TEACHING WITH DYNAMIC GEOMETRY SOFTWARE: A MULTIPLE CASE STUDY OF TEACHERS' TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE

DISSERTATION

Presented to the Graduate Council of
Texas State University-San Marcos
in Partial Fulfillment
of the Requirements

for the Degree

Doctor of PHILOSOPHY

by

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San Marcos, Texas
May 2012

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DEDICATION

To my parents, Wiesława and Lech,

and to my husband, Jeremy.

ACKNOWLEDGEMENTS

First and foremost, I would like to thank God for giving me the ability and perseverance to complete this challenging journey. I have the strength for everything through Him who empowers me (Philippians 4:13).

There are many people who have supported me as I completed this study and no words are adequate to express my gratitude and appreciation for them. I would like to thank my dissertation chair, Dr. Zhonghong Jiang, for his suggestions and feedback. I also would like to express my appreciation to my committee members, Dr. Clarena Larrotta, Dr. M. Alejanda Sorto and Dr. Samuel Obara, for their insights and assistance with this project. I thank my committee for everything they have done.

I would like to thank the participants of this study. I appreciate very much their willingness to participate and the time they spent on providing data for this project. Without them, this project would not be possible, and I cannot thank them enough.

I would like to acknowledge Jessica Taylor Ivy and Julie Riales for creating the TPACK Development Model Self-Report Survey. I thank them for letting me use and reprint the survey.

I would like to thank my family and friends for encouraging me and believing in me. Special thanks to my husband, Jeremy, for his love, encouragement and support throughout this journey.

This dissertation is based upon work supported by the National Science Foundation under Grant No. 0918744. Any opinions, findings, and conclusions or recommendations expressed in this dissertation are those of the author and do not necessarily reflect the views of the National Science Foundation.

This manuscript was submitted on April 16, 2012.

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ABSTRACT

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This qualitative case study investigated how four high school teachers developed and used their knowledge in teaching geometry with technology. In particular, this study focused on teachers' technological pedagogical content knowledge (TPACK) and their integration of dynamic geometry in the classroom instruction. The sources of data included: an initial interview, observations, documents, a closing interview, a survey, implementation questionnaires, professional development attendance records and the researcher's log. Data analysis utilized the TPACK Development Model to describe participants' dynamic geometry integration and to identify their TPACK development levels.

All participants displayed good knowledge of geometry content, although they did not always know how to connect it with their pedagogical and technological knowledge. TPACK development levels were identified through the descriptions of participants' TPACK development and enactment. The levels varied within the themes and their descriptors for each participant; however, overall TPACK development levels were identified for three participants—two at the adapting level and one at the exploring level. The TPACK levels for the fourth participant were consistent only for the teaching theme descriptors and were at the exploring level.

Three unexpected findings surfaced. First, the participant with least teaching experience displayed the highest levels of TPACK. Second, the participant with most teaching experience with dynamic geometry showed the most inconsistency among the TPACK development levels, ranging from recognizing to exploring. Third, ongoing professional development and easy access to computers did not translate to frequent incorporation of dynamic geometry in teaching and learning. The participants claimed the curriculum and standardized testing to be the main barriers to increased technology use. Findings suggested that participants developed their TPACK through attending professional development workshops and implementing what they learned in the classroom instruction. Based on those findings, this study proposed a professional development model designed for teachers interested in integrating dynamic geometry in the classroom instruction.

CHAPTER I

INTRODUCTION

> *If we teach today as we taught yesterday, we rob our children of tomorrow.*
>
> – *John Dewey*

In today's world digital technologies are widely available and young people interact with them on a daily basis (Roblyer & Doering, 2010). Using technology in education not only motivates students by tapping into their comfort zone, but also enhances instructional methods, increases productivity, and allows students to gain required information age skills (Roblyer & Doering, 2010). Use of technology in teaching and learning has been advocated by various professional organizations (ISTE, 2008; NCTM, 2000). The International Society for Technology in Education (ISTE) prepared the National Educational Technology Standards and Performance Indicators for Teachers (NETS). Among them are the following four performance indicators:

> Teachers design or adapt relevant learning experiences that incorporate digital tools and resources to promote student learning and creativity. Teachers develop technology-enriched learning environments that enable all students to pursue their individual curiosities and become active participants in setting their own educational goals, managing their own learning, and assessing their own progress. . . . Teachers demonstrate fluency in technology systems and the transfer of current knowledge to new technologies and situations. . . . Teachers model and

> facilitate effective use of current and emerging digital tools to locate, analyze, evaluate, and use information resources to support research and learning. (ISTE, 2008, p. 1)

These performance indicators clearly show that teachers need to learn about new technologies and be prepared to provide their students with technology-based learning opportunities. The NETS were prepared for all teachers, no matter what grade or what subject they teach.

The National Council of Teachers of Mathematics (NCTM) specifically advocates the use of technology in teaching mathematics in its Principles and Standards for School Mathematics (NCTM, 2000). The technology principle states, “technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning” (NCTM, 2000, p. 373). In more detail, it declares that:

> Students can learn more mathematics more deeply with the appropriate and responsible use of technology. They can make and test conjectures. . . . In the mathematics classrooms envisioned in Principles and Standards, every student has access to technology to facilitate his or her mathematics learning. . . . The teacher must make prudent decisions about when and how to use technology and should ensure that the technology is enhancing students' mathematical thinking. (NCTM, 2000, p. 373)

The NCTM standards advocate for wide access to technology for all students, but they also stress the appropriate use of technology. That indicates technology should be used only when it improves learning and not just for the sake of using it. Technology is not an end, but a means to enhanced learning.

Use of technology is also encouraged by many school districts. Although state standards for mathematics vary across the U.S., on average approximately 25% of content in state standards is on instructional technology (Porter, McMaken, Hwang, & Yang, 2011). The Common Core State Standards mention the use of technology in teaching and learning of mathematics, but their primary focus is on content to be learned and not on instructional strategies to be used (Common Core State Standards Initiative, 2010). The Common Core Standards for Mathematical Practice encourage “use of appropriate tools strategically” and among such tools dynamic geometry software (Common Core State Standards Initiative, 2010). In Texas, the Professional Development and Appraisal System (PDAS), an instrument for evaluating teacher performance, includes the quantity and quality of technology use during class time (TEA, 2005). Therefore, according to PDAS, mathematics teachers not only need to be prepared to integrate technology into teaching and learning of mathematics, but they have to do it effectively by providing good quality instruction in order to obtain a good evaluation.

As a doctoral research assistant, I had an opportunity to work on a research project investigating the use of the *Geometer’s Sketchpad* (Jackiw, 2009), one of the major dynamic geometry software packages, in teaching and learning of geometry at the high school level. My work on the Dynamic Geometry Project (DGP) motivated me to investigate the area of teachers’ knowledge as it relates to integration of dynamic geometry software in teaching and learning of mathematics. Why is this important? The Conference Board of the Mathematical Sciences (CBMS) explains that: “Technology has revolutionized many jobs and substantially increased the mathematical skills needed across the workforce. In contrast, its impact on instructional practices has been more modest and varies greatly from classroom to classroom" (CBMS, 2001, p. 47). As part of my role as a doctoral research assistant, I have observed several teachers in their classrooms, and I have seen first-hand that integration of technology into teaching and learning of mathematics varies considerably.

Educational software such as *the Geometer's Sketchpad* (Sketchpad) can assist in developing students' understanding of mathematical concepts and increasing their reasoning skills (CBMS, 2001). Taking advantage of dynamic features such as dragging, one can see "the universality of theorems in a way that goes far beyond typical paper and pencil explorations" (CBMS, 2001, p. 132). For example, when exploring the exterior angles of a convex polygon, students can measure those angles for different polygons (e.g., a triangle, a quadrilateral, and a pentagon), add them, and notice that the sum equals 360 degrees. Then, they can select any vertex, drag it to change the form of the original polygon (but keeping it convex), and at the same time notice that the sum of the measures of the exterior angles does not change (see Figure 1).

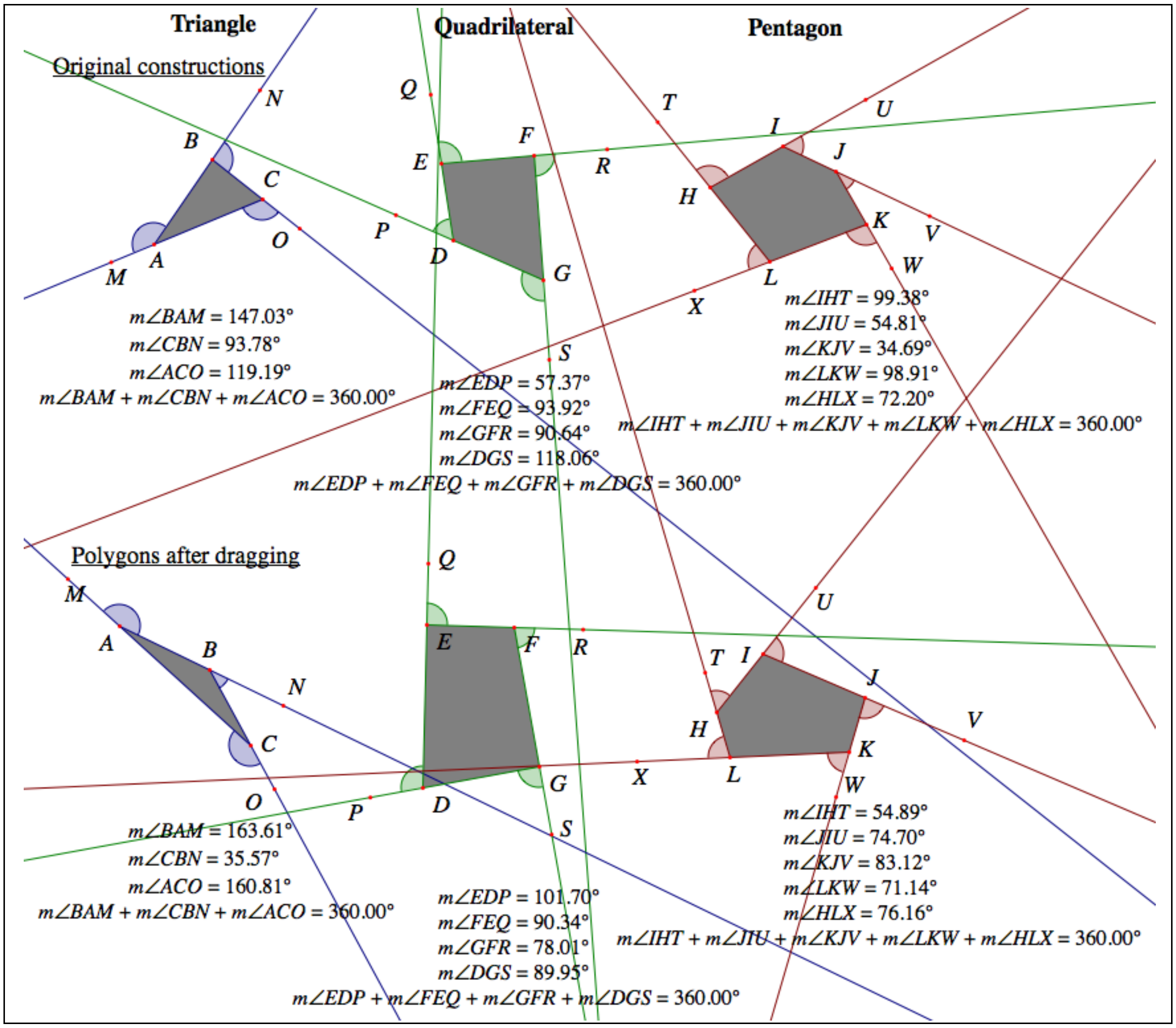


Figure 1. Screenshot of a Sketchpad file showing an activity for exploring the sum of exterior angle measures of convex polygons. The top part of the figure shows the original constructions of three polygons (triangle ABC, quadrilateral DEFG, and pentagon HIJKL), the exterior angle measures of each polygon, and the sums of exterior angle measures for each polygon. The bottom part shows these polygons after their vertices had been dragged.

This activity shows that the exterior angle measures are different from the original construction, but their sum remains the same for each polygon, and it remains constant for all polygons explored here. However, one might argue that a similar exploration can be done with paper and pencil by constructing several different polygons, measuring their exterior angles, and calculating their sum. How is this exploration with Sketchpad different then? And is it better? The use of dynamic geometry tools certainly saves time because only one construction is needed, and the software does all the measurements automatically. The user only has to specify what to measure, drag vertices, and observe what changes and what remains the same. One might argue that the efficiency (the saving of time) when using Sketchpad is good enough because it allows more time for other activities, possibly ones that require more cognitive demand such as writing of a proof. However, there is more that the dynamic tools can offer. Students can continue their exploration of the sum of the exterior angle measures of convex polygons by taking full advantage of the dynamic features of Sketchpad. To do so, they can mark one of the vertices of a polygon as the center for dilation (by double-clicking on a vertex), change the Arrow tool to Dilate Arrow, select the entire construction, and drag it towards the marked center by clicking on any of its part. The final drawing should resemble a star - one point, rays with their vertices at that point, and all angles forming a circle. The number of rays will equal the number of sides of a polygon. Figure 2 represents one such example. Students can see that no matter what kind of convex polygon they have (triangle, quadrilateral, pentagon, etc.), as a result of this exploration, the exterior angles of a polygon will always create a circle, i.e., 360 degrees. This exploratory illustration is extremely powerful and is not possible with paper-and-pencil explorations.

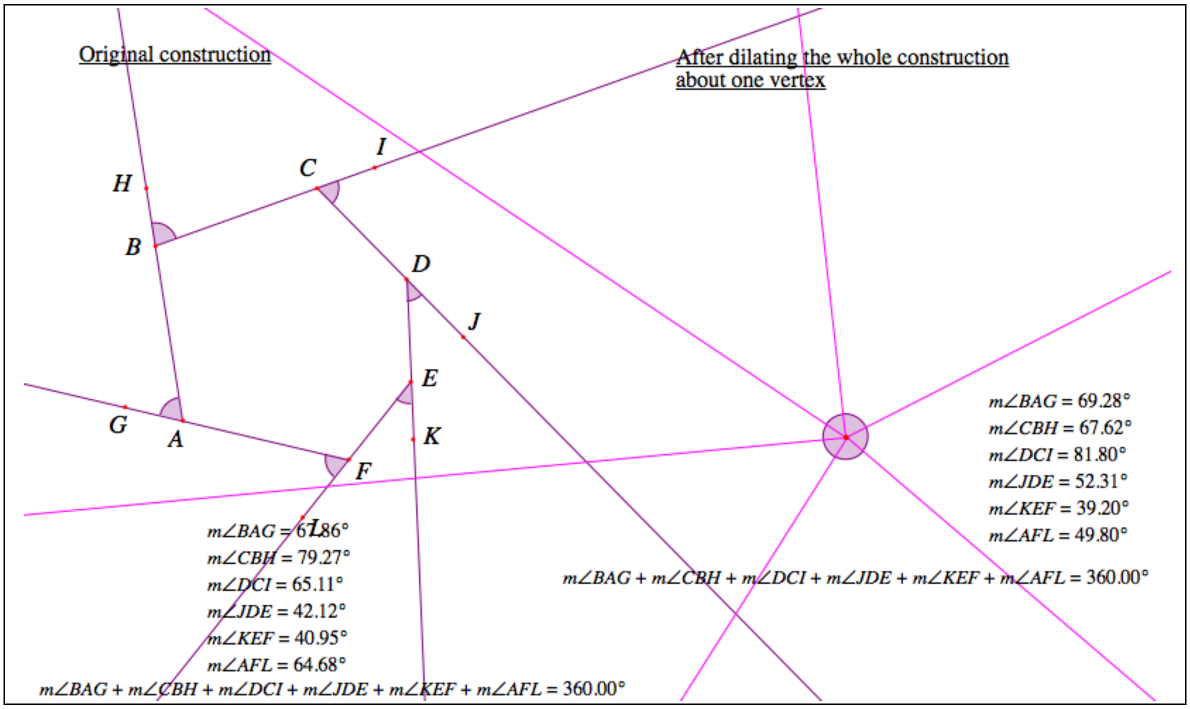


Figure 2. Screenshot of a Sketchpad file illustrating further exploration in the activity for the sum of exterior angle measures of convex polygons.

This kind of investigation also lays a better foundation for students' understanding of proof. Although, this is only one instance of Sketchpad use and how it can enhance student learning and understanding of mathematics, there are many more similar examples described in the literature, e.g., Sinclair and Yurita (2008) and Forsythe (2007, 2009).

Statement of the Problem

Taking into consideration the increase in available technology in schools in the recent years and the requirements placed on teachers to use it in teaching, we still do not know how effective teachers are in integrating technology into teaching (Harris, Grandgenett, & Hofer, 2010). Finding out how teachers learn about technology and how they incorporate it into their teaching is necessary since the way teachers use technology can impact the quality of instruction and student learning (Roberts & Stephens, 1999).

It has always been assumed that teachers need to know the content they teach (Shulman, 1986). They also need to know excellent instructional strategies for teaching that content (Shulman, 1986). Now, with the emergence and wide availability of educational technologies, teachers also need to possess knowledge for effective integration of those technologies into teaching the content. Increasing the effectiveness of technology-supported content area teaching has been a national goal for several years (Riley, Holleman, & Roberts, 2000). According to the most recent national survey (Fast Response Survey System) conducted by the National Center for Education Statistics (NCES), 72 percent of high school teachers feel sufficiently trained in technology usage and 66 percent feel sufficiently trained to integrate technology into classroom instruction (Gray, Thomas, & Lewis, 2010). Additional information reported from this survey included the following: most teachers (93 percent) are interested in using technology in classroom instruction and almost three-quarters of teachers (74 percent) design lessons in which students use a variety of educational technologies (Gray, Thomas, & Lewis, 2010). However, the NCES does not provide statistics specifically for high school mathematics teachers.

Based on data reported by the participants in the DGP during the first year of implementation, teachers did not feel like they were effective in using Sketchpad in teaching geometry. Even though teachers felt sufficiently trained to integrate technology and many of them had created technology-based lessons, it did not necessarily mean they integrated technology into teaching and learning effectively. Based on preliminary observations of the DGP participants, high school mathematics teachers do not have very good knowledge about integrating technology into teaching. Merely learning a new technology is not enough; teachers need to learn how a new technology is connected to the subject they teach and how it can be integrated into their instruction.

Mishra and Koehler (2006) defined a construct of knowledge that teachers possess when teaching with technology, namely technological pedagogical content knowledge (TPACK, previously TPCK). The concept of TPACK and its assessment have been researched (Mishra & Koehler, 2006; Niess, 2005; Niess et al., 2009; Schmidt et al., 2009), although not in the area of geometry or dynamic geometry software. Also, the conceptualization of teacher knowledge related to using technology in teaching mathematics is in the early stages. More research is needed to examine how teachers acquire TPACK and use it to teach geometry with technology effectively. Niess et al. (2009) proposed a TPACK development model for mathematics teachers that can be used in evaluating and describing their TPACK growth. This model consists of five stages (recognizing, accepting, adapting, exploring, advancing) and four themes (curriculum and assessment, learning, teaching, and access) as well as descriptors and examples describing actions in which teachers engage in "while adapting technology in their teaching in order to enhance student learning" (Niess et al., 2009, p. 12). Using this model can assist in identifying the level of teachers' knowledge related to integrating technology into teaching and in providing insights into knowledge growth.

Purpose of the Study

This qualitative case study documented how high school geometry teachers develop and enact knowledge for teaching with dynamic geometry software.

Additionally, this study identified TPACK development levels based on interviews, classroom observations, student activity sheets and a survey. The goal of this study was to contribute to the existing body of knowledge about TPACK and teaching in a dynamic geometry environment.

Researcher's Background

I have four years of experience as a math instructor. Before becoming a high school teacher, I taught developmental mathematics at a community college. Then, I taught mathematics and economics at a large high school in Central Texas. I had an opportunity to work in a program where students' studies were self-paced. This program was designed for students who wanted to accelerate their education (for various reasons). I worked with a diverse population of students and taught algebra 1, geometry, algebra 2, math models, calculus, and TAKS (Texas Assessment of Knowledge and Skills) preparation course. After two years, I decided to become a full-time doctoral student. I began my studies in mathematics education at Texas State University-San Marcos where I had an opportunity to teach developmental mathematics again. After three semesters, I changed my position from teaching to research. As a research assistant, I had the opportunity to work with high school geometry teachers who participated in the DGP (conducted by a research team from Texas State and funded by the National Science Foundation). I established a relationship with those teachers through professional development workshops, classroom visits, interviews, and e-mail communications throughout the 2010-2011 school year. During that year, I became more familiar with the geometry curriculum in Texas high schools, although as a former teacher, I already had a good idea of what it was. Additionally, I learned about instructional strategies used by teachers and the challenges they face when trying to integrate dynamic geometry into teaching and learning of geometry. I also observed that knowledge of geometry, pedagogy, and technology varied substantially among the DGP participants. Therefore, as a mathematics educator, I made my main goal to ensure that teachers have the necessary content knowledge, know appropriate instructional strategies for the subject they teach, and have necessary technological skills to incorporate new technologies effectively into teaching and learning of mathematics.

Researcher's Perspective

As a former high school teacher, I know firsthand that teacher knowledge plays a crucial role in teaching mathematics. I believe that content knowledge (CK), knowledge of mathematics, is the foundation of good quality teaching and that pedagogical content knowledge (PCK) develops as teachers gain more experience. Similarly, I believe that technological pedagogical content knowledge (TPACK) takes time to develop (Olive, 2002). In addition, I think that development of TPACK might take less effort for teachers who already have well-developed PCK than for teachers whose PCK is not well-developed. However, because technology changes all the time, TPACK might not ever be fully developed. Even if we restrict ourselves to one type of technology, such as Sketchpad, continuous learning is necessary because of new features and enhancements in this software.

Having good knowledge of mathematics is very important in today's world. Students who earn their degrees in mathematics are in high demand. Before going to college, students must graduate from high school and in order to do that (in Texas and many other states) they must pass a geometry course. Teachers' geometric knowledge for teaching is essential for students' success and, in today's information-age world, knowledge for teaching in a technology-enhanced classroom is a must. However, teachers face many challenges when trying to integrate technology in their teaching. Some of those challenges include limited access to technology, lack of time for professional development on integrating technology into teaching, lack of support from campus administrators, and lack of knowledge for effective integration of technology into teaching. As difficult as sometimes these challenges might be, I believe teachers should strive to overcome them and improve their TPACK by continuously learning new content, new pedagogies, and developing their technological skills.

Definitions of Terms

The following terms are used throughout this dissertation, and their definitions are provided here:

Technological Pedagogical Content Knowledge (TPACK)

This is "knowledge of the technology-pedagogy-content interaction in the context of content-specific instructional strategies" (Cox & Graham, 2009) and "the total package required for integrating technology, pedagogy, and content knowledge in the design of instruction for thinking and learning mathematics with digital technologies" (Niess et al., 2009).

Technology

Technology consists of "equipment and resources such as calculators, computers, telecommunications, internet, cameras, multimedia, satellites and distance learning facilities, CDROMs, and scanners used for the purpose of instruction when available and appropriate" (TEA, 2005).

Technology Integration

The process of technology integration involves continuous learning about new technologies and resulting in improvement. Technology resources include computers and specialized software. Technology integration entails “the incorporation of technology resources and technology-based practices into the daily routines, work, and management of schools” (NCES, 2002, p. 75).

Dynamic Geometry (DG)

This term refers to active and exploratory geometry carried out with interactive computer software (such as Sketchpad). The characteristic feature of DG software is the continuous and real-time transformation called “dragging” that is not available elsewhere.

Delimitations

This study focused on one type of technology - dynamic geometry software, namely Sketchpad. Also, it involved only high school teachers who taught geometry and integrated Sketchpad in their instruction at the time of this study. In particular, participants in this study were teachers who also participated in the DGP.

Significance of the Study

One of the goals of this study was to contribute to existing research on high school geometry teachers’ TPACK when teaching with dynamic geometry software. Literature in this area of research is limited; therefore, the scholarly body of knowledge is increased by an addition of the case reports offered through this study. Also, the findings along with current literature on TPACK development and professional development offer recommendations for teachers’ professional development in the field of teaching with dynamic geometry software. The findings also offer suggestions for future research in the area of teachers' TPACK. Lastly, this study moves one step closer to developing items that can measure TPACK for teaching with dynamic geometry software. Further development in this area is needed and is left for future studies.

Summary

This chapter identified the need for studying TPACK related to teaching geometry in a dynamic geometry environment. Use of technology has been advocated by many professional organizations and access to technology in schools has increased. Teachers tend to feel sufficiently trained in using technology and integrating it into classroom instruction. However, there is little research on technology integration from the viewpoint of teachers' TPACK. The goal of this study was to investigate how high school geometry teachers integrate dynamic geometry software into instruction and to determine their TPACK development levels.

This dissertation consists of six chapters. The next chapter, Chapter II, discusses literature related to teaching geometry with technology and TPACK. Then, Chapter III describes methodology used for this study; it provides context for the study, introduces the participants, describes the data sources and how they were analyzed. Next, Chapters IV and V present findings of this study. Chapter IV consists of case reports describing TPACK development, TPACK enactment and TPACK developmental levels for each participant. Chapter V provides cross-case analysis. Finally, Chapter VI offers discussion of findings, implications, and suggestions for future research.

CHAPTER II

LITERATURE REVIEW

Those who can, do. Those who understand, teach.

-Lee S. Shulman

This chapter reviews literature relevant to integrating technology into teaching geometry and teachers' technological pedagogical content knowledge (TPACK). The chapter is divided into six sections: pedagogical content knowledge, technological pedagogical content knowledge, assessing teachers' TPACK, use of technology in geometry, the gap in the literature and the theoretical framework. The gap in the literature section provides conclusions from the literature review and offers a design for this research project. The theoretical framework of this study is based on the Mathematics Teacher TPACK Development Model, which describes five stages of how TPACK can grow.

Pedagogical Content Knowledge

In 1986, Shulman introduced the concept of pedagogical content knowledge as a combination of content knowledge and pedagogical knowledge. He claimed teachers not only need to know the subject matter they teach, but they also need to know how to teach it. Pedagogical knowledge is generic, meaning that it can be applied to any subject. For example, a teacher with strong pedagogical knowledge has excellent classroom management skills, knows how to organize student records, etc. Content knowledge is specific, e.g., a mathematics teacher has knowledge of mathematics, whereas a biology teacher has knowledge of biology. Therefore, pedagogical content knowledge is specific because it is related to content knowledge. Pedagogical content knowledge is about knowing how to teach the specific content (e.g., how to teach mathematics); it's about knowing student misconceptions, how to present the content in various ways, etc. (Shulman, 1986).

Technological Pedagogical Content Knowledge

Pierson (2001) investigated how teachers at various levels of technology use (Dwyer, Ringstaff, & Sandholtz, 1991) and teaching abilities (Berliner, 1994) used technology in their classrooms. As a result of her study, she introduced the concept of technological-pedagogical-content knowledge (Pierson, 2001), which defined "effective technology integration" (p. 427) and was built on Shulman's concept of pedagogical content knowledge. Others who discussed the relationships between content, pedagogy, and technology include: Hughes (2004), McCrory (2004), Margerum-Leys and Marx (2002), Niess (2005), and Slough & Connell (2006). In 2006, Mishra and Koehler proposed a conceptual framework for technological pedagogical content knowledge (TPACK), which is comprised of seven domains (see Figure 3).

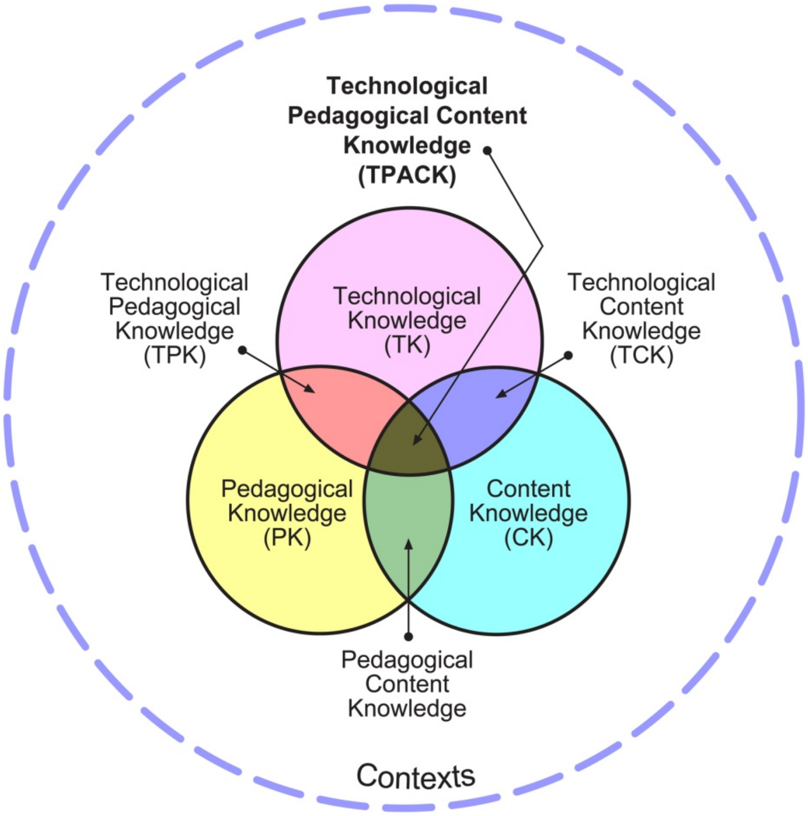


Figure 3. The components of the TPACK framework. Graphic from http://tpack.org/

Content knowledge (CK) is the knowledge about the subject matter to be learned or taught (e.g., knowledge about geometry). Pedagogical knowledge (PK) is a generic form of knowledge (it relates to all subjects) and involves things such as classroom management and student evaluation (Mishra & Koehler, 2006). Technological knowledge (TK) is about knowing standard technologies and skills required for operating particular technologies, e.g., knowing how to install software programs (Mishra & Koehler, 2006). Pedagogical content knowledge (PCK) is about knowing "what teaching approaches fit the content" and "how elements of the content can be arranged for better teaching" (Mishra & Koehler, 2006, p. 1027). Technological pedagogical knowledge (TPK) is the knowledge that can be used with any subject matter, e.g., "knowledge of tools for maintaining class records, attendance, and grading" (Mishra & Koehler, 2006, p. 1028). Technological content knowledge (TCK) is about knowing how to use a technology for a given subject, e.g., MS Excel for statistics, or how to use subject-specific technologies, e.g., Sketchpad (Mishra & Koehler, 2006). Lastly, TPACK is about knowing how to teach a subject with a given technology, "the basis of good teaching with technology" and "pedagogical techniques that use technologies in constructive ways to teach content" (Mishra & Koehler, 2006, p. 1029). Although TCK and TPACK might seem like the same concept, the main difference between the two is that TPACK is about knowing how to *integrate* technology into teaching and learning, and TCK is about knowing how to *use* the technology for a given subject. Students who use technology in learning are likely to possess TCK, but they are not likely to possess TPACK. Teachers, who teach with technology, should have TCK, but they also should have TPACK in order to teach effectively.

