAB + m∠EBA = 90°".

Figure 35. SGK Question (Herbst & Kosko, 2014)

He was asked to recall what he answered on the MKT-G assessment, and he recalled answering C. Daniel seemed to have a hard time understanding what the student was trying to say when he writes 1+2=90 on the board. He did not like answer choice A and B because they were still not correct. He decided not to choose D because it was hard for students to understand that notation. When questioned further about choice D, he said that was the best answer, but students do not know that notation. If a student made this mistake on an exam, Daniel would have counted it wrong before student teaching, but now that he has completed student teaching, he would not count it wrong.

Knowledge of Geometry and Students. During the interview, Daniel was asked to recall a question directly from the MKT-G assessment that addressed the Knowledge of Geometry and Students domain. The following is the question asked in the MKT-G assessment and the interview:

Ms. Jamison asked students to figure out the number of diagonals in an 11-sided polygon. Of the following erroneous solutions, which one might have resulted from using the number of diagonals through one vertex?

A	11
B	72
C	88
D	99
E	121

Figure 36. KGS Question (Herbst & Kosko, 2014)

Daniel recalled that he did not understand this problem when he took the assessment. He drew some pictures to try to reason through the problem, but his original drawing had diagonals going to a center point of an 11-sided polygon. He then drew over that so the diagonals went to the other vertices in his 11-sided polygon. Daniel had to be reminded that the solutions given in the problem are all erroneous, so the correct answer to the intended problem would not be in the solutions. He chose answer A because that was the best answer. If students were taking an exam and many of the students answered 11 for this type of problem, he would go back and try to figure out their reasoning, but if only a few students were incorrect on this type of problem on a test, he would not take the time to see what they did wrong, unless they were a student that usually made good grades. After the interview, he was observed and the lesson he was covering addressed the number of diagonals in polygons. He worked the problem from the assessment again using the information from the lesson, and still stuck with his answer of choice A. When thinking out loud, he came up with 8 diagonals times 11 vertices, divided by 2 to get 44, but still stuck with choice A.

Knowledge of Geometry and Teaching. During the interview, Daniel was asked a question that addressed the Knowledge of Geometry and Teaching domain. The following is the question asked in the MKT-G assessment and the interview:

After teaching her students the base angles theorem (which states that base angles of an isosceles triangle are congruent), Ms. Wellington is pondering what example she could give to illustrate how this theorem can help find new information. Which of the following would be the best example?

A	If you have a triangle that you know is isosceles and you know two of the sides are 8 and 10, then the third side will be 8 or 10.
B	If you have a triangle that you know the sides have length 8, 10, and 12 you know it does not have two angles that are congruent.
C	If you have an isosceles triangle and two of its angles are 30 you can calculate the third angle by subtracting 60 from 180.
D	If you have a kite and draw its minor diagonal, the kite is divided into two isosceles triangles and those have two congruent angles each.

Figure 37. KGT Question (Herbst & Kosko, 2014)

Daniel recalled that he chose C. He did not think choice A was specific enough, he thought B would be alright to choose for his students, and he did not think D was a good choice because the students have not learned quadrilaterals yet. When questioned about new material, he stuck with his answer of C. Since he is using his knowledge of the curriculum, he thinks that triangles are taught prior to quadrilaterals because you can use triangles when talking about quadrilaterals. He drew many quadrilaterals and divided them up into triangles to show how he could connect triangle properties to the quadrilaterals. Since textbooks usually present material in this order, he agreed with this ordering of the material in Geometry courses.

Knowledge of Instructional Techniques and Methods. Daniel was given a survey after completing the MKT-G assessment during the Fall Semester. He was given 10 points to distribute among four instructional methods or techniques and other techniques which he could explain on the survey, giving the most points to the instructional method or technique used most often. He was asked to distribute the points in regards to what he has seen in the Geometry courses he has taken, the education courses he has taken, and how he would teach Geometry in an ideal classroom situation. Table 24 shows how Daniel distributed the points.

Table 24.
Daniel's Distribution of Points among Instructional Methods and Techniques from Survey.

	Seen in Geometry Courses	Seen in Education Courses	Ideal Classroom
Investigations	0	0	5
Compass/Protractor	2	0	0
Computer Software	2	0	5
Manipulatives/Models	1	0	0
Other	5 (Lecture)	10 (Lesson Plans and Lecture)	0

According to self-reported data given in the survey, Daniel saw in his Geometry courses lecture 50% of the time, compass and protractor 20% of the time, computer software 20% of the time, and manipulatives and models 10% of the time. In his Education courses, Daniel had not seen any of the methods or techniques given and said that the primary focus of his Education courses was on lesson plans and lecture. According to the survey, Daniel would spend 50% of his time having his student use investigations and the other 50% of his time having the students use computer software, but when asked in the interview, Daniel would not use software as much because he is not as familiar with the way the programs work. He would prefer to use a compass and protractor instead because students need to put pencil to paper.

Sources of Knowledge. Throughout the interview with Daniel, he referred to his knowledge of student understand from his experiences tutoring students. When asked what he would do if he did not understand material being covered in the course, he first would consult the class textbook and his textbooks from his college Geometry courses. He would then go to online resources, and then he would go to his cooperating teacher. He explained that of all the mathematics courses he would be expected to be able to teach once he graduates, Geometry would be the one that is of most concern, but now that he had student taught Geometry he thought he would have an easier time. He thought that the program he was about to graduate from had prepared him for the classroom with all the foundation that he would need to teach any of the mathematics topics.

Summary of Daniel. Daniel was in the 69.61 percentile for his total score when compared to the other pre-service teachers. He did the best in the Knowledge of Geometry and Teaching portion of the exam. His worst domain was Geometry Content Knowledge and Knowledge of Geometry and Students. His self-evaluation of performance and his actual performance was not statistically significant, so there was not a reliable relationship between the two. Daniel's performance during the interview question addressing Geometry Content Knowledge was weak which is what one would assume from his GCK score. At first he did not believe the conjecture was true for all triangles. He had a hard time starting his proof of the properties of a midsegment. He chose to write a paragraph proof, and he knew that he had similar triangles, but he was unable to prove similarity without help and he was having difficulty with proving for an arbitrary triangle. He also had difficulty proving that the midsegment was parallel to the third side, so he needed help on that portion of the proof because he was not able to use the converse of the parallel lines theorem. His paragraph proof did not end up being a complete proof, more of an outline of the proof of the properties of a midsegment. After completing his proof outline, he said that he would not be able to complete the proof without help. Daniel was correct during the interview on the Specialized Geometry Knowledge question. He did not think that an actual student would be able to answer D because using three points to name an angle is too confusing for students. He would have counted off on an exam if a student wrote an angle incorrectly prior to student teaching, but now that he is in the classroom, he would not count off for incorrect angle notation. On the interview question addressing Knowledge of Geometry and Students, Daniel was incorrect. He chose 11 diagonals, but he did not know where the students would get any of the erroneous solutions. He was asked this question again prior to being observed. He was being observed on a day that he was addressing the number of diagonals in polygons, but he still stuck with his answer of 11. He was able to come up with the number 8, but did not connect that with the answer choice of 88, even though he verbally walked through the process of finding the correct number of diagonals. When asked the question from the MKT-G question addressing Knowledge of Geometry and Teaching, he was incorrect. He did not think the kite extension was a good choice because students have not learned about quadrilaterals yet. From his knowledge of the order of the curriculum he did not think that students had addressed kites yet, so that would be new material. Daniel reported seeing lecture the most in his Geometry courses, seeing lesson plans and lecture in his education courses, and he would spend most of the time in his ideal classroom using investigations and computer software so that students could come up with their own conjectures. When interviewing Daniel, he changed his answer to using the compass and protractor more than computer software because it is more important for them to use pen and paper. Daniel uses his knowledge he has acquired from student teaching and tutoring experiences. The resources Daniel would use if he needed assistance would first be the textbook (student textbooks and his from his college courses), then he would turn to the internet, and then a cooperating teacher.

Case 4: Mrs. Evans. Mrs. Evans is a current high school teacher. She graduated from a central Texas university Spring 2015 with a Bachelor of Science in Mathematics, and she has a mathematics teaching certification for grades 7-12. She prefers to teach high school mathematics, specifically Pre-Algebra, Algebra 1, and Algebra 2. She is currently in her first year of teaching, and teaches Pre-AP Geometry and on-level Geometry. She was chosen to be interviewed and observed due to her being a recent graduate of the same central Texas university as the pre-service teachers participating in this study. She is also in her first year teaching, so this gives a different perspective on her Geometry Teaching Knowledge. Mrs. Evans was observed two times.

Summary of Geometry Teaching Knowledge.

MKT-G Assessment Results. Mrs. Evans took the MKT-G Assessment during the Fall semester of 2015. According to the MKT-G Assessment results supplied from the University of Michigan, Table 25 shows Mrs. Evans' percentile within the in-service teachers in each of the domains.

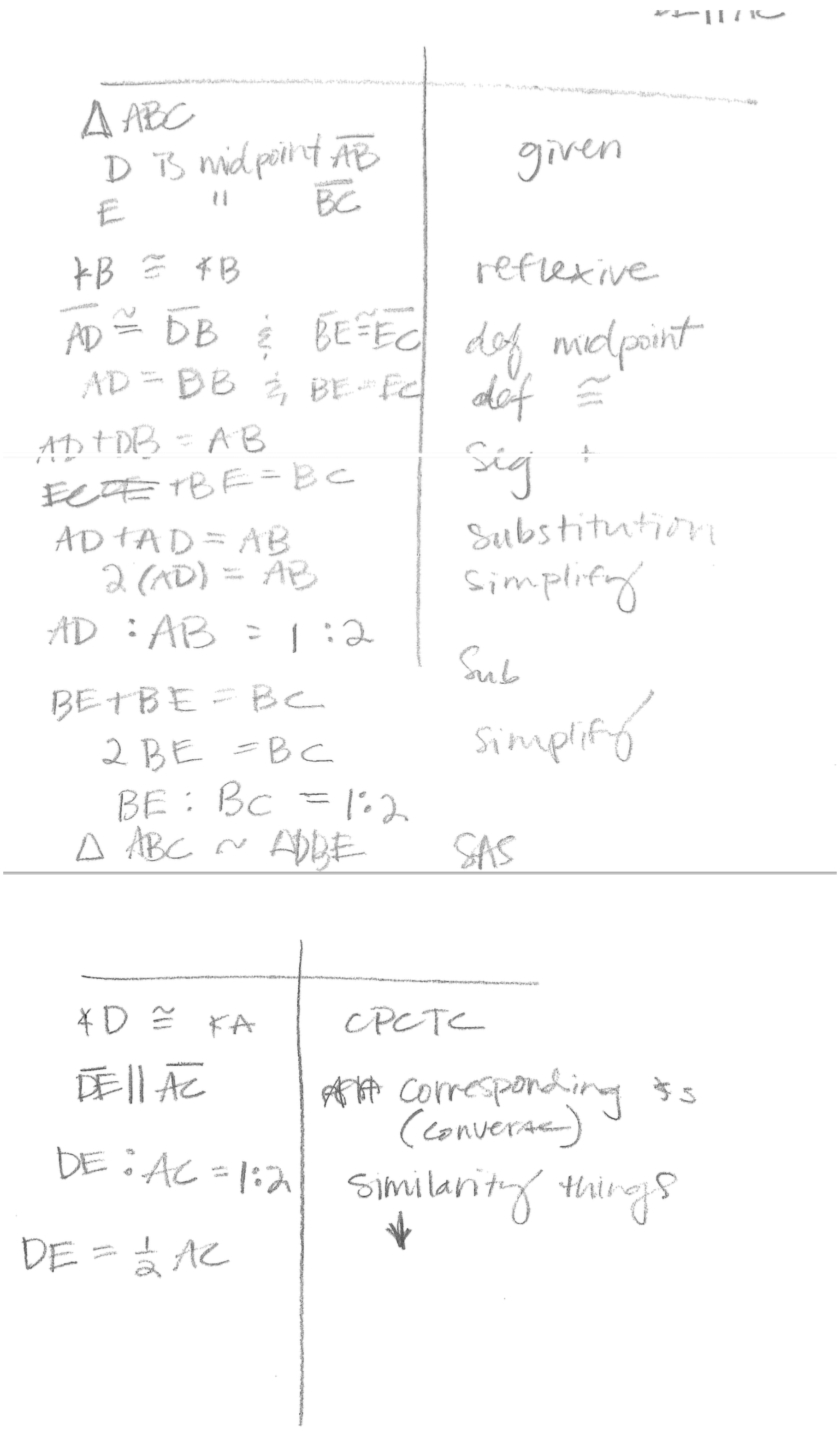
Table 25.
Mrs. Evans' MKT-G Percentile within High School Teachers.

	MKT-G percentile within High School
GCK	27.78
SGK	70.83
KGS	18.06
KGT	12.50
Total	31.94

MKT-G Assessment Self-Evaluation. After Mrs. Evans finished the MKT-G Assessment, she was given a survey. A portion of the survey asked her to rate how she thought she performed on the assessment by giving her a 1 to 5 Likert Scale. The Likert Scale questions only addressed seventeen mathematical concepts that were covered in the assessment. In order to compare how Mrs. Evans thought she performed on the assessment and how she actually performed, I conducted a consistency estimate using Interrater reliability through SPSS. The results of interrater analysis were Pearson's $R=.856$ with $p<.001$ and Kappa=0.197 with $p=0.010$, while statistically significant, there is slight agreement between Mrs. Evan's self evaluation and how she actually performed on the assessment.

Geometry Content Knowledge. During the interview, Mrs. Evans was asked to investigate the midsegment of a triangle and then prove that the midsegment of a triangle was half the length and parallel to the third side of a triangle. The investigation started with having Mrs. Evans draw and arbitrary triangle ABC and find the midpoints of two sides of the triangle, $\overline{AB}$ and $\overline{AC}$, using a compass and a straight edge (protractor) and label those midpoints D and E. She did not have any trouble finding the midpoint of , $\overline{AB}$ and $\overline{AC}$ using a compass, but she did mix up which sides she wanted to find the midpoint of, so she actually found the midpoints of $\overline{AB}$ and $\overline{BC}$. After she constructed the midsegment, she pointed out that the midsegment makes a trapezoid in the figure, and then she recalled that the midsegment is supposed to be half of the third side. She then measured the angles $\angle BDE$ and $\angle BAC$ and saw that they were congruent. Since those were congruent and they were corresponding angles, then $\overline{DE}$ and $\overline{AC}$ were parallel. She was then asked to prove that a midsegment connecting two sides of a triangle was parallel to the third side and half as long. She was given the option to write a paragraph proof or a two-column proof and she chose to write a two-column proof.

When Mrs. Evans began the proof, she saw that she had similar triangles, but asked if she had that information already, or if she must prove that the triangles are similar. When she was told she must prove similarity, she verbally went through all the different similarity theorems and settled on Side-Angle-Side. She then began writing the proof and she was able to complete the entire proof without any help. Below is the proof Mrs. Evans provided during the interview:



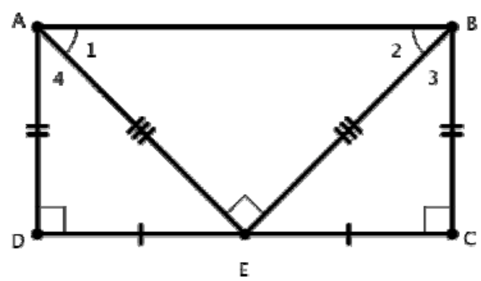
$\triangle ABC$ D is midpoint $\overline{AB}$ E " $\overline{BC}$	given
$\angle B \cong \angle B$	reflexive
$\overline{AD} \cong \overline{DB}$ & $\overline{BE} \cong \overline{EC}$	def midpoint
$AD = DB$ & $BE = EC$	def $\cong$
$AD + DB = AB$ ~~EC DE~~ $+ BE = BC$	Seg +
$AD + AD = AB$	substitution
$2(AD) = AB$	simplify
$AD : AB = 1:2$	
$BE + BE = BC$	Sub
$2BE = BC$	simplify
$BE : BC = 1:2$	
$\triangle ABC \sim \triangle DBE$	SAS

$\angle D \cong \angle A$	CPCTC
$\overline{DE} \parallel \overline{AC}$	~~AIA~~ corresponding $\angle$s (converse)
$DE : AC = 1:2$	similarity things
$DE = \frac{1}{2} AC$	

Figure 38. Mrs. Evan's Midsegment Proof

<u>Specialized Geometry Knowledge</u>. During the interview, Mrs. Evans was asked to recall the assessment she took during the Fall semester. She was asked a question directly from the MKT-G assessment that address the Specialized Geometry Knowledge domain. The following is the question asked in the MKT-G assessment and the interview:

While proving a claim on the board about the figure below, Joe wrote "1 + 2 = 90." Ms. Staples ponders how to correct that statement. Of the following, what is the best alternative?



A	Do nothing. The statement is correct as is.
B	Add the degree symbol so that the statement reads 1 + 2 = 90°.
C	Replace what Joe wrote; write instead that "m∠A + m∠B = 90°".
D	Replace what Joe wrote; write instead that "m∠EAB + m∠EBA = 90°".

Figure 39. SGK Question (Herbst & Kosko, 2014)

She was asked to recall what she answered on the MKT-G assessment, and she could not remember exactly what she answered. She did not choose A because 1+2 does not equal 90, she would not choose B either because adding the degree symbol does not fix the problem. She thought that C was a better than A and B, but it was unclear to what $\angle A$ was referring; angle 1, angle 4, or angle 1 and 4. She knew what the student was trying to say when he wrote 1+2=90 on the board, but she would correct the student. She would probably not use the labeling given in answer choice D, but she would probably write $m\angle 1 + m\angle 2 = 90°$. She was not concerned as much with the degree symbol, but she stresses to her students that the measurement "m" is very important to the problem. If a student were to do this on an exam, she would take off credit because she holds them to a high standard of labeling what they are trying to communicate.

Knowledge of Geometry and Students. During the interview, Mrs. Evan's was asked to recall a question directly from the MKT-G assessment that addressed the Knowledge of Geometry and Students domain. The following is the question asked in the MKT-G assessment and the interview:

Ms. Jamison asked students to figure out the number of diagonals in an 11-sided polygon. Of the following erroneous solutions, which one might have resulted from using the number of diagonals through one vertex?

A	11
B	72
C	88
D	99
E	121

Figure 40. KGS Question (Herbst & Kosko, 2014)

Mrs. Evans could not remember what she answered on the assessment, but during the interview she drew a picture of an 11-sided polygon and then drew the diagonals from one vertex. She originally came up with 9 diagonals through one vertex, but then changed her answer to 8 diagonals through one vertex. She then looked that the answer choices and said, "so if there is 8 through one vertex and there are 11 vertexes, C". She decided she would have chosen C. She saw that students would have chosen 11 because of the 11-sided polygon and then they just multiplied 11 by 11 to get answer E. She did not see how a student would get answer B, but she commented that a student might have gotten choice D by drawing the 11-sided polygon and splitting it up into 9 triangles.

