ASPECTS THAT ARISE IN THE TRANSITION FROM THE MONTESSORI

METHOD TO A TRADITIONAL METHOD: A FOURTH

GRADE MATHEMATICS VIEW

by

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DEDICATION

This entire project is dedicated to all four of my grandparents—Lois King, Boby R. King, Dorothy Hurdle, and Elisha Hurdle III—for telling me I could do anything I wanted, and I believed them.

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LIST OF ABBREVIATIONS

Abbreviation **Description**

CGI – Cognitively Guided Instruction

DISCOVER – Discovering Intellectual Strengths and Capabilities while Observing Varied Ethnic Responses

IB – International Baccalaureate

MTOP – Montessori Teaching Observation Protocol

NCTM – National Council of Teachers of Mathematics

RTOP – Reformed Teaching Observation Tool

TCT-DP – Test for Creative Thinking-Drawing Production

TTOP – Traditional Teaching Observation Protocol

ABSTRACT

The purpose of the dissertation is to investigate three particular aspects that may affect the transition between a third grade Montessori system and a fourth grade non-Montessori system, specifically within the context of teaching and learning mathematics. These aspects are 1) the change in pacing and structure of the classroom, 2) the removal of manipulatives from the learning experience in favor of handwriting methods, and 3) the reversal of roles that teachers and students occupy. The effect of this transition on problem-solving skills is analyzed through a series of problem-solving exercises to determine mathematical understanding about key concepts within the curriculum. Results show that students identify alternative strategies when uncertain how to proceed in a problem. Students revert to previous object-centered methods when a problem is perceived as too difficult. Students also need more exposure with materials for difficult topics during the Montessori ages. The use of manipulatives is one of the most influential aspects of the transition, followed by the shift in student and teacher roles. The pacing and structure of the classroom has minimal effect on the transition.

I. INTRODUCTION

Background

There are a multitude of psychological, educational, and blended theories that all contain various supporters and detractors. For example, works from Piaget, Vygotsky, and Polya have led to many different practices in the mathematics classroom. The purpose of this study is to analyze particular aspects that affect the transition from the Montessori method into another style of learning. The Montessori program focuses on "students [who] can be described as self-regulated to the degree that they are metacognitively, motivationally, and behaviorally active participants in their own learning process" (Zimmerman, 1989, p.329). Often, the Montessori method is intended for a younger audience (Montessori, 1964), and eventually children will have to assimilate into a regular, traditional classroom. Changing from an exploratory setting to a more passive, knowledge-fed environment has been shown to result in differences within a classroom of students as part of the learning process by using the Carnegie Foundation's Turning Points criteria, which elaborates upon the intricacies of community-style educational systems (Rathunde & Csikszentmihalyi, 2005). Researchers seeking to identify what causes students to participate in varying ways between two such contrasting programs should invoke more details and use alternative methods to track progress, because the importance of "the educational setting…can make or break [students]" (Owens & Konkol, 2004, p.176).

While research exists comparing the Montessori program with other styles, there is minimal research documenting the actual student transition. For this study, this gap in knowledge places value on the opportunity to observe fourth graders newly introduced to a different learning style after concluding their third grade year under the school's perceived Montessori practices. Kamii (1994) believes that the main purpose of third grade mathematics is for students to think critically, without the reliance of peers and teachers, which is most efficiently done through problem solving. However, Kamii's work does not generally come from a Montessori perspective. Yet the term "problem solving" has proven difficult for researchers to consistently define throughout various contexts (Schoenfeld, 2009). There is adequate research connecting the significance of problem-solving skills to overall mathematics success, and Chapter 2 will provide clarity toward the use of metacognition as the contextual framework for the study and its conclusions (Zimmerman, 1998; Holton & Clarke, 2006; Flavell, 1976; Noushad, 2008).

The cultural aspects of this study are also important to keep in perspective. Culture "involves situating encounters with the world in their appropriate cultural contexts in order to know 'what they are about'" (Bruner, 1996, p.3). A review of the mathematics education literature suggests that culture is important in effective education, and that self-regulation and problem solving are influenced by such cultural contexts (Butler, 2002; Ferrare & Hora, 2014). While culture was not a main focus of this study, it is imperative to provide some perspective for where the study takes place, given the research location in Central America.

Statement of the Problem

Research has shown there is much potential for disparity between levels of success for students experiencing the Montessori style of schooling versus those of a more traditional style (Boekaerts, 2002; Rathunde & Csikszentmihalyi, 2005; Rathunde, 2003; Copes & Shager, 2003), which reflects the situation at the school in this study. The transition from the Montessori style to other styles is not adequately covered in the existing research. This study aimed to add to the literature by documenting real student transitions. Students that switch from the Montessori style to non-Montessori styles of learning mathematics are exposed to different teaching methods and may require more transitional support to be successful. Students may find difficulty in maintaining knowledge, absorbing new material, and changing their methods according to the new classroom. While post-transition effects are well documented (Hanson, 1998; Owens & Konkol, 2004), research on the shift in learning throughout that transition is limited (Copes & Shager, 2003). Such issues cannot be addressed without first establishing their nature. Many researchers believe the Montessori style may best match how children naturally learn (Illich, 1971; Kazemi & Franke, 2004; Wigfield & Eccles, 1994, Montessori, 1952), while conventional educators may argue "the average child cannot and will not become an independent, self-initiating learner" (Rosanova, 2003, p.12). Previous research on transitions has covered the shift from elementary school to middle school, but has not provided enough evidence pertaining to the shift from Montessori to traditional (Owens & Konkol, 2004; Copes & Shager, 2003). Further, "both the [Montessori] method and the movement remain largely unstudied by mainstream educational researchers" (Cossentino, 2005, p.212). The purpose of this study was to explore three hypothesized aspects related to difficulty in student transitions. This study followed students in primary grades who remain at the same campus, eliminating the need to consider middle school environments and/or changing settings—qualities that uniquely set this situation apart from existing research which must address struggles of changing institutions, regardless of teaching style (Anderman, Maehr, & Midgley, 1999).

Purpose and Significance of Study

At the International Baccalaureate (IB) world school at which this study took place, the administration noticed struggles in performance and skills in the initial year of transitioning from Montessori to a traditional classroom (at this school, from third to fourth grade). The purpose of this study was to investigate three targeted aspects influencing these challenges. The results can be broadly applied to many mathematics students experiencing the same shift in educational environments in other parts of the world. Administrators in this study would like to identify these aspects so they can work on possible future solutions. This study aimed to be applicable for overall program development in similarly structured schools. The ethnic and socioeconomic diversity of the school provided relevant results for many educational systems around the world.

Problem solving was used as a context for this study to observe strategies, cognition, and general learning in the classroom, which has been given credence as a valid method by past research (Pehkonen, Näveri, & Laine, 2013; Zhang, 2014). In particular, problem solving has been found to be indicative of students' levels of understanding. This type of mathematical process points specifically to descriptive student thinking, allowing for the use of metacognition as the theoretical framework for investigating the transition (Zhang, 2014; Zimmerman, 1998). Existing work has discussed the cognitive side of learning from students' perspectives. For example, "in [Cognitively Guided Instruction (CGI)], the emphasis is on what children can do rather than on what they cannot do" (p.14), and students build their own knowledge from what they were given when they are ready to make those connections (Carpenter, Fennema, & Franke, 1996).

Through this framework, the three aspects the study investigates are directly related to the metacognitive characteristics of the students. Teaching at an international school also allows a certain level of cultural influence in how and why students learn mathematics. Chapter 2 will discuss several learning theories involved in how culture can complement education (D'Ambrosio, 1988; Bishop, 1985). The strength of this study is the consistency of campus life and relationships to avoid the influence of unrelated factors. The research questions addressed in this dissertation are as follows:

1. To what teaching practices and learning opportunities are third and fourth grade students exposed? To what extent are these practices and learning opportunities related to the Montessori approach?
2. How are three particular aspects of current teaching practices and learning opportunities in fourth grade mathematics perceived by students and teachers compared to previous exposure in the Montessori style?
3. How, and to what extent, does changing the teaching practices and learning opportunities affect the problem-solving strategies of students?

There are certain aspects of a Montessori program that start to disappear when transitioning out of the style. Based on the related research, three aspects were hypothesized to have the most influence on the transition: 1) the learning pace and its methods of discovery, 2) the changing style of declaring solutions from materials to handwriting, and 3) the role of students and teachers shifting during the transition. First, the change in learning pace was predicted as a strong influencing aspect. In a typical Montessori program, students are tasked with responsibility for their own learning, and discoveries are meant to come naturally at a time when the student realizes connections for themselves. In many other programs, students instead belong to a passive environment where the teacher provides the knowledge directly, regardless of pacing and/or timely understanding (Copes & Shager, 2003). Second, the two styles use different methods for students to express and defend their answers. The Montessori style does not generally include handwritten work or solutions, and students defend their idea verbally (Ward, 1913). However, students who move from a Montessori and into a non-Montessori program are given pencil and paper, often for the first time, to physically write down their thoughts and solutions. This new approach describes a shift in methodology for each student as a learner, whose adjustment time could be substantial. Third, the roles of the teachers and students in the classroom change significantly. In Montessori, students are considered self-regulated and responsible for their learning and the teacher becomes a guide in the classroom. Once students move out of the Montessori style, these roles dramatically shift and can create turmoil. Such classroom relationships may cause chaos in the learning experience (Eccles, J., Wigfield, A., Midgley, C., Reuman, D., MacIver, D., Feldlaufer, H., 1993). Both parties may have trouble adjusting and understanding their new roles in the classroom.

Some definitions are necessary before expanding upon this subject through the literature review in Chapter 2, to promote a quality discussion and explore the idea that "core knowledge, problem-solving strategies, effective use of one's resources, having a mathematical perspective, and engagement in mathematical practices are fundamental aspects of thinking mathematically" (Schoenfeld, 1992, p.5). The significance of this dissertation derives from the education community's work on improving struggling transitions; educators must discover what is causing student issues. There are numerous terms necessary for the discussion of this topic. While the terms will be defined in the literature review, precise definitions beforehand are helpful for context moving forward. The definitions are as follows:

Definitions

Absorbent Mind – The absorbent mind is considered extremely important in early development as a natural ability for students to collect and retain information, and "the fundamental principle of this stage is recognition of the child as a spontaneous learner, driven by an inner drive/energy…" (Isaacs, 2007, p.14). This is a term often used in context with Montessori methods and self-regulated learners (Montessori, 1952). The term has been credited as vital in the socialization, cognitive development, and pattern recognition of students at a young age (Gutek, 2004). The absorbent mind also relates to the Zone of Proximal Development (Vygotsky, 1931).

Autonomy – Autonomy means "the right of an individual or group to be self-governing" (Kamii, 1994, p.59). Autonomy leads to correlations in cooperative ability, imperative for group learning styles, and has been shown to improve problem-solving skills (Frederick, Courtney, & Caniglia, 2014). Research suggests that more self-aware, self-driven students, also referred to as autonomous students, who took the time to discover results for themselves were more likely to retain the information in the future than a student who was given knowledge directly (Grolnick & Ryan, 1987).

Cognitively Guided Instruction – CGI is "focused more directly on helping teachers understand children's thinking by helping [teachers] construct models of the development of children's thinking in well-defined content domains" (Carpenter et al., 1996, p.5). CGI problem sets are open-ended and meant to extract the most knowledge about student understanding. To do this, teachers must consistently question students about their strategies and conclusions to determine how students are interpreting problems and solutions. Gesturing, using objects, verbal discussion, and written work are all viable methods to observe student thinking.

Culture – There are several important aspects of this term. For example, Bruner (1996) defined culture as “assigning meanings to things in different settings on particular occasions,” which “involves situating encounters with the world in their appropriate cultural contexts in order to know ‘what they are about’” (p.149). Two well-known cultural mathematics education researchers have opposing opinions on this matter. On one side, Bishop (1988) states that mathematics can and should be adjusted to fit the culture of the student, while D’Ambrosio (1985) believes culture is what actually creates meaning in mathematics in the first place. Either way, the coexistence of mathematics and culture is important for successfully learning mathematics.

Ethnomathematics – This term, specifically created by D’Ambrosio (1985), discusses how culture and mathematics relate to one another. There are contrasting opinions on the extent of this relationship. The opinions discuss whether culture defines mathematics or whether mathematics exists outside of culture (Bishop, 1988; D’Ambrosio, 1985). Either way, ethnomathematics contributes to the understanding of both points of view, and researchers agree the definition can be muddled. Some phrases related to ethnomathematics include “cultural anthropology,” “environmental activities,” and “mathematics in the community” (Presmeg, 2005).

Intrinsic Motivation – “A pupil displays intrinsic motivation or regulation when she particularly likes or enjoys learning new things with genuine interest, i.e., based on the satisfaction of learning for its own sake in the absence of external rewards” (Ünlü & Dettweiler, 2015, p.676). It is important to note that while self-motivation and self-regulation are different themes, they are often direct results of each other (Zimmerman, 1990).

Knowledge Compilation – Anderson (1982) defines knowledge compilation as “the gradual process by which the knowledge is converted from declarative to procedural form…” (p.370), and this is the definition used in this study. Practice allows new concepts to become routine for students, adding to their ability and knowledge base when tackling future problems—a useful skill in mathematical problem solving.

Manipulatives: This is the term for tangible objects that students can use, often in a mathematics context, for creating physical representations of abstract ideas (Cope, 2015). Commonly found in Montessori classrooms, manipulatives are often used in three main stages: concrete, representational, and abstract (Stein & Bovalino, 2001). The goal is for students to eventually transform the objects into mathematical symbols—a gradual shift that still maintains student understanding.

Metacognition – This abstract and important term in education and psychological learning theory is often paraphrased as “thinking about thinking” (Flavell, 1976). Metacognition allows successful learners to engage and display higher order thinking skills, which often involves active control over their own cognitive processes. Metacognition generally stems from students’ acute awareness of their strengths and weaknesses, allowing them to monitor their own learning experience independently and find strategies that work best in certain situations (Livingston, 1997; Zimmerman, 1998; Pintrich, 2002). For simplicity, the definition many educators use for metacognition relates to the way students self-monitor their goals and understanding (Flavell, 1976), and even simpler, "the ability to know what we know and what we don't know" (Costa & Kallick, 2008, p.5). For this study, a student's ability to acknowledge strengths and weaknesses while continuing to move forward and strategize is the most relevant part of metacognition.

Montessori Method – The Montessori method, "which aims at developing children's senses, academic skills, practical life skills, and character" (Lunenburg, 2011, p.3) relies on self-regulated young learners taking advantage of their environment. This atmosphere involves little teacher direction, instead providing guidance that gives students the chance to make their own discoveries and conclusions in the classroom (Ward, 1913). This responsibility allows the student to shift from a passive learner to an active learner, trusting in self-knowledge rather than automatically turning to a higher authority for direct access to information (Kretchmar, 2016; Koestner, Ryan, Bernieri, Holt, 1984).

Open-Ended Questions – These types of problems are so named "because they have multiple solutions and/or multiple approaches to reach one solution" (Myren, 1995, p.2). Research has shown that these types of problems, rather than closed, simple solution problems, are what mathematically challenge students as learners (Bahar & Maker, 2015).

Problem Solving – Problem solving has been a difficult term for researchers to define. Some researchers assert that educators call nearly anything problem solving, stating that problem solving provides the justification for mathematics in the first place (Wilson, Fernandez, Hadaway, 1993; Stanic & Kilpatrick, 1988). Analysis through problem solving refers to when "an attempt is made to fully understand a problem, to select an appropriate perspective and to reformulate the problem in those terms, and to introduce for consideration whatever principles or mechanisms might be appropriate" (Schoenfeld, 1983, p.298).

Problem-Solving Strategies – For the purposes of this study, the term problem-solving strategies is defined in accordance to the vision of Polya (1949): 1) understand the problem, 2) devise a plan, 3) carry out the plan, and 4) look back. Problem-solving strategies focus on work organization, ability to reason and verbalize methods and solutions, focus, realization of which method to apply, application to future problems, and perseverance after potential failure. This definition of problem-solving strategies was used during analysis, after viewing how the transition from Montessori affected these skills.

Productive Struggle – "When students labor and struggle but continue to try to make sense of a problem, they are engaging in productive struggle" (Pasquale, 2015, p.2). In general, struggle, or productive struggle, is considered an important part of learning new mathematical concepts. Productive struggle is not given negative context, but instead focuses on comprehending obtainable but not yet fully formed ideas (Hiebert & Growus, 2007; Warshauer, 2014).

Self-Determination Theory – In relation to education and self-motivation, this psychological theory assumes that every student has particular tendencies in terms of psychology and growth that need a motivational environment to maximize their potential (Ryan & Deci, 2000). The theory is concerned with internal characteristics and tendencies, and how they dictate people's choice, regardless of outside factors.

Self-Directed Learning – By Knowles' (1975) definition, self-directed learning is "a dynamic process in which the learner reaches out to incorporate new experiences, relates present situations with previous experiences, and reorganizes current experiences based upon this process." Self-regulated learners use this process often, utilizing their motivation and consciousness of their strengths and shortcomings. "Perhaps most importantly, self-regulated learners are aware when they know a fact or possess a skill and when they do not. Unlike their passive classmates, self-regulated students proactively seek out information when needed and take the necessary steps to master it…they find a way to succeed" (Zimmerman, 1990, p.4).

Self-Efficacy – Bandura (1997) defines self-efficacy as the confidence and ability someone has in succeeding at a certain task. Bandura relates this to metacognition and motivation by further explaining that people with self-efficacy can strategize and devise a plan that will allow them to achieve their goals. Zimmerman (2001) elaborated on this point, saying that it is the responsibility of teachers to specify the goal so that students can plan the best method for personal achievement. Research has shown that self-motivation comes from self-perception of ability and self-efficacy, which in turn feeds self-motivation, building into a continuous cycle (Bandura, 1997; Schunk & Rice, 1984).

Self-Reflection – The last of the three steps of self-regulation, this involves reflecting on the result of the assignment and applying the new findings to the current knowledge base (Pajares, 2002). Occasionally, the knowledge may be false and thus should not be included, while correct results are absorbed into the overall content knowledge of the student.

Self-Regulation – Self-regulation has been defined as the child's ability to self-educate and self-direct. Similarly, self-regulation is the child's ability to think about what he or she is in the process of learning (Bandura, 1994). Self-regulation consists of separate phases, where performance is insufficient, and reflection upon the process is imperative.

Task-Based Learning – Also referred to as the structured clinical interview, this research method "lends itself well to the qualitative study and description of mathematical learning and problem solving without the exclusive reliance on counts of correct answers associated with pencil-and-paper tests" (Goldin, 1997, p.40). These types of evaluations discover more about student thinking than a simple right or wrong solution. Often, these questions relate to problem solving, as they are less about direct results and instead focus on the direction students take to reach their conclusions. CGI problem sets are a common example of such an assessment tool.

Traditional Method – In this method, the teacher acts as knowledge holder and uses a form of direct teaching (lecture) to transfer knowledge to their students. Traditional mathematics education focuses on procedural development, rather than inquiry-based development, and has recently come under fire as a possibly outdated form of teaching

(Chapko & Buchko, 2004). Conversely, studies have shown the traditional methods still holds merit in the classroom (Din, 1998).

Understanding – "To understand, students must get inside these topics; become curious about how everything works; figure out how this topic is the same as, and different from a topic they already studied; and become confident that they would handle problems about the topic, even new problems they have not seen" (Hiebert & Wearne, 2003, p.1). To D'Ambrosio (2003), achieving this understanding is the purpose of teaching mathematics through problem solving—learning new material through trial and error of old material. "There may be debate about what mathematical content is most important to teach. But there is growing consensus that whatever students learn, they should learn with understanding" (Hiebert et al., 2000, p.2).

Zone of Proximal Development – Vygotsky (1931) supported the idea of scaffolding in teaching a class under the guise of the Zone of Proximal Development, encouraging teachers to allow their students to eventually take control of the exchange of learning. "Teachers need to feel confident that children have the ability to solve their own problems, in a supportive, trusting atmosphere" (Myren, 1995, p.3). Vygotsky's proximal development is specifically defined as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers" (Vygotsky, 1978, p.86). Finding such a balance between problems that are not challenging enough and problems that are too advanced for worthwhile learning to occur is a challenge many teachers face. Eventually, the goal is for students to take control of their learning experience.

II. LITERATURE REVIEW

Introduction

In order to create a solid, understandable framework to follow when approaching this study, terms will be defined as they appear in the literature review. It is necessary to provide a context for the following questions—to what extent is the transition from a Montessori style to a more traditional style difficult for the student, and how accurately are the influencing aspects identified? When students transition from Montessori into the non-Montessori style, how are their problem-solving strategies and processes affected by the original Montessori method they experienced, specifically at a primary school level? Metacognition is important to a self-directed learner, so how does such a framework affect the traditional education experience? This study's research questions address these ideas, expand upon the details, and explore the definition of the traditional method. When considering these questions collectively, metacognition is a useful interpretation tool, providing a theoretical framework during the progression of the study. The study will examine the process of problem solving as a primary means to observe the mathematical transition between Montessori and non-Montessori. Problem-solving assessments provide considerable evidence to evaluate this transition and appropriately capture what students have learned, particularly in terms of mathematical understanding. The cultural context is also valuable because of the unique situation this study provided. Not only is the school for this study located in Central America, but also the school's entire transitional period occurs at the same campus. The benefit for the study is a more seamless transition that avoids extraneous variables, which is atypical of students who experience such a shift. Many students usually switch institutions entirely as part of the transition to a traditional system after a Montessori system. The structure within the same institution for this study allowed the research to develop without the unintended effects of this non-contributing variable to the study. Instead, the learning style varied while the other factors remained constant.

Problem Solving

Problem solving has long been a part of the mathematics classroom, and “in teaching through problem solving, learning takes place during the process of attempting to solve problems in which relevant mathematics concepts and skills are embedded” (Cai & Lester, 2010, p.3). While the debate continues about how best to implement problem solving, what degree of implementation, and approaches for effective activities, few mathematics teachers would argue against some level of this strategy being used in their classrooms. In 1980, the National Council of Teachers of Mathematics (NCTM) identified one of the primary goals of mathematics courses as the growth of problem-solving skills, demonstrating a need for such abilities. This movement, encapsulated by the manuscript “Agenda For Action,” was a culmination of discourse by NCTM to push for more critical thinking in mathematics classrooms. The publication intended audiences to move past strictly computational mathematics toward more situational mathematics. Strategy, problem-solving methods, and diversity were all stressed aspects of this 29-page document, as well as a movement away from the stress on test taking based on changing times and technology. As a result, NCTM’s version of a national curriculum was published in 1989.

By definition, a problem arises when there is an initial given situation, or given state, and a goal must be reached without a clear or obvious way to get there (Mayer, 1980). However, “when two people talk about mathematics problem solving, they may not be talking about the same thing. The rhetoric of problem solving has been so pervasive…that creative speakers and writers can put a twist on whatever topic or activity they have in mind to call it problem solving” (Wilson et al., 1993, p.1). For example, a word problem may not be an example of problem solving if it is simply disguised as a drill exercise (Hyde, 2003). There is a difference between a word problem and what Lesh and Harel (2003) call a “model-eliciting activity.” While the latter requires organization, interpretation, and manipulation as an overall process, “descriptions and explanations (or constructions) are not just relatively insignificant accompaniments to ‘answers’” (Lesh & Harel, 2003, p.159)—a conclusion reached after extensive 90-minute problem-solving sessions with students. Explanations are equally, if not more, important than the solution itself. Lesh and Zawojewski (2007) argue that educators have yet to find a solid, universal understanding of how to relate problem solving to the classroom. “Historically, problem solving has been included in the mathematics curriculum in part because the problems provide justification for teaching mathematics at all” (Stanic & Kilpatrick, 1988, p.13). Numerous studies have shown that exercising problem-solving techniques often gives students ways to generate and make sense of their own mathematical conjectures—a conclusion that is often not reached without some sort of trial and error method (Mayer, 1980; Siegler, 2003; Chapko & Buchko, 2004, Schoenfeld, 1983). “Tasks that encourage students to use procedures, formulas, or algorithms in ways that are not actively linked to meaning, or that consist primarily of memorization or the reproduction of previously memorized facts, are viewed as placing lower-level cognitive demands on students” (NCTM, 2014). Students who simply memorize steps are thought to be setting themselves up for failure in higher levels of mathematics (Kloosterman & Stage, 1992). Further, if a student immediately wonders which algorithm to use to solve a problem, then the strategies that student has in place are likely faulty. By focusing strictly on the tool rather than the reasoning, the student can lose meaning and context in the "how" of the problem (Finney, 2004). Similarly, Kamii (1994) views algorithms as a way to excuse children from using their own numerical thinking, as they are not required to justify their responses and instead rely on finding solutions formulaically. Thus, "the solution to problems is often the building of a model using particular concepts that are still being developed by the students. In this view, the purpose of the strategies is to help students refine, revise, and extend their ideas, especially through interaction with others" (Hyde, 2003, p.9). In short, the process itself should be the solution rather than focusing on a final numerical answer. Teachers can avoid their students' tendencies to memorize formulas by implementing lessons and activities that are more engaging yet still connected with the material in that lesson. These methods create mathematical problem solving in the classroom, and according to the previous research they are vital to successful long-term mathematics knowledge.

The best way to achieve this goal is still a matter of debate, but Vygotsky's Zone of Proximal Development is one of the more well-regarded methods—"the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers" (Vygotsky, 1978). The goal is to narrow the gap between the information the teacher provides and what the student understands, until the student takes control. Within this context, student reflection during the process is important—deciding whether processes, solutions, and results are correct and worth knowing. Self-reflective students possess metacognition, which allows awareness of their own learning process while going through the stages of problem solving (Zimmerman, 1998). Yet, defining problem solving continues to be difficult. George Polya's (1949) well-established description defines problem solving as the essence of knowing mathematics, and the steps are detailed as follows: 1) understand the problem, 2) devise a plan, 3) carry out the plan, and 4) look back. Similarly, Brownell (1942) stresses this process as the purpose of problem solving, and defined problem solving as perceptual and conceptual tasks exclusively. While a student may or may not have immediate concept understanding, experiencing perplexity, rather than confusion, is an important part of the process (Brownell, 1942). Schoenfeld (2009) notes that problem solving has multiple goals, dependent on the point of view and context of the situation. He defined analysis, synonymous with problem solving, as when "an attempt is made to fully understand a problem, to select an appropriate perspective and to reformulate the problem in those terms, and to introduce for consideration whatever principles or mechanisms might be appropriate" (Schoenfeld, 1983, p.298). Carpenter (1988) supports this definition by arguing that a lesson simply containing what he calls "a collection of problem-solving procedures" does not necessarily constitute problem solving itself. Problem solving can be "a skill which can enhance logical reasoning. Individuals can no longer function optimally in society by just knowing the rules to follow to obtain a correct answer…sometimes [they] need to be able to develop their own rules in a situation where an algorithm cannot be directly applied" (Kaur & Toh, 2011, p.177). Generally, "the goal is for students to learn *how* to construct strategies independently when confronted with academic work" (Butler, 2002, p.92). These last two statements are most appropriate in supporting a definition for use in observing problem solving through this study. Students should be able to do more than just work through a particular problem—they should be able to apply this work to other follow-up problems that may not appear in the exact format as the previous problems (Kaur & Toh, 2011).

The point of problem solving is to avoid memorization and gain true *understanding*—"we use the term 'problem solving' to distinguish this approach to mathematics from the 'memorize-use-forget' approach" (Rusczyk, 2015). Kamii (1994), influenced by a series of third grade classroom observations, found that students taught with methods beyond formulas are encouraged to figure out different directions and methods outside of formulas they are told to memorize. Bahar and Maker's (2015) study made use of the DISCOVER tool (Discovering Intellectual Strengths and Capabilities while Observing Varied Ethnic Responses) among 50+ third grade students, establishing differences between closed and open mathematics problems. The research suggested that enabling creativity in students is an important piece for solid mathematics learning (Bahar & Maker, 2015). Further, allowing students time to problem solve independently has also contributed to more solid understanding (Cozza & Oreshkina, 2013; Butler, 2002; Hiebert & Wearne, 2003), which is defined as the ability for students to take what they have learned and apply it to other situations (Hiebert & Wearne, 2003). With student understanding comes the ability to use abstract and symbolic representations in the future, solidifying cognitive development and creating a positive loop (Hyde, 2003). In this dissertation, viewing the classroom through a broad lens of all mathematics has been narrowed to a more detailed focus on problem solving. Problem solving induces acknowledgement of student understanding, further supporting *problem-solving strategies* as an applicable method of measuring student knowledge. “Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies” (NCTM, 2014). Thus, problem solving is a method of teaching mathematics that allows students to use existing knowledge to figure out a solution and also learn a new concept that is built off the old material (D’Ambrosio, 1987).

Types of Problem Solving in the Classroom

There are many examples of how problem solving is commonly used in the classroom. Anderson (1982) defines *knowledge compilation* as “the gradual process by which the knowledge is converted from declarative to procedural form…” (p.369). Several studies describe *open-ended questions* as useful in providing the most information about the learner (Pehkonen et al., 2013; Lesh & Harel, 2003). Bahar and Maker (2015) recently argued that closed problems are a poor test for a wide variety of significant skills compared with open-ended problem solving, such as general creativity, working memory, and quantitative ability. Another tool they used for evaluation was the TCT-DP (Test for Creative Thinking-Drawing Production), which has been show to have high reliability but is often criticized for validity due to a lack of comparative tools to evaluate against (Bahar & Maker, 2015). The results supported the value of creativity in problem solving and may provide a framework for how to structure such lessons and/or activities, in addition to helping teachers follow student thinking and processes (Mayer, 1980). In general, “changing the task from a ‘closed’ format, or a simple performance task, into an ‘open’ format and into a learning/growing type of task…essentially means that now the problem has many possible answers, instead of just one" (Miller, n. d., p.1). Open-ended problems are questions posed to elicit a discussion about the process rather than the final answer, of which there can be many. "It is important in teaching to use open [ended] problems because they encourage pupils to invent different solutions" (Laine, Näveri, Ahtee, Pehkonen, 2014, p.126). Students' thought processes can be viewed by allowing them freedom to explain themselves, as opposed to the teacher narrowing down the available solutions. Extensive research has shown that students who progress through a more open-ended problem based curriculum retain a better conceptualization of what they accomplished by the end and do not sacrifice test scores in the process, evidenced by scoring the same as other students on standardized tests (Stein, Boaler, Silver, 2003). "In the context of classroom teaching one major advantage of using open problems and investigations is that, because there are multiple solutions, they cater for a wide range of mathematical abilities and stages of development in children" (Way, 2005, p.1). Based on this literature, problem solving was considered the best method to judge mathematical knowledge in this study. Problem-solving exercises ease the process of finding any discrepancies or gaps by better showing student understanding. Research shows that problem solving allows for students to display their skills and shortcomings for teacher support, and that teaching these analytical skills is most efficiently done through elaboration, discussion, and related conversation by asking students to explain their thoughts (Siegler, 2003; Mercer & Sams, 2006).

Problem solving should not be taught as a separate topic in mathematics—it should instead be developed as a lens to view curriculum through the grades (Cai & Lester, 2010; Goos & Galbraith, 1996). Concerning the Montessori program, "problem solving is perhaps the area of mathematics in which self-regulation is most apparent" (Pape & Smith, 2002, p.95). The metacognition framework fits such a situation. Several researchers claim it is harmful for students to rely on adult figures for answers to mathematics problems because students cannot expand internally and retain information for long-term use (Myren, 1995; Kamii, 1994, Lesh & Zawojewski, 2007). While this principle is supported in a Montessori method, it is less supported in traditional teaching methods. As a result, "children become one-dimensional in their thinking and are denied the opportunity to become mathematically powerful problem-solvers" (Myren, 1995, p.3). This statement contributes to the characteristics for both problem solving and problem-solving skills—positive work organization, ability to reason and verbalize methods and solutions, maintaining focus, realization of which method to apply, application to future problems, and perseverance after potential or perceived failure (Myren, 1995). To further detail metacognition, Lesh and Zawojewski's (2007) reviewed the literature and found students that use problem solving are generally considered more aware of their strengths and weaknesses, and look for clear and precise solutions along the way.

The Theoretical Framework: Metacognition

"What constitutes evidence, and therefore, what justifies it, is the result not only of what questions are posed, but of the framework within which they are posed" (Lincoln, 2002, p.4). The theoretical framework becomes clearer when detailing the specifics of students learning to think creatively and discover their own conclusions. "When [students] find they are at a dead end, they must be willing to abandon one strategy for another. When students labor and struggle but continue to try to make sense of a problem, they are engaging in productive struggle" (Pasquale, 2015, p.2). In general, researchers consider *productive struggle* an integral part of learning new mathematical concepts without negative connotation, as comprehending obtainable but not yet fully formed ideas (Hiebert & Growus, 2007; Warshauer, 2014). Many successful students can analyze how, what, and if they have learned a topic sufficiently, giving them a substantial amount of control over their education—out of this situation arises metacognition, which still lacks a consistent definition among all researchers (Lesh & Zawojewski, 2007). Livingston (1997) defines *metacognition* as active control over the cognitive process, allowing successful learners to engage and display their higher order thinking skills—this will be the definition followed for the theoretical framework in this study. The benefits of metacognition are observed in a student's ability to self-motivate and possess behavioral awareness (Zimmerman, 1998). "Metacognitive knowledge includes knowledge of general strategies that might be used for different tasks, knowledge of the conditions under which these strategies might be used, knowledge of the extent to which the strategies are effective, and knowledge of self" (Pintrich, 2002, p.219).

There is an important distinction between cognition and metacognition, which are often discussed as different aspects in the literature (Lesh & Zawojewski, 2007). Flavell (1976) states a more useful description of metacognition as the way that students self-monitor their goals and understanding, a definition that has become more common with educators. "In terms of metacognitive processes, self-regulated learners plan, set goals, organize, self-monitor, and self-evaluate at various points during the process of acquisition" (Zimmerman, 1990, p.4). This is a meaningful distinction, as Bandura and Schunk (1981) found that students' expertise in the ability to set goals had high correlation with improved performance and overall *self-efficacy*, which they defined as the value students puts in themselves and their ability. To reach this conclusion, they placed elementary children in groups according to the extent teachers perceived them as self-sufficient, judging from how well they could do tasks while observers were positioned from varying distances away. The research tools for the study were created and tailored by the researchers, and students were grouped and measured by self-efficacy in order to observe their cognitive results. Additionally, Bandura (1997) later expanded the idea, stating that students who proactively decide on efficient and necessary courses of action are more likely to be successful toward a particular goal. Holton and Clarke (2006) define metacognition as "any thinking act that operates on a cognitive thought in order to assist in the process of learning or the solution of a problem" (p.133). In short, metacognition interprets cognition in the same way cognition interprets aspects of the classroom (Noushad, 2008).

However, Schoenfeld (1992) states that, similar to problem solving, metacognition is not consistently defined. The primarily consistent theme is that creative and critical thinking are regularly stressed in successful metacognition (Papaleontiou-Louca, 2008). Such immeasurable variables are often difficult for researchers to define. The most relevant description of metacognition for this study is found in Lesh and Zawojewski (2007), who describe metacognition as thought and strategy awareness, along with the recognition of the importance of reflection and perseverance. The definition of metacognition used for this study is the student's inner drive, which allows perseverance in thinking critically and includes self-evaluation to determine the level of understanding and conceptualization. "Several projects have focused on teachers' conceptions of mathematical learning as a basis for helping teachers make fundamental changes in instruction. An underlying assumption…is that students construct knowledge rather than simply assimilate some part of what they are taught" (Carpenter et al., 1996, p.4).

Halmos (1980) says that "the mathematician's main reason for existence is to solve problems, and that, therefore, what mathematics *really* consists of is problems and solutions" (p.519). The same can be said of students in the mathematics classroom. "We want students to be exposed, as early as possible, to the idea that beyond the nuts and bolts of mathematics, there are unifying undercurrents that connect disparate pieces" (Wu, 2009, p.6). Teachers are training students to think as mathematicians for one class period at a time, which involves reasoning skills that may take time to evolve (Schoenfeld, 1992). Metacognition can be a helpful tool during this process. "Thus, metacognition has a dual role: (a) It forms a representation of cognition based on monitoring processes; and (b) exerts control on cognition based on the representation of cognition. Yet, metacognition has many facets making difficult the distinction between monitoring and control and the setting of the line between these two functions" (Efklides, 2006, p.4). Research has shown mathematics learners can take responsibility for their own learning when the process is layered through scaffolding—they are both aware of the learning process for themselves as individuals and how to be successful overall in a mathematics course (Pape, Bell, & Yetkin, 2003; Frederick et al., 2014; Graf, Kinshuk, & Lau, 2009; Silver, 1987; Zimmerman, 1998). *Self-regulation* is defined as students' ability to self-educate, self-direct, and understand the context of what they are learning (Bandura, 1994). Self-regulation, simultaneously a cause and also an effect of metacognition, is relevant to the mathematics classroom as “mathematics education teachers and researchers are now trying to understand the impact of classroom contexts on developing mathematical reasoning” (Pape & Smith, 2002, p.93). The Montessori program is founded on ideals and principles that match these comments, a topic that will be explored in a later section of this literature review. Monitoring and regulation are byproducts of the emergence of knowledge and skill, and are part of what makes metacognition—a type of cognition about cognition—an important aspect to study in the classroom. Often the stages of such self-regulation are divided into performance and self-reflection (Zimmerman, 2002; Papaleontiou-Louca, 2008; Bandura & Schunk, 1981). These facets of metacognition have also led to many studies about self-directed learning as a general learning theory (Smith, 2016; Knowles, 1975; Piskurich, 1994; Jennett, 1992; Bandura, 1997). Overall, metacognition relates directly to problem solving because “as children become more aware of appropriate methods and strategies, their problem solving abilities will increase” (McPherson & Payne, 1987, p.77).

The Role of the Teacher

The role of the teacher is part of what defines problem solving, which is relevant to the style of education. Stanic and Kilpatrick (1989) refer to this aspect of teaching and learning problem solving as an artistic quality. While the definition of problem solving has shifted over time (Kaur & Toh, 2011), the role of the teacher in the evolving definition has also been crucial; particularly, to what extent the teacher should help the student. The focus for teachers is “when a child struggles or has the wrong answer, a teacher must determine how and when to intervene to facilitate the child’s moving forward without taking over the child’s thinking” (Jacobs & Ambrose, 2008, p.262).

Generally, the function of the mathematics teacher is to help students become better problem solvers and critical thinkers (Coy, 2001). “Explicit actions by teachers or peers can work to build community understanding and resolve students’ struggle without depriving students of the opportunity to think for themselves” (Warshauer, 2014, p.6). Yet teachers often find difficulty in creating lessons that lead the student to a preconceived destination, but also leave enough of the steps open-ended for a student to struggle getting there without too much guidance (Masingila, Kimani, & Olanoff, 2009; Pape et al., 2003). Simon (1986) promotes the belief that the teacher should act more as a guide, asking questions and sharing in the discovery experience, taking a more supportive role while students explore mathematics for themselves. The guide role exemplifies the qualities of an effective Montessori teacher, but is less representative of a traditional teacher (Montessori, 1964; Zhbanova, Rule, Montgomery, & Nielsen, 2010). Zolkower and Shreyar (2007) establish through their study that the teacher is crucial in keeping a significant mathematical conversation moving forward in the classroom. “The impact of consistent, purposeful listening, especially in a problem-based classroom, can be a powerful way to elicit and understand students’ deeper thinking…” (Driscoll, 2003, p.175). Jacobs and Empson (2016) propose that teachers cannot predetermine how they are going to assist students, but instead must adjust as the situation develops. “Thus, effective teaching involves the considered selection of a teaching approach to attain a desired educational outcome with a particular type of learner…effective teaching involves teacher decision making” (Peterson, 1979, p.48).

Promoting a good discussion helps students build self-motivation, which includes the ability to strategize, planning skills, and positive attitudes (Dimant & Bearison, 1991;

Newman, 2002; Checkley, 2006; Seigler, 2003). Metacognition continues to be valuable in the process. Also, research has shown good mathematical class discussions can create positive effects on each level of thinker in the classroom, allowing advanced students and beginning students alike to find pertinent information from the process rather than becoming passive or being left behind (Kamii, 1994). Further, “teachers who cannot be responsive to individual children’s thinking in an interview setting are unlikely to be able to do so in a class setting” (Jacobs & Empson, 2015, p.196). Research has also suggested that cooperative methods are the most discussed teaching strategies for mathematics (Davidson & Kroll, 1991; Franke, Kazemi, & Battey, 2007), while further studies show such group-style work may be the most effective way of encouraging metacognitive strategies in students. This approach includes additional assistance to help students feel comfortable sharing their findings and supporting their own ideas (Kramarski, Mevarech, & Arami, 2002). For their study, Kramarkski, Mevarech, and Arami (2002) selected students at an average age of 12 years old for separation into different styles of classrooms with the same given tasks; one room included both cooperative and metacognitive conditions while the other room housed only cooperative conditions. Students in the first group were additionally asked to respond to comprehension, connection, strategic, and reflection related questions. The results showed that students with metacognitive conditions reached higher overall achievements than those without. These findings are relevant and encouraging for this dissertation because the Montessori method relies heavily on verbal explanations of methods and solutions. As with many mathematics classrooms, “teachers need to know how to draw students’ identities into the mathematical work, support them to evolve in how they participate, honor different forms of participation, and structure opportunities that allow for different participation forms" (Franke et al., 2007, p.248).