Assessing Teachers' TPACK

During the last century, certification tests for teachers have changed their focus. Towards the end of the 19th century, teacher certification tests focused on the subject matter with only five percent of the total points addressed by pedagogical practice. Towards the end of the 20th century, the focus of those tests reversed and "the emphasis was on how teachers manage their classrooms, organize activities, allocate time, . . . plan lessons, and judge general student understanding" (Shulman, 1986, p. 8). Reviewing how teachers' knowledge has been assessed provides more information on how it has been conceptualized and why it is necessary to be concerned with teachers' knowledge.

The concept of Shulman's pedagogical content knowledge (as previously described) was further developed and measured by Hill, Ball, and Schilling (2008). Hill et al. (2008) broke down subject matter knowledge (what Shulman called content knowledge) and PCK to more specific components. Subject matter knowledge consists of common content knowledge (CCK) (what Shulman meant by his subject matter knowledge (Hill et al., 2008)), specialized content knowledge (SCK), and knowledge at the mathematical horizon. PCK, on the other hand, consists of knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of curriculum. The instruments created by Hill et al. (2008) measure mathematical knowledge for teaching (MKT), which is a combination of subject matter knowledge and PCK. Therefore, measuring teachers' MKT focuses on content and also includes understanding of the content "in particular ways needed for teaching it," "what students are likely to make of the content," and "instruction that takes into account both students and mathematics" (Hill et al., 2007, p. 125). MKT items were multiple-choice and were developed from related literature and the researchers' classroom experiences (Hill et al., 2008) in mathematics education at the elementary and middle school levels.

Currently there are two contrasting paradigms for the assessment of teachers' knowledge; one is mainly quantitative (such as the measures developed by Hill et al.), and the other one is qualitative. The following sections provide an overview of assessment of teachers' TPACK and of their geometric knowledge using both paradigms.

Surveys, which involve self-reported data, largely dominate the quantitative types of TPACK assessment. Archambault and Crippen (2009) developed one such measure. Their study examined 596 teachers from around the U.S. who teach various K-12 online courses. A similar survey was constructed by Schmidt et al. (2009) and was piloted on 124 pre-service teachers. Both surveys were designed to measure teachers' knowledge in all seven domains of TPACK and consisted of 5-point Likert scale items (Archambault & Crippen, 2009; Schmidt et al., 2009). Archambault and Crippen (2009) discovered that online teachers who participated in their study felt well about their knowledge in three domains: content, pedagogy, and pedagogical content; however, they felt less confident about their technological knowledge. The findings also revealed a large correlation between content and pedagogy, and a small relationship between technology and content as well as technology and pedagogy (Archambault & Crippen, 2009).

The main goal of Schmidt et al. (2009) was to develop and validate an instrument that would measure pre-service teachers' development of TPACK. After piloting the instrument and making revisions, the authors believe that their instrument is a reliable measure of TPACK. Both instruments (Archambault & Crippen, 2009; Schmidt et al., 2009) are promising starting points in examining teachers' and pre-service teachers' TPACK; however, instead of knowledge, they measured teachers' TPACK self-efficacy. The limitation of these surveys is that they rely on self-report or self-assessment of knowledge. Research shows that gains measured by such instruments reflect an increase in confidence instead of an increase in knowledge (Lawless & Pellegrino, 2007; Schrader & Lawless, 2004). Also, respondents might provide researchers with inaccurate data and their responses are hard to verify.

Sorto and Lesser (2009) developed a measure related to TPACK that did not rely on self-report; they used it for measuring technological pedagogical statistical knowledge (TPSK) of middle school teachers. The assessment included six multiple-choice and open-ended items; the goal for open-ended items was to convert them later to multiple-choice items after eliciting responses from participants (Sorto & Lesser, 2009). The authors conjectured that years of teaching experience and statistics coursework could predict the level of TPSK, but no significant relationships were found in this study. The authors also found different characteristics of teachers' knowledge related to teaching statistics with technology. This was initial work on developing measures of TPSK. The authors planed on constructing "a longer instrument whose items would span technologies in addition to graphing calculators" (Sorto & Lesser, 2009, p. 7) and participants' responses to open-ended questions informed the researchers about answer choices for creating forced-choice questions.

Harris et al. (2010) created a rubric to aid in assessing pre-service teachers' lesson plans with respect to TPACK. The rubric was tested by fifteen experienced technology-using educators with various years of experience and teaching various subjects across all grades; the instrument found to be reliable and valid. The researchers believe that the rubric can be used with experienced teachers, although it was tested on pre-service teachers only. The researchers also were curious about its use as a teaching observation tool for evaluating the quality of technology integration.

Another way to assess teachers' knowledge is mainly qualitative and "draws upon case descriptions of teachers' classroom practices" (Groth, Spickler, Bergner, & Bardzell, 2009, p. 394). One advantage of this approach is that it allows researchers to explore "contextual factors that contribute to the knowledge that teachers exhibit in their classrooms" (Groth et al., 2009, p. 394). Groth et al. proposed an assessment framework based on the lesson study concept (Lewis, 2002) that is frequently utilized in Japan. They collected data from “teachers’ written lesson plans, university faculty members’ reviews of lessons, transcripts and videos of implemented lessons, and recordings and transcripts of debriefing sessions about implemented lessons” (Groth et al., 2009, p. 392). Strengths of this assessment framework include the close connection of assessment and professional development (which can save time and resources) and learning opportunities for teachers during debriefing sessions. The main weakness of this framework was the inability to measure individual teachers’ knowledge as teachers can influence one another during professional development. One notable advice the researchers gave is “to avoid a purely deficit-oriented approach to describing teachers’ TPACK” (Groth et al., 2009, p. 407), meaning that researchers should not concentrate on teachers’ weaknesses, and their lack of knowledge; instead, they should also include teachers’ strengths when they are apparent.

Use of Technology in Geometry

Educational technologies such as dynamic geometry software can be integrated into teaching not only at the high school level, but also at the middle school level as well as elementary level (Olive, 2002). However, there is a need for more research as the literature in this area contains mainly “personal accounts of the powerful learning . . . with dynamic geometry technology” (Olive, 2002, p. 30) and not research studies. Also, teachers take a long time to adapt their teaching to take advantage of the technology (Olive, 2002). This implies that the development of TPACK takes time. The author mentioned that the full integration of the software into teaching is not immediate, and it requires time as well as willingness from the teachers’ side.

Coffland and Strickland (2004) examined what factors affected teachers' use of technology in high school geometry classrooms. The data was collected through a survey that was mailed to geometry teachers in southeast Idaho. Several significant relationships were found: (a) teachers teaching more geometry sections used less technology, (b) teachers with more awareness of computer capabilities had more positive attitude toward technology, and (c) teachers with technology training "in the integration of subject-specific software into their geometry classes" (p. 357) were more likely to use it (Coffland & Strickland, 2004). This study showed that professional development in the area of instructional technology integration could increase the use of technology when teaching geometry; however, the study did not address the type of training needed for technology integration. It also did not provide an answer to why teachers teaching more sections of geometry (and therefore more students) did not use technology as much as teachers teaching less geometry sections. These questions are left for future studies and can be researched from a TPACK perspective.

Roberts and Stephens (1999) investigated the effects of the frequency of computer use in high school geometry and student achievement. Three groups of students were taught by the same teacher: the first group did not use computers, the second group used Geometry Inventor (Brock, Cappo, Dromi, Rosin, & Shenkerman, 1994) once a week, and the third group used the software two times a week. Geometry Inventor is a software program that allows students to construct, measure, and use dynamic features to explore and discovery geometric concepts (Clements, 1995). All students were given the same chapter tests as well as end-of-the-semester exams. The only significant results were found for the introductory chapter and the transformational geometry chapter; students in the non-technology group performed better than students in the other two groups. This study shows that although "using computer software did improve student interest and participation in geometry" (p. 23), some topics might be taught more effectively without using technology. The authors explained that during the activities for the introductory chapter students focused on becoming familiar with the software, which can give a possible explanation to lower scores in the technology groups. Also, test for the transformational geometry chapter was closely related to paper-and-pencil activities, on which the two technology groups did not spend as much time as the nontechnology group. The authors did not mention the role of the teacher in the instruction and said that when students were using computers they worked individually and did not interact with others. This finding implies that the computer had a role of an instructor, possibly because the teacher did not know how to utilize it in a more beneficial way, further implying the need for TPACK development.

Hannafin, Burruss, and Little (2001) conducted an exploratory phenomenological study whose goal was to identify the roles of participants while they used Sketchpad during their geometry lessons. The participants were one teacher and her twelve students in two 7th-grade classes. The study was conducted over three weeks; during the first week, the teacher received training on the dynamic geometry software and, during the following two weeks, she incorporated the software into her teaching. The researchers made classroom observations, distributed surveys and conducted interviews. The teacher also kept a journal where she recorded her reactions. The teacher's role had to change from a lecturer to a facilitator, which was not easy for her because she did not feel comfortable letting her students take charge of their own learning; the teacher felt more comfortable being in control of the classroom. It was difficult for her to change her role during the three weeks and she was not sure how to handle certain situations that arose when using Sketchpad. The authors suggested providing teachers with a sequence of scaffolding questions to help them transition to this type of teaching and so they know what and when to ask. This study shows we cannot simply give teachers a new technology, train them for a week, and expect a miracle. Also, it is difficult to change teachers' style of instruction (if it can be changed at all) and to ask them to incorporate certain technologies into their teaching might not always have a positive impact on teachers themselves or even on students as a result of teachers' resistance to change. Since the researchers advocate for a more student-centered approach to integrating Sketchpad into instruction, it might be beneficial to study teachers' knowledge as part of the technology integration into teaching and learning of geometry. The following study presents one such example.

Knapp, Barrett, and Kaufmann (2007) studied teachers' mathematical knowledge for teaching (MKT), defined by Hill, Rowan, and Ball (2005) as "the mathematical knowledge used to carry out the work of teaching mathematics" (p. 373) and its development by geometry teachers using dynamic geometry software. The purpose of the study was to find the ways in which teachers develop MKT as they prepared and implemented inquiry-based lessons using dynamic geometry with assistance from a collaborative coach. The authors presented the results from one of the four participants who taught 22 Sketchpad-based lessons in her middle school classes over the period of two years. The results indicated that the teacher "focused largely on 'teaching Sketchpad'" (Knapp et al., 2007, p. 1103) during the first year and "teaching geometry using Sketchpad as a tool" during the second year (Knapp et al., 2007, p. 1103). This finding indicates possibly that the teacher has developed her knowledge related to teaching with Sketchpad as she gained more experience in using it in her teaching.

Gap in the Literature

The literature review of teachers' knowledge and technology use revealed that significant improvements had been made in conceptualizing PCK and TPACK during the past two decades. Also, several attempts had been made in assessing TPACK through quantitative and qualitative approaches. However, more research is needed in the area of TPACK and teaching mathematics with technology. An extensive literature review showed that no studies had been conducted on teacher knowledge and integration of dynamic geometry into classroom instruction. Studies on the use of technology in teaching geometry indicated that teachers who incorporated technology into teaching geometry encounter many challenges. Therefore, it is beneficial to look at this issue through the TPACK lens. Also, because of the lack of research in this area, it is rather impossible to create a TPACK assessment similar to that of Hill et al. (2005). Since "psychometrically sound items are costly to develop" (Harris et al., 2009, p. 402) and their development takes time, exploring in depth specific cases should help further define and clarify the construct of TPACK.

Theoretical Framework

Niess et al. (2009) proposed a Mathematics Teacher TPACK Development Model built on the TPACK framework (Mishra & Koehler, 2006). The model consists of a five-stage process through which teachers might go while developing their knowledge when learning a new technology and integrating it into teaching. The five levels of the model derive from Everett Rogers' (1995) model of the innovation-decision process and are based on researchers' observations of teachers learning to integrate spreadsheets into teaching and learning mathematics (Niess et al., 2009). Level 1 is recognizing (knowledge), where teachers are able to use technology and "recognize the alignment of the technology with mathematics" (Niess et al., 2009, p. 9) but do not integrate it into teaching. Level 2 is accepting (persuasion) and involves forming a favorable or unfavorable attitude towards integrating technology in teaching mathematics. Level 3 is adapting (decision), where teachers decide to use or not to use technology in teaching mathematics. Level 4 is exploring (implementation) and involves active integration of technology and teaching mathematics. Lastly, level 5 is advancing (confirmation) and consists of teachers' evaluating "the results of the decision to integrate teaching and learning mathematics with an appropriate technology" (Niess et al., 2009, p. 9). Furthermore, the model is divided into four themes: curriculum and assessment, learning, teaching, and access. The authors also stated, "a mathematics teacher may be at different levels for different themes" (Niess et al., 2009, p. 13). This model presented a useful framework for studying how teachers gain knowledge related to integration of technology in teaching mathematics and identifying the levels of TPACK they have.

Summary

This chapter reviewed literature related to TPACK and teaching geometry with technology. Through the examination, this chapter identified a gap in literature and a theoretical framework for studying teachers' TPACK. The literature review supported the need for studying teachers' TPACK, how it is acquired, and how it is used in classroom instruction. In particular, more research is desirable in the area of teaching geometry with dynamic geometry software through qualitative research methods. The following chapter, Chapter III, describes the design and methodology implemented in this study. More specifically, it identifies the research questions that guided this study and discusses the research design, pilot study, researcher's roles, context of the study, participants, data collection and analysis, building trustworthiness, and ethical issues.

CHAPTER III

METHODOLOGY

The purpose of this study was (a) to document how high school geometry teachers develop and use their technological pedagogical content knowledge (TPACK) for teaching geometry with dynamic geometry software and (b) to describe teachers' TPACK development levels related to integration of dynamic geometry into classroom instruction. Three research questions guided this study:

1. How do high school teachers develop TPACK while teaching geometry using dynamic geometry software?
2. How do high school teachers enact their TPACK when teaching with dynamic geometry software?
3. How are the five TPACK development levels (i.e., recognizing, accepting, adapting, exploring, and advancing) characterized for high school teachers who incorporate dynamic geometry software in teaching?

The remainder of this chapter describes the methods used to conduct this study and consists of the following sections: research design, pilot study, researcher's roles, context of the study, participant selection, participants and setting, data collection, data analysis, building trustworthiness, and ethical issues.

Research Design

The research questions about teachers' knowledge when integrating dynamic geometry software into teaching indicated a qualitative method of inquiry. Denzin and Lincoln (2005) defined qualitative research as:

> a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that make the world visible. These practices transform the world. They turn the world into a series of representations, including field notes, interviews, conversations, photographs, recordings, and memos to the self. At this level, qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative researchers study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them. (p. 3)

The goal of this study was to provide a rich and descriptive account of teachers' use of dynamic geometry software in their teaching practices. I used the case study methodology (Yin, 2009) to examine the TPACK development levels of high school geometry teachers and how they used their TPACK when integrating dynamic geometry into their teaching. Creswell (2007) defined case study research as:

> a qualitative approach in which the investigator explores a bounded system (a *case*) or multiple bounded systems (cases) over time, through detailed, in-depth data collection involving *multiple sources of information* (e.g., observations, interviews, audiovisual material, and

> documents and reports), and reports a case *description* and case-based themes. (p. 73)

In this study, bounded systems were teachers and their professional practices. Multiple sources of data included interviews, observations, documents, a self-report survey, implementation questionnaires, and informal conversations with participants. Yin (2009) suggested the use of multiple sources of evidence to ensure construct validity because they “essentially provide multiple measures of the same phenomenon” (p. 116). To ensure the reliability of the entire case study (Yin, 2009), I developed a case study database containing all the raw data, so that other researchers can inspect the evidence and “not be limited to the written case study reports” (p. 119).

Pilot Study

I conducted a qualitative pilot case study in spring 2011 with one participant. The study incorporated purposeful sampling and the participant selection criteria used were geographic proximity and a well-established relationship between the participant and me. Data sources consisted of two observations, two interviews, and student handouts. The pilot study helped in refining data collection plans, what data to collect and what procedures to follow. After conducting the initial interview for the pilot case study, I made significant changes to the interview protocol to better elicit participants’ responses related to their TPACK. Based on those revisions, during the closing interview I asked additional questions, which became part of the initial interview protocol for the main study. I also extended the observation protocol to focus more on TPACK enactment. The pilot study also provided an interesting Sketchpad teaching task (see Appendix A); I included this task in the closing interview for the main study. Finally, the pilot study assisted in assessing time and resources needed to conduct the main study.

Researcher's Roles

I assumed several roles while conducting this research study. First, because of the qualitative nature of this study, I was a human instrument for data collection (Merriam, 2009); I interacted with participants through interviews and observations. During observations, I assumed a role of an "observer as participant" (Merriam, 2009, p. 124). Based on Merriam's definition of "observer as participant," the teachers and their students were aware of my activities; my participation in observed lessons was secondary to gathering information through taking notes and videotaping. Second, besides being an instrument for data collection, I was also an instrument for analyzing the collected data. Lastly, I was a learner; as I collected and analyzed the data, I gained deeper knowledge of the concept of TPACK and the integration of dynamic geometry into classroom instruction.

Context of the Study

I conducted this study within the context of the Dynamic Geometry Project (DGP). The DGP is a research project whose main objective was to compare two approaches to the teaching and learning of high school geometry. One approach utilized dynamic geometry (DG) in teaching and learning and the other one did not. The DGP is a four-year research project, which contains two consecutive years of implementation of the DG treatment. I refer to the two years as "the first year of the DG implementation" and "the second year of the DG implementation" respectively in this dissertation. I conducted this study during the second year of the DG implementation. Participants of the DGP were geometry teachers from several school districts, all teaching high school level geometry. There were approximately sixty teachers, half of them in the DG group and half in the comparison group.

The focus of this study was on teachers in the DG group since they were the ones integrating dynamic geometry into classroom instruction. The teachers participated in professional development workshops facilitated by the DGP staff. During the first year of the DG implementation, the professional development consisted of a five-day summer workshop and six half-day Saturday workshops offered throughout the school year. During the second year of the DG implementation, the professional development consisted of one-day summer workshop and three half-day Saturday workshops offered throughout the school year. Teachers received stipends for their participation in those workshops. During the professional development sessions, participants became familiar with Sketchpad, learned about geometric concepts using Sketchpad, created lesson plans, collaborated with peers and shared some of their teaching strategies and activities with one another. Participants received curriculum materials that focused on integration of Sketchpad into learning and teaching of geometry; they also obtained access to the Sketchpad Lesson Link, a website (http://www.keypress.com/x26771.xml) with hundreds of Sketchpad activities aligned to textbooks, state standards, and the Common Core State Standards. The DGP staff asked teachers to use Sketchpad with their students in a lab setting (or in their classroom if laptops were available) twice a week so that students could experience dynamic geometry on a regular basis. The DGP staff also encouraged teachers to use Sketchpad for demonstration purposes whenever students did not have access to laptops or a computer lab.

Participant Selection

I used purposeful sampling to select four participants for this multiple case study. Patton (2002) explained that:

> The logic and power of purposeful sampling derive from the emphasis on in-depth understanding. This leads to selecting information-rich cases for study in depth. Information-rich cases are those from which one can learn a great deal about issues of central importance to the purpose of the research, thus the term purposeful sampling. (p. 46)

To select information-rich cases, I wanted to ensure that each teacher's students have opportunities every week to explore geometric concepts using Sketchpad, and that those teachers displayed a high fidelity of implementing the DG approach in teaching. Therefore, to identify potential participants, I looked at data collected by the DGP staff during the first year of the DG implementation to see which teachers fulfilled the requirements, and I compiled a list of approximately ten teachers. Initially, I sent an invitation letter (see Appendix B) via email to three teachers. One of them agreed to participate, one declined, and one did not respond. A couple of days later, I sent three more invitations. Two teachers agreed to participate and one declined. Once again I sent two more invitations, and both teachers agreed to participate. As a result, there were five participants at the beginning of this study. However, when trying to schedule the initial interview, one of the teachers never responded; therefore, four participants remained for the rest of the study.

Participants and Setting

This study took place in four high schools in a large city in Texas. The participants of this study were four geometry teachers (Brian, James, Susan and Laura, all pseudonyms) who participated in the DGP during both implementation years. Table 1 shows the four participants, their length of Sketchpad use in teaching prior to this study, number of years they had taught, and whether or not they used Sketchpad as college students. All participants were teaching at different schools, and all, except Susan, worked in the same school district. The order in which I listed the participants is the order in which I conducted the initial interviews. I use this order throughout this dissertation.

Table 1

Participants and Their Teaching Experiences

Participant (Pseudonym)	Sketchpad experience as a learner	Total teaching experience (years)	Teaching geometry experience (years)	Teaching with Sketchpad experience (years)
Brian	No	2	2	1
James	No	4	4	1
Susan	Yes	19	9	7
Laura	Yes	2.5	2	1

Table 1 shows that the four participants represented a wide spectrum of experiences. Three teachers (Brian, James and Laura) were novice teachers with less than five years of teaching experience, and Susan was an experienced teacher with more than five years of experience. Susan was also the only teacher with experience in using Sketchpad in teaching for more than one year (meaning she had used it before participating in the DGP). Additionally, Susan and Laura used Sketchpad in learning geometry in college. The following paragraphs describe each participant in more detail.

Brian

He was the least experienced teacher in this study and had taught geometry for two years before the study. However, he was the most experienced geometry teacher at his school. Brian heard about Sketchpad for the first time while he was doing his student teaching, which was also geometry; however, he did not use Sketchpad at that time. During his first year of teaching, he used some sketches provided by his district for demonstrations in the classroom. He did not create any sketches nor had his students do any explorations with Sketchpad before his participation in the DGP.

James

He had taught mathematics (including geometry) for four years before this study. This was his fifth year teaching and his third year working at his current school; he taught for two years in another city in the same state before teaching at his current school. Before signing up for the DGP, James did not know anything about Sketchpad. One of the teachers at his school told him about the project and about Sketchpad. Once he took a look at Sketchpad, he wanted to integrate it into his instruction and thought that signing up for the DGP would be a convenient way to learn about this dynamic geometry software.

Susan

She was the most experienced teacher and had the most experience in using Sketchpad in teaching (8 years). She started her teaching career as a middle school teacher. Later she began teaching geometry at the high school level, and this was her fifth years as the only geometry teacher at her current school. She also used Sketchpad as a graduate student while working on her master's degree, but that was after she used it in teaching.

Laura

She used Sketchpad as a college student; however, she did not use it for teaching until she signed up for the DGP and ended up in the DG group. Laura had computers in her classroom, so her students had easy access to technology (which was not always the case), but she had hoped to be in the comparison group of the DGP because she preferred to use manipulatives with her students instead of using technology.

Data Sources

The main sources of data included interviews and observations. The supporting sources of data included documents, a survey, implementation questionnaires, professional development attendance records, and the researcher's log. All data collection took place in the participants' classrooms during regular school hours. Table 2 summarizes all data sources and the following paragraphs describe them in detail.

Table 2

Summary of Data Sources

Data Sources	Details
Initial interview	30-40 minutes, semi-structured, open-ended
Observations	2-4 per teacher, 45 minutes each, videotaped, focus on teacher
Documents	Student activity sheets used during the observed lessons
Closing interview	20-30 minutes, open-ended questions, different for each participant, teaching-with-Sketchpad task
TPACK Survey	11 groups of 5 statements organized by themes and descriptors from the TPACK Development Model
Implementation questionnaires	Selected questions from the DGP implementation questionnaire related to TPACK
The DGP professional development attendance records	Number of hours/sessions of participation in the DGP professional development
Researcher's log	Ideas, initial thoughts after observations and interviews, and memos to self

Initial Interview

The initial interview was the first step in data collection and coincided with the beginning of a new school year. Each interview lasted approximately 30-40 minutes and took place during the teacher's conference period on a regular school day. Each participant received the interview questions as well as a consent form (see Appendix C) via email the day before the interview. At the beginning of the initial interview, each participant signed the consent form.

The data collected during the interviews provided information on how participants developed their TPACK and how they incorporated it into teaching geometry with Sketchpad. I developed an interview protocol (see Appendix D), which guided the initial interview. The TPACK Development Model framework informed the creation of several interview questions such as: Why did you decide to teach geometry with Sketchpad? In addition to your participation in the DGP, what kind of activities (e.g., professional development, conferences, self-directed study, Internet resources) have you engaged in that lead you to adopt teaching and learning geometry with Sketchpad? The interview protocol developed by Niess, van Zee, and Gillow-Wiles (2010), also based on the TPACK Development Model, suggested additional questions such as: What is your current view and understanding about integrating Sketchpad as a learning tool in geometry? How has your knowledge about students' understanding, thinking, and learning about geometry topics with Sketchpad changed through your work in the DGP (or since you started using Sketchpad in the classroom instruction)? This was a semi-structured open-ended interview. Some characteristics of such interviews include: "interview guide includes a mix of more and less structured interview questions," "usually specific data required from all respondents," and "largest part of interview guided by list of questions or issues to be explored" (Merriam, 2009, p. 89).

Observations

The purpose of the observations was to see how teachers enacted their TPACK when teaching with Sketchpad and to identify their TPACK development levels. Stake (2006) asserted that observations are "the most meaningful data-gathering methods" (p. 4) because "it is important to describe what the case's activity is and what its effects seem to be" (p. 4). It was essential to observe how teachers enacted their TPACK during classroom instruction because that was how they used their knowledge in practice. Guba and Lincoln (1981) said:

> In situations where motives, attitudes, beliefs, and values direct much, if not most of human activity, the most sophisticated instrumentation we possess is still the careful observer - the human being who can watch, see, listen, question, probe, and finally analyze and organize his direct experience. (p. 213)

Therefore, observations of participants in their natural environments (here, teachers in classrooms) provide rich information and assist in triangulation of the data from the interviews and other sources.

I developed an observation protocol (see Appendix E) to assist me in focusing on relevant information during observations. The focus of each observation was on the teacher. The main goal was to identify strengths and areas for improvement of teachers' practices related to their TPACK and integrating Sketchpad in teaching. I observed and videotaped multiple lessons of each teacher, which took place in classrooms and computer labs while teachers were teaching geometry with Sketchpad. Susan was the only participant that I did not videotape because her principal did not allow it. However, I was able to audiotape the observed lessons, and I took more detailed notes during those observations. I observed the same group of students (for each teacher) and took notes on a laptop or an iPad. The length of each observation for all teachers was approximately 45 minutes, the full length of one class period.

Documents

I collected documents related to the observed lessons, which consisted of student activity sheets that teachers either prepared themselves or printed from the Sketchpad

Lesson Link. These documents provided more information about the teacher's intended lesson and how her or his TPACK was reflected in the lesson plan.

Closing Interview

The closing interview took place at the end of the study. The main purpose of this interview was to clarify selected episodes that I noticed during observations. In addition, I wanted to understand better why teachers guided their classroom instruction the way they did when teaching with Sketchpad and how their actions related to their TPACK. The interview questions came from my preliminary analysis of the initial interview and observations. The questions were open-ended and gave an opportunity for each participant to explain how she or he would use her or his TPACK to improve her or his teaching with Sketchpad in the future. I also presented a teaching episode from the pilot study (see Appendix A) to the participants and asked them to reflect on it and explain how they would improve the given learning-teaching situation. Finally, participants were asked to fill out the TPACK Development Model Self-Report Survey.

TPACK Development Model Self-Report Survey

The TPACK Development Model Self-Report Survey (TPACK Survey, see Appendix F) was developed by Ivy (2011) and Riales (2011) and based on the TPACK Development Model (Niess et al., 2009). Margaret Niess reviewed the survey and provided feedback to Ivy and Riales. My purpose for using the survey was to find out what the participants' perceptions were about their TPACK related to Sketchpad integration in teaching. The survey consisted of fifty-five statements organized into eleven groups of five statements. One statement in each group described one of the five TPACK development levels (i.e., recognizing, accepting, adapting, exploring, and advancing), and the eleven groups represented eleven theme/descriptor pairings from the TPACK Development Model. Participants were to choose one statement from each of the eleven groups. Their responses provided data related to their self-perceived TPACK development levels.

Implementation Questionnaires

Since the participants of this study were part of a larger project, the DGP, they completed am implementation questionnaire every 4-6 weeks. The questions on the questionnaires were related to their integration of Sketchpad in teaching and learning. Selected data from the questionnaires were used in this study, in particular the participants' responses to two questions: (a) How many times per week did the students work in a computer lab/classroom using Sketchpad software; (b) How many times per week was the geometry class taught in a classroom with one demonstration computer? Answers to these questions provided information on how often the participants got to use their TPACK in practice.

The DGP Professional Development Attendance Records

I compiled the participants' attendance data from the DGP professional development sessions, which took place over the two years of the DG implementation. Summing up all of hours of attendance for each participant aided in answering the first research question.

The Researcher's Log

After conducting interviews and observations, I recorded ideas, initial thoughts, and memos to self. I also recorded pertinent information from my informal conversations with participants, e.g., e-mail correspondence and conversations before or after the observed lessons.

Data Analysis

Data analysis took place simultaneously with data collection and was a reflective and ongoing process. Since teachers were the primary units of analysis, I analyzed data collected from each participant separately and wrote a rich description of each case. First, I focused on answering the first two research questions, specifically how each participant developed and enacted her or his TPACK. Second, I identified TPACK development levels for each participant based on the collected data and the case descriptions. This portion of data analysis focused on answering the third research question. Then, I performed cross-case analysis (Yin, 2009) to identify common themes among the four cases. Since each case was unique, I also looked for any significant differences among the four cases. The cross-case analysis supplemented the research findings and provided additional information for answering the research questions. Table 3 summarizes data analysis for each data source, and the following paragraphs provide details of the analysis of interviews, observations, and supporting data sources.

Table 3

Summary of Data Analysis

Data Sources	Data Analysis
Initial interview	Transcribed, reviewed for accuracy, used codebook to identify and describe TPACK development levels
Observations	Reviewed observation notes and videos, clipped episodes that involved TPACK, transcribed conversations
Documents	Assisted in describing TPACK enactment
Closing interview	Cross-case analysis of the teaching-with-Sketchpad task
TPACK Survey	Identified self-perceived TPACK development levels
Implementation questionnaires	Provided the frequency of Sketchpad use and challenges to integrating Sketchpad in teaching
The DGP professional development attendance records	Identified the number of hours/sessions of participation in the DGP professional development
The researcher's log	Assisted in the overall analysis

Initial Interviews

The first step into data analysis was to transcribe the audiotaped interviews. After transcribing, I listened to the audio recording again and at the same time read through each transcript carefully to check for accuracy. Next, I read the transcript again in order to identify parts associated with the levels, themes and descriptors of the TPACK Development Model (Niess et al., 2009). I highlighted the relevant quotes and noted the TPACK development level (i.e., recognizing, accepting, adapting, exploring, advancing) and the theme (curriculum and assessment, learning, teaching, access) by using a codebook for content analysis (Patton, 2002) that I created based on the TPACK Development Model (see Appendix G). This was a deductive analysis "where the data are analyzed according to an existing framework" (Patton, 2002, p. 453), where the existing framework was the TPACK Development Model. This process assisted in identifying the correct levels of TPACK for each participant and facilitated in answering the research questions. At times I noticed other intriguing themes emerging from the transcripts, so to keep myself focused on the research questions, I returned to the TPACK Development Model, the five levels, the four themes and their descriptors.