Knowledge of Geometry and Teaching. During the interview, Mrs. Evans was asked a question that addressed the Knowledge of Geometry and Teaching domain. The following is the question asked in the MKT-G assessment and the interview:

After teaching her students the base angles theorem (which states that base angles of an isosceles triangle are congruent), Ms. Wellington is pondering what example she could give to illustrate how this theorem can help find new information. Which of the following would be the best example?

A	If you have a triangle that you know is isosceles and you know two of the sides are 8 and 10, then the third side will be 8 or 10.
B	If you have a triangle that you know the sides have length 8, 10, and 12 you know it does not have two angles that are congruent.
C	If you have an isosceles triangle and two of its angles are 30 you can calculate the third angle by subtracting 60 from 180.
D	If you have a kite and draw its minor diagonal, the kite is divided into two isosceles triangles and those have two congruent angles each.

Figure 41. KGT Question (Herbst & Kosko, 2014)

Mrs. Evans read the question and remembered not liking this problem on the MKT-G Assessment. At first she chose C, but then read choice D and decided she would do either C or D. She said at the very least she would use C because the curriculum throws all the different types of triangles together into one section, and they are using the idea that they have an isosceles triangle. She would have rather used the converse of this example, but then she started to discuss answer choice D. Mrs. Evans reread the problem and then focused on the “new information” portion of the question. She changed her answer to D because she could use that example to introduce quadrilaterals and have the students talk about the difference between major and minor diagonals. She did not mention why she did not choose A or B.

Knowledge of Instructional Techniques and Methods. Mrs. Evans was given a survey after completing the MKT-G assessment during the Fall Semester. She was given 10 points to distribute among four instructional methods or techniques and other techniques which she could explain on the survey, giving the most points to the instructional method or technique used most often. She was asked to distribute the points in regards to what she currently uses in her Geometry class, the professional development she has taken, and how she would teach Geometry in an ideal classroom situation. Table 26 shows how Mrs. Evans distributed the points.

Table 26.
Mrs. Evans' Distribution of Points among Instructional Methods and Techniques from Survey.

	Seen in Geometry Courses	Seen in Education Courses	Ideal Classroom
Investigations	2	6	6
Compass/Protractor	1	0	1
Computer Software	1	0	1
Manipulatives/Models	2	4	1
Other	4 (Direct Teach)	0	1 (Direct Teach)

According to self-reported data given in the survey, Mrs. Evans direct teaches 40% of the instructional time, 20% of the time uses manipulatives and models, 10% of the time uses computer software, 10% of the time uses compasses and protractors, and 20% of the time uses investigations in her current Geometry classroom. In professional development, 60% of the time she has seen investigations and 40% of the time she has seen investigations. If Mrs. Evans had the ideal Geometry classroom, she would spend 60% of the time on investigations, 10% of the time direct teaching, 10% of the time using computer software, 10% of the time using manipulatives and models, and 10% of the time using compasses and protractors. During the interview, she explained that she would love to have the majority of her Geometry course be investigations because that gives students a chance to fully understand end explore the concepts. She would not use Geometer's Sketchpad as much because she thinks they just go through the motions, but if she could have them use Geometer's Sketchpad to make their investigations go a little faster so they can explore easier, then she would like to incorporate the software into her classroom.

Sources of Knowledge. Throughout the interview with Mrs. Evans she referred to her knowledge of student understanding from her experiences as a high school teacher, even though those experiences are limited to one semester. When asked about material, she uses the notes and textbook from last year along with the textbook from this year. Anytime she has issues with Pre-AP Geometry material, she goes to her colleague before doing anything else. Since she is a first year teacher teaching a Pre-AP Geometry course, she is required to got to Gifted and Talented training as well as first year teacher training. These trainings pull her out of the classroom multiple days of the month. She expressed frustration regarding the amount of time she is out of the classroom. She said the trainings are helpful, but they do not seem to be worth being out of the classroom. Since she graduated from the same program as the pre-service teachers in this study, she was asked what she wishes she would have had before she entered the classroom as a full time teacher. She wishes she had another Geometry course that would go deeper into proofs regarding the different Geometry topics. She thinks the connections that are made through proofs are vey important and make it easier to teach the material. She thought that the majority of her mathematics education was focused on Algebra and Calculus topics, and she would have benefitted from an additional Geometry course. Mrs. Evans likes her department and the support system that is in place for her to succeed as a first year teacher. She has a mentor and a team for on-level Geometry and Pre-AP Geometry. She commented that many of her friends that are teachers at other school districts are struggling and she is enjoying her first year teaching. She is grateful for the teams of teachers she has to support her at the high school.

Summary of Mrs. Evans. Mrs. Evans was high in the Specialized Geometry Knowledge domain when compared to the other high school teachers. Her lowest domain was Knowledge of Geometry and Teaching, but she was in the lower half of the high school teachers in the Common Content Knowledge and the Knowledge of Content and Students domain. Her self-evaluation of her performance was statistically significant and she had slight agreement. Mrs. Evan's Geometry Content Knowledge (GCK) was stronger than her score on in the GCK domain would seem to imply. She gave a two-column proof, of the properties of a midsegment of a triangle, and she was able to provide it without much help. Mrs. Evans was correct during the interview on the Specialized Geometry Knowledge question. She would prefer to have the students write the angle $m\angle 1 + m\angle 2 = 90°$. She stresses that the students use the "m" for measure, but she does not stress using the degree symbol. On the interview question addressing Knowledge of Geometry and Students, Mrs. Evans was correct. She chose 88 diagonals, and she was able to reason why students could possibly answer some of the other erroneous solutions. When asked the question from the MKT-G question addressing Knowledge of Geometry and Teaching, she was correct to choose the example of a kite, but did want to answer two of the choices. From her knowledge of the order of the curriculum she did not think that students had addressed kites yet, so that would be new material. Mrs. Evans reported direct teaching the most in her current classroom. She has seen investigations and manipulatives and models in professional development, and she would spend most of the time in her ideal classroom using investigations. Mrs. Evans uses her knowledge she has acquired from being a high school teacher. The resources Mrs. Evans would use if she needed materials would be last years textbook and notes. If she had confusion regarding the material she would go to a fellow teacher before she would do anything else. She is a part of a very supportive group of teachers, so if she has any problems, she has the ability to go to them for advice and help. When asked to reflect on her experiences at the university, she wishes there were more Geometry courses that explored the proofs so that she could better understand how all the material is related.

Case 5: Mrs. Kim. Mrs. Kim is a current high school teacher. She is originally from Korea, and has been teaching mathematics for 8 years. She does not have a degree in mathematics and she has an emergency certification for grades 6-12. She prefers to teach high school mathematics, specifically Algebra 2, Pre-Calculus, and Statistics. She has taught Geometry for 3 years, and currently teaches Pre-AP Geometry at a local high school. Mrs. Kim was randomly chosen to be interviewed and observed because she has taught Geometry for 3 years. Mrs. Kim was observed three times.

Summary of Geometry Teaching Knowledge.

MKT-G Assessment Results. Mrs. Kim took the MKT-G Assessment during the Fall semester of 2015. According to the MKT-G Assessment results supplied from the University of Michigan, Table 27 shows Mrs. Kim's percentile within the high school teachers in each of the domains.

Table 27.
Mrs. Kim's MKT-G Percentile within High School Teachers.

	MKT-G percentile within High School
GCK	27.78
SGK	83.33
KGS	18.06
KGT	80.56
Total	56.94

MKT-G Assessment Self-Evaluation. After Mrs. Kim finished the MKT-G Assessment, she was given a survey. A portion of the survey asked her to rate how she thought she performed on the assessment by giving her a 1 to 5 Likert Scale. The Likert Scale questions only addressed seventeen mathematical concepts that were covered in the assessment. In order to compare how Mrs. Kim thought she performed on the assessment and how she actually performed, I conducted a consistency estimate using Interrater reliability through SPSS. The results of interrater analysis were Pearson's R=.946 with p<.001 and Kappa=0.063 with p=0.190. Since Kappa is not statistically significant, further analysis displayed two mathematics concepts that were outliers, isosceles triangles and rectangle properties. Once those two outliers were removed, the results were Pearson's R=.510 with p=.052 and Kappa=.093 with p=.034, while statistically significant, there is slight agreement between Mrs. Kim's self evaluation and how she actually performed on the assessment.

Geometry Content Knowledge. During the interview, Mrs. Kim was asked to investigate the midsegment of a triangle and then prove that the midsegment of a triangle was half the length and parallel to the third side of a triangle. The investigation started with having Mrs. Kim draw and arbitrary triangle ABC and find the midpoints of two sides of the triangle, $\overline{AB}$ and $\overline{AC}$, using a compass and a straight edge (protractor) and label those midpoints D and E. She seemed a little unsure of herself when she started, but knew exactly how to use a compass to find the midpoint of a segment. She knew right away that $\overline{DE}$ was half of $\overline{BC}$ and the line segments were parallel to one another. She did not need to measure the line segments or the angles in order to know these properties of a midsegment. When asked about why $\angle ABC$ and $\angle ADE$ are congruent, she mentioned the corresponding angle postulate and that parallel lines imply that $\angle ABC$ and $\angle ADE$ are congruent. Mrs. Kim was then asked to prove the conjecture that the midsegment connecting two sides of a triangle is parallel to the third side and is half as long. She was given the option to write a paragraph proof or a two-column proof.

Mrs. Kim decided to draw a picture to refer to throughout the proof and provided a two column proof. Below is the proof she provided:

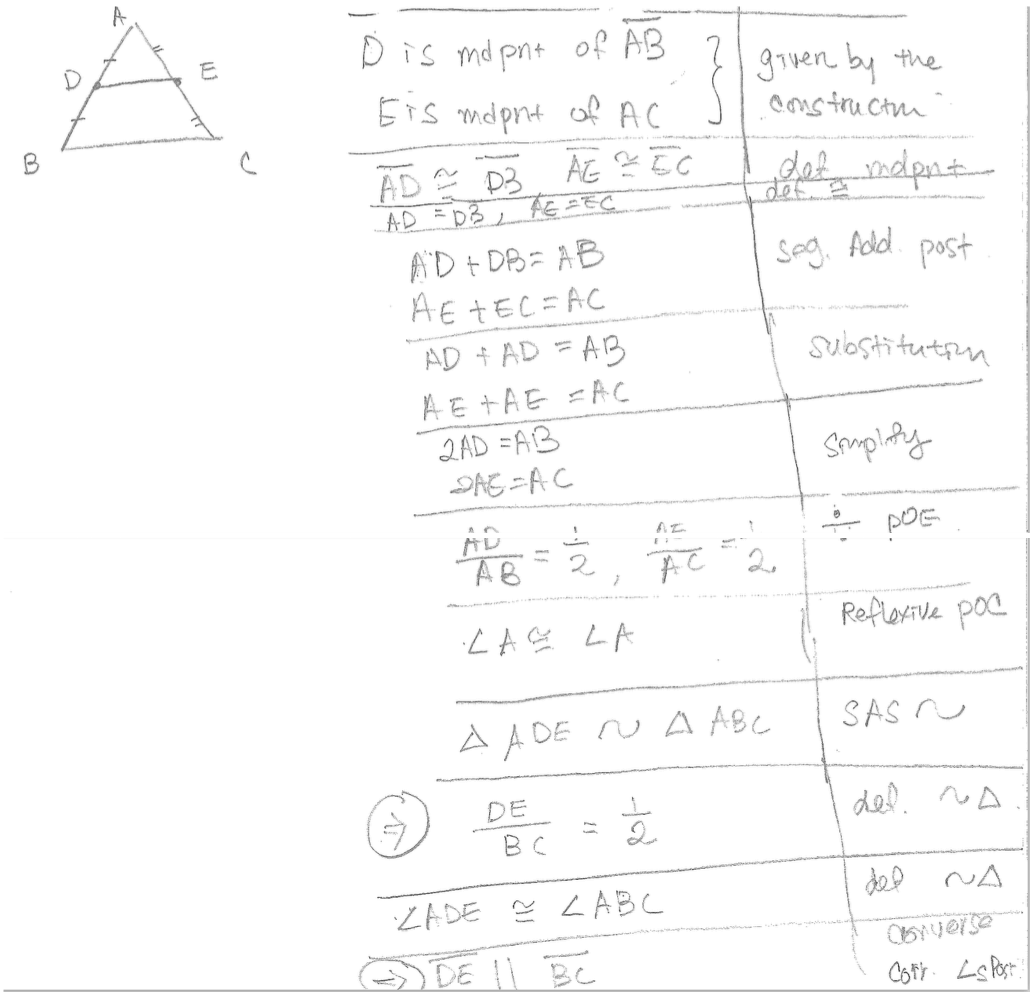
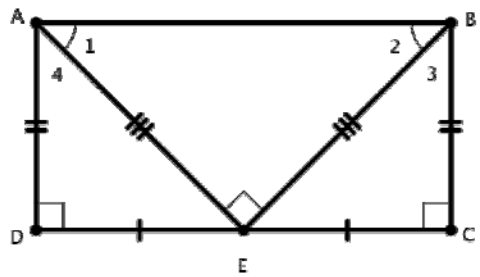


Figure 42. Mrs. Kim's Midsegment Proof

She tried to make the proof as detailed as possible, and used shorthand notation for her reasons. She did not ask any questions about how to prove this conjecture, and she completed the proof at a fast pace. She did not have a problem using the converse of the corresponding angle postulate even though at the beginning of the investigation she stated the reasoning of why $\angle ABC$ and $\angle ADE$ were congruent was due to the segments being parallel.

<u>Specialized Geometry Knowledge</u>. During the interview, Mrs. Kim was asked to recall the assessment she took during the Fall semester. She was asked a question directly from the MKT-G assessment that address the Specialized Geometry Knowledge domain. The following is the question asked in the MKT-G assessment and the interview:

While proving a claim on the board about the figure below, Joe wrote “1 + 2 = 90.” Ms. Staples ponders how to correct that statement. Of the following, what is the best alternative?



A	Do nothing. The statement is correct as is.
B	Add the degree symbol so that the statement reads 1 + 2 = 90°.
C	Replace what Joe wrote; write instead that “m∠A + m∠B = 90°”.
D	Replace what Joe wrote; write instead that “m∠EAB + m∠EBA = 90°”.

Figure 43. SGK Question (Herbst & Kosko, 2014)

Mrs. Kim was asked to recall what she answered on the MKT-G assessment, and she recalled answering D. She understood what the student was trying to show, but she would correct it by changing it to $m\angle 1 + m\angle 2 = 90°$. She tries to correct students using what they have already put on the board, but the best answer of the choices is D. Mrs. Kim would take off points if a student wrote 1+2=90 on an exam, but she would only take off partial credit. She would accept either $m\angle ABC + m\angle ADE = 90°$ or $m\angle 1 + m\angle 2 = 90°$. She would most likely see $m\angle 1 + m\angle 2 = 90°$ from her students because students are lazy and would rather write the shorter equation. She had never seen a student make this mistake before, and she did not think her student would make this mistake, but she knew what the student meant and how she would want them to fix their answer.

Knowledge of Geometry and Students. During the interview, Mrs. Kim was asked to recall a question directly from the MKT-G assessment that addressed the Knowledge of Geometry and Students domain. The following is the question asked in the MKT-G assessment and the interview:

Ms. Jamison asked students to figure out the number of diagonals in an 11-sided polygon. Of the following erroneous solutions, which one might have resulted from using the number of diagonals through one vertex?

A	11
B	72
C	88
D	99
E	121

Figure 44. KGS Question (Herbst & Kosko, 2014)

Mrs. Kim recalled that she chose A. She referred to the choices as ridiculous answers that students would give. When asked why she chose A, she thought that the student misunderstood the definition of a diagonal and jumped to 11 because it is an 11-sided polygon. She stated, "they didn't really understand the definition of a diagonal in the polygon, so it has to be non-consecutive, but they just count all of those together". She did not elaborate on the other choices given on the assessment.

Knowledge of Geometry and Teaching. During the interview, Mrs. Kim was asked a question that addressed the Knowledge of Geometry and Teaching domain. The following is the question asked in the MKT-G assessment and the interview:

After teaching her students the base angles theorem (which states that base angles of an isosceles triangle are congruent), Ms. Wellington is pondering what example she could give to illustrate how this theorem can help find new information. Which of the following would be the best example?

A	If you have a triangle that you know is isosceles and you know two of the sides are 8 and 10, then the third side will be 8 or 10.
B	If you have a triangle that you know the sides have length 8, 10, and 12 you know it does not have two angles that are congruent.
C	If you have an isosceles triangle and two of its angles are 30 you can calculate the third angle by subtracting 60 from 180.
D	If you have a kite and draw its minor diagonal, the kite is divided into two isosceles triangles and those have two congruent angles each.

Figure 45. KGT Question (Herbst & Kosko, 2014)

Mrs. Kim recalled that she chose D, and would still choose that answer. She did not like choice A because it did not go further and only discusses the side lengths of an isosceles triangle. Choice A did not use the base angles theorem. She did not choose B because the question did not have anything to do with isosceles triangles, it refers to scalene triangles so it did not extend to new information. Mrs. Kim did not think C went any further with the base angles theorem information, so she would choose D. In her current classroom, she would teach the base angles theorem and probably relate that to a kite even though they have not covered kites in the curriculum yet. She would show them that they are related, and then bring it back up when they go over kites in the quadrilaterals section. She likes to spiral the curriculum and likes that some textbooks use this strategy to present material.

Knowledge of Instructional Techniques and Methods. Mrs. Kim was given a survey after completing the MKT-G assessment during the Fall Semester. She was given 10 points to distribute among four instructional methods or techniques and other techniques which she could explain on the survey, giving the most points to the instructional method or technique used most often. She was asked to distribute the points in regards to what she currently uses in her Geometry class, the professional development she has taken, and how she would teach Geometry in an ideal classroom situation. Table 28 shows how Mrs. Kim distributed the points.

Table 28.
Mrs. Kim's Distribution of Points among Instructional Methods and Techniques from Survey.

	Seen in Geometry Courses	Seen in Education Courses	Ideal Classroom
Investigations	1	0	3
Compass/Protractor	1	0	1
Computer Software	1	1	2
Manipulatives/Models	1	1	1
Other	6 (Direct Teach)	8 (None)	3 (Discussion)

According to self-reported data given in the survey, Mrs. Kim direct teaches 60% of the instructional time, 10% of the time uses manipulatives and models, 10% of the time uses computer software, 10% of the time uses compasses and protractors, and 10% of the time uses investigations in her current Geometry classroom. In professional development, 80% of the time she has not seen any of these methods, 10% of the time she has seen manipulative and models, and 10% of the time she has seen computer software. If Mrs. Kim had the ideal Geometry classroom, she would spend 30% of the time on investigations, 30% of the time discussing Geometry, 20% of the time using computer software, 10% of the time using manipulatives and models, and 10% of the time using compasses and protractors. During the interview, Mrs. Kim said that in her ideal Geometry classroom she would have them discover the material, but students do not really like that, but she prefers that method of instruction. She used to use Geometer's Sketchpad in her class, but this year, the Geometry team decided to not use Geometer's Sketchpad because there were more new teachers assigned to Geometry and with the new textbooks, it would be too much for them to prepare. Mrs. Kim would rather use Geometer's Sketchpad instead of a compass and protractor.