Vygotsky (1931) supported the idea of scaffolding in the teaching of any class level, under the title of the Zone of Proximal Development. He encouraged teachers to allow their students to eventually take control of the exchange of learning by thinking about their learning process. The *Zone of Proximal Development* is defined as the difference between potential and actual levels of development and uses collaboration to narrow this gap (Greenfield, 1984). Often, teaching strategies require time to eventually change the behavior of the students (Pape et al., 2003). To conclude this, researchers used a Strategy Observation Tool, prompted by probing questions, to track strategies for 54 students labeled as self-regulating. Similarly, Schoenfeld (1992), for his study, constantly and consistently asked difficult questions of his students, requiring a strong defense of their thoughts and reasoning. His stated goal was to create an eventual shift in student thinking, leading them to support their ideas as a natural response—however, he believed it could take up to a semester for this change to occur (Schoenfeld, 1992). A constructivist interpretation may consider this the absolute pinnacle of a successful mathematics classroom. Wilson (1996) explained that problem-based learning, when taken to the extreme, can have little to no teacher involvement at all, instead requiring students to work from prior knowledge and with other students to construct new hypotheses once they have been trained in problem-solving strategies. Cognitive theory suggests that children explore and discover, with parents and teachers acting as encouragers rather than directors (Berger, 2002). Otherwise, they may feel less responsible for their learning (Kamii, 1994), and maintaining metacognition is important for successful problem solving. Teachers should follow students' natural train of thought, as "students' thinking provides a context for teachers to enhance their own understanding of mathematics" (Carpenter et al., 1996, p.5).

While direct teaching is a staple of the traditional methods found in most public schools, hands-off approaches are more prominent in the Montessori method. Many researchers argue that so long as the correct scaffolding is in place, students can learn strictly through cognitive skills and group work with other students (Frederick et al., 2014). Some researchers feel the teacher can practically be nonexistent (Holton & Clarke, 2006). Additionally, Lesh and Harel (2003) claim "simply progressing along ladder-like sequences" is too simplistic a way to look at student learning, and that instead, "communities of relevant concepts tend to be available at any given moment; most of these constructs are at intermediate stages of development, and apparent levels of development vary across tasks as well as across time within a given task" (p.187). The Montessori style of education, which is discussed more in a further section of the literature review, considers the teacher a guide through an environment of exploration for young students (Montessori, 1964; Fang, 2008; Ward, 1913). "Montessori realized that appropriate adult intervention is needed at certain times but should decrease steadily as children learn how to do things for themselves" (Gutek, 2004, p.48). Students transitioning between Montessori and other styles could experience very different levels of teacher intervention—in Montessori, the teacher is a guide as students take eventual responsibility for their own learning processes, while the most rigorous non-Montessori strategies may place the responsibility on the teacher to modify student behavior in pursuit of thinking like a mathematician. Both students and teachers can end up in unfamiliar roles and unaccustomed positions in either classroom style. Yet there are numerous advantages to such unknowns, as the students and teachers learn to create the classroom environment together (Silver, 2003; Masingila et al., 2009).

The Importance of Mathematical Understanding

The domain of mathematics and measures of mathematical competency have changed over the past thirty years (Pape & Smith, 2002; NCTM, 2014). Research shows society rates creative tendencies higher than previously, and the perceived importance of mathematics has dramatically increased (Kilpatrick, 2001). “Problem solving, reasoning, justifying ideas, making sense of complex situations, and learning new ideas independently…are now critical skills for all Americans” (Battista, 1999, p.427). For example, in the opinion of Lumsdaine (1995), increasing value was placed on the cerebral, right side of the brain near the end of the twentieth century. This side of the brain contains the processes for visual, intuitive, and innovative traits. These findings contributed to standards updated for American school systems in the year 2000. These modifications are the focal points of the new mathematics curriculum and place more value on the mathematical process rather than strictly the solution. As a result, NCTM placed more prominence on reasoning, logic, problem solving, and defending hypotheses (NCTM, 2014). “Reasoning is the power that enables us to move from one step to the next” (Wu, 2009, p.14), which is essential to problem-solving strategies.

Overall, researchers have identified a significant impact by metacognitive experiences on self-regulation of learning. Several studies investigated further and discovered that student awareness of knowledge building is dependent on emotions, especially at a younger age, and that this dependency in turn highly affects confidence level (Efklides, 2006; Tornare, Czajkowski, & Pons, 2015). Further, confidence level improves the ability to self-regulate (Pape & Smith, 2002). "Research on self-regulation of problem solving in informal contexts…has demonstrated the importance of self-regulatory processes and associated sources of motivation" (Zimmerman & Campillo, 2003, p.254). Blake and Pope (2008) and Berger (2002) described work by Vygotsky and Piaget as heavily influencing learning methods to achieve self-regulation. While problem solving is the common theme in these discussions, the pathway to the destination can vary. For Vygotsky, group interaction was vital for successful problem solving; for Piaget, adaptation to the environment was more important (Blake & Pope, 2008; Berger, 2002). From either perspective, the student should consciously develop a relationship with the learning environment to achieve high performance. Research also shows that when teachers "high press" their students for conceptual thinking, encourage them to defend their reasoning, and take students' ideas seriously, young elementary students can be completely engaged and drive the conversation forward mathematically (Kazemi & Stipek, 2001; Checkley, 2006). In the 2001 study, the "press for learning" scale was created to reach these conclusions. Four upper elementary teachers of mixed experience taught the same lesson about fractions but approached the experience differently. The four teaching strategies required students to describe their thinking, required teachers to assist students to find more than one path to the solution, required teachers to support student mistakes, and required teachers to give students time to collaborate. The results showed that students thrived as these opportunities were provided more often (Kazemi & Stipek, 2001). Mathematics teaching should be structured around the skills and concepts involved in solving problems (Checkley, 2006), and further clarification of these goals to students helps them to monitor and assess themselves moving forward, leading to self-efficacy (Zimmerman, 2001), a result of keeping track of their learning. Further, studies have shown that students can better improve understanding and knowledge base through work with problem solving methods, rather than working with standard methods often used in direct teaching. This improvement of understanding can also improve students' confidence levels (Carpenter & Fennema, 1989). "If students are to develop conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning abilities, they must believe that mathematics is understandable" (Kilpatrick, Swafford, & Findell, 2001, p.115).

Valuing Problem Solving from a Young Age

Research shows students have difficulty in solving word problems and applying concepts to a situation (Verschaffel, de Corte, & Vierstraete, 1999; Tambychik, 2010; Coy, 2001; Goos & Galbraith, 1996; Kramarski et al, 2002). Yet Polya (1983) claims that this skill is the most important type of mathematics, and that the routine problem does not cognitively grow the student as a mathematics learner. Students who believe computation is the key to mathematics success are less motivated moving forward than those who value problem solving, based on the Effort Can Increase Mathematical Ability Scale and the Usefulness of Mathematics Scale, each previously created for supporting studies (Kloosterman & Stage, 1992). Kilpatrick (2001) shows that these non-routine problems give students a need for the ability to solve basic problems that ask for concepts such as given, unknown, condition, and solution. Similarly, students should see that word problems are an important component of mathematics and that effort is necessary and required, for "students with no motivation to solve problems that they cannot solve quickly will have difficulty in college-level mathematics courses" (Kloosterman & Stage, 1992, p.110). According to Lesh and Harel (2003), model-eliciting activity (or true problem solving) are differentiated from a textbook word problem because converting symbols to meaning is more difficult than transforming meaning into workable symbols. It is plausible that students can apply and calculate through algorithms correctly, but have no understanding of the reasoning or interpretation of the result (McPherson & Payne, 1987). This idea is further supported by research finding that after elementary grades, students consider word problems too difficult to apply a set algorithm (Kloosterman & Stage, 1992). Another study found that "although using a meaningful approach may increase the likelihood of success, students are not being supported to use, or simply will not use, these more transformative behaviors" (Pape & Smith, 2002, p.95). In summary, Pape and Smith (2002) found that although students were more successful when the context made sense and felt personal, not enough students were able to do so. Instead, students needed to learn at a young age to successfully approach a problem with such a method. "Important learning can occur after a correct answer is given when a child is asked to articulate, reflect on, and build on initial strategies" (Jacobs & Ambrose, 2008, p.272).

This portion of the literature supports upper elementary school as a crucial learning period for mathematics students (Wu, 2009), and supports the eventual importance to successful secondary mathematics (Reys & Fennell, 2003). This perceived importance give credence to this dissertation's contribution to elementary mathematics grades. Further, elementary students have been found to possess the ability and interest level to be full participants in mathematics, and such foundational development naturally begins at a young age (Clements & Sarama, 2007). “We have neglected far too long the teaching of mathematics in elementary school. The notion that ‘all you have to do is add, subtract, multiply, and divide’ is hopelessly outdated. We owe it to our children to adequately prepare them for the technological society they live in, and we have to start doing that in elementary school” (Wu, 2009, p.14). Students’ basic views and thoughts about mathematics are shaped in the elementary years, and these views can be difficult to change later in school (Wu, 2009; Reys & Fennell, 2003). Third grade focuses on developing independent thinking and confidence through problem solving (Kamii, 1994), providing value for the transition period (from third to fourth grade) in this dissertation. Research has shown that the transition from upper elementary to junior high provides the most growth in the relationship between attitude toward mathematics and achievement in mathematics, and shown that once students get to the high school level, these emotions and attitudes are more fixed and less apt to change (Ma & Kishor, 1997). Kloosterman and Gorman (1990) believe that motivation is an important characteristic in young learners (elementary ages), and believe this attribute allows students to maintain interest in school over the following years. From birth, children tend to problem solve as a method to understand the world around them, and in so doing, children are considered learners of mathematics and its operations long before entering school (Vygotsky, 1978). Rather than allowing this curiosity in mathematics to decline through the early grades, “the history of mathematics education is replete with attempts to implement powerful ideas in elementary school mathematics” (Carraher & Schliemann, 1993, p.192).

Research has shown that teachers identify self-control and cooperation as big contributors to success in lower grade classrooms (Lane et al, 2014), suggesting a need for studying the elementary years. Through many of the discussed problem solving methods, teachers can highlight these attributes in the elementary grades to begin an earlier process of group learning styles (McPherson & Payne, 1987; Bruner, 1960; Mercer & Sams, 2006). “Young children can actively construct from their everyday experiences a variety of fundamentally important informal mathematical concepts and strategies, which are surprisingly broad, complex, and sometimes sophisticated” (Lee & Ginsburg, 2009, p.39). Research has also suggested that elementary school teachers are best equipped to implement self-motivation and self-paced programs in a successful classroom (Schoen, 1976). Compared to the elementary grade Montessori approach of individualized, personal relationships between students and teachers, the middle school traditional system provides teachers with larger, less personal classes that are designed to master a topic rather than explore. Several researchers have described traditional classroom methods as the “wasteland” of American public schools (Eccles et al., 1993). The location for this dissertation’s research was not a traditional American institution. Instead, as an international school, the third and fourth grades were observed to view the thinking process and overall problem-solving strategies employed by young, creative minds in students, without the bias of a particular school system in mind.

Measuring Students’ Mathematical Thinking

While educators have placed an increased importance on imploring students to reason mathematically, evaluating such a process is difficult. Because this process is not clear, “researchers have explored the noticing of mathematics teachers to understand how they make sense of complex classroom environments in which they cannot be aware of or respond to everything that is occurring" (Jacobs, Lamb, & Philipp, 2010). Educators can bring out the most information about students' methods through task-based learning as a research tool. *Task-based learning* "offer[s] the possibility of obtaining information from students that bears directly on classroom goals and can help answer research questions central to the education reform process" (Goldin, 1997, p.40). Further, the best ways to ensure a high level of challenge and successful implementation are to attach meaning to student work beyond proclaiming a solution, to support teachers in interpreting this meaning, and to facilitate meaning-making from the students themselves (Charalambous, 2010; Jacobs & Empson, 2015), as well as to change the difficulty of the problem by inputting different values according to the ability of the student (Jacobs & Ambrose, 2008). The latter is an important aspect that will often be used in this study during the implementation stage of the problem solving sets. This flexibility provides opportunities that do not present themselves in a group setting, as "teachers may be missing opportunities to advance students' thinking during these [whole-group discussion] moments" (Jacobs & Empson, 2015, p.196).

In designing an effective task-based structure, it is essential to include open-ended questions. This task-based structure allows for pointed questions that will help the interviewer collect the most information about student knowledge (Maher & Sigley, 2014). Scaffolding is unique to this assessment style and is crucial in using a successful task-based process to bring out the most learning evidence from students. First, problems should progressively get more difficult, until the hardest question is a challenge for the highest-achieving student in the class. Second, each round of problems given over time should build from material found in prior questions. Also, the task-based structure should allow unique thinking and provide an opportunity for students to reach outside their comfort zone and apply what they know (Goldin, 1997; Ginsburg, 1997). Reversing the question is a good way to make this type of problem different than the standard; this process relies on the children to explain a problem in their own words, both before and after solving (Jacobs & Ambrose, 2008). Students must be interested in the problem and excited about the process, and "of critical importance is honoring children's approaches to story problems so that they are constructing strategies that make sense to them rather than parroting strategies they do not understand" (Jacobs & Ambrose, 2008, p.261).

Once the task-based interview is underway, many topics can arise for further exploration. The research has discussed that these problems show more about student thinking, but it is also important to question, "Why did the child make that response? What did he mean by that statement? Although it is vital to interpret, don't leap to conclusions" (Ginsburg, 1997, p.141). In making post-interview claims, interpretation is meant to be fully dependent on student thought rather than teacher assumptions. Similarly, Jacobs and Empson (2016) consider four categories when adjusting to student difficulties and train of thought—1) ensure the child knows what the question is asking, 2) bring out details of child thinking, 3) encourage additional strategies, and 4) connect descriptions to mathematical symbols. This may appear to contradict the ideas of Vygotsky, as the Zone of Proximal Development requires teacher guidance to get students to this point (Greenfield, 1984). However, Jacobs and Empson (2016) conducted their study in a system emphasizing responsive teaching methods, which prompted students to analyze, discuss, and conclude facts by themselves. While teachers can adjust based on the situation through predetermined thoughts, through highlighting where the student is going, and through changing stances (Maher & Sigley, 2014), these strategies complement independent student achievement rather than supplement student knowledge that comes directly from the teacher. "Hence, a carefully constructed task is a key component of the task-based interview in mathematics education. It is intended to elicit in subjects estimates of their existing knowledge, growth in knowledge, and also their representations of particular mathematical ideas, structures, and ways of reasoning" (Maher & Sigley, 2014, p.579).

Task-based learning has various interpretations, but the one used for this study was *Cognitively Guided Instruction*. CGI is "focused more directly on helping teachers understand children's thinking by helping them construct models of the development of children's thinking in well-defined content domains" (Carpenter et al., 1996, p.5). The problem discussed is that teachers find it difficult to organize the large amount of data concerning learning strategies and tendencies into a coherent teaching plan. Instead, student thinking should define teacher experience. "CGI teachers have found that students gradually learn to make sense of the context on their own...students learn to look for the mathematical relationships that are a part of the story and use them to get started on a solution" (Carpenter, Fennema, Franke, Levi, & Empson, 2015, p.139). Allowing students to work through their strategy is important because they do not naturally use or copy a method they do not understand (Carpenter et al., 1996). Instead CGI "suggests that students' invented algorithms are constructed through progressive abstraction of their modeling procedures with blocks" and "the manipulation of the blocks become objects of reflection. Eventually, the words that students use to describe their manipulation of blocks become the solutions themselves" (Carpenter et al., 1996, p.12-13). Block use is a simple example, but manipulatives can also take the form of counting out loud, illustrations, and using fingers. For this study, *manipulatives* are defined as mathematical objects that develop motor skills and understanding of the abstract, all through motion (Cope, 2015). While it can be difficult to use these methods for harder-to-model problems in more advanced content areas, this dissertation's research took place among third and fourth grade students who experience a more foundational curriculum. "With CGI the emphasis shifts from teachers finding ways of representing mathematical knowledge for students to students constructing their own representations based on their intuitive problem-solving strategies" (Carpenter et al., 1996, p.14). To successfully implement CGI, providing follow-up questions to students after they arrive at their final solutions gives the interviewer or teacher more insight into students' understanding, and also allows students a chance to stand by their answers as correct or incorrect (Carpenter et al., 2015). The purpose of CGI is to understand student thinking as much as possible. To accomplish this understanding, teachers should not assume the reasoning behind a student's actions or strategies, and instead ask follow-up questions (Ginsburg, 1997; Jacobs & Ambrose, 2008). "Having a student share more details of her thinking engages the student in articulating, explaining, and justifying her thinking and enables the teacher and other students to understand the strategy the student used" (Carpenter et al., 2015, p.140).

The Montessori Method

This section describes a crucial element of the study—the Montessori method. The foundations of Montessori can be properly developed in the literature now that mathematical understanding and problem solving have both been covered. While there are many different teaching strategies, the *Montessori method* “aims at developing children’s senses, academic skills, practical life skills, and character” (Lunenburg, 2011, p.3). The Montessori program was founded and developed in Europe by Maria Montessori in the late 1800s for slower developing children, before expanding to a general learning style across the world (Edwards, 2002; Vardin, 2003). The Montessori structure includes a classroom filled with tangible subject-related objects for the students to manipulate, play with, use, understand, and share, while the teacher is present to foster a curiosity-based environment that keeps the students engaged in discovery (Lunenburg, 2011; Isaacs, 2007; Montessori, 1964; Humphryes, 1998, Pickering, 2004; Bagby & Sulak, 2010; Butler, 2002). These manipulatives are utilized to help students envision abstract ideas and also help students interpret those ideas in multiple ways; to create appropriate understanding by eventually using their words and descriptions as the solution itself (Carpenter et al., 1996).

The Montessori approach is an alternative to the traditional methods that most people consider when discussing education systems. To have a model comparison when discussing the difference between a traditional style and a Montessori program, traditional methods are defined as when “the teacher delivered direct instruction and controlled behavior; students followed directions, recalled knowledge, and worked individually” (Zhbanova et al, 2010, p.251). A non-Montessori structure does not imply a traditional style, and non-traditional structure does not imply a Montessori style. A less positive description of the traditional math teaching system summarize it as “an endless sequence of memorizing and forgetting facts and procedures that make little sense to [students]" while continuing with outdated methods to do so (Battista, 1999, p.426).

A push for Montessori schools in the United States began in the 1960s, with the allure of a child-centered curriculum (Edwards, 2002). Researchers brought attention to mathematical teaching strategies in the early twenty-first century (Clements & Sarama, 2007). The Montessori method focuses on the self-regulated student, who can be described as metacognitively, motivationally, and behaviorally active participants in their own knowledge process (Zimmerman, 1989; Zimmerman, 1990). This relationship further supports metacognition as the appropriate framework for this dissertation, as the Montessori method needs students with such abilities to find more individual success and help them thrive in the system (Montessori, 1952). Research has established sufficient links between metacognition and the Montessori method (Murray, 2011; Vardin, 2003). In addition, Marlowe (2000) identifies Maria Montessori as one of the premiere educators at connecting an education style to metacognition. The Montessori method treats each student as a potential agent of change for the world, and teachers should trust students to take the initiative to learn and to take advantage of the environment they are provided (Isaacs, 2007; Reeve, 2006). "We discovered that education is not something which the teacher does, but that it is a natural process which develops spontaneously in the human being" (Montessori, 1952, p.7). Research has shown that perception of one's own ability and self-efficacy leads to self-motivation, self-motivation drives ability and self-efficacy, and this relationship creates a continuous cycle (Bandura, 1997; Schunk & Rice, 1984).

Teachers must be familiar enough with their class to act as a guide for students to grow intellectually (Fang, 2008; Butler, 2002). Montessori (1964) described this process as letting go of the intuitive notion of a teacher, instead focusing on being a facilitator and strongly embracing the guide role. One case study compared teachers' classroom abilities to their ratings and reviews from previous employment and found that teachers can thrive in a relationship-based classroom, but must rely more on personal intuition than on any past teaching experience (Lockhorst, Wubbels, van Oers, 2009). Another study concluded that teachers require administrative support to have initial success in a new classroom style (Zhbanova et al., 2010). Further, teachers should be willing to adapt to new methods to be successful in the classroom, and "the most effective teacher moves cannot be preplanned. Instead, they must occur in response to a child's specific actions or ideas" (Jacobs & Ambrose, 2008, p.271). Research has shown that if teachers decide to change direction and unit plans in the middle of immersive learning, the results are detrimental for student learning (Edwards, 2002). For example, if teachers influence the class by changing groups or using a different technology mid-unit, students can be interrupted from positive routines and concentration, which impacts meaningful learning. These case studies have shown that interrupting a child's discovery process can have detrimental effects on overall learning, whether the lesson was electronic, pencil-and-paper, or action-based (Greenwald, 1999; Edwards, 2002). "Successful elicitation also requires a teacher who is willing and able to relax intellectual control sufficiently for children to respond with their own solution methods. Furthermore, the teacher must have the ability to leave behind old habits and models of teaching acquired during years of life as a student and as a traditional teacher" (Fraivillig, Murphy, & Fuson, 1999, p.17). Teachers collaborate outside of the class on how to adjust their teaching style to help students benefit most from exploration (Edwards, 2002). This student-centered experience reflects how “Montessori defined education as a dynamic process in which children develop according to the ‘inner dictates’ of their life, by their ‘voluntary work’ when placed in an environment prepared to give them freedom of expression” (Gutek, 2004, p.47). Similarly, Knowles’ (1975) definition of *self-directed learning* is “a dynamic process in which the learner reaches out to incorporate new experiences, relates present situations with previous experiences, and reorganizes current experiences based upon this process.” In short, learning about learning is another way of discussing metacognition.

Classroom relationships have a large influence on education quality (Raschunde, 2003; Eccles et al., 1993). Essentially, the Montessori method gives students complete responsibility for their education, at an earlier age than many would expect. “This respect for a child’s personality has been many times expressed in one form or another, and thinkers from the time of Socrates have urged its importance…We must keep our hands off the child if we wish it to come to its highest self-realization” (Ward, 1913, p.31). For self-directed learners to be most effective, they must make their own strategies and goals—identifying needs, along with where, how, and why they learn a given topic (Jennett, 1992).

Self-Driven Students

Intrinsic motivation is important for this discussion and requires a clearer definition—“a pupil displays intrinsic motivation or regulation when she particularly likes or enjoys learning new things with genuine interest, i.e., based on the satisfaction of learning for its own sake in the absence of external rewards” (Ünlü & Dettweiler, 2015, p.676). Self-regulation and self-motivation are considered different concepts, but are also results of each other (Zimmerman, 1990). There are three phases of self-regulation widely considered by educators—forethought, performance, and *self-reflection* (Pajares, 2002; Pape & Smith, 2002; Borkowski, Carr, Relliger, & Pressley, 1990), also known as "metacognitive, motivational, and behavioral" (Zimmerman, 1990). Research supports motivation and belief as impacting performance, persistence, and creativity (Ryan & Deci, 2000; Zimmerman, 1989). The original goal of the Montessori method was to develop in children the ability to succeed physically and socially in society (Rambusch, 2010). "The absorbent mind is a time of enormous potential in the development of the individual…The fundamental principle of this stage is recognition of the child as a spontaneous learner, driven by an inner drive/energy" (Isaacs, 2007, p.14). The *absorbent mind* is a term often used when discussing the Montessori method, and has been credited as vital in the socialization, cognitive development, and pattern recognition of students at a young age (Gutek, 2004). Self-directed learners generally must be "open, curious, organized, motivated, and enthusiastic" and must be familiar with accepting new ideas, recognize strengths and weaknesses, and be able to judge their progress (Jennett, 1992). Limited research has shown that young students with exposure to early Montessori education can advance more in cognition, social ability, and executive control, as well as score higher on standardized mathematics tests (Lillard & Else-Quest, 2006; Peng & Yunus, 2014; Dohrmann, Nishida, Gartner, Lipsky, & Grimm, 2007). Another study discovered that the differences in learning between autonomous students and directly taught students did not appear immediately—rather, the more self-aware, self-driven students who took the time to discover results for themselves were more likely to retain the information in the future than students who were directly instructed and immediately given the information (Grolnick & Ryan, 1987). Cossentino (2005), unfamiliar with the intricacies of the Montessori program, found through classroom observations that relationships developed at the one-on-one level were incredibly deep and full of emotion in unexpected ways. This perception has merit, as researchers believe there should be more focus "in exploring the association between teacher knowledge and student learning" (Charalambous, 2010, p.275).

The Montessori environment also leads to more natural socialization and can form better culture communities as young students move into middle school when compared to public school institutions (Rathunde & Csikszentmihalyi, 2005; Eccles et al., 1993). These relationships are part of what a teacher strives to accomplish using these methods (Montessori, 1964), and are also highly valued by many alternative education students (Owens & Konkol, 2004). The students' sense of belief in themselves, partially provided by teachers, is one factor for success as students approach the unstable stages of adolescence (Eccles et al., 1993). Self-regulated learning, if not already practiced by a student, can be dependent on the teaching and modeling of others (Zimmerman, 2002). The importance of correct modeling implies the need for strong relationships to exist for the duration of the learning experience. The teacher is also responsible for immersing students in all facets of mathematics education, and Bishop (1988) claims teachers "need to be aware of how their teaching contributes not just to the mathematical development of their pupils, but also to the development of mathematics in their culture" (p.190). Research further supports the ideas that independent problem solving is crucial for a student's understanding (Cozza & Oreshkina, 2013), and that good classroom discussion supports problem solving and promotes continued motivation (Zolkower & Shrayner, 2007; Dimant & Bearison, 1991). Eccles et al. (1993) support growth in the relationships between students and teachers, and recommend that this growth is a vital aspect of the classroom that needs improvement in the traditional setting. Zimmerman (1990) believes self-motivated students show the most proactive need for diligence, thoroughness, and overall explanation of their ideas and reasoning.

Support in this System

There are more factors of the classroom environment, in addition to the responsibility of the teacher. In the Montessori program, there is no separation between the acts of work and play (Isaacs, 2007; Cossentino 2005; Fang, 2008; Montessori, 1964). Therefore, educators critique this lack of separation as creating an ineffective learning experience (Fang, 2008). Rathunde (2003) claims the Montessori students in his study would later benefit both in high school and beyond, after observing the students' hard work and enjoyment coexisting. When researchers examine the Montessori method specifically for applications in mathematics education, teachers may not make other adequate separations, such as concrete versus abstract mathematics, or symbolic understanding versus manipulative comprehension (Uttal, Scudder, & DeLoache, 1997). Children are asked to focus on an object (the manipulative) and use its construction to understand the appropriate mathematics topic (Gutek, 2004; Grolnick & Ryan, 1987). Research has found that if parameters or guidance are required, the careful consideration of how this assistance is implemented becomes imperative to prevent suppressing creativity and growth (Koestner et al., 1984). For example, "if the teacher shows the child how to arrange a set of cubes she should then leave the child free to attempt this himself and not volunteer advice unless asked" (Rambusch, 2010, p.40). This self-directed style is also known as individualized instruction, student-centered learning, and prescriptive learning (Knowles, 1975; Piskurich, 1994). To further build such a setting, administrators heavily emphasize personal teacher evaluations to help provide an environment that raises students' confidence, which leads to students' tenacity to pursue higher-order problems in the future, continuing to build confidence (Murray, 2011; Reeve, 2006). *Self-determination theory* is a resulting idea, which assumes rather than forces the concept that every student has particular psychological and growth tendencies that require a motivational environment to maximize (Ryan & Deci, 2000). "The theory further assumes that students are always in active exchange with their classroom environment and therefore need supportive resources from their environment to nurture and involve these inner motivation resources" (Reeve, 2006, p.226).

Montessori describes a self-determination atmosphere as the basis for her methodology of teaching and learning (Montessori, 1964). Reeve (2006) says that an *autonomy* support style of education does not require controlling student behavior for teachers to have success in this environment, and that the teacher role as motivation nurturer becomes more crucial. When looking at autonomy through Piaget's viewpoints, Kamii (1994) says it means "the right of an individual or group to be self-governing" (p.59), which aligns to an expected Montessori style. "When information is offered in this way, children enjoy taking it in at their own pace, spending as much or as little time as necessary to gain a concrete understanding of the material" (Humphryes, 1998, p.5). Research has shown there is a positive correlation between self-efficacy and several important factors in the Montessori method: learning strategies, self-motivation, behavior, achievement, and persistence (Zimmerman, 1989; Schunk & Rice, 1984, Thomas et al, 1987; Zimmerman, 1998).

Research also supports the idea that students who distinguish between success and failure, while attributing effort as the reason for their success, will then use this realization in the future to have continued success—some believe this process can build self-efficacy in a learner (Borkowski, Turner, & Weyhing, 1986). Other researchers claim there are exclusive ways to build self-regulation in a learner. For example, de Corte, Verschaffel, & Op't Eynde (2000) suggest several ways, such as "modeling of strategic aspects of problem solving by the teacher, guided practice with coaching and feedback, problem solving in small groups, and whole group instruction" (p.720). Students gain an ability to communicate naturally if the teacher structures the classroom to incorporate student explanations (Pape et al., 2003). Many of Montessori's ideas align with Gardner's famous Multiple Intelligences outlook, which describes intelligence as the ability to solve problems in a way that contributes and is defined by cultural environments (Gardner, 1983).

Strengths and Weaknesses of the Montessori Method

While the Montessori method is supported by many researchers, there are also many struggles, transitions, criticisms, and conflicts. One of this study's research questions examines the extent to which the fourth grade non-Montessori style differs from the third grade Montessori style at the studied school. Thus, acknowledging strengths and weakness of the Montessori method is important. Fang (2008) pointed out that some critics view the program's lack of separating work and play as a systemic flaw. Critics also perceive some pedagogies as conflicting, because this teaching "is in many ways directly opposed to some of [the] fundamental pedagogical principles" (Baldwin, 1916, p.149). Montessori is only one example of a self-paced classroom, and teachers may often feel out of place and may struggle with identifying their role within such an environment (Colvin, 1973; Fraivillig et al., 1999). In mathematics education, Clough (1971) and Larsson (1973) found that high-ability students could thrive in this self-paced atmosphere, but other students did not maximize achievement in the same situation. For example, Clough's (1971) study implemented a new type of self-driven curriculum for students, and the high-ability, high-IQ students benefited more from the new system than the rest.

Further, some educators find the lack of direct instruction in Montessori schools unnerving, claiming that "the environment may provide 'the food for mathematical thought', but the existence of mathematical food for thought in a classroom does not guarantee that children will ingest it, let alone digest it" (Lee & Ginsburg, 2009, p.42). Research has shown that while students may be competent at a certain topic in mathematics, or at least believe they are, they do not engage if they perceive no value in the activity or lesson (Wigfield & Eccles, 1994; Boekaerts, 2002). Vygotsky (1978) says students usually possess a ceiling where they can no longer handle a problem individually, regardless of intentions, but need some type of assistance from teachers or peers. Some research has also suggested that the process of writing down mathematics is essential to true understanding (Pugalee, 2001), a task that has little to no use in the Montessori method. There is also much pressure on teachers in the Montessori method, especially in their preparation. "As a small-scale, specialized type of teacher education, Montessori has certain advantages; but that very smallness and specialness may make evaluating our effectiveness even more important to our survival than it is for more traditional modes of teacher education" (Turner, 2001). Zimmerman (1989) and Fraivillig et al. (1999) state that the ideal learning environment is optimized by seeking information and seeking assistance, which are actions less stressed in a Montessori environment. “If students are functioning in their zones of proximal development, then teacher intervention may help them function successfully” (Li & Adams, 1995). Some educational theories stress that adults, not children, are the best-equipped learners to successfully integrate a self-regulated learning style in order to effectively master concepts (Smith, 2016). For example, self-directed learning can also be defined as the “process in which individuals take the initiative, with or without the help of others, in diagnosing their learning needs, formulating learning goals, identifying human and material resources for learning, choosing and implementing appropriate learning strategies, and evaluating learning outcomes” (Knowles, 1975). To many veteran teachers, this explanation may not represent the mindset of the typical elementary school child. For example, Vygotsky’s Zone of Proximal Development requires adult guidance to function “as a scaffold…lead[ing] the child to solve problems collaboratively that could not be solved alone” (Greenfield, 1984, p.119). Noddings (1985) agrees that group-learning environments appear primed for the most efficient teaching of problem solving in the classroom.

However, there is still an abundance of support for the Montessori method, as some believe “no one could educate anyone else, that education must necessarily be self-education” (Savin-Baden & Major, 2004, p.15). Bagby and Sulak (2010) claim that Piaget and Vygotsky believe social constructivism is the best pedagogical theory for Montessori supporters to use as the basis of the program. The Montessori method leads students to be active rather than passive learners, trusting themselves as their own source of knowledge rather than automatically turning to a higher authority (Kretchmar, 2016; Koestner et al., 1984). Montessori methods promote positive work habits and build self-motivation (Ervin, Wash, & Mecca, 2010; Butler, 2002). In Bruner's (1960) opinion, "any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (p.33). This statement suggests that elementary grades are an appropriate sample for the dissertation. Bruner's radical thought opposed many educators' typical thinking at the time, because these educators often utilized specific educational psychological theories to dictate how and when to teach concepts, specifically for mathematics (Steffe & Kieren, 1994). While it may be difficult for educators to move past the idea that teachers are the ultimate information base, "the temptation is to do for children the things they might do for themselves, ignoring the great law of growth, which is that strength comes only through struggle, and that effort often gives greater pleasure than result" (Ward, 1913, p.41). Steffe & Kieren (1994) claim that Piaget's research on cognitive development concluded that manipulatives and play have important places in mathematics education. Lampert (1990) acknowledges that her students should realize that "the warrant for doing [mathematical operations] comes from mathematical argument and not from a teacher or a book" (p.34).

Studies on one specific demographic have shown that "At Risk" students, who have difficulty with symbols, patterns, relationships, and general skills, find success in Montessori-like schools because classroom culture is more accepting. Through a variety of student observations in consistent Montessori settings, these students were found to be less prone to frustration (Pickering, 2004). Rathunde (2003) explains that the schools in one of his studies portrayed reformation toward increased motivation, and that the resulting benefits provide a case for this type of educational reform elsewhere. Further, studies have shown that “students who display initiative, intrinsic motivation and personal responsibility achieve particular academic success” (Zimmerman, 1990, p.14). Also, studies have shown that students from Montessori backgrounds can still perform well on standardized tests (Dohrmann et al., 2007).

Defining Traditional Methods

The American public school system typically employs the traditional view of teaching mathematics—that generally means the direct-instruction approach. *Traditional methods* involve when the teacher directly instructs students and controls behavior, while students follow directions and recall knowledge individually (Zhbanova et al., 2010). The purpose of this study’s first research question is to determine the nature of the non-Montessori fourth grade classroom, and to what extent the class aligns with a direct-instruction style. The traditional style has recently been criticized as possibly being outdated (Chapko & Buchko, 2004), leading to a growing alternative education movement around the world. The traditional style, commonly through direct instruction, differs from Montessori style in terms of how students receive information. Traditional classrooms may sacrifice content and instead focus on simpler tasks such as keeping young students’ attention for an entire class period (Boekaerts, 2002). One study found that the traditional method still improves the student knowledge base, but admitted that “without a curriculum that matched the student’s knowledge and skill level and focused instructions, it is unlikely that the students could gain so much in such a short time” (Din, 1998, p.10). In fact, several studies have found that when students are struggling or have learning difficulties, direct instruction is the best way to quickly correct the situation for standardized testing (Moore, 2014; Al-Makahleh, 2011; Din, 1998). While there is a difference between standardized test results and a conceptual knowledge base, this adjustment is a strength of the traditional style. Research has shown that conceptual knowledge generally improves if either a hybrid direct teaching or a mixed instruction method exists in the classroom instead of the purest form of traditional methods, which is a weakness of traditional teaching styles (Rakes et al, 2010). “Mathematics educators and researchers argue that current mathematics instruction in elementary and secondary school focuses too much on efficient computation and not enough on mathematical understanding, problem solving, and reasoning” (Putnam, Lampert, & Peterson, 1990, p.57). Traditional mathematics education focuses on procedural development rather than inquiry-based development, leading to claims of traditional forms of teaching becoming obsolete (Chapko & Buchko, 2004).

The Transition from Montessori to Traditional

The main focus of this dissertation concerns the transition between a third grade Montessori program and a fourth grade non-Montessori program. This non-Montessori label does not imply completely a traditional, direct-instruction style; part of the study’s purpose is to determine the extent to which the fourth grade classroom follows a traditional style. Traditional methods can vary school by school and may not completely align with direct instruction. Direct instruction in mathematics education focuses on procedural development rather than inquiry-based skills (Chapko & Buchko, 2004; Peterson, 1979). This dissertation focuses on the extent of the fundamental differences between Montessori and traditional styles, and how the transition affects students’ problem-solving skills. “Remarkably, in traditional classrooms there is not much room for self-regulated learning. Students are cognitively, emotionally, and socially dependent on their teachers who formulate the learning goals, determine which type of interaction is allowed, and generally coerce them to adjust to the learning environment they have created" (Boekaerts, 2002, p.594). There is a relationship between autonomous learning and the Montessori method, and "in the intellectual realm…autonomy means the ability to govern oneself by being able to take relevant factors into account, and heteronomy means being governed by somebody else" (Kamii, 1994, p.62). In this context, Montessori aligns with the former and direct instruction aligns with the latter. While the related research has commonly focused on ways students are encouraged to self-discover, some researchers firmly believe that independent, self-initiating learners cannot come out of the traditional schooling system (Illich, 1971).

There is also evidence that there are two shifts in teacher participation when dealing with student work: first attending to the finer details of how a student thinks, and then developing a plan to move ahead with this knowledge in mind (Kazemi & Franke, 2004). In the Montessori style, there is little to no work shown for thought processes like there is in the direct traditional style (Montessori, 1964). Montessori supporters would counter that the environment in a traditional school tends to teach children to fear failure rather than love learning, as there is more pressure to show correct work and solutions (Illich, 1971; Rosanova, 2003). Wigfield and Eccles (1994) argue that traditional middle school does not provide a safe and intellectually challenging atmosphere for students to reach their full developmental potential, a claim that is further supported because "numerous scientific studies have shown that traditional methods of teaching mathematics not only are ineffective but also seriously stunt the growth of students' mathematical reasoning and problem-solving skills" (Battista, 1999, p.424). Research implies students from task-focused elementary instruction experience fewer negative transitions to standard methods in middle school (Anderman et al., 1999). Further, studies suggest that students raised in a Montessori setting have "higher intrinsic motivation, interest, and flow experience in academic work" when compared to students in a traditional middle school setting (Rathunde & Csikszentmihalyi, 2005). "Unlike their passive classmates, self-regulated students proactively seek out information when needed and take the necessary steps to master it" (Zimmerman, 1990, p.4). However, this ability is not limited to Montessori students exclusively, a claim supported in another study using self-efficacy scales, child interviews, and work habit ratings; researchers interviewed students about their thoughts in the classroom, while surveying parents and teachers about how they teach discipline and observe self-efficacy (Ervin et al., 2010). Regardless of the primary school methods that students experience, this motivation has been found to decrease after moving into middle school across all education styles (Anderman et al., 1999), providing possible proof that students struggle with changing schools regardless of the learning method. In this study, changing schools is eliminated as an influential factor, and results can be interpreted accordingly.

