

EXPERIMENTAL ANALYSIS OF EMPIRICAL MODEL FOR FRICTION AND
WEAR IN RUBBER ON DRY CONCRETE

by

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ABSTRACT

The aim of this study is to discuss and analyze the empirical models of friction and wear in viscoelastic materials such as rubber by direct comparison with the experimental result and suggest possible ways of corrections and improvement. This study has great importance as we use rubber in various products such as shoe soles, tires, O-rings, wiper blades, gaskets, artificial joints, and so on. This work analyzes the common and widely used models for friction and wear with the help of experimental data, analyzes the limitation, and helps to improve them. Although friction in rubber is an important topic, its complex nature makes it difficult to understand. The complex nature of this problem makes developing a suitable model an expansive and time-consuming process. In this study, experiments were conducted on dry concrete by changing the normal load and sliding velocity for different rubber samples, both natural and SRB rubber, and found that model proposed by Dorsch and Archard had the best performance for friction and wear, respectively.

1. INTRODUCTION

Viscoelastic material, as its name suggests, has both viscous and elastic properties. For viscoelastic material, the stress and strain depend on time (Lakes, 2017). There are many uses of viscoelastic material from simple to complex structure around us, it is used for tires, for wiper blades, O-ring in complex machinery, it is also used in daily uses product such as shoe soles, flip-flop, and is used even for a more delicate product as artificial joints. Rubber, in particular, does not follow the classic law of friction stated by Coulomb friction law “The friction force is proportional to the normal force or load, however, independent of the magnitude of the velocity.” (Amontons, 1699), this makes the study of friction in rubber a unique field. Since friction and wear play an essential role in the use and its life span, the topic is an essential and important topic of study. For vehicle tire high friction is desirable as it helps to better control the vehicle, an increase in friction of tire-road will cause better handling. Higher friction coefficient decreases the braking distance it makes it easier and faster to stop the vehicle thus, reduces the chance of an accident. Whereas, in artificial joints, the desire is to have less friction as it will lower the heat produced.

This unique and versatile use case makes the study of friction vital, as per our need, we need a higher friction coefficient so that the product will not slip and has a firm grip, however; we need to minimize the wear rate so the product can last longer. In, most material wear rate is directly proportional to friction i.e., higher friction will have a higher wear rate. However, rubber wear is complicated and has multiple factors such as pressure, sliding velocity, shape, temperature material composition. Friction and wear are

very important for practical uses of rubber but are still not fully studied and analyzed.

There have only been few attempts to model these mechanisms theoretically. Due to lack, a concrete theory most industry uses empirical models. The model works well, however, has limits. These models are created by using a set of data in specific conditions under which experiments were performed. These models often cannot work under a vast range of variables. However, these flaws can be addressed if all the conditions are included in the variable, which makes the process not workable. Making a model for a wide spectrum of the condition can be costly and time-consuming.

The complexity of this subject is often simplified with the help of empirical modeling where experimental data are used in creating a simple equation that helps in the prediction of friction coefficient (Radó, 1994). An empirical model is a time and money intensive process, a basic ideology for creating the model is to create the real-time process in a controlled environment as per their specification, for example, the use case, the operating temperature, use environment, and manipulating the variable and factors in a case-by-case stage. The experimental data is used to create the regression model for the analysis of the product specification, the life of the product, and quality. Creating a model is a very expensive task that not all companies can effort or have time to be invested in, so most times, the company uses the model already out there and which fulfills their requirement. All the empirical models have limitation and boundaries so cannot be used in all cases and is a perfect and efficient way (Radó, 1994). Thus, in this study, we will see which models have better performance and ways to improve them.

2. THEORY AND BACKGROUND

Rubber friction is a unique topic of study as the viscoelastic material does not follow the classic theory of friction, which makes analysis furthermore complex. Even though the study is complex, its high practical use makes it an important topic that needs to be explored. Rubber friction and wear are both interrelated, which makes it essential for a proper explanation of friction by the study of wear and friction. The classical theory states that friction is proportional to the normal force or normal load and independent of sliding speed, which has been proven wrong in different phenomena and material. This theory is specially not true for viscoelastic material such as rubber (Schallamach, 1963, Grosch, 1963). There is more variable which affects Rubber Friction such as roughness of the substrate, the material properties of the solids in contact and temperature and sliding velocity. To fully understand friction and wear, multi-scale phenomena that occur between contact need to be understood. Thus, the following topic plays a vital role in this study.

The work of Hertz, 1881, is one of the vital studies of contact mechanics. His basic assumption was there is minimal friction contact between two elastic solids which has a smooth surface. The theory was developed with the assumption of perfect condition however, this theory is still widely being used even to this date. There have been a lot of developed theories based on Hertz for a more complex surface by adding and changing the assumptions so we can group them in the derivatives of Hertz theory. Person, 2001 proposed a novel approach in contact mechanics which differed from Hertz's theory.

Persson's theory could model the multi-scale contact of rough surfaces far more accurately than previous contact theories.

Persson's, 2001, 2007 model is more realistic contact mechanics which was developed based on a multi-scale surface and capable of measurement of surface roughness using different techniques such as stylus instrument, atomic force microscope, and scanning tunneling microscope. Persson et al., 2004 were able to successfully predict the linear relation between the contact area and normal load by using a surface roughness power spectrum got from the surface height profile. Later a corrected factor of order unity was added to fit the theoretical result and was not restricted to a specific condition or material behavior (Persson, Yang, 2008).

2.1 Rubber Viscoelasticity

Rubber is a general example of viscoelastic material; it is composed of long flexible macromolecular chains of high-weight monomer combined as a polymer. The basic propriety of rubber is identical to the property of the monomer such as chemical and mechanical strains. As a stable state, the macromolecular chain stays in the amorphous state, however, when an external force is applied, the polymer chains align along the deformation direction and also result in reduced entropy and generate heat (Saccomandi and Ogden, 2004). For raw rubber, there are no molecular bonds between the chains which cause their polymer chain to slide over each other when temperature above their glass transition is applied. (Goodyear, 1844) studied rubber and Sulphur above its melting temperature to reduce viscosity and called the process as vulcanization, which also helped to improve the mechanical properties.

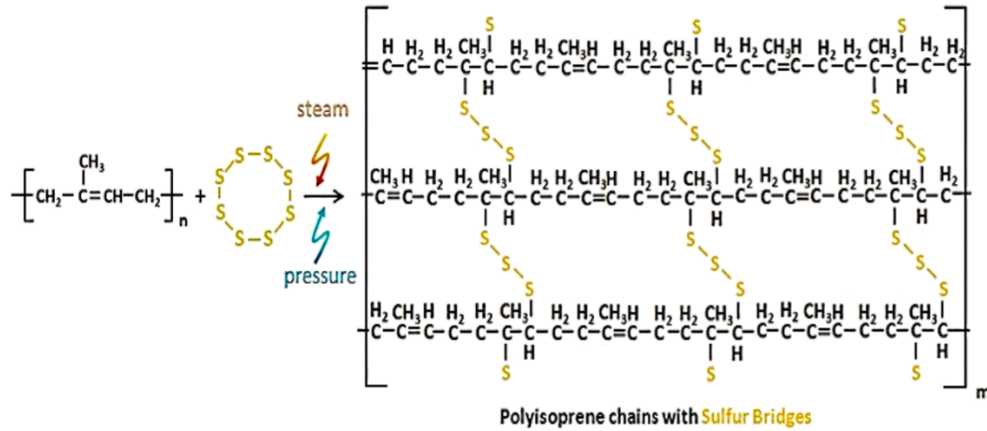


Figure. 1: Vulcanization of Natura Rubber (Zhen, 2015)

Vulcanization made rubber able to undergo large deformation almost up to several hundred percentages and regain the original state, but the rubber was not stiff, which was needed to carry large loads; therefore, carbon black or silica are added in tire treads as filler particles which enhanced the strength. Filler helped in improving the strength but the increase in energy loss because of interfacial friction between fillers and polymers. The energy dissipation is called hysteresis and for large deformations, the polymer detaches from filler resulting in softening of rubber, and this effect is called the Mullins effect (Kumar et al., 2014). Rubber friction is often explained as the combination of adhesion and hysteresis, and depends on the chemical composition of rubber (Bhave et al., 2018).

2.2 Rubber Friction

The surface resistance to a relative motion at the point of contact between multiple surfaces is called Friction and has been subjected to study since the ancient era (Sambursky, 2014). The law of friction was developed by Coulomb and can be summarized as and given as equation 1.

- I. The friction force is independent of the apparent contact area
- II. The friction force is proportional to the normal force
- III. The static friction force is greater than the kinetic friction force.
- IV. The kinetic friction is independent of velocity

$$F_f = \mu F_n \quad (1)$$

where F_n is the normal force, μ is coefficient of friction, and F_f is the frictional force (Amontons, 1699).

Equation (1) is commonly known as the Coulomb friction model and is widely used however, experimental values have a high deviation from the model. The fourth law is often proven wrong or violated in many situations, and the friction coefficient becomes a function of sliding velocity. Based on the transition between static and kinematic friction, several models, such as the Karnopp friction model (Karnopp 1985) and Armstrong's model (Armstrong-Hélouvry et al., 1994) also known as a seven-parameter model, have been proposed. For dynamic friction, different models have been proposed which discuss the stick-slip transition between kinematic and static friction with a sliding velocity as zero. These models were developed for compensation of friction in the control system and have also been used in modeling dynamic friction in tire-road interaction de (Wit et al., 1999). However, it cannot take into consideration the viscoelastic behavior of rubber under different sliding conditions. Thus, models have been developed especially for rubber friction considering the viscoelastic property.

Understanding wear rate can be easy but predicting it is a complex process. However, it is essential that all the factors that impact rubber friction and wear be properly studied and understood. Rubber wear (abrasion) is can vary according to the contact surface.

2.3 Rubber Wear

Wear can be defined as the volume loss from the contact surface of two substances caused because of sliding against each other. Wear is related to the contact area and friction coefficient of both surfaces in contact. In most cases, it is desired to minimize material loss because of wearing while maximizing the friction between those surfaces.



Figure. 2: Needle traces on carbon black filled rubber (Schallamach, 1952)

Wear rate is formulated as volume or weight loss per unit sliding distance. For tires, it is usually expressed as the thickness or height of lost material per traveled distance. There are a lot of factors involved that result in wear rate not limited to material properties and surface topography of the bodies in contact.

Understanding wear rate can be a tough task and predicting it is even more complex however, it is essential that all the factors that impact rubber friction and wear to be properly studied and understood.

Rubber wear can vary according to the contact surface. While sliding on sharp asperities is also known as abrasive creates abrasion lines parallel to sliding direction unidirectional sliding on a smooth and clean surface the adhesive wear results as a ripple perpendicular to the sliding direction and this pattern is known as ridge pattern (Zhang, 1984). Where the sliding direction changes periodically, no pattern can be found.