Sources of Knowledge. Throughout the interview with Mrs. Kim, she referred to her knowledge of student understanding from her experiences as a high school teacher. As far as material for teaching Geometry, she usually asks for help within her Geometry team or department. When she is embarrassed by her questions, she will consult the textbook or the internet before approaching another teacher. She bases all of her instructional decisions on the textbook, but she will bring in outside resources when using Geometer's Sketchpad. She does not tend to write any new curriculum, but uses textbooks and materials that have been developed by other people in her department. She has participated in the College Board AP workshop, and found that to be a useful resource. The main teaching technique that she uses from the AP workshop is the use of patty paper in her Geometry course. Mrs. Kim shared that after she completed student teaching, that she felt as though there was not enough instruction given to her regarding relationship skills and communications skills. She thinks that teachers have a harder time with teaching Geometry than those teaching Algebra because Geometry uses a lot of logical thinking and whenever you write anything in Geometry, you have to make sure that there is a theorem or property that supports whatever you are trying to show. Mrs. Kim thinks that Geometry is very important for people to understand because it teaches you how to think logically and that can be transferred to other situations in peoples' lives.

Summary of Mrs. Kim. Mrs. Kim was high in the Specialized Geometry Knowledge and the Knowledge of Geometry and Teaching domain when compared to the other high school teachers. Her lowest domain was Knowledge of Geometry and Students, but she was in the lower half of the high school teachers in the Geometry Content Knowledge domain. Her self-evaluation of her performance was statistically significant and she had slight agreement. Mrs. Kim's Geometry Content Knowledge (GCK) was stronger than her score on in the GCK domain would seem to imply. She gave a two-column proof, of the properties of a midsegment of a triangle, and she was able to provide it without much help. Mrs. Kim was correct during the interview on the Specialized Geometry Knowledge question. She would prefer to have the students write the angle $m\angle 1 + m\angle 2 = 90°$. She stresses that the students use the "m" for measure, but she does not stress using the degree symbol. The interview question addressing Knowledge of Geometry and Students, Mrs. Kim was incorrect. She chose 11 diagonals, and she thought that the students do not fully understand the definition of a diagonal, so they assume the correct answer is 11. When asked the question from the MKT-G question addressing Knowledge of Geometry and Teaching, she was correct to choose the example of a kite, and is able to fully articulate reasoning behind not choosing the other options. From her knowledge of the order of the curriculum she did not think that students had addressed kites yet, so that would be new material. Mrs. Kim reported direct teaching the most in her current classroom. She has seen some computer software and manipulatives and models in professional development but answers none as well, and she would spend most of the time in her ideal classroom using investigations and discussion. Mrs. Kim uses her knowledge she has acquired from being a high school teacher. If she had confusion regarding the material she would go to a fellow teacher before she would do anything else. She is a part of a very supportive group of teachers, so if she has any problems, she has the ability to go to them for advice and help. She also found the AP Conference helpful in her teaching. When asked to reflect on her student teaching experience, she thinks that she did not have enough instruction over communication skills and relationship skills with students.

Case 6: Mrs. Abbott. Mrs. Abbott is a current high school teacher. She has been teaching mathematics and Geometry for 8 years. She does not have a degree in mathematics and has an alternative mathematics certification for grades 4-8 and 8-12. She prefers to teach elementary school and high school mathematics, specifically Pre-Algebra, Algebra 1, Geometry, Algebra 2, and Pre-Calculus. She currently teaches Pre-AP Geometry and on-level Geometry at a local high school. Mrs. Abbott was chosen to be interviewed and observed due to her having taught Geometry for 8 years. She is also the high school teacher that had Daniel as the student teacher for her class. She was observed three times.

Summary of Geometry Teaching Knowledge.

MKT-G Assessment Results. Mrs. Abbott took the MKT-G Assessment during the Fall semester of 2015. According to the MKT-G Assessment results supplied from the University of Michigan, Table 29 shows Mrs. Abbott's percentile within the high school teachers in each of the domains.

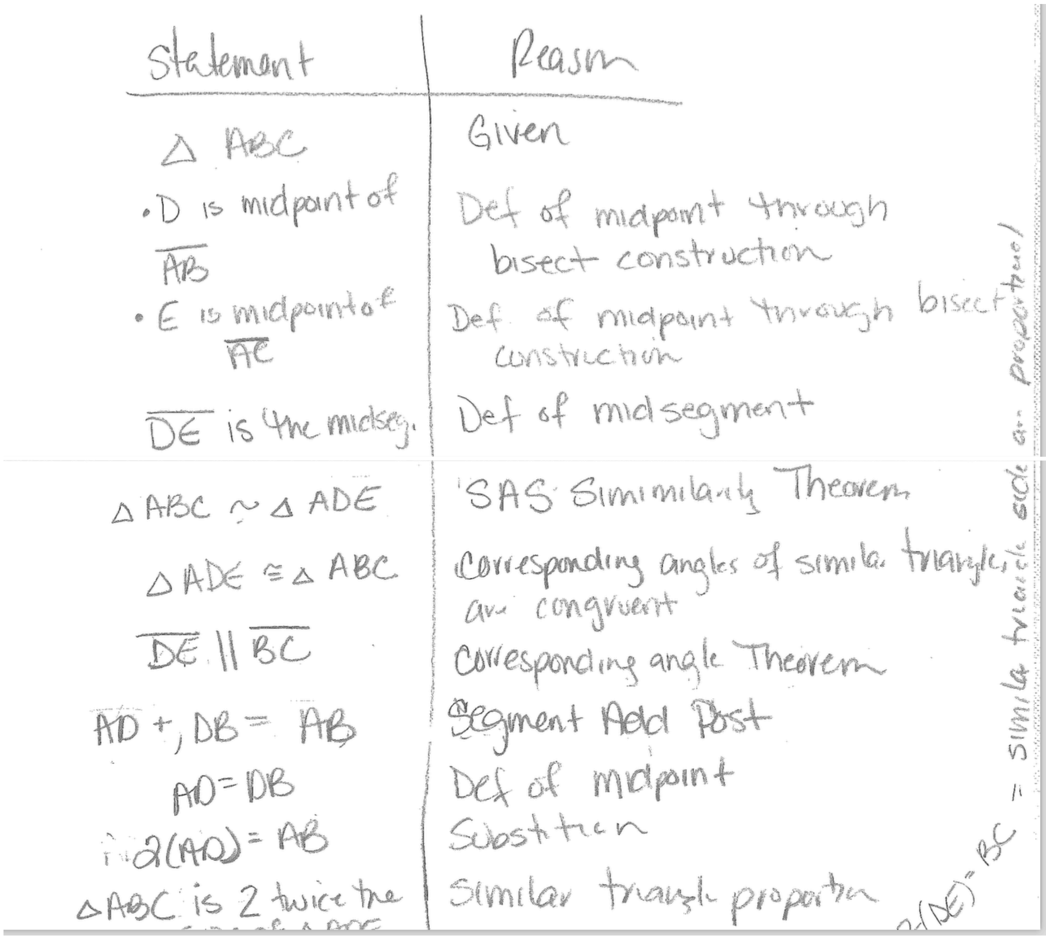
Table 29.
Mrs. Abbott's MKT-G Percentile within High School Teachers.

	MKT-G percentile within High School
GCK	69.44
SGK	26.39
KGS	18.06
KGT	12.50
Total	23.61

MKT-G Assessment Self-Evaluation. After Mrs. Abbott finished the MKT-G Assessment, she was given a survey. A portion of the survey asked her to rate how she thought she performed on the assessment by giving her a 1 to 5 Likert Scale. The Likert Scale questions only addressed seventeen mathematical concepts that were covered in the assessment. In order to compare how Mrs. Abbott thought she performed on the assessment and how she actually performed, I conducted a consistency estimate using Interrater reliability through SPSS. The results of interrater analysis were Pearson's R=.789 with p<.001 and Kappa=0.329 with p<0.001, while statistically significant, there is fair agreement between Mrs. Abbott's self evaluation and how she actually performed on the assessment.

Geometry Content Knowledge. During the interview, Mrs. Abbott was asked to investigate the midsegment of a triangle and then prove that the midsegment of a triangle was half the length and parallel to the third side of a triangle. The investigation started with having Mrs. Abbott draw and arbitrary triangle ABC and find the midpoints of two sides of the triangle, $\overline{AB}$ and $\overline{AC}$, using a compass and a straight edge (protractor) and label those midpoints D and E. She drew the triangle and did not have any trouble finding the midpoints of the two sides using a compass. She stated that $\overline{DE}$ and $\overline{BC}$ were parallel and that $\overline{DE}$ is half of $\overline{BC}$. When asked about $\angle ABC$ and $\angle ADE$, Mrs. Abbott wrote that they are corresponding angles and are congruent, so $\overline{DE}$ is parallel to $\overline{BC}$. She was then asked to prove the conjecture that the midsegment connecting two sides of a triangle is parallel to the third side and half as long. She was given the choice to write a paragraph proof or a two-column proof. She decided she would rather write a two-column proof.

She had a little bit of trouble starting the proof because she was not sure how to set up the situation. She tried to start with $\overline{DE}$ being a midsegment, but then decided to construct the midsegment in the proof. She then wanted to be able to use a ruler to show how to construct the midpoint of the segment, but once I reminded her that this is for an arbitrary triangle, she just wrote that D is a midpoint and E is a midpoint, so $\overline{DE}$ is a midsegment. From here she immediately went to the triangles being similar by Side-Angle-Side Similarity Theorem, but did not show the ratios of the sides and that $\angle A \cong \angle A$. From showing the triangles are similar, she proved that $\overline{DE}$ is parallel to $\overline{BC}$ because of corresponding angle theorem, but she did not refer to it as the converse. As she worked through the proof, she kept trying to talk about the exact measurements of the triangle she drew at the beginning of the investigation. After some guiding questions, she set up the segment addition postulate and then showed that $\overline{DE}$ is half of $\overline{BC}$. Below is the proof she wrote during the interview:



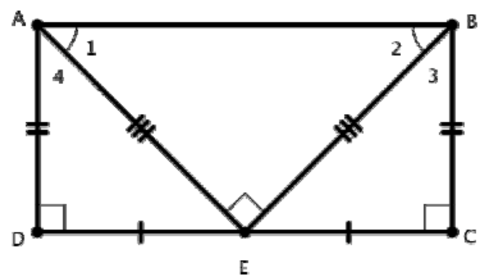
Statement	Reason
△ ABC	Given
• D is midpoint of $\overline{AB}$	Def of midpoint through bisect construction
• E is midpoint of $\overline{AC}$	Def. of midpoint through bisect construction
$\overline{DE}$ is the midseg.	Def of midsegment
△ ABC ~ △ ADE	SAS Similarity Theorem
△ ADE ≅ △ ABC	Corresponding angles of similar triangles are congruent
$\overline{DE} \parallel \overline{BC}$	Corresponding angle Theorem
AD + DB = AB	Segment Add Post
AD = DB	Def of midpoint
2(AD) = AB	Substitution
△ABC is 2 twice the	Similar triangle proportion

(Similar triangle side are proportional) = 2(DE) = BC

Figure 46. Mrs. Abbott's Midsegment Proof

<u>Specialized Geometry Knowledge</u>. During the interview, Mrs. Abbott was asked to recall the assessment she took during the Fall semester. She was asked a question directly from the MKT-G assessment that addressed the Specialized Geometry Knowledge domain. The following is the question asked in the MKT-G assessment and the interview:

While proving a claim on the board about the figure below, Joe wrote "1 + 2 = 90." Ms. Staples ponders how to correct that statement. Of the following, what is the best alternative?



A	Do nothing. The statement is correct as is.
B	Add the degree symbol so that the statement reads 1 + 2 = 90°.
C	Replace what Joe wrote; write instead that "m∠A + m∠B = 90°".
D	Replace what Joe wrote; write instead that "m∠EAB + m∠EBA = 90°".

Figure 47. SGK Question (Herbst & Kosko, 2014)

She was asked to recall what she answered on the MKT-G assessment, and she recalled answering D. She did not choose A because what the student did on the board was not correct. She thought that B should be addressed in the answer, but the degree symbol was not the only concern that she had with regard 1+2=90. She did not choose C because "it couldn't say measure A because $\angle 4$ could also be considered $\angle A$, and $\angle 3$ could be considered $\angle B$, and so therefore, you need to be very specific and use a three letter angle". She has her students write angles in many different ways, but she makes sure they know how to write angles using three points. If a student were to make this mistake on an exam, Mrs. Abbott would count this problem wrong because she would count it wrong on their homework leading up to the exam. She is more lenient with the letter "m" for measure and the degree symbol. She then modifies her response to say that a Pre-AP student would get this problem wrong, but a regular (on-level) student would probably not get this problem wrong on an exam.

Knowledge of Geometry and Students. During the interview, Mrs. Abbott was asked to recall a question directly from the MKT-G assessment that addressed the Knowledge of Geometry and Students domain. The following is the question asked in the MKT-G assessment and the interview:

Ms. Jamison asked students to figure out the number of diagonals in an 11-sided polygon. Of the following erroneous solutions, which one might have resulted from using the number of diagonals through one vertex?

A	11
B	72
C	88
D	99
E	121

Figure 48. KGS Question (Herbst & Kosko, 2014)

Mrs. Abbott at first recalled answering D. 121, but then changed her answer to A.11. When questioned about why she chose A, she stated, “Because kids tend to, like, oh it’s 11 sides, it’s going to be 11 lines, they don’t think.” She then explained that students don’t think about it being two less than 11. Students think that they have 11 points to draw 11 diagonals to, so they will just answer 11. She did not give any other reasoning for why a student would answer D, like she did initially, or any reasoning as to why they would answer any of the other choices.

Knowledge of Geometry and Teaching. During the interview, Mrs. Abbott was asked a question directly from the MKT-G assessment that addressed the Knowledge of

Geometry and Teaching domain. The following is the question asked in the MKT-G assessment and the interview:

After teaching her students the base angles theorem (which states that base angles of an isosceles triangle are congruent), Ms. Wellington is pondering what example she could give to illustrate how this theorem can help find new information. Which of the following would be the best example?

A	If you have a triangle that you know is isosceles and you know two of the sides are 8 and 10, then the third side will be 8 or 10.
B	If you have a triangle that you know the sides have length 8, 10, and 12 you know it does not have two angles that are congruent.
C	If you have an isosceles triangle and two of its angles are 30 you can calculate the third angle by subtracting 60 from 180.
D	If you have a kite and draw its minor diagonal, the kite is divided into two isosceles triangles and those have two congruent angles each.

Figure 49. KGT Question (Herbst & Kosko, 2014)

Mrs. Abbott initially recalled that she chose answer A. She did not choose any of the ones that talked about angles because that was not pulling any new information from the base angles theorem. Since C and D referred to angles, she excluded those from her choices. She reasoned that she did not choose B because the problem was not addressing isosceles triangles, but scalene triangles, so that would confuse the students. She decided that she would use A because the problem is referring to isosceles triangles. After questioning her about why she excluded answer B, she changed her mind and chose B. She then decided that she would do both A and B in her classroom, but B would be a better because it is having the students think about if they have an isosceles triangle in that situation. She elaborated why she decided C would not be a good choice because she focused on the relationship between angle measure and side length information when going over base angles theorem. For answer choice D, she would not choose to use that example because the students have not covered kites yet and they do not really have the knowledge regarding diagonals and how they relate to each other. She would bring back the discussion of base angles theorem when covering kites in the future, but she would not go into kites right after introducing the base angles theorem. When asked why she thought Geometry was structured that way, she said that the curriculum says that they should not cover quadrilaterals until after triangles and that she thinks students should know all about triangles before going into quadrilaterals. She thinks that students should be comfortable with triangles so they can use them to build quadrilaterals and then they can relate other shapes back to the triangle.

Knowledge of Instructional Techniques and Methods. Mrs. Abbott was given a survey after completing the MKT-G assessment during the Fall Semester. She was given 10 points to distribute among four instructional methods or techniques and other techniques which she could explain on the survey, giving the most points to the instructional method or technique used most often. She was asked to distribute the points in regards to what she currently uses in her Geometry class, the professional development she has taken, and how she would teach Geometry in an ideal classroom situation. Table 30 shows how Mrs. Abbott distributed the points.

Table 30.
Mrs. Abbott's Distribution of Points among Instructional Methods and Techniques from Survey.

	Seen in Geometry Courses	Seen in Education Courses	Ideal Classroom
Investigations	2	2	3
Compass/Protractor	2	0	2
Computer Software	1	0	2
Manipulatives/Models	2	4	2
Other	3 (Group Activities)	4 (Activities)	1 (Lecture)

According to self-reported data given in the survey, Mrs. Abbott has group activities 30% of the instructional time, manipulatives and models 20% of the time, investigations 20% of the time, compass and protractor activities 20% of the time, and computer software 10% of the time in her current Geometry class. In her professional development opportunities, she has seen activities 40% of the time, manipulatives and models 40% of the time, and investigations 20% of the time. In Mrs. Abbott's ideal classroom, she would spend 30% of the time on investigations, 20% of the time on compass and protractor activities, 20% of the time using computer software, 20% of the time using manipulatives and models, and 10% of the time lecturing.

Sources of Knowledge. Throughout the interview with Mrs. Abbott, she referred to her knowledge of student understanding from her experiences as a high school teacher. As far as Geometry topics, Mrs. Abbott uses her textbook and classroom activities from previous years to inform and improve her Geometry lessons. Since she has taught Geometry for 8 years, she does not have many issues with the material, and she does not depend on other teachers' knowledge much. When she first started teaching, she was at a discipline center and the only math teacher, so she had to figure many things out on her own. She uses TeachersPayTeachers.com, a website that teachers can contribute lesson plans, handouts, and materials they have made for payment, for many ideas and handouts for her lessons. Mrs. Abbott has experience with the Langford Training through her professional development, where students are able to create the formulas they are using that day and the student is in control of their learning. Since she has experience with this, she brings the knowledge of that method to her ideal classroom and she tries to bring pieces of that into her current Geometry classroom. Mrs. Abbott has also had experience with three different student teachers. She has a unique viewpoint on what the students' strengths and weaknesses are when they come into the high school classroom from the university. The student teachers come in with a large amount of content knowledge, but they have a difficult time teaching the lower level mathematics. Her student teachers have also had a hard time with interactions with students and disciplinary concerns. In her opinion, the student teachers should have more experience in the classroom and take more education courses that discuss ways to interact with students and how to organize a classroom. She thinks that the student teachers should be required to observe many other subjects so they can see strategies that other teachers might use that can be transferred to the mathematics classroom.