Additionally, elementary students in all settings are more ambitious and have higher achievement goals than middle school students (Ruble, 1977). Students who leave a Montessori program usually do so when changing schools and entering middle school. While some may attribute this to puberty and pre-teen attitudes, many researchers claim it is instead a disparity between the development of the child and their learning environment (Rathunde, 2003). Similarly, studies have shown that self-esteem suffers once students have moved into middle school (Wigfield & Eccles, 1994; Simmons, Blyth, Van Cleave, & Bush, 1979). Because self-efficacy is important for student motivation and student success, this is a particularly important conclusion. Owens and Konkol (2004) found that a lack of relationships is the primary problem students point out when moving from a non-traditional style to a traditional style, and students reported this problem more frequently as the number of students in the classroom increased. Since there is definitive value in such relationships, "the educational setting…can make or break [students]" (Owens & Konkol, 2004, p.176). Conversely, one study found that students, teachers, and parents all had similar, positive remarks about the transition from Montessori to a regular middle school program, with only a few discrepancies and complaints about the homework load and pacing of the classroom (Hanson, 1998). Whereas the Montessori style allows students to be responsible for their own knowledge and growth, the traditional style dictates teachers as absolute sources of knowledge and classroom managers (Copes & Shager, 2003). Young students must adjust to this transition while continuing to learn new material. However, there is little research that provides specific evidence regarding which issues arise when dealing with this shift in learning environment. This dissertation aims to determine possible reasons students could have difficulty transitioning. Table 1 provides a quick reference for the many differences between Montessori and traditional styles of education. There are many aspects that the students in this study could experience after moving out of third grade and into fourth grade, and in Chapter 3, the three specific aspects pursued in this study will be explored.

Table 1
Differences Between Montessori and Traditional

Montessori Method	Traditional Method
Hands-off teacher guide	Teacher as direct source of knowledge
Self-paced, eventual learning	Scheduled, structured lessons
Lack of focus on grades	Grade and rank intensive
Manipulatives and objects used more	Pencil-and-paper work stressed
Focus on relationships	Less personalized relationships
Typically elementary ages	Audience of all ages
Discussion and group based approaches	Rigorous exercises and many assessments

Social and Cultural Background

"From age 6 to 12, children are expected to explore a wider world and develop rational problem solving, cooperative social relations, imagination and aesthetics, and complex cultural knowledge" (Edwards, 2002). Accordingly, this dissertation considers several social and cultural aspects because the location is in Central America. "Ethnography is important to educational research as it takes us inside everyday educational context and brings us close to everyday practices and the people involved" (Beach, 2011, p.572). There is limited research linking cognitive abilities to social and learning abilities in students (Cozza & Oreshkina, 2013, Goos & Galbraith, 1996, Sleeter, 2001; Pugalee, 2001). There may be opportunity to use conclusions from this dissertation to address some of this issue through future related research. A recent focus through professional development has led to the creation of teacher education programs that specifically concentrate on cultural aspects of teaching. For example, one study found that there are many benefits to language learning that complement intercultural experiences in education (Kanyaprasith, Finely, & Phonphok, 2015), which provides support for this focus on culture. Recent studies have also focused on verbal communication versus written assignments (Pugalee, 2001; Zolkower & Shreyar, 2007).

Researchers and educators often stress culture as an important aspect of learning mathematics; depending on the background of the student, defining mathematics can be subjective (Brownell, 1942; Bishop, 1988). Torres-Velasquez and Lobo (2004) promote creating culture based on the cultural environment of the classroom, which focuses the content on social, cultural, and real-life experiences of the students. There are two main viewpoints that discuss the role of culture in education. “On the one hand, there is the Piagetian individual who *re*discovers through his/her own actions the rationality of mathematics; on the other hand, there is the (deficient) individual as empty vessel, who comes to be filled with the knowledge that culture makes available” (Roth & Radford, 2011, p.8). Bishop (1988) has mentioned that mathematics curriculum should align with the culture of students in their home lives. “Mathematics curricula, though, have been slow to change, primarily due to a popular and widespread misconception…[where] the conventional wisdom was that mathematics was ‘culture-free’ knowledge” (Bishop, 1988, p.179). Further, D’Ambrossio (1985) claims that culture itself can create mathematics—cognition and culture go hand-in-hand. He also defined the term *ethnomathematics* as the relationship between mathematics and culture (D’Ambrossio, 1985). DiMaggio (1997) echoes some of these thoughts and claims that “meanings are rarely fixed, but instead adapt, diverge, and spread across domains through semantic contagion” (p.284).

Bruner (1996) defines *culture* as meaning making, or “assigning meanings to things in different settings on particular occasions,” and says that culture also “involves situating encounters with the world in their appropriate cultural contexts in order to know ‘what they are about’” (p.3). Several researchers find culture imperative in successful education, and believe self-regulation and problem solving are influenced by such cultural contexts (Butler, 2002; Ferrare & Hora, 2014). “One of the dominant approaches to understanding culture in academia has been to view culture as a unitary set of beliefs, values, and practices that can be ascribed to entire disciplines or institutions” (Ferrare & Hora, 2014, p.795). For teachers to be most effective, they should use context, language, and culture to their students’ benefit (Moschkovich, 1999; Gardner, 1983). “For instance, ‘culturally deprived’ children were slower to develop language, more likely to be physically punished, less likely to do well in school” (Berger, 2002, p.57). This finding supports the inclusion of a cultural discussion for this dissertation, even though it is not the primary research focus. “Cognitive aspects of culture are…a part that we cannot avoid if we are interested in how culture enters into people’s lives, for any explanation of culture’s impact on practice rests on assumptions about the role of culture in cognition” (DiMaggio, 1997, p.285). Ladson-Billings (1995) describes culturally relevant teaching as crucial for encouraging students to increase success, maintain cultural integrity, and think critically about social topics. Presmeg (2005) says researchers agree that culture is present in all facets of learning, and that students need cultural context for continued success. Chapter 3 of this dissertation discusses the setting, context, and population before describing the data-collection stage of the research.

Further, performing research internationally as an American residing in Costa Rica creates unique issues as a researcher. For example, “there is a long-standing debate in ethnography about the analytical tightrope between familiarity and strangeness” (Coffey, 1999, p.47). It is important to realize and accept the differences “between the native culture and the target culture, [as] the unstable new identity has an effect on the learner's language socialization and performance" (Wang, 2010, p.58). For example, nearly all students in this study were bilingual, and half of the students spoke Spanish as their first language. The students in third and fourth grade all spoke fluent English. Yet studies have shown that many researchers in other countries adjust better to their surroundings, and subsequently gain confidence in their experiences as outsiders, when they make efforts to learn the local language, regardless of relevancy to the study or findings (Wang, 2010). Kalinec-Craig (2014) argues that although some researchers have argued against the existence of universal mathematical algorithms, mathematics can still be considered a universal language throughout culture. Kalinec-Craig (2014) interviewed an insightful international researcher, who said "I needed to move away from a first-person perspective of teaching and step into my students' mindset as they solved mathematical problems. As I slowly made this shift, more of my students became successful in mathematics" (p.50). Observing students and interpreting their mathematics work and explanations requires such self-reflection, and these aspects were considered when interviewing teachers for this dissertation. "'Mathematics as a universal language' rests in the beliefs that those who use mathematics should also use similar practices, symbols, algorithms, and problem solving strategies," but evidence also shows that "teaching and learning mathematics is a varied and culturally-specific activity" (Kalinec-Craig, 2014, p.48). The study takes place in a Central American country where many of the elementary students were born and raised, and culture continues to influence their mathematics experience in the classroom.

Additionally, researchers outside of their home country strive for two contradictory goals: to achieve a position of comfort and familiarity within their environment, but also keep professional and personal distance from the new culture (Coffey, 1999). Researchers entering unfamiliar cultural environments in past cross-cultural studies have shown a negative tendency to compare their studied group with their own culture, and several researchers instead believe minorities should be compared and studied within the context of their environment; one example includes comparing minority groups to Caucasian students, sometimes called the "Eurocentric" problem (Padilla, 2004; Coffey, 1999). The school in this study provides the opportunity to use metacognition as the framework, and the culture forms an isolated environment that minimizes the effects of other variables. The methodology is detailed in Chapter 3, which ties the literature to the selected methods moving forward.

III. METHODOLOGY

Appropriateness of the Design

This study used a qualitative methodology that aimed to provide evidence to answer the following research questions:

1. To what teaching practices and learning opportunities are third and fourth grade students exposed? To what extent are these practices and learning opportunities related to the Montessori approach?
2. How are three particular aspects of current teaching practices and learning opportunities in fourth grade mathematics perceived by students and teachers compared to previous exposure in the Montessori style?
3. How, and to what extent, does changing the teaching practices and learning opportunities affect the problem-solving strategies of students?

Table 1 lists contrasting traits between Montessori and traditional styles of education. Table 2 highlights the three specific aspects studied for this dissertation—potential causes for the perception of students struggling in their transition between the third and fourth grades. Three aspects were selected to maintain focus among a large quantity of potential variables. The first aspect is that Montessori creates a self-paced learning style with less structure than many classrooms, while a traditional method contains scheduled, structured lessons. The second aspect focuses on comparing the use of manipulatives in the Montessori classroom with the pencil-and-paper methods of the traditional methods. The third aspect identifies the changing roles of students and teachers; while Montessori gives students control of their learning, traditional methods give teachers power as a direct knowledge source.

Table 2
Selected Aspects for Focus

Montessori Method	Traditional Method
Hands-off teacher guide	**Teacher as direct source of knowledge**
Self-paced, eventual learning	**Scheduled, structured lessons**
Lack of focus on grades	Grade and rank intensive
Manipulatives and objects used more	**Pencil-and-paper work stressed**
Focus on relationships	Less personalized relationships
Typically elementary ages	Audience of all ages
Discussion and group based approaches	Rigorous exercises and many assessments

Research Design

Observation, interview, and documentation were all used to extract the necessary evidence required to fully address each research question. First, it was necessary to determine to what extent the third and fourth grade classrooms aligned to Montessori and non-Montessori methods defined in the literature. Observations and field notes were the typical method used for either mathematics classroom. Additionally, student and teacher interviews were important because they described the intentions, opinions, and implementation of their respective programs. The fourth grade classroom was intended to follow traditional formats, but the deviation from Montessori needed to be documented. Direct instruction was hypothesized as the primary instruction style in the classroom, following the purest forms of traditional definitions of teaching. Specific observation tools and interview templates were helpful to document each classroom. The research instruments were based on relevant studies covered in the literature review. For example, the DISCOVER tool developed by Bahar and Maker (2015) was appropriate for finding the advantages of students thinking about open problems. Bandura and Schunk (1981) studied motivation from varying physical distances to determine the extent to which students motivated themselves when pushed to different degrees by an authority figure.

These ideas were not copied verbatim, but they did help to shape the tools that were used for this study. More detail on the actual structure of the observation and interview templates appear later in Chapter 3.

Interviews were an appropriate method for extracting the most information for the research questions, yet involved their own limitations. First, interviews were much more time-intensive when compared to a survey or written response. Additionally, the interview retained the one-on-one aspect that a survey would have, but potentially covered fewer deep responses. Since the nature of the research questions required deep responses on ideas, attitudes, and emotions from conversations that cannot be documented using a survey alone, the interview was selected as a stronger method. It was very important to press students, particularly more than teachers, to explain their thoughts and reasoning, as younger children may be less focused in their responses than adults. No student in the third or fourth grade classroom was left out of the interviewing process. Four teachers were interviewed three times each because of the value they added to the conversation about the transition. This was the primary method to determine how they embraced the very different roles required between Montessori and non-Montessori classrooms. Overall, 28 student interviews and 12 teacher interviews were conducted—40 interviews altogether. In addition, six fourth graders were selected to proceed through two rounds of CGI problem solving questions during the year; one was administered in October, and the other one given in January/February. Selection was based on teacher perceptions of median mathematical performance from the prior school year. The administered problems are detailed in the instrumentation portion of Chapter 3. The quality and quantity of student and teacher dialogue from interviews provided sufficient evidence to thoroughly address the research questions. Pacing, classroom structure, material difficulty, trends, and styles are all topics could naturally arise through interviews. Since a survey might have ignored some of these topics, interview methods were chosen over survey methods.

The third instrumentation used was documentation of artifacts. One of the three aspects explored in the study was how work was shown in each classroom. One important research consideration was to determine the extent to which students' methods of showing work changed between the two programs. Pictures and videos of students working, discussing, writing, explaining, collaborating, and problem solving were essential when comparing student experiences between the third and fourth grade classrooms. To analyze problem solving in students, the sample of 28 students was narrowed to six students to take part in the CGI assessments, which captured written work. A brief interview was conducted before each problem set to provide the fourth graders more opportunities to share their thoughts. The goal was to ascertain any changes in thinking, methods, and attitudes between the two administered assessment periods. The CGI problems were not simply algorithmic or computational, but instead were open-ended word problems. The problems required students to identify methods and follow through with their beliefs to arrive at not only a correct solution, but also a solution that made sense to them within the context of the problem. Evidence of true understanding could be determined once they were pushed to explain their reasoning further. These task-based problems were mainly used to answer the dissertation's third research question, which addresses how students' problem-solving skills were affected by the transition. Part of the evaluation included determining whether students could make sense of mathematics, outside of formulaic problems that require simple computation and algorithm memorization. This determination provided insight into the effects of the transition stage on students' problem-solving skills. During the assessments, students were not given the same values to calculate, because "when a child does not understand a problem, even after attempts to rephrase or elaborate it, changing the problem itself can be productive. One type of change is to use easier numbers. Specifically, using smaller or friendlier numbers can help a child gain access to the mathematics underlying a problem. After making sense of an easier problem, a child generally gains confidence and, in many cases, can then make sense of the original problem" (Jacobs & Ambrose, 2008, p.263). This process was used to capture what students knew based on each student's individual level, so that they did not strictly focus on the values rather than the context.

Coding was important in finding value from the qualitative data drawn from student and teacher interviews. As recommended by Pino Pasternak (2006), modeling with a framework of metacognition creates three main themes: metacognitive knowledge, metacognitive regulation, and emotional and motivational regulation. Responses by teachers and students were evaluated through these considerations, according to verbal and non-verbal behaviors. For students, one of the goals was to use their descriptions to dictate the extent that the third and fourth grade classrooms followed Montessori and non-Montessori styles, respectively. Coding was more necessary to address the second goal, or theme: interpreting students' stance on mathematics and preference toward the use of manipulatives or handwriting methods. While questions were broad, students often chose specific words and phrases when discussing how they felt about mathematics and manipulatives. Students' terms toward mathematics were categorized as negative, positive, or medium when interpreting their overall stance toward the subject. This strategy included factors such as confidence, workload, emotion, connotation, and experience in mathematics. Similarly, students were asked about feelings toward using manipulatives and handwriting methods for mathematics. Students from both grades were experience with these methods, and comments were coded accordingly to show preference toward one or the other—"no preference" was not an option for this theme, and students rarely found difficulty in stating a preference for one method over another. Teachers were often coded in their responses to questions about their role in the classroom. Often, this coding reflected how teachers' roles aligned to the theory, and notes were also taken regarding their comfort with the appropriate role. In Chapter 4, some trends appeared as these codes separated responses into particular categories, and the conclusions in Chapter 5 reflected these themes.

Setting and Participants

This study took place in a coastal town on the west side of Costa Rica. The school is IB-certified, with a consistent high school curriculum that does not appear in the typical Central American public school system. The lower grades are taught in preparation for this upper level curriculum. The school is private, with approximately 135 students ranging from pre-Kindergarten to twelfth grade (approximate numbers due to international students' tendency to relocate). English is the predominant language, though some students speak it as a second language. Each student is enrolled in both English and Spanish classes from an early age, and is expected to be fully bilingual by graduation. More than twenty countries consistently represent the diverse population of the school across the grades, and there are no comparable schools in the surrounding area. This school uses a trimester system rather than a semester system. The first trimester ran from September 1 to December 14, the second trimester from January 3 to March 31, and the third trimester from April 24 to June 29. This study took place during the first and second trimesters to maximize the opportunity to collect data at the peak of student transition.

Table 3
Participating Students

Name	Grade	Years at the School
Tamaya	Third	Third year
Ruby	Third	Entire schooling
Ziggy	Third	Second year
David	Third	Entire schooling
William	Third	Entire schooling
Holden	Third	Second year
Marlo	Third	Third year
Alena	Third	Third year
Chloe	Third	Third year (with break*)
Cody	Third	Third year
Solace	Third	Entire schooling
Callum	Third	First year
Jayce	Fourth	Third year
Sonny	Fourth	Entire schooling
Merissa	Fourth	Second year
Lilia	Fourth	Entire schooling
Noa	Fourth	Fourth year
Mirabai	Fourth	Second year (with break*)
Norelle	Fourth	Third year
Marysol	Fourth	First year
Jack	Fourth	Fourth year
Cole	Fourth	First year
Felix**	Fourth	Second year
Lior**	Fourth	Entire schooling
Victoria**	Fourth	Fourth year
Romeo**	Fourth	Third year (with break*)
Valentina**	Fourth	Fourth year
Kai**	Fourth	Entire schooling

* Indicates the student went through school elsewhere at some point before the 2016-17 school year.
**Indicates a student who was used during the CGI problem solving exercises.

While the school comprised the population of the study, the sample specifically focuses on third and fourth grade mathematics classrooms. The students in this study transfer out of Montessori after the third grade, earlier than the typical fifth grade transfer out of the Montessori program into a traditional middle school setting. Students had a variety of experience in Montessori settings, and these backgrounds are detailed in Table

3, which lists the information about the student sample. There were 12 third graders and 16 fourth graders involved in the study; six of the fourth graders were selected for participation in CGI assessments to evaluate problem-solving skills, indicated at the bottom of Table 3. The "with break" description indicates students' years at the school were not consecutive. The "entire schooling" description indicates attendance at the school prior to first grade, some students as young as three years old. The last six names in the table are emphasized as students selected for the Cognitively Guided Instruction problems. Participation in CGI exercises required two separate interviews, instead of the standard single interview, and close inspection was given to students' mathematics strategies. Further details on the results of the CGI exercises are detailed in Chapter 4.

The faculty participating in this study included two third grade teachers, two fourth grade teachers, and the head of school. Ms. Julia is a third grade teacher from Spain, who learned English after spending some time in Canada before moving to Costa Rica to teach at the school four years ago, where she has consistently been a Montessori teacher. Her prior employment did not include any teaching beyond the occasional summer camp or nursery setting, and her primary work experience was in social work. Ms. Karla is a third grade teacher from Costa Rica, and has fifteen years of teaching experience at the elementary level (two as an assistant and thirteen as a lead teacher) between the United States and Central America; she is entering her first year as a Montessori teacher. The fourth grade teacher with the most direct student contact is Ms. Vicky, an American who has taught in both Montessori and non-Montessori levels at this school since its inception nearly ten years ago, with twelve years of prior experience teaching in Virginia and Oklahoma. Ms. Verena, another local Costa Rican, is a fourth grade teacher who operates occasionally in the classroom, but focuses more on fourth grade curriculum development—creating activities, games, assignments, and general curriculum for the classroom. She has twenty years of experience in education (not through Montessori), beginning as a Spanish and social studies teacher in Costa Rican and United States public schools before taking administrative roles in the latter half of her career (she also serves as the dean of the school in this study). Mr. Laird is in his third year as the head of the school, and previously taught science in the United States and the Philippines for five years, moved into school administration in China and Venezuela for ten years, and then took the position in Costa Rica. Ms. Julia, Ms. Verena, and Ms. Karla are all bilingual, Ms. Vicky speaks only English, and Mr. Laird has an intermediate Spanish language background on top of his native English.

The Montessori program at this school begins earlier than first grade, in the early childhood ages found in pre-Kindergarten and toddler programs. Ms. Julia and Ms. Karla share responsibilities in third grade, each with their own individual collection of first, second, and third grade students mixed in their classrooms. Students are given the opportunity to work on their choice of activities in an openly structured curriculum. Teachers expect a designated amount of mathematics work to be completed by their third grade students each week in an exploratory learning environment. In fourth grade, Ms. Vicky and Ms. Verena consistently provide direct mathematics lessons in the morning. The non-Montessori classroom observation tools were tailored for the direct, lecture-based learning atmosphere that was present in fourth grade. The fourth grade classroom was adjusted during the first two months of the school year, following unrelated decisions that were made by school administration after the study was already underway. For example, in response to fourth grade struggles identified in the previous school year, more group activities, inquiry-based lessons, and differentiated exercises were initially proposed in the fourth grade curriculum to address the transition issues. Ms. Vicky and Ms. Verena were also positioned as fourth grade teachers for the first time in their careers at this school. The fourth grade class followed the Saxon 4th and 5th grade textbooks as the basis for instruction. The classroom experience and data collection provided in Chapter 4 help determine if these implemented changes were consistent and successful.

The five involved faculty members were asked for their initial thoughts and opinions on the research questions at the beginning of the school year. Through preliminary interviews, teachers shared their thoughts on the challenges faced by students moving into a fourth grade, traditional mathematics classroom after a Montessori program. Ms. Julia believed the cause of struggle was abstractness—the lack of manipulatives moving forward after their predominant use in the Montessori classrooms. Ms. Karla, who was new to the Montessori program, agreed with this opinion, and said that Montessori's focus on objects and hands-on lessons was a huge difference compared to her own teaching experience. Ms. Verena thought the main issue was a lack of differentiation between students in the traditional classroom. She believed that students found difficulty in progressing at the same pace as their classmates, and creating differentiated curriculum was primarily her solution—the follow-through of this implementation is provided in Chapter 4. Ms. Vicky did not consider the effects of changing to handwriting methods in fourth grade from manipulative-focused work in third grade to be important. Mr. Laird offered two possibilities: the differentiation as emphasized by Ms. Verena, and also the shift from visualization to abstract as per the opinions of the Montessori teachers (another case of using manipulatives versus paper and pencil). None of the interviewed faculty mentioned the shift in the roles of the student and teacher when discussing possible factors.

The location of the study also has strengths and weaknesses. For example, "Costa Rican primary teachers in training have much less mathematics and subject matter preparation than secondary teachers in training. Primary teachers are trained to be generalists" (Sorto & Luschei, 2010, p.672). However, these guidelines generally describe the public school system in the country, and the school for this study is a private, international school that operates independently of such limitations. In fact, one study found that private schools in Panama perform better than public schools, and one reason could be that private schools tend to attract more educated and more qualified teachers (Sorto, Marshall, Luschei, & Carnoy, 2009). Awareness of the environment surrounding the school is still valuable, because "understanding the characteristics of the population is critical if we are to generalize the results properly and replicate the findings" (Padilla, 2004, p.130). Further, typical third grade Costa Rican classrooms exhibit behavior similar to a Montessori model in using concrete materials, but differ with Montessori by the inclusion of activity books and handwritten work methods (Sorto et al., 2009). By being so diverse a school, the results of the study are applicable to other school systems in the world. Also, the school is privately owned, which allows more freedom in implementing teaching strategies and curriculum topics.

Instrumentation and Analysis

This dissertation investigates the transition between two different teaching practices and learning opportunities, while maintaining a consistent environment. Tasks, interviews, and observations were used to document changing work methods, emotions, and relationships. There was no experiment implementing a treatment to any group—this was strictly an observational study to view the results over the first two trimesters to determine the extent to which the hypothesized aspects affect the transition. In this section, these three aspects are identified with emphasis on the instruments used to measure intangible variables. To summarize, topics include types of instruments, the method of analysis, the relationship to metacognition, and references of the tools themselves.

The first research question sought to establish the degree to which the third grade classroom aligned to the defined Montessori method, and to what extent the fourth grade classroom fit the traditional, non-Montessori, and direct instruction definitions. Observation was the most effective method of documentation for this research question, and ten observations were conducted in each grade. Once extensive field notes were taken, comparisons and correlations were made between those observations. Combined with what has been found in the literature review, these were deemed enough evidence to satisfy the first research question. The theoretical framework of metacognition was valuable for these observations. One of the main differences between traditional and Montessori classrooms is how active or passive the child is expected to participate in the learning process, one of the three aspects analyzed through the research questions. Additionally, teacher role, dialogue, and manipulatives use also differ between the two styles. An observation tool was created to pinpoint exact relevant characteristics, and this tool included under *Observation Criteria*, on p.246 in the Appendix. The tool was modeled after the RTOP (Reformed Teaching Observation Protocol) and edited to fit the particulars of this study. The edited tools were named the MTOP (Montessori Teaching Observation Protocol) and the TTOP (Traditional Teaching Observation Protocol). The MTOP and TTOP each contained a ratings scale generated from similar statements in the RTOP, combined with unique statements derived from the review of the literature about Montessori and traditional styles of classrooms. These statements were selected because they were most indicative of their respective classroom structures, and additional space was provided to extract the most relevant information for the study.

There were three aspects hypothesized to impact the transition from third to fourth grade. These aspects were the changing structure and pace of the class, the removal of manipulatives in fourth grade with more emphasis on handwriting, and the shifting roles of students and teachers. To assess these aspects accurately, twenty total classroom observations were completed, ten in third grade and ten in fourth grade, over the first seven months of the school year. Ms. Julia and Ms. Karla were observed five times each in the third grade classrooms. In the fourth grade classroom, Ms. Vicky was observed seven times and Ms. Verena was observed three times to account for the disparity in how often each teacher was present in front of the students. Classroom observations followed the MTOP and TTOP forms designed for this study, with twelve provided statements representing Montessori and traditional (specifically direct-instruction) classrooms, respectively. These statements were ranked from never occurred (0), to very common (4), in order to evaluate the alignment of the classrooms to their theoretical formats. A (3) or (4) was considered present in the class, while anything lower portrayed a lack of appearance. Also, many direct quotations from teachers and students in the study have been smoothed for better understanding, without altering the intent of their comments.

The intent of this tool was not to quantify the observations, but instead qualify the instruction as heavily aligned with the respective educational theories.

The second research question focused on the extent of the difficulty transitioning from the Montessori style to the traditional style—actuality versus perception. While school administration recognized a challenge prior to this study, student perception and performance was the preferred method to indicate the degree and context of the struggle. Teacher and student interviews were important for understanding the thought process that each experienced while progressing through the school year, in terms of predisposed thought, qualities noticed, changing ideas, struggles or strengths, and growth of the program. An additional purpose of teacher interviews was to help support observations in determining the extent to which third and fourth grade classrooms aligned to Montessori and non-Montessori styles. Teacher interviews also provided pertinent information such as role identification, opinions on the curriculum and pacing of the students, strengths and weaknesses of the classroom, and details on instruction and scheduling. The student interviews revealed more of their experiences comparing the learning opportunities, thoughts and opinions on the current experienced style, and classroom qualities they found successful. This awareness of the transition and students feelings are also examples of metacognition, the theoretical framework of the study. The interview templates include metacognitive questions covering how mathematics was taught, and student and teacher opinions about the perceived value added by learning mathematics. Student and teacher interviews also provided perspective and comparisons of how their roles changed from the previous school year.

Data was analyzed through coding; many of the words, thoughts, phrases, and expressions that students expressed in these interviews presented patterns and trends, and are described in Chapter 4. To begin the conversation with unfamiliar students, potential introductory interview questions were created as a starting point: third grade students, third grade teachers, fourth grade students, and fourth grade teachers all received unique questions, shown under *Interview Templates*, on p.242 of the Appendix. Within these templates is support for why these questions were asked in order to connect the ideas with the initial literature review. This process highlighted some of the differences between the third grade and fourth grade classrooms. The teaching styles were evaluated utilizing their definitions to maintain relevancy for use in further studies. The purpose of this study was to evaluate three aspects contributing to the perception of a challenging transition from Montessori style to non-Montessori style, so participation was necessary from Montessori teachers, Montessori students, traditional teachers, and traditional students. When the six selected students worked through their CGI task-based problems, this process also provided some evidence about which material they considered more difficult, along with strategies and opinions as to why that was the case.

The third research question focused on how students' problem-solving strategies were affected as they transitioned between programs, and documenting student work was the appropriate evidence of mathematics understanding. For the six students selected to carry out the CGI portion of the study, artifacts provided the tangible proof of how students progressed in material during the transition period. The problems gradually required more challenging strategies and content knowledge, as the literature suggests for obtaining the most understanding about student thinking (Goldin, 1997). Additionally, teachers that utilize CGI instruction have discovered that students gradually learn to make sense of the context of a problem on their own; more importantly, students who struggle to get started on word problems learn to identify the mathematical relationships within the problems and use them to their advantage (Carpenter et al., 2015). All research questions in this dissertation could provide evidence toward analyzing students' problem-solving skills, along with the CGI assessments. The fourth grade teachers recommended which six fourth grade students should participate in the CGI portion of the study. Ms. Vicky and Ms. Verena were asked to collaborate and choose students they perceived as average mathematics performers, based on their prior experience with the students from previous years. The task-based portion of the interview process was given in two rounds.

The first problem set, shown in Table 4, was administered at the beginning of October. The second problem set, shown in Table 5, was administered over the last week

Table 4
First CGI Problem Set

#1: Jennifer has ____ dollars. She earns some more money babysitting over the weekend. Now she has ____ dollars. How much money did she earn over the weekend?
#2: There are ____ kids in the cafeteria. ____ more kids come in for lunch. How many kids are in the cafeteria now?
#3: There are ____ children playing in the park. ____ children had to go home. How many children were left playing in the park?
#4: There are ____ children going to the water park. It costs ____ dollars per person. How much money will it cost for all the children?
#5: There are ____ donuts. ____ donuts fit in a box. How many boxes will be needed for all the donuts?
#6: There are ____ children in P.E. class. The teacher wants to make ____ teams with the same number of kids on each team. How many children can she put on each team?

of February and first week of March. Each problem set contained six word problems that tested student ability to interpret information in the question, identify the correct method, and move forward with the operation. The two CGI problem sets are recreated on p.248 of the Appendix, according to the same format that students received. The first set focuses on addition, subtraction, and multiplication, while the second set emphasizes fractions and division. The numerical values on all but the last question of the second set were available to change from student to student. These values were adjusted at the time of the interview to increase or decrease the difficulty of the questions based on interpreted student understanding and evaluation of concept knowledge.

Table 5
Second CGI Problem Set

#1: ____ children want to share ____ donuts so that everyone gets the same amount. How much can each child have?
#2: There are ____ chocolate brownies at Nina's party. ____ children want to share the brownies so that everyone gets to eat the same amount of brownies. How much can each child have?
#3: Robin went to a party where each person ate ____ of a pizza. If ____ people ate pizza, how many pizzas were there in all so that they each got to eat ____ of a pizza and there were no leftover pieces?
#4: Okhee has a snowcone machine. It takes ____ of a cup of ice to make a snowcone. How many snowcones can Okhee make with ____ cups of ice?
#5: Jorge and Darren are eating brownies that are the same size. Jorge cut his brownie into 3 equal pieces and ate 1 piece. Darren cut his brownie into 12 equal pieces. He wants to each exactly as much brownie as Jorge. Color in the amount of brownie Daren should eat, so that his share is equal to Jorge's share. (Here, two images were provided, where Jorge's was already split up and shaded, and Darren's was already split up).
#6: Jane says that if 6 people are sharing 10 cookies each person gets 1 and 2/3 cookies. John says that each person should get 1 and 4/6 cookies. Who is right? Can they both be right?

Task-based problems were not provided to all sixteen fourth grade students, but these students still impacted the study by providing interview responses and opinions that helped describe their perceived transition experience. Each student was interviewed once, and the CGI students were interviewed twice. Some interview questions were related to the shifting structure in the fourth grade classroom. These responses provided rich, precise conclusions about some of these students' processes, as well as their strengths and weaknesses. "In CGI, the emphasis is on what children can do rather than on what they cannot do" (Carpenter et al., 1996, p.14).

External Validity and Confidentiality

Previous studies have taken place in foreign cultures, and the Latin American context was particularly relevant at the beginning of this study, as evidenced in Chapter 2. Researchers have established young students as reliable and significant sources of information in these types of studies. The results can be used outside of the specific location the research took place, providing external validity. Any findings are useful to many different educational situations, where culture and location may be a factor. This IB world school provides upper level curriculum that is identical to participating countries' high school programs, and the private status of this school minimizes some of the cultural impact typical found in Central American institutions. The elementary classes were created to prepare students for an IB program. This consistency shows the value of this study in terms of education around the world, and the results should be processed and given merit from the qualitative methods. There was no experiment stage and similarly no risk involved in this study for any participants. Individual discussions, opinions, and exercise results were kept confidential, so students need not worry about their comments affecting their overall grade, performance, or teacher perception. At the end of the study, generalized results were shared with the staff to assist in improving the transition. All documents providing consent and permission allowing observations, pictures, and interviews for the study are listed on p.236 in the Appendix. Confidentiality was provided when the school authorized the project (Appendix, p.236), and the proposal was IRB certified (Appendix, p.232). The intended benefit of this study is to improve problem areas that arise in the transition from the Montessori style to the traditional style in a mathematical context for this school alone, but the results can be used for other education models in various countries and cultures. There was enough researched evidence to support the impact of these aspects on students' experiences going through this transition, and detailed recommendations for improvement are available in Chapter 5.

IV. DATA RESULTS

Introduction

The main purpose of this study was to investigate the significance of three aspects of the transition from a Montessori style to a non-Montessori style (hypothesized as direct instruction) that may affect students' problem-solving skills in mathematics. Administration at the school where this study took place identified a struggle during the transitional stage between third and fourth grade, and the goal was to assist in finding improvements for students during this period in their education. This study used qualitative methods to analyze classroom observations, teacher interviews, student interviews, administrative conversations, photographs, and CGI exercises (Heinemann, 2015). Chapter 4 is organized into three sections—Montessori third grade, non-Montessori fourth grade, and the results of the CGI assessments. Together, these sections address the following research questions:

1. To what teaching practices and learning opportunities are third and fourth grade students exposed? To what extent are these practices and learning opportunities related to the Montessori approach?
2. How are three particular aspects of current teaching practices and learning opportunities in fourth grade mathematics perceived by students and teachers compared to previous exposure in the Montessori method?
3. How, and to what extent, does changing the teaching practices and learning opportunities affect the problem-solving strategies of students?

The following sections are organized to present the information covering each of the targeted transitional aspects. The first section addresses third grade and the second section addresses fourth grade. The third section analyzes the CGI results and discusses student thinking and understanding. The first two sections discuss observations, student comments and opinions, teacher perspectives, and a brief summary of the findings. There are many differentiating aspects between the two classroom styles, but this dissertation focuses on three that were hypothesized to have the most influence: 1) the structure and discovery methods, 2) the use of materials versus handwriting, and 3) the shifting roles of students and teachers.

Initial Comments

At the beginning of the school year, five members of administration and faculty offered their opinions regarding why there was a struggle from third to fourth grade. These comments provide insight for teachers' thoughts toward the study at the beginning of the school year. The full dialogue by all five people is available on p.252 of the Appendix. Their hypotheses were then considered when making suggestions after the study concluded.

Ms. Julia is an experienced Montessori teacher, while Ms. Karla is new to the Montessori program, but has a lengthy teaching background. Ms. Verena develops integrated curriculum for fourth grade while acting as dean of the school, and Ms. Vicky has taught in elementary grades at this school for longer than any other active teacher. Mr. Laird is in his third year as head of the school, following administrative positions in other international schools. Each person was asked the following question: "In your opinion, what could be the biggest reason that students have a difficult time transitioning out of third and into fourth grade at this school?" Mr. Laird believed the lack of manipulatives moving into fourth grade could be a very influencing aspect because "those Montessori materials really support the students in reaching and understanding those concepts…because they're seeing it visually." He also suggested a possible pacing issue for higher and lower performing students. Both Montessori teachers also believed the materials were probably the biggest influence in the students' struggling transition. Ms. Julia explained that "after working…everything with manipulative(s), they need to come to this process that is completely abstract and they need to understand things with no materials [in fourth grade]." When creating the curriculum for the fourth grade classroom, Ms. Verena believed the challenge was to establish the effective amount of repetition for incoming students. "You want to develop the math muscle," she said. "[Last year] was too much repetition, and too many concepts that were similar." She identified the difficulty of implementing the correct level of instruction and differentiation required for the class, and this was a direct reflection of her new role of as a fourth grade teacher. Ms. Vicky believed that "the biggest problem we have is going from the Montessori over to whatever we're going to use…because over in lower [elementary] there are certain standards that are not being met with the Montessori materials." She perceived a lack of focus on problem-solving strategies, and Ms. Julia and Ms. Karla reiterated the focus on calculation and fundamentals over improving real mathematical problem solving. "In general, Montessori method is always trying to encourage them to problem solve…so [we need to] give them the strategies to solve those problems," Ms. Julia said. "But that's something that we need to train with them."

Third Grade: Alignment to the Montessori System

Self-Paced, Eventual Learning

Observations

The third grade classrooms followed the Montessori format, but also maintained scheduled blocks of time allotted for certain subjects, lunch, meetings, etc. First, second, and third grade students were combined into two separate classrooms with either Ms. Julia or Ms. Karla as their lead teacher, and both classrooms were found to be nearly identical in structure. This format provided the third grade students the opportunity to work with other children younger than themselves. Students arrived to class at 8:00 AM, but instead of focusing on academics, they began the school day by discussing announcements, personal activities, classroom job assignments, upcoming events, etc., as shown in Figure 1. This generally lasted for thirty minutes before students were instructed to take out their materials and begin working.

Figure 1. Students sit in a group for community meeting at the start of each day.

The students used the next three hours for exploratory learning. The schedule did not provide an exclusive mathematics time period, but rather this time was allotted for students to complete any subject or assignment that they prioritized, including mathematics. The classroom was not focused by age or grade, but instead observations showed that students progressed according to individual ability. After lunch and recess the students focused on reading, Spanish, and cleanup, placing the emphasis on mathematical learning in the exploratory morning block.

During the morning exploratory period, teachers rotated around the room, looking for students that needed extra guidance, more assistance, or new assignments. In eight of the ten observations, the teacher circled the classroom, and all but one observation showed students working in groups or individually with little direct instruction from the teacher. The teachers also provided a brief lesson to selected students once per week, an addition not commonly found in Montessori programs. The lessons covered all subjects for these students on scheduled days that kept the students progressing in the curriculum. For example, both Montessori teachers agreed to use Mondays for brief mathematics lessons to small groups of students at a time, illustrated in Figure 2. During this time, the teachers taught a new concept to students they believed were ready. The lesson was always brief, never lasting more than half an hour. Observations showed that the majority of the classes did not contain an extended lesson for students, but instead were limited of thirty minutes out of the fifteen total work cycle hours available during the week.

Figure 2. Both Karla and Julia take students aside for a brief mathematics lesson.

Teachers assigned written homework specifically for third grade students to turn in each Wednesday. Students said they were often not assigned homework in past school years. Every Wednesday all students are released at 1:45 to give students time for extracurricular activities and teachers time for development meetings. During these Wednesday mathematics classes, the morning meeting was skipped and replaced with a homework review. Students shared their solutions and strategies to the rest of the class, leading with minimal assistance from the Montessori teacher. The homework review was an opportunity provided to solely third grade students, as shown in Figure 3

Figure 3. All third grade students group together for an exclusive review session.

Outside of the special Wednesday schedule, their weekly routine allowed students to focus on schoolwork without the distraction of inconsistency. The classroom continued to be focused yet unstructured, often with some quiet ambient music playing in the background. Students worked quietly while teachers cycled around the room looking for ways to push students further by supporting what they were learning, keeping them on track, and making careful observations. Throughout every observation, teacher and peer encouragement as well as positive relationships pervaded the classroom, both of which are integral to the Montessori program.

The time was not completely unstructured, however. Minor assessments, or follow-ups, were assigned roughly once each week wherein teachers reviewed each student's progress with manipulatives and course material. The follow-ups, which acted as an evaluation of the students' proficiency, required students to perform the practiced task in front of the teacher. Figure 4 and Figure 5 provide examples of follow-ups.

Figure 4. Students perform follow-up tasks in front of the teacher for assessment.

In Figure 4, the teacher is also at the table, out of view of the camera. The teacher provides the instructions and comments while the student works to portray their conceptual understanding. The student uses beads and tiles to demonstrate particular arithmetic facts, and must represent these quickly so the teacher knows they are familiar with the materials from previous assignments. In Figure 5, students are expected to replicate the images on the cards in front of them, and then continually substitute and rearrange the black and white beads in order to create a new strand to match the original. The teacher can choose their amount of involvement given the student's level. Follow-ups are delegated only when students are comfortable to move forward.

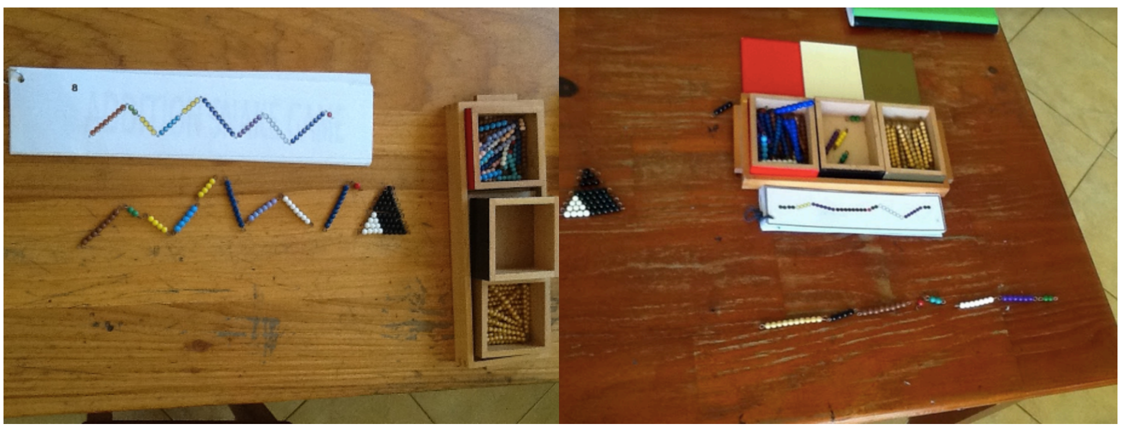

Figure 5. This is another example of a follow-up task given to students for closure.