3. EMPIRICAL MODEL

The empirical model is created with the key principle of William James's radical empiricism which states "the only things that shall be debatable among philosophers shall be things definable in terms drawn from experience" (James,1907) we should not describe theories that incorporate supernatural entities and thus cannot be measured or experienced with sensory perception. For instance, if we see or experience something affected by a cause, we can relate them together, even something as simple as it's cold when the sun is blocked by clouds. The same idea is used for the empirical model but more scientifically and logically.

As mentioned in previous chapters, both the friction and wear of rubber are affected by different variables and this creates studying them a complex task. Thus, this complexity is often simplified with the help of a different empirical model. Over the period, different models are proposed and are used to determine the friction and wear in rubber over different contact substrates. To reduce complexity modeling is done over a small spectrum of variables and with a controlled environment. Because of this, most models cannot predict a good result over a larger range of variables and improve the performance more and more variables need to be introduced in the experiment, thus making it a complex, time and resource-intensive process. Over the period, many models were proposed and our aim for this study is to analyze them by using our experimental data. Our experimental data were obtained by rubber and concrete unidirectional sliding however, the model we are analyzing were modeled over different contact pattern and substrate.

3.1 Empirical Model for Friction

The viscoelastic material has been studied with many contact surfaces over the year as a rubber-like substance does not completely obey Coulomb's fourth law of Friction, the friction in rubber is also strongly dependent on the velocity of sliding (Gorsch, 1963). He found that friction increases with an increase in velocity to a peak value after which it drops and found that temperature influences friction. There has been more research to better see the variable that affects friction in rubber. We are going to analyze the following empirical model for friction:

3.1.1 Rado Empirical Model

Rado proposed a model called the “logarithmic friction model” under the transient condition (Rado, 1994). He developed it for the friction coefficient-slip speed curve, he aimed to develop a model that will describe the entire slip curve, which is shown below:

$$\mu(v) = \mu_p e^{-\left(\frac{\ln(\frac{v}{v_p})}{C}\right)^2} \quad (2)$$

where C is the surface dependent parameter, μ_p is the peak value for friction coefficient, v_p is peak velocity, and v is the velocity.

The model was evaluated by fitting the data measured on 10 different surfaces. The parameters obtained are shown in Table 1.

Table 1. Parameters of Rado Empirical Model (Rado, 1994)

Velocity:56-63 km/h		
μ_p	C	v_p
0.44063	3.013849	13.368123
0.488362	2.936597	20.464507
0.572	2.622164	16.299509
0.571182	3.075427	19.627841
0.661153	2.750673	23.604562
0.9442099	2.484152	13.358819
0.985793	2.477424	17.95673
0.605036	3.057926	20.688211
0.78144	2.981963	-
0.799925	2.535131	13.311376

3.1.2 Savkoor Empirical Model

Savkoor states that the performance of the tire on road depends on the lubrication of the road (if the road is wet or dry). At a low speed, the friction coefficient is not dependent on velocity and the dynamic and static friction coefficient is almost the same (Savkoor, 1966). His model can be shown as equation 3.

$$\mu(v) = \mu_s + (\mu_{\max} - \mu_s) \exp \left\{ -h^2 \log^2 \left(\frac{v}{v_{\max}} \right) \right\} \quad (3)$$

where μ_s is the static coefficient of friction v_{\max} is the velocity at which maximum friction μ_{\max} occurs, h is an experimental constant, and v is the velocity.

3.1.3 Huemer Empirical Model

Huemer used a linear friction Tester by Continental AG, Germany to experiment over a concrete surface (Huemer et al., 2001). They concluded that friction of rubber on the different road surfaces highly dependent on the contact normal pressure and the relative sliding velocity and the environmental temperature. This model can be stated as:

$$\mu(v, p) = \frac{c_1 |p|^{c_2 + c_3}}{c_4 + c_5 ||\vec{v}||^{-\frac{1}{c_6}} + c_7 ||\vec{v}||^{-\frac{2}{c_6}}} \quad (4)$$

where p and v are pressure and velocity respectively and $c_1, c_2, c_3, c_4, c_5, c_6, c_7$, are experimental constant.

3.1.4 Dorsch Empirical Model

Dorsch conducted a linear friction test using abrasion paper (safety walk) to study the friction for tire tread over a rough surface and proposed a quadratic formula shown in equation 5 (Dorsch et al., 2002). Tests were done on abrasive paper and even on snow, the testing condition, sliding velocity ranged from 0.0005m/s to 10m/s, and pressure varied from 10^5 Pa to 10^6 Pa. and the resulting friction coefficient ranged from 0.5 to 2.0.

y.

$$\mu(v, p) = C_1 p + C_2 p^2 + C_3 v + C_4 v^2 + C_5 p v \quad (5)$$

where p and v are nominal pressure and velocity respectively and c_1, c_2, c_3, c_4, c_5 , are experimental constant.

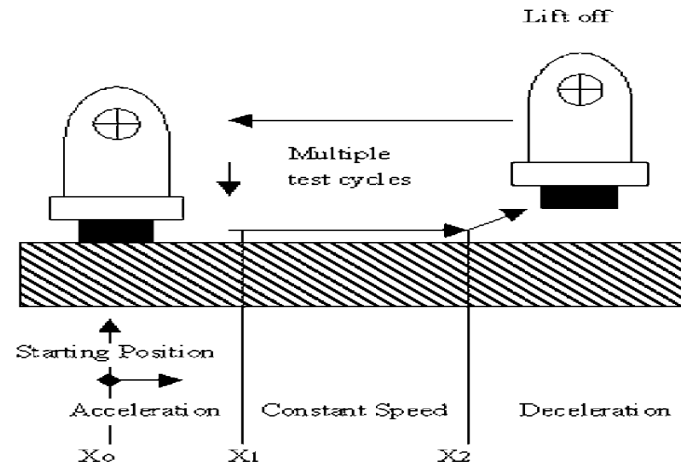


Figure. 3: Schematic Diagram of Linear Friction Tester (Dorsch et al., 2002)

There are more proposed models, but we analyzed these as they are used more often.

Three wear models were studied and are explained next.

3.2 Empirical Model for Wear

Schallamach has investigated rubber wear since the early 1950s and has conducted many experiments to better understand the problem (Gorsch and Schallamach, 1970). He experimented by scratching a needle on different rubber even though this process is not completely applicable, but it has helped in understanding the abrasion in rubber. This process creates a discontinuous series of tear-like tracks on the substrate, which is caused due to stick-slip mechanism. When the needle penetrates the rubber, the tip creates a hole and is then stretches the rubber surface in the sliding direction. During this experiment, Schallamach also found out that the depth and deformation depend on the load applied and hardness of rubber components. Further (Schallamach, 1953) observed a saw-tooth abrasion pattern when rubber was slid over a rough surface. Over the period Schallamach has found out that wear rate has a nonlinear function of nominal pressure (Gorsch and

Schallamach, 1970). There have been many attempts to model the wear of rubber, and we have analyzed some which are listed below.

3.2.1 Ratner empirical model

Ratner considers three stages resulting wear which are the deformation of contact area surface, which is determined by the hardness of the surface, wear caused by relative motion opposed by frictional force and the tear and rupture of material at the junction which is equal to integral of stress-strain relationship (Ratner et al., 1964) and his model can be formulated as:

$$\Delta M = K \frac{\mu \Delta L}{H \sigma_u e_b} \quad (6)$$

where K is a proportionality factor, μ is the coefficient of friction, H is the hardness of the material, e_b is the elongation at break, σ_u is the ultimate tensile stress, ΔL is sliding distance and ΔM is the mass loss.

3.2.2 Archard empirical model

The model proposed by Archard in 1957 is also called general wear law and most models are based on the same idea (Archard, 1957). He shows that the wear is inversely proportional to the hardness of the rubber and proportional to load and the sliding distance. Overall, this was a simple equation, and the proposed model can be formulated as:

$$\Delta M = K \frac{F_N \Delta L}{H} \quad (7)$$

where F_N is normal load, ΔL is the sliding distance, H is hardness, K is the constant and ΔM is the mass loss.

3.2.3 Zhang Empirical Model

In the work by Zhang in 1984, many concepts of elastomers were discussed, and he also proposed a model for rubber abrasion with a razor blade where he correlated the loss was proportional to the work by friction (Zhang, 1984). The model can also be written as equation 9.

$$\Delta M = K_1 W_f^n \quad (8)$$

where ΔM is the mass loss, K_1 and n are the empirical coefficient and exponent respectively and W_f is the work done by friction.

This model is also called the line contact model. In this modeling, tests were conducted using four different rubber samples, namely natural rubber (NR), nitrile rubber (NBR), styrene-butadiene rubber (SBR), and polyurethane (PU), the sliding speeds were (0.06 m/s, 0.10 m/s, 0.16 m/s, and 0.20 m/s) and the normal load (8 N, 10 N and 12 N). The test schematic is shown in Figure 5. In this experiment razor blade moves in a single circular path, thus it is also referred to as the line contact model (Zhang, 1984).

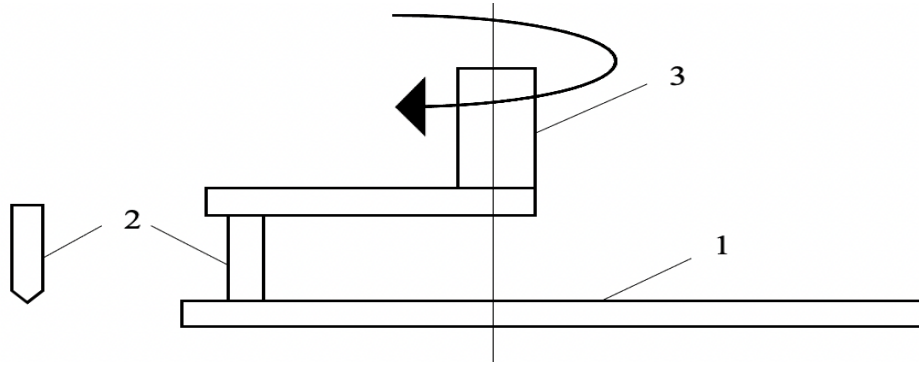


Figure. 4: Schematic Drawing of Friction and Wear Tester (Zhang, 1984)

1-rubber; 2-razor blade; 3-driving shaft.

These are the empirical models been studied and fitted the experimental data.

4. EXPERIMENT

Fisher in 1960 introduced the concept of applying statistical analysis during the planning stages of research rather than at the end of experimentation. When statistical thinking is applied from the design phase, it enables us to build quality into the product by adopting Deming's profound knowledge approach, comprising system thinking, variation understanding, a theory of knowledge, and psychology (Fisher, 1960). Use of one-dimensional research, i.e., one parameter fixed at a time which is time-consuming and is not cost-effective and the other way is fractional design methodology which helps to study the effect of the number of different factors simultaneously. Fractional experimental help in experimenting in an organized manner rather than conducting a series of single-factor experiments.

The experimental process used for this research is going to use the factorial design. In this design, the processor input parameters are intentionally and simultaneously differed according to a predetermined factor. The reason for choosing a factorial design is that it gives us the freedom of adjusting each factor independently and thus matches our idea for data generation. The variables considered for this study are listed below.