Summary of Mrs. Abbott. Mrs. Abbott was high in the Geometry Content Knowledge domain when compared to the other high school teachers. Her lowest domain was Knowledge of Geometry and Teaching, but she was in the lower half of the high school teachers in the Specialized Geometry Knowledge and Knowledge of Geometry and Students domain. Her self-evaluation of her performance was statistically significant and she had fair agreement. Mrs. Abbott's Geometry Content Knowledge (GCK) was weaker than her score on in the CCK domain would seem to imply. She gave a two-column proof, of the properties of a midsegment of a triangle, and she needed help multiple times. She had a difficult time proving the properties for an arbitrary triangle. She could not come up with using the converse of the parallel lines theorem to show the midsegment was parallel to the third side. She completed the proof but skipped some steps. Mrs. Abbott was correct during the interview on the Specialized Geometry

Knowledge question. She would count this kind of a problem wrong on an exam and homework, but she does not stress using the degree symbol. On the interview question addressing Knowledge of Geometry and Students, Mrs. Abbott was incorrect. She chose 11 diagonals, and she thinks that the students do not fully understand the definition of a diagonal, so they assume the correct answer is 11. When asked the question from the MKT-G question addressing Knowledge of Geometry and Teaching, she was incorrect. She would not have chosen the kite as an extension of the theorem because students have not learned about quadrilaterals. From her knowledge of the order of the curriculum she did not think that students had addressed kites yet, so that would be new material that would be too overwhelming for them. Mrs. Abbott reported group activities the most in her current classroom. She has seen some manipulatives and models, investigations, and activities in professional development, and she would spend most of the time in her ideal classroom using investigations. Mrs. Abbott uses her knowledge she has acquired from being a high school teacher. She claims that since she has been teaching Geometry for 8 years, she does not have any confusion, but if she had confusion regarding the material she would go to the textbook before she would do anything else. When asked to reflect on her student teaching experience and the student teachers that she has mentored, she thinks that student teachers do not have enough instruction over interactions and discipline problems. Student teachers also have a hard time teaching lower level mathematics because they are so advanced in mathematics.

Case 7: Mrs. Lane. Mrs. Lane is a current high school teacher. She has been teaching mathematics for 11 years and Geometry for 10 years. She has Bachelors and Masters degrees in Mathematics and a mathematics teaching certification for grades 8-12. She prefers to teach high school mathematics, specifically Geometry, Algebra 2, and Statistics. She currently teaches Pre-AP Geometry at a local high school. She was selected to be interviewed and observed because she has taught Geometry for 10 years. She was observed three times.

Summary of Geometry Teaching Knowledge.

MKT-G Assessment Results. Mrs. Lane took the MKT-G Assessment during the Fall semester of 2015. According to the MKT-G Assessment results supplied from the University of Michigan, Table 31 shows Mrs. Lane's percentile within the high school teachers in each of the domains.

Table 31.
Mrs. Lane's MKT-G Percentile within High School Teachers.

	MKT-G percentile within High School
GCK	69.44
SGK	48.61
KGS	18.06
KGT	80.56
Total	56.94

MKT-G Assessment Self-Evaluation. After Mrs. Lane finished the MKT-G Assessment, she was given a survey. A portion of the survey asked her to rate how she thought she performed on the assessment by giving her a 1 to 5 Likert Scale. The Likert Scale questions only addressed seventeen mathematical concepts that were covered in the assessment. In order to compare how Mrs. Lane thought she performed on the assessment and how she actually performed, I conducted a consistency estimate using Interrater reliability through SPSS. The results of interrater analysis were Pearson's R=.958 with p<.001 and Kappa=0.388 with p<0.001, while statistically significant, there is fair agreement between Mrs. Lane's self evaluation and how she actually performed on the assessment.

Geometry Content Knowledge. During the interview, Mrs. Lane was asked to investigate the midsegment of a triangle and then prove that the midsegment of a triangle was half the length and parallel to the third side of a triangle. The investigation started with having Mrs. Lane draw an arbitrary triangle ABC and find the midpoints of two sides of the triangle, $\overline{AB}$ and $\overline{AC}$, using a compass and a straight edge (protractor) and label those midpoints D and E. She did not have any trouble finding the midpoint of the segments using a compass, and was able to construct the midsegment. She was aware of the properties of a midsegment, and was able to recall this information after she completed the construction. In order to show that $\overline{DE}$ is half of $\overline{BC}$, she used a compass. She then measured $\angle ABC$ and $\angle ADE$ with the protractor. She saw that the angles were congruent "which means these two angles are corresponding angles, which means these two lines are parallel", and she referred to this as the converse. She was then prompted to write the proof of the conjecture that a midsegment connecting two sides of a triangle is parallel to the third side and is half as long, and she was given the option of a paragraph proof or a two-column proof. She chose to write a two-column proof.

When Mrs. Lane began, she stated that she wanted to prove this for any triangle, but then asked if a coordinate proof was wanted. She thought of a coordinate proof because that is what they had been doing in the Geometry class she teaches. Below is a picture of the proof she provided during the interview:

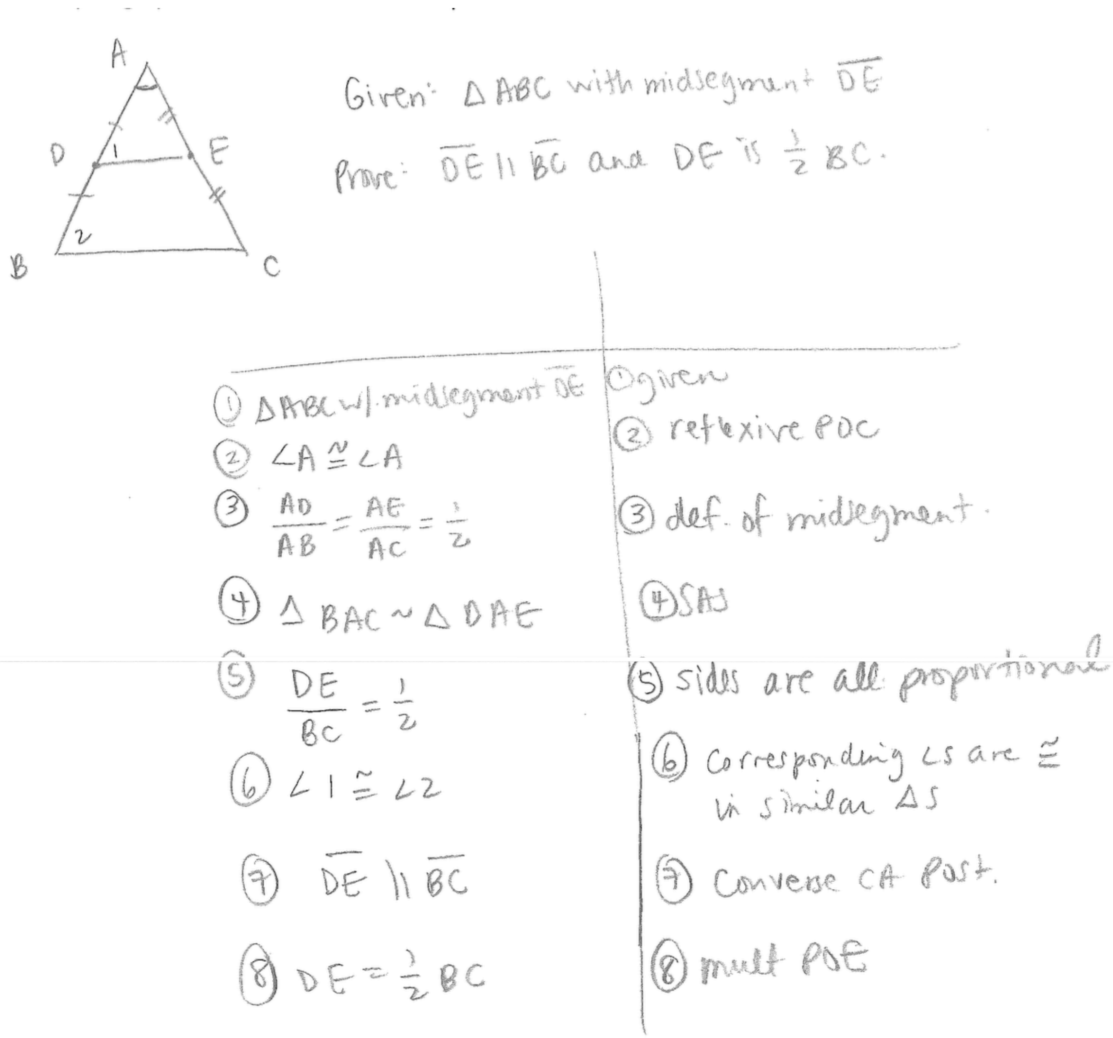
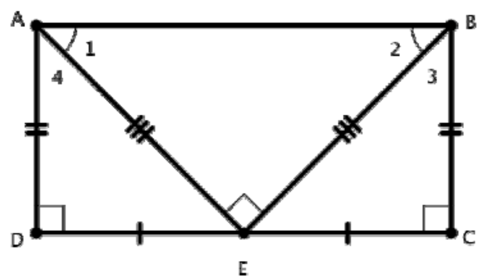


Figure 50. Mrs. Lane's Midsegment Proof

Once she drew an arbitrary triangle, she knew that she needed to prove that the triangles were similar using Side-Angle-Side Theorem. She then went on to show that the sides were proportional. She did not give the steps to show what the proportion was, or why it was one half, but she used that the proportion was one half to show that $\overline{DE}$ is half of $\overline{BC}$. She then proved that $\overline{DE}$ was parallel to $\overline{BC}$ through the converse of the corresponding angles postulate. Her reason for step 8, $\overline{DE} = \frac{1}{2}\overline{BC}$ was mult POE, which stands for multiple parts of equality and congruence. After she completed the proof, she said that she could have written it better, but she believed what she wrote is correct.

Specialized Geometry Knowledge. During the interview, Mrs. Lane was asked to recall the assessment she took during the Fall semester. She was asked a question directly from the MKT-G assessment that addressed the Specialized Geometry Knowledge domain. The following is the question asked in the MKT-G assessment and the interview:

While proving a claim on the board about the figure below, Joe wrote "1 + 2 = 90." Ms. Staples ponders how to correct that statement. Of the following, what is the best alternative?



A	Do nothing. The statement is correct as is.
B	Add the degree symbol so that the statement reads 1 + 2 = 90°.
C	Replace what Joe wrote; write instead that "m∠A + m∠B = 90°".
D	Replace what Joe wrote; write instead that "m∠EAB + m∠EBA = 90°".

Figure 51. SGK Question (Herbst & Kosko, 2014)

She was asked to recall what she answered on the MKT-G assessment, and she recalled answering D. She thought that Geometry was a really specific field of mathematics that required students to be very precise in the notation that they use. She understood what the student is trying to say when they write 1+2=90, but that is incorrect because 1+2=3. Mrs. Lane needed the student to state that they were talking about the measure of the angle, so that would eliminate choice A and B. Choice C would not be correct because $\angle A$ would not be specific enough. $\angle A$ could represent three different angles. That left her with choice D, which was the more specific, even though Mrs. Lane would accept $m\angle 1 + m\angle 2 = 90°$. If a student went up to the board and wrote 1+2=90, Mrs. Lane would try to have the student explain what they meant by that because it looks like the student is saying 3=90. On an exam, she would take off for a student writing 1+2=90 because she understands what they are meaning, but a teacher cannot assume anything about what a student is writing on an exam. She is not as picky about students using the degree symbol, but it is necessary for them to use the measure of an angle symbol.

Knowledge of Geometry and Students. During the interview, Mrs. Lane was asked to recall a question directly from the MKT-G assessment that addressed the Knowledge of Geometry and Students domain. The following is the question asked in the MKT-G assessment and the interview:

Ms. Jamison asked students to figure out the number of diagonals in an 11-sided polygon. Of the following erroneous solutions, which one might have resulted from using the number of diagonals through one vertex?

A 11

B 72

C 88

D 99

E 121

Figure 52. KGS Question (Herbst & Kosko, 2014)

Mrs. Lane did not remember what she answered when she took the assessment. She thought she answered A because if a student misunderstood what a diagonal was, they would come up with 11 diagonals since it is an 11-sided polygon. She said that if there were not any choices she would have put down 9 diagonals because the students would just count the number of diagonals from one vertex. She talked about, in the past, students had been asked to draw all the diagonals of a pentagon and count how many there were, but they usually ended up double counting all of them, which was where a student might get answer E. She then discussed how the students might get C and D by multiplying 8 and 9 by 11, and then choice B by multiplying 8 and 9, but she was unsure about which answer was correct. She did not like the way the question was worded.

Knowledge of Geometry and Teaching. During the interview, Mrs. Lane was asked a question directly from the MKT-G assessment that address the Knowledge of Geometry and Teaching domain. The following is the question asked in the MKT-G assessment and the interview:

After teaching her students the base angles theorem (which states that base angles of an isosceles triangle are congruent), Ms. Wellington is pondering what example she could give to illustrate how this theorem can help find new information. Which of the following would be the best example?

A	If you have a triangle that you know is isosceles and you know two of the sides are 8 and 10, then the third side will be 8 or 10.
B	If you have a triangle that you know the sides have length 8, 10, and 12 you know it does not have two angles that are congruent.
C	If you have an isosceles triangle and two of its angles are 30 you can calculate the third angle by subtracting 60 from 180.
D	If you have a kite and draw its minor diagonal, the kite is divided into two isosceles triangles and those have two congruent angles each.

Figure 53. KGT Question (Herbst & Kosko, 2014)

Mrs. Lane recalls answering D when she took the assessment. She did not choose A because that problem is talking about an isosceles triangle, but it is not addressing the angles of an isosceles triangle which is the emphasis on the base angles theorem. She did not choose B because it is not even dealing with an isosceles triangle and if your emphasis is on isosceles triangle base angles, this questions does not have anything to do with the material being covered in the lesson. Mrs. Lane thought that answer choice C was incorrect because it does at least refer to angle measures, but it does not relate the side lengths to the angle measures that would connect to the base angles theorem. She thinks that D is a nice application of the theorem into a quadrilateral. When asked about the ordering of the curriculum that she was currently teaching, she explained that quadrilaterals were covered after the base angles theorem, but she would jump right into the kite problem after teaching the base angles theorem because it would be a good way to introduce quadrilaterals.

Knowledge of Instructional Techniques and Methods. Mrs. Lane was given a survey after completing the MKT-G assessment during the Fall Semester. She was given 10 points to distribute among four instructional methods or techniques and other techniques which she could explain on the survey, giving the most points to the instructional method or technique used most often. She was asked to distribute the points in regards to what she currently uses in her Geometry class, the professional development she has taken, and how she would teach Geometry in an ideal classroom situation. Table 32 shows how Mrs. Lane distributed the points.

Table 32.
Mrs. Lane's Distribution of Points among Instructional Methods and Techniques from Survey.

	Seen in Geometry Courses	Seen in Education Courses	Ideal Classroom
Investigations	5	6	3
Compass/Protractor	2	0	1
Computer Software	1	0	4
Manipulatives/Models	0	2	2
Other	2 (Lecture)	2 (Discussion/Lecture)	0

According to self-reported data given in the survey, Mrs. Lane has investigations 50% of the instructional time, lecture 20% of the time, compass and protractor activities 20% of the time, and computer software 10% of the time in her current Geometry class. In her professional development opportunities, she has seen investigations 60% of the time, manipulatives and models 20% of the time, and discussions and lecture 20% of the time. In Mrs. Lane's ideal classroom, she would spend 30% of the time on investigations, 10% of the time on compass and protractor activities, 40% of the time using computer software, and 20% of the time using manipulatives and models. She has also become a teacher of a dual credit online statistics course, and this has made her change the format of her Geometry class. This year she started having the students look up vocabulary before they come to class so she can focus on group work and problems in class. Having students work before class has given her more time to have the students fully participate in investigations. She hopes that the school district will be able to provide Geometer's Sketchpad to her students in the future so she can use the technology to better facilitate more advanced investigations. She expressed desire for more manipulatives and models for her classroom, but money is a concern.

Sources of Knowledge. Throughout the interview with Mrs. Lane, she referred to her knowledge of student understanding from her experiences as a high school teacher. As far as the Geometry content, Mrs. Lane has been teaching Geometry for 10 years, so she has many of the topics mastered, but she recalls when she first started teaching, she would go to the other teachers around her for help with the material. At the school she is currently teaching Geometry, she has a team of Geometry teachers and the district supports their collaboration. The district is so supportive that they pay for substitutes for the team to have a workday to collaborate for the coming semester and they have 30-minute meeting once a week to meet and discuss the upcoming lessons. Since she has been teaching at this school for 11 years, she has a network of teachers that she can depend on and collaborate with in order to make the Geometry course successful. If the other teachers were not helpful, she would consult the textbook and then she would look it up online. She has also been to AP workshops and SXSWEdu over Geometry and has found this helpful. She has not had much experience with having student teachers in her classroom, but she does help out with the teacher training program of a local university. She thinks that some of the pre-service teachers have an unrealistic idea of how much a student can retain in one lesson and how much material can be covered in a short amount of time during class. When she thinks back to her student teaching experience, she wishes she would have had more instruction over classroom management.

Summary of Mrs. Lane. Mrs. Lane has been teaching mathematics for 11 years and has taught Geometry for 10 years. She currently teaches Pre-AP Geometry. She was high in the Knowledge of Geometry and Teaching and Geometry Content Knowledge when compared to the other high school teachers. Her lowest domain was Knowledge of

Geometry and Students, but she was in the lower half of the high school teachers in the Specialized Geometry Knowledge domain. Her self-evaluation of her performance was statistically significant and she had fair agreement. Mrs. Lane's Geometry Content Knowledge (GCK) was strong. She gave a two-column proof, of the properties of a midsegment of a triangle, and she was able to provide it without much help. Mrs. Lane was correct during the interview on the Specialized Geometry Knowledge question. She would prefer to have the students write the angle $m\angle 1 + m\angle 2 = 90°$. She stresses that the students use the "m" for measure, but she does not stress using the degree symbol. In the interview question addressing Knowledge of Geometry and Students, Mrs. Lane was incorrect. She chose 11 diagonals, and she thought that the students did not fully understand the definition of a diagonal, so they assumed the correct answer was 11. When asked the question from the MKT-G question addressing Knowledge of Geometry and Teaching, she was correct to choose the example of a kite, and was able to fully articulate reasoning behind not choosing the other options. From her knowledge of the order of the curriculum she did not think that students had addressed kites yet, so that would be new material. Mrs. Lane reported investigations the most in her current classroom. She has seen some investigations and manipulatives and models in professional development, and she would spend most of the time in her ideal classroom using computer software. Mrs. Lane uses her knowledge she has acquired from being a high school teacher. If she had confusion regarding the material she would go to a fellow teacher before she would do anything else. She is a part of a very supportive group of teachers, so if she has any problems, she has the ability to go to them for advice and help. If the other teachers were not helpful, she would consult the textbook and then the internet. When asked to reflect student teaching experiences, she thought that student teachers had an unrealistic idea of how much a student could retain during one lesson and how much a student could cover in one class time. When she thought back to her own student teaching experience, she wished she would have had more instruction over classroom management.

Comparison of Interviews

Throughout the interviews with the seven participants, key differences began to become apparent. Based on the data from the MKT-G, pre-service teachers did not perform as well as high school teachers, but during the assessment this became more obvious. When the pre-service teachers were asked to prove the properties of a midsegment, they tended to have a hard time with believing that the midsegment was parallel to the third side and half the length, while the high school teachers were familiar with the properties. The pre-service teachers tried to give proofs of these properties, but needed more help than the high school teachers. This may be because the high school teachers are more familiar with the properties, but even when given the properties and asked to prove them, the pre-service teachers had a hard time. All of the participants were able to reason through the Specialized Geometry Knowledge question, and they all seemed to understand what the student was trying to communicate. The question addressing the diagonals of an 11-sided polygon (Knowledge of Geometry and Students problem), gave all of the participants difficulty. All of the participants answered incorrectly, and did not change their answer even after discussion of the erroneous solutions. The choice of 11 diagonals stemmed from the participants assuming that the student answering did not understand the definition of a diagonal, or were just answering

11 because of the 11-sided polygon. In the Knowledge of Geometry and Teaching, some of the participants had difficulty, but others were able to correctly answer the problem. It is unclear what may be the cause of this, and more questions in this domain may be needed to be able to truly compare the seven participants, however all of the participants were aware that triangles are generally taught before quadrilaterals.