Students were given a certain number of mathematics activities to complete each week to prepare them for the follow-up tasks. While students could choose the amount of time dedicated to each subject, the follow-ups kept them focused. Seven out of ten observations showed that teachers allowed students to pace themselves through assignments, relying on follow-ups as the incentive for students to continue working—deadlines were more present than a typical Montessori classroom. This tendency differs from a typical Montessori style that assumes students will progress of their own accord. Each observation also showed examples of students working through activities without a certain set of problems they were expected to cover. Instead, the students could create and work through many different questions that required them to create the problems themselves. This structure represents the freedom that students are expected to glean from a Montessori program. With the teacher away from the table, Figure 6 provides an example of students' autonomy as they progress through an open-ended board that allows for exploration of any size problem they want to attempt.

Figure 6. Students first create the problem on this activity board before solving it.

<u>*Student Perceptions*</u>

Based on student remarks, mathematics homework was consistent but not overwhelming. Students described the workload as manageable and not stressful. "Yeah, I've got homework," one student said. "But it's just normal. Like we have to do Spanish and Math and English." Another student agreed that it was not a large load, "because we do different types of homework. So there’s some math homework in there and there’s other types, there’s Spanish and there’s English homework, also known as Reading.” Students did not respond negatively to questions about their workload. Instead of expecting their workload to change, students provided straightforward statements about their strategies to keep up. “It just feels different because it’s hard work,” said another student. “ I don’t know. There’s so [many] lessons that you just have to get quick at.” Mathematics homework, such as worksheets practicing the previous topic, was usually assigned on Wednesdays and was required to be completed outside of class. The interview questions were intended to gauge students’ feelings about the perceived workload change in fourth grade when compared to third grade, including whether students felt overwhelmed or rushed. Homework is generally not stressed in a Montessori system because teachers obtain the most information about student learning from the discovery that occurs in the classroom.

Third grade students expressed various levels of concern about follow-up assessments. The intention of follow-ups was to provide teachers a method to evaluate their students in a heavily differentiated classroom. Some of the comments from third grade students suggested the follow-up implementation seemed to unintentionally create a competitive environment. Students also admitted that they sometimes had trouble performing tasks in a timely manner and occasionally felt like they were falling behind. The following excerpts of dialogue provide insight into the degree to which student focus centered on staying current with their assigned follow-ups, which teachers suggested as weekly goals. These comments also show that the deadlines are flexible, and the goal is more “big picture” than intended to help manage daily schedules.

Z: Okay, do you ever have a lot of work you have to do?

M: Hmm, yeah. I mean, it's fine. It's not too much, it's not too little.

Z: How does it work? How do you turn things in?

M: So, we would get a bunch of different lessons and, so we would get one lesson and then if you want to get another lesson, the teacher would tell you [that] you need to get another lesson, so you would have to finish the work, get another lesson, finish the work, get another lesson. So like, it can be hard sometimes, and sometimes it can be not the hardest. Like, if you were doing, let's say, something that you can't really do in one day, she would give you like, two weeks, or something like that, and if you don't finish in those two weeks you would be a lesson behind.

Z: And then you need to catch up?

M: Yeah.

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Z: I heard her [the teacher] talking about how you had to get so much work in by the end of the trimester, right?

H: Yeah, you need to finish all your follow-up works.

Z: Okay, is that a lot?

H: Well, I only have, like, three.

Z: Like three left?

H: Yeah.

Z: Oh okay. So everyone's got a certain amount they've got to get done. You're just ahead of the game?

H: Yeah. Well, I don't…well, it probably is normal to get three. It's kind of good to get three, but I've also missed some follow-ups. So I'm probably going to end up with a normal amount.

Z: Does the scheduling, like the follow-ups and all of that, ever get overwhelming? Do you know what I mean by overwhelming?

H: Yeah, like, you have so many?

Z: Right. Does that ever happen?

H: Yeah it's happened before. You just need to have, like, a day where you just do follow-ups. Or like three days where you just do follow-ups.

Z: Is there a lot of work that you have to do? I heard her [the teacher] talking about what you have to get done before the end of the—

W: I don't have a lot of lessons.

Z: Okay, so you don't get stressed out about it or anything?

W: No.

Z: Good, good. Is that normal? That you don't get stressed out the whole trimester?

W: Uh…I did have, I did have like ten lessons but I had to finish them all before the break.

These third grade students described the follow-up procedure as primarily focusing on staying punctual with assignments, which potentially loses value in students' academic goals. The comments reflected students' need to stay on schedule and complete the given assignments in a timely manner. While the teachers may have initially required follow-ups to provide some structure to the classroom, the students' appeared to overly focus on time maximization and pacing. However, the students did not speak negatively in their responses, but instead approached the workload as the reality they faced in the classroom. When pushed to determine if they felt stressed or not, many students commented the workload seemed "normal," "good," or "fine." Based on the literature, this was a divergence from the Montessori style in perhaps unintentionally placing more focus on task-completion rather than self-improvement (Murray, 2011). Research shows elementary school is the best equipped for self-motivation and self-paced programs to thrive and succeed (Schoen, 1976).

Teacher Perceptions

Research has shown freedom is important in the Montessori program (Montessori, 1964). Teachers were observed allowing this freedom in their classrooms. "They have three freedoms," Ms. Julia said. "The freedom to choose which job they want to do, the freedom to choose where they want to sit down, and the freedom to choose with who they want to work." However, the teachers still placed limited structure within the parameters of the free environment, such as the brief weekly mathematics lessons. "It's a math check-in every Monday with a new lesson to move them up a little bit farther with the curriculum," Ms. Julia said. "But they can keep working on their math skills depending how they consider what they need to practice more or what they really like. Like math, they can keep working on math, but we do math check-in new class lessons every Monday." Teachers kept these lessons short to allow students time to discover concepts without teacher supervision, and to allow teachers time to move around the room to assist other students.

When teachers included lessons in their Montessori classrooms, they diverted from the typical Montessori system by providing a structured schedule for students. The lessons were based on ability rather than age or grade, requiring students to work on various levels of mathematics material each day. This concept of differentiation is fundamental to Montessori programs. “The biggest strength is the differentiation,” Ms. Karla said. “I feel that you can actually adapt to all the kids at their different stages.” The Montessori learning environment is, by definition, open-ended and discovery-focused, as education “is a natural process which develops spontaneously in the human being” (Montessori, 1952, p.7). Ms. Julia and Ms. Karla believed that building this independence in their students took time, and therefore as teachers they needed to supply a structured work regimen for all Montessori students during this process. Students were required to complete three subject assignments each day, called “works”, including a follow-up (for all subjects, not limited to mathematics).

> Julia: Yeah, they need to complete three works every morning, because the work cycle is so open that we need to put any limits about—
>
> Z: Three of anything.
>
> Julia: Yeah, three of anything. So they can choose what to do. The thing is that every time they receive a lesson they have a follow-up. So mostly, when they are doing works, they are doing follow-ups from the lessons that they’ve received.
>
> Z: That’s different than the three things.

Julia: No, the follow-ups could be considered as a work, but if you are really good with your follow-ups maybe you just work with one follow-up and then you can choose your two favorite materials and do other, two different works.

Z: And a follow-up is when you guys have already done something and then it's like they're going to do it on their own now.

Julia: Yes, so when we do a lesson we always deliver some follow-up to keep doing practicing during the week.

Z: So that means there's like fifteen works a week.

Julia: Yes, yes. Like follow-ups will be like five per week, but then means like fifteen works per week, yeah.

Z: How many of those have to be math? Is there a certain amount that have to be math?

Julia: For sure it will be one related with the lesson that they did, and then we always try them to do, at least you need to do every day is one math, one language, and the third one is the one that they can pick wherever they want to do.

Z: So you push for like, five.

Julia: Yes, yes. Yes.

Critics of the Montessori method believe a weakness of the program is educators' assumption that students are self-motivated (Lee & Ginsburg, 2009). While the Montessori method relies on students' internal drive and intrinsic motivation, follow-ups were assigned in the studied classrooms as checkpoints to encourage students to work.

These follow-ups usually happened at the end of the week but time periods were adjusted based on student achievement. Teachers intended follow-ups as a closure for the latest topics each student had covered, with teachers present to evaluate student understanding. Ms. Karla emphasized the number of works students needed to complete, rather than the amount of topics students needed to cover. Ms. Julia and Ms. Karla said they would adjust a student's schedule if the student needed to repeat a follow-up. Students who struggled or needed extra practice were therefore given that opportunity. Ms. Karla explained if students did not understand a concept, the teachers did not allow them to move forward until they had practiced and mastered the difficult concept, and that "in order to reach the next lesson they need to have completed the follow-up for the prior lesson." The mini-lesson that teachers provided was considered the introduction to each topic, while the follow-up served as the conclusion. Teachers selected which students would participate in a lecture-based instruction aided by manipulatives. When Ms. Julia was asked about the overall schedule for mathematics, she stressed the importance of the adjustment periods by explaining that if students are not ready to move forward, they do not include that particular student in the mathematics lesson:

> Our normal day, when we are talking about math, is we deliver lessons based by level and not by age or grade, so I will group the kids based on their level and then they receive the lesson. In Montessori the lessons are never longer than twenty minutes or something like that, and every lesson you present them a new material which is related with a new content, and then you give them a follow-up and they can work over their follow-up for a week until the next lesson that will be next Monday. Then they receive

the next lesson according to the curriculum if they are ready and they've been working enough with the material that you present them the week before.

<ins>*Summary*</ins>

Student assessment is an important characteristic of any classroom environment. The third grade teachers in this study said they needed to establish an assessment method that would be appropriate for a differentiated program. These follow-up activities provided students the opportunity to showcase their learning by working through their new skills under teacher supervision. The follow-ups emphasized the use of manipulatives, but also occasionally included written work. The next section covers more about the balance between manipulatives and handwritten work. Teachers used manipulatives as the teaching foundation, and used the follow-up results to determine the level of student understanding. Students developed a predictable but flexible daily schedule. The teachers also discussed the advantages of including a small lesson each week. Observations consistently portrayed a Montessori environment that contained structure, but that also kept the open atmosphere dictated by the philosophy. Teachers monitored the classroom while students worked on differentiated material according to individual need, and students were challenged and monitored based on their performance on the assigned activities.

Use of Manipulatives and Objects

<ins>*Observations*</ins>

The most defining characteristic of a standard Montessori classroom is the use of manipulatives as primary learning materials. A Montessori system allows students to utilize tangible objects, with limited or no handwriting necessary (Rosanova, 2003; Montessori, 1964). Gesturing, discussion, and visualization are the main forms of learning in the Montessori system (Fang, 2008). The Montessori classroom in this study was filled with mathematics manipulatives, as shown in Figure 7. Students were provided with a wide variety of materials, and often the activities required two or more students to participate. Observations showed that materials were the primary source of information, but also found that handwriting methods were included.

Figure 7. One of the large shelves of materials that students have to work with.

Students worked on assignments with manipulatives, but once they concluded the activity, copied the problems and solutions in written form in notebooks. Even though students' knowledge did not come from writing the problems, teachers implemented the handwriting requirement to give students the opportunity to experience mathematics in the written form seen in future grades. This handwriting stage strictly focused on copying, not problem solving. In Figure 8, students use this combination of materials and notebooks to work on mathematics problems. In eight out of ten observations, third grade students used manipulatives as the primary source of learning new concepts. In the other two observations, third grade students were combined in groups for Wednesday homework reviews that focused strictly on handwritten practice. For example, students used whiteboards to practice handwriting addition during one review session.

Figure 8. Addition tiles, multiplication facts, BINGO cards, and bead chains.

Observations showed that descriptions, gesturing, materials, and figures were the main forms of learning in the Montessori classrooms. Manipulatives were imperative to students' learning processes, and several examples of these mathematics materials are shown in Figure 9. On the top left of Figure 9, students fill out an operation fact table using only tiles, similar to completing a puzzle. Students match the rows and columns for addition or multiplication and fill the intersecting square with a board piece containing the correct solution. The top right of Figure 9 shows a BINGO game board testing single-digit multiplication fact recollection. The bottom left of Figure 9 shows an example of transferring manipulative knowledge from counting beads into handwritten form, specifically for third grade students. The student performs an indicated operation with the beads, and then writes down the operation fact in algorithmic form in a notebook. The bottom right of Figure 9 shows an example of an activity focused on place-value, where colored tiles represent the tens, hundreds, and thousands values.

Figure 9. These are some examples of Montessori materials in use around the room.

While materials were the main source of knowledge in these lessons, handwritten work was also a method students were expected to eventually learn. Observations showed that there were various forms of Montessori materials in the classroom, and students could hold, move, place, and remove these objects during mathematics activities. According to the literature in Chapter 2, the use of manipulatives is a fundamental part of the Montessori system. Figures 10 and 11 provide more examples of the classroom activities to further convey both the environment of the classroom and the style of mathematics materials available to students. On the left side of Figure 10, the strip board activity permits students to place a blue bar of set length and a red bar of set length to model an addition fact on the board. This provides students the opportunity to learn addition facts, while also visualizing the commutative property by creating different addition problems. On the right side of Figure 11, students use bead chains to represent counting by multiples and visualizing area or volume.

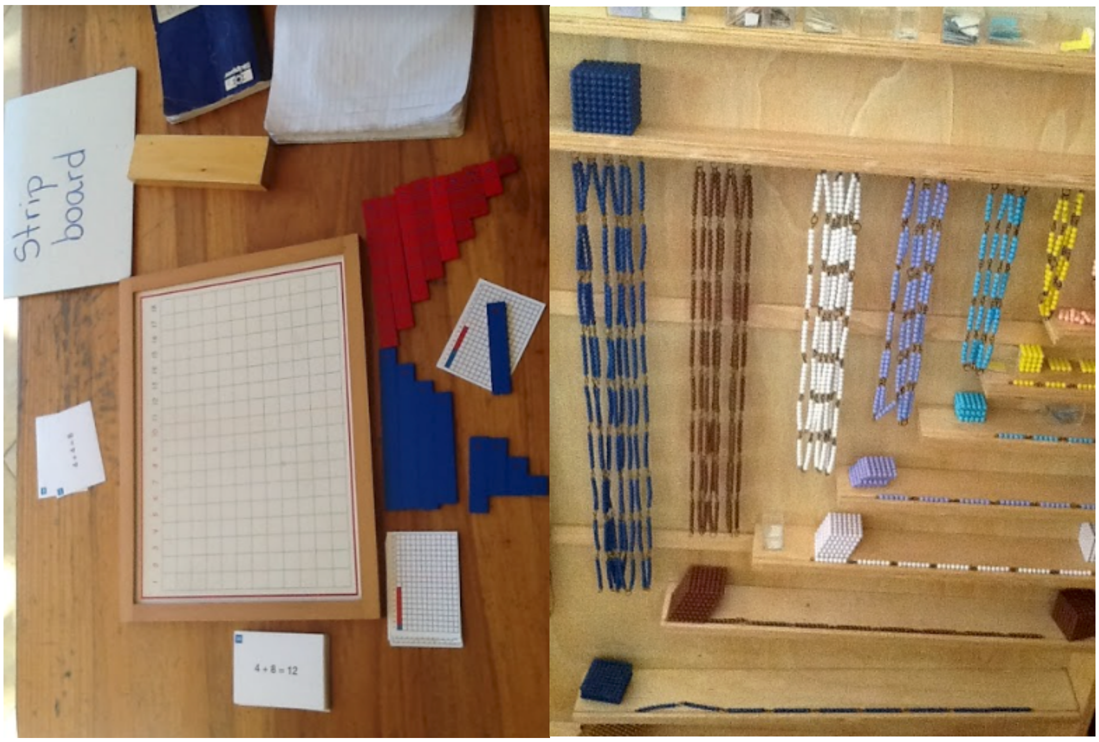

Figure 10. The addition facts board and the multiples bead chains are commonly used.

Each colored string of beads is associated with both a two-dimensional square and a three-dimensional cube with similar dimensions and colors. On the left side of Figure 11, a student works through addition facts using tiles to match problems to accurate solutions without using paper and pencil. On the right side of Figure 11, students can use geometric figures to create basic shapes, such as circles, triangles, squares, and rectangles. Students are able to become familiar with these shapes without ever drawing them on paper.

Additionally, third grade students experienced consistent methods in other subjects. The students used visual representations for each subject before implementing handwriting methods. For example, one observation showed a matching game with science flashcards where students did not handwrite the statements. Another observation showed students working on a vocabulary assignment in which they drew and colored shapes above words in a sentence, such as red circles above nouns and blue rectangles above verbs.

Figure 11. *Addition fact tiles and geometry manipulatives.*

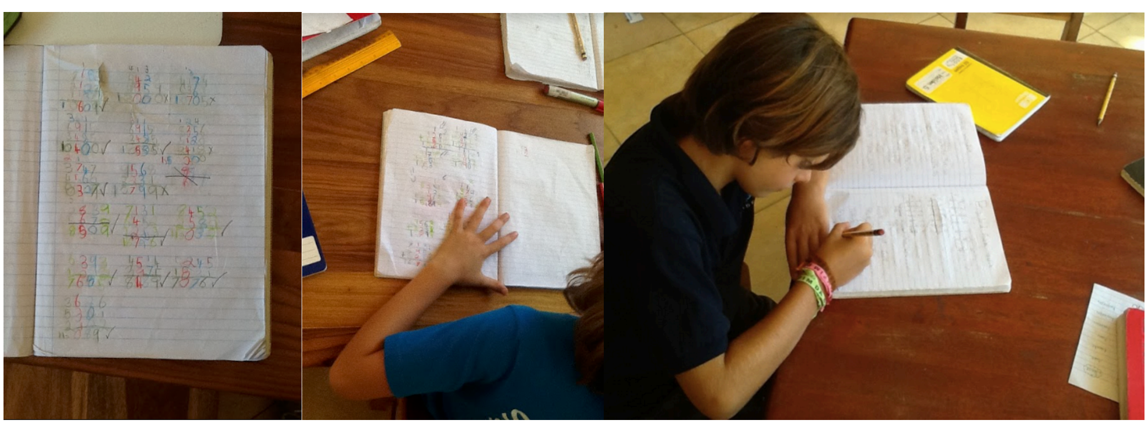

Figure 12. *One third grade student works on multi-digit addition with colors, then pencil.*

When asked about an equivalent mathematics example, teachers explained that before getting to the pencil-and-paper stage, students used colored markers or pencils, where color-coding depicted different place values, until they understood this concept enough to move on to non-colored pencil, as shown in Figure 12. It was vital that students experience a similar style across subjects, and this gradual process allowed full understanding at each stage. Figure 13 shows an example of one student working on this type of classwork, documenting their use of manipulatives with what they had copied down during and after the process. A middle step was included in this situation, in which the tiles, color-coding, and matching symbols in the notebook preceded handwriting as the end result. This was a concept reflected in many content areas, including mathematics. Third grade differed from first and second grade in that students eventually phased out manipulatives and used handwriting as the primary way to show evidence of their understanding.

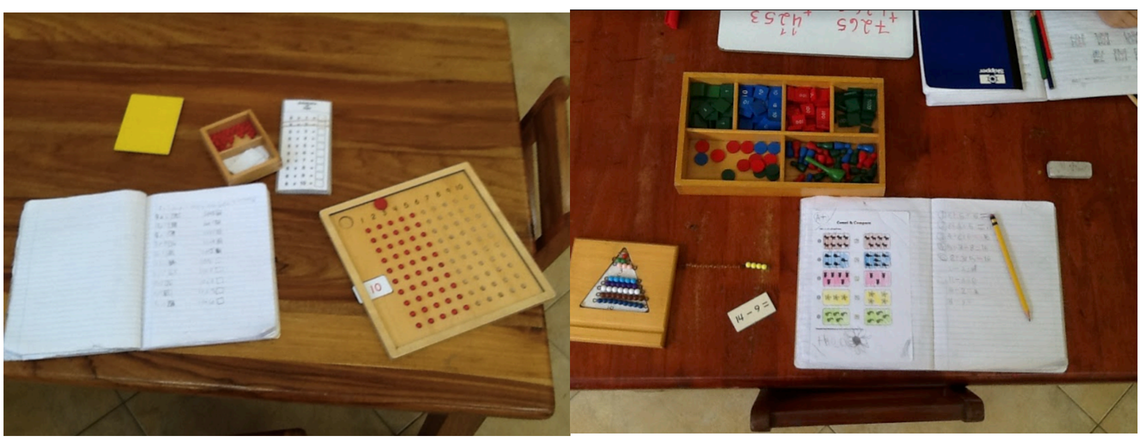

Figure 13. Grouping boards with objects, then colors, then algorithms are shown.

Figure 14 provides an example of the copied work that students write in their notebooks after using manipulatives. Observations showed that teachers emphasized writing methods as a goal solely for third grade students. Eventually, students were expected to perform their mathematical tasks by hand without the use of manipulatives. This divergence is another example of the classroom aligning differently from the pure Montessori system.

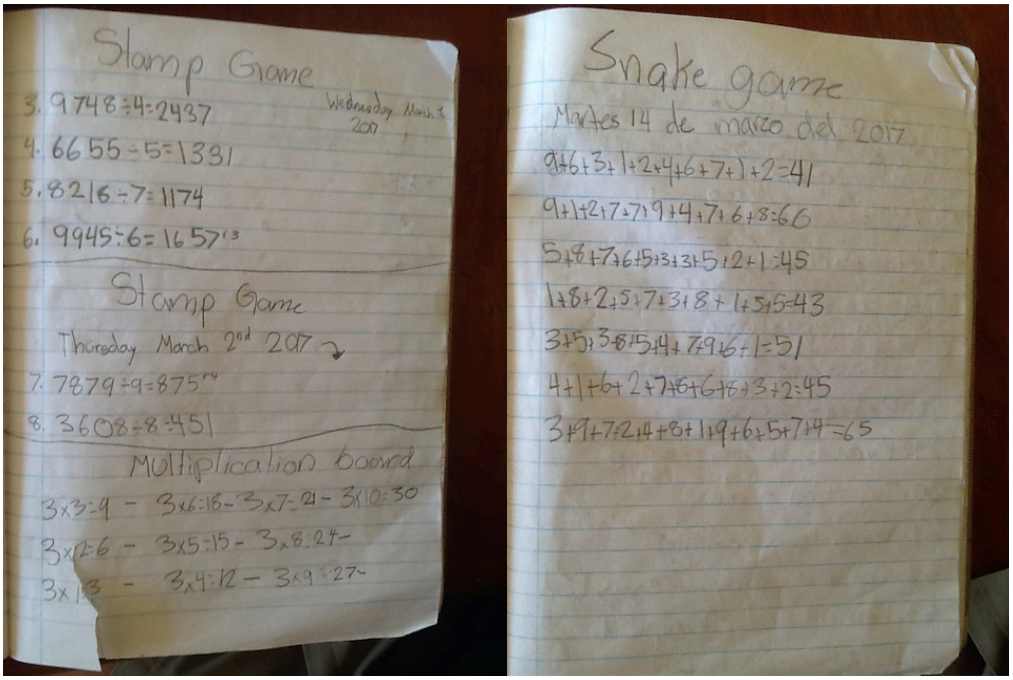

Figure 14. *Students document the problems they created/worked through on activities.*

Figures 15-17 portray the three different stages of material usage in the Montessori classroom. To show their concept mastery, third grade students phased out materials from their mathematical process. These images show students with manipulatives only (Figure 15), followed by a combination of manipulatives and handwritten work (Figure 16), and conclude with handwritten-only (Figure 17). These images exemplify what the teacher sees in the classroom as the day progresses, as they walk around the classroom guiding and monitoring student progress.

Figure 15. *Students use tiles and boards to practice basic operations with materials.*

On the left side of Figure 15, a student works at filling out a 12-by-12 addition chart by placing the correct addition solution tile at the intersection of each row and column. On the right side of Figure 15, students practice memorizing multiplication facts by playing a version of BINGO. They draw cards with given values and find whether they have matching multiplication pairs on their boards. Many students and teachers claimed this activity was a class favorite, and the teachers said the students benefited from the extra multiplication memorization practice.

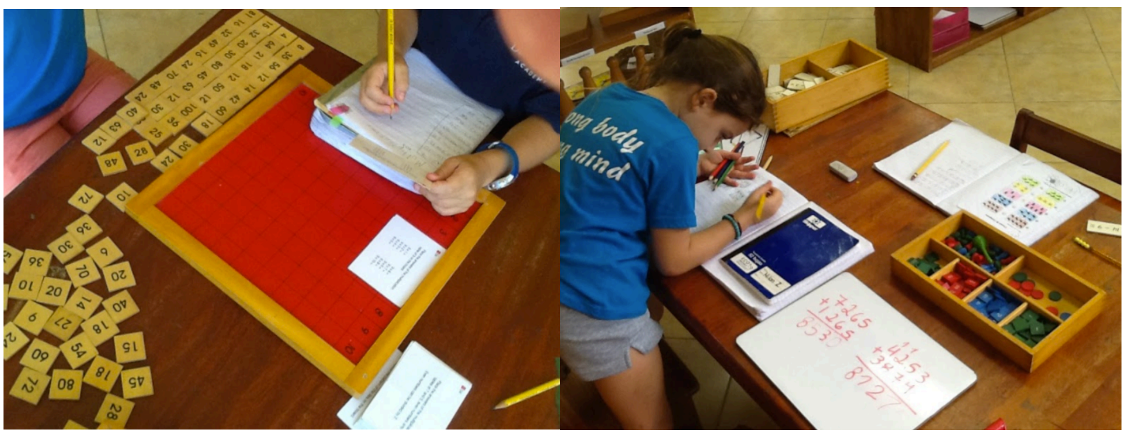

Figure 16. Students write the material down once they are comfortable with objects.

The left side of Figure 16 shows a student filling out a full multiplication board. Similar to the addition board in Figure 15, students place a tile in the row and column intersection that matches the multiplication fact. Next, students copy that fact onto notebooks, horizontally writing out problem, but without any real work shown—the building of knowledge came from manipulating the materials. The right side of Figure 16 shows a multi-digit addition activity, with different colored tiles representing place values. Once finished, the student copies the problem onto a notebook or whiteboard. The thinking process and work shown were represented in tile form prior to handwriting.

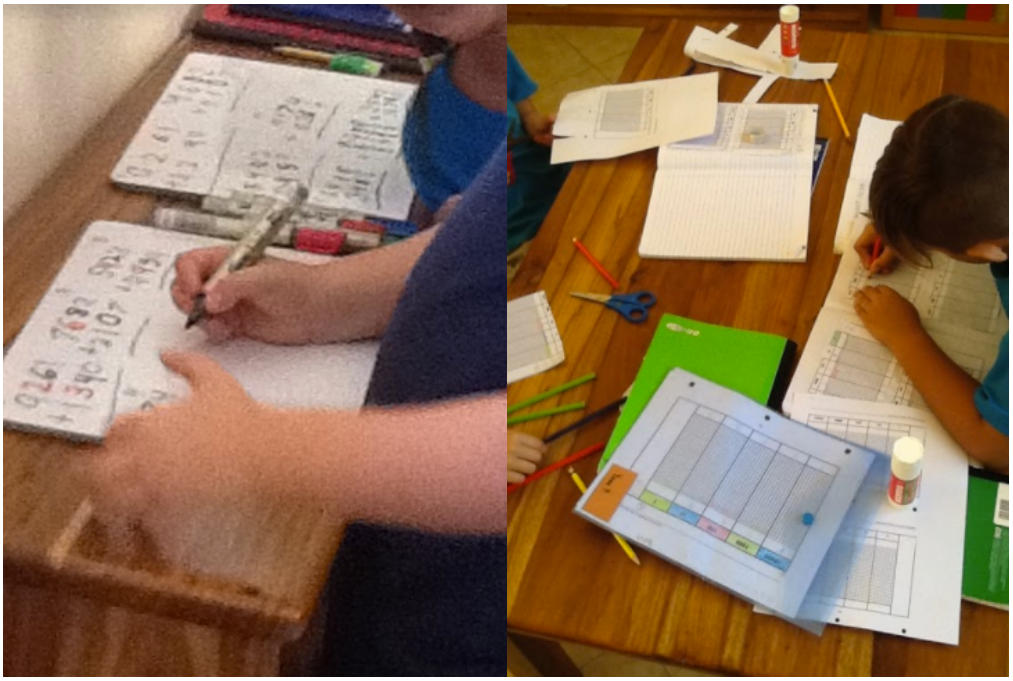

Figure 17. Students rely solely on handwriting (some color-codes for assistance).

The left side of Figure 17 shows an example of a third grade student using whiteboards and different colored markers to represent the different place values when adding or subtracting multi-digit whole numbers. The right side of Figure 17 shows a similar activity with paper and colored pencil. This second activity maintains the creativity involved in the color-coded process of writing, using colors and markers with whiteboards and grids as a direct substitute for regular pencils and notebook paper. In both examples, students derive information from handwriting when performing these tasks.

In third grade, the images shown in Figure 17 are the finished product of a lesson, as third grade students differentiated themselves from first and second grade students by prioritizing their transition to handwriting methods. Once the teacher believed students understood a concept through the follow-ups, they moved students to handwriting as the sole method of showing work. Students then proceeded without manipulatives before moving to a new concept and restarting the process once again. Observations provided classroom evidence that while teachers implemented the use of manipulatives for students correctly and effectively, their emphasis on students transitioning to handwriting methods was a divergence from a pure Montessori style. Figures 15-17 provide evidence that third grade students experienced other methods besides manipulatives to prepare for future grades. This made the use of materials only a portion of the mathematics curriculum rather than the entire experience as dictated by the literature. In Figure 17, a third grade student actively solves four-digit addition and subtraction problems through the handwriting process. In contrast, Figure 16 shows a student who has already performed the task through materials and is copying work into written form in a notebook, without performing the operation by hand. At this point, the student did not have the algorithmic knowledge to progress through the problem by hand.

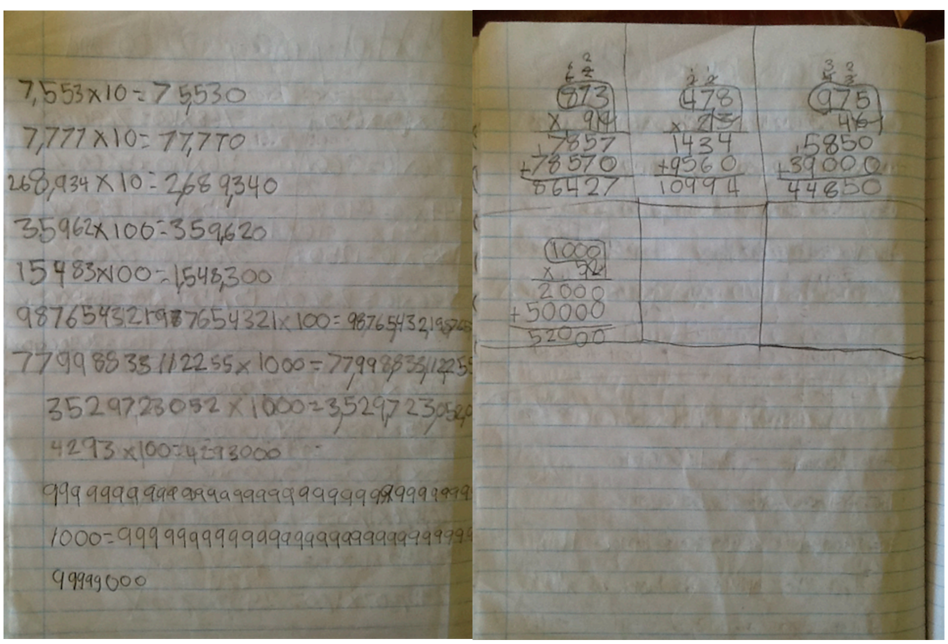

Figure 18. Students first copy down the process, before later working with the process.

The left side of Figure 18 shows further evidence of this process, in which students write down operations originally performed through materials. By comparison, the right side of

Figure 18 shows an example of a student working on multiplication problems through the handwriting process alone. Once students gain this ability, materials are no longer the main source of information and students can proceed algorithmically.

Figure 19. *Division boards are one of the last manipulatives that students see.*

There were particular differences in how the manipulatives were used at various stages. Some of the activities required students to perform physical representations of algorithms. For example, students replaced a collection of tiles at a certain point in their problems to show the process of grouping in addition, or perhaps carrying for different values places in multiplication problems. In other examples, there were no algorithmic representations, and the students were using the manipulatives as a memorization aid. For example, the snake game allowed students to learn counting by certain factors, and the multiplication or addition boards played more as a memory game than a tool for understanding.

Student Perceptions

Students were aware of the connection between the manipulative and handwriting processes, but some viewed the handwriting method as simply an extra step before being finished. For example, one student commented that he had no preference toward either method, but that “writing can be a lot shorter because all you need to really do is just do it with the beads really fast and then you can just write down the problem,” with extra emphasis on solving the problem with materials, not during the handwriting stage. Other students supported this thinking by describing their processes from manipulatives to handwriting, and often stated preference for the latter. In contrast, a different student showed his self-motivation by saying that he “liked the snake multiplication game better because it’s harder,” and preferred to challenge himself. However, some students appeared to have no choice because of their limitations in perceived mathematics ability, and used the materials as a fallback system quite often. The students were aware that differentiation continued to exist in the Montessori classroom. While some students were able to use handwriting or materials depending on the situation, some students relied more on materials. One student admitted she liked using the boards because it made mathematics operations easier, while another student needed the materials in order to count correctly toward a successful solution. The responses toward using materials were a mix of positive, negative, and apathetic responses. A few students shared that they believed manipulatives made the process take longer, describing it as extra work required to get the same end result, seemingly for no reason. “I prefer a lot more writing,” one third grade student said about looking forward to future school years. “Because when we graduate to fourth grade we’ll have to, there’s no nothing you can use with math, no symbols or nothing like that. Only books or papers for the problem." The responses to these questions varied dramatically across the third grade students.

Table 6
Comparing Stance on Mathematics to Learning Style Preference (3rd Grade)

Name	Stance on Mathematics	Learning Style Preference
Tamaya	Negative	Materials
Ruby	Negative	Materials
Ziggy	Medium	Handwriting
David	Positive	Handwriting
William	Positive	Handwriting
Holden	Positive	Materials
Marlo	Positive	Handwriting
Alena	Negative	Materials
Chloe	Medium	Handwriting
Cody	Medium	Materials
Solace	Positive	Materials
Callum	Positive	Handwriting

Table 6 organizes the specific thoughts from third grade student responses during interviews. Thoughts, phrases, statements, and physical gestures portrayed by each third grade student were used to determine that student's stance on mathematics (negative, medium, or positive) and their preference for learning mathematics (materials or handwriting). Two potential associations stood out from the table. First, the opinion toward handwriting versus material showed no overall classroom preference for either method. Second, the three students who had a negative stance on mathematics preferred the use of materials to handwriting methods.

Teacher Perceptions

The Montessori teachers were adamant about using the materials in the classroom a particular way. The teachers wanted the entirety of student knowledge to come directly from the use of manipulatives, but then required students to practice handwriting the process by writing the content algorithmically after completing with materials. "Of course they were using materials, and then they copy in their notebooks whatever the material is saying," Ms. Julia explained. "But the original thing is coming from manipulatives. They are not really understanding what they are doing on paper, so the next step will be teaching them to do it on paper with no manipulatives." The teachers' goal was to give students advanced exposure to the handwriting process, but only after using materials as the direct source of information, as Montessori-implemented instruction suggests. Teachers implemented this strategy in response to the identified issues of transitioning to fourth grade in previous school years. Ms. Julia and Ms. Karla emphasized the handwriting process at the end of each activity, to an extent that students were expected to perform the task completely without the use of manipulatives so they would be ready for future grades. "Well, they handwrite that, because they are copying from the math cards that we have for Montessori so they are going to copy, of course, the addition or whatever fact operation that they are doing, but they are using materials to get the answer," Ms. Julia said. "Thinking process is through materials, and it's in fourth grade when you show them the thinking process with paper and pencil." According to the literature, manipulatives are intended to give a focus point for students to build a knowledge base, and eventually transfer that knowledge from the concrete to the abstract (Carpenter et al., 1996; Uttal et al., 1997). Ms. Julia continued to explain this as the goal for future grades after they leave the Montessori system:

> Yeah, they repeat that but with no material, and the process that I saw that is beautiful. They been working a lot with materials, once you give them the key to do on paper and pencil, they are able to transfer 'ah, okay, so I understand this, that now I need to carry the units in here because I was doing the change with the beads, blah blah blah [sic].' So in fourth grade

> they then use materials, but they are going to repeat the same process we have been studying, just that no materials, with paper and pencil.

Manipulatives were available for all students in the Montessori classrooms to assist in their learning process. Observations and discussions showed that while first and second grade students may have experienced a more pure Montessori method, third grade students worked in a highly adapted Montessori environment, one that emphasized transference of knowledge from manipulatives to a paper-and-pencil method. “There is writing, but when they actually only do paper and pencil is at the end of third grade,” Ms. Karla clarified. “But they start with all materials. They write it up but after, like a follow-up of their activity once they have done it hands on.” Eventually, teachers removed materials once a third grade student had mastered a concept with the manipulatives, and the student was then required to process the problem entirely by formulas and handwriting. Once students reach fourth grade and beyond, they have already received exposure to performing mathematical processes without the use of the Montessori materials. “Hopefully it won’t [be a factor], because from what I’ve heard this hasn’t been worked on as much during past years,” Ms. Karla confirmed in her initial interview at the start of the year. “The paper-pencil, and the solving without the use of manipulatives, that’s like our specific goal this year. We’re working on that in order to prepare them to go to fourth grade where they don’t have all the materials and that’s like the biggest difference.” Ms. Vicky, the fourth grade teacher in this study with prior experience in Montessori classrooms at this school, said that she could tell when a student was no longer using the manipulatives as support, but were instead performing the tasks superficially until they were allowed to move on. “At first, they are very organized with their pieces," Ms. Vicky said. "As they are using the materials to think, they are very organized. But eventually you can see their process becomes messier, and that means they are not really using them anymore."

Summary

Observations and discussions with both students and teachers revealed that manipulatives were a focal point in the Montessori classroom. Students were mixed in their responses toward manipulatives as learning tools. Students who felt negatively toward mathematics admitted their preference of using materials over handwriting skills, but the class overall was mixed in their feelings toward manipulatives versus handwriting methods. All Montessori students, regardless of grade, had access to the same materials, but third grade students were pushed to move toward handwriting methods by the end of their activities more than the first or second grade students. Each activity culminated in the ability to perform the task by hand, until at the end of the school year, materials were no longer considered essential for student learning. Teachers emphasized handwriting in response to reports of difficulties transitioning out of the Montessori program. A later section will discuss some of the fourth grade students' thinking toward manipulatives to determine whether the strategies in the Montessori classroom worked as intended.

Hands-off Teacher Guide

Observations

In the structure of the classroom, teachers assigned students three "works" per day in the content area of their choice. Teachers suggested students prioritize mathematics as one of the content areas as often as possible. Students were responsible for their own priorities and pace when determining which content area to cover. Students then had the freedom to roam the classroom to find materials, assessments, partners, or anything else they decided was necessary to succeed during the designated three-hour time block, as shown in Figure 20. This process gave students power in the classroom, as the literature suggests should be the case in a Montessori system (Myren, 1995; Checkley, 2006; Montessori, 1964; Zimmerman, 2001). Teachers then adopted a guide role in the classroom to facilitate differentiated learning, and they kept students moving forward in the curriculum without taking the focus as a direct source of knowledge. Eight out of ten observations showed that the teachers assumed this role successfully, and nine out of ten observations showed students relying on classmates or working individually rather than approaching the teacher. Three out of ten observations showed teachers directing students to focus by preventing side-discussions and other distractions.

Figure 20. The free-flowing environment of Montessori is unique to typical structures.

According to the literature, environments that stresses student control lead to students and teachers working cohesively (Jacobs & Empson, 2015; Lockhorst et al., 2009; Silver, 2003). Observations showed evidence of personal relationships and open dialogue, which also followed suggested literature. Students were typically found working in pairs or groups when working with concrete objects. Observations showed students asking friends for help, but rarely seeking assistance from the teacher. Some of the activities in the room, such as the BINGO or memory card games, required at least two people to collaborate, as shown in Figure 21.

Figure 21. BINGO cards help students make a game of memorizing multiplication facts.