Table 2. Input Parameters

Sample type	Eight rubber types
Normal Load	45.70 N, 67.94 N, 90.18 N, 134.66 N
Velocity	0.025 m/s, 0.041 m/s, 0.057 m/s, 0.073 m/s, 0.089 m/s

4.1 Test Setup

For the experiment data, “Sliding Friction and Wear Test Setup” designed and developed at the ROBOCO lab at Texas State University. A test setup was developed for linear sliding motion which will induce the sliding motion, of rubber on a different substrate. For this experiment, concrete is used as the test substrate. This setup was developed for precise unidirectional movement of the sample with the controlled speed with minimum noise and low friction sliding. The rubber sample is in the lower part of the test block, the sample has a longitudinal direction of the setup during the test. The setup can conduct an experiment which is as close to as the real-life situation and the experimental data obtained can be used to analyze and study the Friction, Wear, Abrasion Pattern, Contact Noise of Different samples(rubber) on the different surfaces including asphalt and concrete, however, for this study concert and rubber was used and the specification are listed in table below.

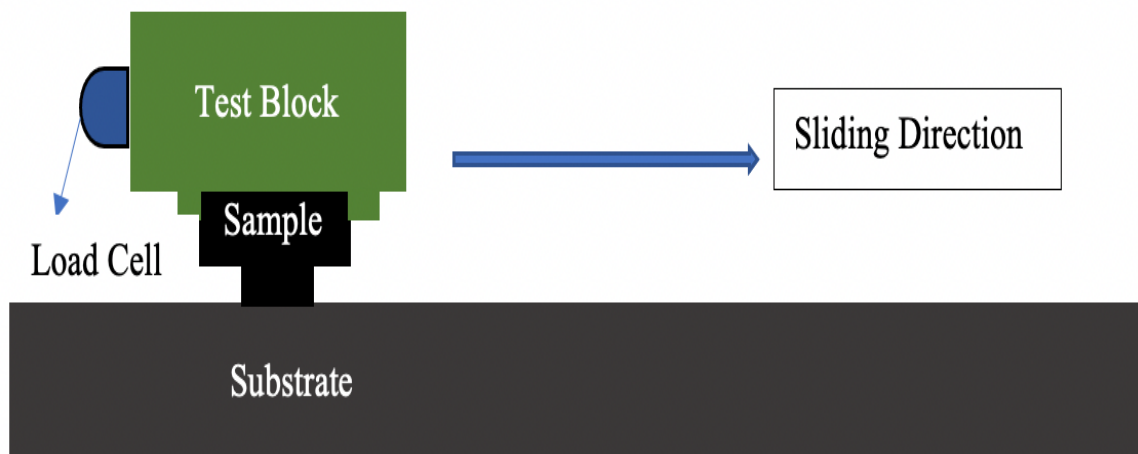


Figure. 5: Schematic Diagram of Test Setup

Table 3. Test Setup Specification

External dimensions	Length = 2,960mm, Wide = 590mm, Height = 415 mm
Substrate dimension	Wide = 127mm, Length = 2,150mm, Height = 50.8mm
Maximum linear speed	0.2 m/s

4.2 Rubber Sample

Experiments were conducted using eight different rubber types that were received from Sumitomo tires. We tested two types of rubber Styrene-butadiene rubber (SBR) and natural rubber. Samples A-D was SBR with different compositions of SRB polymer and are reinforced with 20% Carbon black. Samples E-H were natural rubber with a low percentage of SBR and were reinforced with 50% Carbon black. The hardness and Glass transition (T_g) for different rubber samples are shown in Table 4.

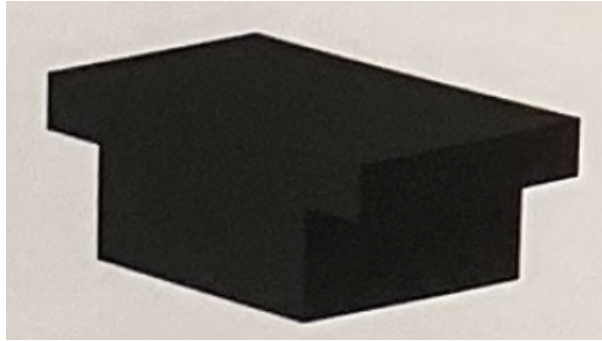


Figure. 6: Rubber Sample

Table 4. Hardness, Glass Transition & Hysteresis of Rubber

Sample	A	B	C	D	E	F	G	H
Hardness	58	58	59	58	60	60	59	60
T _g	-8.2	-21.3	-21.7	-36.0	-52.5	-42.5	-51.6	-50

4.3 Experimental Data

Before any experiments were done, all the samples are first rounded by trimming the edges. This was done because the sample which was supplied has sharp edges. These edges can rip during the experiment, causing outliers in the data. After the samples were rounded, the test was performed with all eight samples simultaneously. The combination for the test run is given in Table 6 below. Each test setup is run 4 times and with each run before and after the weight of the rubber sample is measured to obtain the mass loss. 5 different speeds for the test run and 4 different loads, including the sample holder which is considered as no load is being used. In total, there are 160 different testing conditions, which include eight different rubber types, 5 different sliding speeds, and 4 different normal loads. There are 640 different data as each test setup is run 4 times.

The reason for this extensive data size is the rubber wear is dependent on both normal loads and the sliding speed, so to analyze these different variables and their effect on the wear rate, the test size is necessary. Different rubber composition plays a vital role in the wear rate so, the tire company had provided us with these different rubber and better understand what types of rubber composition can help the company in producing a quality tire.

Table 5. Testing Conditions

Velocity (m/s)	Distance (m)	Run Time (S)	Load (N)	Pressure(kpa)
0.025	1.63	65.5	45.70	70.79
0.041	1.67	40.3	45.70	70.79
0.057	1.65	29	45.70	70.79
0.073	1.63	22.4	45.70	70.79
0.089	1.66	18.7	45.70	70.79
0.025	1.63	65.5	67.94	105.24
0.041	1.67	40.3	67.94	105.24
0.057	1.65	29	67.94	105.24
0.073	1.63	22.4	67.94	105.24
0.089	1.66	18.7	67.94	105.24
0.025	1.63	65.5	90.18	139.69
0.041	1.67	40.3	90.18	139.69
0.057	1.65	29	90.18	139.69
0.073	1.63	22.4	90.18	139.69
0.089	1.66	18.7	90.18	139.69
0.025	1.63	65.5	134.66	208.59
0.041	1.67	40.3	134.66	208.59
0.057	1.65	29	134.66	208.59
0.073	1.63	22.4	134.66	208.59
0.089	1.66	18.7	134.66	208.59

Table 6. Experimental Run

Load/ Velocity	45.699 N	67.94 N	90.182 N	134.6649 N
0.025 m/s	A to H	A to H	A to H	A to H
0.041 m/s	A to H	A to H	A to H	A to H
0.057 m/s	A to H	A to H	A to H	A to H
0.073 m/s	A to H	A to H	A to H	A to H
0.089 m/s	A to H	A to H	A to H	A to H

4.4 Data Collection

The value for wear was obtained by measuring the weight of the sample after each test run using a measuring scale, and the mean of all four tests was calculated as average weight loss for each sample. For the friction coefficient, the data collected by the load cell during each run was used. The load cell collects the friction force in terms of voltage over time with a sample size of 20,000 per second.

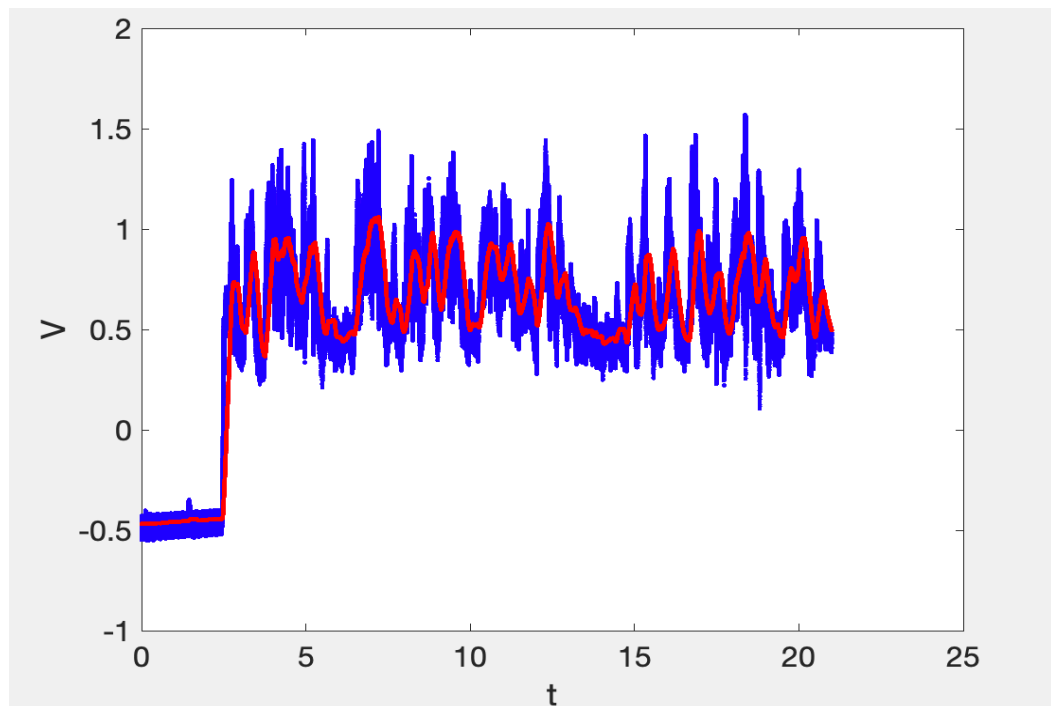


Figure. 7: Filtered Data After Application of Moving Average

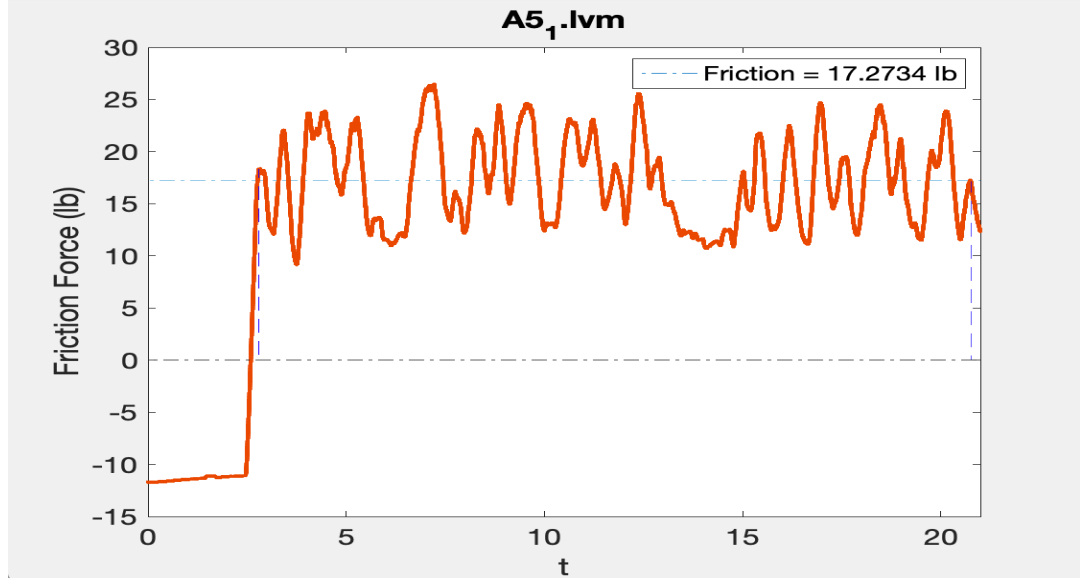


Figure. 8: Data with Average Friction Force

The data then are filtered using a moving average, which is a common tool used to smooth the time series data. For this process, the filter function of MATLAB with a window size of 5000 was used. A sample of filtered data is shown in figure.8. Further, the voltage is converted into Friction Force by interpolation and the friction force for the test run is obtained. The mean for all the local peaks between the first and last local max points of the friction force is calculated to obtain the average, as shown in Figure 8.