The knowledge that the participants were drawing from to answer these questions primarily came from their interactions with students, whether that be through tutoring in the pre-service teachers or through being in the classroom in the high school teachers. It is hard to tell where their knowledge of instructional techniques and methods comes from because all of their reported time seeing or using the different types of techniques and methods do not correspond. If the participants encountered any difficulty with the material or with presenting material, they would use different resources. The pre-service teachers would rely on their textbook primarily, while the high school teachers would go to a colleague for help. Pre-service teachers seemed reluctant to go to other teachers for help first, but would eventually go to other teachers for help if the textbook did not help them. Pre-service teachers would also rely on the internet for lesson and presentation ideas, and high school teachers would rely on the previous years of notes, textbooks, and then would go to the internet.

CHAPTER 5

DISCUSSION

Introduction

To investigate the Geometry Teaching Knowledge of pre-service and high school teachers, a mixed methods study was designed to focus on pre-service teachers from the university and high school teachers from central Texas. During the course of one semester, pre-service and high school teachers were given an assessment and surveyed regarding their Geometry Teaching Knowledge. A selection of these participants was interviewed in order to understand where Geometry Teaching Knowledge is developed. Those participants were also observed in order to verify that their survey responses and interview response were accurate. Geometry Teaching Knowledge is comprised of four domains: Geometry Content Knowledge, Specialized Geometry Knowledge, Knowledge of Geometry and Students, Knowledge of Geometry and Teaching, and knowledge of instructional techniques and methods used in the Geometry classroom. The MKT-G and survey results were used to see what pre-service and high school teachers know, and then the results were used to compare the differences between the two groups. This chapter summarizes the results of the study and positions the findings within the body of research in the field. Implications of the findings, limitations of the study, and future research will be discussed.

Summary of Findings

MKT-G Assessment. Results from this study's MKT-G assessment were compared to the results presented by Herbst and Kosko (2014). The correlations between the four domains, Geometry Content Knowledge (GCK), Specialized Geometry Knowledge

(SGK), Knowledge of Geometry and Students (KGS), and Knowledge of Geometry and Teaching (KGT) were not exactly the same as the original paper, but they were very similar. Due to the similarity of the correlations between the domains of this study as a whole, the data regarding the MKT-G results are comparable to the findings from Herbst and Kosko (2014). Correlations between each domain, pre-service or high school teacher, the number of years teaching mathematics, and the number of years teaching geometry were also significant. This result of increased scores based on the number of years teaching Geometry is mirrored in the Herbst and Kosko (2014) paper.

Through t-test analysis, pre-service teachers had significantly lower scores in each of the domains; Geometry Content Knowledge (GCK), Specialized Geometry Knowledge (SGK), Knowledge of Geometry and Students (KGS), and Knowledge of Geometry and Teaching (KGT), and the total scores. The primary domains in which pre-service and high school teachers had the biggest difference were SGK and KGT. All of the domains and total scores had significant effect sizes as well, with very large effect sizes in the SGK domain, KGT domain, and the total scores. From the interviews and observations, one reason for this difference may be due to the pre-service teachers only having experience with Common Geometry Knowledge (CGK) through their Geometry courses and possible experiences with Knowledge of Geometry and Students (KGS) through tutoring or being a student themselves in a Geometry course.

Knowledge of Instructional Techniques and Methods. The Post-Assessment Survey contained questions regarding the knowledge of instructional techniques and methods. Participants were asked to distribute 10 points to 5 different instructional methods; investigations, compass and protractor, computer software, manipulatives and models, and an “other” category that asked them to describe what they considered other. Pre-service teachers were asked what they have seen in their Geometry courses, what they have seen in their Education courses, and what they would use in their ideal classroom. High school teachers were asked what they use in their current classroom, what they have seen in their professional development, and what they would use in the ideal classroom. Chi-squared tests for independence were run in order to compare the responses.

When comparing pre-service and high school teachers’ ideal Geometry courses using a chi-squared test, the responses to these questions were statistically different. Pre-service teachers thought that more compass and protractor activities and manipulatives and models were important to their ideal classes when compared to the high school teachers. The high school teachers thought more investigations and computer software would be important to their ideal classrooms, as well as a larger portion dedicated to other, which most of the high school teachers described as lecture or direct teach. A possible reason for the differences could be due to pre-service teachers not having the opportunity to be in the high school classroom, they are unaware of what can and cannot be done with the students. During the interview with Mrs. Lane, she noticed that pre-service teachers have a difficult time with knowing how much a student can do in a limited amount of time and how much material can be covered in one class period.

When comparing pre-service teachers’ experiences in their education courses and their geometry courses using a chi-squared test, these responses were statistically different. They reported to have seen more compass and protractor activities and manipulatives and models in their geometry courses and more investigations and other in their education courses. They reported other as including lesson plans, PowerPoints, projects, and lecture. They reported seeing computer software approximately the same amount of time in geometry and education courses. One possible reason for this difference could be due to the requirement that the geometry courses the pre-service teachers must take are content courses, not methods courses. The methods courses that pre-service teachers take are not usually specific to mathematics.

When comparing pre-service teachers' geometry courses and their ideal classroom using a chi-squared test, these responses were statistically different. They reported more compass and protractor activities in their geometry courses, but thought investigations and computer software were more important in their ideal classroom than the amount they saw these techniques and methods in their geometry courses. Manipulatives and models were given about the same weight for the ideal classroom and their geometry courses. One reason for this difference could be due to pre-service teachers seeing different methods in their geometry and education courses. They may be trying to combine what methods they have seen to make a successful geometry class.

High school teachers' reports of instructional techniques and methods used in their ideal geometry class and current geometry class were analyzed using chi-squared and were statistically different. They would use investigations and computer software more in their ideal classroom than they do in their current classroom. They reported to lecture more in their current geometry class than they would in the ideal class. They would use manipulatives and models and compass and protractor activities about the same amount in their ideal classroom as they do in their current classroom. One reason for the difference could be that teachers do not have unlimited budgets for the materials needed to use the instructional techniques and methods they would prefer to use in their geometry courses. High school teachers usually have to pay for any materials used in the classroom themselves.

High school teachers' reports of instructional techniques and methods used in their ideal geometry class and what they have seen in professional development were analyzed using chi-squared and were statistically different. They would use investigations and compass and protractor activities more in their ideal classroom than they have seen in their professional development. They would use computer software about the same amount in their ideal classroom as they see in their professional development. They see more manipulatives and models, teaching strategies, classroom management, project based instruction, and direct teach/lecture in professional development than they would use in their ideal classroom. During the interviews, when professional development was mentioned, the high school teachers tended to think their professional development was not helpful in their geometry classes. This may be due to professional development rarely targeting math courses, specifically geometry.

When comparing high school teachers' professional development and their current geometry classroom using a chi-squared test, these responses were statistically different. They reported more investigations, computer software, and manipulatives and models in their professional development, but use compass and protractor activities and lecture in their current geometry classroom than the amount they saw in their professional development. This difference could be due to there being a lack of geometry specific professional development for teachers. Also, professional development instructors may not be in the high school classroom and be aware of the methods required to teach with new textbooks or district curriculum.

Pre-service teachers' geometry and education courses, as a whole, were compared to the professional development opportunities for high school teachers using a chi-squared test and the responses are statistically different. Pre-service teachers have seen more compass and protractor activities and manipulatives and models in their geometry and education courses, but high school teachers have seen more investigations, computer software, teaching strategies, classroom management, project based instruction, and direct teach/lecture in their professional development. This difference could be due to pre-service teacher geometry and education courses being taught by instructors who are not in contact with those teaching the professional development opportunities to high school teachers.

A final comparison was made between pre-service geometry and education courses, as a whole, and current high school geometry classrooms. A chi-squared test was conducted and the responses were statistically different. Pre-service teachers reported having more experience with compass and protractor activities and manipulatives and models, and high school teachers report spending more time on lecture than pre-service teachers have experienced. There is a disconnect between what pre-service teachers see and how the geometry class is actually conducted. A possible explanation for the differences could be that there is no communication between the instructors of the pre-service courses and the current teachers of high school mathematics.

Self-Assessment of Performance on MKT-G. Based on the kappa analysis performed in Chapter 4, there is not much of a statistically significant reliable relationship between the self-assessment over geometry topics in the survey and how the participants, both pre-service and high school teachers, actually performed on the MKT-G assessment. The few topics that were statistically significant, angle relations and altitudes of triangles in pre-service teachers, had very low Kappa values, so there is poor agreement between self-assessment and their actual performance. A possible reason for the poor agreement between the predicted performance and actual performance may be because pre-service teachers and high school teachers think that they know more than they actually do. The participants did not want to seem confused or uncertain about their answers when being interviewed, so this might be the same situation. Pre-service and high school teachers do not want to admit where they may be weak in their chosen profession.

Interviews of Selected Participants. Maria was a student teacher in the Spring semester of 2016. She has had tutoring experience and has passed Modern Geometry. Maria performed well on the MKT-G assessment. She was high in the Geometry Content Knowledge domain when compared to the other pre-service teachers. Her lowest domain was Knowledge of Geometry and Students. Her self-evaluation of her performance was statistically significant and she had fair agreement. Maria's Geometry Content Knowledge (GCK) was not as strong as her score on in the GCK domain would seem to imply. She had a difficult time formulating a proof of the properties of the midsegment of a triangle. She did not provide all the steps in the proof and it was not a paragraph proof or a two-column proof. She attempted a two-column proof, but it ended up being a list of steps. Maria was correct during the interview on the Specialized Geometry Knowledge question. She did have some confusion with regard to the correct way for labeling an angle. On the interview question addressing Knowledge of Geometry and Students, Maria was incorrect. She chose 11 diagonals, but did not give much reasoning as to why a student would choose 11 besides them thinking about an 11-sided polygon. She recalled giving up on this question when answering it during the MKT-G assessment. When asked the question from the MKT-G question addressing Knowledge of Geometry and Teaching, she was correct to choose the example of a kite. From her knowledge of the order of the curriculum she did not think that students had addressed kites yet, so that would be new material. Maria reported seeing Geometry software the most in her Geometry courses, seeing none of the techniques and methods in her education courses, and she would spend most of the time in her ideal classroom using computer software and manipulatives and models because she is a visual learner. Maria uses her knowledge she has acquired from tutoring. The resources Maria would use if she needed assistance would first be the textbook, then a cooperating teacher, and then she would turn to the internet.

Jason finished student teaching in the Fall of 2015. He was assigned to teach 8^{th} grade mathematics, has experience tutoring, and he has completed Modern Geometry. Jason did very well on the MKT-G. He did the best in the Knowledge of Geometry and Teaching portion of the exam. His worst domain was Geometry Content Knowledge. His self-evaluation of performance and his actual performance were statistically significant, but he had slight agreement. This implies that Jason’s self-evaluation was related to his actual performance, but he was only correct in his evaluation of himself a few times. Jason’s performance during the interview question addressing Geometry Content Knowledge was stronger than one would assume from his GCK score. At first he did not believe the conjecture was true for all triangles. He had a hard time starting his proof of the properties of a midsegment. He chose to write a paragraph proof, and he knew that he had similar triangles, but he was unable to prove similarity without help and he was having difficulty with proving for an arbitrary triangle. He also had difficulty proving that the midsegment was parallel to the third side, so he needed help on that portion of the proof because he was not able to use the converse of the parallel lines theorem. Jason was correct during the interview on the Specialized Geometry Knowledge question. He did have some doubt in what he might have answered on the MKT-G exam, but he was confident in his answer during the interview. He thought the students were being “nonchalant” about things and not being specific enough. On the interview question addressing Knowledge of Geometry and Students, Jason was incorrect. He chose 11 diagonals, but he did not know where the students would get any of the erroneous solutions. He is able to come up with the number 8, but does not connect that with the answer choice of 88. When asked the question from the MKT-G question addressing Knowledge of Geometry and Teaching, he was correct to choose the example of a kite. From his knowledge of the order of the curriculum he did not think that students had addressed kites yet, so that would be new material. Jason reported seeing compass and protractor activities the most in his Geometry courses, seeing none of the techniques and methods in his education courses, and he would spend most of the time in his ideal classroom using manipulatives and models and foldables so that students can come up with their own conjectures. Jason uses his knowledge he has acquired from student teaching. The resources Jason would use if he needed assistance would first be the textbook, then a cooperating teacher, and then he would turn to the internet.

Daniel finished student teaching in the Fall of 2015. He was assigned to teach Geometry, has experience tutoring, and he has completed Informal Geometry and Modern Geometry. Daniel was in the 69.61 percentile for his total score when compared to the other pre-service teachers. He did the best in the Knowledge of Geometry and Teaching portion of the exam. His worst domain was Geometry Content Knowledge and Knowledge of Geometry and Students. His self-evaluation of performance and his actual performance was not statistically significant, so there was not a reliable relationship between the two. Daniel's performance during the interview question addressing Geometry Content Knowledge (GCK) was weak which is what one would assume from his GCK score. At first he did not believe the conjecture was true for all triangles. He had a hard time starting his proof of the properties of a midsegment. He chose to write a paragraph proof, and he knew that he had similar triangles, but he was unable to prove similarity without help and he was having difficulty with proving for an arbitrary triangle. He also had difficulty proving that the midsegment was parallel to the third side, so he needed help on that portion of the proof because he was not able to use the converse of the parallel lines theorem. His paragraph proof did not end up being a complete proof, more of an outline of the proof of the properties of a midsegment. After completing his proof outline, he says that he would not be able to complete the proof without help. Daniel was correct during the interview on the Specialized Geometry Knowledge question. He does not think that an actual student would be able to answer D because using three points to name an angle is too confusing for students. He would have counted off on an exam if a student wrote an angle incorrectly prior to student teaching, but now that he is in the classroom, he would not count off for incorrect angle notation. On the interview question addressing Knowledge of Geometry and Students, Daniel was incorrect. He chose 11 diagonals, but he did not know where the students would get any of the erroneous solutions. He was asked this question again prior to being observed. He was being observed on a day that he was addressing the number of diagonals in polygons, but he still stuck with his answer of 11. He is able to come up with the number 8, but does not connect that with the answer choice of 88, even though he verbally walked through the process of finding the correct number of diagonals. When asked the question from the MKT-G question addressing Knowledge of Geometry and Teaching, he was incorrect. He did not think the kite extension was a good choice because students have not learned about quadrilaterals yet. From his knowledge of the order of the curriculum he did not think that students had addressed kites yet, so that would be new material. Daniel reported seeing lecture the most in his Geometry courses, seeing lesson plans and lecture in his education courses, and he would spend most of the time in his ideal classroom using investigations and computer software so that students can come up with their own conjectures. When interviewing Daniel, he changed his answer to using the compass and protractor more than computer software because it is more important for them to use pen and paper. Daniel uses his knowledge he has acquired from student teaching and tutoring experiences. The resources Daniel would use if he needed assistance would first be the textbook (student textbooks and his from his college courses), then he would turn to the internet, and then a cooperating teacher.

Mrs. Evan's is a first year teacher at a local high school. She is a graduate of the same program in which the pre-service teachers involved in this study are enrolled. She currently teaches Pre-AP Geometry and on-level Geometry. She was high in the Specialized Geometry Knowledge domain when compared to the other high school teachers. Her lowest domain was Knowledge of Geometry and Teaching, but she was in the lower half of the high school teachers in the Geometry Content Knowledge and the

Knowledge of Geometry and Students domain. Her self-evaluation of her performance was statistically significant and she had slight agreement. Mrs. Evan's Geometry Content Knowledge (GCK) was stronger than her score on in the GCK domain would seem to imply. She gave a two-column proof, of the properties of a midsegment of a triangle, and she was able to provide it without much help. Mrs. Evans was correct during the interview on the Specialized Geometry Knowledge question. She would prefer to have the students write the angle $m\angle 1 + m\angle 2 = 90°$. She stresses that the students use the "m" for measure, but she does not stress using the degree symbol. On the interview question addressing Knowledge of Geometry and Students, Mrs. Evans was correct. She chose 88 diagonals, and she was able to reason why students could possibly answer some of the other erroneous solutions. When asked the question from the MKT-G question addressing Knowledge of Geometry and Teaching, she was correct to choose the example of a kite, but does want to answer two of the choices. From her knowledge of the order of the curriculum she did not think that students had addressed kites yet, so that would be new material. Mrs. Evans reported direct teaching the most in her current classroom. She has seen investigations and manipulatives and models in professional development, and she would spend most of the time in her ideal classroom using investigations. Mrs. Evans uses her knowledge she has acquired from being a high school teacher. The resources Mrs. Evans would use if she needed materials would be last years textbook and notes. If she had confusion regarding the material she would go to a fellow teacher before she would do anything else. She is a part of a very supportive group of teachers, so if she has any problems, she has the ability to go to them for advice and help. When asked to reflect on her experiences at the university, she wishes there were more Geometry courses that explored proofs so that she could better understand how all the material is related.

Mrs. Kim has been teaching mathematics for 8 years and has taught Geometry for 3 years. She is originally from Korea. She currently teaches Pre-AP Geometry. She was high in the Specialized Geometry Knowledge and the Knowledge of Geometry and Teaching domain when compared to the other high school teachers. Her lowest domain was Knowledge of Geometry and Students, but she was in the lower half of the high school teachers in the Geometry Content Knowledge domain. Her self-evaluation of her performance was statistically significant and she had slight agreement. Mrs. Kim's Geometry Content Knowledge (GCK) was stronger than her score on in the GCK domain would seem to imply. She gave a two-column proof, of the properties of a midsegment of a triangle, and she was able to provide it without much help. Mrs. Kim was correct during the interview on the Specialized Geometry Knowledge question. She would prefer to have the students write the angle $m\angle 1 + m\angle 2 = 90°$. She stresses that the students use the "m" for measure, but she does not stress using the degree symbol. On the interview question addressing Knowledge of Geometry and Students, Mrs. Kim was incorrect. She chose 11 diagonals, and she thinks that the students do not fully understand the definition of a diagonal, so they assume the correct answer is 11. When asked the question from the MKT-G question addressing Knowledge of Geometry and Teaching, she was correct to choose the example of a kite, and is able to fully articulate reasoning behind not choosing the other options. From her knowledge of the order of the curriculum she did not think that students had addressed kites yet, so that would be new material. Mrs. Kim reported direct teaching the most in her current classroom. She has seen some computer software and manipulatives and models in professional development but answers none as well, and she would spend most of the time in her ideal classroom using investigations and discussion. Mrs. Kim uses her knowledge she has acquired from being a high school teacher. If she had confusion regarding the material she would go to a fellow teacher before she would do anything else. She is a part of a very supportive group of teachers, so if she has any problems, she has the ability to go to them for advice and help. She also found the AP Conference helpful in her teaching. When asked to reflect on her student teaching experience, she thinks that she did not have enough instruction over communication skills and relationship skills with students.