Observations

After coding the initial interview, I reviewed my notes from the observations and the videos several times. I clipped parts of the videos related to TPACK and transcribed the associated conversations. I coded them using the same codebook (see Appendix G) as for the initial interview.

Supporting Data Sources

Recording participants' responses to the TPACK Survey identified their self-perceived TPACK development levels. Because each response identified a TPACK development level for one theme-descriptor pairing, and the order of the statements (for each group of five statements) corresponded to the five TPACK development levels, from the lowest (recognizing) to the highest (advancing), no further analysis was needed of the survey alone. Later, I compared the self-perceived levels of TPACK reported through the survey with the TPACK development levels identified through other data sources.

Next, I compiled the responses to the two Implementation Questionnaire questions related to the frequency of computer use for students and teachers. The data spanned from the end of August 2011 to the end of February 2012 for a total of twenty-two (22) weeks. The data assisted in identifying the TPACK development levels for the access theme, usage descriptor. In addition, I looked at participants' open-ended responses and identified those that were related to TPACK. They provided information about the challenges in incorporating Sketchpad in the classroom instruction.

Student activity sheets contributed in describing the observed lessons. The DGP professional development attendance records assisted in finding the total number of hours and sessions that participants took part in. The researcher's log provided initial thoughts and analysis that contributed to the overall analysis.

Finally, I triangulated the data from all data sources to crosscheck themes that emerged from several sources and to strengthen the research findings. As a result of the data analysis, I was able to identify TPACK development levels across different themes for all participants and consequently answered the three research questions. Additionally, I performed cross-case analysis to compare and contrast TPACK development and enactment among the participants. The cross-case analysis was especially helpful in identifying challenges in Sketchpad integration and providing suggestions for TPACK professional development.

Identifying TPACK Development Levels

To answer the third research question, I identified TPACK development levels for each participant. As mentioned earlier, I utilized a codebook to perform this part of data analysis and to identify the correct TPACK development levels across the eleven descriptors from four themes of the TPACK Development Model (Niess et al., 2009). I began this process after performing the preliminary analysis for the first two research questions. I reviewed the case descriptions carefully and identified TPACK development levels that were present. Next, I checked for which descriptors TPACK development levels were still not identified. I returned to the interview transcripts and the observation data to look for the specific instances where the missing TPACK development levels were present. This process aided in completing the case reports.

After identifying all of the TPACK development levels for each participant, I checked across the four cases to make sure that the same levels were assigned for the equivalent actions. Additionally, I double-checked that the corresponding actions had the same levels assigned across the four cases. Finally, I compiled a table that indicates which data sources contributed in identifying TPACK development levels for each theme-descriptor pairing (see Table 4 for details). The collected data, with the exception of the TPACK Survey, did not identify TPACK development levels for the curriculum and assessment theme, assessment descriptor.

Table 4

Sources of Data that Identified TPACK Development Levels by Themes and Descriptors

Theme (descriptor)	Interviews	Observations	Supporting data sources
Curriculum & Assessment (curriculum)	✓	✓	✓
Curriculum & Assessment (assessment)			
Learning (mathematics learning)	✓	✓	
Learning (conception of student thinking)	✓	✓	✓
Teaching (mathematics learning)	✓	✓	
Teaching (instruction)		✓	
Teaching (environment)	✓	✓	
Teaching (professional development)	✓		✓
Access (usage)	✓	✓	✓
Access (barrier)	✓	✓	✓
Access (availability)	✓	✓	

Note. Data sources that provided evidence for TPACK development levels for each theme-descriptor pairing are indicated by a check mark.

Building Trustworthiness

When conducting this research study, I adopted a stance of *neutrality* (Patton, 2002); I attempted "become aware of and deal with selective perception, personal biases, and theoretical predispositions" (Patton, 2002, p. 51). Because of my role as an instrument in data collection and analysis, I engaged in careful reflections on potential sources of bias and dealt with them. My main predisposition was to focus on participants' flaws in instruction during observed lessons. To deal with it, I constantly reminded myself that it was not my goal for this study and kept referring to the TPACK Development Model as my lens for identifying accurate and reliable findings. I also returned to the research questions to ensure I was staying on track and not looking at irrelevant issues.

To produce high-quality data that are credible and trustworthy, I conducted a pilot study to refine the data collection instruments. In addition, I collected data from different sources, triangulated the data, and performed member checking. Triangulation of the data, also known as "the most well known strategy to shore up the internal validity of a study" (Merriam, 2009, p. 215), took place in many ways: through checking what I learned through interviews with what I noticed during observations, comparing findings from interviews and observations with those from the TPACK Survey, conducting follow-up interviews and multiple observations of the same participant. Member checking is "the most critical technique for establishing credibility" (Lincoln & Guba, 1985, p. 314). Member checking involves "taking data, analyses, interpretations, and conclusions back to the participants so that they can judge the accuracy and credibility of the account" (Creswell, 2007, p. 208). According to Stake (1995), participants should "play a major role directing as well as acting in case study" research (p. 115). I took preliminary analyses to participants in this study and asked them for their opinions, and if they would add anything that is noteworthy that I omitted.

Ethical Issues

Before beginning the study, I obtained formal approval from the Institutional Review Board at Texas State to conduct this research project. I informed all the participants that their participation was voluntary and that they could withdraw at any time. I also asked them to sign a consent form, which explained the study in more detail. I assured them that all the data collected was going to be kept confidential and that I would protect their privacy. I masked their names in all of my reports and used pseudonyms instead. Through the participants and their school districts, I obtained parents' permissions to videotape their students. One participant's principal did not grant permission to videotape; therefore, I only recorded the audio during observations of that teacher.

Summary

This chapter introduced three research questions that guided this qualitative case study. A pilot study assisted in refining data collection plans and provided additional suggestions for data sources. The DGP was the context of this study and four of its teachers were participants in this study. The participants had varying experience with respect to teaching geometry and teaching with Sketchpad. Sources of data included: initial interview, observations, documents, closing interview, a survey, implementation questionnaires, professional development attendance records and the researcher's log. A deductive content analysis based on the TPACK Development Model aided in data analysis. The pilot study, multiple data sources, triangulation of the data, and member checking assisted in producing high-quality data and assuring credibility and internal validity. The data collected through this study aided in answering the research questions and in identifying the participants' self-perceived and enacted TPACK development levels. Findings are presented in the following two chapters, Chapter IV and Chapter V. Chapter IV consists of four case reports describing TPACK development, TPACK enactment and TPACK developmental levels for each participant. Chapter V provides cross-case analysis.

CHAPTER IV

CASE REPORTS

The main goal of this study was to provide a rich and descriptive account of teachers' technological pedagogical content knowledge (TPACK), i.e., the knowledge related to their use of dynamic geometry software in classroom instruction. This study employed a case study methodology, and multiple data sources assisted in answering the research questions. This chapter describes the findings of this study through four case reports. Each report is divided into three sections: TPACK development, TPACK enactment and TPACK developmental levels. The three section titles correspond to the three research questions:

1. How do high school teachers develop TPACK while teaching geometry using dynamic geometry software?
2. How do high school teachers enact their TPACK when teaching with dynamic geometry software?
3. How are the five TPACK development levels (i.e., recognizing, accepting, adapting, exploring, and advancing) characterized for high school teachers who incorporate dynamic geometry software in teaching?

Case 1: Brian

Brian was in his third year as a teacher at the time of this study. He has taught geometry since he started teaching. At the time of this study, he was in the DGP for the second year, and was teaching three sections of geometry and three sections of algebra 1. During the previous year, he had six sections of geometry. Although he was a novice teacher, he had the most experience in teaching geometry at his school and was serving as a geometry leader. In the geometry syllabus, he mentioned the use of dynamic geometry in the classroom. Other than Sketchpad, he had used only one other dynamic geometry program; he mentioned using Cabri Junior on a graphing calculator only once or twice in the past.

TPACK Development

This section describes Brian's TPACK development journey with respect to Sketchpad integration in classroom instruction. All quotes in the following paragraphs come from the initial interview with Brian.

Brian used Sketchpad during his first year of teaching "just a small handful of times" by incorporating the sketches provided by his school district for demonstration. This indicated an overall recognizing level of TPACK. Just before signing up for the DGP, he attended a conference session where he learned more about Sketchpad. However, it was not until the DGP that he learned "what the program [Sketchpad] really had to offer," indicating an overall accepting level of TPACK. After one year in the DGP, he felt confident when teaching with Sketchpad, indicating an overall adapting level of TPACK—"not a hundred percent, but confident enough."

According to the DGP professional development attendance records, Brian attended all of the sessions offered by the DGP except one half-day session, for a total of approximately 69 hours. The fact that Brian focused on learning only one type of technology suggests the adapting level of TPACK for the teaching theme, professional development descriptor. As a geometry leader at his school, Brian has been actively promoting the use of Sketchpad for learning geometry and sharing what he learned during the professional development sessions with his colleagues at work, indicating on overall exploring level of TPACK.

Brian decided to use Sketchpad in the classroom because:

> Once I figured out what the program had to offer—that it was going to be useful in the classroom—and also the availability—here they basically gave me my own computer lab, so I have access to it all the time. They made it really easy for me.

This statement shows that Brian saw the potential of incorporating Sketchpad into classroom instruction. Also, he had easy access to a computer lab, so he could take his students there anytime. However, Brian realized that knowing how to use Sketchpad and how to integrate it in classroom instruction were not the same thing as he stated in the initial interview:

> Being good with the program and being able to teach are two separate things, and I want to make sure I focus on that—that I can teach with it. I think I want to integrate it into the classroom more, not just the computer lab. It has a lot more application than I am using it for. I definitely use it more in the computer lab than I do in the classroom. I think it has just as much place in the classroom. That is what I need to look at doing more.

Brian expressed in this statement a desire to learn how to teach with Sketchpad and develop his TPACK in addition to knowing how to use Sketchpad. Also, he mentioned a goal for integrating Sketchpad as a teaching tool in the classroom, not just as a learning tool in a computer lab. This showed that when he incorporated Sketchpad into classroom instruction, it was in a learner-centered environment (computer lab). However, he wanted to start using it in the classroom, too, as a demonstration tool.

Brian's conceptions about teaching geometry have not changed through incorporating Sketchpad into classroom instruction because he started using it only one year after he had started teaching, i.e., he was "still building a lot of my conceptions of my teaching of different topics." This indicateed that he was developing his TPACK at the same time as he was developing his pedagogical content knowledge.

Brian mentioned, "teaching with Sketchpad is helpful to students and I want to continue to use it." At the same time, he saw that not all geometry topics were easily compatible with Sketchpad: "Some of the lessons or some of the units cater to it [Sketchpad] a little bit more." He thought that lessons on lines and angles accommodated the integration of Sketchpad better than lessons on three-dimensional figures.

Sketchpad integration was important to Brian; he was glad that he decided to use it in teaching because "it allows me to teach things that I think would be difficult to teach in the classroom or difficult to show" without Sketchpad, indicating an overall adapting level of TPACK. In addition, he mentioned in the initial interview that his students' parents enjoyed the fact that their children got to use technology in the classroom. Furthermore, school and district administrators liked to see parents happy and technology in math classrooms.

TPACK Enactment

This section describes how Brian enacted his TPACK—i.e., how he put his combined knowledge of Sketchpad, geometry, and pedagogy into practice. Although this section focuses on TPACK enactment, it also provides additional information about Brian's TPACK development through his DG implementation. All quotes that appear in this section, with the exception of quotes and dialogs that appear in the triangular thinking lesson subsection, come from the initial interview.

According to the initial interview, during the first year of the DG implementation, Brian incorporated Sketchpad in teaching in the following two ways: introducing new topics in the classroom first and then going to a computer lab to explore them or introducing new topics in a computer lab first and then debriefing them in the classroom. Through this experimentation, he found that he preferred the latter:

> Yes, generally I like to do it that way. They are introduced to it, they find it on their own, and then we discuss it. I think it is better when they find it first on their own. Although it is a little harder, I think it is better for them to see it for the first time by discovering it versus being told.

He also brought this up in one of his implementation questionnaire responses. He stated that if there was a concept that could be investigated with Sketchpad, he preferred to allow his students to discover it in the lab and then "refine and define the idea as a class." The observation on the triangle congruence lesson confirmed Brian's belief. Students were exploring concepts new to them in a computer lab, and after the lesson, Brian mentioned to me that he was going to review students' work and discuss it further with the class the following day.

Although Brian tried to take his students to a computer lab approximately twice a week during this school year, according to the implementation questionnaire data, he got to do it on average once every other week. The infrequent use of Sketchpad indicated the adapting level of TPACK for the access theme, usage descriptor. Even though he had good intentions and access to a computer lab at any time, there were other things that he had no control over that prohibited him from fulfilling his goal. One of them was the implementation of a new curriculum in which the order of topics was different from what he was accustomed to. In one of his implementation questionnaire responses, Brian expressed his dislike for the new pacing calendar by saying "it has an odd order" and that it did not accommodate the DG implementation. The new scope and sequence required more time spent on planning lessons, which in turn made it challenging to plan ahead and to integrate Sketchpad in any lessons. Because of the challenges with the new scope and sequence, Brian integrated Sketchpad into teaching only in some units. This suggested the adapting level of TPACK for the access theme, barrier descriptor. Still, he had noticed that students were "learning better" from this new approach "because it is allowing them to discover, so it is benefiting their learning, it is getting them more engaged in it than if they are sitting at their desks."

At the beginning of each observed lesson, Brian gave a handout to his students with instructions for each exploration. He created his own activity sheets for those Sketchpad-enhanced lessons, which indicated the exploring level of TPACK for the curriculum and assessment theme, curriculum descriptor. Originally those activities did not incorporate any technology, so Brian thought about how he could integrate Sketchpad into them and created technology-based lessons.

Brian described his current view about integrating Sketchpad as a learning tool in geometry as follows:

> In the lab, they find it on their own, they come up with some idea, and we decide "Yes, that is actually a theorem" and we go on proving from there. So instead of writing theorems in the classroom and accepting them, we discover the relationships, come up with conjectures, and actually prove it. It allows them to understand geometry much better than just know geometry. I think that has been a big difference.

He saw the benefits of having his students explore geometric concepts and discover relationships instead of just telling them what those relationships were. What he stated in the initial interview was clearly visible during the observed lessons. Students were exploring, measuring, observing, finding relationships, and forming and justifying conjectures. Creating an environment where students were engaged and self-directed in learning geometry suggested the advancing level of TPACK for the teaching theme, environment descriptor.

As his students worked on Sketchpad activities during the observed lessons, Brian circulated throughout the room and answered students' questions if they had any. He also asked guiding questions if he noticed that students were not following the instructions on the handout carefully. His actions during the observed lessons confirmed what he said during the initial interview, "I like to think that when they are in the lab, they are learning it more on their own, and I just make sure they stay on track." The following lesson description provides several dialogs that Brain had with his students during class and confirms what he stated in the initial interview about "discovery" and guiding his students in their learning.

Triangular thinking lesson. During the triangular thinking lesson, students were examining the relationships formed by constructing midsegments of a triangle. Because the goal of the lesson was to come up with different conjectures related to the midsegments of a triangle, Brian integrated Sketchpad to help students with their thinking and understanding of this topic. This use of Sketchpad indicated the advancing level of TPACK for the learning theme, mathematics learning descriptor.

Based on the student activity sheets created by Brian, students were to construct an arbitrary triangle, its midsegments, and make conjectures about the angles, segments, and any shapes that they formed. While students were working on this activity, Brian noticed that some students were measuring many things, but they were not coming up with any conjectures (see Figure 4 for example), so he asked guiding questions, e.g., “Do you notice any relationships? What conjectures can you make?” These questions helped students focus on the objective of the lesson, i.e., forming conjectures.

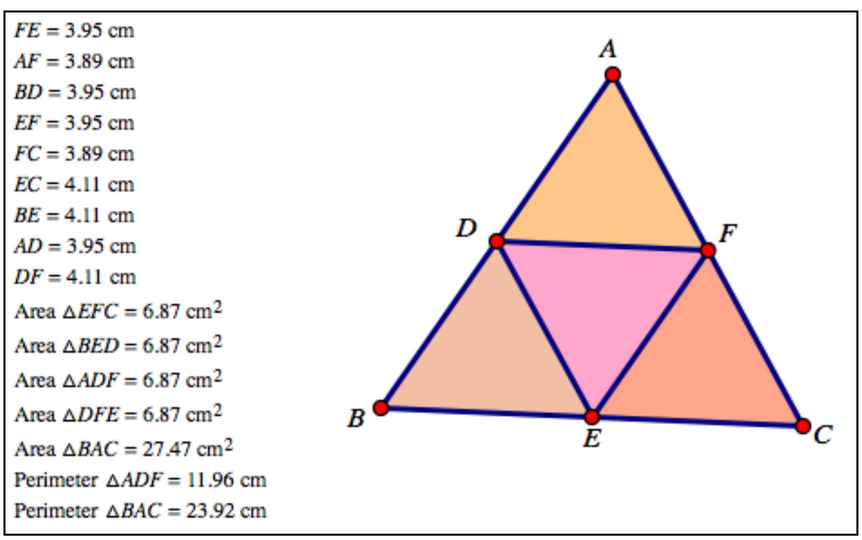


Figure 4. Screenshot of one student’s measurements during the triangular thinking lesson.

Brian noticed that a student, Anna, calculated the areas of the original triangle and one of the smaller triangles formed after constructing the three midsegments. The following is the dialogue between him and Anna:

Brian: How do they compare?

Anna: We are going to do the rest.

Brian: Well, there are a lot of things that you can do. But how do those two compare? [He was pointing to the two area measures of triangles.]

Anna: It is smaller. [Referring to one of the four smaller triangles.]

Brian: Just smaller? Randomly smaller?

Anna: Because there are four of them.

Brian: There are four of them! You think if you had four of these [small triangles], what should it come up to?

Anna pointed to the area measurement of the original triangle and then multiplied the area of the small triangle by four. The product was equal to the area of the original triangle allowing Anna to form a conjecture with a justification.

This episode illustrates how Brian steered his students' learning and conjecture making by questioning. By prompting Anna, Brian guided her in making a conjecture and providing a justification for that conjecture. A similar conversation happened with another student, Beth, who measured areas of the original triangle and one of the smaller triangles. However, Beth already had a conjecture—"The area of one of the smaller triangles is 1/4th the area of the big triangle"—but she did not calculate the ratio of the two area measurements. Brian asked her how she knew that her conjecture was true.

Brian: You said that is one fourth, but you have not checked it. Go to Number, Calculate, click [the measure of the small triangle's area] and divide by [the measure of the large triangle's area]. So is that one-fourth? (see Figure 5)

Beth: Yes.

Brian: Is it true always? Try dragging it around.

Beth dragged vertex B and the ratio of the areas remained constant.

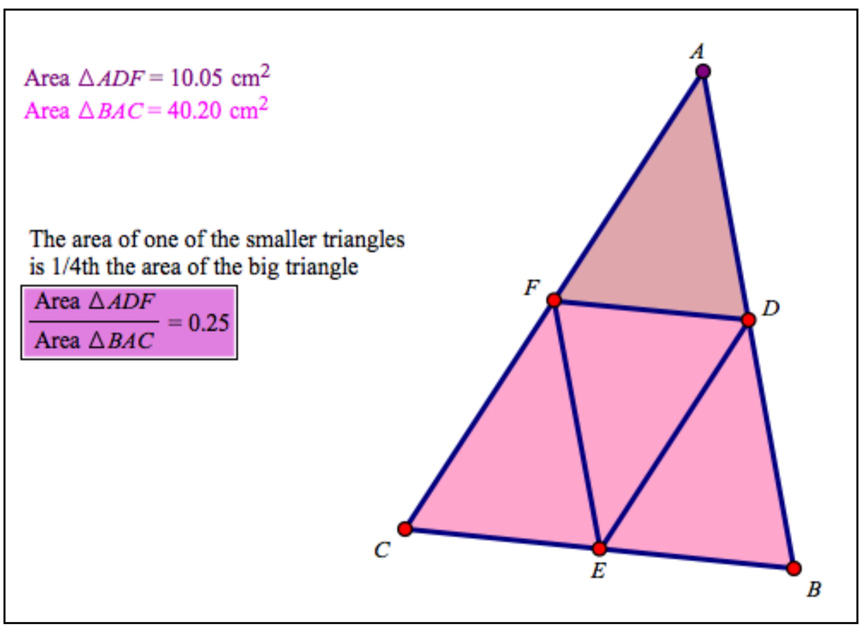


Figure 5. Screenshot of Beth's conjecture with a measurement justification.

This episode shows that Brian incorporated some of the main features of the dynamic geometry, i.e., measuring and dragging. By calculating the ratio of the triangle area measurements and dragging a vertex of the large triangle, Beth could see that the ratio did not change although the figure and the area measurements changed. This extra step assisted in justifying her initial conjecture. By questioning and guiding his students in explorations of geometric concepts, Brian displayed the exploring level of TPACK for the learning theme, conception of student thinking descriptor.

Brian continued to monitor his students' work and assisted them if they needed any help as he did with Anna and Beth in the aforementioned episodes. Brian made use of his TPACK in many more conversations with his students during this lesson. Dialogs not described here were of a similar nature to those conducted with Anna and Beth where his TPACK was unquestionably "visible." At the end of the lesson, all students came up with at least one conjecture, while many had three or four conjectures. One student's conjectures included the following:

- The area of one of the smaller triangles is 1/4th the area of the big triangle.
- Both the large and small triangles have the same size angles.
- The midsegment is parallel to the opposite side of the original triangle.
- The midsegment of the smaller triangle is half of the opposite side of the original triangle.

Brian summarized all of the conjectures that students came up with during a whole-class discussion. He had a sketch of a triangle and its midsegments; on this sketch he also had buttons for five possible conjectures and corresponding measurements for justification (see Figure 6). He went through each one by asking students what conjectures they came up with and revealing those conjectures on his sketch one at a time. Table 5 summarizes four conjectures Brian discussed with his class; he did not reveal the one for the area (which was included in the episode with Beth) because none of the students mentioned it in the limited time they had for the summary at the end of this lesson.

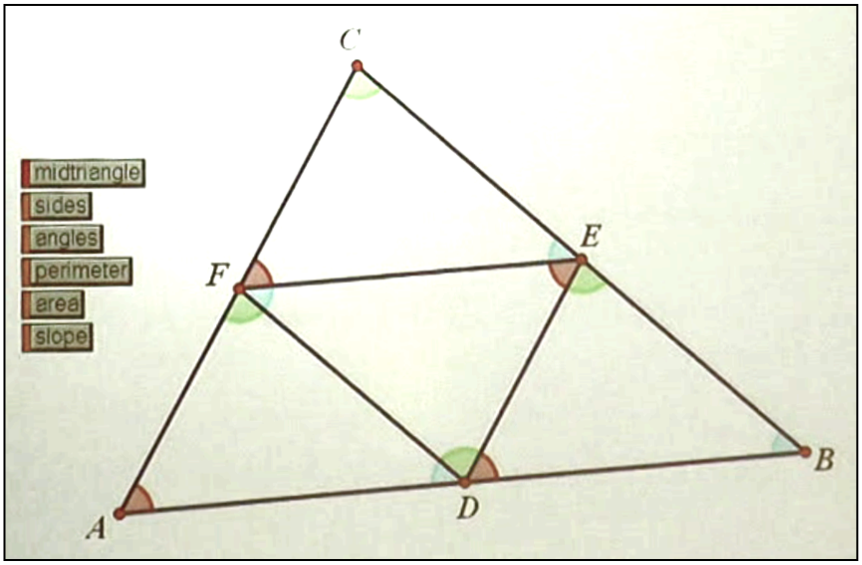


Figure 6. Screenshot of Brian's triangular thinking lesson summary sketch.

Table 5

Summary of Conjectures for the Triangular Thinking Lesson

Conjecture	Measurement justification
The measures of the sides of the smaller triangle are half that of the larger.	Ratios of the sides
The [corresponding] angles of all the triangles are congruent.	Measures of angles
The perimeter of the smaller triangle is half that of the larger triangle.	Ratio of the triangle perimeters
The midsegment is parallel to the opposite side.	Measures of slopes

TPACK Development Levels

Brian's responses to the TPACK Survey (see Figure 7) indicated that he considered his TPACK development levels to range from adapting to advancing, with most of them (six descriptors out of eleven) being "in the middle" at the exploring level. The following paragraphs describe Brian's TPACK development levels derived from the other data sources (e.g., interviews, observations) and are split into four sections for the four themes of the TPACK development model, i.e., curriculum and assessment, learning, teaching, and access (Niess et al., 2009).

Theme (descriptor)	TPACK Development Levels				
	Recognizing	Accepting	Adapting	Exploring	Advancing
Curriculum & Assessment (curriculum)				▤	
Curriculum & Assessment (assessment)				▤	
Learning (mathematics learning)					▤
Learning (conception of student thinking)				▤	
Teaching (mathematics learning)					▤
Teaching (instruction)				▤	
Teaching (environment)					▤
Teaching (professional development)				▤	
Access (usage)			▤		
Access (barrier)			▤		
Access (availability)				▤	

▤ Responses to the TPACK Survey

Figure 7. Brian's responses to the TPACK Survey.

Curriculum and assessment theme. The observed lessons and the accompanying student handouts provided evidence that Brian created his own Sketchpad-enhanced activities. This implied that Brian was at the exploring level for the curriculum descriptor. The finding agreed with his ranking for this descriptor. On the TPACK Survey, he also reported to be at the exploring level for the assessment descriptor; however, no other data provided any information to where his TPACK actually is for this descriptor.

Learning theme. Based on the observed lessons, Brian was at the advancing level for the mathematics learning descriptor. In the triangular thinking lesson, it was apparent that Brian seriously thought about how to integrate Sketchpad. Because the goal of the lesson was to come up with different conjectures related to the midsegments of a triangle, Brian knew that Sketchpad could help students with their thinking and understanding of this topic and their learning would be enhanced through the use of Sketchpad.

Based on the observed lessons, initial interview, and implementation questionnaire responses, Brian was at the exploring level for the conception of student thinking descriptor. He guided his students in understanding by giving “hints as to where they should be looking” as he stated in one of the implementation questionnaires. His response to the TPACK Survey also indicated this level.

Teaching theme. Based on the observed lessons, it was clear that Brian used Sketchpad for higher-level thinking activities (e.g., his students form and justify conjectures) placing him at the exploring level for the mathematics learning descriptor. He ranked himself to be at the advancing level in this area; however, the infrequent use of Sketchpad that he reported through implementation questionnaires suggested his Sketchpad integration was not “active and consistent” (indicator of the advancing level).

The data collected through observations suggested that Brian was at the exploring level for the instruction descriptor. He engaged students in explorations using Sketchpad where students took control of their learning, and he was in the role of guide. He probed students with questions and answered questions from his students.

Brian created an environment where students could explore and discover geometry. He managed the Sketchpad activities so that his students were engaged and self-directed in their learning of geometric concepts. Therefore, he was at the advancing level for the environment descriptor. His response to the TPACK Survey also indicated this level.

According to data from the initial interview and the DGP professional development attendance records, Brian was at the adapting level for the professional development descriptor because he only attended professional development designed to focus on Sketchpad and, as a geometry leader, he shared ideas with other teachers at his school about incorporating Sketchpad in teaching.

Access theme. According to data collected through the initial interview and implementation questionnaires, Brian was at the adapting levels for the usage and barrier descriptors; these levels agreed with his responses to the TPACK Survey. Brian allowed his students to use Sketchpad for explorations and discovery of new geometric topics only in specific units (indicator of the usage descriptor). For example, he did not use Sketchpad for teaching and learning of 3-dimensional figures, but he used it for triangle congruence and geometric transformations. He experienced challenges with the DG implementation because of the new scope and sequence, so he integrated it to enhance geometry learning only in some units (indicator of the barrier descriptor).

Brian was at the adapting level for the availability descriptor. He taught geometric concepts differently because students got to discover them instead of finding out about them through lecture. For example, during one of the observed lessons, students investigated and made connections between different options for triangle congruence. Brian indicated his TPACK to be at the exploring level for this area descriptor; however, there was no other evidence to support his statement. His students did not explore concepts using multiple representations, which would indicate the exploring level of TPACK.

Summary of TPACK development levels. Brian's perceptions about his TPACK were close to the TPACK development levels extracted from the non-survey data (see Figure 8). The TPACK development levels identified through interviews, observations, implementation questionnaire responses and documents aligned with his TPACK development levels reported through the TPACK Survey for seven descriptors: curriculum and assessment theme/curriculum, learning theme/mathematics learning and conception of student thinking, teaching theme/instruction and environment, and access theme/usage and barrier. His TPACK development levels were one step lower than the levels reported on the survey for three descriptors, i.e., teaching theme/mathematics learning and professional development, and access theme/availability. Interviews, observations, implementation questionnaire responses and documents indicated four adapting levels, four exploring levels and two advancing levels; therefore overall, Brian's TPACK development level could be described as exploring.