Third grade students seemed happy when helping other students, evidenced by their repeated requests to help other students, and their comments showed this process gave them a sense of pride. Teachers supported this initiative, reflected in their remarks in the following sections. Some examples of collaboration are provided in Figure 22.

Figure 22. Students work on the Snake Multiplication and an abacus-like manipulative.

The left side of Figure 22 shows a second grade student working on building multiplication patterns with third grade students. This group of students was observed working on this activity often, at times using the entire three-hour block. The right side of Figure 22 shows an example of one student assisting another student struggling with understanding the instructions of an activity. Figure 23 shows two third grade students mentoring younger students on their activities.

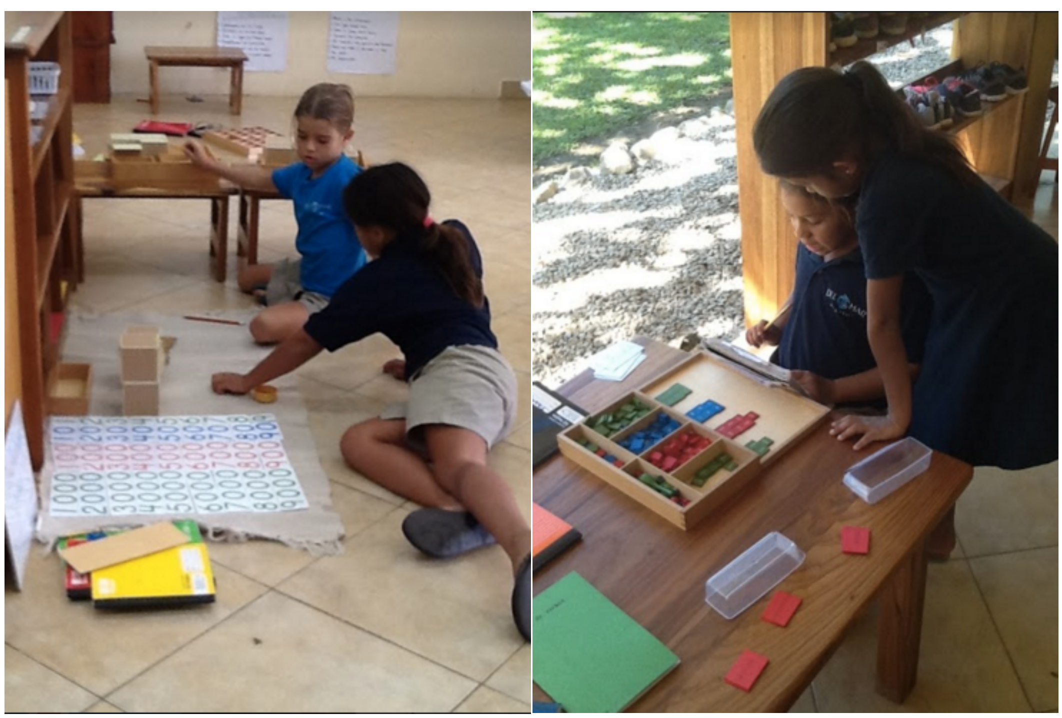

Figure 23. As students got more comfortable, younger kids often want help from older.

Students were encouraged to support each other, and their comfort with each other became evident quickly. In fact, students often approached the teacher to ask if they could teach their friends, even when their friends were from lower grades, and teachers usually supported this use of class time.

Observations also provided examples of the Montessori curriculum content. Curriculum design and implementation was not a focus of this study, but observations showed that the first half of the school year was devoted to addition and subtraction of large quantities and multi-digit values. While teachers taught addition and subtraction to third grade students as if students had some prior knowledge of the content, multiplication and division were not introduced until after addition and subtraction were completed. While multiplication and division were scheduled to receive equal emphasis in the curriculum, teachers admitted difficulties in managing this expectation because of the large quantity and time intensive nature of the activities for each subject. Occasionally, third and fourth grade teachers debated the quantity and quality of the exposure to division presented at the end of third grade. Figure 24 shows students working through the division portion of the curriculum in March (on the left side), which commonly lasted through May (on the right side). Students used manipulatives before handwriting their process, eventually learning long division before the end of school. However, not all of the activities provided the opportunity for students to learn the meaning behind the handwriting process they would later learn (for all operations, not limited to division). The initial copying phase was implemented for students to gain insight into what mathematics looks like in written form, but the meaning behind the symbols was often based on memorization and visual experience, rather than connecting the manipulative to the operation.

Figure 24. Students use division manipulatives before learning long division.

Montessori (1964) envisioned students as responsible for their own learning, and this self-pacing was evident in the classrooms observed for this study. Observations, such as Figure 24, provided evidence that curriculum details may have hampered student control in the classroom. The Montessori curriculum at this school was entirely focused on the four basic operations: addition, subtraction, multiplication, and division. The availability of fifty activities for each observation limited the extent to which differentiation was present in the classroom. For example, when the entire third grade class was working on subtraction activities, some students completed thirty to forty lessons while other students only managed fifteen to twenty lessons. However, teachers dedicated portions of the curriculum to each operation. Students were allowed to progress through these activities and topics at their own pace, but there were so many activities included within each overarching concept that even the highest performing students were often unable to reach the final activities or move past the four basic operations. The Montessori curriculum at this school may not matchup particularly well to other schools based on the restrictions to the curriculum. The teachers expressed fears that students had potential to explore more difficult concepts beyond the four main operations, but were hindered by the designed classroom structure. These ideas are discussed with the Montessori teachers in another section of Chapter 4.

Student Perceptions

In interviews, third grade students discussed whether they preferred receiving help from classmates, from teachers, or persevering individually. Most students had positive reactions to working with others, and many preferred this strategy to individual work. Many students expressed their tendencies to work with others or work alone, but generally did not mention a desire to approach the teachers. These comments appeared to support Ms. Julia's claim that the teachers were working on building students' independence, and also add to the existing literature that finds students can take ownership roles within their classrooms at elementary ages (Lunenburg, 2011; Kazemi & Stipek, 2001; Wu, 2009). "Montessori realized that appropriate adult intervention is needed at certain times but should decrease steadily as children learn how to do things for themselves" (Gutek, 2004, p.48).

Students had freedom when covering mathematics concepts, and in certain extreme cases, could choose to dedicate an entire day to mathematics and then neglect those assignments for the remainder of the week. Most students criticized this approach as a weak strategy, instead preferring to consistently explore a portion of each subject every day. "In our work cycle it's better to do like one math, and one science, and then another thing," one student said. "Because if you just do math all day, all the rest of the stuff you're not going to be that good at, and then if you're just good at math, then all the other stuff, like English, you have to learn your English or else you can't do math that well." The third grade students appeared cognizant enough to balance their workload, to connect concepts across the disciplines, to maintain awareness of their weaknesses, and to emphasize certain subjects based on individual goals. Very few students admitted to asking teachers for help, which supported students' responsibility for their own knowledge base. "I just kind of put the answer that I think it is, and then check it," one student said. Another student added, "Sometimes I do it on my own, and sometimes I ask my friends for help," emphasizing that he would rather ask a classmate than approach the teacher. Another student also confessed that he purposely avoided the teacher, and worked with friends every day. However, students were still aware of the extent of assistance they received from their Montessori teachers when they did interact. As third grade students, they understood the teacher's role as someone to provide ideas for the process, but not as someone to give the solution directly. "When I asked help for let's say, a multiplication, she'll give me tricks for it, or just what to do in it," one student said when describing the amount of help the teacher provided. "Say I don't understand a word...she would almost, like, give me a lesson on that word...so I learn it." This student description of teacher assistance was often not the same in the fourth grade classroom, as another section of Chapter 4.

Teacher Perceptions

Teachers fully supported the roles of students as collaborators and sources of knowledge. At the beginning of the school year, Ms. Karla described looking forward to the discovery process happening around the classroom. "I think [third grade students] will tend to be together but also they are role models for the rest, so what I've heard is that they help out a lot, especially when there's something that somebody else doesn't understand. They already know how to use the materials and know how things work so they go and help out a lot with smaller kids." Third grade students had become leaders in the classroom from the first day of the school year, and they had to deal with more difficult material and preparation for showing their work while also leading by example for younger students. "[Third grade students] can be working even in the same spot next to them because some of them are really good friends between second and third graders, so that really doesn't affect separation," Ms. Karla explained about students working together. "Actually they enjoy it. Sometimes they have new things that they have learned, and they come up and say 'Can I teach this to my friend that is in second grade?' and I go 'Sure! Yeah!'" At the same time, teachers were aware of how they and students needed to adjust to their roles in the classroom, with students taking control of the experience. Ms. Julia and Ms. Karla, respectively, described the balance between their mini-lessons and their guiding role by explaining their ideas on differentiation and self-pacing.

> J: The role of the teacher is more a guide more than delivering lessons for everybody no matter their age or their level. So for example, if I realize that someone is not ready to work on multiplication because they didn't work enough in their addition skills, you are not going to deliver that lesson because you want to make sure that they really accomplish all the stuff before that. I think that's different from traditional, that usually you just deliver lessons because it's on a book and it's on a curriculum and here you respect the path of the student.
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> K: I think that with the Montessori approach I'm actually looking and observing and going straight to the needs of each one of the children and not just to an entire group and, like if someone is having difficulty I can work directly with that person and if somebody can do it by themselves I can challenge them to do other things that are higher level, instead of everybody at the same pace which is what actually happens in traditional.

The literature suggests that success in a Montessori environment is contingent upon teachers fully embracing their roles, as "the most effective teacher moves cannot be preplanned. Instead, they must occur in response to a child's specific actions or ideas" (Jacobs & Ambrose, 2008, p.271). For new Montessori teachers, this program is usually different from previous education experiences. For example, this school year was Ms. Karla's first in a Montessori program, and she admitted that "it's very different. It's like the same topics but a different approach completely, so it's basically get to know the new materials and be able to present the math in a different way." Montessori defines the important role that a third grade teacher should take in a Montessori classroom as a guide, rather than as a direct knowledge source. Ms. Julia discussed the difficulties of maintaining this goal consistently:

> Another very important part for Montessori is that you are not supposed to be always sitting down just delivering lessons. You need to be walking around the classroom, you need to be working one-on-one with those kids that need a review, or they were struggling with something. So you really need to know how to combine that thing about the delivering the lessons but being present in the classroom also. Sometimes I need to stop myself

> and remind myself it's not all about delivering lessons. So today for example, I didn't do any lessons because I sat down with three or four kids specifically to work on something. So sometimes I need to remind myself that this is not always about lessons, lessons, lessons. And that's very important in the Montessori part, because you are not working with the entire group all of the time, so you need to know how they are doing.

Students were given primary control over the subjects they wanted to cover in the classroom, and for mathematics they were required to follow a curriculum guideline that varied from student to student. Independence was not a trait that teachers expected to appear naturally in students at the beginning of their Montessori education. "The biggest strength in the classroom is that generally [the students] are hard workers—they are making efforts to improve things, and so far they are motivated to do the things they are doing," Ms. Julia said. "The weakness right now is the independence. It is very important. Extremely important."

Teachers adjusted their approaches and student activities based on positive results. Despite teachers continuing to strive for self-improvement and maintaining their role in the classroom, students perceived teachers as someone they could approach for help when necessary, not as someone who would simply supply the solution. Simultaneously, teachers constantly sought to improve the environment, motivation, and curriculum. This willingness to embrace new teaching strategies is vital for a successful role as a Montessori instructor (Fraivillig et al., 1999). As a new Montessori teacher, Ms. Karla discussed the challenges of implementing an unfamiliar curriculum, and how to promote hard work and collaboration in students.

> I have been really guided by Ms. Julia this year because I had no idea how it worked, but sometimes the kids will come to the lesson that we had planned, and they'll be like 'Ugh, but we already learned this.' So I feel that we could be moving forward and giving all these fractions and other topics that we could be emphasizing, and instead we are just focusing on the four operations.

When asked for more details about the curriculum design during the second trimester, Ms. Karla explained her fear that the reason the following grades struggled with student success could also be their lack of familiarity with the more difficult material. "In the case of third graders…we should do a review of addition, subtraction, multiplication, maybe focus more on multiplication and division," Ms. Karla said. "But addition and subtraction, it took us the whole first trimester to work on those...because Montessori has fifty addition lessons and fifty subtraction lessons." She also talked about how their goal during the school year was to ensure the students received activities and lessons for all mathematics topics, with the hope of moving into more difficult concepts. She described her optimism in covering topics after division—"hopefully we will finish division and be able to start with something else"—but that there were still another fifty division lessons to first complete. Ms. Karla further confirmed that pushing third grade students to move past division and having third grade students practice handwriting were not considered priorities in previous years at the school. The second round of CGI assessments later used to assess fourth grade students progressively emphasized division and supported Ms. Karla's beliefs that teaching division was not given enough focus in the Montessori classrooms. "At the very end of March we start division," Ms. Karla said. "It's the same [strategy]. First, it’s all materials with the different Montessori materials, different strategies for division…all hands-on and once they’re good with that, then we move on to paper.” Ms. Julia also supported emphasizing division, but believed that multiplication was just as important in developing that skill. “The idea will be we want to keep practicing because I think for me it is very important to focus on multiplication,” she said. “We want to do division with them also, but I want to make sure they go to fourth grade with multiplication facts properly, so the idea will be January and February is still multiplication on paper.”

Summary

Teachers were consistent in following the intention of Montessori for maintaining student roles and teacher roles, though the teachers admitted difficulties in the process. The students took control of their education and were enthusiastic about helping others, preferring to approach each other instead of the teacher when they had questions. Students consistently displayed self-motivation and perseverance, and the teachers continually worked to develop strategies to challenge them without directly teaching. Teachers facilitated dialogue and encouraged cooperation in the classrooms. The curriculum was not evaluated as a transitional aspect for this study, but it did influence teachers’ perception of their roles. Third grade students are only introduced to the four basic operations in this school’s Montessori program, and despite students deeply covering these concepts, there is minimal reference to other mathematics topics such as fractions or geometry.

Fourth Grade: Achieving a Non-Montessori Approach

Scheduled, Structured Lessons

Observations

There were many characteristics for what comprised a fourth grade students' school day in a non-Montessori setting. As the teachers adjusted to best account for the difficulty in transition that administration acknowledged, the structure that students initially worked through changed over the first two months of the year. Initially, Ms. Vicky and Ms. Verena split the duties of teaching the fourth grade. Ms. Vicky taught Monday, Wednesday, and Friday classes, while Ms. Verena taught Tuesday and Thursday classes. Observations showed that the classrooms varied in atmosphere depending on the present teacher. Ms. Verena was originally in charge of developing the fourth grade curriculum while simultaneously filling an administrative role. She implemented an exploratory environment while she was in the classroom, and Ms. Vicky strictly followed the textbooks for the course—Saxon Math Course 4 and 5—following lesson by lesson and varying the emphasis depending on how the students responded to the material. Ms. Vicky assigned practice worksheets after the instruction period, and handwritten work was the primary performance method. In eight out of ten observations, Ms. Vicky taught by direct instruction, while Ms. Verena was never observed using such methods. Ms. Vicky addressed all students collectively, and then afterwards the students had thirty to forty-five minutes to practice the lesson. Students did not have the opportunity to revisit their mathematics lesson after the class was finished.

After just two months, however, both teachers agreed that this classroom structure should be changed based on the development of student needs. The first fifteen minutes of class was set aside for small tasks, such as attendance, lunch orders, and upcoming events. Next, students sat in a circle on the floor similar to the Montessori morning meeting. Unlike third grade, students were combined for the lesson period, regardless of ability, so that the entire class was addressed at the same time. The routine for each textbook lesson included a short warm-up set called the Power-Up and a quick review of previous material, which also involved a Mental Math section that implied students could perform the algorithms without handwriting or using manipulatives. Figure 25 shows the consistency of the classroom setup over four separate days between September and February. After the introduction was finished, Ms. Vicky used both 4th and 5th grade Saxon material, as required by administration, in an attempt to prepare students for future grades. In the new schedule, the lesson was completed in roughly twenty minutes. All ten observations showed that regardless of the teacher present in the room, the material was still focused on one specific mathematics topic for the day.

Classes moved in unison from topic to topic at a consistent pace. Fitting the new structure, students used the three hours after the lesson to work on various assignments not limited to mathematics. According to the fourth grade teachers, this new routine was designed to maintain similarities to previous grades. One of the most apparent differences from the previous schedule was that the teachers began detailing a schedule and prioritizing assignments on the whiteboard. Teachers provided daily tasks to motivate students to complete a checklist for each day, unlike the more open-ended atmosphere that students experienced in the Montessori classrooms. Students were consistently assigned a mathematics task each day in addition to their practice set, and mathematics was the only subject where students were routinely assigned classwork.

Figure 25. Students sit together for mathematics lessons with Vicky on a daily basis.

Students chose the order to work on these different assignments, but were required to complete all assignments on that day. Observations showed that one mathematics worksheet was assigned for homework after nearly every class. While students were tasked with organizing their own work schedule, the majority of students did not describe the new workload schedule as daunting or overwhelming. Students were required to cover many content areas and assignments during the first half of the day, but the mathematics lesson was always the first subject they covered. Most students chose to work on their mathematics assignment directly after the lesson and practice portion of the class.

The mathematics assignments usually followed the corresponding Saxon workbook. These worksheets did not vary from student to student, and the exercises were usually drill and repeat. Nine out of ten observations showed these worksheets were the typical form of assignments for students. Observations also showed that students followed along with the lesson and were assigned the same lessons and practices regardless of how much they actually retained during the lesson. However, differentiated curriculum was still briefly valued and emphasized in the early stages of scheduling at the beginning of the year, when the teachers were still determining the best methods. For example, one early observation followed a lesson, and then the practice exercises, which were kept in red, yellow, and green folders. All exercises during the observed practice session were related to the same topic, but varied in difficulty, with the hardest material in the red folders. Students could choose the difficulty level with which they felt most comfortable. At first, the students highlighted the option of attempting only green problems each day, but observations showed that they wanted to compete and strengthen themselves with the most difficult material they could, and instead chose mostly yellow and red folders. However, these color-coordinated folders were discontinued approximately two months into the school year because of the time intensive nature of their creation. Also, teachers were still shifting their ideas to fit into the ideal role for success in the classroom at this time. Instead, students were assigned worksheets from the matching Saxon workbook that followed the textbook lesson by lesson. The perceived lack of differentiation in the classroom was thus renewed, as all students spread out to work on the same material, shown in Figure 26.

Many students indicated during interviews that they preferred to work on their mathematics assignments directly after the lesson and practice period, while the concepts were still fresh in their minds, before moving onto other assignments.

Figure 26. Students work on the mathematics assignment after their morning lesson.

After lunch and recess, students worked on other content areas unrelated to the morning block that contained mathematics, such as Spanish. The morning block was the time to work individually, and while some students described their preference for working with others, much less group work was present in the fourth grade classroom than in the Montessori classrooms. Traditional classrooms are defined as those in which “the teacher delivered direct instruction and controlled behavior; students followed directions, recalled knowledge, and worked individually” (Zhbanova et al., 2010, p.251). This description matched many of the observations, showing a shift away from Montessori methods entirely.

Another characteristic of the fourth grade classroom was the split nature that was created by two separate teachers integrating their own styles on the days they were responsible for the class. Ms. Vicky implemented direct, lecture-style instruction to students, followed by drill and repeat exercises. Ms. Verena kept some of the Montessori aspect in terms of exploration and provided students more variety in the mathematical exercise activities. Ms. Verena's beliefs were clear from early classroom observations. For example, Figure 27 shows a more creative activity through a colored system to portray decimal place values. One hundred squares were available for students to color equivalent decimal portions on the grid, and the visualization was intended as an alternative method of understanding. This type of activity did not represent the normal practice methods in the fourth grade classroom, but was observed on a few occasions. Analyzing curriculum was not a focus of the study, but provided insight on the implementation and consistency of the fourth grade teaching methods. The students saw less and less of Ms. Verena as the school year progressed, until she worked completely behind the scenes and Ms. Vicky took control of the classroom approximately two months into the school year.

Figure 27. Students color hundredth pieces to fill out visual representations of decimals.

The format of the assignments became more standard and consistent, with repetitive exercises using drill and repeat, such as those shown in Figure 28. Eight out of ten observations gave strong evidence of drill exercises as the main form of practice, and handwriting methods were consistently used in the classroom. The left side of Figure 28 shows an example of weekly mathematics assignments, and the right side of Figure 28 shows a student working through a multiplication practice worksheet. Nine out of ten observations also reflected that students used repetitive formats to engage in recollection of facts and methods from earlier lessons, rather than focusing on forming new ideas through problem-solving skills. Further, homework was consistently assigned nearly every night—an increase workload over third grade students.

The class quickly transformed to direct instruction and lesson practice for the remainder of the school year, and the hands-on activities and differentiated curriculum were almost entirely phased out.

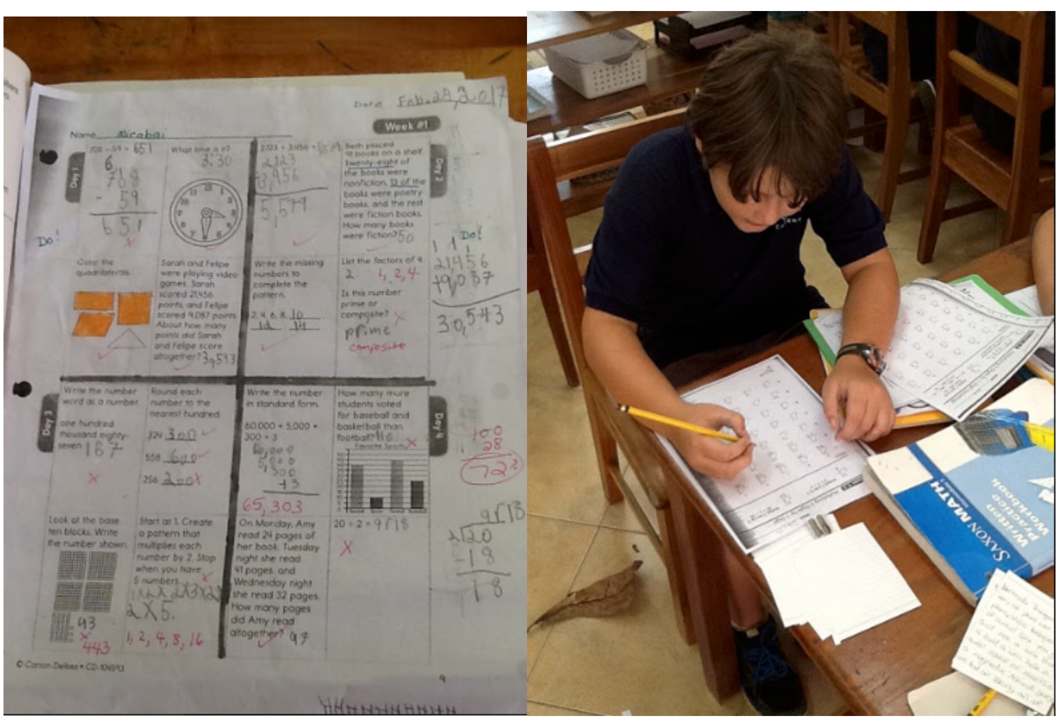

Figure 28. *Weekly and daily assignments asked students to repeat what they learned.*

Figure 29. Students check their own homework with the solution manual in a group.

Students continued to look to each other for support as a fallback, as they were observed relying more on the teacher as the direct source of knowledge in the classroom. While some aspects of group work were maintained initially, students began to choose individual work more often.

<u>*Student Perceptions*</u>

Students coming from Montessori backgrounds acknowledged the change in lesson structure in fourth grade. "It's [different] because, in math last year just a little group would go up and do it," one student said. "But now the whole class, all the fourth graders go up and take a math lessons, but when I was in third grade, only four people would go sit down and take the math lesson." Students noticed the lack of differentiation in many mathematics classes, but many responded apathetically to this situation. Students began the school year reluctant to use the instructor as a knowledge base after coming from a style that encouraged them to be self-reliant. In comparison, a newly transferred student from another institution said she initially felt it difficult to ask questions. “You have to do this on your own more,” she said. “If you have a question you have to come to her, like you have to go to her. It’s different.” Her comments suggest the teacher was not walking around the room, as was the case in the Montessori style. Instead, the teacher was a source that students would approach for help with the intention of implementing specific strategies they did not create on their own. Another student, transferred mid-year from a public school, described her perspective of the different structures between Ms. Vicky’s and Ms. Verena’s classes. “Ms. Verena, she would do more things altogether and she would come to you to help,” she said. “Ms. Vicky likes everyone to do more things separately, it’s different than Ms. Verena. She teaches more like public, while Ms. Verena is more feel, and games.”

After the scheduled mathematics portion of the class, teachers assigned a handful of tasks for students to work on during the first half of the day, including more mathematics. These assignments could be research projects, science exhibits, mathematics worksheets, map-making, spelling lists, or reading chapters in a book. To keep some Montessori values present, Ms. Vicky allowed an open room for a few hours for students to decide which material to prioritize. Mathematics was consistently the first item on the agenda in the mornings with time for homework checks, lessons, and practices. Whereas third grade required weekly deadlines, fourth grade required daily deadlines. As a result, students continued to take part in a routine in the fourth grade classrooms. “We have a list of things we need to do that very same day,” one student said. “So we need to finish that all. It’s time management.” Another student said she could not have side conversations with her friends because she had so much assigned work. Her strategy was to work through difficult mathematics material first and leave the easier non-mathematics assignments for the end. “Some people do the easiest thing first, and then the hard stuff last, and that doesn’t give you enough time,” she explained.

Students switched from one mathematics assignment per week in third grade to one mathematics assignment per night in fourth grade. Students confirmed that the workload was longer and harder, but did not appear to feel negatively toward this increase. Instead, students suggested they valued the increased pressure that seemed to come with the fourth grade stature. One student claimed the homework load currently felt “more proportional and balanced” compared to third grade, and another said “it’s kind of more work, which is better because I don’t get time to rest or...I [would] get bored.” The students have been trained to handle their own workload and to see the value in what they learn. “Sometimes [math] is hard and sometimes it’s okay and sometimes it’s easy,” one student said. “But I like when sometimes it’s hard because that’s how you learn, by making errors.” Further, some students described their appreciation for using teachers as a knowledge source, because they felt they could grow stronger from it. One student unknowingly confirmed the worries of the Montessori teachers on curriculum limits when she said, “Last year we didn’t advance that much, but with Ms. Vicky, each day sometimes there’s something that I don’t understand so I ask for a little help, so she just tells me what it is.” When prompted, she explained that while third grade students had to figure out material on their own, in fourth grade someone was available to explain the problem so students could immediately know what to do. “It’s just harder because in third grade we didn’t do as much,” one student admitted. “Like a lot of time you didn’t know what to do, or you didn't have that much to do. I like [fourth grade] better." Another new student felt like he had to approach the teacher rather than classmates. "Normally it's independent work so you can't really copy off each other, so if someone has already done something you haven't, you probably shouldn't work together," the student said. ""You just go to the teacher and ask for help." These responses suggest that students either felt positively toward the new workload and schedule, or that the workload was a non-factor in determining their success. The comments also suggest that there may be various strategies for students to keep up with the expected assignments. Student comments further suggest that the students were equipped to succeed with such a workload during this transition. Students also described the workload with terms such as "balanced," "concentrated," and "individual" while further mentioning they were glad there was enough material to work on, so they could avoid "having time to rest" that would make them "bored."

Teacher Perceptions

The two fourth grade teachers in this study had very different strategies and mindsets for teaching, covered further in another section of Chapter 4. The clash of different teaching styles created the structure used in fourth grade mathematics. Ms. Verena described her vision as "20% Montessori" when describing her plan at the beginning of the school year. "We are leaving [off] a little bit of the traditional part because traditional—I teach, you receive, and then you do the practice—we want it to be a little more interactive," she said. "So we want to introduce activities that are not just filling out the blanks. We want them to play games, we want them to live the math in a different way. So that is, if you're thinking traditional, it's not as traditional as we think."

One example of these activities was a mathematics dictionary that groups of students shared responsibility in creating. Another example was when students created their own lesson plan to teach their classmates. Ms. Verena's activities were more group-based and student-driven than the Ms. Vicky's version of the classroom, especially when considering the structure that became the routine, and she discussed how the teachers balanced their different styles when creating curriculum.

> We are collaborating in the way that will be something new. Vicky is a person of structure, and she was our past math teacher. She's used to the Saxon, but we want to take that out, like this is a tool that we want to use, but we want to teach math skills. This is the basic thing, so what I am doing is helping her with curriculum, putting some activities in a different way...And for her it's to see differentiation too, not everyone will be on the same lesson at the same time. Kids will be in different things, doing differently according to their skills and that is something that is different from her, so I am there to support that area too.

This point of view did not reflect the eventual, non-differentiated structure that was observed in the classroom, because the follow-through only lasted for two months. This structure did not follow for the remainder of the year because the required workload to maintain the differentiated curriculum for the classroom structure was too time-intensive. Instead, Ms. Vicky took charge of a new scheduled work cycle. "What I'm trying to do is, mornings will be focused on math," she said. "But other than that their follow-up lesson can be done immediately after, or...they don't have to go immediately and do that follow-up work." The term "follow-up" varies from the Montessori definition at this school. Instead, follow-ups in fourth grade were defined as normal classwork assignments, not assessments. A routine schedule was frequently present in the fourth grade classroom. While the third grade classroom had minimum schedule requirements, fourth grade students were given a much stricter, repetitive schedule. “I keep going back to Saxon,” Ms. Vicky said. “Because the children can look at their book and say ‘Oh, so next Tuesday I’ll have this homework, and the following Thursday’ and so on, because it’s all routine.” As the school year progressed and the fourth grade teachers settled on a beneficial system, teachers admitted the difficulties in achieving their goals. “It’s definitely a collaboration,” Ms. Vicky said. “Verena’s designed everything so I’ll be orchestrating it. But she wants me to take the position of the lead teacher with that, and not just be a collaborating teacher.” Eventually, the material was taken directly from the Saxon textbook series, and Ms. Verena maintained the students were independent in the classroom. “There will be some pieces that are traditional when we explain to the kids the concept or something,” she said. “But they will be more independent.”

Conversations with Ms. Vicky and Ms. Verena provided insight into what transpired in the previous fourth grade classes. According to the teachers, instruction time was more limited, and students consistently worked from workbooks on a wide variety of student-chosen lessons that were extremely difficult to track or maintain. Implementation and curriculum goals appear to be issues for concern in the fourth grade classroom. Ms. Vicky and Ms. Verena also found it difficult to consistently identify their roles in the structure of the classroom—a possible issue to be raised in another section of Chapter 4. Meanwhile, there was no differentiation in the classroom for the majority of the year. Observations showed that students consistently received a direct lesson from the teacher, covering a specific topic each class period. The lack of differentiation was a concern to the teachers. They actively discussed fixing this issue with particular strategies, but these strategies were not consistently maintained. “I feel both of us are concerned. Vicky is concerned for the lower [students], I’m concerned for the higher [students],” Ms. Verena expressed. “I feel the higher ones are not receiving enough challenge, and I feel the lower ones are okay because she’s doing this repeating, and she’s doing this over and over and over, so they are receiving that support.” Both teachers agreed differentiation was not occurring in the classroom, but had different opinions as to which students were being most affected.

Summary

The final resulting mathematics environment that fourth grade students experienced was one of structure, routine, and lectures. Ms. Vicky was the direct source of knowledge for each new topic of the day, following a textbook series that also allowed the students appropriate practice. The routine was so ingrained that students had the ability to look ahead at future lessons. Many students believed they could be more efficient in this classroom compared to the third grade Montessori classroom. Ms. Verena and Ms. Vicky had two different ideas and styles for the classroom, but the structure eventually settled upon a direct-instruction approach. While differentiation was initially stressed, the classroom transformed into a setting where students moved together from topic to topic. The fourth grade classroom was classified as non-Montessori based on multiple observations, and while the direct-instruction style was not the sole cause of the struggling transitions for students, the format laid a foundation for some of the miscommunications and problems as part of the targeted aspects.

Pencil-and-paper Work Stressed

Observations

Observations showed the use of handwriting numerical expressions was a primary means of showing work and turning in assignments. Pencil and paper was dominant in all facets of the fourth grade mathematics class, from practice to explanations to class assignments to homework. Manipulatives were all but nonexistent during every mathematics period. Nine out of ten observations provided evidence of the lack of concrete objects or gesturing as part of the explanations. Figure 30 provides some examples of students working individually on mathematics problems using handwriting as the primary method of work.

Figure 30. Students work through Saxon workbooks to perform mathematics tasks.

Figure 31 provides more evidence of handwritten work, with an added emphasis on seating arrangements that separated students and allowed for more individual focus in the classroom. Reducing the focus on collaboration is a major point of the learning methods in this section. Any comments in this section about the structure of the classroom were focused on the majority of the school year, when differentiation had become less prevalent in the classroom. Ms. Vicky provided examples, and students copied down what they felt was necessary and important into their notebooks. This was the first instance that note-taking was observed as a form of communication in the mathematics classroom between students and teachers.

Figure 31. Students work on their own in workbooks, following methods from the lesson.

The assignments that followed after the lesson were almost always in handwritten format. Workbooks were the primary source of assignments, with pre-printed pages containing problem sets that were modeled after the corresponding lesson in the teacher textbook. Figure 32 demonstrates the use of student notebooks as a reference (on the left side) and as an assignment sheet (on the right side).

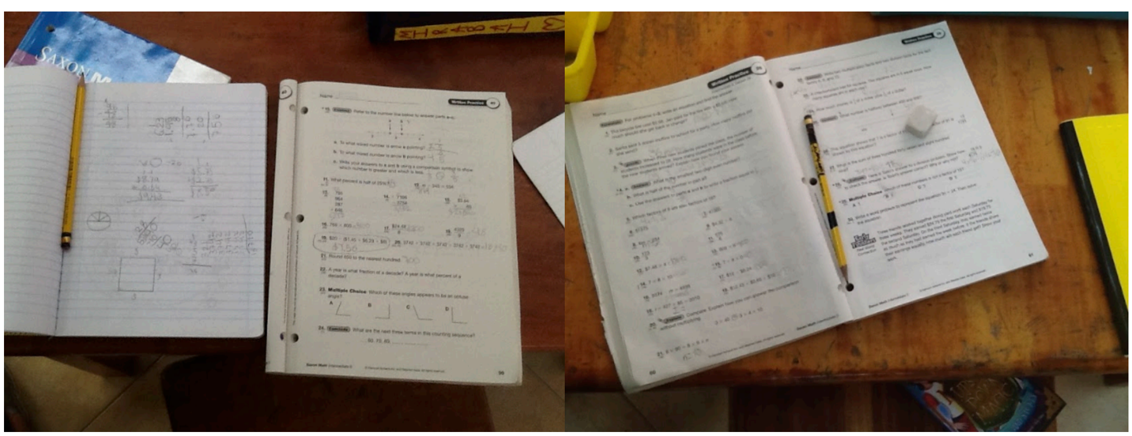

Figure 32. Notebooks and workbooks were the standard method of mathematics practice.

These problems mirrored the strategies taught in the lesson, so students repeated much of the information previously provided to them. Seven out of ten observations reflected this drill-and-repeat strategy. The three observations that did not show these methods took place earlier in the year, when Ms. Verena spent more time in the classroom. While Ms. Vicky fully implemented the use of these workbooks, Ms. Verena did not use them when she was in charge of the classroom.

Student Perceptions

Fourth grade students were asked about the differences between focusing on manipulatives and instead working by handwriting mathematics. A majority of fourth grade students' responses indicated students did not mind manipulatives being absent in the classroom. Many comments suggested the students actually felt positive about the change, and described the use of manipulatives negatively. The reasoning for these statements varied from student to student. For example, because these students had been taught to control their learning process earlier in the Montessori program, they continued to be cognizant enough to state how they thought their time should be used most efficiently. "They're annoying," one student said of manipulatives. "I mean, they did [help], but I still didn't like them at all. Not one bit. I don't like using, like, little materials. It takes so much longer than writing on a paper." Many students concurred with this thought, and one said he simply "like[s] it better without all the materials, [which] just [made] everything way longer." Another student agreed about the time-intensive nature of manipulatives. "It was harder, I think, because I had to learn how to use them," she said when reflecting on using manipulatives in the past. "But here Ms. Vicky just explains it to me and I think that's easier. [Before] it felt like I had way more to do." One of the fourth grade students insisted she did not miss manipulatives, but admitted, "Yes, they were [helpful]. But now I don't really need them." Other comments from students continued to reference students challenging themselves. "I liked [the materials], but I like this better," another fourth grade student said about handwriting. "It's faster. I like writing and doing all the other stuff we didn't get to do in third grade. I don't know, it's just harder because third grade was way too easy."

Student responses followed metacognition, the theoretical framework of this study, as expected. Students were aware of their effective and ineffective strategies, and discussed the reasons why they felt manipulatives might have been helpful to their initial processes. Namely, many students believed they were more "mature" in fourth grade and "ready to move on" from the use of mathematics materials. In this school, grades K-3 are designated "lower elementary" while grades 4-5 are labeled "upper elementary." Some students appeared happy to set themselves apart from third grade because they had moved on into the upper elementary level. Students' comments suggested they were proud of using different methods and working through more difficult mathematics. The majority of fourth grade students did not identify the lack of manipulatives as a hindrance to a successful mathematics experience, despite teachers' assumption that this aspect would be problematic. "Using the beads and stuff is like—I don't like it when you have to use it all the time," a student explained. "I like doing it in my head. Like, doing it by heart instead of just putting beads down to count them. They weren't [helpful]. Sometimes they were." Generally, students appeared to respect the availability and assistance that manipulatives provided in third grade, but believed they were ready to move on quicker than teachers realized.

Table 7 organizes the specific thoughts from fourth grade student responses during interviews. Thoughts, phrases, statements, and physical gestures portrayed by fourth grade students during the interview process were used to determine students' stances on mathematics (negative, medium, or positive) and their preferences for learning mathematics (materials or handwriting). Three students were transfers during the school year and had no prior experience with manipulatives.

Table 7
Comparing Stance on Mathematics to Learning Style Preference (4th Grade)

Name	Stance on Mathematics	Learning Style Preference
Jayce	Positive	Handwriting
Sonny	Medium	Handwriting
Merissa	Medium	Handwriting
Lilia	Positive	Handwriting
Noa	Positive	Handwriting
Mirabai	Negative	Handwriting*
Norelle	Negative	Handwriting
Marysol	Positive	Handwriting*
Jack	Positive	Handwriting
Cole	Positive	Handwriting*
Felix	Medium	Handwriting
Lior	Medium	Handwriting
Victoria	Positive	Handwriting
Romeo	Positive	Handwriting
Valentina	Medium	Materials
Kai	Medium	Materials

*Indicates students who never experienced the Montessori program, so only knew handwriting methods.

Three students were transfers during the school year and had no prior experience with manipulatives. While third grade results from Table 6 showed a mix of opinions toward learning methods, the comments in Table 7 nearly unanimously supported handwriting as the superior method. The few students who preferred the use of manipulatives did not have positive stances toward mathematics. Coding of student comments included non-verbal communication such as confidence, perceived ability, environment, enjoyment, workload, stress level, and environment. Fourth grade students listed many reasons for preferring handwriting methods: an increased challenge, more fun class, overall efficiency, less annoying steps, general speed, and workload ease. The students' perspective of their transition showed they were less affected by the lack of manipulatives than teachers believed. The interviews instead suggested that students found manipulatives helpful at one time, but would have liked to transition to new methods sooner to achieve more in their mathematics classes. Student comments showed they appeared ready to move on from materials, and many suggested using manipulatives was more a chore than a helpful tool. The CGI assessments were important when evaluating student mathematical thinking against student beliefs to determine if they indeed preferred to use handwriting in a problem-solving setting rather than pictures, objects, or materials—that is, whether their actions reflected their comments.

Teacher Perceptions

The fourth grade teachers did not focus on the lack of manipulatives in the classroom to the degree of the third grade teachers. However, Ms. Verena agreed with some of the fourth grade students' comments, that "it's a point too, like with the manipulatives, that enough is enough. The thing is for the kids that are like 'are you kidding me? Doing this again?'" She agreed that fourth grade students could simply grow tired of a concept, regardless of the utilized method. She also believed that the fourth grade students might have disguised their desire to downplay manipulatives behind the need to appear older and more mature:

> I think it's like a stage. I believe that when you play with dolls or play with your trucks, there's a moment that 'I don't want to play with dolls or trucks anymore.' That's what I think is with the kids. 'Oh, I'm mature enough, I don't want to play…' They see those as toys more than materials for learning, you know what I'm saying? When you have a paper and a pencil, like 'Oh, I am old enough,' you know?