Mrs. Abbott has been teaching mathematics for 8 years and has taught Geometry for 8 years. She does not have a mathematics degree, but she has a teaching certification in mathematics. She currently teaches Pre-AP Geometry and on-level Geometry. She is also the high school teacher that had Daniel as the student teacher for her class. She was high in the Geometry Content Knowledge domain when compared to the other high school teachers. Her lowest domain was Knowledge of Geometry and Teaching, but she was in the lower half of the high school teachers in the Specialized Geometry Knowledge and Knowledge of Geometry and Students domain. Her self-evaluation of her performance was statistically significant and she had fair agreement. Mrs. Abbott's Geometry Content Knowledge (GCK) was weaker than her score on in the GCK domain would seem to imply. She gave a two-column proof, of the properties of a midsegment of a triangle, and she needed help multiple times. She had a difficult time proving the properties for an arbitrary triangle. She could not come up with using the converse of the parallel lines theorem to show the midsegment was parallel to the third side. She completed the proof but skipped some steps. Mrs. Abbott was correct during the interview on the Specialized Geometry Knowledge question. She would count this kind of a problem wrong on an exam and homework, but she does not stress using the degree symbol. On the interview question addressing Knowledge of Geometry and Students, Mrs. Abbott was incorrect. She chose 11 diagonals, and she thinks that the students do not fully understand the definition of a diagonal, so they assume the correct answer is 11. When asked the question from the MKT-G question addressing Knowledge of Geometry and Teaching, she was incorrect. She would not have chosen the kite as an extension of the theorem because students have not learned about quadrilaterals. From her knowledge of the order of the curriculum she did not think that students had addressed kites yet, so that would be new material that would be too overwhelming for them. Mrs. Abbott reported group activities the most in her current classroom. She has seen some manipulatives and models, investigations, and activities in professional development, and she would spend most of the time in her ideal classroom using investigations. Mrs. Abbott uses her knowledge she has acquired from being a high school teacher. She claims that since she has been teaching Geometry for 8 years, she does not have any confusion, but if she had confusion regarding the material she would go to the textbook before she would do anything else. When asked to reflect on her student teaching experience and the student teachers that she has mentored, she thinks that student teachers do not have enough instruction over interactions and discipline problems. Student teachers also have a hard time teaching lower level mathematics because they are so advanced in mathematics.

Mrs. Lane has been teaching mathematics for 11 years and has taught Geometry for 10 years. She currently teaches Pre-AP Geometry. She was high in the Knowledge of Geometry and Teaching and Common Geometry Knowledge when compared to the other high school teachers. Her lowest domain was Knowledge of Geometry and Students, but she was in the lower half of the high school teachers in the Specialized Geometry Knowledge domain. Her self-evaluation of her performance was statistically significant and she had fair agreement. Mrs. Lane's Geometry Content Knowledge (GCK) was strong. She gave a two-column proof, of the properties of a midsegment of a triangle, and she was able to provide it without much help. Mrs. Lane was correct during the interview on the Specialized Geometry Knowledge question. She would prefer to have the students write the angle $m\angle 1 + m\angle 2 = 90°$. She stresses that the students use the "m" for measure, but she does not stress using the degree symbol. On the interview question addressing Knowledge of Geometry and Students, Mrs. Lane was incorrect. She chose 11 diagonals, and she thinks that the students do not fully understand the definition of a diagonal, so they assume the correct answer is 11. When asked the question from the MKT-G question addressing Knowledge of Geometry and Teaching, she was correct to choose the example of a kite, and is able to fully articulate reasoning behind not choosing the other options. From her knowledge of the order of the curriculum she did not think that students had addressed kites yet, so that would be new material. Mrs. Lane reported investigations the most in her current classroom. She has seen some investigations and manipulatives and models in professional development, and she would spend most of the time in her ideal classroom using computer software. Mrs. Lane uses her knowledge she has acquired from being a high school teacher. If she had confusion regarding the material she would go to a fellow teacher before she would do anything else. She is a part of a very supportive group of teachers, so if she has any problems, she has the ability to go to them for advice and help. If the other teachers were not helpful, she would consult the textbook and then the internet. When asked to reflect student teaching experiences, she thinks that student teachers have an unrealistic idea of how much a student can retain during one lesson and how much a student can cover in one class time. When she thinks back to her own student teaching experience, she wishes she would have had more instruction over classroom management.

Discussion of Findings

The primary domains where pre-service and high school teachers had the biggest difference was in Specialized Geometry Knowledge (SGK) and Knowledge of Geometry and Teaching (KGT). Specialized Geometry Knowledge is "mathematical knowledge and skill unique to teaching" (Ball et al., 2008). SGK is the knowledge of mathematics that is not necessarily used in any other field. Knowledge of Geometry and Teaching is the category that "combines knowing about teaching and knowing about mathematics" (Ball et al., 2008). KGT primarily focuses on the planning of the teacher, the sequencing of topics so that students are the most successful, or what examples the teacher decides to show the students. These results are not surprising when SGK is knowledge of Geometry that would not be used in any other field besides teaching Geometry and KGT would require the pre-service teachers to have some idea of how to present material to students. Geometry Content Knowledge and Knowledge of Geometry and Students are still lower in the pre-service teachers, but they are stronger in these domains. Geometry Content Knowledge is what they would get from their Geometry courses and the Knowledge of

Geometry and Student could come from them interacting with students through tutoring or remembering being a student themselves.

The knowledge of the different instructional techniques is statistically different between pre-service teachers and high school teachers. This was unexpected, but this is a problem that needs to be addressed. One can understand teachers not being able to teach their ideal Geometry class because of budgetary restrictions and time, and it seems that professional development would introduce current teachers to other instructional techniques that they may not use in their current classroom, but the techniques presented in professional development would seem to transfer over to the teacher's ideal geometry class. It seems strange that pre-service teachers are being taught Geometry and are in education courses, but their methods of teaching their ideal Geometry class do not relate. Where are these pre-service teachers getting these ideas? It seems that there would be differences between the pre-service ideal classroom and the high school teachers' classroom because the pre-service teachers do not have as much classroom experience, and current high school teachers are drawing from their experiences being a geometry teacher. This also could relate to the MKT-G results showing that pre-service teachers have a lower score on the Knowledge of Geometry and Teaching. One surprising result form these comparisons is the difference between the pre-service Geometry and education courses and the professional development opportunities for high school teachers. It would seem that both of these types of teacher education would correspond in some way, but statistically they are different. The comparison between the pre-service teachers' geometry and education courses and the current high school geometry classroom is also interesting. If pre-service teachers are not being introduced to what the current high school teachers do in the Geometry classroom, is this setting them up for failure? The self-assessment data and the performance on the MKT-G do not seem to be reasonably related. So this either means that the Likert Scale portion of the survey needs to be modified, or that participants in both pre-service and high school do not understand what they know and what they do not know. They are unable to self-evaluate their knowledge. This would make it difficult for teacher education and professional development to address certain issues in the way mathematics is taught because teachers would think that they know the issues, but have no way of understanding if they do or not.

Through the data collected in the interviews, it would seem as though the MKT-G assesses the abilities of the participants, but there are some differences in performance. This could possibly be due to the MKT-G assessment being multiple choice. Participants have the ability to guess at an answer, so when asked in person, it can become more evident that they do not know the answer. This could also mean that those who did poorly on the assessment, but did better in the interview, did not adequately convey their knowledge when taking the online assessment. Mrs. Kim’s performance on the MKT-G was surprising, but due to the amount of reading, I wonder if English being a second language could have contributed to her performance. The pre-service teachers primarily wanted to give paragraph proofs, but the high school teachers all preferred a two-column proof. This might be due to the familiarity that they must have with the two-column proof method because they are required to teach that method of proof writing in the curriculum. As far as where the participants’ knowledge comes from, all of the participants attributed their knowledge of student understanding to either tutoring or their experiences in the classroom, either student teaching or currently teaching. Some of the high school teachers mentioned things they had learned about teaching through professional development, but many of them did not even reference it. Pre-service teachers never said that their knowledge of student understanding came from their geometry courses or education courses. As far as resources that pre-service teachers use, the first instincts of all the pre-service teachers interviewed were to consult a textbook. After they had exhausted that resource, then they would go to another teacher. The high school teachers were the opposite. When they were confronted with uncertainty, three of the four would go directly to another teacher, and then their second choice, if still confused, would be to go to the textbook or the internet.

Implications

Geometry is a field in mathematics that every student in the United States is required to study in order to fulfill high school graduation requirements. According to the National Center for Education Statistics (2012), two content areas in mathematics are consistently behind in performance: Geometry and Measurement. The literature shows three possible reasons for poor performance in Geometry and Measurement are: not enough exposure and emphasis in K-12 curriculum implemented by the teacher, challenges associated with implementation of Geometry and Measurement in the classroom, and limited knowledge of the teacher (Steele, 2013). This subject is required for mathematics teachers to teach successfully, and this study investigates where pre-service teachers are with respect to their Geometry Teaching Knowledge when being compared to high school teachers. Since the group of teachers that did the best on the MKT-G Assessment were current high school teachers, specifically in the SGK and the

KGT domains, there is knowledge that the current high school teachers have in these domains. When interviewing the current teachers, they attributed their knowledge of the class to their experiences in the classroom. The pre-service teachers that were interviewed relied on tutoring, so student teachers would probably benefit from more exposure to the classroom in the instructor role.

This study has shown that pre-service teacher Geometry courses, education courses, ideal classrooms, high school teachers' current classrooms, professional development, and ideal classrooms are independent of one another. This is a serious problem and the following questions arise from this data. What is being taught in the geometry and education courses if the pre-service teachers do not include that in their ideal Geometry classroom? What is being taught in the professional development if those techniques are not being taught to pre-service teachers in their geometry or education courses? Are the techniques being addressing in professional development and pre-service geometry and education courses applicable to the current geometry classroom? Why aren't these techniques being incorporated in the ideal classrooms for both groups? These are all concerns that the results from this study highlight.

The self-assessment data and the performance on the MKT-G do not seem to be reasonably related. If teachers are unable to self-evaluate their own knowledge, this would make it difficult for teacher education and professional development to address certain issues in the way mathematics is taught because teachers would think that they know the issues, but have no way of understanding if they do or not.

A pattern that was evident throughout the interviews was that pre-service teachers tend to rely on textbooks for information rather than fellow colleagues. This could be due to the uncertainty that pre-service teachers seem to exhibit when confused, so they do not want to depend on another teacher. They may also be new to collaborating with other teachers in order to be a successful teacher themselves. One way that this could be alleviated is to have the pre-service teachers work together in their geometry or education courses, so they are comfortable asking others for help. They could also be given opportunities to work with current high school teachers so they can see how some departments help one another.

Limitations

This study focused on a group of pre-service teachers from a single university in central Texas. The structure of this university's pre-service teacher training program could be different than other universities in Texas and in other states. This study also focuses on current high school mathematics teachers in Texas. The knowledge level of Geometry may be different depending on the state in which the teachers work. The high school teachers selected to be interviewed and observed were a part of a supportive department, so they felt that they had many resources within their schools. This may be different with schools that do not have supportive departments or administrations. While some of the results may be extended beyond the scope of this particular university, any generalizing must be done cautiously.

The survey given to all the participants was developed by the researcher. The intention for the survey was to gather information about the knowledge of instructional methods and strategies of the participant, as well as their beliefs on how they performed on the MKT-G. There is no guarantee that the survey accurately gathered all of the knowledge of the participants because no method was utilized to corroborate this knowledge.

Due to the limited amount of participants selected to be interviewed and observed, the data collected and conclusions might not represent the population of all pre-service teachers at one university and current high school teachers in Texas. The pre-service teacher that was selected based off being assigned Geometry may not necessarily reflect all the knowledge of other pre-service teachers assigned to teach Geometry.

Lastly, the researcher solely conducted the interviews and observations and the analysis of the data. As any mixed methods analysis, there was some level of personal bias due to the researcher making decisions on the interpretation of data.

Future Research

This study brought up issues of the differences in Geometry Teaching Knowledge between pre-service and current high school teachers. Pre-service teachers were weaker in all domains, but primarily in Specialized Geometry Knowledge (SGK) and Knowledge of Geometry and Teaching (KGT). There is a need for future research that focuses on these domains, specifically to target what can be done to increase scores in these domains for pre-service and high school teachers.

This study has shown there are differences in pre-service and high school teachers' experiences with instructional techniques and methods. Further research is needed to investigate the different instructional techniques and methods used in pre-service courses and professional development courses. It would seem that these two forms of teacher education courses would correspond, and that knowledge would be transferred to the teachers' ideal Geometry class. There is also a need for more research into ways one can implement the teacher education courses into their current or future classroom.

When participants in this study were asked how they thought they performed on the assessment, they were unable to assess their knowledge accurately. Research into this aspect of pre-service and high school teachers is necessary in order to target what current or future teachers will need in their pre-service teacher courses and professional development.

Further research is needed to elaborate on the origin of Geometry Teaching Knowledge in pre-service and high school teachers. If we can pinpoint where the majority of this knowledge is obtained, then we can make sure pre-service teachers have those experiences in their training programs.

While this study is focused on Geometry Teaching Knowledge, there is a need to extend this type of research into other secondary mathematics courses and even into post-secondary education. These results provide some insight into how this could be extended to other subjects, but specialized assessments will need to be developed.

APPENDIX SECTION

APPENDIX A: Terms of Use Contract for MKT-G Assessment

APPENDIX B: IRB Approval Form

APPENDIX C: Pre-Service Teacher Consent Form

APPENDIX D: High School Teacher Consent Form

APPENDIX E: Demographic Information included in MKT-G Assessment

APPENDIX F: Released Items from the MKT-G Assessment

APPENDIX G: Geometry Pre-Service Teacher Survey

APPENDIX H: Geometry High School Teacher Survey

APPENDIX I: Teacher Interview

APPENDIX J: Teacher Interview Protocol

APPENDIX K: Observation Protocol

APPENDIX A:

Terms of Use Contract for MKT-G Assessment

SCHOOL OF EDUCATION
UNIVERSITY OF MICHIGAN

GRIP
GEOMETRY REASONING AND INSTRUCTIONAL PRACTICES

610 E. UNIVERSITY AVE.
ANN ARBOR, MICHIGAN 48109-1259
PROJECT E-MAIL: gripmail@umich.edu
FAX: 734-763-1368

Mathematical Knowledge for Teaching - Geometry (MKT-G) Measures

Terms of use between Patricio Herbst and Shawnda Smith
Date: 03/10/2015

When using our items or instruments, Shawnda Smith (hereafter, the researcher) agrees to:

1. Administer the items through Lesson*Sketch*, by directing participants to Lesson*Sketch* Experiences created by our team for the administration of those items. This helps to ensure the security of the measures and allows for pooling of responses to improve the measures over time.

2. Refrain from using these measures to publicly demonstrate teachers' ability or lack of ability in mathematics. This helps secure teacher participation in future studies. Specifically, this includes:

- Not publicly discussing raw frequencies or number correct by an individual, though could report the number correct privately to an individual;
- Not comparing your participants to other participants in any way that reveals raw frequencies or number correct for either sample.

Researchers may calculate descriptive statistics, z-scores, or IRT (item response theory) scores when reporting results.

3. Refrain from using these items to evaluate individual teachers for tenure, pay, hiring, or any other use with high-stakes consequence. Refrain from using scores on these items as basis for course grades. These measures are not validated for these purposes. An instructor is allowed to award credit to their students for completion of the instrument.

4. Safeguard the privacy of participants in research as outlined by the host institution's Institutional Review Board procedures.

5. Refrain from distributing or orally complementing or administering any non-released item item in any presentation, debrief, lesson, paper, article, or other public forum. Refrain from showing, discussing, or distributing items in the teaching of classes for practicing or prospective teachers. Released items are provided for this purpose.

6. Refrain from making any changes to any item without our prior permission. Changes should be suggested in letters to Pat Herbst.

7. Refrain from using descriptions of items not authored or vetted by GRIP in presentations and publications. When in need of item descriptions, this need can be communicated to Pat Herbst.

8. Abide by provided administration protocol

9. Report any user difficulties (e.g. misunderstanding, discomfort) that they are privy to.

- This allows for the continual improvement of the Lesson*Sketch* platform in general and administration of the MKT-G measures specifically.

11. Abide generally by the standards put forward in the *Standards for Educational and Psychological Testing* (AERA/APA 1999).

12. Share ideas with GRIP researchers for revision of items.

13. Communicate about special needs that may require transformations in the form in which the items are provided (e.g., translations).

In the collection of data around these items, GRIP agrees to:

1. Provide a PDF of the MKT-G publicly-released items for the researcher to use in their own research dissemination.

2. Share access with the researcher to disaggregated data (how individuals answered particular questions). Specifically, researchers will receive disaggregated reports.

3. Share access with the researcher to aggregated data (how groups of individuals scored on questions and on groups of questions) to use with others. This includes aggregated reports from other groups that have taken the instrument.

4. Share with the researcher the ability to present and publish about particular user's responses to items and their responses to a background questionnaire (while not disclosing the item and honoring confidentiality of the human subjects who produced those responses).

5. Share with other researchers the ability to use the aggregated information.

Shawnda Smith	
Shawnda Smith, Researcher	Patricio Herbst, Principal Investigator

Lesson*Sketch* MKT Administration Protocol

Prior to administration of instrument,

1. Identify instrument administrators (could be yourself or another) who will facilitate participants' experience as they move through the instrument.
2. Ensure instrument administrators have read this document prior to delivering the instrument with participants.
3. Ensure instrument administrators have access to the instrument to review it for any bugs that might present locally to trouble shoot those bugs with Vu Minh Chieu (vmchieu@umich.edu) prior to the delivery of the instrument with participants. Instrument should be delivered using a Chrome or Firefox browser if possible.
4. Ensure that instrument administrators have the knowledge to make appropriate technological accommodations for participants prior to delivering the instrument. Two common accommodations we recommend instrument administrators are prepared for are the provision of external mice and browser magnification.
5. Ensure that all associated researchers, instrument administrators, and participants understand there is no copying, printing, or screen captures of any of the items found within the instruments.

During the administration of the instrument, instrument administrators will

1. Monitor the completion of instruments in order to avoid compromised responses.
2. Ensure that participants, researchers, or administrators do not copy, print, or screen capture any of the items included in an instrument, regardless of whether they are publicly released items or not. This includes the collection and disposal of any scrap paper participants used to complete the item.
3. Avoid providing any assistance interpreting items within the instrument. Responses to participants' questions about the content within the item should only be responded with encouragement to do the best they can as an individual to make sense of the item.
4. Provide assistance to technical questions and respond to the need for technical accommodations that may present themselves (two to anticipate include the need for external mice and magnifying the browser).