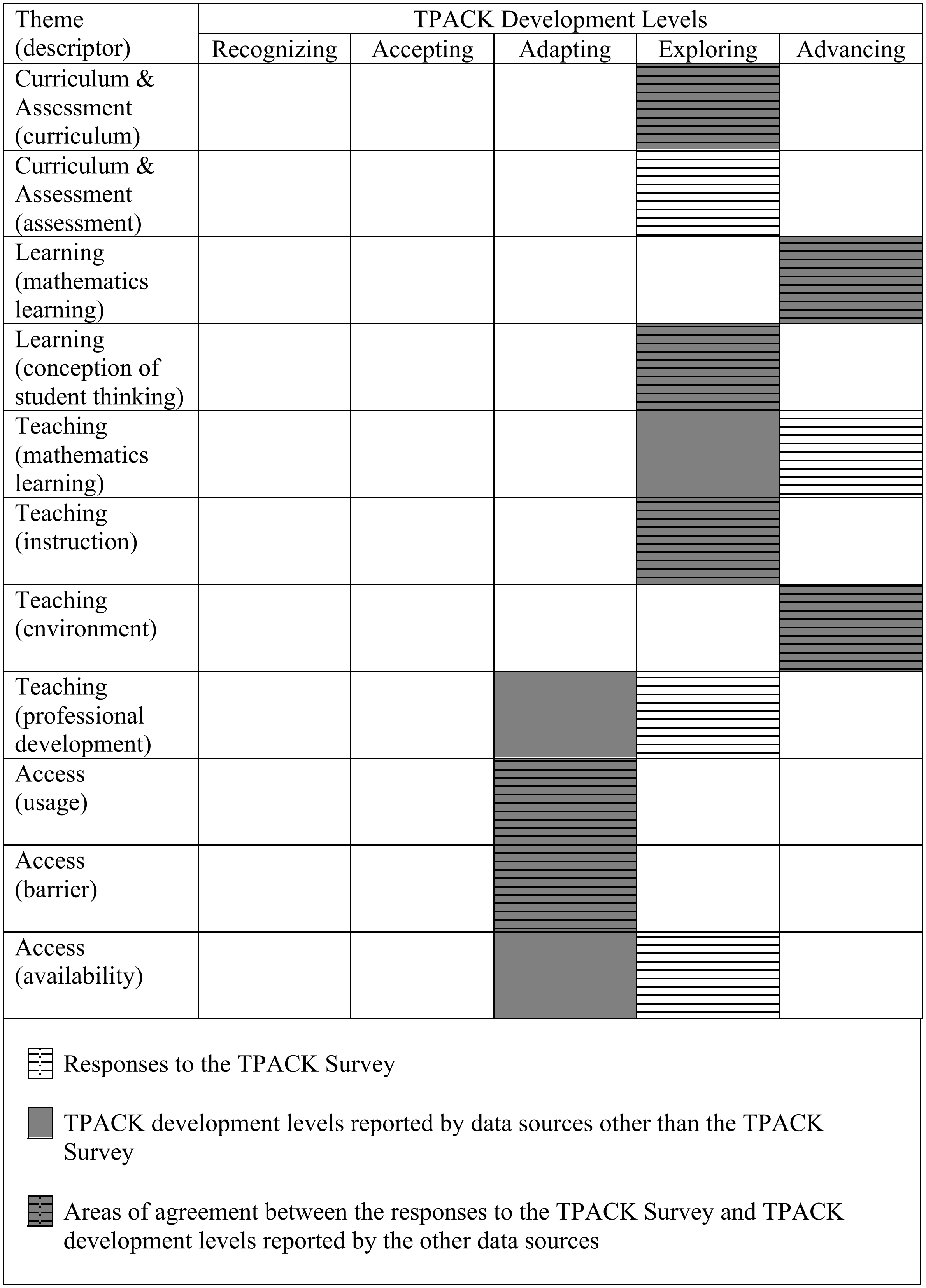


Figure 8. Summary of Brian's TPACK development levels.

Summary

Brian recognized the need for professional development and learning how to teach with Sketchpad because knowing how to use Sketchpad was not enough to teach with it effectively. He preferred to use Sketchpad for introducing new geometric topics instead of reinforcing already-learned topics. He saw that students got to discover different concepts this way, were more engaged in a lesson, and “learned better.” He displayed good knowledge of Sketchpad and geometry content through creating his own Sketchpad-enhanced lessons. While students explored geometric concepts with Sketchpad, Brian circulated throughout the classroom and took on the role of a guide by prompting his students through questioning and facilitating their learning. He assisted them with forming their conjectures and integrating Sketchpad whenever possible to clarify their statements. Brian’s overall TPACK was at the exploring level; he created his own Sketchpad activities, used Sketchpad for higher-level thinking activities, and took on the role of a guide while students explored and discovered geometric concepts.

Case 2: James

James has been teaching geometry since the beginning of his teaching career, although he had never taught geometry solely. At the time of this study, he was teaching two sections of geometry and four sections of math models, a course that uses mathematics to solve real-life applied problems. This was his fifth year teaching and his third year working at his current school; he taught for two years in another city in the same state before his current school.

TPACK Development

This section describes James's TPACK development journey with respect to Sketchpad integration in classroom instruction. All quotes in the following paragraphs come from the initial interview.

Before signing up for the DGP, James did not know Sketchpad. One of the teachers at his school told him about the project and Sketchpad, indicating an overall recognizing level of TPACK. Once he took a look at Sketchpad, he wanted to integrate it into his instruction and thought that signing up for the DGP would be a convenient way to learn about this dynamic geometry software, indicating an overall accepting level of TPACK.

James was interested in learning about Sketchpad and incorporating it in his instruction. Once he attended the first DGP summer professional development workshop (five days), he began using Sketchpad in the classroom and became "comfortable integrating it to the class certainly within the first semester," indicating an overall adapting level of TPACK. According to the DGP professional development attendance records, he attended all of the workshops offered by the DGP, a total of approximately 73 hours. However, since he concentrated on learning only one type of technology, he was at the adapting level of TPACK for the teaching theme, professional development descriptor.

James considered himself to be proficient with Sketchpad. During the initial interview, he made several references to using animations in Sketchpad, incorporating movement and observing what changes and what stays the same:

> Sometimes, when we are talking about areas of shapes, like of a trapezoid or a kite or rhombus, I use Sketchpad to create a sketch that you can figure out where those formulas are derived from. If you take a kite, take one-half the product of the diagonals—you take Sketchpad, and you can take the triangles formed by the diagonals. You can rotate them outwards to form a rectangle (see Figure 9), and then show students that you essentially have one-half of the rectangle. The diagonals become your base and height, and so that gives them at least a visual reference as to where that formula is being derived from as opposed to just simply—"here's the formula for a kite." So I like to use it for some of those animations.

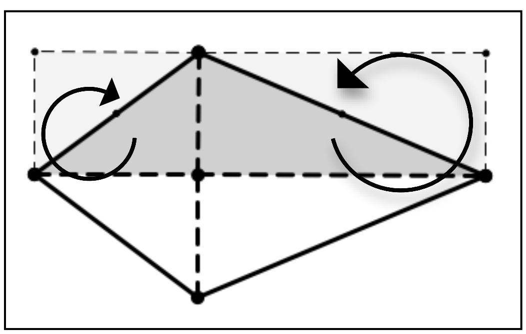


Figure 9. Illustration of rotation of two triangles in a kite to form a rectangle.

Based on this statement, the classroom observations and the work James did and shared during the DGP professional development sessions, he could be considered an expert when it comes to knowing how to use Sketchpad to present mathematical concepts.

TPACK Enactment

This section describes how James enacted his TPACK—i.e., how he put his combined knowledge of Sketchpad, geometry, and pedagogy into practice. Although this section focuses on TPACK enactment, it also provides additional information about James's TPACK development through his DG implementation. All quotes that appear in this section, with the exception of quotes and dialogs that appear in the descriptions of the observed lessons, come from the initial interview.

James was comfortable with using Sketchpad in the computer labs while students did their explorations as well as in the classroom as a demonstration tool. He viewed learning with Sketchpad as more efficient because:

> You can look at so many more examples in a short period of time than you can on paper. You can burn a whole class period on some really simple constructions. It takes a lot of time, so when you can construct the same thing in Sketchpad quickly, if you have that computer, you can actually get through stuff a lot quicker and have them explore it with a greater understanding.

His statement suggested the accepting level of TPACK for the access theme, availability descriptor, because James saw that the use of Sketchpad allowed covering a greater number of examples in a limited time. However, he also made several references to incorporating Sketchpad as a tool for discovery. He viewed the use Sketchpad during classroom instruction as "a natural extension or a better way to get students to understand what you teach them." Without Sketchpad, "you are just lecturing about some sort of postulate or theorem and you cannot discover or explore any [postulate or theorem]."

This sort of Sketchpad use was visible during the observed lessons, and it indicated the adapting level of TPACK for the access theme, availability descriptor, because geometric concepts were taught differently with Sketchpad.

James described his typical classroom instruction as “traditional,” consisting mainly of lectures. On the other hand, when he took his students to a computer lab, they got to explore and discover different geometric concepts by incorporating Sketchpad. He believes that it is “important to have the two methods: the traditional approach, how we have been teaching it without Sketchpad, as well as using Sketchpad.” After one year of teaching with Sketchpad, he noticed that most of the students in “regular” geometry classes “get lost” when they did explorations using Sketchpad while students in Pre-AP (Advanced Placement) classes did well and “flourish with it.” He believes that, for those students who struggle with Sketchpad explorations, it is best to use a traditional approach as he stated:

> When you lay it down with a traditional approach—“Here is the property. Here is how we use it to solve a problem. You go do it.”—You have modeled, you asked them, you showed them exactly what you want them to do, and they are able to mimic. So can they mimic you? Yes. Can they think for themselves and make the connection when they explore in Sketchpad? They struggle.

Based on this statement, James was more apt to introduce the key concepts to students in regular classes without Sketchpad, which indicated the accepting level of TPACK for the learning theme, mathematics learning descriptor. At the same time, he let students in

Pre-AP classes explore some geometric concepts with the Sketchpad, which indicated the adapting level in the same category.

Because of difficult access to a computer lab, James tended to take his students there only when he taught topics in “units that we are supposed to do in the lab,” meaning the units in the DGP-developed curriculum materials. The five units were (a) points, lines, and angles, (b) triangles and similarity, (c) transformations, (d) polygons, and (e) circles. James felt that the units on polygons and circles lend themselves to Sketchpad as a learning tool the most. Since properties related to circles were typically taught at the end of the school year, there was not much time to explore them, “but with Sketchpad, we can actually explore and learn.” The fact that he incorporated Sketchpad into teaching and learning only in specifically designed units suggested the adapting level of TPACK for the access theme, usage descriptor. Additionally, he would still limit some of the lab time even if he had access to a computer lab, and he would “pick and choose” the topics where his students struggled and where the use of Sketchpad could heighten their ability to make connections. This indicated the adapting level of TPACK for the access theme, barrier descriptor.

At the beginning of each observed lesson, James gave an activity sheet to his students with directions for each exploration. He obtained the activities from the Sketchpad Lesson Link website. Each handout had step-by-step instructions for students. As students worked on the activities, James walked around and answered their questions. Most of the questions revolved around how to do something or find something in Sketchpad. It was apparent that James had an excellent knowledge of the software while assisting his students as well as when demonstrating something on Sketchpad for the whole class. However, students also had some questions related to the geometric concepts being explored; the following subsections describe a few examples.

Triangle inequalities lesson. During the lesson on triangle inequalities, students were to construct a triangle, measure the lengths of the three sides, calculate the sum of any two side lengths, and then drag a vertex of the triangle to try to make the sum they calculated equal to the length of the third side. Next, they had to answer the question—"Is it possible for the sum of two side lengths in a triangle to be equal to the third side length?" As students dragged one triangle vertex around, they discovered that the sum of two side lengths was equal to the third side length when the vertex they were dragging landed on that third side. However, some of the students seemed to have trouble answering the question. James noticed that and decided to do this part of the exploration on the demonstration computer so that the whole class could see it on the screen.

James: Ok, class, look up on the main board if you are struggling with number one [the first question]. You should have constructed a triangle. Find the length of all three sides and add two sides together. In this case, I have AB and BC. We are going to try to make that [the sum of AB and BC] the same length as the third side. We begin to move it over [James was dragging vertex B until it landed on segment AC - see Figure 10]. They are now equal. Do I have a triangle?

Class: No.

James: No. That's where you should be at with number one.

One student, Donna, was still confused.

Donna: But…

James: Do I have a triangle?

Donna: No.

James: So then, is it possible?

Donna: No.

James: Are you asking me or telling me?

Donna: I don't know… I'm telling you! I think. I'm making sure I'm right. So it's not possible.

James: Is it?

Donna: No! It's impossible. Then there would be no triangle.

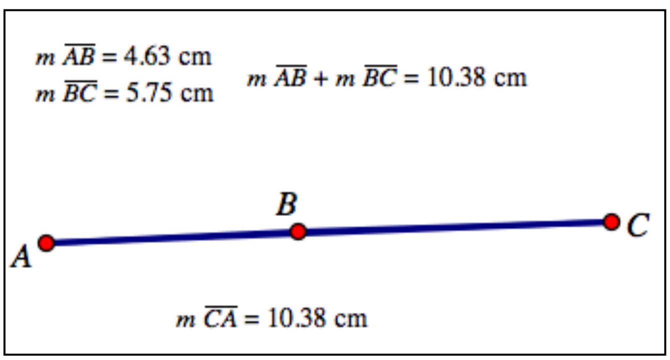


Figure 10. Screenshot from the triangle inequality lesson.

It was obvious that students saw that there was no triangle when the sum of two side lengths was equal to the third side, but they had trouble in answering the first question—"Is it possible for the sum of two side lengths in a triangle to be equal to the third side length?"—perhaps because it did not clearly ask if the triangle still existed.

Trigonometric ratios lesson. During this lesson, students were exploring the trigonometric ratios in a right triangle. According to the activity sheet from the Sketchpad Lesson Link website that James provided, they were to construct a right triangle, measure the ratios of various sides, and then use Sketchpad's calculator to find the sine, cosine, and tangent for a given angle. Although students had clear instructions for constructing a right triangle, many of them did not construct it correctly. Instead of constructing a line perpendicular to a given segment, they constructed two segments that appeared to be perpendicular, but did not pass a drag test. (A drag test is a form of assessment used to determine if a construction has been done properly. One drags parts of the construction to see if it holds true in different instances.) Because it was crucial to have a right triangle for this exploration, James decided to show this construction to the whole class after noticing that students were not following the instructions. Several minutes after showing the class how to construct a right triangle (as well as how to label its sides and find ratios of its sides), James was walking around and noticed that some students were still working with arbitrary triangles. So he showed them individually how to complete that construction. During the closing interview, he stated that if he were to teach this lesson again, he would probably provide his students with a pre-made right triangle so that they did not have to construct it themselves. This seemed like a good idea since the objective of this lesson was to explore trigonometric ratios and the construction of a right triangle was not the main objective.

Summary of the observed lessons. As lessons progressed and more students started asking the same questions, James tended to take more control of how the activities progressed. When students were not sure how to answer a question, or how to construct or calculate something, he either showed the whole class how to do it, or did the required construction or calculation for students who struggled instead of guiding them in explorations. This indicated the adapting level of TPACK for the teaching theme, environment descriptor. However, the way James reflected on the trigonometric ratios lesson and the fact that he acknowledged that it could be improved showed that he was developing his TPACK. Also, because of his original intent to engage students in examining geometric concepts by using Sketchpad, his TPACK could be classified at the exploring level for the teaching theme, environment descriptor. In addition, James's instructional purposes for his students were clear; he wanted them to explore new geometric concepts on their own while he was walking around answering their questions. This indicated the exploring level of TPACK for the teaching theme, instruction descriptor.

In the initial interview, James mentioned creating his own curriculum materials that incorporate the use of Sketchpad, especially the ones that used animations. This was not evident in the observed lesson because James used existing activities from the Sketchpad Lesson Link website every time, indicating the adapting level of TPACK for the curriculum and assessment theme, curriculum descriptor.

TPACK Development Levels

James's responses to the TPACK Survey showed that his TPACK development levels spanned from accepting to advancing, although the majority of his responses were at the adapting level and the exploring level (see Figure 11 for details). The following paragraphs describe James's TPACK development levels derived from the other data sources (e.g., interviews, observations) and are split into four sections for the four themes of the TPACK development model, i.e., curriculum and assessment, learning, teaching, and access (Niess et al., 2009).

Theme (descriptor)	TPACK Development Levels				
	Recognizing	Accepting	Adapting	Exploring	Advancing
Curriculum & Assessment (curriculum)				▤	
Curriculum & Assessment (assessment)			▤		
Learning (mathematics learning)			▤		
Learning (conception of student thinking)				▤	
Teaching (mathematics learning)					▤
Teaching (instruction)				▤	
Teaching (environment)				▤	
Teaching (professional development)			▤		
Access (usage)			▤		
Access (barrier)			▤		
Access (availability)		▤			

▤ Responses to the TPACK Survey

Figure 11. James's responses to the TPACK Survey.

Curriculum and assessment theme. James identified his TPACK at the exploring level for the curriculum descriptor. Although the initial interview provided some evidence for that level, data from the observed lessons suggested the adapting level because he understood that there was a benefit of using Sketchpad in teaching and learning geometry curriculum:

> I think it [Sketchpad] is a benefit to a fair enough group of students. It [integration of Sketchpad] is a worthy cause. So it is worth going to the lab to help students that will benefit from it [using Sketchpad to learn geometry].

Learning theme. According to the interview and observation data, James was at the adapting level for the learning theme, mathematics learning descriptor. He introduced geometric concepts to students and used Sketchpad as a learning tool only in some units. James's response to the TPACK Survey also indicated this level.

James indicated the exploring level for the conception of student thinking descriptor; however, it was difficult to identify one level of TPACK from the non-survey data sources. In fact, James provided evidence for four TPACK development levels in this category. He stated in the initial interview:

> If there was no project… Honestly if there was no project [DGP], I would probably have a hard time getting to the lab and using Sketchpad. I would still give class demonstrations, showing the different topics, but taking a class to a lab to explore… Honestly, if I were not in the project, it would probably not happen.

His statement indicated the recognizing level of TPACK. Immediately after that he added, “I am not sure how beneficial it was to any of the students majority of the time. So I am not sure if it was worth the time spent versus [using a] traditional approach.” He was referring to the first year of the DG implementation, and then he also added, “we have been learning geometry without it [Sketchpad] for a long time.” These comments suggested the accepting level of TPACK because James expressed his concern for the lack of development of appropriate geometric thinking skills in his students when they used Sketchpad for explorations. During the closing interview, James mentioned that he was planning to switch classrooms, permanently, with another teacher who had computers in the classroom but was not going to use them the following year. This showed a change in how James viewed the integration of Sketchpad as a learning tool because he said that by having computers in the classroom, he would be able to make it an integral tool for learning rather than as something additional as it was at the time of this study. If James is successful in this endeavor, then it will become easier for him to transition to the advancing level of TPACK for the conception of student thinking descriptor. Based on the observation and implementation questionnaire data, James was at the adapting level, because he allowed his students to explore selected geometric concepts with Sketchpad.

Teaching theme. James indicated the advancing level for the mathematics learning descriptor. However, his acceptance of Sketchpad as a tool for learning was not active and consistent. Based on the interview and observation data, he was at the exploring level. He incorporated Sketchpad into classroom instruction by engaging his students in high-level thinking activities. Also, during the DGP professional development sessions, he shared his Sketchpad-based lessons and ideas with peers.

James's responses to the TPACK Survey indicated the exploring level for the instruction and environment descriptors. The data collected through observations confirmed the exploring level for those descriptors. James engaged students in explorations using Sketchpad where students took control of their learning, and he was in the role of a guide (the instruction descriptor). He provided his students with various instructional strategies to engage them in thinking about the geometric concepts under investigation (the environment descriptor). However, James tended to take more control of the activities and often did geometric constructions for his students instead of guiding them. This fact indicated the adapting level for the environment descriptor.

Data collected through the initial interview and the DGP professional development attendance records indicated that James was at the adapting level for the teaching theme, professional development descriptor because he attended professional development focusing only on Sketchpad (one type of technology).

Access theme. According to the data collected through the initial interview and implementation questionnaires, James was at the adapting level for the usage descriptor because he incorporated Sketchpad only into specific units and lessons. Additionally, the initial interview, the observed lessons and the implementation questionnaire responses indicated the adapting level for the barrier descriptor. James allowed his student to explore geometric concepts with Sketchpad infrequently because of limited access to a computer lab. His responses to the TPACK Survey indicated the adapting level for the usage and barrier descriptors as well. For the availability descriptor, James indicated the accepting level of TPACK, and the initial interview data confirmed that. James viewed the incorporation of Sketchpad as more efficient and allowing him to demonstrate more examples. However, the initial interview and the observed lessons also suggested the adapting level for the availability descriptor because geometric concepts were taught differently with Sketchpad than without it, and students got to explore and discover them instead of learning about them through lecture.

Summary of TPACK development levels. Combined results from all data sources indicated that James's TPACK development levels were closely aligned to his perceptions about his TPACK (see Figure 12). The TPACK development levels identified through interviews, observations, implementation questionnaire responses and student handouts aligned with his TPACK development levels reported through the TPACK Survey for seven descriptors: learning theme/mathematics learning, teaching theme/ instruction, environment, and professional development, and access theme/ usage, barrier, and availability. His TPACK development levels were one step lower than the levels reported on the survey for three descriptors, i.e., curriculum and assessment theme/curriculum, learning theme/conception of student thinking, and teaching theme/mathematics learning. James did not use the Sketchpad for any student assessments, but he mentioned that if he did then he would make sure that the tests consisted of different types of questions (conceptual as well as procedural), which indicated the adapting level. Interviews, observations, implementation questionnaire responses, and documents indicated one accepting level, eight adapting levels, and three exploring levels; therefore overall, James's TPACK was at the adapting level.

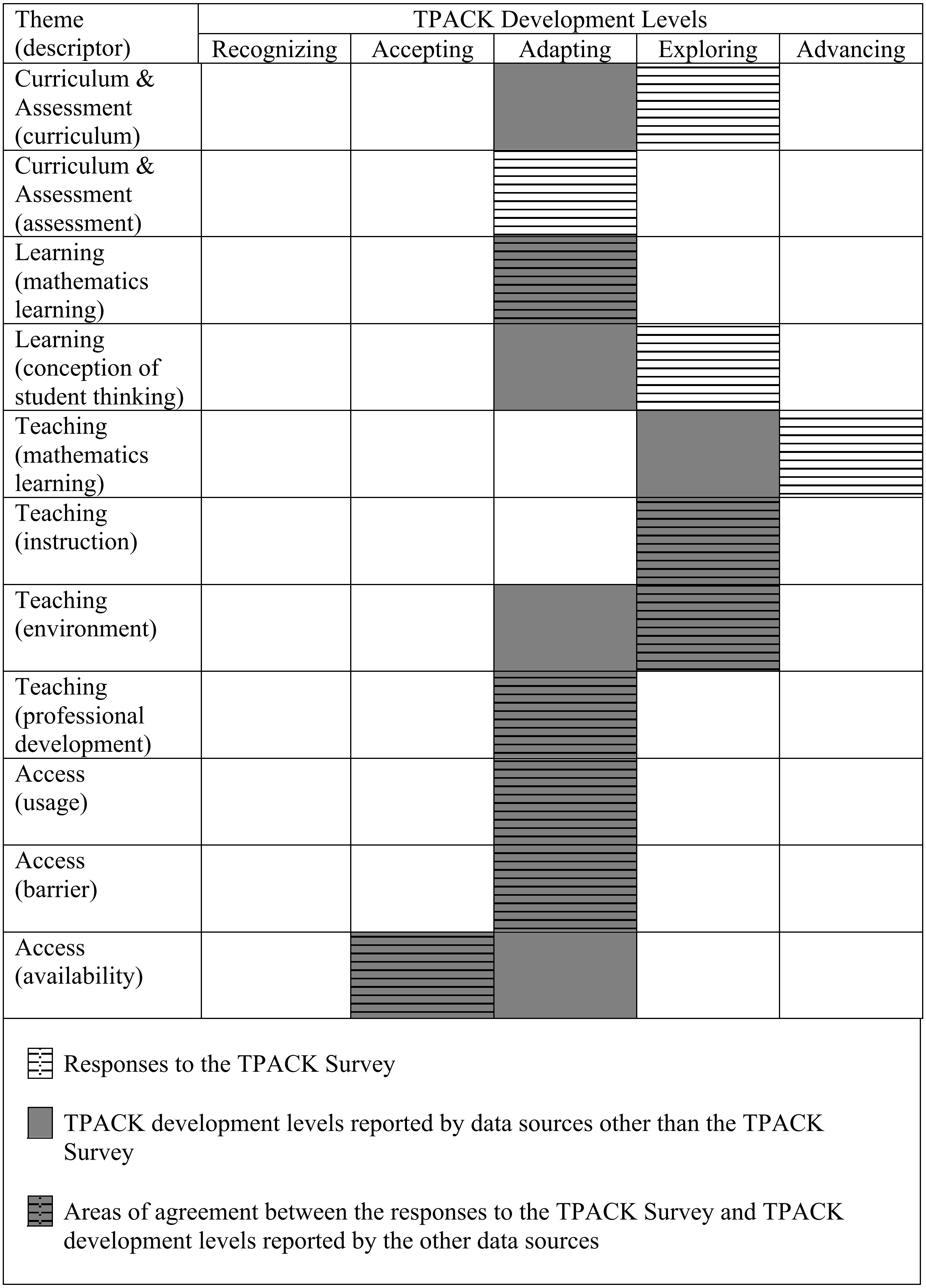


Figure 12. Summary of James's TPACK development levels.

Summary

James was an excellent example of how quickly one could learn to use Sketchpad and become highly proficient with it. He did not know about Sketchpad before the DGP, and after only one year of Sketchpad professional development and usage, he was able to create elaborate sketches. He also knew shortcuts and different ways of doing things in Sketchpad (e.g., measuring an angle). He used Sketchpad as a demonstration tool in the classroom as well as a learning tool in a computer lab. He believes in having two instructional approaches, one more "traditional" with lectures and one more learner-centered when students explore geometric concepts. His main reason for doing so was to assist students in making connections between what they "discover" with Sketchpad and what they need to do "on paper." Because sometimes students had difficulties with Sketchpad explorations, James adjusted his lessons appropriately for "next time." While students explored geometric concepts with Sketchpad, he took on the role of a facilitator and assisted students with any technical problems and with answering questions related to the content of a given lesson. James's overall TPACK was at the adapting level; he incorporated Sketchpad as a teaching and a learning tool, and enabled his students to discover geometric concepts through the use of Sketchpad.

Case 3: Susan

Susan began teaching twenty years ago. She started her teaching career at the middle school level, and later switched to teaching geometry at the high school level. At the time of this study, she was in her tenth year of teaching geometry and her fifth year of teaching at her current school. She was teaching geometry solely and was the only geometry teacher at her school. Susan was tremendously enthusiastic about incorporating technology into instruction. She had laptops, graphing calculators, clickers, an interactive white board, a document camera and a projector in her classroom.

TPACK Development

This section describes Susan's TPACK development journey with respect to Sketchpad integration in classroom instruction. All quotes in the following paragraphs come from the initial interview with Susan.

Susan began developing her TPACK with regard to Sketchpad approximately eight years ago when she was working on her master's degree. She was part of a teacher quality grant with other in-service teachers during one summer. Her overall TPACK was at the recognizing level then as she learned about Sketchpad. In the following fall, she was elected to be the math teacher in a new program at her school. As part of that program, hundreds of freshmen received laptops, and Susan had to use some kind of educational computer program with her students. She chose to use Sketchpad (indicating an overall accepting level of TPACK), although she was teaching algebra at that time. She also had training in Cabri Junior, a dynamic geometry application for graphing calculators, but she preferred to use Sketchpad because she had laptops in her classroom.

As part of her participation in the DGP, Susan attended all of the professional development meetings offered by the DGP, for a total of approximately 73 hours. She also attended several conferences with Sketchpad sessions throughout her teaching career, indicating an overall adapting level of TPACK. She enjoyed learning more about Sketchpad; however, she stated:

> I have been doing it long enough up to where I am usually ahead, so that is a disappointment. But I love going. That is why I love being part of this

> program because I am learning more to do that. But you got me thinking when I am using [Sketchpad] that I am able to point out to students more what they are supposed to see and not just see the magic. I think that I have improved a lot just by being in those [professional development sessions]. Any time you are with your coworkers and you can discuss ideas, it works better.

This statement indicates the exploring level of TPACK for the teaching theme, professional development descriptor, because Susan enjoyed learning more about Sketchpad and cooperating with colleagues on incorporating Sketchpad into teaching geometry. Susan's comment also revealed that she already knew a lot about Sketchpad; however, she still learned more through interacting with her colleagues using pedagogical dialogs.

TPACK Enactment

This section describes how Susan enacted her TPACK—i.e., how she put her combined knowledge of Sketchpad, geometry, and pedagogy into practice. Although this section focuses on TPACK enactment, it also provides additional information about Susan's TPACK development through her DG implementation. All quotes in this section, with the exception of quotes and dialogs that appear in observed lesson subsection, come from the initial interview.

Susan decided to use Sketchpad in teaching and learning because she was fascinated with it. Based on the initial interview, observed lessons and implementation questionnaire data, she incorporated Sketchpad into instruction as a learning tool for students' explorations and as a teaching tool for demonstrations. Exploring different instructional strategies with Sketchpad indicated the exploring level of TPACK for the teaching theme, environment descriptor. At the same time, her implementation questionnaire responses suggested that she used it as a teaching tool more so than a learning tool. Therefore, her TPACK for the learning theme, conception of student thinking descriptor, was at the recognizing level.

Susan liked Sketchpad because it is accurate in measurements as opposed to students' measurements on paper. This indicated the recognizing level of TPACK for the access theme, availability descriptor, because Susan saw Sketchpad as a useful tool for replacement of paper–and–pencil activities where student tended to make more mistakes on their measurement, and with Sketchpad their measurements were accurate. So when working on any activity that involved measuring, she preferred to use Sketchpad. This, in turn, suggested the adapting level of TPACK for the access theme, usage descriptor, because she incorporated Sketchpad only in specifically designed units. Also, Susan noticed that using Sketchpad was more efficient in teaching some geometric topics—"in a way it is quicker to get some points across—like angles—just something simple like to name an angle, at the beginning of the year—that the vertex has to be the middle letter." This indicated the accepting level of TPACK for the access theme, availability descriptor, because Susan saw Sketchpad as a valuable tool simply for its efficiency.

Susan decided to do Sketchpad activities with her students every Thursday this year; since "time is a big barrier," she set that one day (Thursday) aside for Sketchpad activities, and it was her "big goal." This indicated the exploring level of TPACK for the access theme, barrier descriptor, because Susan recognized that it took additional time to incorporate Sketchpad activities. By dedicating one day a week for Sketchpad activities, she overcame this challenge. In addition, implementation questionnaire data indicated that, in addition to engaging students in Sketchpad activities once a week, Susan used it for demonstration every day. This pointed to the exploring level of TPACK for the curriculum and assessment theme, curriculum descriptor, because Susan tried to integrate Sketchpad in a more integral role. However, at the same time she does not believe that students should use Sketchpad every day in class. Previously when she was teaching and had to use laptops every day, she "had a little issue with that." At that time, she was mainly teaching algebra and thought that students needed more "hands-on" activities as she stated:

> I think you still need to do a little pencil and paper. A little bit of both is good, but you cannot just go completely immersed. I think [Sketchpad] is great to use. Again, I still feel that it does not need to be—I do not believe it can be used every single day, but I think that it is a great tool to use.

Susan's comment indicated that she recognized Sketchpad as a great tool for learning geometry, even though she did not use it every day. She incorporated it as a tool to facilitate learning of such geometric topics as angles and triangles. This indicated the exploring level of TPACK for the learning theme, mathematics learning descriptor, because she used Sketchpad "to facilitate the learning of specific topics in the mathematics curriculum" (Niess et al., 2009).

Susan's students used Sketchpad during all of the observed lessons except a follow-up lesson to the parallel-lines-with-a-transversal lab; Susan used Sketchpad then for demonstration. At the beginning of each lesson, Susan instructed her students to take a laptop from the cart located in the back of the classroom. Then she introduced the lesson and necessary vocabulary for that lesson, and distributed handouts with instructions for the activity. Students were working alone for the most part, talking to their nearby classmates about the activity from time to time. Susan acted as a guide walking around the classroom and answering any questions they had, which indicated the exploring level of TPACK for the teaching theme, instruction descriptor.