These same steps are used for all the tests run. Finally, all the friction force is converted into kg by using a multiplication factor of 0.45392.

The coefficient of friction is then calculated using the frictional value and normal load as the equation as:

$$f = \mu N \quad (9)$$

where f is the frictional force, μ is the coefficient of friction and N is the normal force.

5. RESULTS AND DISCUSSIONS

We used average weight loss and the average friction coefficient of all the replication of each test condition to study the result of sliding velocity and normal load. Table 7 has the friction coefficient value for all the testing conditions. For each sample first-five test run is performed with increasing velocity, however, the normal load for that run is kept constant. And it is carried out for four different dead weights. In Table 7 we can see that with each increment in normal load the friction coefficient decreases, and we can all see there is some increase in friction coefficient with an increase in velocity causes peaks.

Table 7. Values of Friction Coefficient of All Testing Conditions

V (m/s)	Load(N)	μ A	μ B	μ C	μ D	μ E	μ F	μ G	μ H
0.025	45.70	1.78	1.81	1.76	1.75	1.7	1.74	1.66	1.67
0.041	45.70	1.83	1.91	1.88	1.86	1.8	1.78	1.69	1.74
0.057	45.70	1.86	1.88	1.85	1.84	1.8	1.71	1.71	1.79
0.073	45.70	1.79	1.87	1.86	1.84	1.9	1.75	1.72	1.87
0.089	45.70	1.86	1.88	1.87	1.87	1.8	1.74	1.74	1.9
0.025	67.94	1.61	1.63	1.6	1.59	1.5	1.46	1.46	1.53
0.041	67.94	1.67	1.7	1.66	1.66	1.5	1.49	1.51	1.55
0.057	67.94	1.71	1.7	1.64	1.64	1.6	1.5	1.49	1.55
0.073	67.94	1.71	1.73	1.66	1.63	1.6	1.49	1.49	1.57
0.089	67.94	1.73	1.75	1.64	1.63	1.6	1.48	1.5	1.6
0.025	90.18	1.5	1.52	1.47	1.45	1.4	1.34	1.34	1.44
0.041	90.18	1.56	1.61	1.52	1.48	1.4	1.36	1.38	1.45
0.057	90.18	1.53	1.6	1.51	1.51	1.4	1.36	1.36	1.45
0.073	90.18	1.59	1.57	1.51	1.52	1.4	1.36	1.39	1.45
0.089	90.18	1.55	1.56	1.53	1.55	1.4	1.34	1.38	1.46
0.025	134.66	1.34	1.34	1.31	1.27	1.2	1.21	1.2	1.28
0.041	134.66	1.36	1.37	1.32	1.31	1.2	1.22	1.2	1.28
0.057	134.66	1.39	1.37	1.33	1.35	1.3	1.22	1.21	1.31
0.073	134.66	1.37	1.37	1.35	1.33	1.2	1.18	1.2	1.26
88.5061	134.66	1.38	1.38	1.39	1.32	1.2	1.18	1.2	1.28

Table 8. Data for Weight Loss

V (m/s)	F (N)	Weight Loss for unit distance (g/m)							
		A	B	C	D	E	F	G	H
0.025	45.7	0.002	0.00168	0.0034	0.00208	0.000673	0.00092	0.00083	0.00037
0.041	45.7	0.0037	0.00127	0.0019	0.00169	0.000753	0.00093	0.00078	0.00041
0.057	45.7	0.0025	0.00112	0.0016	0.00109	0.000726	0.001	0.0007	0.00039
0.073	45.7	0.0018	0.00087	0.0019	0.00117	0.000736	0.0011	0.00083	0.00038
0.089	45.7	0.0012	0.00098	0.0018	0.00109	0.000695	0.00103	0.00085	0.00042
0.025	67.9	0.0045	0.00428	0.0044	0.00251	0.000918	0.00119	0.00098	0.0005
0.0412	67.9	0.0056	0.00413	0.0039	0.00181	0.00094	0.00111	0.00096	0.00051
0.057	67.9	0.005	0.00442	0.004	0.00224	0.00109	0.00121	0.00112	0.00042
0.073	67.9	0.0052	0.0046	0.0039	0.00319	0.001043	0.00135	0.00123	0.00042
0.089	67.9	0.006	0.00399	0.0042	0.0035	0.001118	0.00139	0.00124	0.00048
0.025	90.2	0.0084	0.00731	0.0063	0.00532	0.001224	0.00144	0.00113	0.00046
0.041	90.2	0.0072	0.0067	0.0061	0.00494	0.001265	0.00139	0.00108	0.00054
0.057	90.2	0.0066	0.00617	0.0062	0.00521	0.001271	0.00151	0.00121	0.00056
0.073	90.2	0.0068	0.00569	0.0061	0.00595	0.001288	0.00151	0.00129	0.00053
0.089	90.2	0.0058	0.00579	0.0066	0.00634	0.001269	0.0016	0.00133	0.00066
0.025	134.66	0.0173	0.01132	0.0113	0.00857	0.001346	0.0015	0.00119	0.0007
0.041	134.66	0.0149	0.01048	0.0114	0.00807	0.001356	0.00147	0.00108	0.0006
0.057	134.66	0.0151	0.00902	0.012	0.00841	0.001271	0.00163	0.00124	0.0007
0.073	134.66	0.0137	0.00873	0.0116	0.00859	0.001227	0.00159	0.00117	0.00061
0.089	134.66	0.0144	0.00846	0.0122	0.00876	0.001178	0.00169	0.00112	0.00066

Table 8 contains all the experimental data for weight loss in grams. Preliminary analysis of the data from table 7 and 8 shows that an increase in normal load decreases the coefficient of friction however, the change in velocity gives some peak values.

Furthermore, we can say both normal load and velocity have an impact on friction and wear.

5.1 Friction Coefficient Analysis

As previously mentioned, for this study we have considered normal load and sliding velocity factor for the friction and wear analysis. And from the data collected, we can say

that both components impact the friction coefficient. The figure below will help to better see what happens with the change in variables. And for isolating each of that one component is kept constant. Figure 9 shows the friction coefficient versus normal load the velocity is kept constant, and the analysis is done in a separate set. Similarly, in figure 10 the data with the same normal load is plotted to see only the impact of velocity.