APPENDIX B:
IRB Approval Form

TEXAS STATE UNIVERSITY
SAN MARCOS
The rising STAR of Texas

Institutional Review Board

Request For Exemption

Certificate of Approval

Applicant: Shawnda Smith

Request Number : EXP2015U801251Y

Date of Approval: 06/03/15

Assistant Vice President for Research and Federal Relations

Chair, Institutional Review Board

Return to IRB Home

APPENDIX C:
Pre-Service Teacher Consent Form

TEXAS STATE UNIVERSITY
CONSENT STATEMENT
Geometry Teaching Knowledge: A Comparison of Pre-Service and High School Geometry Teachers

I have the opportunity to participate in the research study titled "Geometry Teaching Knowledge: A Comparison of Pre-Service and High School Geometry Teachers" conducted by Ms. Shawnda Smith from the Department of Mathematics at Texas State University. I understand that my participation is voluntary. I can stop taking part without giving any reason, and without penalty.

PURPOSE
The purpose of the project is to compare the Geometry Teaching Knowledge of Pre-Service and current High School Geometry teachers. Mathematics teachers entering into the high school classroom for the first time are expected to be able to teach all levels of high school mathematics, however there are few courses in traditional teacher education programs that address Geometry topics. The main goals of this project are to compare the Geometry Teaching Knowledge of pre-service and high school teachers, knowledge of Geometry teaching methods, and where this knowledge is developed. This study is part of a dissertation and will be included in the dissertation.

PROCEDURES
Data will be collected through an online assessment, a post-assessment survey, classroom observations, and interviews. Still photos and audio recordings may supplement the field notes from the observations and interviews.

BENEFITS
The benefit to me is an opportunity show my abilities regarding Geometry Teaching Knowledge and to help showcase the Texas State Teacher Education Program. If I participate in the online assessment, I will be entered into a drawing for 3 $50 Amazon gift cards. If I participate in the interviews, I will receive a $50 Amazon Gift Card.

CONFIDENTIALITY
Any data gathered, including observation notes, audiotapes, and copies of students' written work will be stored in a locked office. The audiotapes will be used for research purposes, and may be included in research presentations. All tapes will be destroyed three years after the dissertation is completed. No information that identifies me will be shared with those outside of the research team, and my participation will not affect my grade positively or negatively.

FURTHER QUESTIONS
The researcher will answer any further questions I have about this research, now or during the course of the project. The primary contact person is Shawnda Smith (srh100@txstate.edu). This project (IRB Exemption Number: EXP2015U801251Y) was approved by Texas State IRB on 06/03/15.

CONSENT
I consent to participate in this study.

Student's Name ____ (please print)

Student's signature____

If you are willing to be interviewed for this project please include the information below:

Phone _________________________________ School
email____

Major/Minor_______________________________________ Projected Graduation Date:
____

APPENDIX D:
High School Teacher Consent Form

TEXAS STATE UNIVERSITY
CONSENT STATEMENT
Geometry Teaching Knowledge: A Comparison of Pre-Service and High School Geometry Teachers

I have the opportunity to participate in the research study titled "Geometry Teaching Knowledge: A Comparison of Pre-Service and High School Geometry Teachers" conducted by Ms. Shawnda Smith from the Department of Mathematics at Texas State University. I understand that my participation is voluntary. I can stop taking part without giving any reason, and without penalty.

PURPOSE
The purpose of the project is to compare the Geometry Teaching Knowledge of Pre-Service and current High School Geometry teachers. Mathematics teachers entering into the high school classroom for the first time are expected to be able to teach all levels of high school mathematics, however there are few courses in traditional teacher education programs that address Geometry topics. The main goals of this project are to compare the Geometry Teaching Knowledge of pre-service and high school teachers, knowledge of Geometry teaching methods, and where this knowledge is developed. This study is part of a dissertation and will be included in the dissertation.

PROCEDURES
Data will be collected through an online assessment, a post-assessment survey, classroom observations, and interviews. Still photos and audio recordings may supplement the field notes from the observations and interviews.

BENEFITS
The benefit to me is an opportunity show my abilities regarding Geometry Teaching Knowledge and to help showcase the current Geometry Teaching Knowledge necessary to teach Geometry successfully. If I participate in the online assessment, I will be entered into a drawing for one of 3 $50 Amazon gift cards. If I participate in the interviews, I will receive a $50 Amazon Gift Card.

CONFIDENTIALITY
Any data gathered, including observation notes, audiotapes, and copies of students' written work will be stored in a locked office. The audiotapes will be used for research purposes, and may be included in research presentations. All tapes will be destroyed three years after the dissertation is completed. No information that identifies me will be shared with those outside of the research team, and my participation will not affect my grade positively or negatively.

FURTHER QUESTIONS
The researcher will answer any further questions I have about this research, now or during the course of the project. The primary contact person is Shawnda Smith (srh100@txstate.edu). This project (IRB Exemption Number: EXP2015U801251Y) was approved by Texas State IRB on 06/03/15.

CONSENT
I consent to participate in this study.

Teacher's Name ____ (please print)

Teacher's signature_______________________________________

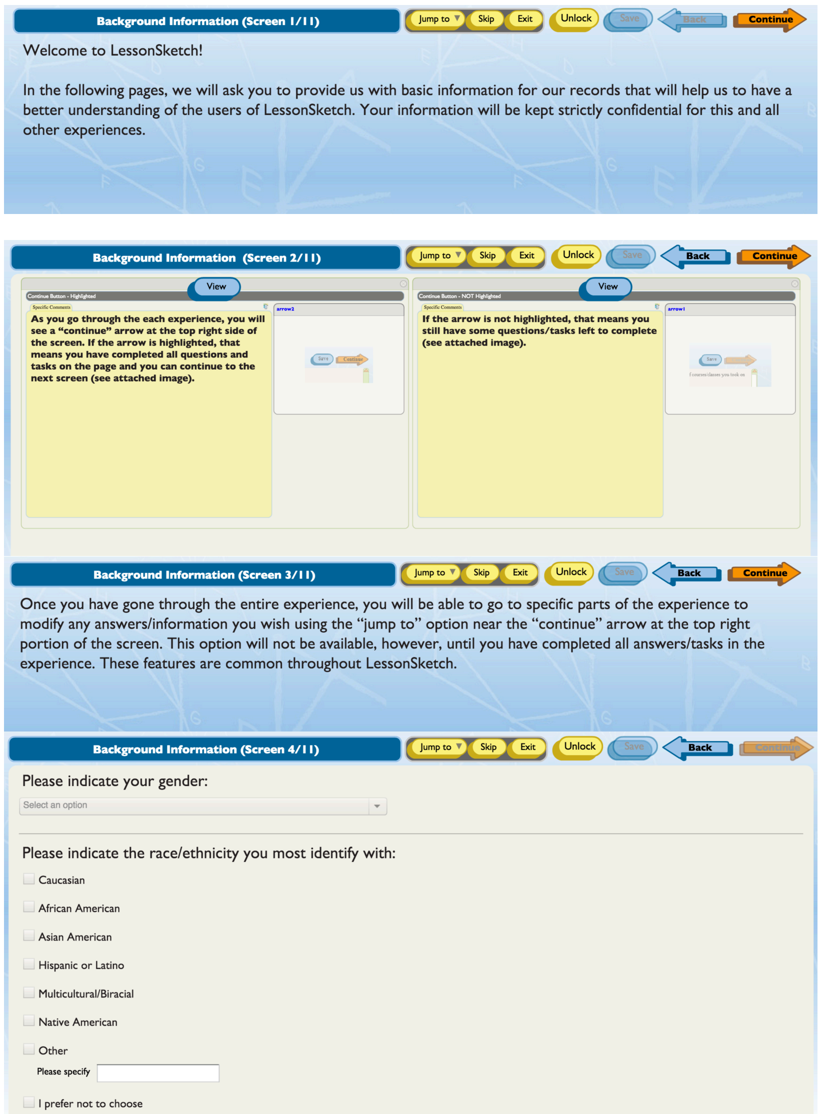
If you are willing to be interviewed for this project please include the information below:

Phone ________________________________ School
email____

School District____

APPENDIX E:

Demographic Information included in MKT-G Assessment



Background Information (Screen 4/11)

Jump to | Skip | Exit | Unlock | Save | Back | Continue

Please indicate your gender:

Select an option

Please indicate the race/ethnicity you most identify with:

- [ ] Caucasian
- [ ] African American
- [ ] Asian American
- [ ] Hispanic or Latino
- [ ] Multicultural/Biracial
- [ ] Native American
- [ ] Other

Please specify

- [ ] I prefer not to choose

Background Information (Screen 6/11)

Jump to | Skip | Exit | Unlock | Save | Back | Continue

Which of the following best describes your **employment experience**? (Check all that apply)

- [ ] I've worked summers or part time while going to school
- [ ] I've worked full time for less than 5 years
- [ ] I've worked full time for 5 years or more
- [ ] I've had management responsibilities
- [ ] None of the above apply

Background Information (Screen 7/11)

Jump to | Skip | Exit | Unlock | Save | Back | Continue

Please indicate the number of courses focused on **mathematics teaching** or **mathematics learning** that you have taken in college so far.

Select an option

Please indicate the total number of **college-level mathematics** courses you have taken and passed:

Select an option

How many college-level mathematics courses focusing on **Algebra** topics have you taken?

Select an option

How many college-level mathematics courses focusing on **Geometry** topics have you taken?

Select an option

How many **mathematics courses** have you taken that cover **topics that are taught in the K-12** education system (include all)?

Select an option

Background Information (Screen 10/11)

Jump to | Skip | Exit | Unlock | Save | Back | Continue

What mathematics classes would you like to teach?

Please write your response here.

Please indicate all the mathematics courses you would feel able to teach given your current level of preparation.

- Kindergarten Mathematics
- Grade 1 Mathematics
- Grade 2 Mathematics
- Grade 3 Mathematics
- Grade 4 Mathematics
- Grade 5 Mathematics
- Grade 6 Mathematics
- Grade 7 Mathematics
- Grade 8 Mathematics
- Algebra I
- Geometry
- Algebra II
- PreCalculus or Trigonometry
- Calculus
- None of them

Background Information (Screen 9/11)

Jump to | Skip | Exit | Unlock | Save | Back | Continue

In which grade-levels have you had experiences with mathematics learners?

- Grades K - 2 (*Early Elementary*)
- Grades 3 - 5 (Upper elementary school)
- Grades 6 - 8 (Middle School / Junior High)
- Grades 9 - 12 (High School)
- College
- None apply

Please indicate in which grade levels you expect to be able to teach mathematics

- Grades K - 2 (*Early Elementary*)
- Grades 3 - 5 (*Upper Elementary*)
- Grades 6 - 8 (*Middle School / Junior High*)
- Grades 9 - 12 (*High School*)
- College
- Not applicable

Background Information (Screen 8/11)

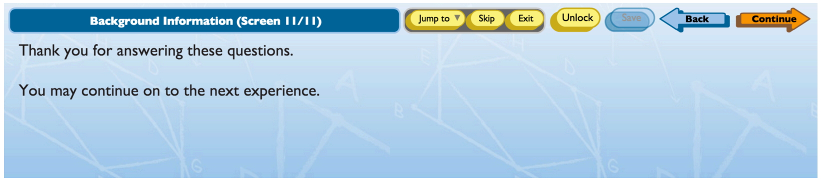
Jump to | Skip | Exit | Unlock | Save | Back | Continue

We'd like to know the extent of your current experience in mathematics classrooms. Including time spent as a substitute teacher, volunteer, pre-student teacher (intern), or student teacher, **how many different days** do you estimate having been in a K-12 mathematics class?

Select an option

How many **different mathematics teachers** have you been able to **work with** (as volunteer, intern, tutor, pre-student teacher, or student teacher)?

Select an option

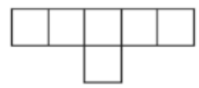
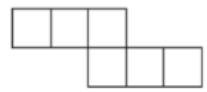

Background Information (Screen 11/11)
Jump to
Skip
Exit
Unlock
Save
Back
Continue
Thank you for answering these questions.
You may continue on to the next experience.

APPENDIX F:

Released Items from the MKT-G Assessment

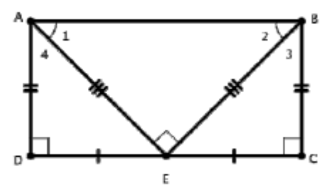
CCK released item

Students in Mr. Wingate's class have been creating nets that they want to be able to fold to create a cube. For each of the student-created nets shown below, identify whether it can be successfully folded into a cube?

		Yes	No
i			
ii			

SCK released item

While proving a claim on the board about the figure below, Joe wrote "1 + 2 = 90." Ms. Staples ponders how to correct that statement. Of the following, what is the best alternative?



A	Do nothing. The statement is correct as is.
B	Add the degree symbol so that the statement reads 1 + 2 = 90°.
C	Replace what Joe wrote; write instead that "m∠A + m∠B = 90°".
D	Replace what Joe wrote; write instead that "m∠EAB + m∠EBA = 90°".

KCS released item

Ms. Jamison asked students to figure out the number of diagonals in an 11-sided polygon. Of the following erroneous solutions, which one might have resulted from using the number of diagonals through one vertex?

A	11
B	72
C	88
D	99
E	121

KCT released item

After teaching her students the base angles theorem (which states that base angles of an isosceles triangle are congruent), Ms. Wellington is pondering what example she could give to illustrate how this theorem can help find new information. Which of the following would be the best example?

A	If you have a triangle that you know is isosceles and you know two of the sides are 8 and 10, then the third side will be 8 or 10.
B	If you have a triangle that you know the sides have length 8, 10, and 12 you know it does not have two angles that are congruent.
C	If you have an isosceles triangle and two of its angles are 30 you can calculate the third angle by subtracting 60 from 180.
D	If you have a kite and draw its minor diagonal, the kite is divided into two isosceles triangles and those have two congruent angles each.

APPENDIX G:

Geometry Pre-Service Teacher Survey

1. Name:____
   Email:_______________________________

2. Are you willing to be interviewed during the Fall 2015 Semester? Yes No

3. Male/Female (circle one)

4. Expected Graduation Date: _________________________ (semester and year)

5. Have you ever tutored mathematics? (Circle one) Yes No

   a. If yes, what level or course of mathematics have you tutored?

      ____

      ____

      ____

6. Choose one of the following:

   a. I completed student teaching in _________________ semester.

   b. I am currently student teaching.

   c. I plan to student teach in _________________ semester.

7. Prior Math Courses: Please check all the math courses you have taken at Texas State University or another institution.

☐ Pre-College Algebra	☐ Number Systems
☐ Basic Mathematics	☐ Introduction to Advanced Mathematics
☐ College Algebra	☐ Deterministic Operations Research
☐ A Survey of Contemporary Mathematics	☐ Calculus III
☐ Plane Trigonometry	☐ Engineering Mechanics
☐ Mathematics for Business and Economics I	☐ Linear Algebra
☐ Mathematics for Business and Economics II	☐ Analysis I
☐ Principle of Mathematics I	☐ Discrete Mathematics II
☐ Informal Geometry	☐ Principles of Mathematics II
☐ Calculus for Life Sciences I	☐ Capstone Mathematics for Middle School Teachers
☐ Elementary Statistics	☐ Math Understandings
☐ Calculus for Life Sciences II	☐ Probability and Statistics
☐ Discrete Mathematics I	☐ Fourier Series and Boundary Value Problems
☐ Pre-Calculus Mathematics	☐ Modern Algebra
☐ Calculus I	☐ Introduction to the History of Mathematics
☐ Calculus II	☐ Analysis II
☐ Introduction to Probability and Statistics	☐ General Topology
☐ Modern Geometry	☐ Studies in Applied Mathematics
☐	☐

8. Teaching Preference: (Circle as many that apply)

    a. Pre-School
    b. Elementary School
    c. Middle School
    d. High School
    e. Post-Secondary School

9. Subject Matter Preference: (Circle as many that apply)

    a. Pre-Algebra
    b. Algebra 1
    c. Geometry
    d. Algebra 2
    e. Pre-Calculus/ Trigonometry
    f. Calculus
    g. Statistics
    h. Other________________________

10. What instructional techniques have you used in your Geometry courses (including those you were exposed to in High School)?

Read the following techniques and consider which ones you have seen in your Geometry courses. You are given a total of 10 points to distribute among 5 techniques however you would like based on how often you have used these techniques in your Geometry courses (assign a value between 0 and 10 to all items), with the number of points assigned to the topic reflecting the frequency these techniques were used in your Geometry courses. You must use all 10 points. Please make sure the points add up to 10 by including a total count at the end. If you have never seen any of these, please circle f. None of the above or g. I have never taken a Geometry course.

a. Investigations (Example: Discovery lessons) ________

b. The use of a compass and protractor to construct figures ________

c. Computer Software (Geometer's Sketchpad, Geogebra, etc)________

d. Manipulatives/Models ________

e. Other: (please describe) ____________________________________

Total: ________

f. None of the above

g. I have never taken a Geometry course

11. What instructional techniques have you seen in your education courses?

Read the following techniques and consider which ones you have seen in your education courses. You are given a total of 10 points to distribute among 5 techniques however you would like based on how often you have used these techniques in your Geometry courses (assign a value between 0 and 10 to all items), with the number of points assigned to the topic reflecting the frequency these techniques were used in your education courses. You must use all 10 points. Please make sure the points add up to 10 by including a total count at the end. If you have never seen any of these, please circle f. None of the above.

a. Investigations (Example: Discovery lessons) ________

b. The use of a compass and protractor to construct figures ________

c. Computer Software (Geometer's Sketchpad, Geogebra, etc)________

d. Manipulatives/Models ________

e. Other: (please describe) ____________________________________

Total: ________

f. None of the above

12. If you had unlimited access and budget, what instructional techniques would you use in your own Geometry Classroom?

    Read the following techniques and consider which ones you would use in your own Geometry Classroom. You are given a total of 10 points to distribute among 5 techniques however you would like based on what you would think would be best for your students (assign a value between 0 and 10 to all items), with the number of points assigned to the topic reflecting the importance of these techniques in your classroom. You must use all 10 points. Please make sure the points add up to 10 by including a total count at the end.

    a. Investigations (Example: Discovery lessons) ________

    b. The use of a compass and protractor to construct figures ________

    c. Computer Software (Geometer's Sketchpad, Geogebra, etc)________

    d. Manipulatives/Models ________

    e. Other: (please describe) ___________________________________

    Total: ________

    f. None of the above

13. Why did you distribute the points the way you did in question 12?

14. Read the following topics addressed in the online assessment and consider those in which you are most knowledgeable.

    Please rate on a scale from 1 to 5 (1 being "I have never seen the topic" and 5 being "I am very knowledgeable of this topic i.e. I feel confident that I correctly answered the questions regarding this topic")

Angle Bisectors	1	2	3	4	5
Angle Relations	1	2	3	4	5
Altitude of Triangles	1	2	3	4	5
Isosceles Triangle Properties	1	2	3	4	5
CPCTC	1	2	3	4	5
Quadrilateral Properties	1	2	3	4	5
Diagonals of quadrilaterals/rectangles	1	2	3	4	5
Rectangle Properties	1	2	3	4	5
Quadrilateral Similarity	1	2	3	4	5
Cube Properties	1	2	3	4	5
Definition and properties of a Kite	1	2	3	4	5
Polygon Diagonals	1	2	3	4	5
Distance Formula	1	2	3	4	5
Circle Properties	1	2	3	4	5
Construction of Tangent Circle	1	2	3	4	5
Inscribed Angle Theorem	1	2	3	4	5
Proof Validity	1	2	3	4	5

APPENDIX H:

Geometry High School Teacher Survey

1. Name:____

2. Male/Female (circle one)

3. School District:____

4. Years teaching at above School District:___________________

5. Total years teaching mathematics:__________________

6. Total years teaching Geometry:_________________

7. Do you have a degree in Mathematics?___________

   If yes,
   a. What degree do you have?___________
   b. What school did you receive your degree?