Parallel lines with a transversal lesson (lab). In this lesson, students were exploring angle relationships between two parallel lines and a transversal. This was an introductory lesson, and they had not explored this concept before. Susan introduced the lesson and pointed out a few things on the handout to make sure the students understood everything and that they followed the instructions closely. One of the things that she mentioned was the use of the construct menu:

> Susan: When it says, “construct,” you need to be really careful and make sure you use the construct menu. You all like to just draw. It says, “construct parallel lines.” It is not going to stay parallel [if you just draw]. You want to construct parallel lines. You have to use the construct menu. You have to use this. It is not going to work [if you just draw]. If you do not use this, I will know. I will come around, and I will check it, and you will end up redoing it so pay attention. If you do not use this, you have to redo it.

It was clear that Susan had experience with students simply drawing lines that appeared to be parallel, but when dragged the lines did not remain parallel. Susan probably witnessed this in several other lessons, and that is why she addressed it at the beginning of this activity.

As students began to work on the activity, one student constructed the transversal so that it was perpendicular to the parallel lines, making all angles 90 degrees. Susan noticed that, and she said to the whole class, “If you did perpendicular, please undo. We do not want perpendicular lines. Parallel lines, but not perpendicular.” Later on she kept noticing that some students still had their transversal line perpendicular to the parallel lines. She addressed that issue with one of her students, Craig:

Susan: Do not construct a perpendicular [transversal].

Craig: I can’t make right angles?

Susan: You can later. But it’s not fun. That makes it boring. Math should be fun.

All students ended up having a correct construction, although Susan did not explain why they should not have a perpendicular transversal except that it was “boring.” After students had measured all angles, Susan directed them to move the angle measurements next to the corresponding angles because it makes it “more visual” (see Figure 13). The rest of the lesson concentrated on identifying congruent, complementary, supplementary, alternate interior, same side interior, alternate exterior, same side exterior and corresponding angles.

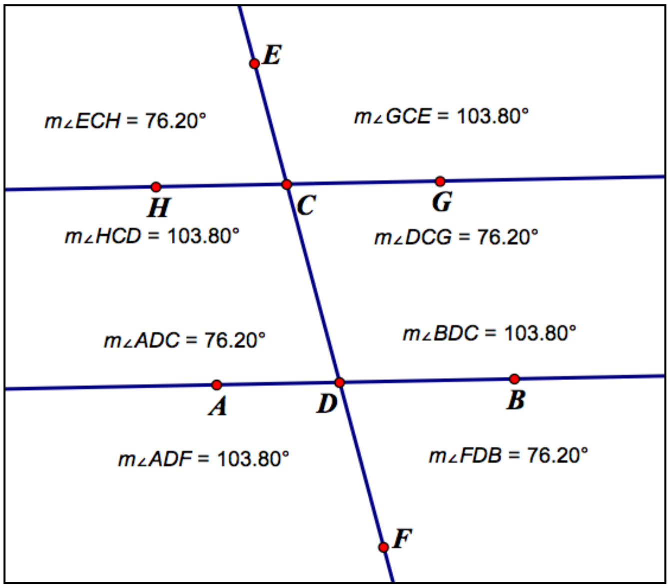


Figure 13. Screenshot of one student's work during the parallel lines with a transversal lesson.

Parallel lines with a transversal lesson (follow–up lesson). This lesson was a follow–up to the lab activity, which was four days before (there was a test day and a weekend in between the two). Susan was using Sketchpad and an interactive white board to review what the students did during the lab (see Figure 14). This confirmed that Susan recognized that geometric ideas were easily presented with Sketchpad and useful for making sense of topics in the curriculum and confirmed what she stated in the interview—the use of "different colors" makes it more visual and "it is quick to show." The fact that she used Sketchpad for the visual effect, with different colors, indicated the recognizing level of TPACK for the curriculum and assessment theme, curriculum descriptor. The rest of the lesson consisted of reviewing answers to questions from the lab activity and working on problems that did not involve Sketchpad.

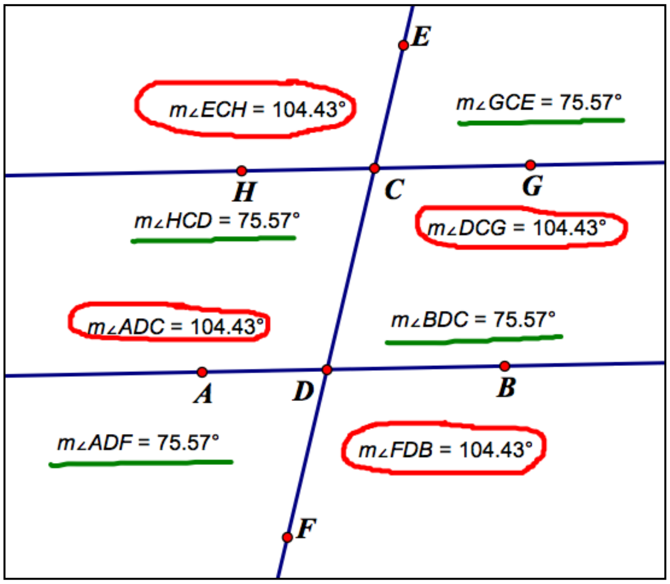


Figure 14. Screenshot of Susan's review of the lab activity during the parallel lines and a transversal lesson.

Properties of isosceles triangles lesson. During the break between classes and before this lesson, Susan told me that the Sketchpad activity that her students were going to work on was very similar to what they did the day before on paper. She tried to do a more challenging activity with her first period class that day, but it was too difficult for them. She stated that her student simply were not able to do it, so she decided to switch it with another activity. In the original and more challenging activity, students were to develop different ways of constructing isosceles triangles. In the "new" activity, students were to "discover" that the base angles of an isosceles triangle were congruent, which they already learned the previous day. Because the activity was reinforcing already known topic, Susan's TPACK indicated the adapting level for the teaching theme, mathematics learning descriptor. At the same time, the fact that Susan had a difficulty identifying a topic for including Sketchpad as a learning tool indicated the accepting level for the curriculum and assessment theme, curriculum descriptor.

TPACK Development Levels

Susan's responses to the TPACK Survey showed that she perceives her TPACK to be at the exploring and advancing levels except for the assessment descriptor (see Figure 15). She indicated she did not allow her students to use Sketchpad on tests:

> not because "I don't like to allow..." [but because] 1) Designing a Sketchpad test would be difficult. 2) Students would have more ways to cheat while on the computer so I would have to watch them very closely.

Susan's statement suggested that she assessed her students' learning through asking procedural questions where students were more likely to cheat. This showed an opportunity for TPACK development and the incorporation of assessment questions that examine conceptual understanding rather than procedural one. Asking conceptual questions could eliminate cheating on tests and make the inclusion of Sketchpad even more integrated with learning.

Although Susan indicated high levels (exploring and advancing) of TPACK through the TPACK Survey, the non-survey data indicated that her TPACK development levels range from recognizing to exploring. The following paragraphs describe Susan's TPACK development levels derived from the data sources and are split into four sections for the four themes of the TPACK development model, i.e., curriculum and assessment, learning, teaching, and access (Niess et al., 2009).

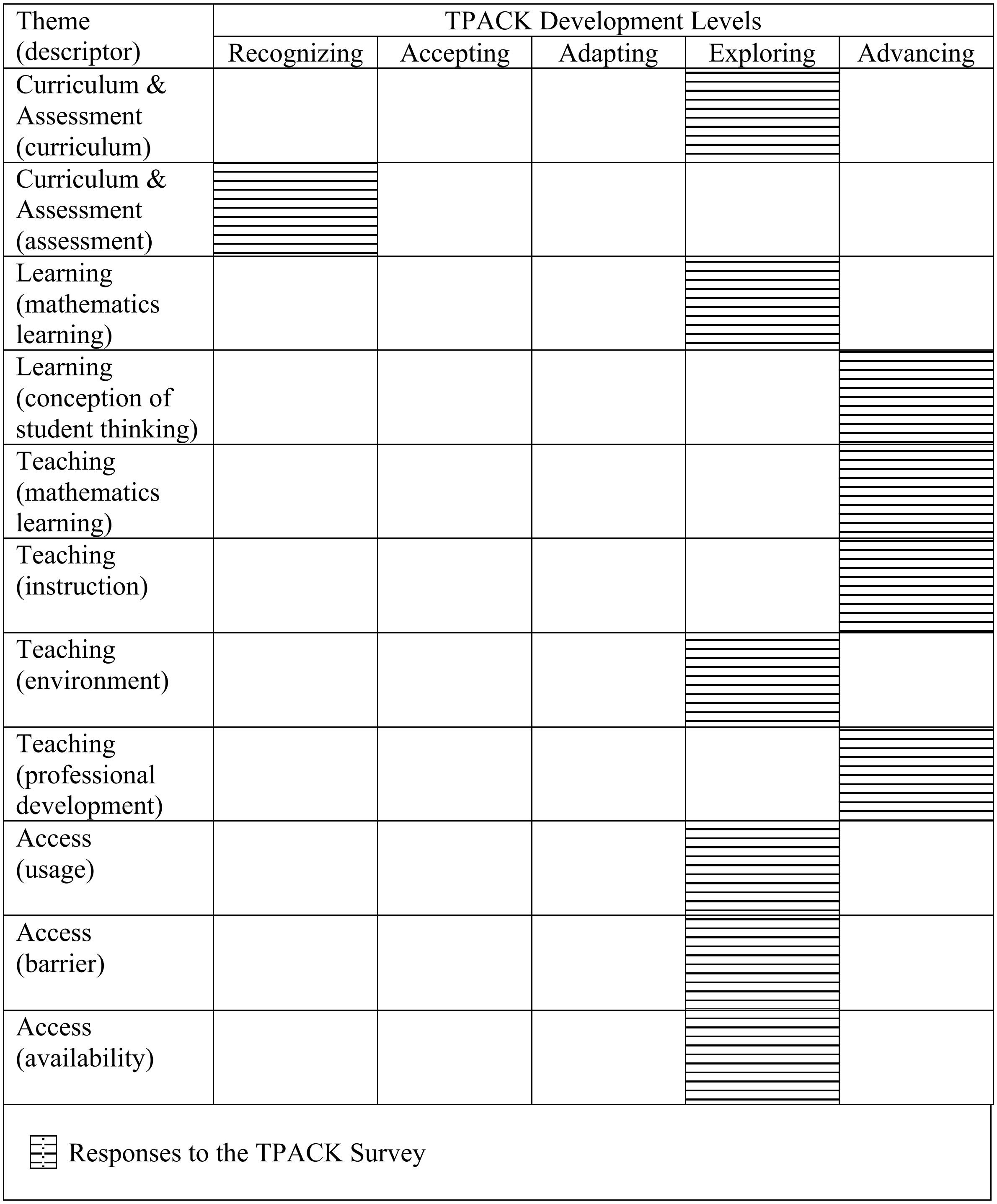


Figure 15. Susan's responses to the TPACK Survey.

Curriculum and assessment theme. The initial interview and the second observed lesson—the parallel lines with a transversal follow-up lesson—indicated that

Susan's TPACK development level was extremely low (recognizing) for the curriculum and assessment theme, curriculum descriptor, because she focused on how things were displayed with Sketchpad (e.g., the use of different colors). Additionally, the third observed lesson—properties of isosceles triangles lesson—indicated the accepting level because Susan tried one Sketchpad activity with her first period class, but then changed it to a less-challenging activity, which in turn was a repetition of the lesson from the previous day. This showed that Susan might have had a difficulty identifying appropriate topics in the geometry curriculum for her students to explore with Sketchpad. However, the initial interview and implementation questionnaire responses suggested the exploring level for the curriculum descriptor; Sketchpad integration had "more integral role for the development of the mathematics that students are learning" (Niess et al., 2009) because Susan tried to use it every day in addition to students' weekly explorations.

Learning theme. Susan knew that integration of Sketchpad in learning was a worthwhile effort. Throughout all the years she had been using it in the classroom instruction, she identified topics, for which Sketchpad facilitates better learning and understanding of geometric concepts (e.g., parallel lines with a transversal). This indicated the exploring level of TPACK for the mathematics learning descriptor, which agreed with her self-reported TPACK development level. However, her TPACK development level was low (recognizing) for the conception of student learning descriptor because she preferred to use Sketchpad as "a teaching tool rather than as a learning tool" (Niess et al., 2009). She reported the advancing level for this descriptor, indicating that Sketchpad integration in her classroom was integral "to development of the mathematics students are learning" (Niess et al., 2009). This was not evident in the observed lessons, and other data sources did not suggest it either.

Teaching theme. Based on the observed lessons, Susan's TPACK was at the adapting level for the mathematics learning descriptor. She indicated the advancing level through the TPACK Survey; however, it was not evident that her students engaged in high-level thinking activities when using Sketchpad. Instead, she incorporated Sketchpad for enhancing or reinforcing already-learned topics as well as for activities outside the curriculum. Based on the observed lessons, Susan's TPACK was at the exploring level for the instruction descriptor because she guided her students in Sketchpad explorations. The initial interview data indicated the same level for the environment descriptor because Susan incorporated a variety of instructional strategies when using Sketchpad. According to data from the initial interview, Susan was at the exploring level for the teaching theme, professional development descriptor, because she associated and worked with other geometry teachers (through the DGP) who integrated Sketchpad in geometry instruction.

Access theme. The initial interview and observation data indicated the adapting level for the usage descriptor because Susan used Sketchpad only in specific units such as those that require accurate measuring. By recognizing challenges in teaching geometry with Sketchpad (such as the extensive curriculum), Susan explored strategies to overcome them (e.g., setting one day a week to allow students to use Sketchpad), which indicated the exploring level for the barrier descriptor. Lastly, Susan's TPACK was low (at the recognizing and accepting levels) for the availability descriptor because she preferred to use Sketchpad mainly for its accurate measurements and efficiency instead of using it to explore more complex geometric topics.

Summary of TPACK development levels. Combined results from all non-survey data sources indicated that Susan's TPACK development levels were not aligned to her perceptions about her TPACK (see Figure 16). The TPACK development levels identified through interviews, observations, implementation questionnaire responses and student activity sheets aligned with her TPACK development levels reported through the TPACK Survey only for three descriptors (all at the exploring level): learning theme/mathematics learning, teaching theme/environment, and access theme/barrier. Her TPACK development levels were lower for the remaining themes/descriptors than the TPACK development levels identified through the TPACK Survey. Because Susan's TPACK development levels ranged widely (from recognizing to exploring), it was difficult to identify one TPACK development level for her. Among all four themes, the teaching theme showed most consistency, with one adapting and three exploring levels; therefore, Susan's TPACK was at the exploring level for this theme.

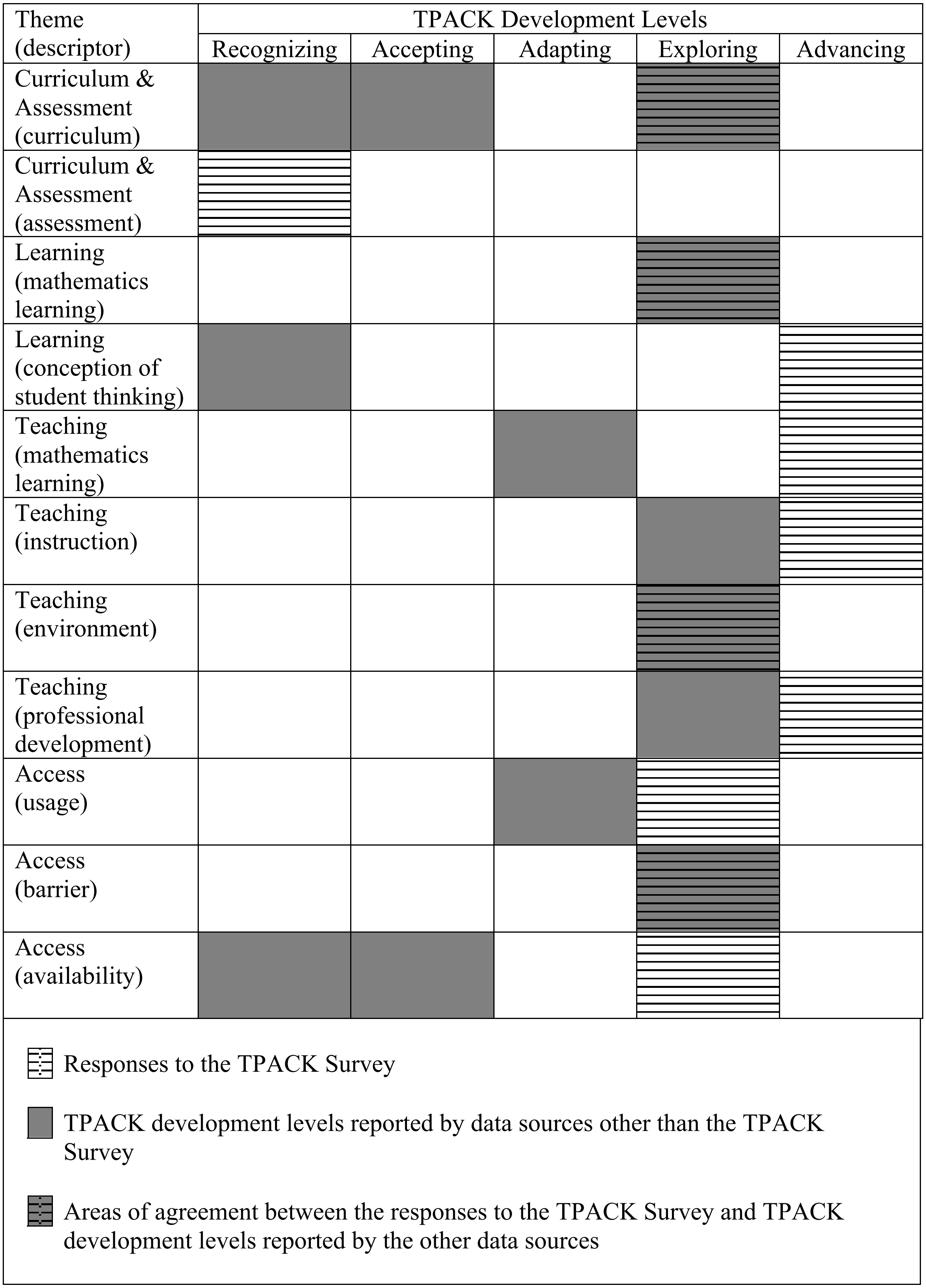


Figure 16. Summary of Susan's TPACK development levels.

Summary

Susan learned about Sketchpad approximately seven years before this study and has been using it since. She had used it for teaching algebra and geometry. She does not believe students should use it all the time, but she believes it is an excellent tool for visualizing some difficult-to-learn concepts (such as angles). She incorporated Sketchpad into her lectures on a daily basis and her students used it for explorations about once a week. During the observed lessons, it was clear that Susan had experience in teaching with Sketchpad because she anticipated many areas where students had troubles (e.g., using the construct menu to construct parallel lines instead of drawing lines that appear to be parallel). However, she tended to use Sketchpad because it was easier and quicker to present geometric ideas with it. If her students had difficulties with Sketchpad activities (or if those activities were challenging and students struggled with the concepts), she decided to change them to something easier, i.e., something that she knew students would be able to do (e.g., the lesson on properties of isosceles triangles). Susan's TPACK for the teaching theme was at the exploring level; she incorporated Sketchpad as a learning tool on a weekly basis, and enhanced her students' learning through the use of Sketchpad. Based on the collected data, it was impossible to identify one TPACK development level for the remaining themes (i.e., curriculum and assessment, learning, and access).

Case 4: Laura

Laura was in her third year of teaching geometry at the time of this study. She taught geometry solely during the previous year, but during this study she was teaching three sections of geometry and three sections of math models, a course that uses mathematics to solve real-life problems. Laura went to college after raising five children.

As a Girl Scout and a Boy Scout leader for thirty-five years, Laura gained a significant amount of pedagogical knowledge prior to becoming a mathematics teacher. She had computers in her classroom, so her students had easy access to Sketchpad.

TPACK Development

This section describes Laura's TPACK development journey with respect to Sketchpad integration in classroom instruction. All quotes in the following paragraphs come from the initial interview.

Laura used Sketchpad as a college student in her teacher preparation program (indicating an overall recognizing level of TPACK) approximately four years before this study. She "felt very comfortable with Sketchpad" when she began her work in the DGP. Even though she used Sketchpad in college, she did not use it for teaching until the DGP because she preferred to do hands-on activities with her students. Laura used the term "hands-on" to describe activities, in which students "use their hands" or engage in tactile learning. Therefore, the term "hands-on" is used in this context in the remaining parts of this case report. The fact that she chose not to use Sketchpad in teaching indicated an overall accepting level of TPACK—in this case, an unfavorable attitude towards integrating technology in teaching mathematics (Niess et al., 2009). When she signed up for the DGP, she wanted to be in the comparison group, but because of the random assignment she ended up in the DG group. She attended all of the professional development sessions offered by the DGP, except for a one-day summer workshop, for a total of approximately 66 hours. This indicated the adapting level of TPACK for the teaching theme, professional development descriptor, because she focused on learning only one type of technology. Through professional development and practice, her knowledge about teaching with Sketchpad changed for the better:

> The main way that [my knowledge] has changed is just that experience of learning what students are going to make a mistake on, and what students are going to struggle with doing. Knowing that is extremely helpful to circumvent the flaw in your lesson because you understand how students are going to make a mistake.

Laura's comment suggested the adapting level of TPACK for the learning theme, conception of student thinking descriptor, because she "begins developing appropriate mathematical thinking skills when technology is used as a tool for learning" (Niess et al., 2009). Furthermore, by incorporating Sketchpad in teaching and learning, she noticed how little students understand about geometry.

> What [Sketchpad] has shown me was how little [students] truly understand about geometry. When you tell them something, they are trying to memorize something to do, and they do not actually understand anything. I think that was the most eye-opening thing for me last year—trying a little harder to get more feedback from them, explaining to me what is going on, so that I can see if they know or they are just mimicking back.

Laura's statement indicates that when she incorporated Sketchpad, student learning and understanding of geometry was more "visible," provided her with more feedback about her students, and allowed her to adjust her instructional strategies accordingly. She also mentioned that she saw "much more deeper thinking" and "more understanding of what is going on with the Sketchpad." By seeing how her students benefit from using

Sketchpad, Laura started to change her attitude towards incorporating it into classroom instruction. Before the DGP, she did not use it in teaching, although she had used it as a college student and had access to it at her school. Through her participation in the DGP, her conception about incorporating Sketchpad changed so that she “probably will still use Sketchpad” if she has access to it after the DGP.

Laura based her above-mentioned reflections on the first year of the DG implementation. During the second year of the DG implementation, at the time of this study, Laura struggled immensely with her students’ attitude and lack of motivation towards learning geometry with Sketchpad. She mentioned in one of the implementation questionnaire responses, “This year has been very challenging to engage students in experimentation. Students have a very lazy attitude towards learning and wait for information to be given.” This also created a barrier to Sketchpad explorations; however, approximately two months after making that comment, Laura found a solution:

> Students are terribly impacted if they believe they will be receiving credit for something. Having them turn their work in electronically certainly seems to motivate them, and I enjoy it much more. They are becoming much more receptive of the work part of the dynamic geometry tools. They have always been receptive, but they thought it meant a free day.

This comment shows that Laura was developing her TPACK in the barrier descriptor area. First, she did not feel like students were learning much from using Sketchpad because “they have been very lazy” and “they think they are just there to play.” However, she started exploring strategies, such as having them turn in their work electronically, to “minimize the impact of those challenges” (Niess et al., 2009).

TPACK Enactment

This section describes how Laura enacted her TPACK—i.e., how she put her combined knowledge of Sketchpad, geometry, and pedagogy into practice. All quotes that appear in this section, with the exception of quotes and dialogs that appear in the subsections with descriptions of the observed lessons, come from the initial interview.

Laura preferred to do hands-on activities with her students, so she used Sketchpad only in a few units. She thought Sketchpad was perfect for exploration of some, but not all, topics. This indicated the adapting level of TPACK for the learning theme, mathematics learning descriptor. She described one lesson where the incorporation of Sketchpad did not work well:

> If you try to do an exploration with parallel lines cut by a transversal, it seems like a terrific idea—"Let's measure all the angles and see how they change." Number one, with the current version of Sketchpad, you cannot anchor the measurement to the angle. You can do it if you know how, but my students are not going to do that, so by moving they loose the angle measurement, and then they cannot seem to get the angles measured correctly. So some things that require accurate measurement, especially of angles, for them to explore is almost setting them up for failure because they are not going to see the relationship if they are measuring incorrectly.

Laura's description indicated that she did not like using Sketchpad for exploring parallel lines cut by a transversal; however, she liked using it for introducing her students to the "building blocks" of geometry because "students have absolutely no concept of what a point, a line, an angle are, especially angles." This indicated the adapting level of TPACK for the access theme, usage descriptor, because she used Sketchpad to teach geometric topics in specific units. At the same time, Laura incorporated Sketchpad to give her students "access to connections formerly out of reach" (Niess et al., 2009). This indicated the adapting level of TPACK for the access theme, availability descriptor. So Laura saw the benefit of letting her students use Sketchpad for explorations because they could "really understand the topic" that way. At the same time, it was difficult to find enough time for those explorations because of the extensive curriculum:

> Sometimes I tell myself, "Just forget the curriculum if you need three days to explore, so [students] understand the topic." But then it is scary because students are held responsible, so I do not want them to be disadvantaged in any way.

Laura's statement indicateed that it was challenging for her to incorporate Sketchpad into teaching and learning with the current curriculum. However, she strived to provide her students with a new way to approach geometry—"I see much more, much more deeper thinking about what is going on and more understanding of what is going on" when using Sketchpad. This suggested the adapting level of TPACK for the access theme, barrier descriptor. She understood benefits of integrating Sketchpad into classroom instruction, although she did it rather infrequently. Therefore, she was at the adapting level for the curriculum and assessment theme, curriculum descriptor.

During the observed lessons, Laura guided her students through geometric explorations with Sketchpad. This indicated the exploring level of TPACK for the teaching theme, instruction descriptor, because she did not direct their every move. She started off the lessons with students sitting in their desks while she introduced the lesson topics. Then students moved to computers to do Sketchpad explorations. Before the end of the lessons, students came back to their desks for a lesson summary, a lecture, or a problem-solving session directed by Laura, with or without Sketchpad, exactly as she described it in the initial interview:

> I have them explore on computers, and they make notes. But then I do not get much input from them until they come back to their desks at the end of the class, and we talk about it, and they start hearing other people's ideas…. It has to have that final push of either people putting something on the board or sharing something with the class or it never closes, never comes to a conclusion.

This statement suggested the adapting level of TPACK for the teaching theme, environment descriptor, because she tried to save time and "maintain control of how the activity progresses" (Niess et al., 2009).

At the end of each lesson, Laura reflected on her teaching with Sketchpad and asked herself "Was that good? Or was that bad [about this lesson]?" By planning, implementing, and reflecting on Sketchpad-enhanced lessons she exemplified the exploring level of TPACK for the learning theme, conception of student thinking descriptor. The following two subsections present parts from the observed lessons related to Laura's TPACK.

Triangle congruence lesson. During this lesson, students were investigating different conditions for triangle congruence. After listing all possible three-item combinations of sides and angles—i.e., SSS, AAA, SAS, SSA, AAS, ASA—students were to explore on Sketchpad which of them guarantee triangle congruence and which do not. The Sketchpad file had one already-constructed triangle for each of the six conditions and students' goal was to construct a noncongruent triangle given the same condition (see Figure 17 for an example). Students could manipulate parts of the second figure, but they could not change lengths of the congruent segments or sizes of the congruent angles. The guiding question was: Can you make a different triangle?

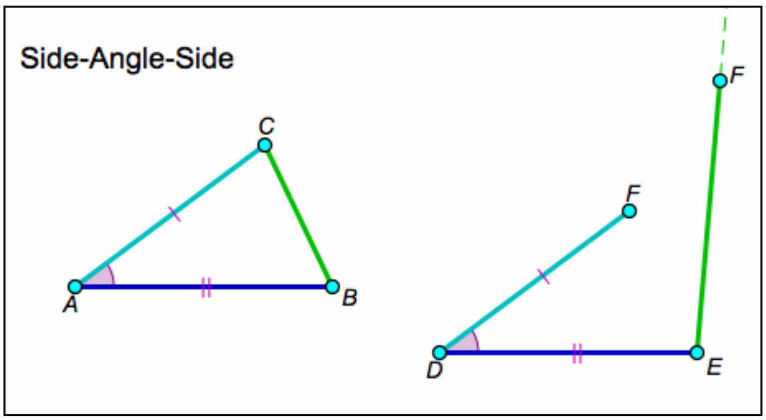


Figure 17. Sample screenshot from the triangle congruence lesson.

This lesson showed that students engaged in a higher-level thinking activity, and it implied the exploring level of TPACK for the learning theme, mathematics learning descriptor. At the end of the lesson, students posted their votes on the board as to which conditions proved congruence and which ones did not. Laura discussed each one with the whole class and made clarifications for three conditions where the students' votes were divided between "proves congruence" and "does not prove congruence", i.e., AAA, SSA, and AAS (see Figure 18 for details). Using the condition AAA, Laura constructed another triangle in Sketchpad and dragged one of its vertices to change its size; students saw that the angles remained the same, but the side lengths did not, therefore showing that AAA did not prove congruence. For SSA, Laura showed that a noncongruent triangle could be created. So SSA did not prove congruence. For AAS, Laura divided it into two cases—AAS(corresponding) and AAS(noncorresponding)—because students had these two cases to explore, which were listed in Sketchpad as A-A-(Corresponding)S and A-A-(Noncorresponding)S. Laura concluded that AAS(corresponding) proved congruence and AAS(noncorresponding) did not, and she mentioned that this was probably why the votes were divided for AAS. Laura used a “smiley face” to indicate that a condition proved congruence and an “X” to indicate that it did not.

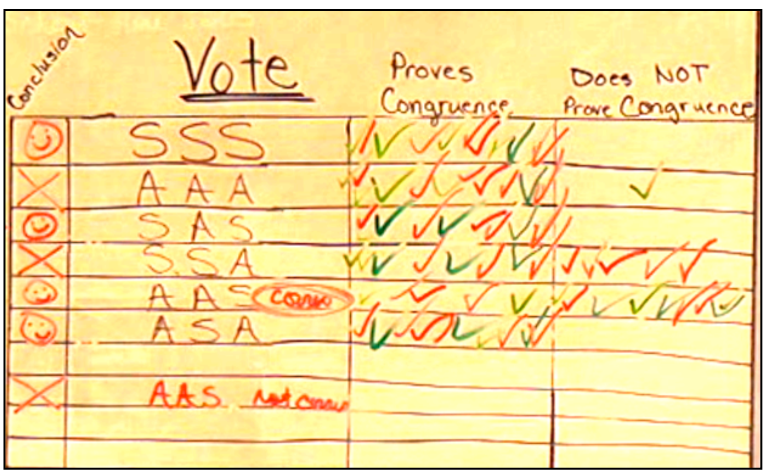


Figure 18. Students’ votes on triangle congruence and Laura’s conclusions.