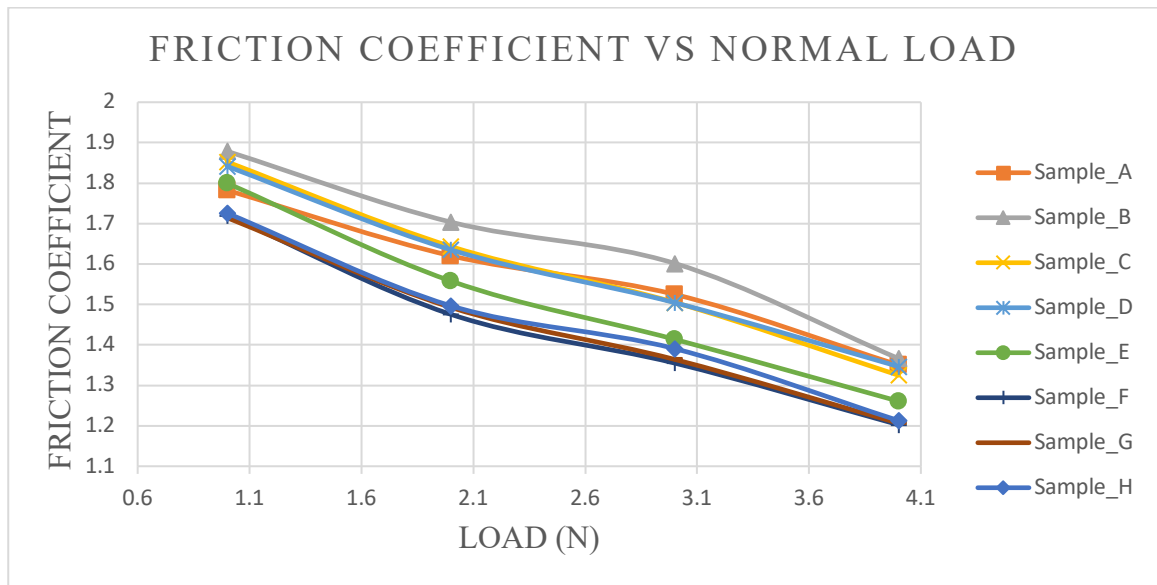


Figure. 9: Friction Coefficient vs Multiple Load for All Samples

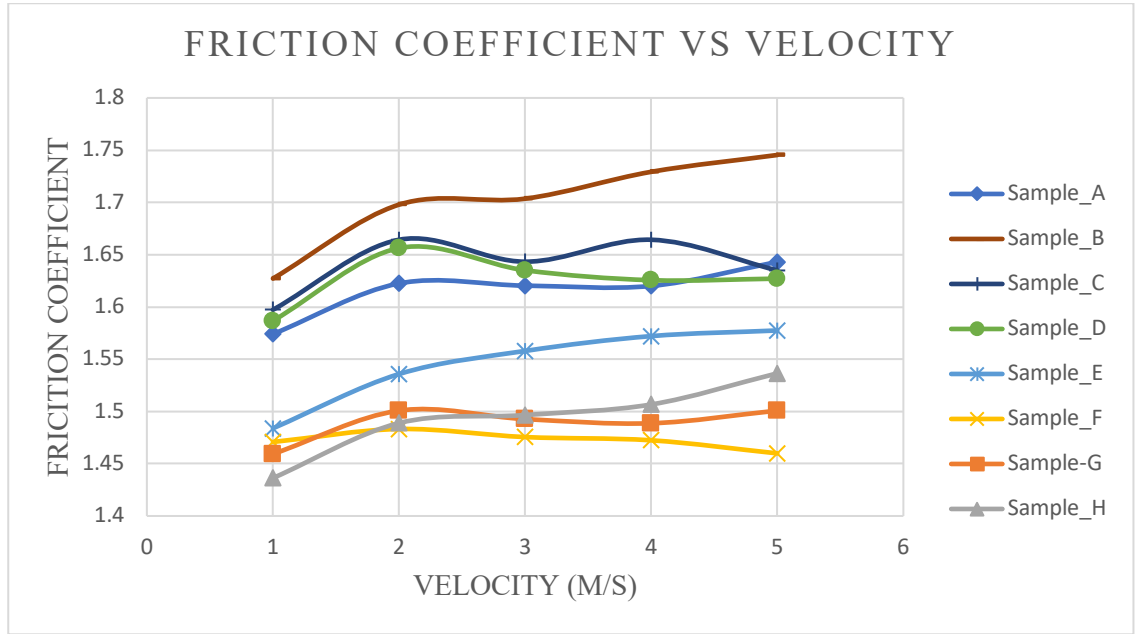


Figure. 10: Friction Coefficient vs Velocity for All Samples

Figure 9 shows that there is a clear decrease in coefficient of friction with an increase in the normal load and this is true for all the rubber types and it can be said that regardless of the material composition, a normal load has an inverse relationship on coefficient. The coefficient of friction versus velocity graph shows there is a change in the coefficient of friction with increasing velocity however, this change is not decisive. Overall, there is an uptrend in friction coefficient, but for some velocity, the coefficient drops and rises for a different velocity. The velocity vs coefficient graph shows that velocity does play a role in the variation of the coefficient of friction. We can say all the factors used for the experiment have an impact on friction, even though the changes caused by velocity are not high, the velocity used for the experiment was low and a wider range can make a more decisive analysis.

5.2 Wear Analysis

As discussed earlier, two types of rubber samples were used, samples A to D and E to H are SBR and Natural rubber, respectively. So, we looked into the data and the wear rate is different for these two. SBR rubber shows higher wear than natural rubber has minimal wear. We captured the image of the contact surface for both rubber types. We observed a greater wear pattern on the samples SBR rubber (A-D) but for the contact surface on natural rubber, there was a light wear pattern and seems to appear plain in comparison to samples A Sample G had very less wear. Figures 11 and 13 show the contact surfaces of samples A and G respectively with the same testing condition. And from the figure, it is clear that natural rubber was less prone to wear than SBR rubbers. After each test, the sample is weighed and the difference in weight before and after the test is calculated gives us the mass loss during a run, and this is the same for all the testing conditions.

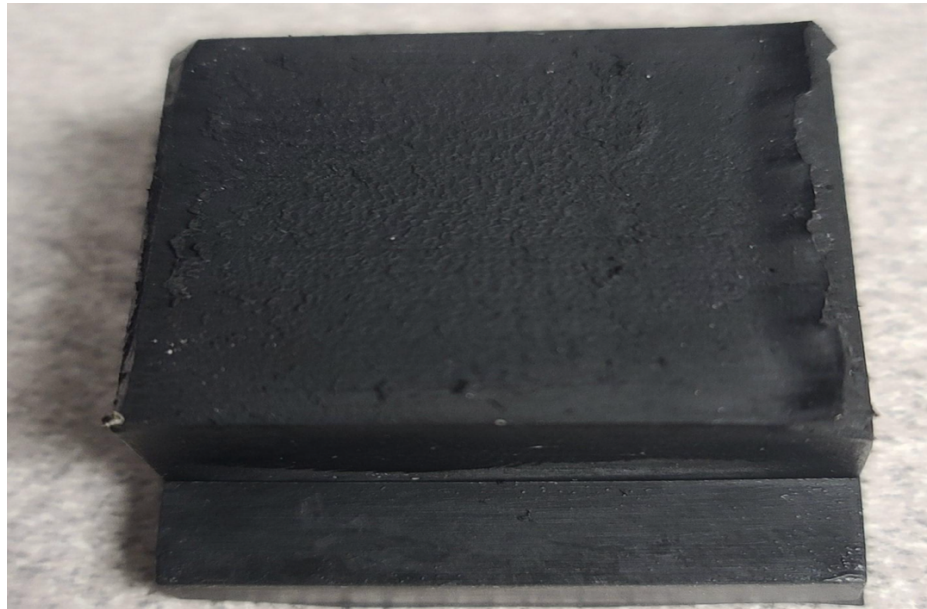


Figure. 11: Sample A After Four Test Run

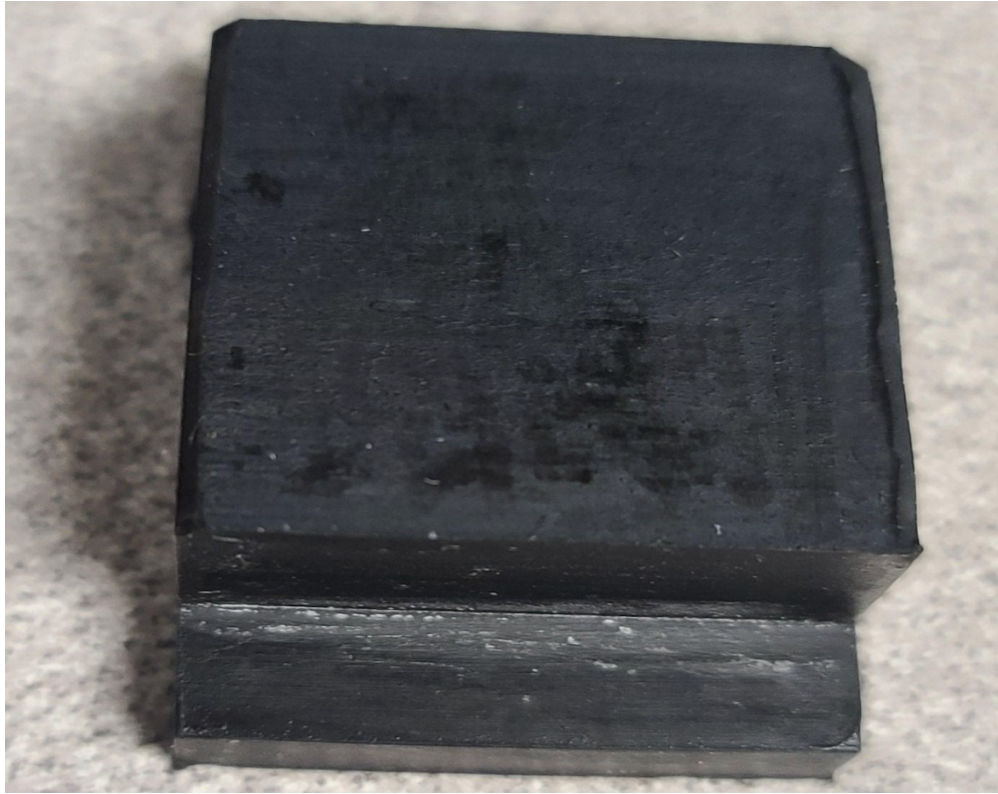


Figure. 12: Sample G After Four Test Run

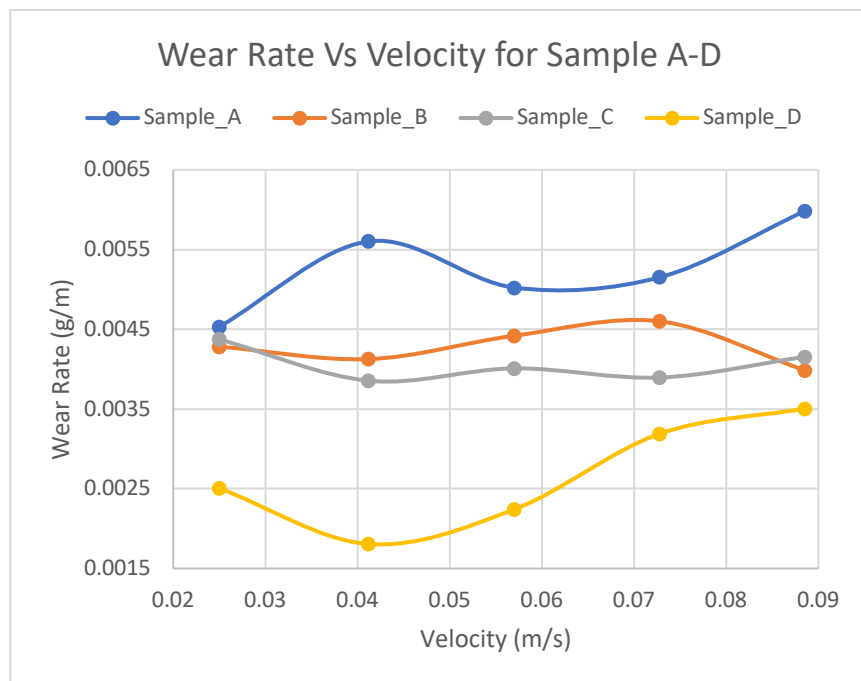


Figure. 13: Wear Rate vs Velocity for SBR Rubber

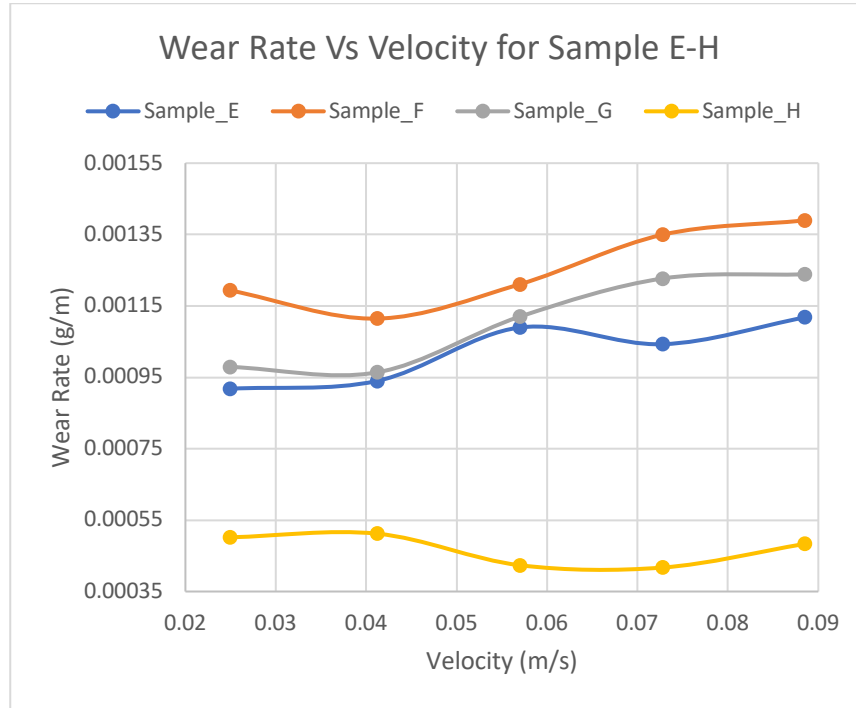


Figure. 14: Wear Rate vs Velocity for Natural Rubber

Figures 14 and 15 show the graph of wear over the change in velocity and despite the magnitude, both types show a similar increasing mass loss creating local peaks in the mass loss.