Ms. Vicky did not think the lack of manipulatives was a problem, but did believe students were not using the manipulatives to full potential in lower elementary. “Actually, because I taught lower, I know that there are gaps in what they are learning,” she said. “I did give the lower teacher a big chart full of word problems that they could use, and then the answer is actually, like visually there.” She continued to talk about how routine was more important than the new use of handwriting in fourth grade, and that “over in lower [elementary] there are certain standards that are not being met with the Montessori materials.” She admitted that the handwriting issue was not something she had considered. Both fourth grade teachers promoted word problems as a priority because fourth grade is the first time students see these types of questions in handwritten form. Vicky stressed her belief in word problems in her classes.

> I think that once they get the hang of it they’re fine. I think that for some, it’s new coming from lower, so they probably haven’t had a lot of word problems...I can have a whole basket of word problems that they can do, so that will help. I mean there’s other things I can add to that, all those Montessori materials down there, but once they’re finished, they need to access other things.

Summary

Fourth grade student comments revealed that students did not find the focus on handwriting methods particularly difficult, and many students described manipulatives as helpful in the beginning, but also as eventually devaluing the effectiveness of time management in class. Students comments confirmed that most fourth grade students preferred the new strategies of handwriting to solve problems, but students with lower confidence levels felt more positively toward manipulatives. Teachers shared similar views, claiming that fourth grade students may mentally have set themselves apart from the lower grades because they felt more advanced and therefore past the stage of using materials to aid in mathematics. Fourth grade students were completely reliant on handwritten methods during lessons, practices, and homework, and manipulatives were very rarely implemented in the fourth grade classroom. While third grade teachers assumed taking away materials from students might be harmful, the fourth grade teachers saw more issues with filling gaps in knowledge that third grade teachers were not completing in terms of standards and curriculum with the use of materials.

Teacher as Direct Source of Knowledge

Observations

Observations showed that each class period began with a mathematics lesson. The teacher directly instructed, while students took a passive role and received the information all at once. For example, during one class Ms. Vicky provided examples of two-digit by two-digit multiplication and showed all steps involved numerous times before allowing students a moment to volunteer their answers. During individual practice, students copied the methods they had observed, following the patterns and strategies on their assignments. It was not clear how much understanding was gained during these lessons, because students did not typically ask questions during direct lessons. Eight out of ten observations showed that the teacher took on a lecture role for instruction. Nine out of ten observations provided evidence of student engagement in the form of recollection of facts, formulas, and definitions, along with students following algorithms at various levels of understanding. Students' pre-printed workbooks had examples that complemented the teacher textbook, and students could also copy problems from the board in their notebooks. Students also used notebooks for various side assignments or scratch work. When students needed assistance, they knew to walk up to Ms. Vicky on one side of the classroom when Ms. Vicky was present, while Ms. Verena would walk around looking for students that needed help on her days in the classroom, as shown in Figure 33. On the left side, Ms. Vicky is shown waiting for students to approach when they needed assistance. On the right side, Ms. Verena actively looks for students who appear to be struggling. This created a sense of the teacher as the solution to any problem—the source of knowledge available at any moment.

Figure 33. Vicky waits for students and Verena seeks out struggling students.

Often, Ms. Vicky would invite a student to come up and explain a problem or ask a question on the whiteboard, as shown in Figure 34. Another tool that students used was a miniature whiteboard or clipboard for extra practice and quick exercises, as shown in Figure 35. These strategies allowed for individual work but also provided students a more creative outlet for showing and explaining their processes, sharing their thoughts with the class. Students participated in a more individualized education compared with how they received lessons in the Montessori program, especially how the class moved together as a unit.

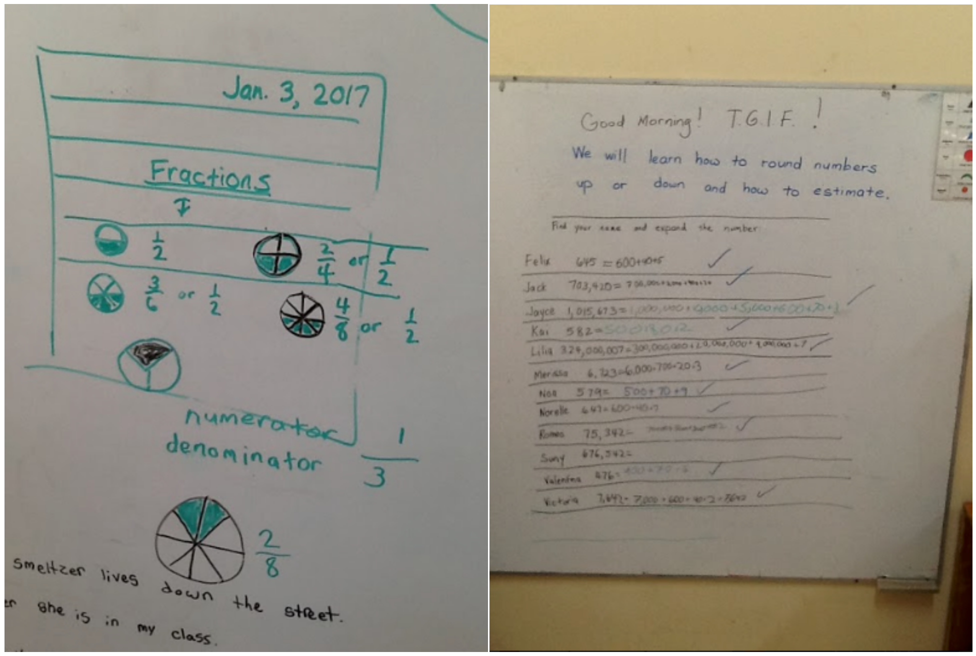

Figure 34. Students have another outlet to show work, ultimately judged by the teacher.

Figure 35. Students continue to imitate methods from the board during solo practice.

Seven out of ten observations showed a lack of differentiation in the classroom as students were not given time to discover their own findings, and instead copied what they heard or saw from the teacher, skipping the reflection step entirely. The students were generally separated around the room as they worked, as shown in Figure 36.

Figure 37 illustrates the difference between learning styles in third grade and fourth grade. On the left side, third grade students collaborate to present what they know, and on the right side fourth grade students are sectioned off by folders to keep them focused on their own work. The fourth grade students did not always use these table dividers, but there was an observed trend that fourth grade students were less likely to work together as the year progressed.

Figure 36. Students are separated to practice individually, with minimal collaboration.

While third grade students show their skills through the follow-up method of assessment, fourth grade students only used handwritten exams for assessment. This focus on test scores created a competitive atmosphere where individualization and private work were apparent when observing the class.

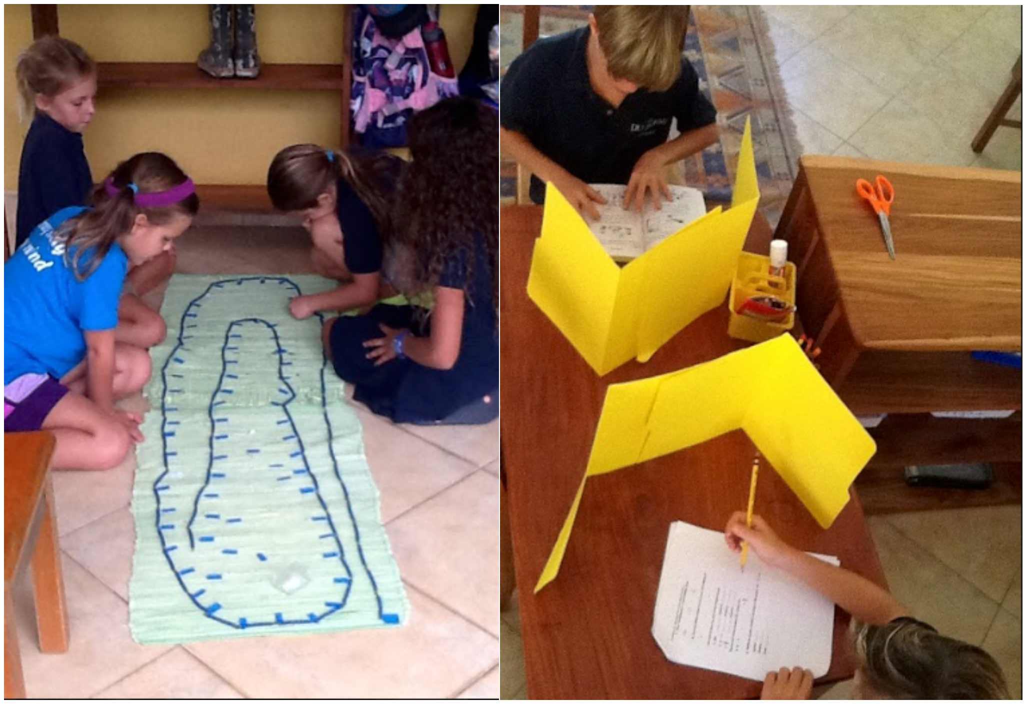

Figure 37. A comparison of third and fourth graders when working on mathematics.

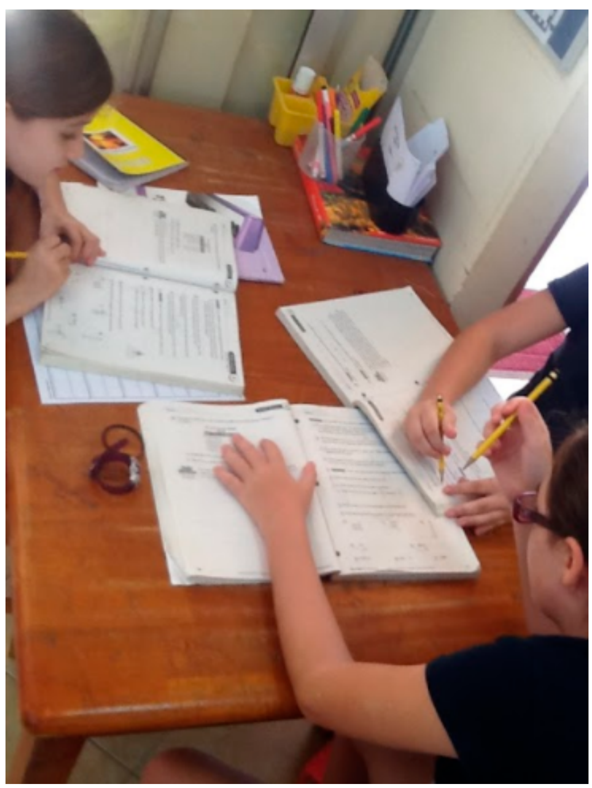

Figure 38. Brief instances of collaboration were still found in the room early on.

Observations also showed that the teacher was in control of classroom pacing, and students did not move ahead or stay behind as the lessons progressed. Fourth grade students instead moved forward together, regardless of strengths or weaknesses among students in the class. The teacher dictated when to move forward and which concepts to focus on, including the length of time given to those topics. Five out of ten observations showed that students were still willing to collaborate early on, but collaboration lessened as the year continued and the students gradually secluded themselves into a more individualized system.

Student Perceptions

Third grade students expressed a high level of reliance on working with friends in the Montessori classroom, and many fourth grade students admitted they still liked working with others but often found themselves going directly to teachers for help instead. The most common response was simply, “you just go to the teacher for help,” from multiple students. One transfer student said she found it difficult to ask anyone questions, and described this classroom as more “separated individually” than her old school. Fourth grade student comments did not negatively reflect going to the teacher for help, and many students admitted using the teacher for assistance before attempting to persevere on their own. In third grade, most students talked about pushing through on their own or using their friends for guidance. “Here, it’s individual,” one fourth grade student said. “In third, you all had to work together to do something, and I just like it to be individual more, there’s more concentration [now]. They treat you like you’re more mature and stuff. You know what you are doing.” The time period at the beginning of the day was meant specifically for mathematics, not for exploring other content areas. While other content areas may have partially followed the Montessori openness, students were not responsible for their schedule in mathematics. One student pointed out that this seemed to provide the teacher more time to help them. “If I get stuck, just ask the teacher,” he said. “In third when you go ask the teacher, she’s always doing something else because there’s so many things to do in that class.” This response implied that working with friends or persevering individually in third grade was a necessity, while in fourth grade students could get teachers’ attention at any time.

Additionally, the teachers filled their roles differently based on their beliefs and experiences. The students picked up on these differences. As one student described, “Ms. Verena, she would show us videos, like how to teach them [videos], and Ms. Vicky would explain it to us on the board. Ms. Verena wouldn’t really explain it to us—we would watch videos and then if we didn’t understand it, we need to ask a friend or go to Ms. Vicky and ask her.” She then admitted that Ms. Vicky’s approach was more helpful. Student comments reflected that students appreciated the roles teachers took as a source of information, and often regarded Ms. Vicky’s role more positively than Ms. Verena’s, citing the simplicity of just asking the teacher when they got stuck on a problem.

Teacher Perceptions

The teachers did not believe students were prepared enough by fourth grade to think critically. Their comments indicated that the past curriculum prepared them for calculations, but not necessarily problem-solving skills, which was part of the goal of the earlier differentiated activities. Conversations with teachers showed that classes became more focused on separating the students during work time as the school year progressed, and teachers admitted that students needed to be turned away from collaboration and come to the teacher for help instead. “As a community we know always that they have to help each other,” Ms. Verena said in agreement about more individual, challenging work. “But at the same time you need to create that challenge for all of them.” When Ms. Vicky gave a lesson, instead the process involved writing and direct lecturing as the primary method of instruction. “Now I’m doing a lot more of the math, it’s basically on Saxon,” she said of her greater role in the direct-instruction classroom. “So the children are getting a lot more of that input through the Saxon math and not through the games and activities through Verena. Our schedule kind of shifted a little and so I’ve been teaching math a little bit more.” Ms. Vicky also mentioned that the lesson period could not be more than fifteen or twenty minutes or she would start to lose students’ attention:

> The attention span of these children can only be so long or they’re going to tune out, some start misbehaving, you know you want to have a quick lesson. That’s the beauty of Saxon, you have a short lesson, some vocabulary, and then they either get it or they don’t. So it’s really a quick way to do whatever the skill is for math. The follow-up work can be done any time. If they don’t get the skill of course I’ll sit with them, that way when they’re working independently I’m going to have time to work with them individually to grasp whatever we’re doing. I also want to see independent work, so they should have at least four independent practices per week.

Ms. Verena had her own thoughts about her struggle to fill her teaching role in the classroom. Her belief was that differentiation gave students more passion and control of their mathematics education, and she was afraid they would lose it otherwise. “If you have a kid who is pretty good in math doing something over and over and over, this kid will lose the passion for math, and that is my biggest concern," she said. "I don't want the kids to lose that passion." This differentiation required a lot of time to prepare, and the fourth grade teachers were challenged in keeping up with preparation time. "I think we've had a couple of complications with what we started out to do," Ms. Vicky said. "Verena was making the curriculum, and it was always coming in late. Like, Monday mornings I didn't know what I was teaching because the curriculum wasn't available yet." They decided Ms. Vicky would teach the way she felt more comfortable and Ms. Verena would continue her methods on her selected days in the classroom. However, Verena's days in the classroom were eventually phased out altogether, with the curriculum integration described as "hard to blend together." Ms. Verena's role moved away from curriculum developer and settled as a guide for Ms. Vicky. The majority of the class periods therefore allowed students to experience the teacher as the giver of knowledge in a direct-instruction style through Ms. Vicky. "I need to know what I'm teaching for the week and kind of have a mindset before the children arrive and I already know where it's leading," Ms. Vicky explained. "So having Saxon I just feel more comfortable, knowing where it's going."

The two teachers settled into their roles in less than two months. Ms. Vicky was focused on routine and scheduled bookwork, stressing practice exercises in the classroom. Multiple classroom observations showed that she treated the beginning of class as a solid, mathematics-only period. For Ms. Vicky, it was important for students to be "thinking mathematically first thing in the morning." In an early interview she stated that the class still retained some elements of the Montessori program for all subjects except mathematics. "But the children will get used to that routine. Because I really like and think that the children learn well with the way the Saxon is," she advised. Ms. Vicky's focus was on the way she would design the classroom structure and how the students would operate within those parameters to fill their roles. Ms. Verena began as a co-teacher who developed differentiated activities, hands-on games, and less-directed activities for students to explore in her two classes per week. However, her role gradually shifted to more behind-the-scenes, and the students noticed her complete absence by December. Teacher collaboration is often considered the best way for teachers to learn the most about student thinking, but student collaboration is considered important as well (Edwards, 2002).

Summary

While either teacher could easily argue her viewpoint, both struggled to identify their correct roles in the fourth grade system. Whereas differentiation was highly stressed in the Montessori style, this was difficult to maintain in the teacher-led direct-instruction style fourth grade classrooms used. The fourth grade classroom promoted individual-based working styles, but this did not automatically translate to an individualized curriculum. The student-led discovery environment in the Montessori program shifted greatly to what fourth grade students experienced by being directly given new information daily. Students appeared positive about having the teacher available to ask for help, but may have relied too much on their presence rather than persevering alone or with a classmate. Routine and schedule were habits in the fourth grade classroom, and students did not react negatively to a heavier workload.

Results of the CGI Problem Solving Exercises

Introduction with Criteria for Student Selection

The final research question of this dissertation addresses any impact the transition from third grade Montessori to fourth grade non-Montessori has on students' problem-solving skills at this school. Cognitively Guided Instruction (CGI) has been supported through the literature as an excellent guide in observing student thinking and mathematical processing (Carpenter et al., 1996). Ms. Verena, one of the fourth grade teachers, believed a major problem in the format of fourth grade direct teaching methods was the extended focus on computational skills, while mathematical exploration opportunities were hardly present. Students relied on working by themselves and using the teachers as direct sources of information rather than thinking through true problem solving. Students' comments reflected their preference to have a teacher present to answer questions immediately upon request rather than having to figure it out themselves.

The CGI assessments for this study were administered in October 2016 and February/March 2017 and aligned with curriculum objectives and topics for those portions of the school year. The problems also increased in difficulty enough to observe strength and growth of students' mathematical mindsets over similar concepts as they were challenged, a necessity when maximizing the takeaways from observing students (Goldin, 1997). In these word problem questions, the number portion is left blank to allow the interviewer the freedom to change the difficulty of the values according to the level of ability the student portrays during the exercise (Jacobs & Empson, 2015; Jacobs & Ambrose, 2008). Further, the fourth grade teachers in this study suggested which six students would participate in the CGI portion of the study. The two teachers chose students with average mathematical performance, given their prior experience with the students during past school years. Because the two fourth grade teachers were some of the longest-tenured faculty, they knew the students much better than a newcomer. This prior experience, combined with some brief conversations with the students, led to the selection of these six fourth grade students for the CGI assessments. Additionally, while the selected students had attended this particular school for different lengths of time, all had experience in the Montessori program to some degree. Two of the students, Lior and Kai, attended the school since before first grade. Felix was the least experienced, with one year in the third grade Montessori system before moving into fourth grade. Romeo was in his third year at the school, but had only attended in first, third, and now fourth grade (he lived out of the country for second grade). Victoria and Valentina attended the school since first grade.

Ms. Vicky, a more traditional-inclined fourth grade teacher, believed the fourth grade students would do fairly well in the CGI assessment, but admitted the initial transition period is the most difficult. "So [fourth grade] starts out real simplistically. We look at different words like altogether, product, sum, so they know what the conversation [or] operation is going to be," Ms. Vicky said. "So I think that once they get the hang of it they're fine. I think that some, it's new, coming from lower, so they probably haven't had a lot of word problems." While all students received the same question and were allowed time to write and think out loud, the numbers could be adjusted based on how the student was responding. All four teachers involved in the study considered problem solving a major goal of the academic year, but they also all admitted the students struggled with the concept. "In general Montessori method is always trying to encourage them to problem [solve], and not giving the [problems] done already, so [we] give them the strategies to solve those problems," Ms. Julia, one of the Montessori teachers, said. "But that's something that we need to train with them...I hope that it will get better." In fourth grade, Ms. Verena agreed and thought the focus was too much on calculation rather than problem solving. The third grade experience that the fourth grade students previously came from should focus on developing independent thinking and confidence through problem-solving strategies (Kamii, 1994).

For this section, each question from each problem set is addressed and analyzed individually in terms of student responses, approaches, and general ideas toward student strategies—the appropriate method of constantly questioning students to gain insight into their true understanding (Ginsburg, 1997). According to the literature, an important facet of this process is to compare how students perceive the problem in terms of original approach in order to fully interpret student understanding (Charalambous, 2010). Following these guidelines, the results of each problem are organized into a table to compare the student-identified strategies, along with a description of students' actual processes. The first section covers the October problem set, with six questions that focused on subtraction, addition, subtraction, multiplication, division, and division, respectively. The second section covers the February/March problem set that focused completely upon the idea of multiplication/division and equivalent fractions. The two problem sets were given four months apart to try and capture the greatest changes in student thinking and sense making. According to the students and teachers, they had not yet reached division in fourth grade by the time the first round of problems were administered. Although students had limited experience with division by the end of the third grade curriculum, all four teachers debated the degree of experience. For the second round, students confirmed that they were in the middle of division and fractions (typically only fractions with a 1 in the numerator). The problems were already created as previously tested and published examples of CGI instruction (Empson, Turner, & Junk, 2006), but the interpretation method was altered for this analysis. Rather than providing questions based on students' responses as a type of mapping that changed according to student success, all six problems in each set were consistently given to all students. However, the problems themselves were not altered in any way besides varying the numbers for students. If the situation required, follow-up questions and/or alternate numerical values would be used to continue to test student thinking. Method identification was based on definitions from Carpenter et al. (2015). Two columns are provided to demonstrate student thinking in alignment to valid descriptions according to the CGI framework. Further, analysis dictated whether the identified strategy was relevant to the problem, and if the correct solution was obtained. Students could potentially identify a correct strategy but not answer the problem, or vice versa.

First Round of Assessment Implementation

First Question

Jennifer has ____ dollars. She earns some more money babysitting over the weekend. Now she has ____ dollars. How much money did she earn over the weekend?

Source: Empson, S., Junk, D. & Turner, E. "Formative Mathematics Assessments for Use in Grades K-3."

All six students were given values that required them to show their borrowing and regrouping ability. Four of the six students arrived at the correct solution (Lior, Victoria, Romeo, and Valentina), and they all identified the appropriate operation as subtraction. Three students subtracted vertically with one number over the other and one student

(Victoria) subtracted with the numbers next to each other, yet still with the standard borrowing technique. The two students who gave incorrect answers approached the problems from different directions. One student (Felix) preferred to set up the problem as 23 + ___ = 41, and counted by tens and then subtracted back to get back to 41. While this is a standard strategy for a missing addend problem, he assumed the solution started with a 2 because 2+2 put 4 in the tens place, without accounting for regrouping. Aside from this error, the method was longer and while he avoided using subtraction by identifying a correct alternative method, he still did not arrive at the correct solution. The other student (Kai) decided to use subtraction for a similar problem, but incorrectly identified the setup as 17 - 31 = 26, which gave the incorrect solution (a common mistake in multi-digit subtraction). Table 8 summarizes the information and shows that the two students who tried alternative strategies were incorrect with their final results.

Table 8
Round 1, Question 1 Results: Join—Change Unknown

Name	Identified Operation	Method	CGI Strategy	Validity/Result
Felix	Subtraction	Building addition from zero	Counting on from first	Valid/Incorrect
Lior	Subtraction	Vertical subtraction with borrowing	Algorithmic	Valid/Correct
Victoria	Subtraction	Horizontal subtraction with borrowing	Algorithmic	Valid/Correct
Romeo	Subtraction	Vertical subtraction with borrowing	Algorithmic	Valid/Correct
Valentina	Subtraction	Vertical subtraction with borrowing	Algorithmic	Valid/Correct
Kai	Subtraction	Reversed order with starting amount minus resulting amount	Algorithmic	Valid/Incorrect

Second Question

There are ____ kids in the cafeteria. ____ more kids come in for lunch. How many kids are in the cafeteria now?

Source: Empson, S., Junk, D. & Turner, E. "Formative Mathematics Assessments for Use in Grades K-3."

All six students correctly answered this question, with minimal variance in their chosen methods. Four students (Felix, Lior, Valentina, and Kai) used vertical addition. Lior at first identified subtraction, but quickly realized his mistake without prompting. The other two students (Victoria and Romeo) used horizontal addition in different ways. Victoria still performed the traditional carrying over the tens place as she regrouped. Romeo broke down his problem of 33+18 into simpler numbers without the need for carrying: 33 + 10 + 8. One of the Montessori activities for addition involved place value in the form of beads to create large numbers, and Romeo's strategy appeared to be an abstract version of this strategy. Victoria's method represented more mental operations, while Romeo's strategy reflected the abstract use of manipulatives in a new context. Table 9 summarizes the strategies, in which students mostly followed similar methods.

Table 9
Round 1, Question 2 Results: Join—Result Unknown

Name	Identified Operation	Method	CGI Strategy	Validity/Result
Felix	Addition	Vertical addition with carrying	Algorithmic	Valid/Correct
Lior	Addition	Vertical addition with carrying	Algorithmic	Valid/Correct
Victoria	Addition	Horizontal addition with carrying	Algorithmic	Valid/Correct
Romeo	Addition	Horizontal expanded addition	Derived Fact	Valid/Correct
Valentina	Addition	Vertical addition with carrying	Algorithmic	Valid/Correct

Table 9 continued

Name	Identified Operation	Method	CGI Strategy	Validity/Result
Kai	Addition	Vertical addition without carrying	Counting on from first	Valid/Correct

Third Question

There are ____ children playing in the park. ____ children had to go home. How many children were left playing in the park?

Source: Empson, S., Junk, D. & Turner, E. "Formative Mathematics Assessments for Use in Grades K-3."

All six students identified subtraction as the most efficient operation to use for this problem, and all but one (Kai) arrived at correct solutions. This problem tests the ability to find an unknown result through subtraction (or missing addend). A problem that requires borrowing challenges students to regroup or find alternative methods. Felix, Romeo, and Valentina chose to use the standard vertical subtraction algorithm. Kai operated differently by counting backwards on his fingers to arrive one away from the correct solution, but still showed his reliance on physical objects. Lior decided to calculate his problem mentally without writing down the problem or showing any work, while Victoria did write down the problem, but did not show any work. Students mostly showed their ability to move past materials to carry out this problem, and Table 10 summarizes their strategies.

Table 10

Round 1, Question 3 Results: Separate—Result Unknown

Name	Identified Operation	Method	CGI Strategy	Validity/Result
Felix	Subtraction	Vertical subtraction with borrowing	Algorithmic	Valid/Correct
Lior	Subtraction	Mental subtraction, no writing	Derived Fact	Valid/Correct
Victoria	Subtraction	Horizontal subtraction, no work shown	Number Fact	Valid/Correct

Table 10 continued

Name	Identified Operation	Method	CGI Strategy	Validity/Result
Romeo	Subtraction	Vertical subtraction with borrowing	Algorithmic	Valid/Correct
Valentina	Subtraction	Vertical subtraction with borrowing	Algorithmic	Valid/Correct
Kai	Subtraction	Repeated subtraction using fingers, off by 1	Counting Down	Valid/Incorrect

Fourth Question

There are ____ children going to the water park. It costs ____ dollars per person. How much money will it cost for all the children?

Source: Empson, S., Junk, D. & Turner, E. "Formative Mathematics Assessments for Use in Grades K-3."

The purpose of this question is to test identification of a multiplication problem. Students could potentially recall the multiplication facts by memory, so questions were asked that went outside of the 12-by-12 multiplication tables. Instead, each student's problem was formatted to ask for a two-digit number of people multiplied by a one-digit amount of money. Valentina incorrectly identified division as the necessary operation, formatting her work as vertical division and showing a lack of knowledge of the operation, as shown in Figure 39. After trying to continue the division process by drawing seventeen tally marks and unsuccessfully grouping them into fours, she admitted, "I don't know how to do division. Dollars and cents I just started so it's kind of hard." Her drawing represented the beginning of her tendency to try to create an image of the situation, which did not work out in this case because she incorrectly identified the strategy. Three students (Felix, Lior, and Kai) decided to count by factors. Two of these students identified the problem as multiplication, and one student (Felix) labeled it as building addition. While Kai and Felix counted by the one-digit amount of money, Lior knew to hasten the process by counting by the larger factor, the number of people, to arrive at the solution quicker. He confirmed this intent afterward, when asked about the approach. Kai was assigned a simpler, adjusted problem than the rest of the students after initially appearing overwhelmed by the given values. He used a combination of counting backwards, fingers, and multiplication facts, to produce an incorrect result. The two students (Victoria and Romeo) that appeared, early on, as the strongest of the group, each used multiplication. Romeo's examples are provided in Figure 39 and Victoria's examples are provided in Figure 40. They displayed the problem horizontally rather than stacking the numbers on top of each other, and did not use a traditional algorithmic multiplication. Instead, these two students again performed a Montessori multiplication process of breaking the problem down by tens and ones: for Romeo, 26×4 was written as 20+20+20+20+12+12, and Victoria vocally made 18×4 into 10+10+10+10+8+8+8+8. Each of these students obtained correct results working this way, which they voiced as the more comfortable method. All six students avoided the standard two-number algorithmic multiplication process of multiplying the ones digit to each above digit, involving regrouping. This was the first multiplication problem in the CGI set, and the strategies varied more than they had to this point, as shown in Table 11.

Table 11
Round 1, Question 4 Results: Multiplication—Result Unknown

Name	Identified Operation	Method	CGI Strategy	Validity/Result
Felix	Addition	Building addition by smaller factors	Skip counting all	Valid/Correct
Lior	Multiplication	Building addition by larger factors	Skip counting all	Valid/Correct
Victoria	Multiplication	Horizontal expanded addition instead of multiplying	Measurement division	Valid/Correct

Table 11 continued

Name	Identified Operation	Method	CGI Strategy	Validity/Result
Romeo	Multiplication	Horizontal expanded addition instead of multiplying	Measurement division	Valid/Correct
Valentina	Division	Vertical division and illustration of grouping	Written form of direct modeling partitive division	Invalid/Incorrect
Kai	Multiplication	Repeated subtraction by smaller factors using fingers, off by 1	Skip counting down	Valid/Incorrect

Fifth Question

There are ____ donuts. ____ donuts fit in a box. How many boxes will be needed for all the donuts?

Source: Empson, S., Junk, D. & Turner, E. "Formative Mathematics Assessments for Use in Grades K-3."

The intent of this problem, according to the authors, is to prompt students to use division, but only two students identified this method. However, the wording of this problem invites many strategies for solving, and every student avoided performing division operations for this problem. Still, the results showed a success rate of four out of six students. The two students that got this problem wrong (Kai and Lior) struggled to begin the problem, but once they could start, each used the strategy of repeatedly subtracting the number of donuts in one box from the total number of donuts. They did not complete the process, as they only subtracted one iteration and incorrectly believed that was enough to arrive at a conclusion. Overall, this showed a lack of confidence in the division algorithm, which was consistent with student remarks and with earlier teacher descriptions of how division in the curriculum is split between the end of third grade and beginning of fourth grade. The concept of division did not appear beyond students' abilities, as their strategies reflected some understanding and interpretation of how division works. For example, Romeo said, "Oh I'm not good at division, I was going to do division, but I hate division" before changing his strategy to multiplication facts and quickly using the word "groups" often. Much like the students used addition/subtraction strategies for the previous multiplication problem, here they knew their multiplication facts were an alternative to division, which is how three students (Felix, Victoria, and Romeo) decided to operate. For example, Victoria wrote down and knew that 42÷6 is 7 "because six times seven is forty-two, that's easier." Felix provided his answer after mental calculation. Valentina counted by sixes rather than using multiplication facts, as shown in Figure 39. These CGI division strategies from early in the school year, shown in Table 12, can be compared to the strategies of the many CGI division questions that appear in the second round of problems, which were administered after students had four more months in the classroom to practice the operation.

Table 12
Round 1, Question 5 Results: Measurement Division

Name	Identified Operation	Method	CGI Strategy	Validity/Result
Felix	Multiplication	Mental multiplication	Multiplication	Valid/Correct
Lior	Subtraction	Repeated subtraction, stopped after one iteration	Skip counting down	Valid/Incorrect
Victoria	Division	Multiplication facts recollection	Multiplication	Valid/Correct
Romeo	Division	Multiplication facts recollection	Multiplication	Valid/Correct
Valentina	Multiplication	Building addition	Skip counting all	Valid/Correct
Kai	Subtraction	Repeated subtraction, stopped after one iteration	Skip counting down	Valid/Incorrect

Sixth Question

There are ____ children in P.E. Class. The teacher wants to make ____ teams with the same number of kids on each team. How many children can she put on each team?

Source: Empson, S., Junk, D. & Turner, E. "Formative Mathematics Assessments for Use in Grades K-3."

At first, this problem appears very similar to the prior division problem, searching for equal groups, or "equal share," as a form of division. However, students identified division here much more readily than in the previous question. While the identification was clearer, the success rate remained the same with three out of six students correctly answering the question by completely following through with their own methods, and much of the students' processes were verbal instead of written. The students again wanted to avoid performing a division operation, instead looking for alternative methods. This tendency confirmed students' uncomfortable feelings toward operating strictly with division algorithms. For example, Felix counted factors with building addition until reaching values he was unfamiliar with, before attempting repeated subtraction away from the final number, losing confidence the further he went. Lior adopted a similar strategy, but rather than counting by fours in his problem, he grouped the fours into sets of sixteen. However, his strategy of continually adding sixteens did not perfectly reach seventy-two, so he became discouraged by his process. The most common strategy students used was counting by the number of teams until they reached the number of children, using the missing factor approach as a faster way to reach the solution. Multiplication fact knowledge was an advantage to a certain extent. For example, in Victoria's case, she only knew up to $3\times12=36$. She did not recognize the option to double this number, so became stuck and could not continue with this approach. She said that multiplication facts were very helpful to her, but since she did not know them after the twelves, the facts could not help the rest of the way. The other two students (Romeo and Valentina) counted by their divisors to find their missing factor, again verbally. The results are shown in Table 13.

Table 13
Round 1, Question 6 Results: Partitive Division

Name	Identified Operation	Method	CGI Strategy	Validity/Result
Felix	Division	Repeated subtraction and building addition as comfortable	Skip counting all and skip counting down	Valid/Incorrect
Lior	Division	Building addition with the faster assistance of grouping	Direct modeling multiplication	Valid/Incorrect
Victoria	Division	Multiplication facts recollection	Multiplication	Valid/Incorrect
Romeo	Division	Building addition	Skip counting all	Valid/Correct
Valentina	Division	Building addition	Skip counting all	Valid/Correct
Kai	Addition	Building addition for solution, repeated subtraction to check	Skip counting all and skip counting down	Valid/Correct

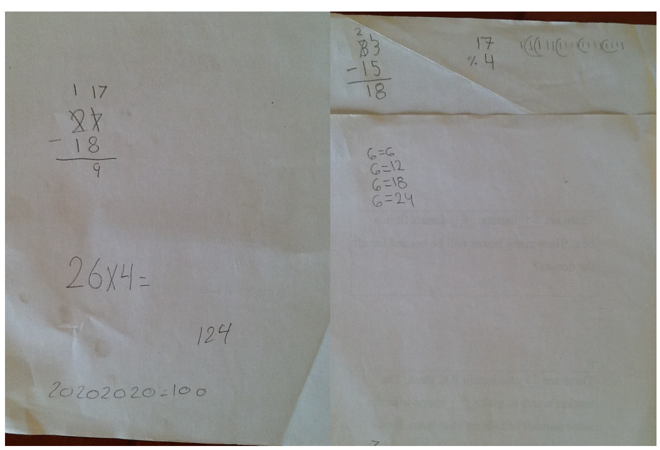

Figure 39. Romeo's work is shown on the left, and Valentina's on the right.

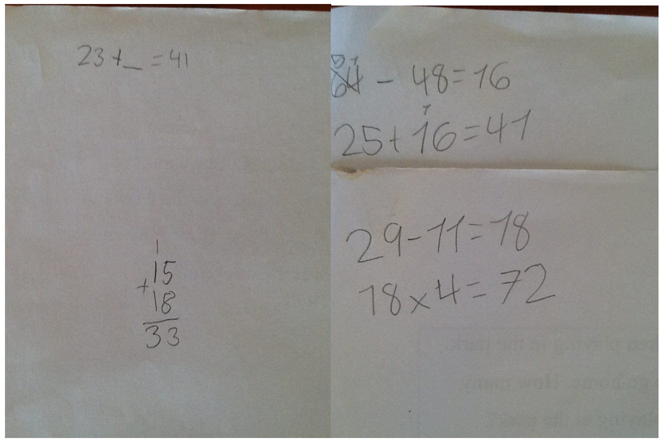

Figure 40. Felix's work is shown on the left, and Victoria's on the right.

<u>Summary of Round 1</u>

In general, the CGI students did not have trouble with the addition and subtraction problems. They were able to target a method and, outside of the occasional small mistake, consistently used the selected method until they arrived at the correct solutions. Students algorithmically solved the addition and subtraction problems by traditional methods of carrying and borrowing in approximately half of the occasions. Multiplication problems showed similar results, but students had more trouble when the problem required them to operate outside of the 12-by-12 multiplication facts that most of them had memorized. Several other strategies became apparent, but the students most commonly used building addition, breaking down the numbers into more manageable values, a style that was reflected in the Montessori materials observed in third grade. By the end of the problem set, strategies began to vary from student to student for the last two division problems. While the students clearly grasped the concept of division, they were not comfortable with the algorithms of standard division techniques. Instead, they employed a variety of methods, including building addition, multiplication fact memorization, repeated subtraction, and missing factor multiplication. However, one student (Valentina) also misidentified a multiplication problem as division. She attempted to draw a picture to model grouping for the situation. Not only was this strategy incorrect, it was indicative of how she would approach future problems in the next set that focused more on division and fractions.

As the material became more difficult, students stopped using algorithms and instead reasoned with modeling or counting methods. Most noticeably, students admitted being uncomfortable with division at this time (October) in the school year. Students and teachers admitted that division was a weakness during the transition between third grade and fourth grade. The results of the two division problems in this first problem set showed that students either obtained the correct answer, or initiated an effective strategy without following through to the end. In the next section, these results are compared to the second round of CGI assessments that took place four months later in the school year.

Second Round of Assessment Implementation

First Question

____ children want to share ____ donuts so that everyone gets the same amount. How much can each child have?

Source: Empson, S., Junk, D. & Turner, E. "Formative Mathematics Assessments for Use in Grades K-3."

This problem is intended to test division strategies, but students proceeded with the multiplication process. This round of questions was administered during a week in which students were working on division and fractions in the classroom. During class, students were learning about fractions with a one in the numerator. Students indicated that either division was not an operation they were comfortable with, or that they were inclined to find alternative strategies to avoid using division. Students used strategies such as counting through multiplication or building factor addition, but their methods continued to be limited by their multiplication facts. For example, Felix quickly answered the problem with 36 donuts among 9 students, but when prompted with a follow-up question of 144÷4, he was not familiar enough to move forward. Victoria was able to answer similar questions correctly with more confidence than she exhibited in the previous problem set. While Valentina and Romeo continued to use multiplication facts with missing factors as their basis of understanding, Kai continued to use building addition as his strategy. Five of the six students correctly answered this problem, and the student who answered incorrectly (Lior) made a mathematics error in the process rather than failing to understand the problem. The results of this division question, shown in Table 14, indicate improvement over the original set.

Table 14
Round 2, Question 1 Results: Partitive Division

Name	Identified Operation	Method	CGI Strategy	Validity/Result
Felix	Multiplication	Multiplication facts recollection	Multiplication	Valid/Correct
Lior	Multiplication	Building addition by an easier factor, then extra to compensate	Estimated skip counting all	Valid/Incorrect
Victoria	Multiplication	Multiplication facts recollection	Multiplication	Valid/Correct
Romeo	Division	Multiplication facts recollection	Multiplication	Valid/Correct
Valentina	Multiplication	Multiplication facts recollection	Multiplication	Valid/Correct
Kai	Multiplication	Building Addition	Skip counting all	Valid/Correct

Second Question

There are ____ chocolate brownies at Nina's party. ____ children want to share the brownies so that everyone gets to eat the same amount of brownies. How much can each child have?

Source: Empson, S., Junk, D. & Turner, E. "Formative Mathematics Assessments for Use in Grades K-3."

This problem also tests division strategies, and students identified and processed the problem the same way as before, but all students were correct this time around. Students who seemed to have less trouble with these problems were then given larger numbers afterwards to see what they could achieve. While Victoria succeeded at both such attempts, Felix found 63÷3 too overwhelming to proceed after successfully solving 56÷7. Similar to the first round of CGI problem set, if the question used values outside of students' multiplication fact knowledge, many students changed their strategy to building addition from the ground up, particularly Kai. The students still avoided performing straight division, but they were able to connect the similarity of multiplication as another method to achieve the same result. Valentina used straightforward algorithmic multiplication to find her answer immediately, and expressed her comfort in the procedure. All six students continued to perform better at division problems as they found the correct solution while reflecting on similar class problems, shown in Table 15.