The $30^\circ - 60^\circ$ right triangle lesson. In this lesson, according to the activity sheet provided by Laura, students were to construct the $30^\circ - 60^\circ$ right triangle by constructing an equilateral triangle and one of its medians. Then, they were to use half of the equilateral triangle for the rest of the exploration. Students constructed squares on the three sides of the $30^\circ - 60^\circ$ right triangle and calculated two ratios—the ratio of the area of the square on the hypotenuse to the area of the square on the shorter leg and the ratio of the area of the square on the longer leg to the area of the square on the shorter leg. Students discovered that these ratios were constant and equal to four and three, respectively.

During this lesson, Laura mentioned to one of her students, Donna, that they discussed the $30° - 60°$ right triangle together with the $45° - 45°$ right triangle the day before. That meant that students already knew what the ratios of sides were, and it indicated the adapting level of TPACK for the teaching theme, mathematics learning descriptor, because students were using Sketchpad to reinforce already-learned concepts. The following conversation with Donna provides evidence of this.

Laura: Remembering from yesterday, what is the relationship between the smallest side [the shorter leg] and the biggest side [the hypotenuse]?

Donna mentioned the Pythagorean Theorem.

Laura: Okay, so what you are remembering is that, for every right triangle, areas of the two smaller ones [squares] add up to equal the area of the biggest one [square]. That is right, but then from yesterday, when we were looking at the side measurements of the special $30° - 60° - 90°$ triangle, what did we say was the relationship between the smallest side [the shorter leg] and the biggest side [the hypotenuse]?

Dave: It was half.

Laura: Right, it was half. From here [the shorter leg] to here [the hypotenuse] was times two, correct?

Dave: Yes.

Laura: Why do you think the ratio of areas is four instead of two.

Dave: Times two.

Laura: Did you times it by two? Or what did you do to it?

Dave: Squared it.

Laura: So maybe the reason why the relationship of the areas is four is because the relationship between the sides is two.

It was not clear what the goal of the lesson was as far as the use of Sketchpad since students already knew what the ratios of sides were. For the rest of the lesson, students moved back to their desks and were to find the missing lengths in $30° - 60° - 90°$ triangles, where one of the side lengths was given. Laura worked a few such problems on the board and reminded her students how to set up a proportion to solve for a missing value.

TPACK Development Levels

Laura's responses to the TPACK Survey spanned across all levels but most of them were at the adapting level (see Figure 19 for details). The remainder of this section describes Laura's TPACK development levels derived from the interviews, observations and implementation questionnaire responses. They are split into four subsections for the four themes of the TPACK development model, i.e., curriculum and assessment, learning, teaching, and access (Niess et al., 2009).

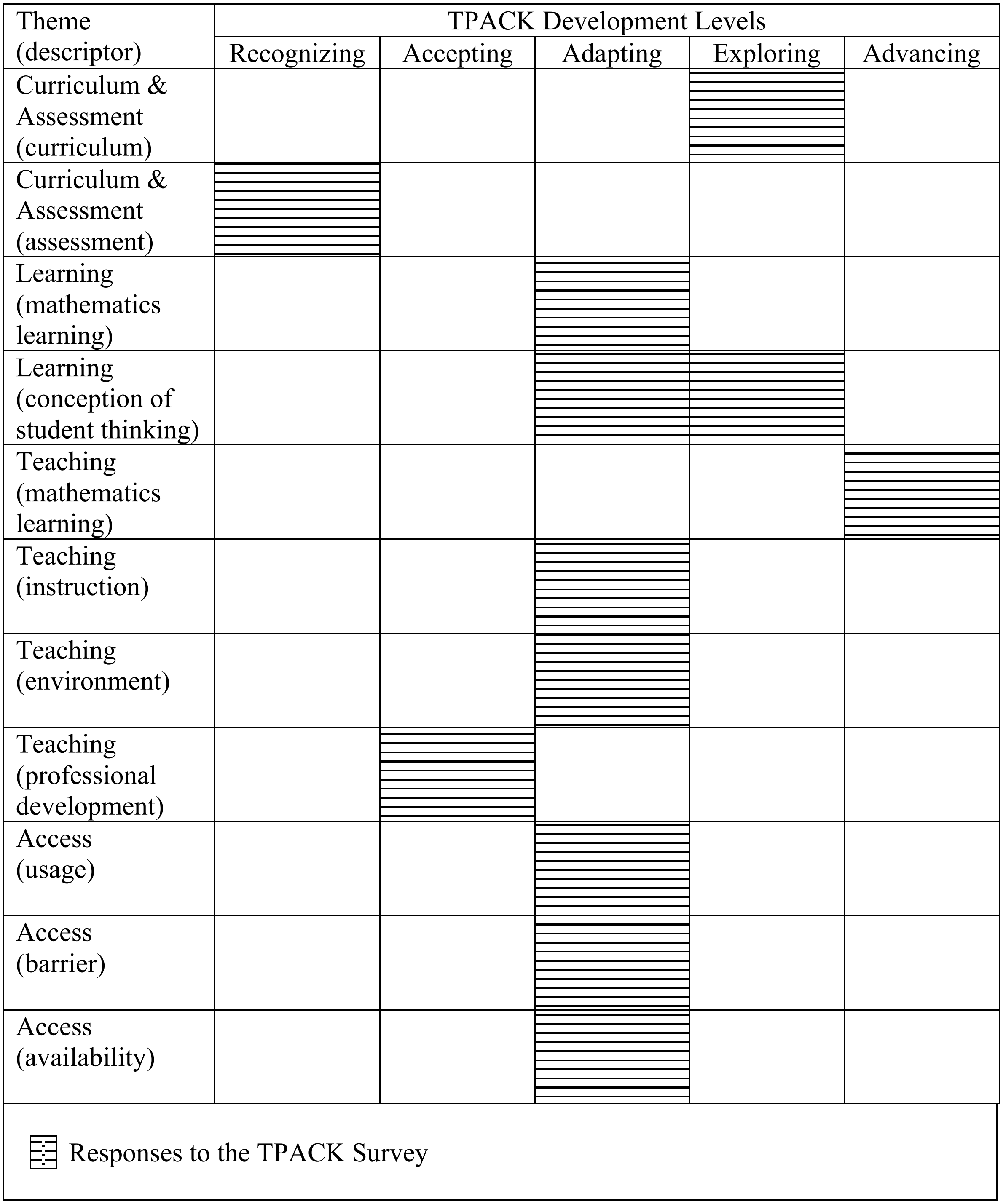


Figure 19. Laura's responses to the TPACK Survey.

Curriculum and assessment theme. Based on the initial interview, Laura set a goal for herself to have her students use computers every day as they come into class, at least for a few minutes, because she wanted to make the technology a more integral part of the learning. This would indicate the exploring level of TPACK for the curriculum and assessment theme, curriculum descriptor, which was also the level she indicated in the TPACK Survey. However, according to data reported through the implementation questionnaire, she did not get to implement her goal. Her TPACK for the curriculum descriptor was at the adapting level, based on the interview and implementation questionnaire data.

Learning theme. According to the interview and observation data, Laura was at the adapting level for the learning theme, mathematics learning descriptor. She introduced geometric concepts to students and used Sketchpad as a learning tool only in some units. Her response to the TPACK Survey also indicated this level.

For the conception of student thinking descriptor, Laura indicated the adapting and the exploring levels through the TPACK Survey. The initial interview and observation data also provided evidence for both of those levels. Laura’s students used Sketchpad as a learning tool, but she did not assess their thinking by incorporating Sketchpad, which indicated the adapting level. At the same time, she reflected “on teaching and learning with concern for guiding students in understanding” (Niess et al., 2009) indicating the exploring level.

Teaching theme. Laura classified herself to be at the advancing level for the mathematics learning descriptor. The observation data, however, provided evidence for the adapting and the exploring levels. During the lesson on the $30° - 60°$ right triangle, she was using Sketchpad for an already-learned concept, suggesting the adapting level.

During the lesson on triangle congruence, she engaged her students in a higher-level thinking activity, indicating the exploring level.

Laura classified her TPACK to be at the adapting level for the instruction and the environment descriptors. However, the observation data suggested the exploring level for the instruction descriptor because she engaged students in explorations using Sketchpad where students took control of their learning, and she was in the role of a guide. The observation data confirmed the adapting level for the environment descriptor; Laura provided extensive introductions and summaries to the lessons, during which students used Sketchpad. She made sure to tell students at the end of the lesson what they were supposed to discover during their Sketchpad explorations.

Data collected through the initial interview and the DGP professional development attendance records indicated that Laura was at the adapting level for the teaching theme, professional development descriptor because she only attended professional development designed to focus on Sketchpad (one type of technology).

Access theme. Laura's responses to the TPACK Survey indicated the adapting level for all three descriptors for the access theme. Data collected through interviews, observations and implementation questionnaires also suggested the adapting level for all descriptors. She used Sketchpad in specific units (the usage descriptor), e.g., to introduce points, lines, and angles. Also, finding time to integrate it was difficult because of the extensive curriculum (the barrier descriptor). When she got to use Sketchpad in her instruction, Laura taught geometry differently then and enabled her students to understand geometric concepts through investigations (the availability descriptor).

Summary of TPACK development levels. The findings show that Laura's TPACK development levels were at the adapting and the exploring levels, with the majority of evidence at the adapting level, which was consistent with her perceptions about her TPACK (see Figure 20 for details). The TPACK development levels identified through interviews, observations, implementation questionnaire responses and documents aligned with her TPACK development levels reported through the TPACK Survey for seven descriptors: curriculum and assessment theme/curriculum, learning theme/mathematics learning and conception of student thinking, teaching theme/environment, and access theme/usage, barrier, and availability. Her TPACK development levels were one step higher than the levels reported on the survey for two descriptors, i.e., teaching theme/instruction and professional development. Her TPACK development level for the teaching theme, mathematics learning descriptor, however, was two steps lower than the one reported on the TPACK Survey. Interviews, observations, implementation questionnaire responses and documents indicated eight adapting levels and three exploring levels; therefore overall, Laura's TPACK development level could be described as adapting.

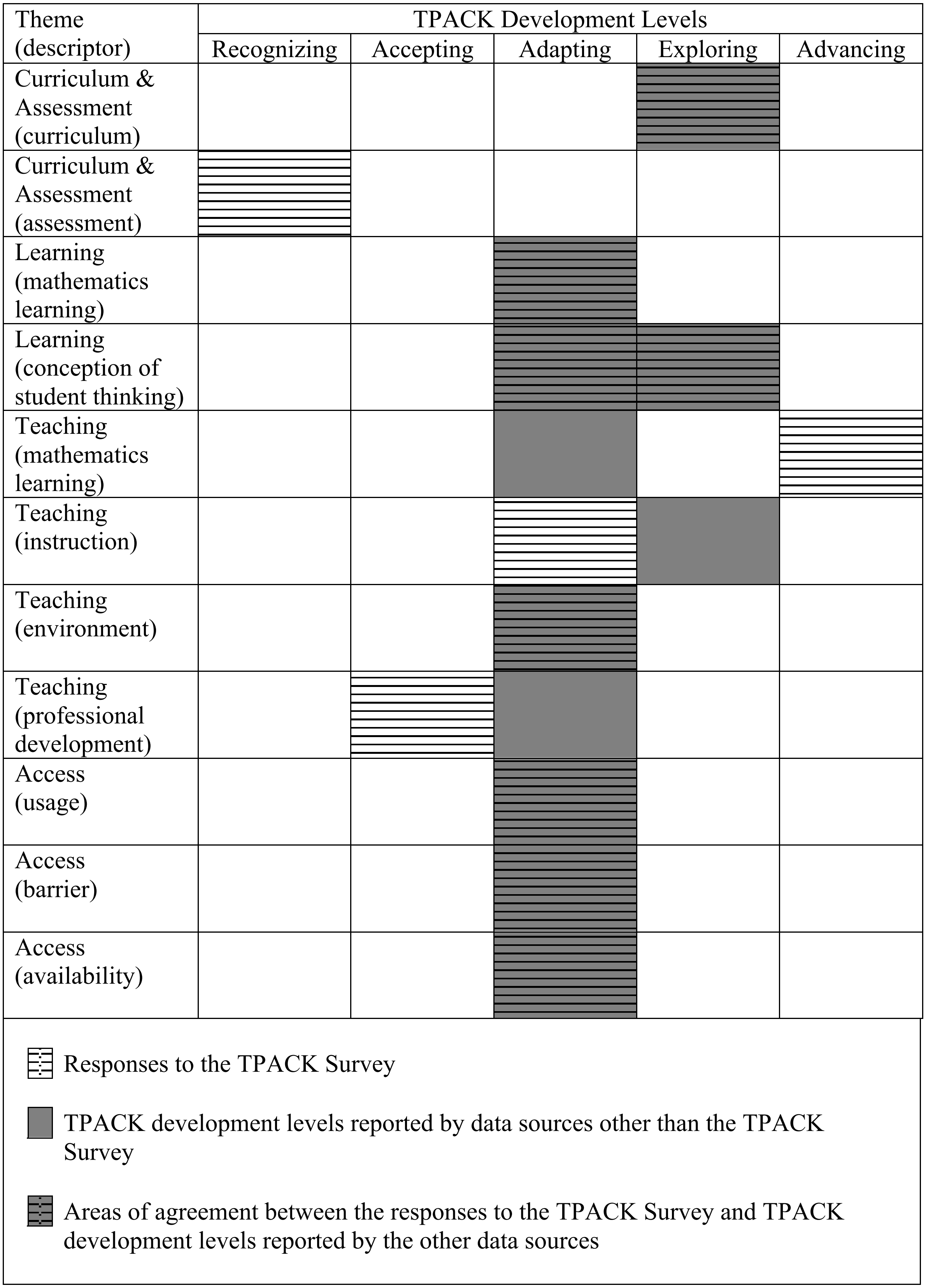


Figure 20. Summary of Laura's TPACK development levels.

Summary

Laura learned about Sketchpad in college and used it as a student for learning geometry. She did not want to use it with her students because she preferred to do hands-on activities before the DGP. Once she signed up for the project, she was asked to use Sketchpad on a regular basis because she was part of the DG group. She had no problem with incorporating it into her classroom instruction. During the observed lessons, Laura introduced the lesson topic and then sent her students to computers for Sketchpad explorations. Because she had computers in her classroom, incorporating Sketchpad in learning was seamless. At the end of each lesson, students went back to their desks, and Laura reviewed the lesson with them as a class. She believes that students need a summary of the completed exploration; this gives them a chance to compare their findings with their classmates' findings and to come to a common conclusion. Laura's overall TPACK was at the adapting level; she incorporated Sketchpad as a learning tool and enabled her students to better understand geometric concepts they investigated.

Summary

This chapter described how the participants of this study developed their TPACK and how they enacted it in the classroom instruction. Additionally, TPACK development levels were identified for all participants within the four themes and its descriptors (except for curriculum) from the TPACK Development Model (Niess et al., 2009). Four case reports provided answers to the three research questions. Each case report offered a rich description of a given participant's TPACK development and enactment related to teaching geometry with Sketchpad.

Susan had the most Sketchpad training and used it in the classroom instruction often. Laura learned geometry with Sketchpad in college but did not use it in teaching until the DGP. Brian was aware of Sketchpad before the DGP but did not have training until he signed up for the project; he started integrating it into teaching then. James did not know about Sketchpad until the DGP, and he became proficient in it within a year. All teachers acted as guides while their students explored geometric concepts with Sketchpad; they circulated throughout the room and answered students' questions. All teachers displayed sound knowledge of geometry content, although they did not always know how to connect it with their pedagogical and technological knowledge. The findings also revealed that easy access to computers does not always result in frequent Sketchpad use; the participants of this study claimed that the curriculum and standardized testing were responsible for that.

Through the descriptions of participants' TPACK development and enactment, TPACK development levels as defined by the TPACK Development Model (Niess et al., 2009) were identified. The levels varied within the themes and their descriptors for each participant; however, an overall TPACK development level was identified for Brian (exploring), James (adapting) and Laura (adapting). For Susan, an overall TPACK development level (exploring) was identified only for the teaching theme, as the levels for the remaining descriptors varied significantly.

This chapter provided findings for individual participants. The next chapter, Chapter V, continues reporting the findings for this study through cross-case analysis.

CHAPTER V

CROSS-CASE ANALYSIS

Chapter IV provided four case reports describing participants' TPACK development, TPACK enactment and their TPACK development levels. This chapter provides cross-case analysis and findings for the four participants as a group, although individual references are present at times. Therefore, this chapter augments the findings for individual participants and offers further answers to the research questions. In addition to providing findings for TPACK development and TPACK enactment, this chapter provides analysis of a teaching-with-Sketchpad task. This task was developed as a result of the pilot study; it provides an avenue for assessing TPACK related to the same geometric content for different participants. Therefore, it allows for comparison of different participants' TPACK using the same context.

TPACK Development

All teachers participated in all or almost all DGP professional development sessions. Even though they received stipends for attending these sessions, they expressed willingness to learn more about Sketchpad and to teach with it effectively. Susan and Laura had previous experience with Sketchpad as graduate/college students. In addition, Susan used it in the classroom instruction for six years before the DGP. Brian and James had limited or no knowledge of Sketchpad before the DGP, therefore, they developed most of their knowledge through the DGP professional development sessions and through their own explorations and lesson planning. At the time of this study, out of all participants, James seemed to have the best knowledge of Sketchpad, interestingly enough he was the only teacher who did not know Sketchpad when signing up for the DGP; all other teachers had some experience with it before the DGP. Although James appeared to have the best knowledge of Sketchpad, Brian had the best TPACK related to teaching with Sketchpad. This was an unexpected finding since Brian was the least experienced teacher—in his third year of teaching at the time of this study. The reason why this finding was unexpected is the fact that the literature suggests that teachers with more developed pedagogical content knowledge (PCK) are more likely to develop their TPACK quicker. This was only the second year of the DG implementation for Brian, and he was still developing his PCK. However, based on the initial interview, he was developing his TPACK at the same time he was developing his PCK.

TPACK Enactment

All teachers enjoyed using Sketchpad, but they did not use it in a computer lab setting often (see Table 6 for frequency of Sketchpad use) even though all of them, except James, had easy access to computers—Brian had a devoted computer lab, Susan had laptops in her classroom, and Laura had desktop computers in her classroom. This was an unexpected finding—the fact that easy access to technology did not guarantee its (increased) integration into the classroom instruction.

Table 6

Frequency of Sketchpad Use

Teacher	Frequency of student Sketchpad use per week	Frequency of teacher Sketchpad use per week	Total Sketchpad use per week
Brian	0.5	0.5	1.0
James	0.6	0.9	1.5
Susan	1.2*	4.0*	5.2*
Laura	0.7	1.0	1.7

Note. Data for Susan's Sketchpad use cover only the first nine (9) weeks of school while for the rest of the teachers it spans across twenty-two (22) weeks.

Each participant had a different reason for limited Sketchpad integration. James had a difficult access to a computer lab; he had to plan his lab days weeks or months ahead. Brian struggled with incorporating Sketchpad into his lessons because of new scope and sequence of the curriculum. Laura preferred to do hand-on activities that did not involve technology. Susan believes, "you can't just go completely immersed."

Brian had easy access to a computer lab, so he was able to take his students there whenever he wanted to; the main obstacle with access was that the computer lab was located far away from his classroom and he had to switch in between the two very quickly on the days he took his geometry students to the lab because he had to teach his algebra students in the classroom. In the lab, Brian's students were able to have one computer per student. The computers were located on three walls of the room so that Brian could see his students' monitors as he walked around the room.

James's situation was similar to that of Brian's in a way that he had to take his students to a different room in order to use computers. The main challenge for James, however, was access to a computer lab. He had to share a computer lab with other teachers at his school and had to sign up for the lab time weeks in advance. During the first two lessons, students were working on computers (one computer per student) in computer labs. As in Brian's case, the computers were located on three walls of the room so that James could see his students' monitors as he walked around the room. During the third lesson, James took his students to another teachers' classroom, which had a similar setup as the computer labs (computers on three walls of the room), but it also had individual student desks in the middle of the room making it a little more difficult to walk around to monitor student work on computers.

Susan and Laura had easy access to computers. Susan had a cart with laptops in her classroom at all times so whenever her students were working on Sketchpad activities, they took one laptop from the cart and sat at their regular student desk in the classroom. Laura, on the other hand, had computers along the three walls of her classroom. Whenever her students were working on Sketchpad activities, they moved from their student desks, which were located in the middle of the room, to the desks with computers on the edges of the room.

Brian and James preferred using Sketchpad instead of manipulatives. They acknowledged that Sketchpad and manipulatives could accomplish the same learning goals; however, there were some issues associated with the use of manipulatives, e.g., lack of availability of manipulatives. Also, Brian mentioned in the initial interview that the use of manipulatives "wastes just as much time as learning the program," so going to a computer lab was a much better option given that he had easy access.

Based on their experience from the first year of the DG implementation, Brian and James decided that the topics that did not work well with Sketchpad during the previous year would be covered in class first this year. James and Laura both noticed that, when teaching with Sketchpad, they were able to see how little their students know about geometry because when students were exploring concepts on their own (in Sketchpad), it was easier to see what they knew and did not know. Students' learning and understanding were less visible during class lectures when students sat in their desks and took notes.

All teachers displayed good knowledge of mathematical content taught in the observed lessons. They provided activity sheets to their students for the concepts being explored with Sketchpad. Brian made his own handouts, while James, Susan and Laura used pre-made handouts from the Sketchpad Lesson Link website or other Sketchpad-related resources available to them.

The class instruction was similar among the four teachers, but it also varied in some ways. All teachers monitored their students during Sketchpad explorations and answered their questions. James provided the least instruction at the beginning of each lesson. During the lesson, if he saw that students were struggling with something, he jumped in and explained to the whole class. Brian and Susan introduced each lesson briefly and then let their students explore. James, Brian and Susan, did not summarize any lessons with the whole class (except one time for Brian). Laura, on the other hand, provided a thorough introduction to and summary for each lesson; in between, her students got to explore with Sketchpad for about half of the class time. Laura believes that without the whole-class summary students never arrive at a conclusion and "it never closes."

Teaching-with-Sketchpad Task

Exploration. Part of the closing interview involved reflecting on a teaching-learning episode that took place during the pilot study:

> Ms. Johnson's geometry class is covering the unit on transformations. Yesterday they discussed translations and today they are discussing reflections. Students are using Sketchpad for an exploration. They are asked to create a pentagon in one of the quadrants and reflect it over the x-axis and then over the y-axis. The picture below (Figure 21) is a screenshot of one of Ms. Johnson's students, Ellen.

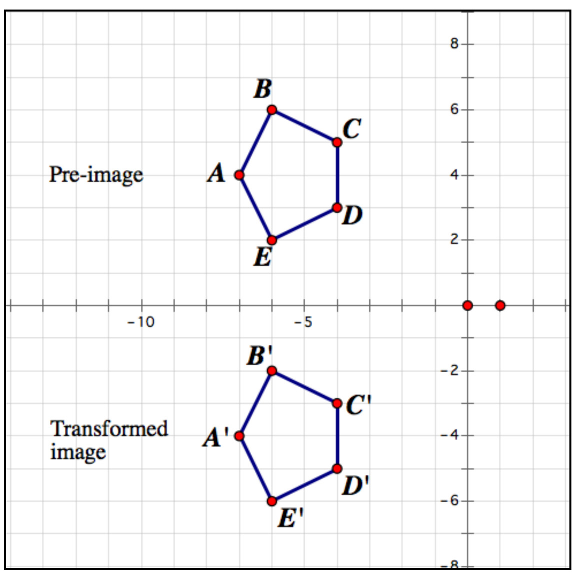


Figure 21. Transformation from the teaching-with-Sketchpad task.

What type of transformation do you think it is? Brian, Susan and Laura concluded that it was a translation. James's first response was that the presented transformation was a reflection over the x-axis, but the labels did not correspond. He kept looking at the two figures seeing a reflection, but after a few moments he concluded that it was a translation because of how the vertices were labeled. This was in fact a reflection over the x-axis; however, just like James noticed, the labels of the vertices were not corresponding, making it look like a translation. The only possible explanation for this mismatch was that Ellen changed the labels of the reflected figure to those illustrated in Figure 21 because Sketchpad labels vertices of transformed figures correctly.

If the student reflected the pre-image and then changed the labels of the reflected image to those shown in the figure, what would you do to help this student? Brian would change the shape of the pre-image so that it was not "symmetrical." James would construct a more "abstract" shape, maybe a triangle that "does not look like it can be translated." Therefore, Brian and James had similar ideas and would use the dynamic features of the software to "correct" this situation. Laura's response was along the same lines; she would change the pre-image figure. Susan, on the other hand, did not mention using Sketchpad at all. She would try to explain to Ellen that the corresponding vertices had to be labeled appropriately, e.g., B' would have to be E', C' would have to be D'. When I asked her to explain more, she mentioned folding the paper along the x-axis to make it more visible to Ellen which vertices were corresponding to which vertices. When reminded that Ellen was using Sketchpad for this activity, Susan repeated that she would explain to Ellen that the corresponding vertices had to match. Susan's response and the lack of Sketchpad use was intriguing because during the initial interview she mentioned the topic of geometric transformations as one that lend itself to Sketchpad use as a learning tool—"I like doing rotations and reflections [in Sketchpad] because those are hard concepts. It is harder than I thought for some students." The lack of Sketchpad incorporation in correcting Ellen's misconception was an unexpected finding since Susan had many years of experience in Sketchpad integration and she mentioned that she could not teach without it. It was surprising that she did not incorporate it in this task. At the same time, it showed that she still had room for developing her TPACK and the experience of technology use in the classroom instruction did not necessarily translate into increased TPACK suggesting that a sustained professional development is necessary.

Conjecture. The second part of the task involved reflecting over Ellen's conjecture about reflections over the x-axis and the y-axis:

> Later in the exploration, students were asked to find coordinates of all the points (see Figure 22) and write a conjecture about the relationship between coordinates of the pre-image and coordinates of the image reflected over the x-axis as well as about the relationship between coordinates of the pre-image and coordinates of the image reflected over the y-axis. Ellen wrote the following conjecture, "When I move one of the corners of the pre-image, the corresponding corner of the image reflected over the x-axis also moves. The same thing happens with the image reflected over the y-axis. The coordinates of the reflected images change from positive to negative and vice versa."

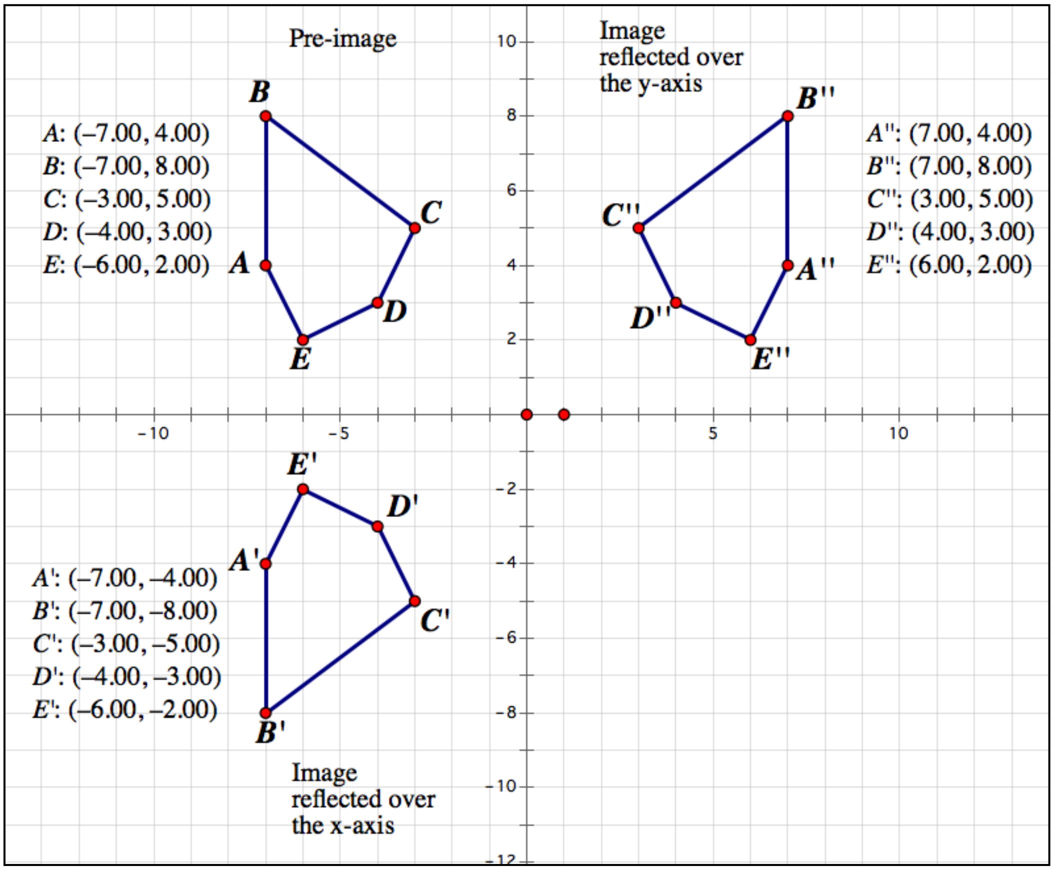


Figure 22. Sample screenshot illustrating coordinates of the pre-image and the reflected images.

What would you do to help Ellen improve her conjecture? How would you use sketchpad to help you accomplish this goal? All teachers understood what Ellen was trying to say and agreed that the first part of the conjecture was right; the only thing they would change was the last sentence—"The coordinates of the reflected images change from positive to negative and vice versa." They thought that it was not clear enough. Brian said, "I think the student probably knows what's happening but is having trouble putting it into words when saying that the coordinates change from positive to negative." He would tell the student to look at the coordinates and ask, "Is everything changing?

What is changing? And which axis?" This showed that when teachers know their students, they understand their train of thought and are good translators of students' ideas/expressions.

James's response was similar to Brian's in a sense that he could see what the student was trying to say, "I think I know where they're going with it," and he would continue to talk with the student so that she could better articulate the last sentence of the conjecture and be more specific. He would also spend a few minutes with the whole class to recap and make sure that they understood what happened when the image was reflected over the x-axis and the y-axis.

Susan's response was also similar to that of Brian's and James's and she would make sure that the student revised the last sentence and was more specific about what exactly happened. Laura's response was a little different and she said she would have her students log the coordinates and look at more instances because:

> If a student is looking just at her image, it is hard for her to see the big picture of what is happening to the x and y very specifically. So it is more general. Maybe if it was tabled with several people's numbers, then it is easier to see more specific. This is what is happening: across the x-axis, x stays the same, and y changes; across the y-axis, y stays the same, and x changes. I think more data and other people's [data] would clarify that because they have much more confidence in other people's data than their own, too.

Even though Ellen already had five points with coordinates, Laura thought it would be better to have even more examples; however, she did not mention using Sketchpad in any way to achieve that. By using Sketchpad, students can drag any of the vertices to obtain "more points" or they could construct a data table (see Figure 23 for example).