The graph for wear vs normal load and wear versus friction force is shown in figures 15, 16, 17, and 18, and it can be said that an increase in normal load and friction force results in higher wear. The plot for both friction force and normal load shows that when a higher normal load is applied on the sample, it causes the rubber to wear more. This also shows that friction force and normal load have a similar role for rubber abrasion however the magnitude of mass loss in natural rubber seems to be lower in either case which highlights an important aspect that not only velocity and normal load are vital for friction and wear, but types of rubber also need to be studied.

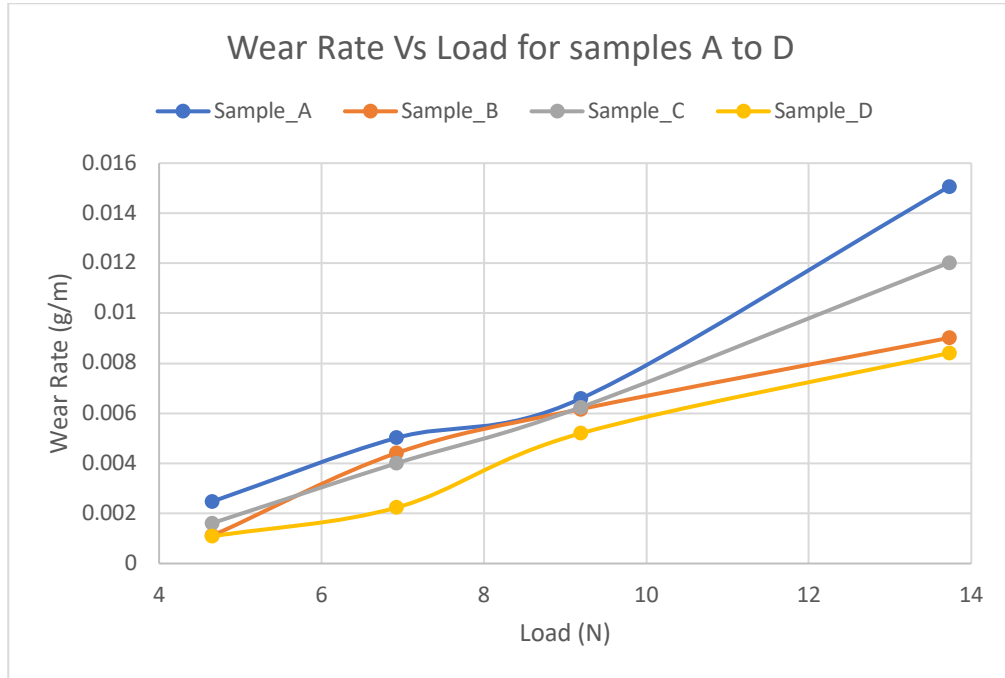


Figure. 15: Wear Rate vs Normal Load for Samples A to D

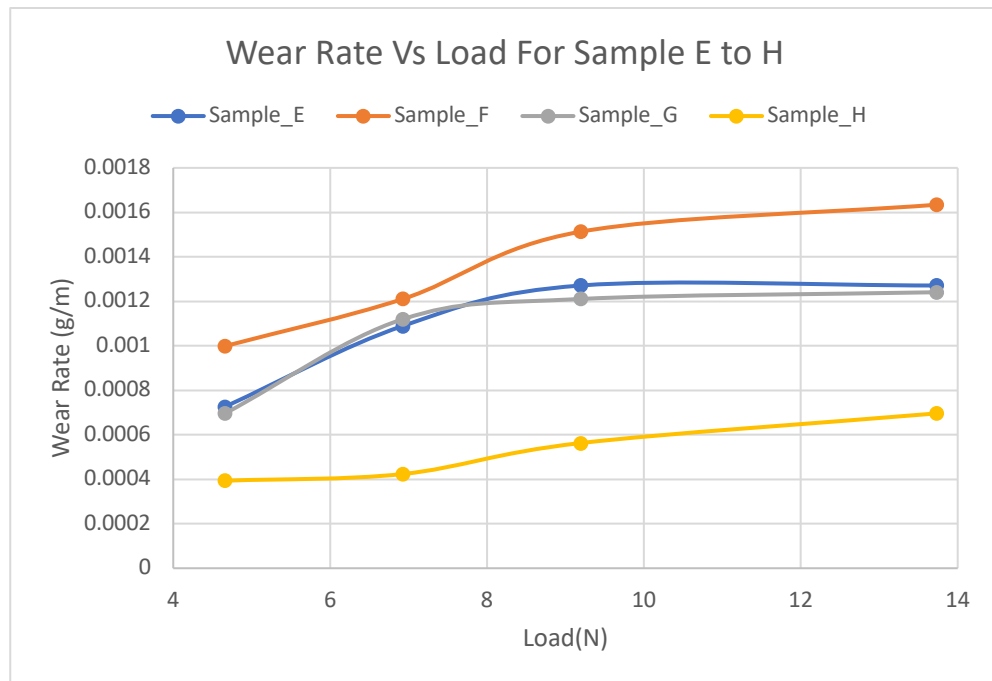


Figure. 16: Wear Rate vs Normal Load for Samples E to G

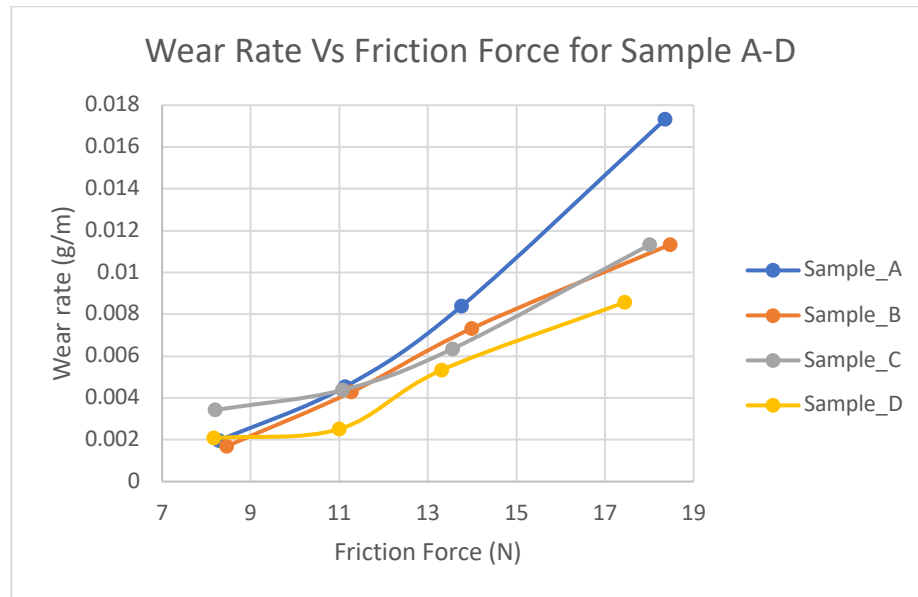


Figure. 17: Wear Rate vs Friction Force for SBR rubber

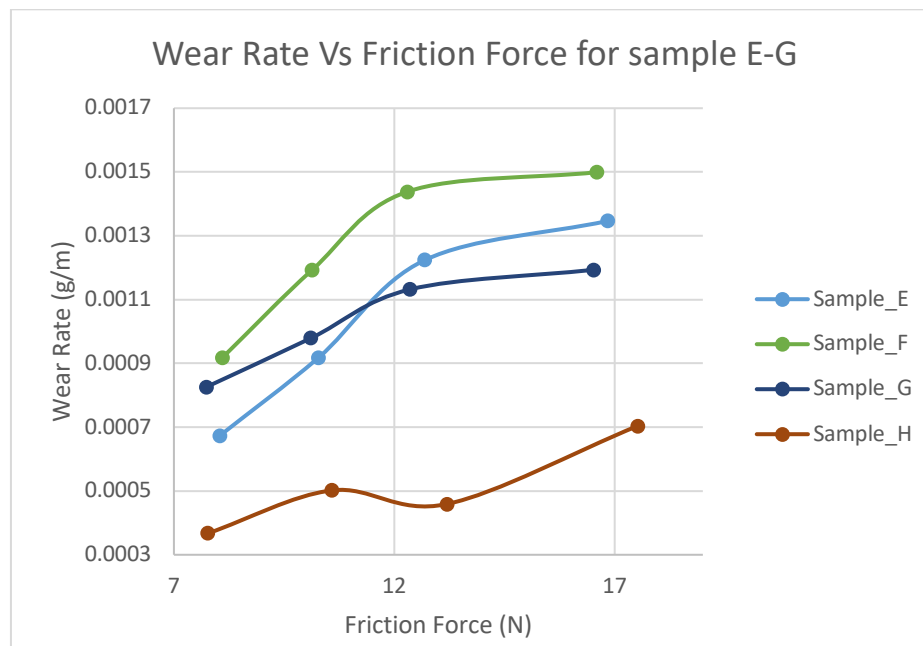


Figure. 18: Wear Rate of Natural Rubber vs Friction Force

It is seen all the factors used for the experiment have an impact on friction and wear, even though the changes caused by velocity are not high, the velocity used for the experiment was low and a wider range can make a more decisive result. And small increment in

velocities can produce a clear result in terms of peaks as local peaks were seen for different velocities. After seeing the relation of both velocity and normal load, the data obtained were fitted to the empirical models.

5.3 Friction Model Analysis

For the model analysis, all the sample types are then analyzed separately using the curve fitting toolbox of MATLAB into all empirical models. The fitting tool was used for the set of each sample type on all the models. As discussed previously, we analyzed four different models for our study and this selection allows us not just to analyze the model but also to see if the experimental data that we collected correlates with the different experimental setup used for the construction of these models. Table 8 shows the performance of fit for all the models in terms of R-square.