Table 15
Round 2, Question 2 Results: Partitive Division

Name	Identified Operation	Method	CGI Strategy	Result
Felix	Multiplication	Multiplication facts	Multiplication	Valid/Correct
Lior	Multiplication	Building addition	Skip counting all	Valid/Correct
Victoria	Multiplication	Multiplication facts	Multiplication	Valid/Correct
Romeo	Division	Building addition	Skip counting all	Valid/Correct
Valentina	Division	Vertical multiplication with carrying representing grouping	Written form of direct modeling partitive division	Valid/Correct

Table 15 continued

Name	Identified Operation	Method	CGI Strategy	Validity/Result
Kai	Multiplication	Building addition	Skip counting all	Valid/Correct

Third Question

Robin went to a party where each person ate ____ of a pizza. If ____ people ate pizza, how many pizzas were there in all so that they each got to eat ____ of a pizza and there were no leftover pieces?

Source: Empson, S., Junk, D. & Turner, E. "Formative Mathematics Assessments for Use in Grades K-3."

This problem may have been the most challenging for the students. Fractions were used to imply division in this question, and fractions appeared to help students identify this problem as division with greater success than they had in prior situations. While it was not a straightforward division problem like the first and second questions, the one in the numerator of the provided fraction allowed students to think in terms of division. Students said that, to this point, the only fractions they had covered in class had ones in the numerators. One obstacle that was discussed with most students was their automatic assumption that pizzas always have eight slices, and that this fact helped them in the problem. Once this extraneous information was established as irrelevant to the problem, students expressed different ways of thinking for this fraction-based problem, particularly those that solved it correctly. For example, Valentina quickly decided to draw a circle to represent each pizza, and then divided it into the appropriate fraction (in her case, fourths). She then meaningfully used the picture by physically counting each piece that represented each person, as shown in Figure 41. Kai also used the concept of material objects, using his fingers to count by three to represent the people each eating ⅓ of a pizza, rather than writing it down. Both of these strategies involved visualization rather than algorithms or formulas to achieve the correct answer. The students created their own objects to use in a difficult, unfamiliar scenario. Victoria was quick to realize that 24 people, each eating ⅓ of a pizza, could be represented as 24÷3. She said, out loud, that this problem used her division facts rather than multiplication facts, but when pushed to do the problem again with ⅔ as the fraction instead, she admitted less confidence with anything besides a one in the numerator. However, she proceeded with the number sense to realize the solution should be doubled, and correctly answered 16. Felix and Lior struggled to start on the problem, but persevered for nearly ten minutes, despite admitted weakness in the topic. This problem resulted in a three-three split on the final results that are summarized in Table 16.

Table 16
Round 2, Question 3 Results: Measurement with Fractions

Name	Identified Operation	Method	CGI Strategy	Result
Felix	Subtraction	Repeated subtraction, miscounted in process	Skip counting down	Valid/Incorrect
Lior	Division	Building addition	Skip counting all	Valid/Incorrect
Victoria	Division	Division facts	Memorization	Valid/Correct
Romeo	Multiplication	Algorithmic multiplication	Algorithmic	Valid/Incorrect
Valentina	Division	Draw a picture	Written form of direct modeling for grouping	Valid/Correct
Kai	Addition	Building addition through finger counting	Counting all	Valid/Correct

Fourth question

Okhee has a snowcone machine. It takes ____ of a cup of ice to make a snowcone. How many snowcones can Okhee make with ____ cups of ice?

Source: Empson, S., Junk, D. & Turner, E. "Formative Mathematics Assessments for Use in Grades K-3."

This problem stresses fraction-based strategies. In this problem, students should differ from the third problem by identifying multiplication as the appropriate method. Three students assumed that the presence of a fraction automatically meant division was necessary to solve the problem. Felix began correctly this time, but erred in his reasoning. He said, "¼ is kind of like a quarter, if it was full of ice then it would be three quarters, and then times 20 [cups]." Because of this, he was short on his final solution as he did not account for the full 4/4 of the cup of ice and only provided ¾ of what he presumed was the correct answer. Lior and Victoria identified the problem as division due to the inclusion of fractions. Values were chosen that easily divided to see if students would select the method because it "looked right" rather than made sense, and these two students selected division fairly quickly. Lior and Victoria then analyzed the problem and strictly took the fractional part of all snowcones, arriving at an incorrect solution that was extremely small and did not make sense in context. Victoria, who had performed strongly in both problem sets to this point, looked at her solution with a somewhat confused expression that showed uncertainty, but moved on into the next question. Romeo and Kai each ultimately decided to use multiplication for their solution. Romeo realized this strategy immediately by exclaiming, "that's a lot of snowcones!" Kai took longer, at first saying "I think this is division." As shown in Figure 41, Valentina again preferred to draw a picture, this time cutting each shape into fifths. She did not hesitate or confuse the strategy with the slightly different pizza problem, and proceeded with a completely different mindset. Whereas before she drew pizzas as necessary until she reached her goal, this time she knew to draw 15 boxes before cutting them into fifths. Romeo, Kai, and Valentina provided the three correct solutions, as shown in Table 17.

Table 17

Round 2, Question 4 Results: Partitive with Fractions

Name	Identified Operation	Method	CGI Strategy	Result
Felix	Multiplication	Algorithmic multiplication	Algorithmic	Valid/Incorrect
Lior	Division	Division facts	Memorization	Invalid/Incorrect
Victoria	Division	Division facts	Memorization	Invalid/Incorrect
Romeo	Multiplication	Algorithmic multiplication	Algorithmic	Valid/Correct
Valentina	Multiplication	Draw a picture	Written form of direct modeling for grouping	Valid/Correct
Kai	Division	Recollection of multiplication facts	Multiplication	Correct

Fifth Question

Jorge and Darren are eating brownies that are the same size. Jorge cut his brownie into 3 equal pieces and ate 1 piece. Darren cut his brownie into 12 equal pieces. He wants to eat exactly as much brownie as Jorge. Color in the amount of brownie Darren should eat, so that his share is equal to Jorge's share. *Note: Here, two images were provided, where Jorge's was already split up and shaded, and Darren's was already split up, requiring shading.*

Source: Empson, S., Junk, D. & Turner, E. "Formative Mathematics Assessments for Use in Grades K-3."

Students did not have trouble with this question, which pertained to fractions and provided pictures as part of the question. Whereas the third problem took longer for many students, and the fraction problems in general required some use of counting fingers or drawing objects, this problem included the diagram for students to use. The provided visual aid prompted all students to identify the problem as division. All students were accurate by the end of their methods, and four of them proceeded the same way. Students acknowledged that twelve split into three parts results in four per part. Some students knew that 12÷3 was 4, and some said out loud that 1/3 of 12 was 4. Kai continued his fairly consistent method of building addition, counting by factors until he reached his goal. Felix was the only student that appeared to visually guess by measuring the width of the shaded rectangle between his index finger and thumb without mathematics, then moving to the second picture, lining up his fingers, and stating that it could be four or five given the estimated measurement, before finally resting on the correct final answer of four. Valentina was asked if she was glad the picture was already present in the question, or if the problem would be different without it. She responded that without the picture the problem would have been harder, but she could easily draw a similar picture. She continued by revealing that whenever she sees a division problem, she always strives to make it multiplication instead. The results are shown in Table 18.

Table 18
Round 2, Question 5 Results: Measurement Division with Equivalent Fractions

Name	Identified Operation	Method	CGI Strategy	Result
Felix	Division	Use of picture/estimating portion size with thumb, index finger	Guessing	Invalid/Correct
Lior	Division	Fractional parts knowledge	Memorization	Valid/Correct
Victoria	Division	Division facts	Memorization	Valid/Correct
Romeo	Division	Division facts	Memorization	Valid/Correct
Valentina	Division	Multiplication facts	Multiplication	Valid/Correct
Kai	Division	Building addition	Skip counting all	Valid/Correct

Sixth Question

Jane says that if 6 people are sharing 10 cookies each person gets 1 and 2/3 cookies. John says that each person should get 1 and 4/6 cookies. Who is right? Can they both be right?

Source: Empson, S., Junk, D. & Turner, E. "Formative Mathematics Assessments for Use in Grades K-3."

The results of this final problem confirmed that students had the ability to understand what fractions represent. Five of the six students quickly decided that the students could each have one cookie but not quite two. They identified the fact that the remaining four cookies would be split six ways, and believed John's statement to be correct with 4/6 as a representation of this idea. Felix appeared to only select John's statement because it was given in fraction form for him, and otherwise could not have derived the form himself. The other four students who selected John's statement knew that 4/6 was correct because they worked it out independently of the provided statements. Valentina thought that ten people were unable to share six objects. She struggled for a long time, going back and forth with her thoughts and ideas, and Figure 41 shows why this was the case: she had a very difficult time figuring out how to split apart the images with the given information, particularly because of the non-integer final solutions. "This one is hard to draw," she said after nearly ten minutes of silently drawing and erasing. Kai confessed that division was not something he felt he was good at, and guessed, "I think they're both wrong." While many recognized 4/6 as a possibility, no student recognized that 2/3 was the same as 4/6, which was one of the goals for this type of question. While they realized that fractions represented division, they were not strong in fraction equivalency. While students said that four cookies could be split among six people, discussing two cookies split among three people sounded either unrelated or impossible to them. Students also confirmed they were more familiar with remainders than fraction notation to express leftovers in a division problem. No one correctly identified Jane as also being correct in this situation. In fact, many students did not simply state they were unsure of Jane's accuracy, but instead showed confidence in denouncing her answer as incorrect. Students were therefore partially, rather than fully, accurate with their final answer, as shown in Table 19.

Table 19
Round 2, Question 6 Results: Measurement Division with Equivalent Fractions

Name	Identified Operation	Method	CGI Strategy	Result
Felix	Division	Confirming fractions given in the problem	Guessing	Invalid/Partially Correct
Lior	Division	Using remainders to form fractions	Measurement division	Valid/Partially Correct
Victoria	Division	Using remainders to form fractions	Measurement division	Valid/Partially Correct
Romeo	Division	Using remainders to form fractions	Measurement division	Valid/Partially Correct
Valentina	Division	Draw a picture	Written form of direct modeling for grouping	Invalid/Incorrect
Kai	Division	Guessing	Guessing	Invalid/Partially Correct

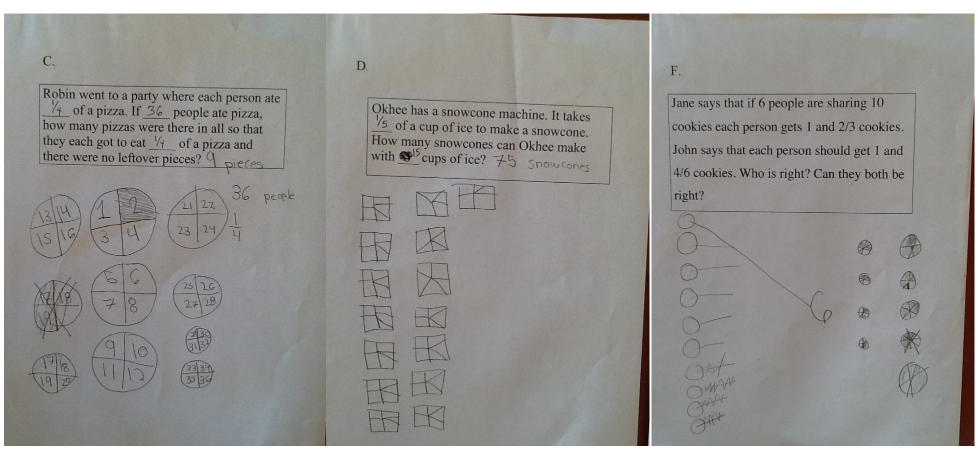

Figure 41. Valentina's approach when she worked on fraction-focused problems.

Summary of Round 2

Students showed more confidence in understanding division than understanding fractions. Students strategized a variety of ways to achieve their goal in division problems, which showed conceptual understanding rather than simply working through an algorithm. Their comments reflected their knowledge of why they were choosing that particular strategy. The students who leaned toward manipulatives did so more during the second round of problems because they covered more unfamiliar topics such as division and fractions. The students that downplayed the importance of manipulatives to their learning struggled more with how to proceed on these new, challenging concepts. Students were more able to identify problems as division in the second round than they were in the first round. The overall results of the division-based questions were better in the second round than in the first round, and students declared fractions a new concept both to themselves and in the classroom curriculum. Operations with fractions appeared to be a concept that students could mostly understand as a context for division. Fraction equivalency was present in the final problem and students not only failed to recognize the equivalency, but also denounced the proposed equivalent fraction as incorrect. If an illustration was present in the question, students did not have much issue with the problem, although one student still relied on using his fingers to measure the distance rather than calculating fractional portions. Sometimes students thought that the presence of a fraction automatically meant division was the appropriate strategy, and even one of the stronger students fell into this assumption when the problem actually called for multiplication based on the correct interpretation of the situation.

Overall Summary of Results

The final results of both CGI problem sets are shown in Table 20. The results are separated into the two rounds. The first round focused more on addition, subtraction, and multiplication, with hints at division despite students acknowledging lack of confidence, and the second round focused on fractions, division, and multiplication. In the second round, fractions appeared conceptually solid for the students, but operations including fractions were more inconsistent. This mirrored the results in the first round, where students appeared to understand the foundations of division, but the strategies involved were more inconsistent. During the second round, students appeared to have a better grasp of division than before. The total score was given to show overall results.

Table 20
CGI Results for Both Rounds

Name	1st Round	2nd Round	Total Score
Felix	4 out of 6	3.5 out of 6	7.5 out of 12
Lior	4 out of 6	2.5 out of 6	6.5 out of 12
Victoria	5 out of 6	4.5 out of 6	9.5 out of 12
Romeo	6 out of 6	4.5 out of 6	10.5 out of 12
Valentina*	5 out of 6	5 out of 6	10 out of 12
Kai*	2 out of 6	5.5 out of 6	7.5 out of 12

*These students showed a preference toward using objects, materials, or pictures as a visual aid.

Overall, the students did fairly well on these assessments. Out of the six students tested, four of them did better on the first round than the second round. Two students did not experience lower scores in the second round, and these were also the students that expressed the most interest in visual aids such as pictures or objects. Kai had earlier expressed in interviews that he liked both manipulatives and handwriting equally, but many times during the assessments he relied on tapping his pencil, counting his fingers, or thinking in groups—all of which represent physical objects in his mind. His improved scores may show that these skills are helpful in learning new, complicated concepts.

Similarly, Valentina had previously described her feelings toward manipulatives as helpful in the past, but using them repetitively became so “boring” and “annoying” that she preferred handwriting in fourth grade. However, more than any other student in this assessment, Valentina preferred drawing pictures to help with many of the second round division and fraction problems that she admitted were more difficult. When she felt uncomfortable with a problem, she reverted back to visualization methods, and when she already knew how to approach a problem, she used algorithmic handwriting methods. Valentina particularly struggled when she could not identify a way to illustrate a difficult problem through direct modeling. This may have been one of the more significant conclusions from the second set of problem-solving questions, and it may best serve in understanding more about how these students experience the manipulatives-to-handwriting stage of their transition process in the classroom from third grade to fourth grade. In short, the students who preferred materials and direct modeled the situation either improved or maintained success when the topics covered division and fractions, which many students perceived as the most difficult problems. Ms. Vicky was observed teaching fourth grade students the long division algorithm throughout the year, such as lessons observed during October and December, so it is natural that students would invent their own strategies based on their general inexperience with division. Students also showed that they could fairly consistently choose relevant, or valid, strategies to work on the problems. With more difficult material, students struggled to fully use those strategies to the correct solution. Students conceptually understood a large amount of the problems, and often identified valid strategies to move forward, but struggled more when relying on an algorithm rather than creating the problem how it made sense to them instead of how they believed they were expected to perform.

V. CONCLUSIONS, LIMITATIONS, AND FUTURE WORK

Summarizing the Research Findings

This chapter summarizes the research findings, draws important conclusions pertaining to the research questions, discusses any limitations contained in the study, and considers the broader impact to the mathematics education research agenda domestically and globally. Based on classroom observations at this school, the third grade classroom had several aspects that aligned to a theoretical Montessori approach, and the fourth grade classroom appeared to represent a more traditional, direct-instruction based classroom. The three aspects expected to cause a perceived difficult transition were 1) the discovery methods and class structure, 2) the use of materials versus handwriting, and 3) the shifting roles of students and teachers. These important aspects are addressed in the research questions, which are as follows:

1. To what teaching practices and learning opportunities are third and fourth grade students exposed? To what extent are these practices and learning opportunities related to the Montessori approach?
2. How are three particular aspects of current teaching practices and learning opportunities in fourth grade mathematics perceived by students and teachers compared to previous exposure in the Montessori method?
3. How, and to what extent, does changing the teaching practices and learning opportunities affect the problem-solving strategies of students?

It is appropriate to begin the conclusions by covering the aspects that were hypothesized as major turning points during the transition. The perceptions by students and teachers regarding the transition will become clear as these aspects are explored. The interpretation of the results of the problem-solving sessions will be an appropriate conclusion to this discussion. Afterward, the context of this study will be addressed within the broader scope of mathematics education, along with admitted limitations to the study and how the results could be used in the future.

The First Aspect: Changing the Pace and Structure

The students experienced a substantial change in classroom structure and pacing when they left third grade and entered fourth grade. While the third grade classroom exemplified much of the Montessori values and philosophies, the fourth grade classroom was structured very differently, especially when considering the curriculum development changes from the first two months. The third grade students experienced an exploratory style of education that required a certain amount of work to keep students on track, but did not accelerate nor hinder the academic progress of any student based on ability. First, second, and third grade students were mixed in the same classroom. If a student needed to repeatedly review a particular topic, their classwork could reflect that concept alone while fellow classmates advanced further. This allowed students the freedom to cover topics at their own pace, in ways that made sense to them individually. The teachers continually stressed that the division of students in terms of content was solely based upon ability rather than grade level or age. One major divergence from the Montessori system was the inclusion of a weekly mini-lesson for each subject. While the lesson groups were small, this idea is not normally part of the Montessori style. By definition, lessons are not considered characteristic of the typical Montessori curriculum, which instead allows the student to consistently take the lead. Also, the teachers instituted the follow-up strategy to provide students more short-term goals. The addition of follow-up assessments and weekly homework and classwork goals also kept the students moving forward at a pace that was not entirely set by their own motivation. Students were encouraged to work in groups and assist others; occasionally even tutoring their classmates on topics they felt most confident in. This group mentality was fully encouraged from the morning meeting, to various lessons throughout the classroom, to follow-ups that showcased what students had learned as a form of assessment. The concept of teamwork and encouraged dialogue is strongly representative of the typical Montessori style, and this was one of the most consistent aspects observed in the classroom.

Comparatively, fourth grade students were directly instructed, together as a unit, and teachers implemented lessons consecutively and repetitively to establish a routine for student practice and exposure. Lessons were delivered to the entire class at the beginning of the period, and afterward students were expected to drill and repeat the strategies they observed during the lesson. Teachers described differentiation as being diminished in the fourth grade classrooms, and for students the classwork was focused on the individual rather than the group. Students were not actively encouraged to help others, and instead more emphasis was placed on advancing students' own prerogatives. Observations also showed that students were called on individually to share their solutions or strategies, rather than communicating in pairings or groups. These descriptions are consistent with the definition in the literature for a direct-instruction style classroom (Zhbanova et al., 2010), which uses routine teacher lectures and student written practice as the consistent structure of each day.

The pacing and overall style of learning clearly changed from third grade to fourth grade at this school. Traditionally in the Montessori method, students focus on group work, often helping their peers and engaging in productive struggle to obtain a stronger feeling of achievement and discovery toward a new topic. Many times, third grade teachers encouraged students to help others learn a new topic, but fourth grade teachers separated students, sometimes even with folders, to prevent any collaboration. The third grade classroom provided students a way to work through topics one at a time, with a chance to build a foundation that made sense to them, as the Montessori literature suggests (Varnin, 2003; Lunenburg, 2011). While typical Montessori methods provide minimal structure in terms of what needs to be achieved, a certain workload was expected in the third grade classrooms at this school. Fourth grade students expressed that the workload was not particularly heavier than third grade, and even when there was more to accomplish, they did not provide negative responses to the new amount of work they needed to complete. In fact, many students appreciated more the direct instruction and leadership because they perceived they were learning more and progressing at a faster rate than they did in third grade. The limited inclusion of lessons, assessments, due dates, and written assignments in the Montessori-aligned classroom appeared to have some preparatory effects on the fourth grade students, as many of them felt the workload may have changed in fourth grade, but not significantly nor enough to induce stress. Research shows that if students know what is expected of them, they can grow and learn more in the situation (Zimmerman, 2001). Simultaneously, the addition of non-Montessori traits into the third grade classroom lessened the Montessori atmosphere.

Differentiation was not emphasized in the fourth grade classroom, but student responses suggested that they did not feel too shy or intimidated to ask the teacher questions repeatedly during the initial lesson if they did not understand. Some students were bold enough to inquire in front of their peers, rather than waiting until the end of the lesson. At the same time, the Montessori program was clearly effective at instilling in students a sense of individual pride in their work. This was evident from students' repeated expressions of their desire to work on their own and to persevere through the challenging times. The most striking conclusion was the extent to which students were aware of their own shortcomings and strengths, with a genuine desire to better themselves as learners, confirming their status as self-motivated learners as the literature suggests (Jennett, 1992). By acting this way, students showed awareness of their pacing and awareness of how successfully they were moving through the assignments and general curriculum. Group work still appeared occasionally in the fourth grade classroom, which is not typical of a direct-instruction style, but mostly took place at the beginning of the year as the teachers solidified the structure of the class. However, individualism was stressed much more in fourth grade than in the Montessori style in third grade, which was a major difference for students during the transition. Facilitated conversation and cooperation were always present in the third grade classrooms, and are generally considered characteristic of an effective learning environment (Checkley, 2006; Franke et al., 2007). During interviews, students did not express intimidation after comparing themselves to other students, which suggested that the pacing did not bother the students—they were not trying to compete with their classmates. Students seemed to enjoy the direct scheduling of assignment completion, and most students responded positively about this type of structure and schedule. The fourth grade teachers often debated the level of direct instruction required for the class, but the overwhelming tendency was for more direct instruction rather than allowing an exploratory environment. The two fourth grade teachers each believed that the lack of differentiation was a problem, but they disagreed in terms of which students were most detrimentally affected—students with low achievement performance or high achievement performance. Observations showed students worked in pairs or groups far more often in third grade, confirmed by fourth grade students in interviews. Many of the fourth grade students spoke positively about the freedom to avoid group work if they chose, but others had very mixed opinions about their willingness to work alone before asking a friend or the teacher. The group versus the individual was an important factor because of the implications toward the different structures for students.

Third and fourth grade classrooms each mostly aligned with their respective theoretical instructional modalities: Montessori (third grade) focused on the group and eventual learning, and traditional (fourth grade) focused on promoting the individual with a more rigid schedule and heavier workload. Promoting individuality in the traditional classroom was observed less often than expected from the literature (especially early on), which was a slight deviation from the source material. However, the individualism was still stressed much more than in the Montessori classrooms, and students did not express a common sentiment toward this change. Fourth grade had a far stricter day-to-day schedule compared to third grade, and mathematics was the most routine of all subjects, according to the fourth grade teachers. The fourth grade lead teacher believed that mathematics students benefit most from a repetitive, traditional, non-Montessori classroom, which was why mathematics was always presented in the exact same structure every day.

The findings led to a few conclusions about the influence of the pacing and structure aspect on the struggles in transitioning from third grade to fourth grade. At the very beginning of the school year, the structure of the fourth grade classroom was not different enough from the third grade classroom to warrant a comparison of this aspect. However, the teachers came to an agreement in allowing a direct-instruction approach to be the dominant form of teaching, specifically for mathematics. This aspect then became much more prominent, and the structure differed from the Montessori approach to a higher degree. Traditional styles of education typically lead to a less personalized classroom, one that puts the teacher in absolute control (Eccles et al., 1993). The lecture and workbook style did not vary despite the spectrum of abilities that students portrayed in the classroom, but this lack of differentiation is not considered a major aspect of this study. The goal of the school's Montessori style was to provide students the ability to be aware of their own needs in the classroom, and many of the students' remarks expressed this ability in a positive light. Many students were confident in continuing a workload that they were only somewhat accustomed to after the edited Montessori environment in third grade. According to the students, the classwork and homework loads were not overwhelming, and many expressed the confidence to ask questions in front of others if they felt behind. Because students expressed no concern with helping other students or feeling held back or hurried, the structure and pacing aspect did not strongly influence the transition from third grade to fourth grade. In fact, students were impressive with their ability to self-regulate and push themselves forward if they struggled with a particular topic. This idea leads to another aspect, in which fourth grade students found themselves in a role very different than they were used to in the Montessori program. The changing roles will be addressed in a later section of this chapter.

The Second Aspect: Taking Away the Montessori Materials

Observations in the third grade classroom showed that objects and materials were an essential part of the Montessori experience, but they were not the only learning method. First, second, and third grade students used the manipulatives in the classroom as the original source of knowledge, but third grade students were also deemed responsible for transitioning their learning into handwritten methods. According to the literature, connecting from the concrete manipulatives to the abstract symbols is a process that usually involves drawing visual aids as a transitional stage; the use of manipulatives themselves is more for mathematical understanding rather than algorithmic proficiency (Stein & Bovelino, 2001). Students were required to copy over much of their material-based assignments into written format, and eventually handwriting was the primary method they were expected to utilize. The goal of the handwriting stage was ultimately to use symbols for mathematics operations, instead of copying what had already been accomplished through manipulatives. “In fact, mathematics can be said to be *about* levels of representation, which build on one another as he mathematical ideas become more abstract” (Kilpatrick et al., 2001, p.21). However, many third grade students complained they had to learn material twice, because they had to understand the instructions of the activity before also learning the mathematics involved in the lesson. They believed that the goal in later mathematics classes was to handwrite, and so they saw the value in practicing handwriting in third grade—they viewed manipulatives, at times, as a hindrance to their future goals of mastering handwriting quickly. To the students, manipulatives were not gradually phased out as concepts became more abstract, but were instead replaced as more efficient. Third grade comments were mixed concerning the use of manipulatives, in which half of the class preferred manipulatives and the other half preferred to write their work. Third grade students conceptually organized their information with a manipulatives-only approach, followed by handwriting, which they observed from middle and high school students in the upper grades. The fourth grade students had more extreme opinions about the Montessori materials. Many of their comments were negative toward manipulatives, stating they did not miss using Montessori materials. While a few students appreciated the use of manipulatives, the vast majority of fourth grade students believed they achieved more in class when not required to use them. As a result, many of the fourth grade students said they preferred fourth grade because they were not required to use materials, and also said they could more efficiently learn as a result. One of the purposes of manipulatives is for students to gain confidence by coming up with methods to achieve solutions, but when students neared the handwriting stage, they were persuaded to follow new algorithms. Nearly all of the early remarks by teachers and administration included the diminishing use of Montessori materials as a primary source of conflict. However, only a few fourth grade students reflected upon materials positively, and many stated they preferred the new methods of pencil-and-paper, individual work, and direct teacher instruction to the methods they experienced in Montessori. Students may have felt that teachers would eventually give them the fastest method regardless, so they were happy to have the opportunity to approach the teacher in the fourth grade classroom.

While student comments about manipulatives were mixed in the third grade classroom, fourth grade heavily favored the handwriting methods. However, third and fourth grade students who felt negatively toward mathematics and found the subject more difficult voiced their desire to continue using materials rather than handwriting. Also, many were willing to admit that manipulatives were helpful in the past, but now felt they were ready to move on without them. Once the students believed they had mastered their activities, they were not as studious and focused when completing them, particularly when manipulatives were included. When the students found the exercises uninteresting, the activities no longer made as much of an impact toward their learning, which is supported by the literature (Boekaerts, 2002). Also, when encountering new and less comfortable concepts, some of the students were more prone to draw pictures or simulate counting objects with their fingers, pencil eraser, or imaginary items. Many of the fourth grade comments indicated that handwritten work takes less time, often because they felt using the materials required effort in learning how to perform that method before being allowed to move on and learn the handwritten method. Many students perceived handwriting as more mature and helpful, while also less boring and repetitive. Other comments indicated that handwriting made learning mathematics easier, which could be interpreted to mean they enjoyed learning an algorithm and working through those motions rather than taking the time to explore and discover the ideas for themselves. This lack of curiosity does not promote successful problem solving (Rusczyk, 2015). Other students admitted that the new topics and methods in fourth grade were challenging, but further clarified their statements by saying the challenge was best for them as a learner. Some students remarked that they needed to work through more difficult problems to be a balanced learner, and improving on their methods made the overall subject less boring and more fun.

The fourth grade students were the focus for the actual transition stage in this study, and they often reflected on their experiences in the Montessori third grade classrooms. Many teachers and administrators expected the diminishing use of manipulatives to be a very influential aspect of the difficult transition. In fact, manipulatives did have an impact on the students' perception of learning mathematics, but in a different way than the educators in the study believed. Teachers believed students faced a difficult challenge in the move from concrete mathematics learning to abstract mathematics learning. Instead, students actually valued the decrease in material usage in the classroom from the first day of fourth grade. Often, students expressed their displeasure in being required to connect the two different methods (one with materials and one without) for the same task. The use of materials is usually continued through fifth grade in typical Montessori classrooms. While the debate between the two fourth grade teachers in this study on what makes an effective direct methods classroom reflected the same debate in the literature, students appeared to prefer handwriting to using objects. Students did not enjoy the requirement of interpreting rules and instruction for manipulative-based activities to help them learn abstract concepts—they perceived mathematics positively despite, or even in response to, manipulatives' absence in the classroom. Only a few students described missing the presence of mathematics materials. Further, the students who consistently found mathematics difficult and struggled with new concepts voiced their desire to lean toward using manipulatives. Many students, in third and fourth grade, said that they got tired or frustrated from using manipulatives repetitively once they had already achieved success in a concept—they no longer found value in the exercises, which is considered vital for learning mathematics (Bahar & Maker, 2015).

One of the third grade teachers agreed with students that there was too much repetition in the Montessori curriculum, requiring students to continually remain on the same content rather than pushing forward, which left less time for students to proceed further into more difficult topics. Teachers believed that manipulatives were effectively used to master the abstract before moving on to handwriting methods, but the students found manipulatives less engaging long before reaching this stage. Further, the CGI problem-solving assessments showed evidence that over half of the students assessed simply worked algorithmically toward the solution, with little hesitation once they had identified which operation should be used. This tendency showed a comfort level with certain problem scenarios, but division problems still brought out the largest variety of strategies from these students because of the unfamiliarity. For example, instead of working through a typical division problem algorithmically, they still chose to use building addition, repeated subtraction, or multiplication strategies. The second round of CGI assessments showed student improvement in division once they had practiced their strategies enough in the classroom to achieve better understanding. No student had memorized the long division process at the time of the CGI exercises, and the students instead used strategies that made sense to them; this is the sign of a strong mathematics learner (Kloosterman & Stage, 1992; Kaur & Toh, 2011).

Many observations showed that third grade students only used objects as they worked through their learning process, and initial assessments and evaluations were given strictly in this form. In the fourth grade classroom, note taking and handwriting every problem became the standard. For this particular school, the third graders experienced much more handwriting than is usual in a Montessori program. Several researchers have found handwriting to be essential for learning mathematics (Wigfield & Eccles, 1994; Boekaerts, 2002). While manipulatives still formed the basis of students' learning, they did not continue past the topic until they also learned the way to formulate handwritten tasks covering the same content—intended to be their only strategy moving forward. Teachers appeared to see value in the method of using objects first and then handwriting repeatedly within a concept, but the students themselves found the strategy repetitive and inefficient. These student opinions provided evidence that students were educated in a system that gave them much of the power in the classroom, leading to more drive and awareness of the learning experience—an important goal of the Montessori method. Some fourth grade students believed they had to work double to move past a mathematics topic, and they valued being in fourth grade because they could avoid what they perceived as pointless extra work. The words and phrases used by these students did not suggest laziness or a low work ethic, but rather that they were aware of their own priorities and efficiencies. The students placed emphasis on the importance of mathematics as a subject, and wanted to further challenge themselves, a trait that the literature suggests is important for learners (Bahar & Maker, 2015).

This evidence supports a claim that while manipulatives served their purpose initially, the students were more ready to move on from them than administration and teachers realized or planned. The three stages of using manipulatives are the concrete stage (materials), the representational stage (drawing), and the abstract stage (symbolic representation). Students and teachers confirmed in interviews that the materials were helpful with early problems, but quickly lost their appeal upon repetition. Students only achieve mathematical understanding if they use the manipulatives as a tool rather than a requirement (Cope, 2015). In the third grade curriculum, students were usually expected to make the leap from the concrete stage to the abstract stage when handwriting problems. However, results of the CGI problem-solving assessments showed that students naturally reverted back to some version of visualization to tackle a new problem when the topic was less familiar, such as division or fractions. This tendency showed students' natural inclination to use the representational stage that was not emphasized during the process of moving to handwriting skills. Fourth grade student comments also reflected this fallback, as those who had more negative emotions toward mathematics exhibited a preference for continuing manipulatives. Students need to see value in what they are learning to maximize the activity (Wigfield & Eccles, 1994; Boekaerts, 2002). Students appeared to prefer algorithmic calculation in nearly all cases for addition, subtraction, and multiplication, because they were past the manipulatives stage. Student comments reflected that these operations were the most repetitive with manipulatives. While many students considered the manipulatives inefficient after a certain point, they consistently saw the value in handwriting to prepare for future mathematics classes. The six students who participated in the CGI assessments admitted that addition, subtraction, and multiplication were much easier than division and fractions. For example, five of the six students, in the first round of assessments, consistently identified the appropriate operation and worked out the mathematics by hand. Only one student relied on basic drawing of pictures and finger counting when the content was difficult. However, students used these visualization methods more often in the second round of problems, when two students, who admittedly struggled more with division and fractions, used these more creative methods to approach the problems. These two students, who relied on pictures and counting imaginary objects, either improved or were consistent in their second round scores compared to their first round scores. In fact, these students did not face as many challenges in the second round as the students who quickly used patterns and algorithms. These results may indicate that students had a solid foundation with the concepts from the first round, past the point of needing help from materials. However, when more recent topics were addressed, such as division and fractions, the findings were consistent with prior studies, in which students relied on their knowledge of manipulative strategies to identify appropriate methods for new concepts moving forward (Carpenter et al., 1996). For example, some students wanted to revert to the process of using the concrete materials for the more unfamiliar concepts, such as division and fractions. Third grade teachers confirmed that these topics were given less time and formed the weaker portion of the third grade curriculum. Their comments suggested that reducing the amount of time spent on the first three operations (addition, subtraction, and multiplication) to assure students still found value from their practice would be beneficial, by allowing more time for difficult topics such as division or fractions in preparation of fourth grade. Topics should be available to students based on readiness, not simply as part of a checklist (Lesh & Harel, 2003).

Manipulatives were a strong presence in the Montessori classroom, but did not affect the transition period in the way that many hypothesized at the beginning of the school year. In fact, three of the five educators interviewed at the beginning of the study stated that the disappearance of manipulatives was probably the most crucial issue in the transition from third grade to fourth grade. Rather than students' inability to handle the lack of materials, much of the evidence expressed through student and teacher comments pointed to an overuse of manipulatives at certain stages near the end of the third grade year. Students need to build upon what they learn to grow mathematically (D'Ambrosio, 1987), and the repetitive methods did not reflect this value. The style of learning in third grade at this school did align to the Montessori method in many ways, and the inclusion of manipulatives should continue to be part of that environment. However, because the third grade teachers wanted to prepare third grade students for their fourth grade traditional experience, the teachers implemented the handwriting phase early in anticipation of future transition problems. The students often expressed their appreciation for handwriting methods, which supported the teachers' decision. The best future course of action for this school is to limit the number of activities that third grade students are required to complete. Students' understanding should dictate their places in the curriculum, in anticipation of practicing more challenging concepts in the future. Manipulatives should remain part of the third grade curriculum as a cornerstone of the Montessori style of teaching. There should be more focus on the connection between using the manipulatives and the procedural development they were taught in handwritten exercises. Drawing pictures is a version of this bridge, but was a method not implemented often in the classroom despite students using it during difficult CGI problems. Similarly, the direct classroom need not include manipulatives to help ease the transition period. Instead, in the same way that a Montessori approach should adjust to the student, the third grade curriculum should adjust to give students more expansive use of different materials in new and creative ways. Varying the quantity and depth of the activities should prevent students' apparent burnout from using manipulatives. In summary, the Montessori materials' effect on the transition had a large impact, but not in the way educators at the school expected.

The Third Aspect: Reversing the Teacher and Student Roles

The third aspect involved the reversal of student and teacher roles as the students moved out of a Montessori style and entered a direct teaching style. In the third grade classroom, evidence showed that the implemented Montessori program provided students the opportunity to control the pace of their education and to approach the teacher for help as necessary. Meanwhile, teachers drifted around the room and, outside of the brief lessons once per week, never lingered for an extended period of time with one particular student or group of students. According to the literature, the teacher's role in a Montessori classroom is defined as a guide, not an instructor (Montessori, 1964), but this school deviated from the source material when teachers also took on the additional responsibility of providing brief lessons once per week. These lessons should continue to have a place in the classroom to both prepare students for fourth grade and provide some structure for young students. Still, educators at the school should follow the advice of Koestner et al. (1984), in that mathematics lessons should be carefully constructed to not impede student creativity and discovery. The majority of classes showed teachers creating a significant Montessori learning environment, where they took a secondary role in the class and students were allowed to approach them for help only when the student needed it, as the literature suggests (Fang, 2008). When students did not ask for assistance, they completely reached their own conclusions through discovery involving other students and repeated efforts to master a particular concept. Interviews with the Montessori teachers showed that they consciously decided to draw back and not become too involved, because their natural tendency was to take an authoritative teaching role. By moving confidently forward in the guide-role, the third grade teachers maintained their roles in fostering student-led learning very well, and students actively gained the ability to be both self-motivated and aware of the importance of their education. Students also showed characteristics of an improving learner, by openly admitting their own strengths and weaknesses.

Once the students entered fourth grade, there was a reversal of roles and structure. Due to the changing schedule, students initially experienced different teachers with drastically different teaching styles on alternating days. One teacher used a direct form of instruction three days per week (that eventually became five days per week), with a thirty to forty-five minutes mathematics lesson to start the day. This schedule included warm-ups, homework review, new material, and group or individual practice. Alternatively, the other teacher tried to instill more Montessori values into the fourth grade classroom through the use of activities and games. Both teachers assumed leadership roles in the classroom, in which they were sources of information for students, as direct-instruction styles dictate (Copes & Shager, 2003). While students appreciated both styles, they reacted more positively to the direct teaching style, because they felt it was more direct, efficient, and productive. Observations showed that teachers struggled to settle into their roles and to define what they were looking for in the classroom during the first two months of the school year. Teachers often need to change their instruction methods according to the students involved (Carpenter et al., 1996). Student comments revealed that they were fortunately willing and able to adjust to these uncertain periods as teachers found their place in a system for former Montessori students. However, the students were also aware of the differences between the teachers, and expressed their preferences for either teacher. These acknowledgements showed that the ongoing switch between styles in the classroom indeed affected the transition—including adjustment to new roles and finding a consistent rhythm or schedule.

In the Montessori program, students were taught to be active learners and to be in control of their own experiences in the classroom. Once they entered fourth grade, the students were abruptly placed in a system where all students were taught with the same concept expectations for each student, and where they received information from the same source: the teacher (Kretchmar, 2016). For the majority of the time, students were in the role of passive learner to the teacher's authoritarian role, while teachers privately debated the effectiveness of this style, as educators often have in the past (Lesh & Zawojewski, 2007). Observations of the classroom structure showed the implemented Montessori system was aligned with the theory and literature, with an additional emphasis on structured schedule that is not typically found in such a system. Meanwhile, fourth grade observations showed the new system was not only non-Montessori, but was also a mostly direct-instruction style, specifically for mathematics. Other content areas were given more of a free, exploratory structure, but the final mathematics schedule strictly held to a routine, lecture-based structure. Most students said they elected to focus on their mathematics classwork directly after the mathematics lesson, even though it was not technically required. This choice gave students a solid hour of direct instruction, drill and repeat, and practice with the teacher's methods. This schedule summarized their typical mathematics experience each day. Research supports that the adult in the classroom is a necessary knowledge base for students to effectively learn more complex mathematics (Smith, 2016; Li & Adams, 1995). Students did not appear to feel negatively toward the new direct-instruction style, but instead embraced the teacher as a source of knowledge for more efficient learning. As interviews and observations showed, the teachers turned out to have more struggles than students with identifying their own place in the classroom, and this supported role reversal as an impactful aspect of student transition. While it is not uncommon for teachers to struggle with identifying their roles in the balance of discovery and direct learning in the classroom (Fraivillig et al., 1999), consistency is essential for student success as well.