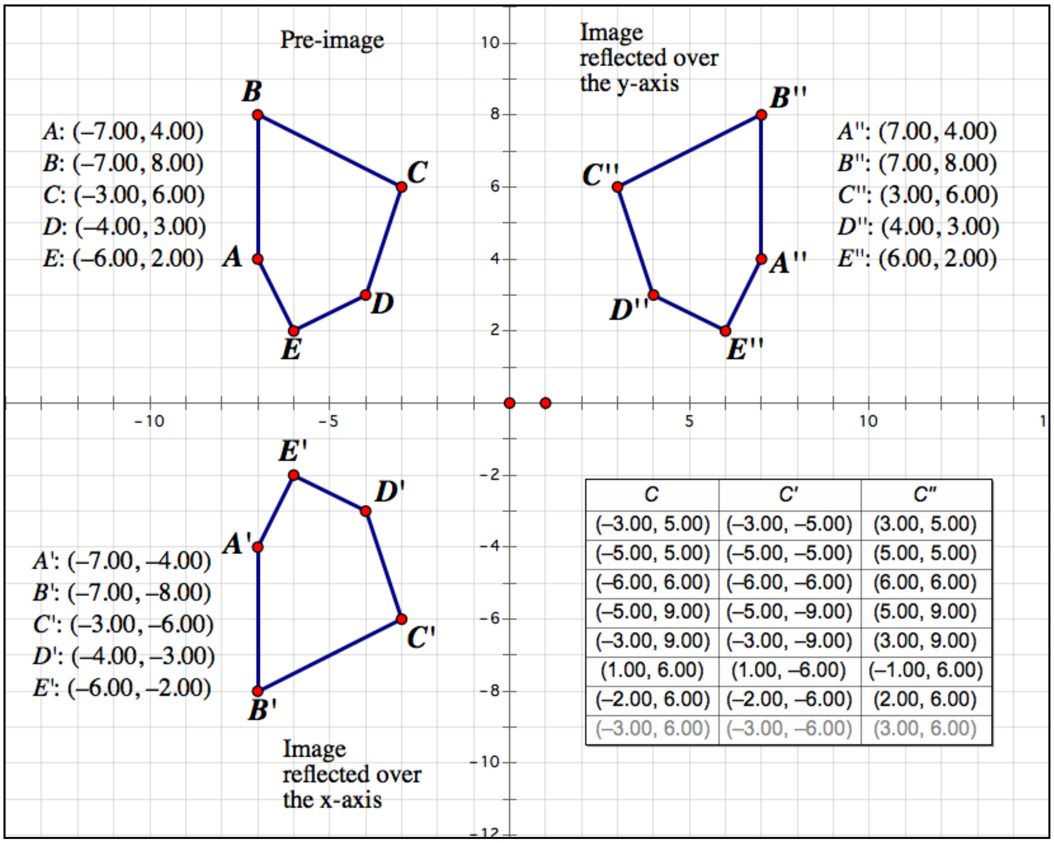


Figure 23. Screenshot of a sample data table constructed in Sketchpad for the coordinates of point C and its reflected images over the x-axis and the y-axis, C' and C'' respectively, after dragging point C.

TPACK Development Levels

Combined TPACK development levels were created after identifying individual TPACK development levels for all participants (see Figure 24 for a tabular representation and Figure 25 for a graphical representation).

Theme (descriptor)	TPACK Development Levels				
	Recognizing	Accepting	Adapting	Exploring	Advancing
Curriculum & Assessment (curriculum)	1		2	2	
Curriculum & Assessment (assessment)					
Learning (mathematics learning)			2	1	1
Learning (conception of student thinking)	1		2	2	
Teaching (mathematics learning)			1	4	
Teaching (instruction)				4	
Teaching (environment)			2	2	1
Teaching (professional development)			3	1	
Access (usage)			4		
Access (barrier)			3	1	
Access (availability)	1	2	3		
Level totals	**3**	**2**	**22**	**17**	**2**

Figure 24. Combined TPACK development levels for all participants from all non-survey data sources.

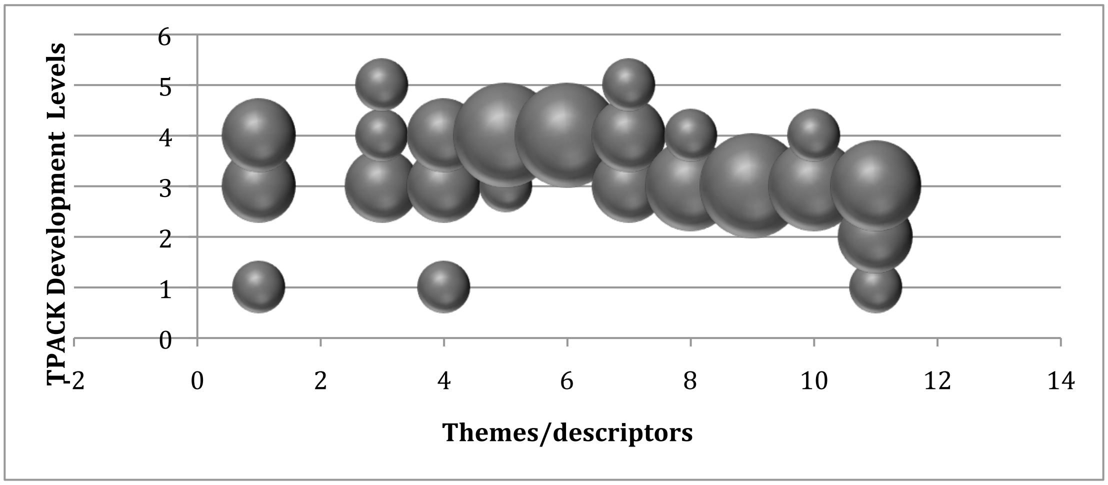


Figure 25. Combined TPACK development levels for all participants. Themes (descriptors) are: 1. Curriculum & Assessment (curriculum), 2. Curriculum & Assessment (assessment), 3. Learning (mathematics learning), 4. Learning (conception of student thinking), 5. Teaching (mathematics learning), 6. Teaching (instruction), 7. Teaching (environment), 8. Teaching (professional development), 9. Access (usage), 10. Access (barrier), 11. Access (availability).

It is clear that the combined TPACK concentrated around the adapting and exploring levels, which aligned with the TPACK development levels identified for each participant in Chapter IV—Brian (exploring), James (adapting), Susan (exploring for the teaching theme) and Laura (adapting). Although there is room for growth in knowledge, these figures are promising, considering this was only the second year of the DG implementation for most of the participants.

As mentioned earlier, none of the participants used Sketchpad for assessment, although they might have had beliefs about how they would incorporate Sketchpad in assessment. This finding offered an opportunity that can be explored in future studies.

Combined TPACK for the learning theme was identified at all levels except accepting. The TPACK development levels for the teaching theme were the most consistent and the highest—adapting (six indicators), exploring (eleven indicators) and advancing (one indicator). Combined TPACK for the access theme was also consistent for the usage and barrier descriptors— adapting (seven indicators) and exploring (one indicator).

Summary

This chapter provided cross-case analysis, common themes and differences among the four cases. It also identified several unexpected findings. Teachers can develop their TPACK at the same time they develop their PCK, so that even novice teacher can have high TPACK development levels. Easy access to technology does not guarantee technology integration in the classroom instruction. Also, prolonged incorporation of technology in teaching does not guarantee high TPACK development levels.

The next chapter, Chapter VI, presents discussion based on the findings of this study. Chapter VI also discusses tensions related to teaching with dynamic geometry software, implications for technology-enhanced professional development and recommendations for future research.

CHAPTER VI

DISCUSSION

This was a qualitative case study investigating high school teachers' technological pedagogical content knowledge (TPACK). Participants of this study were four high school geometry teachers who participated in the Dynamic Geometry Project (DGP) and were incorporating Sketchpad, a dynamic geometry software program, into their instruction. Three research questions guided this research study:

1. How do high school teachers develop TPACK while teaching geometry using dynamic geometry software?
2. How do high school teachers enact their TPACK when teaching with dynamic geometry software?
3. How are the five TPACK development levels (i.e., recognizing, accepting, adapting, exploring, and advancing) characterized for high school teachers who incorporate dynamic geometry software in teaching?

To answer these questions, I gathered data from multiple sources: initial interview, observations, documents, closing interview, a survey, implementation questionnaires, professional development attendance records and the researcher's log. I presented the findings in the individual case reports in Chapter IV and cross-case analysis in Chapter V. In this chapter, I address the following topics: discussion of findings, tensions related to curriculum and standardized testing, implications for technology-enhanced professional development and recommendations for future research.

Discussion of Findings

TPACK Development

All participants displayed strong knowledge of Sketchpad during the interviews and observations. They participated in all or almost all professional development sessions provided by the DGP. Susan was the only participant who used Sketchpad in teaching before the DGP; therefore, she had additional training and experience in using Sketchpad. Laura also used Sketchpad as a college student, but she did not use it in teaching before the DGP because she preferred doing hands-on activities that did not involve technology with her students. All participants valued the DGP professional development sessions; not only did they learn how to use Sketchpad, they also learned and shared implementation strategies with their colleagues. That collaboration was valuable to them and assisted them in incorporating Sketchpad in their classroom instruction. For Susan, the collaboration was especially valuable because she was the only geometry teacher at her school; meeting with other geometry teachers from local schools on a regular basis helped her learn how to better integrate Sketchpad into her geometry lessons. Even though she used Sketchpad for six years before the DGP, and she knew how to use it, she improved as a teacher as she mentioned in the initial interview—she was "able to point out to students more of what they were supposed to see." She mentioned several times that her students saw "magic" while using Sketchpad and did not understand where those things that they were exploring came from. Based on her comments, her TPACK had increased and she can teach more effectively with Sketchpad now. However, as Susan's

TPACK development levels and the closing interview suggested, her TPACK was not fully-developed. Even though she was the most experienced teacher in this study, overall, her TPACK development levels were inconsistent, except for the teaching theme where her TPACK development level was at the exploring level. Moreover, Brian, James and Laura did not use Sketchpad in teaching before the DGP, and they developed their teaching strategies with Sketchpad through the modeled lessons from the DGP professional development sessions. That was clearly visible in the observed lessons when teachers were guiding students in their explorations and creating learner-centered environments. All the cases implied that a sustained and collaborative professional development was necessary in order for teachers to develop and improve their TPACK. Such professional development also should be connected to practice directly as all the cases in this study indicated that their TPACK development occurred through attending the DGP professional development sessions and through their teaching practice. This dissertation offers a model for TPACK professional development that is based on the findings of this study. It is designed to be ongoing and connected to practice. I discuss it in detail in the section titled "The Total PACKage Professional Development Model (TPACK PD Model) for Teaching Geometry with Dynamic Geometry Software."

TPACK Enactment

One of the main themes that emerged from analyzing the data was the surprisingly low usage of Sketchpad by participants' students (see Appendix H). On average, Brian's and Laura's students used Sketchpad once every other week while having easy access to computers; James's students used Sketchpad with the same frequency while meeting many challenges with computer lab access. Based on the interviews with participants and other conversations, curriculum and standardized testing played a key role in deciding whether they took their students to a computer lab or not. This "barrier" to technology integration is discussed in more detail in the forthcoming section titled "Curriculum and Standardized Testing."

Susan used Sketchpad primarily for demonstrations even though her students had opportunities to do Sketchpad explorations every week. This suggested that she preferred to use it in a more teacher-directed instruction than in a student-centered instruction. This finding also agrees with findings from Hannafin et al. (2001); teaching style is difficult to change and it might take a long time for a teacher to adjust to a different approach to teaching and learning.

TPACK Development Levels

Since there are eleven descriptors across the four themes of the TPACK Development Model (Niess et al., 2009), participants had different TPACK development levels for the different descriptors. TPACK development level for the assessment descriptor was not identified for any of the participants. The overall TPACK development levels for the participants were: exploring for Brian, adapting for James and Laura, and exploring for Susan for the teaching theme only. Taken as a whole, the combined TPACK was at the adapting and exploring levels, which was highly promising. At the same time, the findings showed that the participants needed to further develop their TPACK through an ongoing professional development.

For Brian, James and Laura, the TPACK development levels aligned closely with their self-perceived TPACK development levels for most descriptors. For Susan, most of the TPACK development levels were below her self-perceived TPACK development levels. One explanation for this could be that Susan felt more comfortable and confident with Sketchpad since she was using it for a long time, and that possibly resulted in her higher perceptions about her TPACK. This confirms findings from other studies, which concluded that gains measured by self-report surveys reflect an increase in confidence instead of an increase in knowledge (Lawless & Pellegrino, 2007; Schrader & Lawless, 2004). The rest of the participants, who were using Sketchpad in teaching only for the second year, felt like they still had a lot to learn and expressed lower TPACK development levels than Susan did in the TPACK survey.

Curriculum and Standardized Testing

Based on the interviews and implementation questionnaire responses, two tensions surfaced—one of them was related to the curriculum and the other one to standardized testing. Although all participants were open to using Sketchpad with their students and had no problems with identifying topics in the curriculum where Sketchpad could be used, they did not let their students use Sketchpad often. The main reasons for that were the extensive curriculum and the standardized testing accountability. In Texas, most students take a geometry course in 10^{th} grade, but at the end of the school year they take a test that largely consists of algebra. In addition the geometry objectives tested in 10^{th} grade consist of middle school geometry content; in 11^{th} grade, the Exit Level assessment consists of high school level geometry content (see Table 7). The teachers in this study needed to prepare their students to do well on this test and had to follow the scope and sequence of curriculum that consisted of algebra in addition to geometry. That made it more challenging for integrating Sketchpad and having students explore and "discover" geometric concepts because these activities took more time. At the same time, the teachers wanted their students to be successful and they wanted to ensure they covered all topics in the curriculum (including algebra) so that their students were ready to do well on the test.

Table 7

Texas Assessment of Knowledge and Skills (TAKS) Blueprint for Grade 10 and Exit Level Mathematics

TAKS Objectives	Number of Items: Grade 10	Number of Items: Exit Level
1: Functional Relationships	5	5
2: Properties and Attributes of Functions	5	5
3: Linear Functions	5	5
4: Linear Equations and Inequalities	5	5
5: Quadratic and Other Nonlinear Functions	5	5
6: Geometric Relationships and Spatial Reasoning	5	7
7: 2-D and 3-D Representations	5	7
8: Measurement	7	7
9: Percents, Proportions, Probability, and Statistics	5	5
10: Mathematical Processes and Tools	9	9
Total number of items	56	60

On the other hand, the standardized testing in Texas is currently changing, which can mean fantastic news for geometry teachers and students. The new testing system includes end-of-course exams instead of grade-level exams and students will take a geometry exam at the end of the school year in which they take the geometry class. The spring 2012 is the first time for the new tests; at the high school level, only freshmen will take them in 2012. The new testing system brings changes into how geometry is taught and focuses fully on the geometry content (see Table 8).

Table 8

State of Texas Assessments of Academic Readiness (STAAR): Geometry Blueprint

Category	Number of Standards	Number of Items
1: Geometric Structure	9	10
2: Geometric Patterns and Representations	5	8
3: Dimensionality and the Geometry of Location	6	10
4: Congruence and the Geometry of Size	12	16
5: Similarity and the Geometry of Shape	4	8
Total	36	52

As the curriculum changes, teachers need to adapt to these changes and possibly adjust their teaching practices. The new test in Texas and related curriculum might create more favorable conditions for integrating dynamic geometry programs into instruction because of the full focus on the geometry content (see Table 9). Out of the four participants in this study, Brian was the only teacher who was teaching freshmen this year, and so he was the only one implementing new curriculum. Brian exhibited the highest TPACK development levels out of the four participants, which partially supports the hypothesis that the new testing system and curriculum focusing solely on geometry content facilitate more effective technology integration. At the same time, however, adjusting to the new curriculum requires time and, just as in Brian's case, adjusting to the new curriculum can take away time from planning and incorporating technology into instruction.

Table 9

High School Level Geometry Content in Texas Assessments

Assessment	Number of High School level Geometry items	Percentage of High School level Geometry Items
Grade 10 TAKS	0	0
Exit Level TAKS	21	35
STAAR	52	100

Because teachers have little or no influence on the curriculum or standardized testing, it is difficult for them to overcome this barrier to technology integration. With the current state of affairs, they can still integrate dynamic geometry into instruction; however, it might take a significant amount of time for them to readjust to the new curriculum and figure out how dynamic geometry can be integrated in it. Nonetheless, continued professional development can ease some of the readjustment challenges.

Implications for Technology-Enhanced Professional Development

Three participants in this study indicated that the main source of their Sketchpad knowledge development were the professional development workshops offered by the DGP. Since Susan had taught with Sketchpad before the DGP, she was already familiar with the software; however, she had an opportunity to learn about the new features of the software as well as how to integrate it into instruction better. Additionally, Susan's TPACK development levels indicated that, even with many years of incorporating Sketchpad into the classroom instruction, she had not developed her TPACK completely.

The second main source of TPACK development was practice, i.e., planning Sketchpad activities and teaching with Sketchpad. This source of knowledge development suggested how TPACK development could be structured. In addition, since the combined TPACK development levels were around adapting and exploring, the participants could still improve their knowledge for teaching with Sketchpad; therefore more professional learning opportunities for in-service teachers are needed. The following sections, TPACK-in-Practice and (Virtual) Lesson Study, briefly review two contemporary efforts for enhanced professional development.

TPACK-in-Practice

The TPACK-in-Practice Framework (Figg & Jaipal, 2012) offers a professional learning opportunity for teachers wanting to integrate technology in their instruction. This professional development model is closely related to content taught and does not focus merely on learning the technology. The model consists of four stages "(a) modeling a tech-enhanced activity type (learning WITH the tool), (b) integrating 'pedagogical dialog' in a modeled lesson, (c) developing TK (in context) through tool demonstrations, and (d) applying TPACK-in-Practice to design an authentic learning task" (Figg & Jaipal, 2012, p. 4685).

(Virtual) Lesson Study

Lesson study (LS) is a form of professional learning that is popular in Japan and other Asian countries; it is also gaining recognition in the United States and so far is mainly used for research purposes. The major benefits of LS are collaboration, teacher knowledge improvement and instructional improvement (Lewis, Perry, & Hurd, 2009). The features of LS include investigation, planning, research lesson and reflection (Lewis et al., 2009). Yursa and Silverman (2012) proposed a virtual model for LS, which creates an increased access to the LS community for teachers in rural and urban districts. Since the virtual LS is facilitated online, it allows for participation from remote locations and becomes more accessible.

The Total PACKage Professional Development Model (TPACK PD Model) for Teaching Geometry with Dynamic Geometry Software

Based on the findings of this study and the professional development provided by the DGP, and drawing from the two models of professional development, TPACK-in-Practice and (Virtual) Lesson Study, I propose a new type of professional development specifically designed for teachers interested in integrating dynamic geometry into their instruction. This new model is called the Total PACKage Professional Development Model or TPACK PD Model and consists of seven stages: technological knowledge (TK) development, technological content knowledge (TCK) development, technological pedagogical knowledge (TPK) development, developing a technology-enhanced lesson, teaching, observing and reflecting. As Figure 26 illustrates, TPACK PD is a cyclical process; simply going through all stages once does not guarantee total TPACK development. Instead, TPACK PD participants continue learning and developing their knowledge by visiting all of the stages multiple times. This kind of professional development can facilitate teacher change because it uses additional sessions and collective participation (Penuel, Fishman, Yamaguchi, & Gallagher, 2007). By creating a professional learning community and devoting more time to professional development, teachers are more likely to integrate new knowledge into practice (Brown, 2004; Penuel et al., 2007).

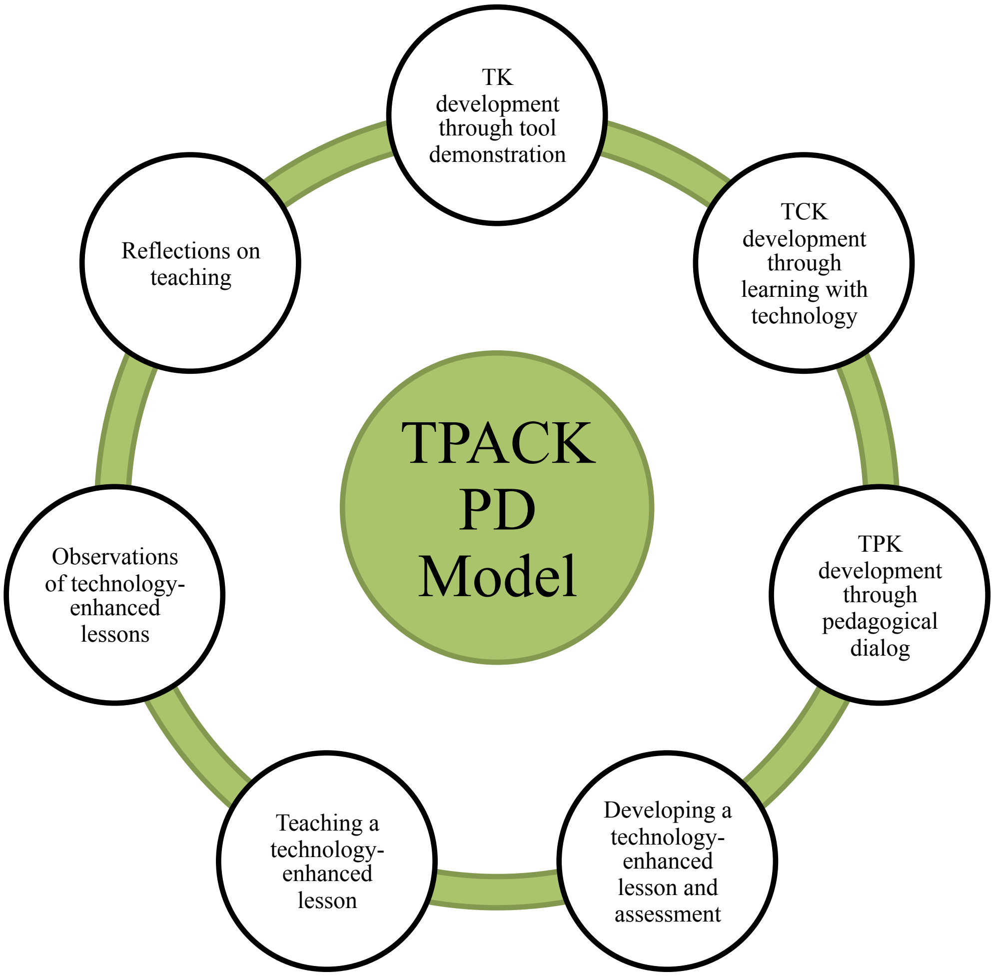


Figure 26. TPACK PD model.

The first four stages of the TPACK PD Model are adapted from the TPACK-in-Practice framework; however, they have been rearranged to better suit the purpose of developing TPACK for teaching with dynamic geometry software. In the TPACK-in-Practice model, TK development is the third stage; in the TPACK PD model, it is the first stage. The fourth stage from the TPACK-in-Practice model is also part of the LS model. The remaining three stages are adapted from the (Virtual) LS model. The following are descriptions of all the stages in the TPACK PD Model:

- TK development through tool demonstration. Participants learn how to use the software in context. Since dynamic geometry software is very user friendly, participants can quickly learn the basics. Additionally, since the software is specifically designed for being used for teaching and learning of geometry, participants can quickly move to the next stage of knowledge development, TCK development. As mentioned earlier, TPACK development is cyclical, therefore, participants will be coming back to this stage to learn more about the software at later times.
- TCK Development through learning with technology. The next step is to introduce workshop participants to the tool (Sketchpad) by learning with the tool. This is accomplished by a technology-enhanced model lesson where participants are learners and a workshop designer facilitates the lesson. “The experience provides participants with context for how the tool is useful in instruction” (Figg & Jaipal, 2012, p. 4685).
- TPK development through pedagogical dialog. Following the model lesson, participants engage in discussion about the lesson, which allows them to learn what decisions are involved in “designing and implementing technology-enhanced activities” (Figg & Jaipal, 2012, p. 4685). This stage is especially valuable for novice teachers who are still developing their pedagogical content knowledge.
- Developing a technology-enhanced lesson and assessment. Participants develop a lesson or an activity that incorporates Sketchpad. They will later teach and

videotape this lesson. Participants also think of how Sketchpad can be used for assessment of topics taught in a given lesson.

- Teaching a technology-enhanced lesson. Each participant teaches the designed lesson with Sketchpad and videotapes it. Later, videotaped lessons will be shared with other teachers for watching and feedback.
- Observing technology-enhanced lessons. All participants watch their own, videotaped lessons as well as videotaped lessons of their colleagues. These lessons can be shared through an online management system, to which all participants have access. This virtual component of LS allows participants to watch the research lessons at their own time; it also allows them to observe a larger number of lessons.
- Reflecting on the observed lesson. Participants discuss the observed lesson, "draw out implications for lesson redesign, for teaching-learning more broadly, and for understanding of students and subject matter" (Lewis et al., 2009). Participants revise the lesson so that the dynamic geometry program plays a more integral part in learning. In the virtual LS, this can be accomplished through asynchronous online discussion boards, synchronous chat rooms or video-conferencing.

Summary

Many experienced teachers did not have a chance to experience technologies such as Sketchpad in their teacher preparation. Also, many new teachers did not go through a traditional teacher preparation education; instead, they received their teacher certification through an alternative certification program. That usually means that they were not required to take a geometry course in college. And if they did not take such a course in college then they were not likely to know about Sketchpad. Therefore, providing high-quality in-service professional development that focuses not only on TK development, but also on total TPACK development is imperative. As the findings of this study suggested, experienced teachers needed an ongoing and collaborative professional development as well. To facilitate a change in teacher practices, professional development should take place at the school or district levels (Penuel et al., 2007). Also at the school level, teachers can form groups teaching the same subject with technology so they can collaborate, share ideas, and support each other throughout the process of learning a new technology and integrating it into practice.

Recommendations for Future Research

This study fills in the gap in literature on TPACK and teaching geometry with dynamic geometry software by providing four case reports on how geometry teachers develop and use their TPACK. Researchers should look to see how teachers' TPACK affects student learning in a dynamic geometry environment. Evaluating students' learning, in addition to teacher's TPACK, will help in detecting to what extent teachers' TPACK impacts students' learning.

This study was a first step towards gaining more resources for future studies that wish to explore teachers' TPACK quantitatively, especially with respect to dynamic geometry. However, more studies in this area are needed in order to start developing items that measure TPACK related to teaching with dynamic geometry software.

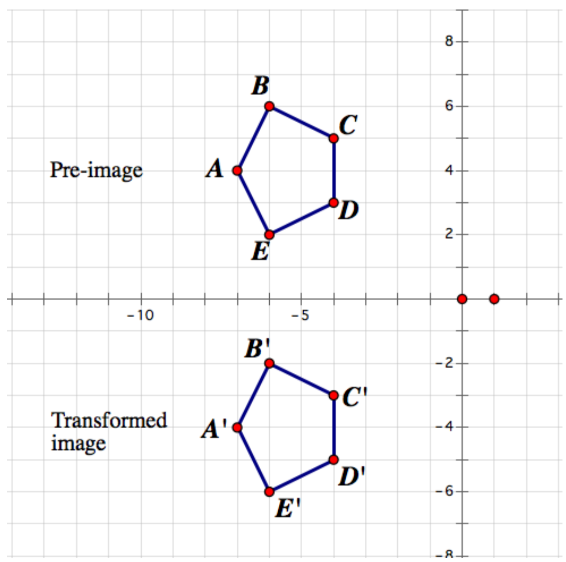
As a result of this study, I proposed a new TPACK PD Model. Future studies can put it into practice and evaluate its effectiveness. Additionally, because participants of this study were part of a grant project, they received external motivation and support (e.g., professional development, monetary stipends) in integrating dynamic geometry in the classroom instruction. Since most teachers do not participate in such projects and do not receive that extra support, it would be worthy to investigate TPACK of such teachers by employing the TPACK Development Model (Niess et al., 2009).

APPENDIX A

TEACHING-WITH-SKETCHPAD TASK

Ms. Johnson's geometry class is covering the unit on transformations. Yesterday they discussed translations and today they are discussing reflections. Students are using Sketchpad for an exploration. They are asked to create a pentagon in one of the quadrants and reflect it over the x-axis and then over the y-axis. The picture below is a screenshot of one of Ms. Johnson's students, Ellen.

Do you think that Ellen might have any misconceptions when doing this exploration or that she might form some misconceptions based on what you see on her screen?



Later in the exploration, students were asked to find coordinates of all the points and write a conjecture about the relationship of coordinates of the pre-image and coordinates of the image reflected over the x-axis as well as about the relationship of coordinates of the pre-image and coordinates of the image reflected over the y-axis. One student wrote the following conjecture, "When I move one of the corners of the pre-image, the corresponding corner of the image reflected over the x-axis also moves. The same thing happens with the image reflected over the y-axis. The coordinates of the reflected images change from positive to negative and vice versa."

What would you do to help this student improve her conjecture?
How would you use Sketchpad to help you accomplish this goal?

APPENDIX B

INVITATION LETTER TO POTENTIAL PARTICIPANTS

I would like to invite you to participate in my dissertation study (which is part of the Dynamic Geometry Project). You are a perfect candidate because of the high level of implementation of dynamic geometry in your teaching in the past year. Below are some details about my study so you can decide if you'd like to participate.

Background Information:
The purpose of this study is to describe how high school teachers acquire knowledge of geometry, pedagogy, and technology, and how this knowledge affects their use of dynamic geometry software in teaching geometry.

Procedures:
If you agree to be in this study, you will be asked to:

- Participate in two audiotaped interviews (approximately 50 minutes in length each). The first interview will be conducted at the beginning of the school year and the second interview will be conducted after all observations.
- Allow me to observe (and videotape if possible) three of your geometry classes while you teach with Sketchpad.
- Provide me with lesson plans and/or handouts that accompany the observed lessons.

All data will be collected August through December this year. We can schedule the interviews and observations at times most convenient for you. My schedule is flexible.

Voluntary Nature of the Study:
Your participation in this study is voluntary. This means that everyone will respect your decision of whether or not you want to be in the study and if you decide to join the study, you are free to withdraw at any time.

Risks and Benefits of Being in the Study:
There are no known risks and/or discomforts associated with this study. The expected benefit associated with your participation is that you will gain a deeper insight of your teaching practice with Sketchpad and of how to teach geometry with Sketchpad more effectively.

Please let me know if you have any questions. My phone number is 000-000-0000 if you prefer to call. I appreciate you taking the time to read this and considering participating in my study. I hope to hear from you soon.

APPENDIX C

CONSENT FORM

Teaching geometry with dynamic geometry software

You are invited to take part in a case study that will describe how teachers learn about dynamic geometry for teaching and how they use it in the classroom. You were chosen for the study because you use the Geometer's Sketchpad in teaching geometry and you showed a high level of implementation in the past year. The following information is provided for you to decide whether or not you wish to participate in this study.

Background Information
The purpose of this study is to describe how high school teachers acquire knowledge of geometry, pedagogy, and technology, and how this knowledge affects their use of dynamic geometry software in teaching geometry.