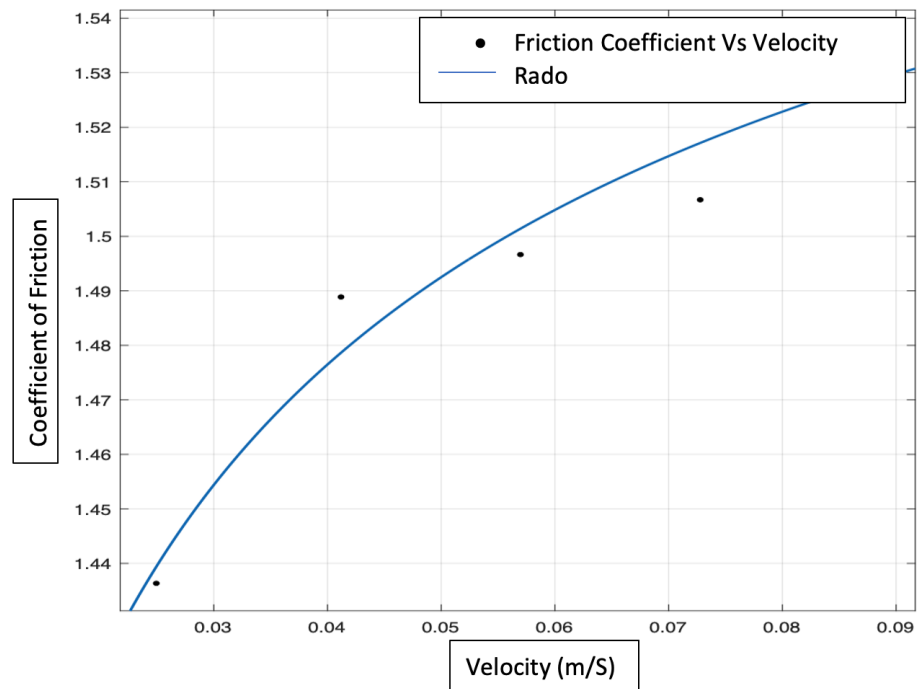


Figure. 19: Rado's Model Curve Fitting for Sample H

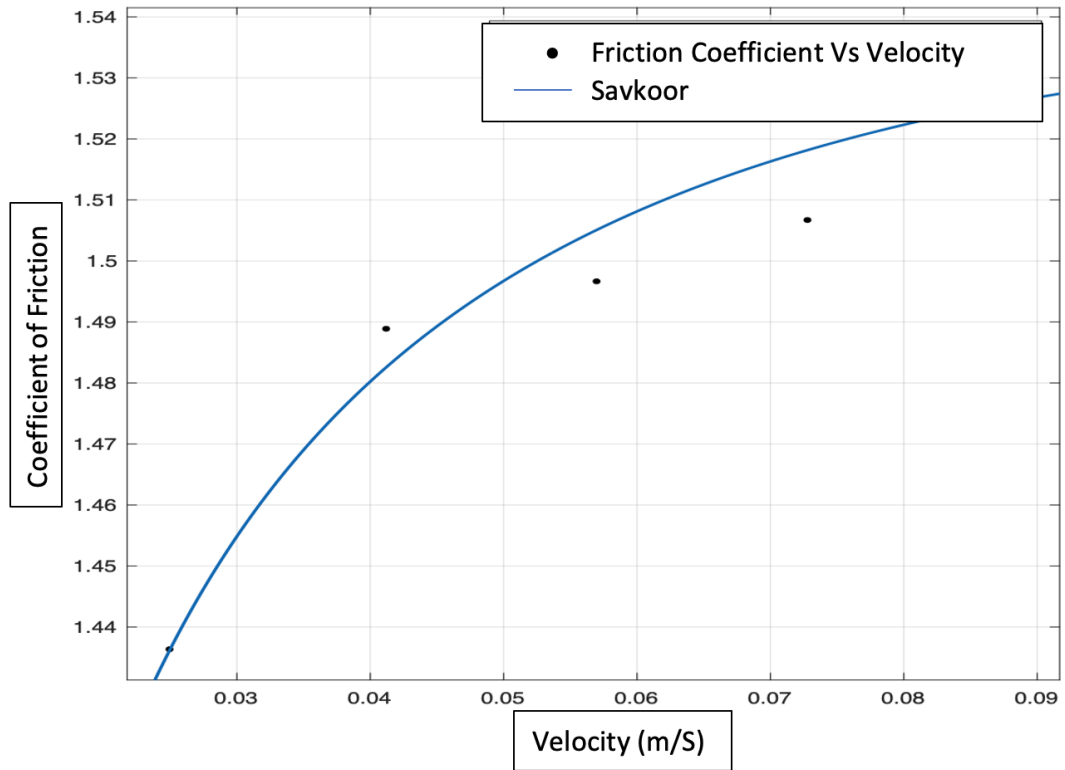


Figure. 20: Savkoor's Model Curve Fitting for Sample H

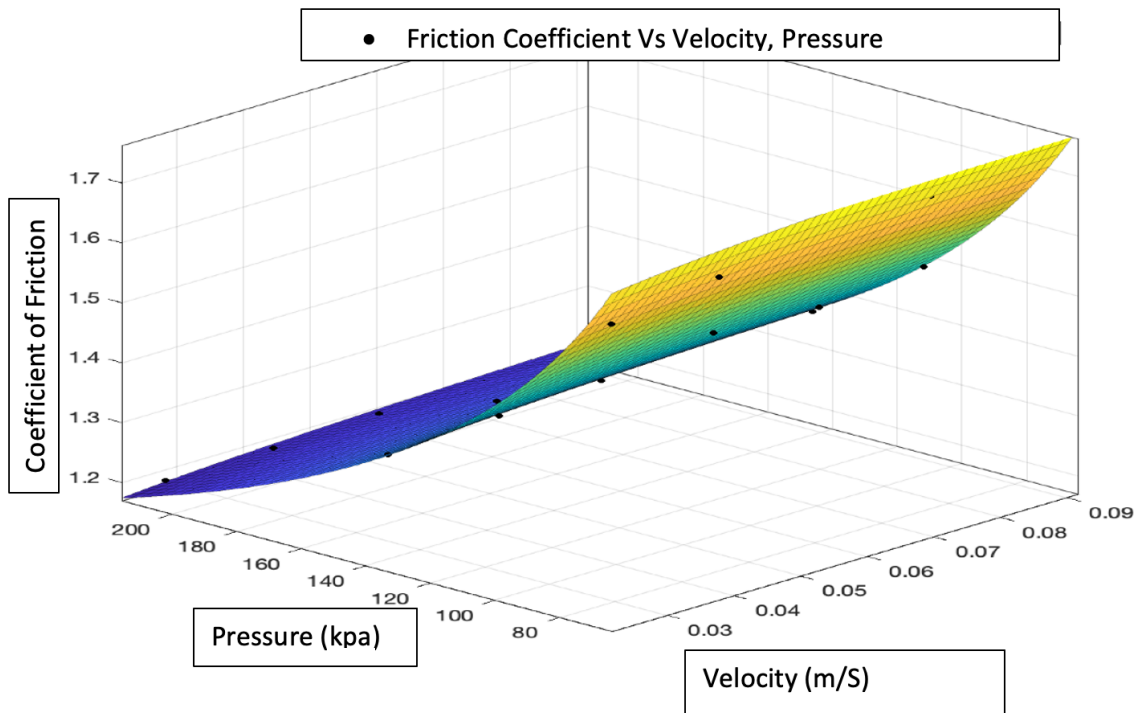


Figure. 21: Huemer's Model Curve Fitting for Sample G

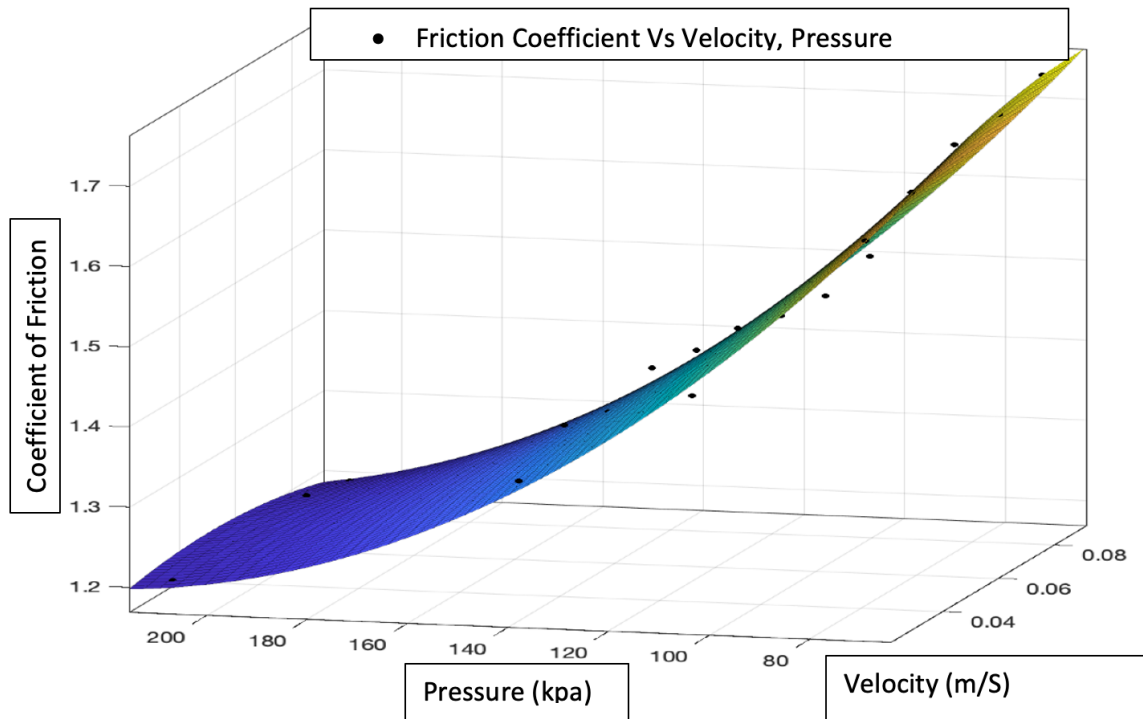


Figure. 22: Dorsch's Model Curve Fitting for Sample G

The Value of R-Square is the proportion of variation in the dependent variable with a response to the independent variable and RMSE is the deviation to the fitted curve or line from the data point, or simply RMSE is the distance between the actual data and the regression (Arkes, 2019). Since concluding with just RMSE can be complicated as there is no actual good value, for instance, the same value can mean a good fit in a case but can be interpreted differently because it is scale-dependent, so we consider R-squared value too to see the goodness of fit. An R-squared closer to 1 means the fit is good and inverse.

We can see the values for R-squared from table 8. Since Rado's and Savkoor's models only consider velocity as the variable and normal load as constant, we have four different sets of data. Overall, both these models fit well to our data however we have only five different velocities, curve fitting for some sets of data does not have a good fit, and also

from our preliminary analysis of data we saw that normal load has more impact than velocity. Huemer and Dorsch's model considers velocity and pressure for the model to have a better performance. When comparing the performance for both these models, Dorsch's Model has a better fit over the other three models.