Overall Conclusions

Observations in both the third and fourth grade classrooms showed their practices were aligned to theory. Not every feature of the classrooms was completely accurate, but the majority of either style in third and fourth grade reflected that of Montessori and non-Montessori, respectively. In fact, observations and interviews supported the conclusion that fourth grade actually followed a direct-instruction format, the classic form of a traditional system that is taught in many school systems. Even though the teachers, and possibly the structure, have changed from the previous year when the problem was identified, that should not be considered a limitation of the results. The administration continued to actively seek a solution to a pressing issue they perceived as still present during the course of the study and school year, and the conclusions taken from the research focused on solving some of those issues. The problem identified in this study existed both before and after changes were made to teachers or structure. There were three identified aspects of the problematic transition: changing the pace and structure, taking away the Montessori materials, and reversing the teacher and student roles. Changing the pace from third grade to fourth grade did not appear to have a significant impact on the transition. Managing the use of manipulatives between the two grades appeared very significant. Shifting roles between students and teachers was somewhat significant, as students experienced inconsistencies in teaching styles. Further, curriculum details in third grade created a transition that could be confusing for students who had not reached all of the available material before moving into fourth grade. Fourth grade teachers believed standards were not being met in the Montessori classrooms. Fourth grade students also experienced inconsistencies with how the fourth grade curriculum was presented when teachers could not agree on a set method of instruction.

Changing the pace and structure did not appear to be impactful, and instead was more of a self-contained issue within the context of the third grade curriculum covering standards effectively. Teachers agreed, citing examples of curriculum standards not being met. Students in fourth grade actually preferred the new pacing because they found it more efficient. They appreciated being able to easily get help when needed, but were also initially willing to assist their classmates when needed. However, the individualistic nature of the classroom environment became more apparent over time.

Taking away the Montessori materials turned out to be a very important aspect, but in a different way than was expected. The absence of Montessori manipulatives in the fourth grade classroom originally seemed like the obvious problem to most educators involved in the study. However, taking away manipulatives affected the transition differently than expected. Most students were happy to use Montessori materials in the lower grades, but in fourth grade they were quick to move on and denounce the materials as no longer necessary. However, more detailed questioning showed that while students and teachers alike admitted the materials helped students initially, opinions became more negative once students had to use them after their perceived mastering the concept. One example of the reluctance to repeat understood topics in the third grade classroom was student and teacher perception that addition and subtraction received too much focus, allowing less time for difficult concepts such as multiplication and division. In reality, the issue was that students were required to continue using manipulatives longer than necessary, and were much more receptive of using manipulatives with newer, admittedly unfamiliar topics, such as fractions. Student comments consistently showed they were not afraid to make negative remarks about which features of the classroom they did not like, yet comments about Montessori did not reflect a negative association with the materials themselves. Instead, the students talked about the materials' implementation as frustrating. To conclude, students often appreciated the materials and how they helped learn new topics even if they were not the preferred method, but they did not like the way materials were repeatedly applied. When challenged with particularly difficult material, the CGI students sometimes reverted to direct modeling with a written form of the manipulatives they were familiar with, which showed a reliance on them more than they may have expressed in interviews.

When changing roles, fourth grade students moved into their new roles easily and did not portray negative statements about either moving together at the same pace with the rest of the class or being instructed to work individually. Students also thought they learned more efficiently when they were allowed to handwrite and work individually, asking the teacher for assistance when needed. Students preferred going to the teacher for help instead of waiting for limited guidance because they felt they could achieve more in the class period. Teachers were identified as struggling to determine the correct method for the new classroom, and struggling how to balance students coming from a Montessori system to a more direct method of learning. In short, fourth grade teachers had trouble finding their place in the classroom.

In terms of ranking the impact of the three aspects, evidence pointed to the handling of manipulatives as the strongest aspect, followed by the shifting of student and teacher roles, with pacing and structure as the weakest. The third grade classroom aligned strongly with the Montessori values, and the fourth grade classroom showed enough evidence to be classified as a direct-instruction style. The problem-solving CGI sessions provided insight into student understanding, and showed that the more comfortable students were with a topic, the more likely they were to correctly calculate algorithmically with handwritten strategies, consistent with findings in the literature (Lesh & Harel, 2003). The students showed strong understanding in addition, subtraction, and multiplication, but had trouble with division and fractions. Their concept of division improved from the first to second problem sets, particularly in terms of identifying when division was necessary. However, the students never implemented long division algorithms, and instead they used a variety of other strategies to work out the problems. This strategy showed a strong understanding rather than copying a memorized algorithm. The more students drew pictures in unfamiliar situations, the more they could progress in difficult problems.

Conceptually, students recognized the meaning behind fractions and division as the school year continued, despite difficulties in calculations. Teachers agreed that students often initially struggle with these concepts, and many believed this was because of the amount of time allotted to those topics. However, fourth grade teachers had positive remarks about the students' progress in their classrooms. Teachers from both grades had the same goal to push their students to become problem solvers, and in third grade students needed to focus more on calculation because their mathematical problem solving was not yet strong enough. The continued perception in the Montessori classroom was that students had the calculation ability but were not able to use those strategies in more open problem scenarios where the required operation was not specified. Fear of becoming too focused on calculation in the classroom is typical among teachers, based on prior studies (Putnam et al., 1990). The students in the CGI exercises portrayed experience in problem solving, suggesting that the group setting in Montessori provided more opportunity for these students to grow as problem solvers than teachers originally believed (Noddings, 1985).

There are five main strands of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick et al., 2001). The CGI assessment results, teacher discussions, and student opinions showed some weaknesses in these areas that may translate to a difficult exit from a Montessori mathematics program. Students in this study showed conceptual understanding very well, but had a more difficult time with procedural fluency, particularly if they did not create the knowledge (and understanding) themselves. For example, Kai made a classic procedural mistake in the first question of the first round, described in Table 8, which showed his struggle to understand the procedure that matched the topic that he grasped conceptually. When translating from the use of manipulatives to pencil-and-paper methods, some of the connections appeared to be lost as students concluded activities that represented mathematical procedures. While students appeared to make the connection between direct modeling and counting strategies, the translation to algorithmic formulas (beyond fact memorization) was not as apparent during the study.

Context and Limitations, and Moving Toward the Future

The school in this study provided a somewhat different experience than typical Montessori programs because of the classroom and curriculum formats. For example, when students leave a Montessori program, they also usually switch institutions entirely. Because the transition at this school was contained within the elementary grades, there was an added benefit to the study in viewing students that were consistently familiar with their classmates, faculty, and the environment. However, this is not the case for all schools, and the results of this study should be interpreted accordingly. This self-contained transition was also a benefit to the reliability of the study, compared to other studies in which outside factors may influence the results. Also, this school is an international school located in Central America. While research has documented struggles with many schools in the Latin American region (Sorto et al., 2009), this private school has more diverse student and teacher populations to compare to other schools in the world. The language barrier was not very impactful to the study, as all students involved were bilingual from the beginning of the school year. The observed Montessori program was very similar to what the literature suggests, but there were noticeable differences. Specifically, deadlines and instruction time were included in the system. These are features not typically defined as Montessori, but may vary from school to school in terms of implementation.

A limitation to this study is that the studied group of fourth grade students is not the same set of fourth grade students from the previous year, when the problem was originally identified. Therefore, the fourth grade students from the previous year, in fifth grade during the study, were never observed. Similarly, this study did not have the two-year time span to follow a set group of students transitioning out of Montessori third grade and into non-Montessori fourth grade. Instead, fourth grade students described their previous exposure to mathematics, and the third grade students were only exposed to the Montessori methods as the school intended. The teaching staff also changed this year compared to last; only one of the four teachers involved in the study had been in their same position the previous school year. The context of the study specifically addressed two different styles of teaching and learning mathematics. The study evaluated the degree of Montessori style that the third grade style program represented, and similarly evaluating the non-Montessori program as direct-instruction based. Similar methods may aid in evaluating other transitions, even when considering different programs at various levels. Many schools around the world employ a Montessori program, and even more implement direct-instruction approaches. The results of this study can have implications not just for other school systems attempting such a transition, but also program transitions at other grade levels. For example, while the transition from high school to college does not rely on the manipulatives aspect, the other two aspects may be relatable.

The discussion of the results can lead to the most efficient, positive implementation of Montessori in similar schools. The results analyze student and teacher perceptions of the strengths and weaknesses of Montessori and traditional programs. The conclusions reached in this study may provide a guiding point for those attempting to implement these programs. Applications of this study do not require that students come from a Montessori system, as much of the provided evidence about the fourth grade experience carries significance for other traditional systems. Evaluating student opinions on a lack of differentiation, listening to teachers discuss their struggles identifying their most comfortable teaching methods, and observing the classroom routine and structure are all discussed in the findings. The results may give teachers and administrators ideas to spark discussions about the improvement and efficiency in their own classrooms. These issues can be expounded upon in further studies. The results of the study contribute to the existing knowledge of teaching mathematics in a Montessori system, and also help fill the void in the literature regarding the transition stage out of Montessori and into non-Montessori systems. Another important conclusion is to draw attention to the best implementation of the manipulatives stage of learning mathematics. In this study, there were issues translating what students conceptually understood to algorithmic handwritten procedures, which masked some of their problem solving abilities in the process. Montessori programs may provide students with solid mathematical understanding with physical objects and even direct modeling through drawing pictures. The issues arise when moving into the procedural stages, and future studies may explore this stage more fully.

Two specific recommendations can be made for this specific school after collecting and analyzing the data. First, the Montessori materials should continue to be used in third grade, but their implementation should be altered. The manipulatives curriculum should provide more time for students to cover advanced material, as well as future topics if the students are able to progress enough to reach those goals. If the students have understood a concept, there needs to be a system in place to ensure that they are not bored or discouraged from using manipulatives. One method in particular would be to focus on the bridge between using manipulatives and algorithmic handwriting. Students showed a tendency to lean on drawing pictures for their own direct modeling, but in the classroom there exists a gap from using manipulatives to handwriting their procedures in formulaic ways. Second, teachers in fourth grade should agree upon their determined role in the classroom. The students were receptive of the differences between the two styles of fourth grade teaching, but teacher comments reflected frustration about identifying the exact style they wanted to implement in the classroom. The shift in pace, structure, and differentiation did not appear problematic in the transition, so continuing to allow students to move into the traditional system should remain effective. These recommendations can be made specifically for this institution after reaching conclusions in this study. Further, suggests can be given to the general education community concerning the biggest takeaways from the study and what could be done moving forward to improve on implementation strategies and classroom perceptions between the two teaching styles. Both lists of recommendations are given below.

Recommendations for the School

- Allow the third grade teachers to make executive decisions in letting students move past the four basic operations if they have the ability and motivation.
- Determine how teachers should more consistently fill their role in the fourth grade so that students know what to expect.
- Continue the recent strategy of teaching handwriting methods to third grade students so they are prepared going into fourth grade.
- Include fractions in the Montessori program, and also significantly alter the stress that each of the basic operations receives in the school year.
- Give third and fourth grade teachers the curriculum expectations for both grades so they know what is covered as students transition, particularly the points where teachers agree gaps exist.
- If third grade students show they are past the point of using manipulatives for a particular activity, teachers should allow them to move past that form of activity or past that topic, rather than devalue the experience with materials due to lack of interest.
- Decide whether or not to focus on differentiation. The students are able to adjust to their roles in fourth grade, so once a strategy is picked, they appear to handle it.
- Provide more exposure to problem solving, instead of just calculation, in the third grade classrooms.
- Officially assess students' experience with problem solving. Their alternative strategies when faced with unfamiliar situations in the CGI problem sets showed they were better at problem solving than teachers believed.

- Focus less on compartmentalizing the use of manipulatives and procedural form of handwriting, and emphasize the bridge connecting the two strategies.

Recommendations for the Education Community

- Educators should explore the idea of bringing handwritten methods into the Montessori classroom as students finish their time in the program; there are benefits that may not become apparent through a manipulatives-only approach.
- Teachers may want to implement CGI assessments in their own classrooms; particularly over topics they find students have the most trouble.
- Manipulatives provide a very solid conceptual understanding of the four basic operations, including awareness of when to appropriately use them in various situations.
- There may need to be further research covering the connection between the conceptual understanding stage of learning and procedural fluency and strategic competence phases.
- Judging students' problem solving capabilities as a class can be difficult; assessing student strategies may provide clearer, and surprising, insights for teachers.
- When students are uncomfortable with a new concept, they tend to fall back on old strategies regardless of stated opinions; teachers should be aware of student arguments as legitimate or not during such cases.
- When teachers do not come to a consensus about pedagogy, students can sense the turmoil and may be affected in their learning; teachers should be aware of their outward appearance to students in such cases.

APPENDIX SECTION

IRB Protocol Approval

COLLABORATIVE INSTITUTIONAL TRAINING INITIATIVE (CITI PROGRAM)
COURSEWORK REQUIREMENTS REPORT*

* NOTE: Scores on this Requirements Report reflect quiz completions at the time all requirements for the course were met. See list below for details. See separate Transcript Report for more recent quiz scores, including those on optional (supplemental) course elements.

- **Name:** Zachariah Hurdle (ID: 5546692)
- **Email:** zbh4@txstate.edu
- **Institution Affiliation:** Texas State University - San Marcos (ID: 741)
- **Institution Unit:** Math Education

- **Curriculum Group:** Information Privacy Security (IPS)
- **Course Learner Group:** IPS for Faculty, Students, Staff
- **Stage:** Stage 1 - Basic Course

- **Report ID:** 19492497
- **Completion Date:** 05/09/2016
- **Expiration Date:** N/A
- **Minimum Passing:** 80
- **Reported Score*:** 86

REQUIRED AND ELECTIVE MODULES ONLY	DATE COMPLETED
Basics of Health Privacy (ID: 1417)	05/09/16
Basics of Information Security, Part 1 (ID: 1423)	05/09/16
Basics of Information Security, Part 2 (ID: 1424)	05/09/16

For this Report to be valid, the learner identified above must have had a valid affiliation with the CITI Program subscribing institution identified above or have been a paid Independent Learner.

CITI Program
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Phone: 305-243-7970
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COLLABORATIVE INSTITUTIONAL TRAINING INITIATIVE (CITI PROGRAM)

COURSEWORK REQUIREMENTS REPORT*

* NOTE: Scores on this Requirements Report reflect quiz completions at the time all requirements for the course were met. See list below for details. See separate Transcript Report for more recent quiz scores, including those on optional (supplemental) course elements.

- **Name:** Zachariah Hurdle (ID: 5546692)
- **Email:** zbh4@txstate.edu
- **Institution Affiliation:** Texas State University - San Marcos (ID: 741)
- **Institution Unit:** Math Education

- **Curriculum Group:** Human Research
- **Course Learner Group:** Social and Behavioral Research Students
- **Stage:** Stage 1 - Basic Course

- **Report ID:** 19492496
- **Completion Date:** 05/09/2016
- **Expiration Date:** 05/09/2018
- **Minimum Passing:** 80
- **Reported Score*:** 82

REQUIRED AND ELECTIVE MODULES ONLY	DATE COMPLETED
Belmont Report and CITI Course Introduction (ID: 1127)	05/09/16
History and Ethical Principles - SBE (ID: 490)	05/09/16
Defining Research with Human Subjects - SBE (ID: 491)	05/09/16
The Federal Regulations - SBE (ID: 502)	05/09/16
Assessing Risk - SBE (ID: 503)	05/09/16
Informed Consent - SBE (ID: 504)	05/09/16
Privacy and Confidentiality - SBE (ID: 505)	05/09/16
Texas State University - San Marcos (ID: 1131)	05/09/16

For this Report to be valid, the learner identified above must have had a valid affiliation with the CITI Program subscribing institution identified above or have been a paid Independent Learner.

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AVPR IRB [ospirb@txstate.edu]

To: Hurdle, Zachariah B

Actions

Monday, June 06, 2016 10:36 AM

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The reviewers have determined that your IRB Application Number 2016G2321 is exempt from IRB review. The project is approved.

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Institutional Review Board

Office of Research Compliance

Texas State University-San Marcos

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DEL MAR

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May 20th, 2016

TO WHOM IT MAY CONCERN

Zach Hurdle will be observing a Third Grade Montessori Math classroom at Del Mar Academy, and interviewing/documenting a traditional Fourth Grade Math classroom.

His goal is to determine the underlying causes of the challenges students face when moving from one to the other.

Please let me know if you have any questions.

Kind regards,

Laird MacDonald

Head of School

Del Mar Academy

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mep

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Consent Forms

VERBAL CONSENT

Study Title: FACTORS THAT ARISE IN THE TRANSITION FROM THE MONTESSORI METHOD TO A TRADITIONAL METHOD: A FOURTH GRADE MATH VIEW

Principal Investigator: Zach Hurdle **Co-Investigator/Advisor: Alejandra Sorto**
Sponsor:

My name is Zachariah B. Hurdle and I am a graduate student at Texas State University. I am doing this study to investigate which classroom factors contribute to the challenges of taking a traditional approach to mathematics when compared to a Montessori education. I am asking you to take part because you are a student at Del Mar Academy. I'm going to tell you a little bit about the study so you can decide if you want to be in it or not.

Students in the research project could be videotaped in his/her mathematics class, have pictures taken of some classwork/homework, asked about mathematical ideas, and possibly interviewed.

If you want to be in this study, I'll ask you about math, and your opinions about math. The questions aren't going to be to find the correct answer, but more about what you were thinking about, how difficult it was, explaining your work, and so on. You do not have to answer any question you don't want to or you can stop at any time.

I talked to your parents about this study and they said you could do it if you wanted to. But you can still say "No" if you don't want to be in the study or you can start and then if you want to stop being in the study at some point, that's okay. No one will be mad at you.

Do you have any questions for me?
Do you want to be in the study?
Do you understand what was said to you?

PARENT/GUARDIAN INFORMED CONSENT

Study Title: FACTORS THAT ARISE IN THE TRANSITION FROM THE MONTESSORI METHOD TO A TRADITIONAL METHOD: A FOURTH GRADE MATH VIEW
Principal Investigator: Zach Hurdle   Co-Investigator/Faculty Advisor: Alejandra Sorto
Sponsor:
Dear Parent/Guardian:

My name is Zach Hurdle and I am a PhD candidate student in the Math Education Department at Texas State University. I am asking for your permission to include your child in my research. This consent form will give you the information you will need to understand why this study is being done and why your child is being invited to participate. It will also describe what your child will need to do to participate as well as any known risks, inconveniences or discomforts that your child may have while participating. I encourage you to ask questions at any time. If you decide to allow your child to participate, you will be asked to sign this form and it will be a record of your agreement to participate. You will be given a copy of this form to keep.

- **PURPOSE AND BACKGROUND**

Del Mar Academy is participating in an effort, through a Texas State dissertation project, to investigate which classroom factors contribute to the challenges of taking a traditional approach to mathematics when compared to a Montessori education.

- **PROCEDURES**

The purpose of these study is to evaluate observe students' tendencies in their first year of a traditional math education setting when coming out of a Montessori style. To do this, there will be recordings of classrooms sporadically throughout the semester, as well as interviews of select students and teachers. In addition, there will be some pictures taken of student work that can be further analyzed, to get a clearer picture as to what is challenging and what is easier in the classroom, and to also look at the problem solving skills of students.

- **RISKS/DISCOMFORTS**

Your child may feel uncomfortable being videotaped, but the camera will be placed in a matter that should not distract them. You can ask for your child not to be taped at any time. Your child may also ask not to be taped at any time. You are able to remove your child from the study at any time and your child will continue to receive quality math instruction in this classroom.

- **EXTENT OF CONFIDENTIALITY**
- All video will be coded with no personally identifying information on them.
- The tapes will be kept secure and out of reach of other students/teachers.
- To make possible future analysis the investigator will retain the recordings.
- Classwork pictures will not in any way affect grades or outcomes of the study or class.

- Images of work will not be shown to other students.
- The data resulting from your participation may be made available to other researchers in the future for research purposes not detailed within this consent form. In these cases, the data will contain no identifying information that could associate your child with it, or with their participation in any study.
- I may want to show video clips of the classroom interactions at conferences or workshops. I will ask for a separate consent for that.

The **records** of this study will be stored securely and kept confidential. Authorized persons from Texas State University, members of the Institutional Review Board, and (study sponsors, if any) have the legal right to review your child's research records and will protect the **confidentiality** of those records to the extent permitted by law. All publications will exclude any information that will make it possible to identify your child as a subject. If any new information becomes available that could affect your decision to remain in the study, I will let you know.

- **BENEFITS**

There are no tangible benefits for participating in this study. The **benefits** to society and to the school your child attends involve increased understanding of how to support students to learn mathematics.

- **PAYMENT/COMPENSATION**

There will be no payment to you or your child as a result of your child taking part in this study.

- **QUESTIONS**

If you have any questions or concerns about your participation in this study, you may contact the Principal Investigator, Zach Hurdle, at zbh4@txstate.edu or making an appointment at the school.

This project #2016G2321 was approved by the Texas State IRB on June 6, 2016. Pertinent questions or concerns about the research, research participants' rights, and/or research-related injuries to participants should be directed to the IRB Chair, Dr. Jon Lasser 512-245-3413 – (lasser@txstate.edu) or to Monica Gonzales, IRB Regulatory Manager 512-245-2314 - (meg201@txstate.edu).

DOCUMENTATION OF CONSENT

I have read this form and decided that my child will participate in the project described above. Its general purposes, the particulars of involvement and possible risks have been explained to my satisfaction. I will discuss this research study with my child and explain the procedures that will take place. I understand I can withdraw my child at any time.

____

Printed Name of Child

Printed Name of Parent/Guardian **Signature** of Parent/Guardian Date

Signature of Person Obtaining Consent Date

ASSENT CONSENT

Study Title: FACTORS THAT ARISE IN THE TRANSITION FROM THE MONTESSORI METHOD TO THE TRADITIONAL METHOD: A FOURTH GRADE MATH VIEW
Principal Investigator: Zach Hurdle   Co-Investigator/Faculty Advisor: Alejandra Sorto
Sponsor:

My name is Zachariah B. Hurdle, and I am a graduate student (PhD candidate) at Texas State University. I am conducting a research study titled "Factors That Arise in the Transition from the Montessori method to the Traditional Method: A Fourth Grade Study". I am doing this study because I want to make such a transition as smooth as possible, and want to find out what works well and what is still challenging. I am asking you to be a part of this study because you are a student at Del Mar Academy. This form will tell you a little bit about the study so you can decide if you want to be in the study or not.

The purpose of these study is to evaluate observe students' tendencies in their first year of a traditional math education setting when coming out of a Montessori style. To do this, there will be recordings of classrooms sporadically throughout the semester, as well as interviews of select students and teachers. In addition, there will be some pictures taken of student work that can be further analyzed, to get a clearer picture as to what is challenging and what is easier in the classroom, and to also look at the problem solving skills of students. Remember, you can also stop being in this study at any time.

There are no tangible benefits for participating in this study. The **benefits** to society and to the school your child attends involve increased understanding of how to support students to learn mathematics.

Please talk about this study with your parents before you decide if you want to be in it. I will also ask your parents to give their permission. Even if your parents say you can be in the study, you can still say that you don't want to. It is okay to say "no" if you don't want to be in the study. No one will be mad at you. If you change your mind later and want to stop, you can.

You can ask me any questions about this study the next time you see me. You can also talk to my advisor, Alejandra Sorto, or your mom or dad about this study. After all your questions have been answered, you can decide if you want to be in this study or not.

If you want to be in this study, please sign.   If you don't want to, please do not sign.

____________________________________   ____________
PRINT your name   Date

SIGN your name | Date

Signature of Person Obtaining Consent | Date

Interview Templates
Teacher (Montessori)

Potential Question	Reasoning Provided by Literature
How do you perceive that your role fits into this classroom environment?	*Teacher's roles in Montessori classrooms are extremely important to success, as they have to balance leading the student with a hands-off approach (Lester et al., Schoenfeld, Berger, Franke et al., Lockhorst)*
What types of social cues do you look for in your students each day, and how do established relationships assist with this?	*While the teachers are fulfilling their role, they need to know when students are pushing positive and negative limits (Gutek, Rambusche, Montessori), and the strong bond with students between themselves and teachers is indicative of this (Cossentino, Montessori, Kamii, Kramarski et al., Siegler)*
How does your view of your teaching match/differ from your students' perception?	*The way the teacher views themselves and their role in the process is just as important as the student's roles (Driscoll, Pape et al., Silver)*
What do you find is the biggest weakness of the classroom?	*It has been said that young students need more guidance than older students, and this can be considered a drawback to the method (Zimmerman, Fravillig et al., Pugalee, Lin & Ginsburg)*
What is your interpretation of the biggest strength of the classroom?	*Research shows learners gain more understanding through self-realization of concepts, like problem solving (Bagby & Sulak, Lampert, Checkley, Carpenter & Fenema)*
How important is a student's ability to self-regulate and self-motivate in this class?	*A key ability shown throughout the research has been for the student to take control of their learning, which led to the decision for a metacognitive framework (Montessori, Zimmerman, Livingston, Rathunde, Schoenfeld)*
How would you describe the role of problem solving in the classroom?	*In order to get a sense of students' understanding in one semester, problem solving lessons/activities are the best way to draw out the desired display, and match with metacognition (Hiebert & Wearne, Chapko & Buchko, Polya, Flavell, B. D'Ambrosio)*

Teacher (Traditional)

Potential Question	Reasoning Provided by Literature
How do you perceive that your role fits into this classroom environment?	*The traditional teacher is considered a direct source of information, more of a knowledge-giver that students are expected to take in (Wigfield & Eccles, Lesh & Harel, Copes & Shager, Boekaerts)*
What types of social cues do you look for in your students each day, and how do established relationships assist with this?	*Focus becomes an issue with students, and passive reactions may be more prominent in this method (Kazemi & Franke, Illich, Boekaerts), while relationships are not heavily stressed in traditional classrooms (Owens & Konkol, Koestner et al.)*
How does your view of your teaching match/differ from your students' perception?	*The way the teacher views themselves and their role in the process is just as important as the student's roles (Driscoll, Pape et al., Silver)*
What do you find is the biggest weakness of the classroom?	*Traditional methods promote passivity in accepting facts, and this tends to push students along regardless of problem solving ability or general understanding (Din, Punam et al., Moore)*
What is your interpretation of the biggest strength of the classroom?	*Results, through exercise and repetition, have data to back them up, and has a strong influence on self-efficacy (Ruble, Hanson, Al-Makahleh, Ervin et al.)*
How important is a student's ability to self-regulate and self-motivate in this class?	*The research suggests that, while the environment doesn't encourage it, students still have the choice to become active learners (Rathunde & Csikszentmihalyi, Kretchmar, Zimmerman)*
How would you describe the role of problem solving in the classroom?	*In order to get a sense of students' understanding in one semester, problem solving lessons/activities are the best way to draw out the desired display, and match with metacognition (Hiebert & Wearne, Chapko & Buchko, Polya, Flavell, B. D'Ambrosio)*

Student (Montessori)

Potential Question	Reasoning Provided by Literature
Can you describe a typical math day at school here?	*The Montessori method has its set format, so we need to discover how much the classroom at this school lines up with the theories behind such a learning environment (Humphryes, Knowles, Piskurich).*
How comfortable are you in describing your ideas and explaining thoughts to other students or the teacher?	*Mathematical discussion, in the research, is found to be a positive influence; it is important to see conversation's role in the traditional classroom (Mercer & Sams, Siegler, Schoenfeld)*
What does your teacher do to help you with questions and problems? Do you like to figure it out yourself?	*Students may hesitate to suddenly rely on the teacher for guidance, but also may become too dependent; either way it is an adjustment (Masingila et al., Stanic & Kilpatrick, Ward, Copes & Shager)*
What do you like about using objects to learn about math?	*In the 3rd grade class, students don't show work, but it is a vital piece of the process in the their new 4th grade circumstance (Montessori, Ward, Sorto et al.)*
Do you like to figure problems out on your own, prefer teacher help, or want to work in groups?	*Productive struggle is one method, along with collaboration, or asking for assistance; but students have to find what works for them (Warshauer, Myren, Frederick et al., Kramarski et al., Livingston, Zimmerman)*
What do you do when you don't understand how to do something?	*The literature shows that self-awareness of struggle and success is an important aspect of proceeding through a grade school math course (Grolnick & Ryan, Lesh & Zawojewski, Zimmerman, Savin-Baden & Major)*
Do you like working with materials or writing your work down more?	*A major difference between Montessori and traditional is the emphasis on manipulatives (Montessori; Zhbanova et al.; Carpenter et al.)*
Do you ever feel like there is too much work to handle in class? Do you ever get stressed out?	*The goal here is to determine if the classroom truly is self-paced. In theory, students should be attended to by individual need (Piskurich, Ward, Pickering)*
What is your favorite part of math class, overall?	*Self-efficacy is a strong byproduct of the Montessori methods, and one that students need to succeed in such an environment (Bandura, Zimmerman, Schunk & Rice)*

Student (Traditional)

Potential Question	Reasoning Provided by Literature
What is the biggest difference between your math class this semester compared to last school year?	*Studies show that going from active learning to passive learning may be the biggest change in the classroom between the two methods (Zhbanova et al., Eccles et al., Montessori, Cossentino)*
How comfortable are you in describing your ideas and explaining thoughts to other students or the teacher?	*Mathematical discussion, in the research, is found to be a positive influence; it is important to see conversation's role in the traditional classroom (Mercer & Sams, Siegler, Schoenfeld)*
What does your teacher do to help you with questions and problems? Do you like to figure it out yourself?	*Students may hesitate to suddenly rely on the teacher for guidance, but also may become too dependent; either way it is an adjustment (Masingila et al., Stanic & Kilpatrick, Ward, Copes & Shager)*
What do you think about showing your work with pencil and paper?	*In the 3rd grade class, students don't show work, but it is a vital piece of the process in the their new 4th grade circumstance (Montessori, Ward, Sorto et al.)*
Do you like to figure problems out on your own, prefer teacher help, or want to work in groups?	*Productive struggle is one method, along with collaboration, or asking for assistance; but students have to find what works for them (Warshauer, Myren, Frederick et al., Kramarski et al., Livingston, Zimmerman)*
What do you do when you don't understand how to do something?	*The literature shows that self-awareness of struggle and success is an important aspect of proceeding through a grade school math course (Grolnick & Ryan, Lesh & Zawojewski, Zimmerman, Savin-Baden & Major)*
Do you like working with materials or writing your work down more?	*A major difference between Montessori and traditional is the emphasis on manipulatives (Montessori, Zhbanova et al., Carpenter et al.)*
Do you feel there is more work to do in this class than before? How do you feel about math overall?	*Self esteem, or self-efficacy and motivation, are extremely fragile points at a young age (Illich, Rosanova, Schunk & Rice, Bandura)*
When someone tells you to follow a pattern or a formula, is this more or less helpful than how you wanted to work a problem out?	*Research shows that it may be detrimental for students to simply memorize algorithms without understanding (Kamii, Kloosterman & Stage, Schoenfeld, Kaur & Toh)*

Observation Criteria
Montessori

Montessori Teaching Observation Protocol (MTOP)

Dissertation Project

Texas State University

I. BACKGROUND INFORMATION

Name of teacher ____________________________

Class specifics ____

Number of students observed ____________ Grade
Level__________________________________

Observer __________________________________ Date of observation
____

Start time _________________________________ End time
____

Observation number ______________________________
(First, second, third, fourth)

II. DESCRIPTION OF TEACHING CONTEXT

In the space below please give a brief description of the lesson observed, the classroom setting in which the lesson took place (regular classroom, computer lab, setting arrangements, etc.) Capture, if you can, the defining characteristics of this situation that you believe provide the most important context for understanding what you will describe in greater detail in later sections. Use diagrams if they seem appropriate.

III. DESCRIPTION OF EVENTS

Record here events which may help in documenting the ratings.

Time	Description of Events

IV. DESCRIPTION OF *IMPLEMENTED* MONTESSORI CLASS PERIOD

	Never Occurred				Very Common
1. Students are working by themselves or in groups, with little assistance from the teacher.	0	1	2	3	4
2. The math class includes tasks that involve the use of movement and/or manipulatives.	0	1	2	3	4
3. Students do not need to be pushed and reminded to focus in order to continue with what they are working on.	0	1	2	3	4
4. The activities are meant for a student to keep curious and make their own findings.	0	1	2	3	4
5. The teacher is guiding the students rather than feeding them direct information.	0	1	2	3	4
6. Students are relying on descriptions, gesturing, and figures rather than pencil and paper to make realizations.	0	1	2	3	4
7. The class period does not contain a true, direct lesson.	0	1	2	3	4
8. Relationships seem to be personal in this setting.	0	1	2	3	4
9. Dialogue is encouraged and facilitated throughout the room when the teacher deems it necessary.	0	1	2	3	4
10. Assessments are focused on open-ended questions that could result in multiple solutions, rather than concentrating on actual scores.	0	1	2	3	4
11. Students are encouraged to share questions, hints, ideas, and/or progress with other students.	0	1	2	3	4
12. Teacher circulates, observes (to monitor progress), ask questions, and provides necessary help as students work.	0	1	2	3	4

Additional comments you may wish to make about this observation.

Traditional

Traditional Teaching Observation Protocol (TTOP)

Dissertation Project

Texas State University

I. BACKGROUND INFORMATION

Name of teacher ________________________

Class specifics ____

Number of students observed __________ Grade Level_____________________________

Observer ______________________________ Date of observation ______________________

Start time _____________________________ End time _______________________________

Observation number ___________________________
(First, second, third, fourth)

II. DESCRIPTION OF TEACHING CONTEXT

In the space below please give a brief description of the lesson observed, the classroom setting in which the lesson took place (regular classroom, computer lab, setting arrangements, etc.) Capture, if you can, the defining characteristics of this situation that you believe provide the most important context for understanding what you will describe in greater detail in later sections. Use diagrams if they seem appropriate.

III. DESCRIPTION OF EVENTS

Record here events which may help in documenting the ratings.

Time	Description of Events

IV. DESCRIPTION OF *IMPLEMENTED* TRADITIONAL CLASS PERIOD

	Never Occurred				Very Common
1. For the majority of the instruction time, the teacher directly instructs students.	0	1	2	3	4
2. Students engage in recollection of facts, formulas, or definitions.	0	1	2	3	4
3. Students follow algorithms, whether they understand the reasoning and logic behind them or not.	0	1	2	3	4
4. The majority of questions are directed toward the teacher as the primary source of information rather than peers.	0	1	2	3	4
5. Pencil and paper work is the primary method of showing logic, reasoning, and understanding.	0	1	2	3	4
6. Students are not relying on descriptions, gesturing, objects, or manipulatives.	0	1	2	3	4
7. The class period is focused on one particular topic to cover for that day.	0	1	2	3	4
8. The concept of grading, scoring, and general assessment come up often in the class as a primary means of motivation.	0	1	2	3	4
9. There is some working in pairs and potential for collaboration, but more of a shift toward individual. performance.	0	1	2	3	4
10. Homework is assigned and students are expected to keep up with the workload.	0	1	2	3	4
11. The class consists of exercises of a drill and repetition nature.	0	1	2	3	4
12. Students are not generally given time to discover and conclude their own findings.	0	1	2	3	4

Additional comments you may wish to make about this observation.

Teacher Initial Comments Full Transcript

Question: “In your opinion, what could be the biggest reason that students have a difficult time transitioning out of third and into fourth grade at this school?”

Julia (3rd Grade): A hard time? I would say it’s a big change after working three years, everything with manipulative, they need to come to this process that is completely abstract and they need to understand things with no materials and no manipulative things. I think that’s the big challenge. But I think that if they’ve been really working and they’ve been into Montessori materials they are able to understand that the things they were doing before with materials is the same that they are doing right now on paper without manipulatives. So I will say that it’s challenging because they’ve been working with materials for six years coming from CASA (pre-K), and now everything is going to be books, paper, and pencil, so they don’t have that not abstract part anymore. Everything coming will be more abstract than before.

Karla (3rd Grade): Probably because it’s so hands on up to third grade and when they actually change from the Montessori approach to the traditional approach that’s probably where the difficulty may arise. That’s why we’re probably going to be working on a more traditional approach for the third graders, trying to prepare them for the transition into fourth grade.

Laird (Head of School): Well my feeling is that the Montessori program does an excellent job at getting students to reach pretty sophisticated mathematical operations through the manipulatives. They get to visualize division and multiplication and addition and they can complete complex problems using those. When they go to fourth grade, then they step into a more traditional textbook series that's based on the Common Core and it's all abstract, it's all done with pencil and paper, and so there's a big division. So in some ways what we see is our third graders doing more advanced math than our fourth graders for a couple of reasons. First, the Montessori program is individualized and advanced students have the opportunity to excel, and second, those Montessori materials really support the students in reaching and understanding those concepts, or at least a kind of gut feeling about what they're doing when they're doing that particular mathematical operation, because they're seeing it visually. It doesn't necessarily translate those when you present them the same problem on pencil and paper, and in the fourth grade they have to work a little more lock step together and so I think the high end and the low end students perhaps suffer a little bit because there's not the differentiation that is naturally inherent to the Montessori program.

Verena (4th Grade): Because it is a big challenge right now. Like, we are talking about the skills that they have in fourth grade, the kids that are coming from third grade, and these books are a little bit hard for the kids, so I feel that the math teacher…This is my perception: the math teacher last year saw the program, and it was a little repetition because the program's like that. You want to develop the math muscle. That was too much repetition, and too many concepts that were similar there, so they felt that pushing a little bit they can integrate the algebra and the geometry, but you have to push the lower levels. We will try this year, it's our experiment this year to see if that will work. It's a challenge.

Vicky (4th Grade): Speaking mathematically? I mean, socially, you’re looking at three classrooms coming together where last year they’re the top of the class, they’re in the third year, all of the children below them are younger. So they’re like seniors in high school, and then they transition to where they’re all the same age, and they haven’t been together in a classroom, so socially that’s a little bit difficult, um, you’ve got those dynamics going on. But I see the biggest problem we have is going from the Montessori over to whatever we’re going to use, whether it’s Saxon, or...mathematical learning, um, because over in lower there are certain standards that are not being met with the Montessori materials.

CGI Problem Sets
First Set: October

A.

Jennifer has ___ dollars. She earns some more money babysitting over the weekend. Now she has ___ dollars. How much money did she earn over the weekend?

B.

There are ___ kids in the cafeteria. ___more kids come in for lunch. How many kids are in the cafeteria now?

C.

There are ___ children playing in the park. ___ children had to go home. How many children were left playing at the park?

D.

There ___ children going to the water park. It costs ___ dollars per person. How much money will it cost for all the children?

E.

There are ___ donuts. ___ donuts fit in a box. How many boxes will be needed for all the donuts?

F.

There are ___children in P.E. class. The teacher wants to make ___teams with the same number of kids on each team. How many children can she put on each team?

Second Set: February/March

A.

___ children want to share ___ donuts so that everyone gets the same amount. How much can each child have?

B.

There are ___ chocolate brownies at Nina's party. ___ children want to share the brownies so that everyone gets to eat the same amount of brownies. How much can each child have?

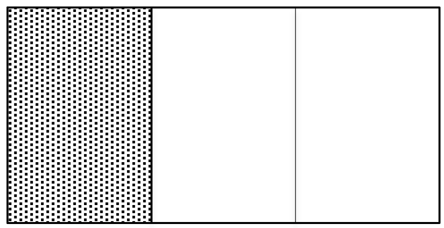
C.

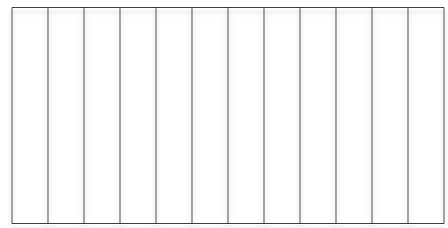
Robin went to a party where each person at ____ of a pizza. If ____ people ate pizza, how many pizzas were there in all so that they each got to eat ____ of a pizza and there were no leftover pieces?

D.

Okhee has a snowcone machine. It takes ____ of a cup of ice to make a snowcone. How many snowcones can Okhee make with ____ cups of ice?

E. Jorge and Darren are eating brownies that are the same size.





Jorge cut his brownie into 3 equal pieces and ate 1 piece.

Darren cut his brownie into 12 equal pieces. He wants to eat exactly as much brownie as Jorge. Color in the amount of brownie Darren should eat, so that his share is equal to Jorge's share.

F.

Jane says that if 6 people are sharing 10 cookies each person gets 1 and 2/3 cookies. John says that each person should get 1 and 4/6 cookies. Who is right? Can they both be right?

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