Procedures
If you agree to be in this study, you will be asked to:

- Participate in two audiotaped interviews (approximately 60 minutes in length each).
- Allow me to observe and videotape 3-5 of your geometry classes while you teach with Sketchpad.
- Provide me with lesson plans and/or handouts that accompany the observed lessons.

All data will be collected September through December this year.

Voluntary Nature of the Study
Your participation in this study is voluntary. This means that everyone will respect your decision of whether or not you want to be in the study and if you decide to join the study, you are free to withdraw at any time.

Risks and Benefits of Being in the Study
There are no known risks and/or discomforts associated with this study. The expected benefit associated with your participation is that you will gain a deeper insight of your teaching practice with Sketchpad and of how to teach geometry with Sketchpad more effectively.

Compensation
There will be no compensation for participating in this study.

Confidentiality
Any information you provide will be confidential. Your name will not be associated with the research findings in any way. I would be happy to share my findings with you after the research is completed.

Contacts and Questions
My name is Ewelina McBroom and my faculty advisor is Dr. Zhonghong Jiang. You may ask any questions you have about this study by contacting me via phone at 000-000-0000, via email as XXXX@txstate.edu, or Dr. Jiang via email at XXXX@txstate.edu.
Texas State University's approval number for this study is EXP2011R4108.

Please sign your consent with full knowledge of the nature and purpose of the procedures. A copy of this consent form will be given to you to keep.

____
Name of Participant

________________________________ ______________________
Signature of Participant Date

APPENDIX D

INITIAL INTERVIEW PROTOCOL

The purpose of this interview is to develop an understanding of how you have developed your knowledge for teaching geometry with the Geometer's Sketchpad.

Background questions:

How long have you been teaching?
How long have you been teaching geometry?
How long have you been teaching with Sketchpad?
Have you ever used another dynamic geometry software program either in learning or teaching? If so, for how long?
Have you assumed additional roles/positions (e.g., department chair, math specialist, etc.) during your teaching career? If so, briefly describe your responsibilities.

Main interview questions:

1. When you hear the words "dynamic geometry" what comes to your mind?
2. How and when did you first hear about Sketchpad? Briefly describe this experience.
3. Why did you decide to teach geometry with Sketchpad?
4. What is your current view and understanding about integrating Sketchpad as a learning tool in geometry?
    a. How do you define "integration" of Sketchpad in teaching and learning of geometry?
    b. Describe a lesson you taught that represents your current view.
5. What specific geometry topics lend themselves to Sketchpad as a learning tool?
6. What do you see as barriers for integrating Sketchpad in teaching and learning of geometry?
7. How did your knowledge and skills with Sketchpad changed through your work in the DGP?
8. How has your knowledge about students' understanding, thinking, and learning about geometry topics with Sketchpad changed through your work in the DGP?
9. How has your conception of incorporating Sketchpad in teaching specific topics in geometry changed through your work on the DGP project? Give an example from a lesson you taught last year.

10. When you start using Sketchpad with your students (at the beginning of a school year), what strategies do you use to guide them in learning about the software? (Learning about Sketchpad as students are learning about a specific geometry topic or focusing on learning about the technology and later as the geometry context?)
11. In addition to your participation in the DGP, what kind of activities (e.g., professional development, conferences, self-directed study, Internet resources) have you engaged in that lead you to adopt teaching and learning of geometry with Sketchpad? Briefly describe what influenced your choice.
12. How important is (to you) the integration of Sketchpad into teaching and learning of geometry?
13. Do you evaluate the results of your decision to integrate teaching and learning of geometry with Sketchpad? How do you do it?

APPENDIX E

OBSERVATION PROTOCOL

<table>
<tr><td colspan="2">Date: __________ Time: __________
School: ___________________________
Teacher: __________________________</td></tr>
<tr><td>Time</td><td>Descriptive Notes</td></tr>
<tr><td colspan="2">The physical setting
How is space allocated? What objects, resources, technologies are in the setting?

The participants
How many students are present? Are there any other people present?

Activities and interactions
What is going on? How do students and the teacher interact with the lesson and with one another?

Conversation
What is the content of conversations? Who speaks to whom? Who listens?

What is the role of technology in this lesson?

What are the benefits of using technology in this lesson?

Are there any apparent disadvantages of using technology in this lesson?

How does teacher manage the classroom when teaching this lesson?

How does the teacher assess student learning in this lesson?</td></tr>
</table>

APPENDIX F

TPACK DEVELOPMENT MODEL SELF-REPORT SURVEY

From *Secondary Mathematics Teachers' Perceptions of Their Integration of Instructional Technologies* (Doctoral dissertation), by J. T. Ivy, 2011, pp. 127-131, and from *An Examination of Secondary Mathematics Teachers' TPACK Development Through Participation in a Technology-Based Lesson Study* (Doctoral dissertation), by J. W. Riales, 2011, pp. 230-234. Copyright [2011] by J. T. Ivy and J. W. Riales. Reprinted with permission.

<table>
<tr><td colspan="2">TPACK Development Model Self-Report Survey

Specific to _________ (technology)

Please place a check in the box to the left of each statement that describes your beliefs and/or integration of technology in your classroom. You may give additional information in the spaces provided to clarify your selections or if none of the statements describe your beliefs/integration.</td></tr>
<tr><td></td><td>1. I can see how this technology might be useful with some of the topics in my curriculum, but I am not convinced its use will make much of a difference for my students' learning.</td></tr>
<tr><td></td><td>2. I believe this technology would make a difference in my students' learning and would like to use this technology with my students, but I'm not really sure how to integrate its use with the topics in my curriculum.</td></tr>
<tr><td></td><td>3. I believe this technology is beneficial to students' learning. I have allowed my students to use this technology for investigation of a few topics.</td></tr>
<tr><td></td><td>4. I believe this technology facilitates students' learning. I have allowed my students to use this technology for investigation of several topics. I have changed some of my lessons to integrate the technology and am searching for more ways to integrate the technology into the curriculum.</td></tr>
<tr><td></td><td>5. I am convinced that this technology is essential to promote learning for my students. My students use this technology on a regular basis. I extend the objectives in my curriculum by allowing my students the opportunities to develop deeper mathematical thinking through the technology use.</td></tr>
<tr><td colspan="2">Use this space for any additional information related to the statements above.</td></tr>
</table>

	6. I don't like to allow my students to use this technology on tests because I want to know what they know about mathematics, not what the technology can do.
	7. I allow my students to use this technology only on certain parts of tests or only on certain tests.
	8. If I allow my students to use this technology on tests, I make sure that the test questions measure what my students understand (concepts) along with what they know how to do (procedures).
	9. I allow my students to use this technology on tests. I make my tests to involve a variety of questions (some that require the technology, some that they could use the technology but it is not required, and some in which the technology use has no impact).
	10. I design my assessments so that the students must demonstrate the understanding of the mathematics through the technology use.
Use this space for any additional information related to the statements above.	
	11. I believe that if my students use this technology too often, they will not learn the math for themselves.
	12. I am afraid that if I try to introduce a new topic with this technology, that my students will be too distracted by the technology use to really learn the mathematics. I want them to learn how to do it on paper first, and then they can use the technology.
	13. I have allowed my students to explore a few topics using this technology even before the topics are discussed in class.
	14. My students explore several topics for themselves using this technology to help them develop a deeper understanding. Sometimes the students' thinking guides their explorations in directions other than what I had planned.
	15. I design my own technology lessons. When I plan my lessons, I really think about how to integrate the technology to help the students better understand the mathematics. After the lesson, I reflect on the lesson and how it could be changed to increase student understanding using this and/or other technologies.
Use this space for any additional information related to the statements above.	

	16. I might show my students how this technology relates to the topic, and I don't mind if my students use this technology outside of class, but I do not plan to allow class time for the students to use this technology.
	17. If my students use the technology to explore a new topic, they won't think about and develop the mathematical skills for themselves.
	18. I try to use this technology to promote my students' thinking, but have not had a lot of success.
	19. I often use pre-made technology activities to engage my students in their learning. I reflect on my students' thinking, communication and ideas during the technology use to make decisions about any changes that need to be made in the design of the lesson.
	20. I cannot imagine my classes without this technology! Using this technology is a vital piece of facilitating my students' learning and helps promote their thinking to more advanced levels.
Use this space for any additional information related to the statements above.	
	21. This technology might be useful, but before I could use this technology, I would have to teach my students about the technology and how it works. I have too many objectives to cover to do that.
	22. I use this technology occasionally, such as between units or at the end of the term. The technology use doesn't necessarily tie with the mathematical goals of the class.
	23. I use this technology to reinforce concepts that I have taught earlier or that my students should have learned in a previous class. I do not use it regularly when teaching new topics.
	24. I use this technology as a learning tool to engage my students in high-level thinking activities (such as projects or problem-solving).
	25. I use this technology to present mathematical concepts and processes in ways that are understandable to my students. I actively accept and promote use of this technology for learning mathematics. Other teachers come to me as a resource for ideas of how to help their students use the technology to promote understanding.
Use this space for any additional information related to the statements above.	

	26. My students and I use this technology for procedural purposes only.
	27. I have led my students through a few simple ideas of how to use this technology that I learned during professional development.
	28. I have led my students through uses of this technology that I learned during professional development, but I changed the activities to meet the needs of my students.
	29. When my students explore with this technology, I serve as a guide. I do not direct their every action with the technology.
	30. On a regular basis, I use a wide variety of instructional methods with this technology. I present tasks for my students to engage in both deductive and inductive strategies with the technology to investigate and think about mathematics to deepen their understanding.
Use this space for any additional information related to the statements above.	
	31. In my class, the focus is on the mathematics first. I can imagine that perhaps this technology might be used to reinforce those mathematical ideas only after the students have shown they can perform the skills on paper.
	32. I allow my students to use this technology to assist them with their skills. I direct my students step-by-step to use this technology.
	33. I use some exploration activities with this technology, but I usually guide my students through the steps to save class time.
	34. I have explored a variety of instructional methods with this technology, to allow my students to engage both inductively and deductively.
	35. I use this technology in a student-led environment, where the students explore with the technology both individually and in groups. When working in groups, all members of the group are actively involved.
Use this space for any additional information related to the statements above.	
	36. I would consider attending a workshop demonstrating the use of this technology, but only if it is local.
	37. I am interested and would be likely to attend workshops or professional developments to learn more about how to use this technology to further mathematics education.
	38. I am likely to attend professional developments related to technology use in mathematics education and to share those ideas with other teachers in my building, but I am likely to focus on learning one type of technology integration at a time.
	39. I have made contact with others who are using this technology and plan to meet and work with them throughout the year to integrate this and other technologies appropriately into our mathematics curriculum.
	40. I believe it is time to transform our mathematics curriculum to one that utilizes 21st century technologies! I have found organizations and workshops that I can attend to learn more about how to integrate this and other technologies into my math curriculum. I plan to share what I learn with others in my district.
Use this space for any additional information related to the statements above.	

	41. My students can use this technology only after they have mastered the pencil-and-paper skills.
	42. I allow my students to use this technology on a regular basis, usually just for skill purposes and under tightly controlled circumstances.
	43. I have a few units in which I allow students to explore new topics with this technology.
	44. I encourage my students to use this technology during most class meetings. They often explore new topics using this technology.
	45. I allow my students to use this technology in every aspect of the class and encourage the technology use to challenge the boundaries of what they can learn and understand.
Use this space for any additional information related to the statements above.	
	46. Mathematics has not changed just because we have more technologies available. Students still need to know how to do everything they've always been taught. For example
	47. It takes too much time and hassle to allow the use of this technology every day. I will let my students use it from time to time
	48. Using this technology will present some management issues
	49. I know that using this technology presents some new management issues
	50. Using this technology presented some issues
Use this space for any additional information related to the statements above.	
	51. I see the use of this technology tool for simplifying some "messy math" problems (problems with "unfriendly" real-life numbers for example). I make this technology available on the rare occasion that we encounter those type problems (maybe for extra credit).
	52. Using this technology allows me to demonstrate more examples.
	53. I take a different approach to teaching using this technology. Through its use, my students not only explore and apply key concepts using multiple representations, but they are also able to examine more complex mathematics topics making mathematical connections than they would be able to without the technology use.
	54. Using this technology allows my students access to explore and apply key concepts using multiple representations (such as symbols, graphs, tables, and/or data lists) and making important connections among representations and concepts.
	55. My students regularly explore and apply key concepts of more complex mathematical topics than normally outlined for this class using multiple representations and connections.
Use this space for any additional information related to the statements above.	

APPENDIX G

CODEBOOK FOR TPACK DEVELOPMENT LEVELS AND THEMES

The codes were created for each TPACK development level in each of the eleven categories formed by themes and descriptors. The code descriptions are based on the TPACK Development Model (Niess et al., 2009).

Theme (descriptor)	Recognizing	Accepting	Adapting	Exploring	Advancing
Curriculum & Assessment (curriculum)	C1c	C2c	C3c	C4c	C5c
Curriculum & Assessment (assessment)	C1a	C2a	C3a	C4a	C5a
Learning (mathematics learning)	L1m	L2m	L3m	L4m	L5m
Learning (conception of student thinking)	L1c	L2c	L3c	L4c	L5c
Teaching (mathematics learning)	T1m	T2m	T3m	T4m	T5m
Teaching (instruction)	T1i	T2i	T3i	T4i	T5i
Teaching (environment)	T1e	T2e	T3e	T4e	T5e
Teaching (professional development)	T1p	T2p	T3p	T4p	T5p
Access (usage)	A1u	A2u	A3u	A4u	A5u
Access (barrier)	A1b	A2b	A3b	A4b	A5b
Access (availability)	A1a	A2a	A3a	A4a	A5a

Code	Code description
C1c	Acknowledges that mathematical ideas displayed with the technologies can be useful for making sense of topics addressed in the curriculum.
C2c	Expresses desire but demonstrates difficulty in identifying topics in own curriculum for including technology as a tool for learning.
C3c	Understands some benefits of incorporating appropriate technologies as tools for teaching and learning the mathematics curriculum.
C4c	Investigates the use of topics in own curriculum for including technology as a tool for learning; seeks ideas and strategies for implementing technology in a more integral role for the development of the mathematics that students are learning.
C5c	Understands that sustained innovation in modifying own curriculum to efficiently and effectively incorporate technology as a teaching and learning tool is essential.
C1a	Resists idea of technology use in assessment indicating that technology interferes with determining students' understanding of mathematics.
C2a	Acknowledges that it might be appropriate to allow technology use as part of assessment but has a limited view of its use (i.e., use of technology on a section of an exam).
C3a	Understands that if technology is allowed during assessments that different questions/items must be posed (i.e., conceptual vs. procedural understandings).
C4a	Actively investigates use of different types of technology-based assessment items and questions (e.g., technology active, inactive, neutral or passive).
C5a	Reflects on and adapts assessment practices that examine students' conceptual understandings of the subject matter in ways that demand full use of technology.
L1m	Views mathematics as being learned in specific ways and that technology often gets in the way of learning.
L2m	Has concerns about students' attention being diverted from learning of appropriate mathematics to a focus on the technology in the activities.
L3m	Begins to explore, experiment and practice integrating technologies as mathematics learning tools.
L4m	Uses technologies as tools to facilitate the learning of specific topics in the mathematics curriculum.
L5m	Plans, implements, and reflects on teaching and learning with concern and personal conviction for student thinking and understanding of the mathematics to be enhanced through integration of the various technologies.
L1c	More apt to accept the technology as a teaching tool rather than a learning tool.
L2c	Is concerned that students do not develop appropriate mathematical thinking skills when the technology is used as a verification tool for exploring the mathematics.
L3c	Begins developing appropriate mathematical thinking skills when technology is used as a tool for learning.
L4c	Plans, implements, and reflects on teaching and learning with concern for guiding students in understanding.
L5c	Technology-integration is integral (rather than in addition) to development of the mathematics students are learning.

Code	Code description
T1m	Concerned that the need to teach about the technology will take away time from teaching mathematics.
T2m	Uses technology activities at the end of units, for “days off,” or for activities peripheral to classroom instruction.
T3m	Uses technology to enhance or reinforce mathematics ideas that students have learned previously.
T4m	Engages students in high-level thinking activities (such as project-based and problem solving and decision making activities) for learning mathematics using the technology as a learning tool.
T5m	Active, consistent acceptance of technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students.
T1i	Does not use technology to develop mathematical concepts.
T2i	Merely mimics the simplest professional development mathematics curricular ideas for incorporating the technologies.
T3i	Mimics the simplest professional development activities with the technologies but attempts to adapt lessons for his/her mathematics classes.
T4i	Engages students in explorations of mathematics with technology where the teacher is in role of guide rather than director of the exploration.
T5i	Adapts from a breadth of instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics.
T1e	Uses technology to reinforce concepts taught without technology.
T2e	Tightly manages and orchestrates instruction using technology.
T3e	Instructional strategies with technologies are primarily deductive, teacher-directed in order to maintain control of the how the activity progresses.
T4e	Explores various instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics.
T5e	Manages technology-enhanced activities in ways that maintains student engagement and self-direction in learning the mathematics.
T1p	Considers attending local professional development to learn more about technologies.
T2p	Recognizes the need to participate in technology related PD.
T3p	Continues to learn and explore ideas for teaching and learning mathematics using only one type of technology (such as spreadsheets).
T4p	Seeks out and works with others who are engaged in incorporating technology in mathematics.
T5p	Seeks ongoing PD to continue to learn to incorporate emerging technologies. Continues to learn and explore ideas for teaching and learning mathematics with multiple technologies to enhance access to mathematics.

Code	Code description
A1u	Permits students to use technology 'only' after mastering certain concepts.
A2u	Students use technology in limited ways during regular instructional periods.
A3u	Permits students to use technology in specifically designed units.
A4u	Permits students to use technology for exploring specific mathematical topics.
A5u	Permit students to use technology in every aspect of mathematics class.
A1b	Resists consideration of changes in content taught although it becomes accessible to more students through technology.
A2b	Worries about access and management issues with respect to incorporating technology in the classroom.
A3b	Uses technology as a tool to enhance mathematics lessons in order to provide students a new way to approach mathematics.
A4b	Recognizes challenges for teaching mathematics with technologies, but explores strategies and ideas to minimize the impact of those challenges.
A5b	Recognizes challenges in teaching with technology and resolves the challenges through extended planning and preparation for maximizing the use of available resources and tools.
A1a	Notices that authentic problems are more likely to involve 'unfriendly numbers' and may be more easily solved if students had calculators.
A2a	Calculators permit greater number of examples to be explored by students.
A3a	Concepts are taught differently since technology provides access to connections formerly out of reach.
A4a	Through the use of technology, key topics are explored, applied, and assessed incorporating multiple representations of the concepts and their connections.
A5a	Students are taught and permitted to explore more complex mathematics topics or mathematical connections as part of their normal learning experience.

APPENDIX H

FREQUENCY OF SKETCHPAD USE

How many times per week did the students work in a computer lab/classroom using GSP software?

Week	1	2	3	4	5	6	7	8	9	10	11	12	13
Brian	1	3	0	0	1	0	0	0	0	0	0	1	1
James	0	1	1	1	1	0	0	1	0	0	1	0	1
Susan	1	1	1	1	1	1	1	2	2				
Laura	0	0	0	1	1	0	0	0	2	2	0	1	1

Week	14	15	16	17	18	19	20	21	22	Average
Brian	0	0	2	0	2	0	0	0	0	0.5
James	0	0	0	0	2	2	1	2	0	0.6
Susan										1.2
Laura	1	1	1	2	0	1	1	0	0	0.7

How many times per week was the geometry class taught in a classroom with one demonstration computer?

Week	1	2	3	4	5	6	7	8	9	10	11	12	13
Brian	2	1	0	0	1	0	0	1	1	0	0	1	1
James	0	1	1	1	1	1	0	1	1	1	0	1	0
Susan	4	4	4	4	4	4	4	4	4				
Laura	0	0	0	1	1	2	0	2	1	1	1	1	1

Week	14	15	16	17	18	19	20	21	22	Average
Brian	0	0	2	0	2	0	0	0	0	0.5
James	2	0	1	0	2	3	1	2	0	0.9
Susan										4.0
Laura	2	2	2	1	0	1	1	2	1	1.0

REFERENCES

Archambault, L. & Crippen, K. (2009). Examining TPACK among K-12 online distance educators in the United States. *Contemporary Issues in Technology and Teacher Education, 9*(1), 71-88.

Berliner, D. C. (1994). Expertise: The wonder of exemplary performances. In J. N. Mangieri & C. C. Block (Eds.), *Creating powerful thinking in teachers and students: Diverse perspectives* (pp. 161-186). Fort Worth, TX: Harcourt Brace College Publications.

Blumenfeld, P. C., Soloway, E., Marx, R., Guzdial, M., & Palincsar, A. (1991). Motivating project-based learning: Sustaining the doing, supporting the learning. *Educational Psychologist, 26*(3/4), 369-398.

Brock, C. F., Cappo, M., Dromi, D., Rosin, M., & Shenkerman, E. (1994). *Tangible math: Geometry Inventor [computer software]*. Cambridge, MA: Logal Educational Software and Systems.

Brown, J. L. (2004). *Making the most of Understanding by Design*. Washington, DC: Association for Supervision and Curriculum Development.

Clements, D. H. (1995). Geometry Inventor (software review). *Teaching Children Mathematics 1*(6), .

Coffland, D. A., & Strickland, A. W. (2004). Factors related to teacher use of technology in secondary geometry instruction. *Journal of Computers in Mathematics and Science Teaching, 23*(4), 347-365.

Common Core State Standards Initiative (2010). *Common Core State Standards for mathematics*. Retrieved from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Conference Board of the Mathematical Sciences (2001). *The mathematical education of teachers*. Providence, RI: American Mathematical Society, in cooperation with Mathematical Association of America, Washington, D.C.

Cox, S., & Graham, C. R. (2009). An elaborated model of the TPACK framework. In I. Gibson et al. (Eds.), *Proceedings of Society for Information Technology & Teacher Education International Conference 2009* (pp. 4042-4049). Chesapeake, VA: AACE.

Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five approaches* (2nd ed.). Thousand Oaks, CA: Sage.

Denzin, N. K., & Lincoln, Y. S. (2005). *The Sage handbook of qualitative research* (3rd ed.). Thousand Oaks, CA: Sage.

Dwyer, D. C., Ringstaff, C., & Sandholtz, J. H. (1991). Changes in teachers' beliefs and practices in technology-rich classrooms. *Educational Leadership, 48*(8), 45-52.

Figg, C., & Jaipal, K. (2012). TPACK-in-Practice: Developing 21st Century Teacher Knowledge. In P. Resta (Ed.), *Proceedings of Society for Information Technology & Teacher Education International Conference 2012* (pp. 4683-4689). Chesapeake, VA: AACE.

Forsythe, S. (2007). Learning geometry through dynamic geometry software. *Mathematics Teaching Incorporating Micromath 202*, 31-35.

Forsythe, S. (2009). Centers and circles of a triangle. *Mathematics Teaching 215*, 9-13.

Gray, L., Thomas, N., & Lewis, L. (2010). *Educational technology in U.S. public schools: Fall 2008* (NCES 2010-034). Washington, DC: National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education.

Groth, R., Spickler, D., Bergner, J., & Bardzell, M. (2009). A qualitative approach to assessing technological pedagogical content knowledge. *Contemporary Issues in Technology and Teacher Education, 9*(4), 392-411.

Guba, E. G., & Lincoln, Y. S. (1981). *Effective evaluation*. San Francisco: Jossey-Bass.

Hannafin, R. D., Burruss, J. D., & Little, C. (2001). Learning with dynamic geometry programs: Perspectives of teachers and learners. *Journal of Educational Research*, *94*(3), 132-144.

Harris, J., Grandgenett, N. & Hofer, M. (2010). Testing a TPACK-based technology integration assessment rubric. In D. Gibson & B. Dodge (Eds.), *Proceedings of Society for Information Technology & Teacher Education International Conference 2010* (pp. 3833-3840). Chesapeake, VA: AACE.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372-400.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.

Hill, H. C., Sleep, L., Lewis, J. M., & Ball, L. B. (2007). Assessing teachers' mathematical knowledge. What knowledge matters and what evidence counts? In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 111-155). Reston, VA: National Council of Teachers of Mathematics.

Hughes, J. (2004). Technology learning principles for preservice and in-service teacher education. *Contemporary Issues in Technology and Teacher Education, 4*(3), 345-362.

International Society for Technology in Education (2008). *National Educational Technology Standards for Teachers* (2nd ed.).

Ivy, J. T. (2011). *Secondary Mathematics Teachers' Perceptions of Their Integration of Instructional Technologies* (Doctoral dissertation). Retrieved from ProQuest Dissertation and Theses. (Order No. 3461290)

Jackiw, N. (2009). *The Geometer's Sketchpad V5*. Emeryville, CA: Key Curriculum Press.

Knapp, A. K., Barret, J. E., & Kaufmann, M. L. (2007). Prompting teacher knowledge development using dynamic geometry software. In T. Lamberg & L. Wiest (Eds.) *Proceedings of the 29th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 1098-1105. Stateline, NV: University of Nevada (Reno).

Lawless, K. A., & Pellegrino, J. W. (2007). Professional development in integrating technology into teaching and learning: Knowns, unknowns, and ways to pursue better questions and answers. *Review of Educational Research, 77*(4), 575-614.

Lewis, C. (2002). *Lesson study: A handbook of teacher-led instructional change*. Philadelphia: Research for Better Schools.

Lewis, C., Perry, R., & Hurd, J. (2009). Improving mathematics instruction through lesson study: A theoretical model and North American case. *Journal of Mathematics Teacher Education*, *12*(4), 285-304.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Beverly Hills, CA: Sage.

Margerum-Leys, J., & Marx, R. (2002). Teacher knowledge of educational technology: A study of student teacher/mentor teacher pairs. *Journal of Educational Computing Research, 26*(4), 427-462.

McCrory, R. (2004). A framework for understanding teaching with the Internet. *American Educational Research Journal, 41*(2), 447-488.

Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation*. San Francisco, CA: Jossey-Bass.

Mishra, P. & Koehler, M. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record, 108*(6), 1017-1054.

National Center for Education Statistics. (2002, November). *Technology in schools: Suggestions, tools, and guidelines for assessing technology in elementary and secondary education.* Washington, DC: Author.

National Council of Teachers of Mathematics (2000). *Principle and standards for school mathematics*. Retrieved from http://www.nctm.org/standards/content.aspx?id=26809

Niess, M. L. (2005). Preparing teachers to teach science and mathematics with technology: Developing a technology pedagogical content knowledge. *Teaching and Teacher Education, 21*(5), 509-523.

Niess, M. L., Ronau, R. N., Shafer, K. G., Driskell, S. O., Harper S. R., Johnston, C., Browning, C., Özgün-Koca, S. A., & Kersaint, G. (2009). Mathematics teacher TPACK standards and development model. *Contemporary Issues in Technology and Teacher Education*, *9*(1), 4-24.

Niess, M. L., van Zee, E. H., & Gillow-Wiles, H. (2010). Knowledge growth in teaching mathematics/science with spreadsheets: Moving PCK to TPACK through online professional development. *Journal of Digital Learning in Teacher Education, 27*(2), 42-52.

Olive, J. (2002). Implications of using dynamic geometry technology for teaching and learning. In: M. Saraiva, J. Matos, I. Coelho, (Eds.) *Ensino e Aprendizagem de Geometria*. Lisbon: SPCE. Retrieved from http://www.spce.org.pt/sem/JO.pdf

Patton, M. Q. (2002). *Qualitative research and evaluation methods* (3rd ed.). Thousand Oaks, CA: Sage.

Penuel, W. R., Fishman, B. J., Yamaguchi, R., & Gallagher, L. P. (2007). What makes professional development effective? Strategies that foster curriculum implementation. *American Educational Research Journal, 44*(4), 921-958.

Pierson, M. E. (2001). Technology integration practices as function of pedagogical expertise. *Journal of Research on Computing in Education, 33*(4), 413-429.

Porter, A., McMaken, J., Hwang, J., & Yang, R. (2011). Common Core standards: The new U.S. intended curriculum. *Educational Researcher*, *40*(3), 103-116.

Riales, J. W. (2011). *An Examination of Secondary Mathematics Teachers' TPACK Development Through Participation in a Technology-Based Lesson Study* (Doctoral dissertation). Retrieved from ProQuest Dissertation and Theses. (Order No. 3461312)

Roberts, D. L., & Stephens, L. J. (1999). The effect of the frequency of usage of computer software in high school geometry. *Journal of Computers in Mathematics and Science Teaching, 18*(1), 23-30. Charlottesville, VA: AACE. Retrieved from http://www.editlib.org/p/15199

Roblyer, M. D. & Doering, A. H. (2010). *Integrating educational technology into teaching* (5th ed.). Boston: Allyn & Bacon.

Rogers, E. (1995). *Diffusion of innovations*. New York, NY: The Free Press of Simon and Schuster Inc.

Riley, R.W., Holleman III, F.S., & Roberts, L.G. (2000). *E-learning: Putting a world-class education at the fingertips of all children* (Report). Washington, DC: U.S. Department of Education.

Schmidt, D., Baran, E., Thompson, A., Mishra, P., Koehler, M., & Shin, T. (2009). Technological pedagogical content knowledge (TPACK): The development and validation of an assessment instrument for preservice teachers. *Journal of Research on Technology in Education, 42*(2), 123-149.

Schrader, P. G., & Lawless, K. A. (2004). The knowledge, attitudes, and behaviors (KAB) approach: How to evaluate performance and learning in complex environments. *Performance Improvement, 43*(9), 8-15.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14.

Sinclair, N., & Yurita, V. (2008). To be or to become: How dynamic geometry changes discourse. *Research in Mathematics Education 2*(10), 135-150.

Slough, S., & Connell, M. (2006). Defining technology and its natural corollary, technological content knowledge (TCK). In C. Crawford, D. Willis, R. Carlsen, I. Gibson, K. McFerrin, J. Price, & R. Weber (Eds.), *Proceedings of Society for Information Technology and Teacher Education International Conference, 2006* (pp. 1053-1059). Chesapeake, VA: AACE.

Sorto, M. A., & Lesser, L. (2009). Towards measuring technological pedagogical content knowledge in statistics: A pilot survey of middle school teachers using graphing calculators. *Proceedings of the International Association of Statistics Education Satellite Conference*, Durban, South Africa. [Online: http://www.stat.auckland.ac.nz/~iase/publications.php].

Stake, R. E. (1995). *The art of case study research*. Thousand Oaks, CA: Sage.

Stake, R. E. (2006). *Multiple case study analysis*. New York: The Guilford Press.

Texas Education Agency (2005). *Professional development and appraisal system: Teacher manual*. Austin, TX: Author. http://www5.esc13.net/pdas/docs/PDASTeacherManual.pdf

Yin, R. K. (2009). *Case study research: Design and method* (4th ed.). Thousand Oaks, CA: Sage.

Yursa, M. H., & Silverman, J. (2012). Seeking Change in Mathematics Teacher Development: Supporting the Immergence of a Virtual Lesson Study Community. In P. Resta (Ed.), *Proceedings of Society for Information Technology & Teacher Education International Conference 2012* (pp.). Chesapeake, VA: AACE.

VITA

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