Table 9. Performance of Model Over Friction Coefficient

Model	Pressure	Sample	A	B	C	D	E	F	G	H
Rado	70.79 kpa	R-Square	0.95	0.74	0.83	0.78	0.90	0.89	0.98	0.98
	105.24 kpa	R-Square	0.86	0.97	0.81	0.78	0.99	0.95	0.72	0.94
	139.69 kpa	R-Square	0.92	0.93	0.78	0.98	0.74	0.95	0.79	0.89
	208.59 kpa	R-Square	0.86	0.84	0.73	0.93	0.77	0.76	0.90	0.85
Savkoor	70.79 kpa	R-Square	0.94	0.74	0.82	0.77	0.89	0.85	0.98	0.98
	105.24 kpa	R-Square	0.86	0.97	0.81	0.78	0.99	0.94	0.71	0.94
	139.69 kpa	R-Square	0.90	0.93	0.78	0.97	0.74	0.95	0.79	0.87
	208.59 kpa	R-Square	0.86	0.84	0.99	0.97	0.79	0.76	0.90	0.85
Huemer	R-Square		0.98	0.98	0.97	0.98	0.98	0.93	0.99	0.98
Dorsch	R-Square		0.99	0.99	0.98	0.98	0.99	0.96	0.99	0.98

From the result for the curve fitting, we see that Dorsch empirical model is the best fit for dry concrete and rubber friction. Next, we fitted that mass loss in Matlab to see the fit of the wear empirical model.

5.4 Wear Model Analysis

For the wear model, models proposed by Ratner, Archard, and Zhang were fitted with the experimental data. As Archard and Ratner have considered, wear rate highly depends on rubber type as we can see from our mass loss. The SBR rubber has a large mass loss than

natural rubber. All the model has a good fit for rubber A to D however, for samples E to H the performance of the models is not well.

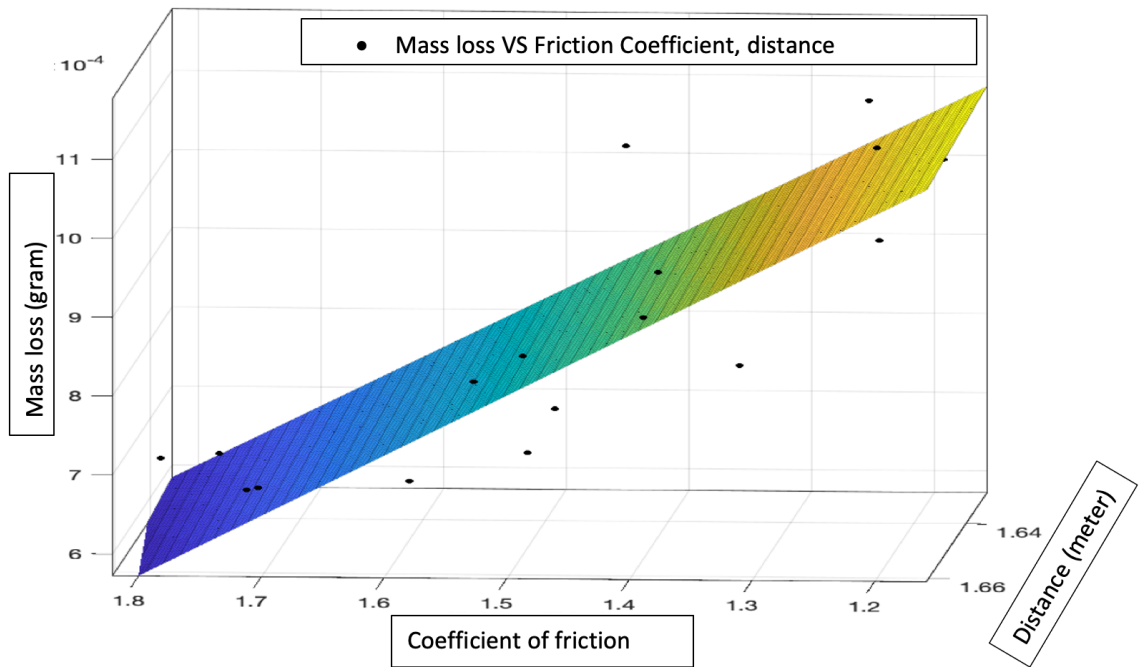


Figure. 23: Ratner Model Curve Fitting for Sample G

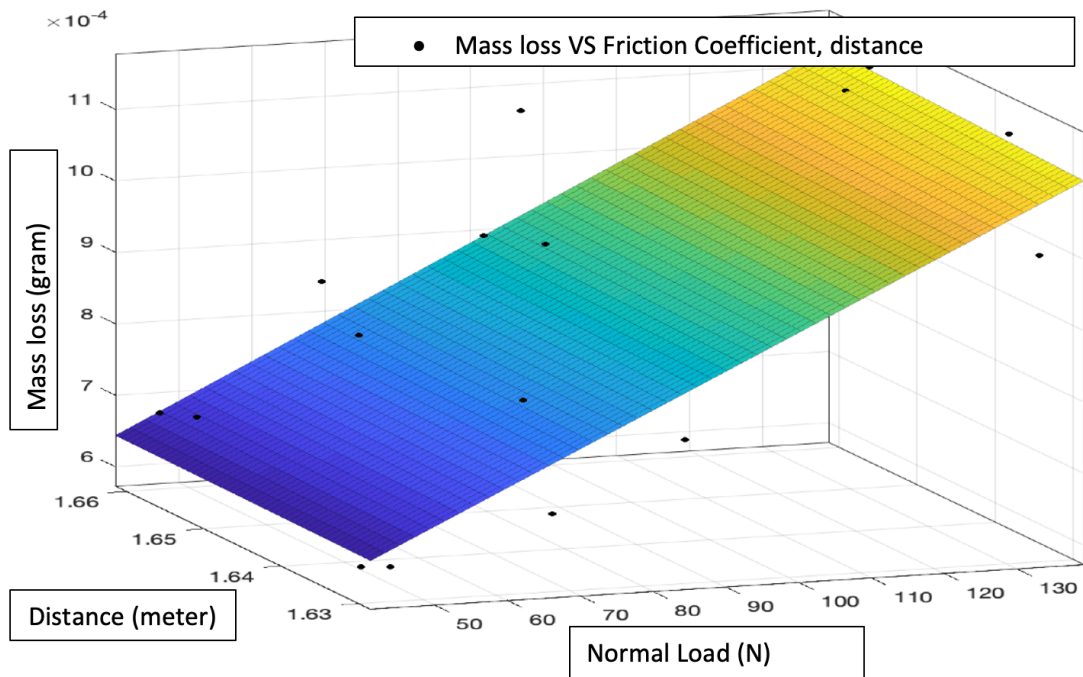


Figure. 24: Archard Model Curve Fitting for Sample G

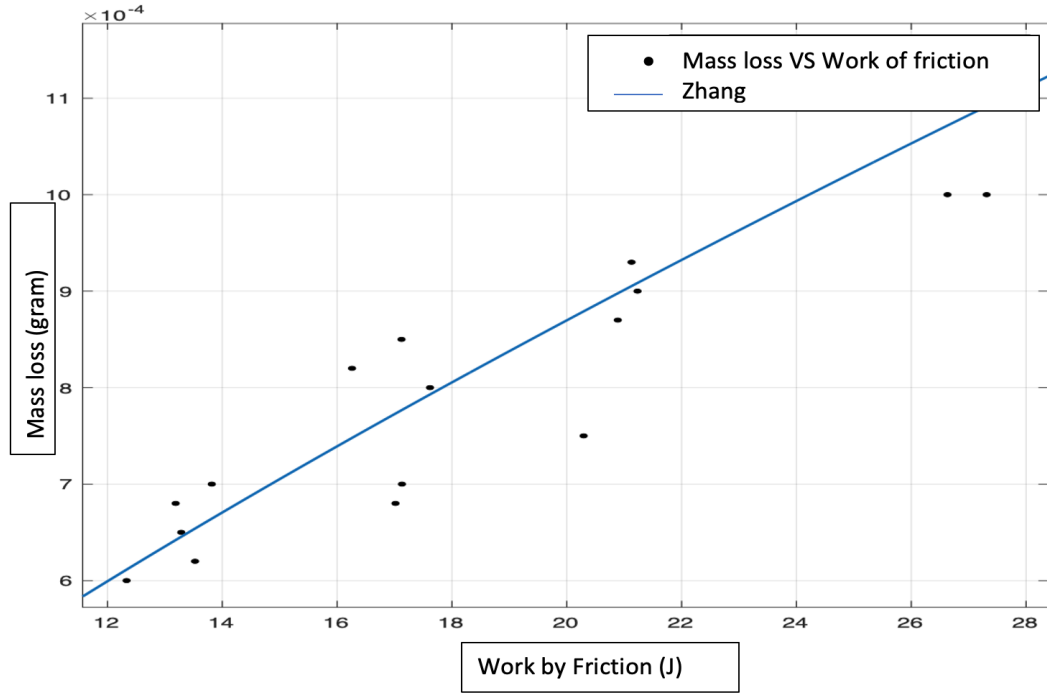


Figure. 25: Zhang Model Curve Fitting for Sample G

The SBR rubber has a good fit and as we see in table 10, the performance is well for all models. Overall, all the model performance was great however, for natural rubber the data was indecisive, and require more research to better analyze wear in rubbers E to H.

Table 10. Performance of Model for Mass Loss

Model	Sample	A	B	C	D	E	F	G	H
Ratner	R-Square	0.91	0.93	0.89	0.89	0.81	0.85	0.58	0.70
Archard	R-Square	0.97	0.93	0.98	0.95	0.71	0.76	0.42	0.81
Zhang	R-Square	0.96	0.91	0.98	0.94	0.77	0.79	0.51	0.84

We see that Archard's empirical model had the best performance on the wear data we collected, and we can say that this model is the best for predicting the wear for rubber on dry concrete. Even though Ratner and Archard considered rubber type as an influential factor, they only considered hardness however, other properties of rubber were not considered in their models.

6. CONCLUSION

The data collected and its analysis have provided a lot of insight into rubber friction and wear we can say all the proposed models are well constructed and have a good performance. The study also made it clear that not just physical factors, but chemical properties of rubber have a relationship to both friction and wear. And we can see that natural rubber has a low wear rate compare to SBR rubber. Among all the findings it was clear that normal load has an inverse relation to friction coefficient and proportional to wear and velocity impact it in an oscillating pattern creating peak values at some velocity and would need more research to make a conclusion. Rubber types play a vital role in friction and wear and the model could perform better if it was more inclusive of rubber type.

The friction model proposed by Rado and Savkoor performs well however, it does not interpret both velocity and pressure, and the model proposed by Huemer and Dorsch has the best performance. The wear model proposed by Ratner had the best performance. All the models can be improved if they included the impact of rubber type. This study helped in shading some lights on the wear and friction in better understanding the topic and the data collected is going to be an asset for expanding the range for variables. And can be developed by adding onto it as we introduce more factors such as humidity, temperature as they too play a role in the topic.

7. FUTURE WORK

This study aims to evaluate the most used models of friction and wear and analyze the limitations of each one. In future work, the limitations of these models need to be addressed by proposing a new model that also considers other parameters such as surface roughness and mechanical behavior of rubber. For this purpose, more experiments need to be conducted on different surfaces such as various types of pavement. Moreover, future research should focus on understanding different mechanisms involved in rubber friction and wear for different operating conditions. The future models may be proposed based on the dominant mechanisms of friction and wear for each range of normal load and sliding velocity considering the types of rubber and substrate surface.

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