8. What type of training did you have to gain your certification? (circle one)
   a. Traditional 4-year University
   b. Emergency Certification
   c. I do not have a teaching certification of any kind

9. Do you have a teaching certificate in Mathematics?____________

   If yes, what grade levels are you certified?_____________

   If no, what teaching certification do you have?_________

10. Teaching Preference: (Circle as many that apply)

a. Pre-School
b. Elementary School
c. Middle School
d. High School
e. Post-Secondary School

11. Subject Matter Preference: (Circle as many that apply)

a. Pre-Algebra
b. Algebra 1
c. Geometry
d. Algebra 2
e. Pre-Calculus/ Trigonometry
f. Calculus
g. Statistics
h. Other________________________

12. What instructional techniques have you used in your Geometry classroom?

Read the following techniques and consider which ones you have seen in your Geometry classroom. You are given a total of 10 points to distribute among 5 techniques however you would like based on how often you have used these techniques in your Geometry classroom (assign a value between 0 and 10 to all items), with the number of points assigned to the topic reflecting the frequency these techniques were used in your Geometry classroom. You must use all 10 points. Please make sure the points add up to 10 by including a total count at the end. If you have never seen any of these, please circle f. None of the above

a. Investigations (Example: Discovery lessons) ________

b. The use of a compass and protractor to construct figures ________

c. Computer Software (Geometer's Sketchpad, Geogebra, etc)________

d. Manipulatives/Models ________

e. Other: (please describe) ____________________________________

Total: ________

f. None of the above

13. What instructional techniques have you seen in your professional development?

Read the following techniques and consider which ones you have seen in your professional development courses. You are given a total of 10 points to distribute among 5 techniques however you would like based on how often you have seen these techniques in your professional development courses (assign a value between 0 and 10 to all items), with the number of points assigned to the topic reflecting the frequency these techniques were used in your professional development courses. You must use all 10 points. Please make sure the points add up to 10 by including a total count at the end. If you have never seen any of these, please circle f. None of the above.

a. Investigations (Example: Discovery lessons) ________

b. The use of a compass and protractor to construct figures ________

c. Computer Software (Geometer's Sketchpad, Geogebra, etc) ________

d. Manipulatives/Models ________

e. Other: (please describe) ____________________________________

Total: ________

f. None of the above

14. If you had unlimited access and budget, what instructional techniques would you use in your Geometry Classroom?

    Read the following techniques and consider which ones you would use in your own Geometry Classroom. You are given a total of 10 points to distribute among 5 techniques however you would like based on what you would think would be best for your students (assign a value between 0 and 10 to all items), with the number of points assigned to the topic reflecting the importance of these techniques in your classroom. You must use all 10 points. Please make sure the points add up to 10 by including a total count at the end.

    a. Investigations (Example: Discovery lessons) ________

    b. The use of a compass and protractor to construct figures ________

    c. Computer Software (Geometer's Sketchpad, Geogebra, etc)________

    d. Manipulatives/Models ________

    e. Other: (please describe) ____________________________________

    Total: ________

15. Why did you choose what you did in question 14?

16. Read the following topics addressed in the online assessment and consider those in which you are most knowledgeable.

    Please rate on a scale from 1 to 5 (1 being "I have never seen the topic" and 5 being "I am very knowledgeable of this topic i.e. I feel confident that I correctly answered the questions regarding this topic")

Angle Bisectors	1	2	3	4	5
Angle Relations	1	2	3	4	5
Altitude of Triangles	1	2	3	4	5
Isosceles Triangle Properties	1	2	3	4	5
CPCTC	1	2	3	4	5
Quadrilateral Properties	1	2	3	4	5
Diagonals of quadrilaterals/rectangles	1	2	3	4	5
Rectangle Properties	1	2	3	4	5
Quadrilateral Similarity	1	2	3	4	5
Cube Properties	1	2	3	4	5
Definition and properties of a Kite	1	2	3	4	5
Polygon Diagonals	1	2	3	4	5
Distance Formula	1	2	3	4	5
Circle Properties	1	2	3	4	5
Construction of Tangent Circle	1	2	3	4	5
Inscribed Angle Theorem	1	2	3	4	5
Proof Validity	1	2	3	4	5

APPENDIX I:

Teacher Interview

Midsegments of Triangles

Introduction

In this activity, you will use compass, protractor, and straight edge to investigate *the midsegment, a segment that connects the midpoints of two sides of a triangle. First, you will construct and investigate one midsegment and the relationship of the new small triangle to the original triangle. Then, all three midsegments will be constructed and this figure will be explored.*

One midsegment investigation

1. Construct/draw arbitrary ΔABC.

2. Use a compass and straight edge to construct the midpoints of AB and AC, and label them D and E respectively. Construct DE. DE is a midsegment of ΔABC.

3. What relationship do you observe between DE and BC ?

4. Measure the lengths of DE and BC . What do you observe?

5. Measure $\angle ABC$ and $\angle ADE$. What do you observe? What does your finding imply?

6. Based on your findings in the above steps, state a conjecture about the relationship between *DE* (a midsegment connecting two sides of a triangle) and *BC* (the third side)?

Another teacher (Mary) did this activity as well. Her conjecture was that "A midsegment connecting two sides of a triangle is parallel to the third side and is half as long." Do you think her conjecture is true? Why or why not?

10. Prove or disprove Mary's conjecture. You might write your explanation as an argument in paragraph form or as a two-column proof.

APPENDIX J:

Teacher Interview Protocol

Midsegments of Triangles

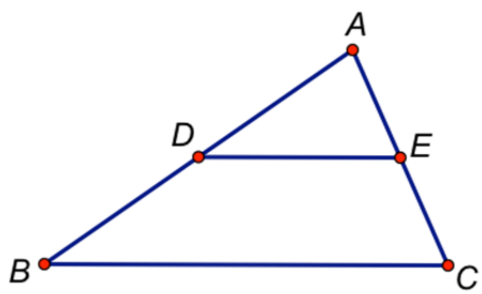
Introduction

In this activity, you will use compass, protractor, and straight edge to investigate *the midsegment, a segment that connects the midpoints of two sides of a triangle. First, you will construct and investigate one midsegment and the relationship of the new small triangle to the original triangle. Then, all three midsegments will be constructed and this figure will be explored.*

One midsegment investigation

1. Construct/draw arbitrary ΔABC.

2. Use a compass and straight edge to construct the midpoints of AB and AC, and label them D and E respectively. Construct DE. DE is a midsegment of ΔABC.
   a. Give teacher some time to see if they can construct midsigment.
   b. Notice if teachers use ruler to measure the lengths of side, to determine location of midpoint. If so, ask them if they know how to find it using a compass. Only show figure below if teacher has trouble constructing the figure on his/her own.



3. What relationship do you observe between DE and BC ?

   a. We are looking for “A midsegment connecting two sides of a triangle is parallel to the third side and is half as long.” But teacher may not jump to this until questions 4 and 5 below.

4. Measure the lengths of DE and BC . What do you observe?

   a. We are looking for “DE is half as long as BC.”

5. Measure ∠ABC and ∠ADE. What do you observe? What does your finding imply?

    a. We are looking for the measures of the angles are equal, hence the midsegment and *BC* are parallel.

    b. If the teacher stops at "angles are equal", ask them what "kind" of angles these are. Guiding them towards "corresponding angles". Ask them what the relationship between the angles says about the relationship between the segments *DE* and *BC*.

6. Based on your findings in the above steps, state a conjecture about the relationship between *DE* (a midsegment connecting two sides of a triangle) and *BC* (the third side)?

    a. Allow the teacher to state the conjecture in informal language if he/she want to. Have the teacher write this down. Then have the teacher state the conjecture in formal mathematical language "as it would in the textbook." Have them write this down if it is different.

    b. Give the teacher 10 minutes total to measure, observe and conjecture. If the teacher cannot come up with a conjecture proceed to 7.

Another teacher (Mary) did this activity as well. Her conjecture was that "A midsegment connecting two sides of a triangle is parallel to the third side and is half as long." Do you think her conjecture is true? Why or why not?

10. Prove or disprove Mary's conjecture. You might write your explanation as an argument in paragraph form or as a two-column proof.

If the teacher has difficulty producing such as a proof, consider the following questions. Give the hints one a time, letting the teacher consider it to see if this is sufficient to let the teacher proceed.

a) What is the relationship between ΔADE *and* ΔABC? *(If you need help, see question 2)*

b) *Do these two triangles have the same shape? So* what is the relationship between *them?*

c) Can you prove that $\Delta ADE \sim \Delta ABC$? *If so, please do so. (If you need help, see questions 4-6)*

d) Observe ΔADE *and* ΔABC. Based on the given conditions, what do we already know? (What is the relationship between angle DAE and angle BAC? What is the ratio $\frac{AD}{AB}$? What is the ratio $\frac{AE}{AC}$?)

e) *Can the AA Similarity postulate be used to do the proof? If yes, why? If not, why not?*

f) *Can the SAS Similarity postulate be used to do the proof? If yes, why? If not, why not?*

g) Based on you answers to questions 1-6, explain/prove that $\Delta ADE \sim \Delta ABC$.

h) *What are the properties of similar triangles*

i) *What can you say about* the ratio *DE* to *BC*

j) *What can you say about angle ADE and angle ABC?*

k) *What relationship between DE* and *BC* does Question 10 imply?

l) Based on your answers to questions 7-11, *write a proof explaining that DE is parallel to BC , and is half as long.*

APPENDIX K:

Observation Protocol

Geometry Teaching Observation Protocol

MKT-G Dissertation

Texas State University

I. BACKGROUND INFORMATION

Name of Teacher ____________________________ Announced Observation? ________________

Location of class____

Number of students observed___________ Type of Class__________________________

Observer_____________________________________ Date of observation__________________

Start Time___________________________________ End time________________________________

Observation number______________________

II. DESCRIPTION OF TEACHING CONTEXT

In the space below please give a brief description of the lesson observed, the classroom setting in which the lesson took place (regular classroom, computer lab, seating arrangements, etc.) Capture, if you can, the defining characteristics of this situation that you believe provide the most important context for understanding what you will describe in great detail in later sections. Use diagrams if they seem appropriate.

III. DESCRIPTION OF EVENTS

Record here events that may help in documenting the ratings.

Time	Description

IV. DESCRIPTION OF GEOMETRY LESSON

	Never				Frequently
1. The lesson plan has appropriate objectives for the concept being explored	0	1	2	3	4
2. The lesson plan includes tasks that involve the use of dynamic geometry software.	0	1	2	3	4
3. The activities in the lesson plan include tasks that involve a compass and straightedge/protractor.	0	1	2	3	4
4. The activities in the lesson plan include tasks that involve manipulatives/models.	0	1	2	3	4
5. The activities in the lesson plan include tasks that are investigations or discovery based.	0	1	2	3	4
6. The activities in the lesson plan develop the notion of "figure" rather than "drawing- attending to underlying relationships rather than particulars of a specific drawing.	0	1	2	3	4
7. The activities in the lesson plan are designed to move students from initial conjecture, to investigation, to more thoughtful conjecture, to verification.	0	1	2	3	4

V. DESCRIPTION OF IMPLEMENTED GEOMETRY LESSON

The lesson leads the class to:

	Never				Frequently
1. Measure.	0	1	2	3	4
2. Construct.	0	1	2	3	4
3. Observe.	0	1	2	3	4
4. Investigate mathematical relationships in multiple ways.	0	1	2	3	4
5. Form conjectures.	0	1	2	3	4
6. Test conjectures.	0	1	2	3	4
7. Receive immediate feedback from the teacher about conjecturing.	0	1	2	3	4
8. Be motivated to think mathematically.	0	1	2	3	4
9. Prove (or disprove) their conjectures.	0	1	2	3	4

VI. ASSESSMENT OF QUALITY OF TEACHING

	Never				Frequently
1. Students engage in recollection of facts, formulae, or definitions (memorization).	0	1	2	3	4
2. Students engage in performing algorithmic type problems and have no connection to the underlying concept of meaning (procedures without connections)	0	1	2	3	4
3. Students engage on the use of procedures with the purpose of developing deeper levels of understanding concepts or ideas. (Procedures with connections)	0	1	2	3	4
4. Students engage in complex and non-algorithmic thinking, students explore and investigate the nature of the concepts and relationships (Doing Mathematics)	0	1	2	3	4
5. The teacher has a solid grasp of the geometry content at the level he or she is teaching (grade level geometry knowledge)	0	1	2	3	4
6. The teacher has knowledge of the use of instructional techniques specifically to teaching geometry (mathematical pedagogical knowledge)	0	1	2	3	4
7. The teacher has a deep understanding of geometry to appropriately integrate the use of instructional techniques with concepts inherent in the lesson (mathematical knowledge for teaching)	0	1	2	3	4
8. The teacher guides the teachers through an exploration of a geometric situation.	0	1	2	3	4
9. The teacher assists students with the organization of deductive reasoning.	0	1	2	3	4
10. The teacher leads students to produce a statement of conjecture.	0	1	2	3	4
11. The teacher guides students to a proof of the conjecture.	0	1	2	3	4

VII. ASSESSMENT OF ENGAGEMENT AND DISCOURSE

	Never				Frequently
1. Students are encouraged to share questions, hints, and progress reports with their neighbors.	0	1	2	3	4
2. Students are asked to cooperate with peers by offering help (even when working at their own computer).	0	1	2	3	4
3. Students are asked to cooperate with peers by requesting help (even when working at their won computer)	0	1	2	3	4
4. Teacher circulates, observes (to monitor progress), asks questions, and provides necessary help as students work.	0	1	2	3	4
5. Teacher initiates class discussion when necessary.	0	1	2	3	4

REFERENCES

Conference Board of the Mathematical Sciences (2012). *The Mathematical Education of Teachers II*. Providence RI and Washington DC: American Mathematical Society and Mathematical Association of America.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: what makes it special?. *Journal of Teacher Education*, 59(5), 389-407.

Begel, E. G. (1972). Teacher knowledge and student achievement in algebra. *SMSG Reports*, 9.

Brophy, J. E. (1979). Teacher behavior and its effects. *Journal of Educational Psychology*, 735-750.

Charalambous, C. Y., Hill, H. C., & Ball, D. L. (2011). Prospective teachers' learning to provide instructional explanations: How does it look and what might it take?. *Journal of Mathematics Teacher Education*, 14(6), 441-463.

Charalambous, C. Y., Hill, H. C., & Mitchell, R. N. (2012). Two negatives don't always make a positive: Exploring how limitations in teacher knowledge and the curriculum contribute to instructional quality. *Journal of Curriculum Studies*, 44(4), 489-513.

Chinnappan, M., & Lawson, M. (2005). A framework for analysis of teachers' geometric content knowledge and geometric knowledge for teaching. *Journal of Mathematics Teacher Education*, 8(3), 197-221.

Clements, D. H. (1999). Teaching length measurement: Research challenges. School Science and Mathematics, 99(1), 5–11.

Creswell, J. W. (2013). *Qualitative inquiry and research design: Choosing among five approaches* (3rd ed.). Thousand Oaks, CA: Sage Publications Inc.

Even, R. (1993). Subject-matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education*, 94-116.

Harrell, P. & Jackson, J. *Teacher knowledge myths: An examination of the relationship between the Texas examination of educator standards and formal content area coursework, grade point average, and age of coursework.* Unpublished manuscript. University of North Texas.

Herbst, P., & Kosko, K. (2014). Mathematical knowledge for teaching and its specificity to high school geometry instruction. *Research Trends in Mathematics Teacher Education*, 23-45.

Hill, H. C., Ball, D. L., Blunk, M., Goffney, I. M., & Rowan, B. (2007). Validating the economical assumption: The relationship of measure scores to classroom teaching and student learning. *Measurement*, 5(2-3), 1-7-118.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 372-400.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371-406.

Knuth, E. J. (2002). Secondary school mathematics teachers' concept of proof. *Journal for Research in Mathematics Education*, 33 (5), 379-405.

Krauss, S., Brunner, M., Kunter, M., Baumert, J., Blum, W., Neubrand, M., & Jordan, A. (2008). Pedagogical content knowledge and content knowledge of secondary mathematics teachers. *Journal of Educational Psychology*. 100, 716-725.

McCrory, R., Floden, R. E., Ferrini-Mundy, J., Reckase, M., & Senk, S. (2012). Knowledge of algebra for teaching: A framework of knowing and doing. Journal for Research in Mathematics Education, 43(5), 584-615.

Mitchell, R. & Barth, P. (1999). Not good enough: A content analysis of teacher licensing examinations how teaching licensing tests fall short. *Thinking K-16*, 3(1), n1.

National Center for Education Statistics. (2012). Common core of data. Retrieved from http://nces.ed.gov/ccd/

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Washington, DC: Authors.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher*, 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57(1), 1-23.

Silverman, J., & Thompson, P. W. (2008). Toward a framework for the development of mathematical knowledge for teaching. *Journal of Mathematics Teacher Education*, 11(6), 499-511.

Simon, M. (2006). Key developmental understandings in mathematics: A direction for investigating and establishing learner goals. *Mathematical Thinking & Learning.* 8, 359-371.

Sowder, J. T. (2007). The mathematical education and development of teachers. *Second Handbook for Research on Mathematics Teaching and Learning*, 157-223.

Steele, M. D. (2013). Exploring the mathematical knowledge for teaching geometry and measurement through the design and use of rich assessment tasks. *Journal of Mathematics Teacher Education*, *16*(4), 245-268.

Texas Education Agency. (2010). Texas Examinations of Educator Standards Preparations Manual. Retrieved from http://cms.texes-ets.org/texes/prepmaterials/texes-preparation-manuals/.

Tatto, M. T., Schwille, J., Senk, S. L., Ingvarson, L., Rowley, G., Peck, R., Bankov, K. Rodriguez, M. & Reckase, M. (2012). *Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries. Findings from the IEA teacher and development study in mathematics (TEDS-M)*. International Association for the Evaluation of Educational Achievement.

Tooke, D. J. (1993). Student teachers' mathematical backgrounds and attainment of their secondary students. *Clearing House*, 66(5), 273-277.

University of Michigan. (2008). Mathematical knowledge for teaching (MKT) measures. Retrieved from http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf.

U.S. Department of Education, Office of Planning, Evaluation and Policy Development, & Policy and Program Studies Service. (2010) State and Local Implementation of the No Child Left Behind Act. Washington D.C.

Usiskin, Z. (2001). Teachers' mathematics: A collection of content deserving to be a field. *The Mathematics Educator*, 6(1), 81-99.

Wilson, P. S., & Heid, M. K. (2010). *Framework for mathematical proficiency for teaching.* Unpublished manuscript.

Yow, J. A. (2009). Two mathematics teachers talk about their teacher preparation experiences: One an undergraduate degree and one a masters degree. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, 4. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ859287&site=eds-live.